Framework for the study of DDVCS phenomenology at NLO

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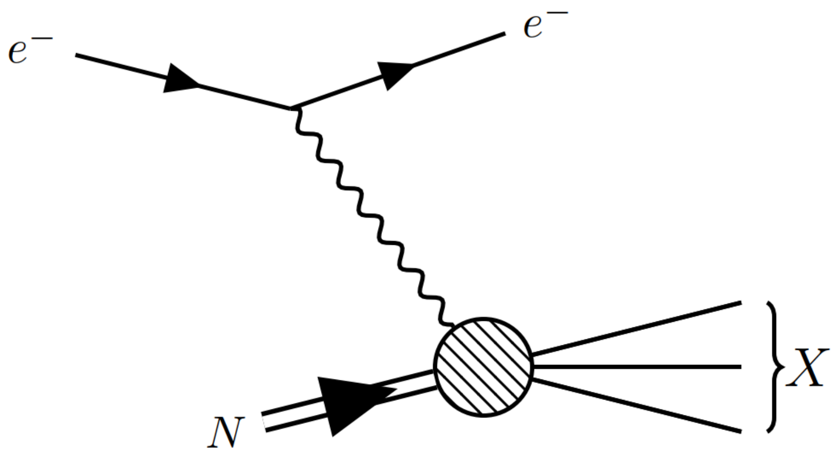
Why NLO corrections?

- Precise determination of GPDs (and so nucleon tomography)
- Universality testing in processes that at Born level look alike
- Apparent divergences of the form $\ln 0$

Refreshing Deep Inelastic Scattering (DIS)



Factorization in cross-section

$$d\sigma_{\text{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

pQED

Non-perturbative QCD

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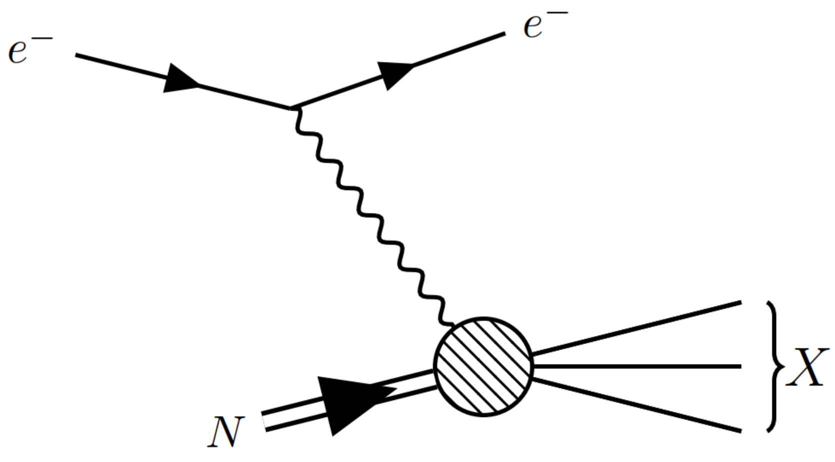

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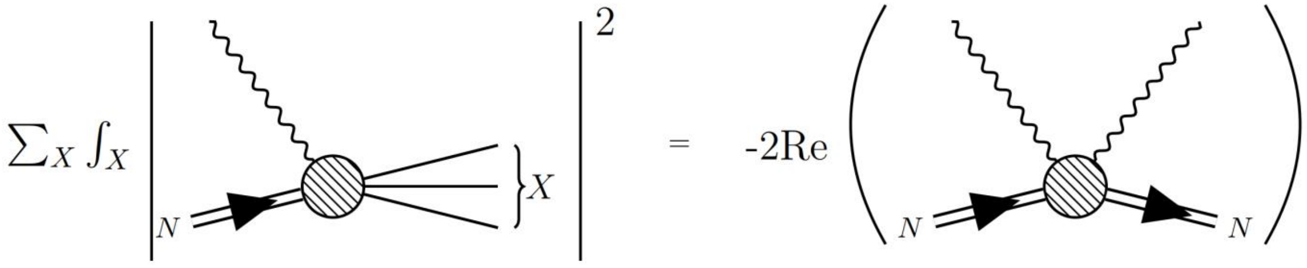
Optical theorem to DIS



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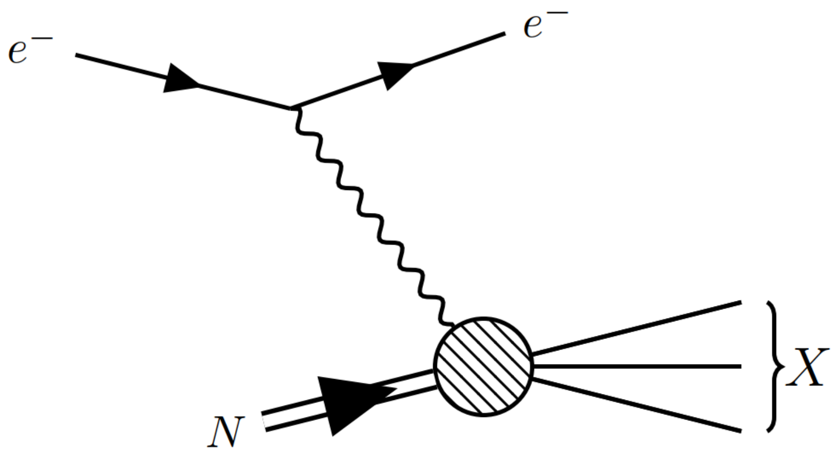

