Istituto Nazionale di Fisica Nucleare
SAPIENZA UNIVERSITÀ DI ROMA

Small-x resummation at the LHC:

differential cross sections and beyond the leading logs

Federico Silvetti, Sapienza università di Roma and INFN
federico.silvetti@uniroma1.it

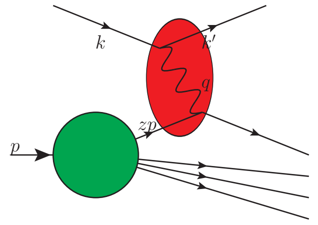
QCD Masterclass 2021, Saint-Jacut-de-la-Mer
06/09/2021

Table of contents

Collinear factorisation

$$\sigma\left(x, Q^2\right) = \sum_{i\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, Q^2\right) f_i\left(z, Q^2\right)$$

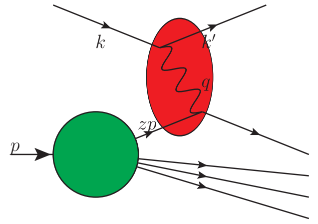
$$x = \frac{Q^2}{2(q\cdot p)}\,, \quad Q^2 = -q^2$$



Collinear factorisation

$$\sigma\left(x, Q^2\right) = \sum_{i\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, Q^2\right) f_i\left(z, Q^2\right)$$

$$x = \frac{Q^2}{2(q\cdot p)}, \quad Q^2 = -q^2$$



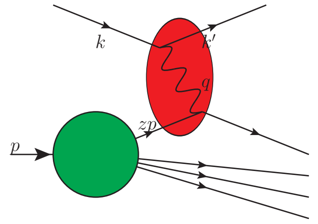
Coefficient function C:

- Computed from QCD in perturbation theory

Collinear factorisation

$$\sigma\left(x, Q^2\right) = \sum_{i\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, Q^2\right) f_i\left(z, Q^2\right)$$