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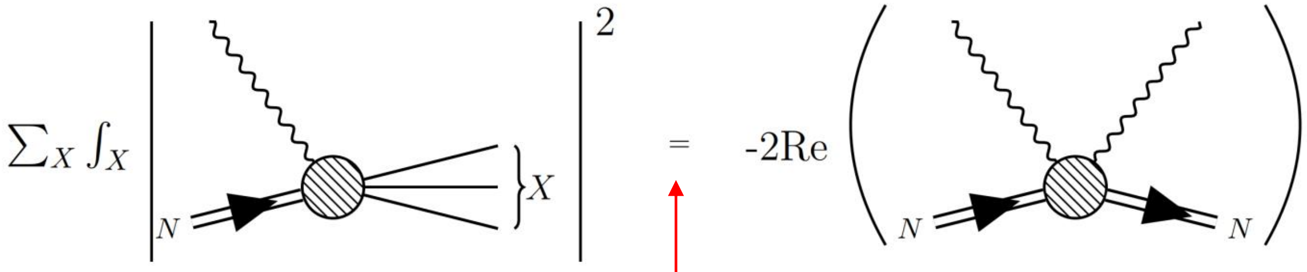

$$\sum_X \int_X |X\rangle\langle X| = 1$$ → DIS is inclusive process

Parton Distribution Functions (PDFs)

- Extracted from the hadronic tensor $W_{\mu\nu}$, they determine the internal nucleon structure

$$\mathrm{PDF}(x) = \frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixp^- z^+} \langle P|\bar{q}_f(-z/2)\gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2)|P\rangle\Big|_{z_\perp = z^- = 0}$$

Wilson line definition

$$\mathcal{W}[z_1^+, z_2^+] = \mathbb{P}\exp\left[ig\int_{z_2^+}^{z_1^+} d\eta^+ A^-\right]$$

Samuel Wallon. Hard exclusive processes in perturbative QCD: from medium to asymptotical energies. Doctoral Schools ED 107 (Physique de la Région Parisienne) and ED 517 (Particules, Noyaux et Cosmos). 2014.

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$$\gamma^+ = \frac{\gamma^0 + \gamma^3}{\sqrt{2}}$$

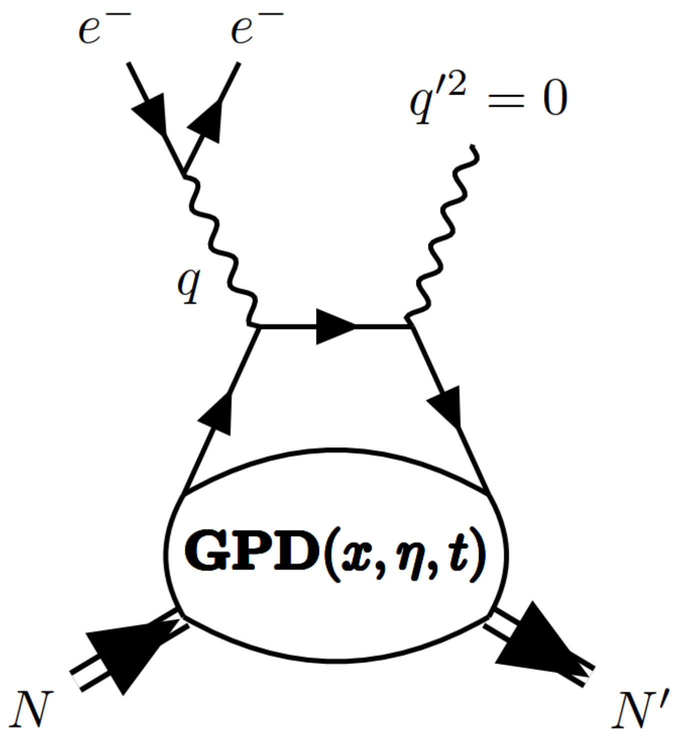
PDF is 1D

x = longitudinal parton momentum

Improving PDF's 1D picture

- In 1997, Ji introduces Generalized Parton Distributions (GPDs) through Deeply Virtual Compton Scattering (DVCS) process

- The point now is to study the conversion of a virtual photon into a real one



Xiang-Dong Ji. Gauge-Invariant Decomposition of Nucleon Spin. *Phys. Rev. Lett.*, 78:610–613, 1997

DVCS = exclusive process = factorization in amplitude

Sketch of DVCS amplitude

$$\begin{aligned}\mathcal{A}_{\text{DVCS}} &\sim \int_{-1}^{1} dx \, \frac{1}{x-\eta+i0} \text{GPD}(x,\eta,t) + \cdots \\ &= \text{PV}\left(\frac{1}{x-\eta}\right)(\text{GPD}(x,\eta,t)) - \int_{-1}^{1} dx \; i\pi\delta(x-\eta)\text{GPD}(x,\eta,t) + \cdots\end{aligned}$$

So we can measure GPDs at $x=\eta$ only, i.e., we can access $\text{GPD}(\eta,\eta,t)$

GPD definition: 3D distribution

$$\mathrm{GPD}(x,\eta,t)=\frac{1}{2}\int\frac{dz^+}{2\pi}e^{ixp^-z^+}\langle P'|\bar{q}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]q_f(z/2)|P\rangle\Big|_{z_\perp=z^-=0}$$

Measure the difference between P and P'

$$\eta=-\frac{(q-q')(q+q')}{(P+P')(q+q')}$$

$$t=(P-P')^2$$

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Measure the difference between P and P'

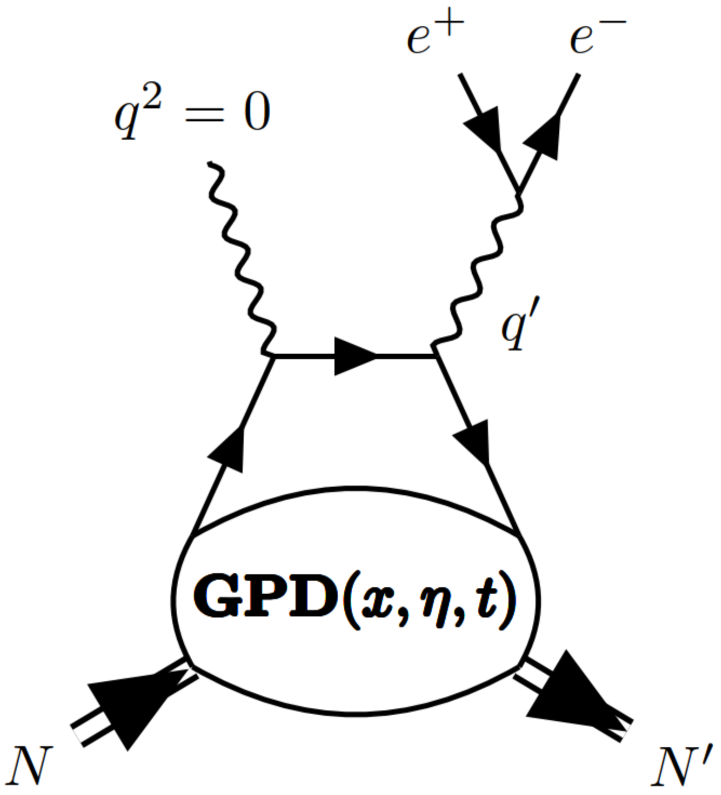
$$\eta = -\frac{(q - q')(q + q')}{(P + P')(q + q')}$$

$$t = (P - P')^2$$

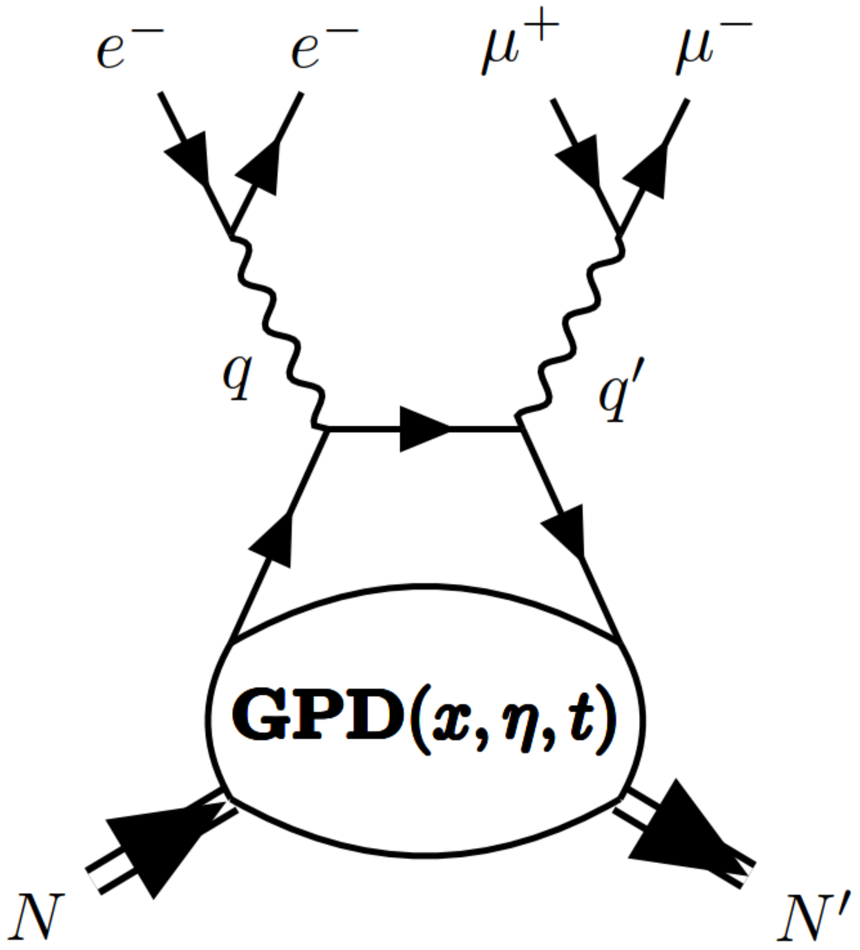
Nucleon tomography via Fourier transform in the plane transverse to proton motion

Other 2 “golden channels”: TCS & DDVCS

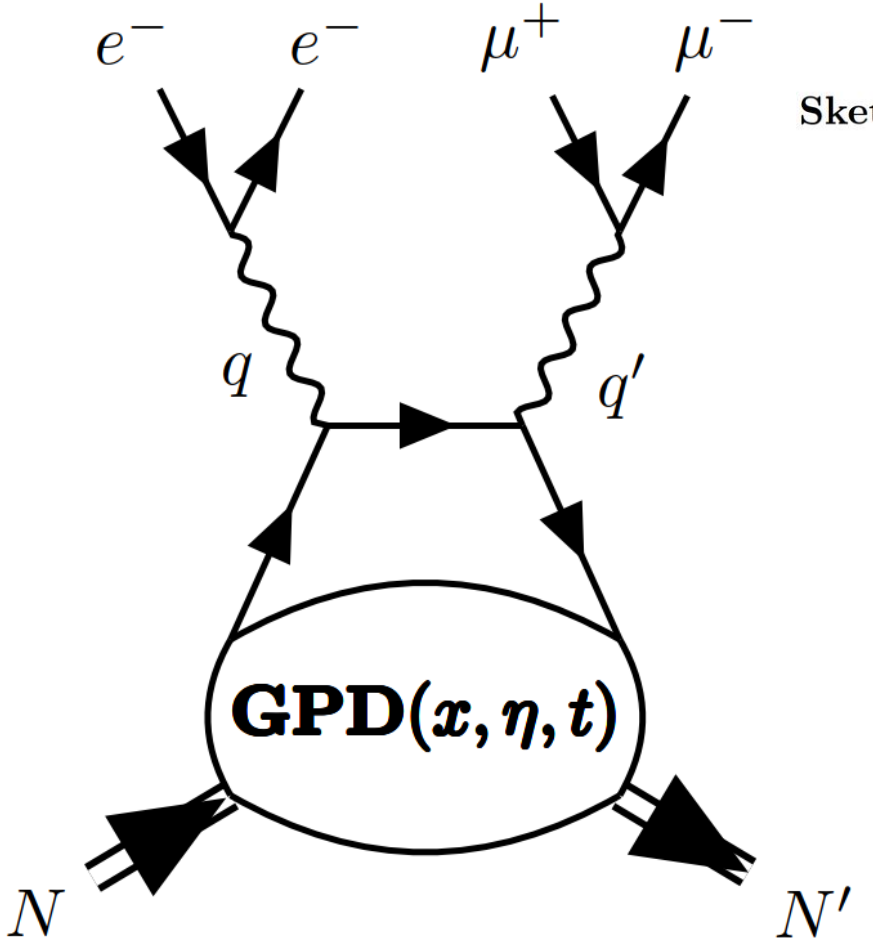
- TCS or timelike Compton scattering
- Counterpart of DVCS
- A real photon transforms into a virtual one (lepton photo-production)



- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)



- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)
- **Allows to measure GPDs outside $x = \eta$**



Sketch of DDVCS amplitude

$$\begin{aligned}\mathcal{A}_{\mathrm{DDVCS}} &\sim \int_{-1}^{1} dx\, \frac{1}{x - \xi + i0} \mathrm{GPD}(x, \eta, t) + \cdots \\ &= \mathrm{PV}\left(\frac{1}{x-\xi}\right)(\mathrm{GPD}(x, \eta, t)) - \int_{-1}^{1} dx\; i\pi\delta(x-\xi)\mathrm{GPD}(x, \eta, t) + \cdots\end{aligned}$$

So now we can access $\mathrm{GPD}(\xi, \eta, t)$

Details in DDVCS

Here, ξ is the *generalized* Bjorken variable,

$$\xi = \frac{-\bar{q}^2}{2p\bar{q}}, \quad \bar{q} = \frac{q+q'}{2}, \quad p = \frac{P+P'}{2}$$

q, q' are the 4-momenta of incoming & outgoing photon
P, P' are the 4-momentum of incoming & outgoing proton

Experimentally, DDVCS is very demanding: x-sec smaller than DVCS' → EIC will have enough luminosity to accurate measurements

B. Pire, L. Szymanowski, and J. Wagner. NLO corrections to timelike, spacelike and double deeply virtual Compton scattering. *Phys. Rev. D*, 83:034009, 2011.

Renormalization

- Need to renormalize both GPDs and hard part. Amplitude:

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} \tilde{T}^q(x)\tilde{F}^q(x) + \tilde{T}^g(x)\tilde{F}^g(x) \right]$$

Bare GPD

Bare hard-part coefficients

- We can work in DDVCS and use

$$\begin{aligned} \mathrm{DDVCS}|_{\xi=\eta} &\quad\rightarrow\quad \mathrm{DVCS} \\ \mathrm{DDVCS}|_{\xi=-\eta} &\quad\rightarrow\quad \mathrm{TCS} \end{aligned}$$

GPDs at NLO

Renormalize the associated Green function

$$G^{qq'} = \int d^4x_1 d^4x_2\, e^{-i(p_1x_1+p_2x_2)} \langle\Omega|\mathbb{T}\left\{\bar{\psi}_q(z_1)\mathcal{W}[z_1,z_2]\gamma^+\psi_q(z_2)\bar{\psi}_{q'}(x_1)\psi_{q'}(x_2)\right\}|\Omega\rangle\Big|_{z_{1,2}=z^+_{1,2}} \,, \quad \gamma^+ = \frac{\gamma^0+\gamma^3}{\sqrt{2}}$$

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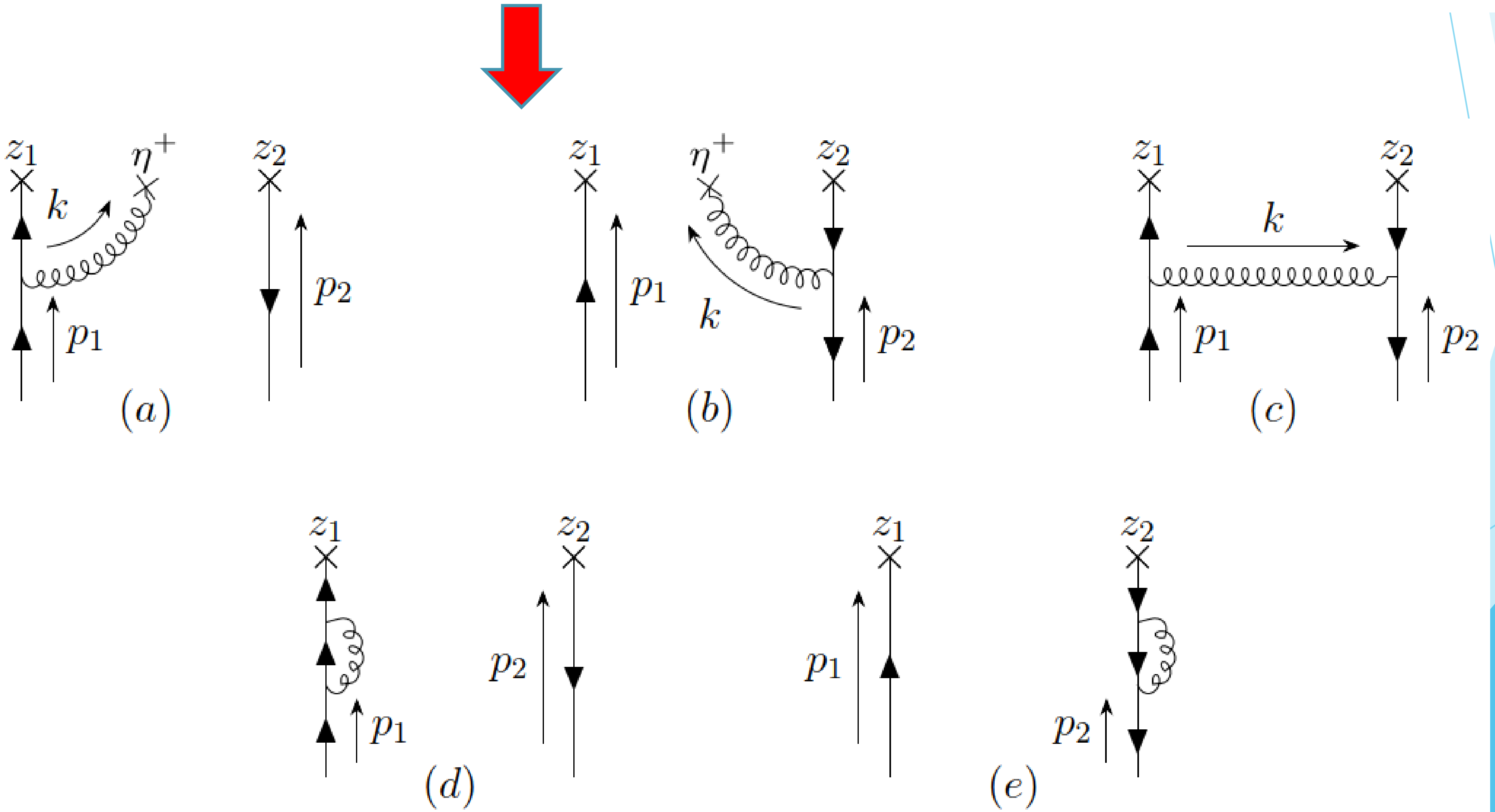

Final GPDs at NLO

$$
\begin{aligned}
\tilde{F}^q(x) &= F^q(x) - \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qq}(x,x') \otimes F^q(x') \\
&\quad - \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qg}(x,x') \otimes F^g(x')
\end{aligned}
$$

Kernels $K^{qq,qg}$ can be read from

M. Diehl, Phys. Rept. **388** (2003) 41;
A. V. Belitsky and A. V. Radyushkin, Phys. Rept. **418**, 1 (2005)

Corrections and $\ln 0$

- When renormalizing hard part you find expressions that can be squematized as

$$\widetilde{T}^q \sim \text{prefactors} \times \ln\left(\frac{\bar{Q}^2}{\xi\mu_F^2}(\xi - x) - i0\right) + \cdots \qquad \bar{Q}^2 = \frac{Q^2 - Q'^2}{2}$$

$$\text{If } Q^2 \sim Q'^2, \text{ then } \bar{Q}^2 \to 0 \text{ and } \ln \text{ is divergent}$$

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More details in

Phys Rev D, vol 83, 2011

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire and L. Szymanowski and J. Wagner

$$\frac{\bar{Q}^2}{\xi} = \frac{Q^2}{x_B} - \hat{Q}^2 \xrightarrow[Q^2 \to Q'^2]{} 2Pq - Q^2 = s - P^2 = s - M^2 > 0$$

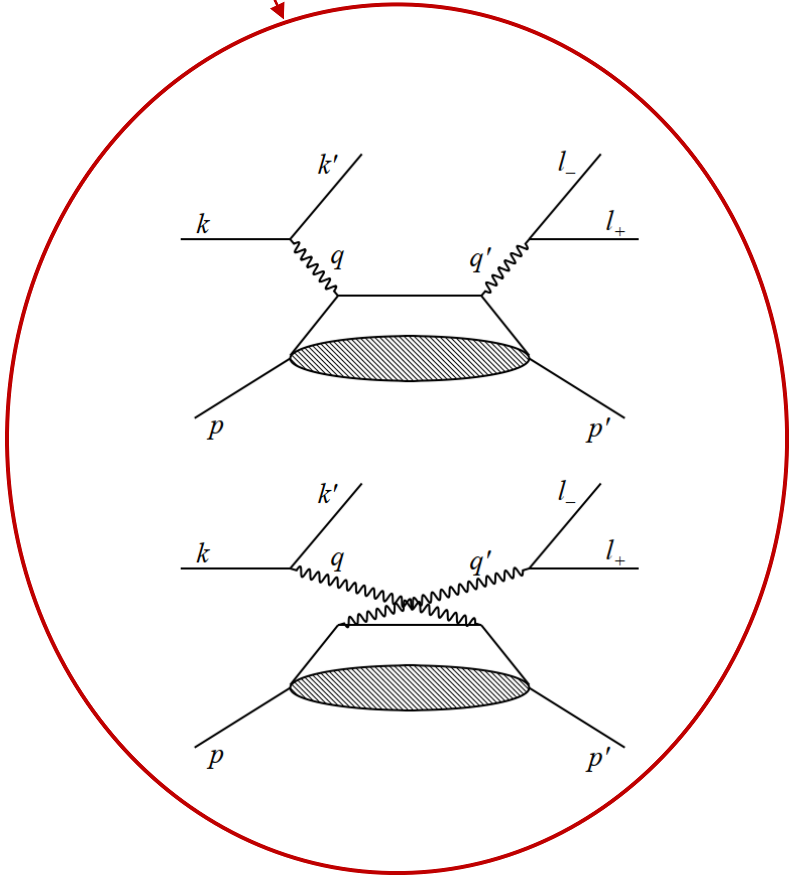
$$\Rightarrow \frac{\bar{Q}^2}{\xi} \not\to 0$$

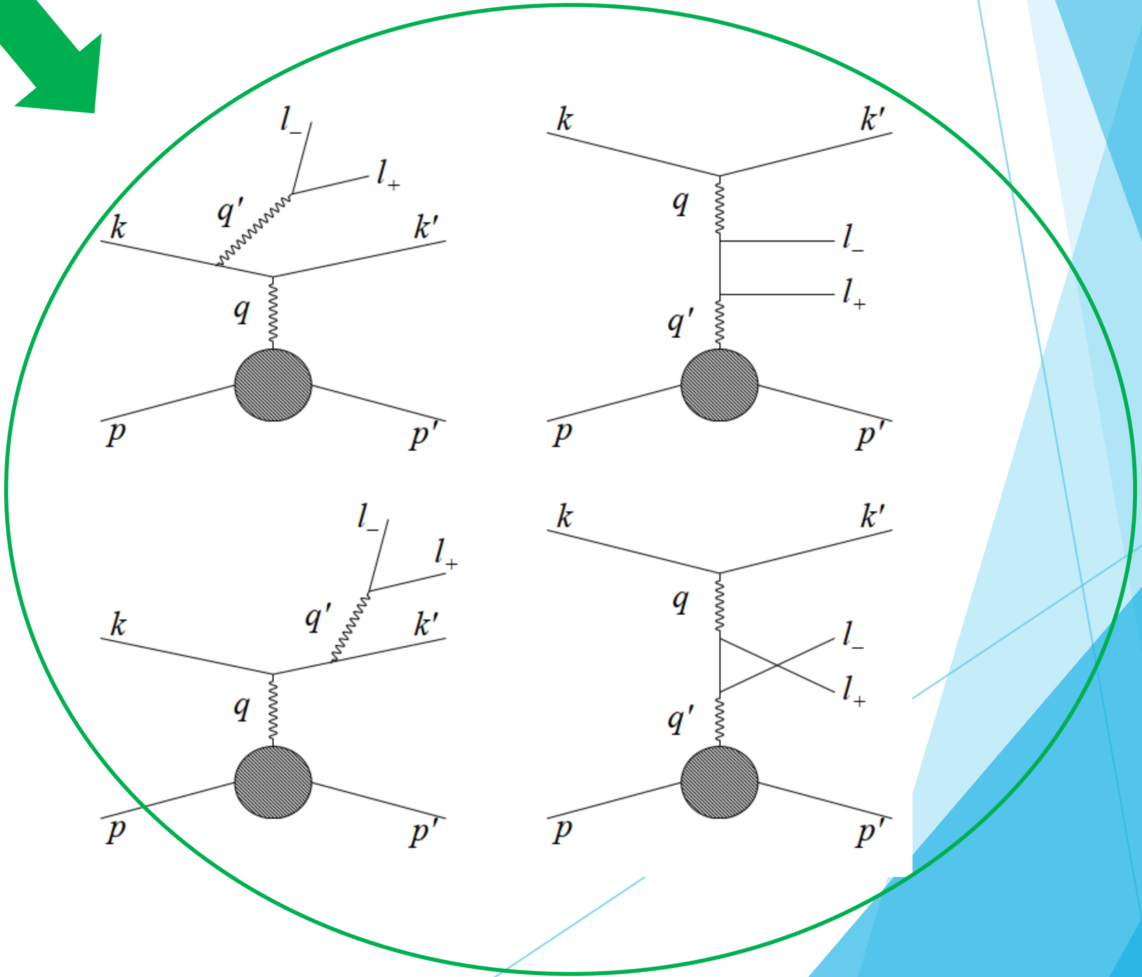
Actually,

$$\frac{\bar{Q}^2}{\xi} = \frac{\hat{Q}^2}{\eta} > 0 \Rightarrow \textbf{no} \text{ divergent ln}$$

Not only Compton scattering counts…

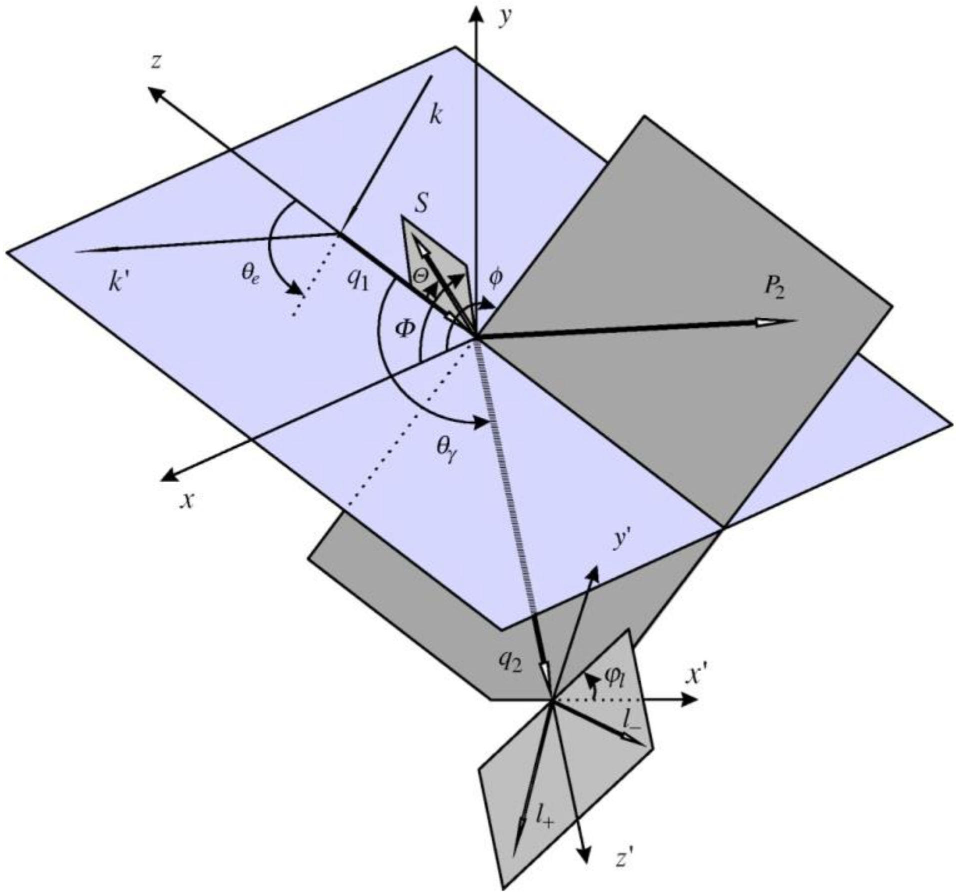
- Compton scattering + BH (Bethe-Heitler diagrams) is a precise representation of DDVCS





Amplitude structure

- DDVCS is a 2 → 4 problem, i.e., complex amplitude dependent on several angles
- Set-up:



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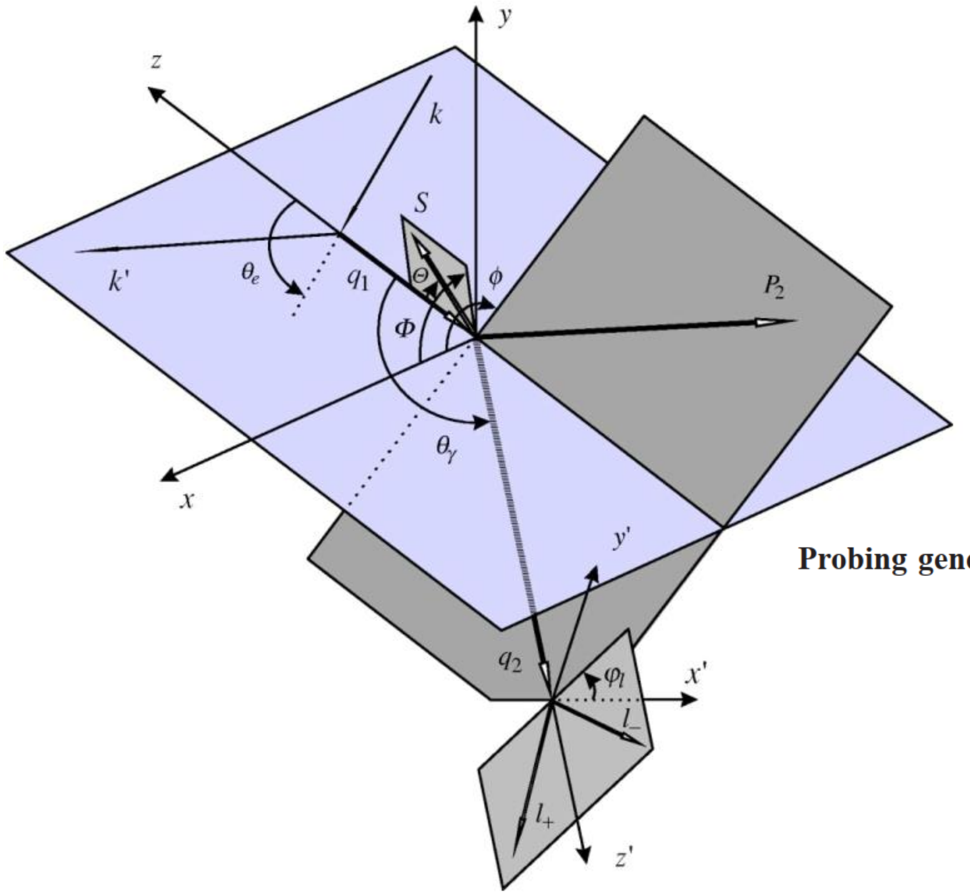

Amplitude can be written as a Fourier expansion in the angles.

LO expressions can be found in

PHYSICAL REVIEW D **68**, 116005 (2003)

Probing generalized parton distributions with electroproduction of lepton pairs off the nucleon

A. V. Belitsky D. Müller

Specialized software for this study

- PARTONS platform: open-source C++ program
  - Contains several GPD models
  - Leading twist… but higher twists will be included in near future
  - Can be used by theorists and experimentalists
  - Provides x-secs, Compton form factors, etc

PARTONS

PARtonic Tomography Of Nucleon Software

To download and for tutorials
http://partons.cea.fr

For detail description of architecture see:
Eur. Phys. J. C78 (2018), 478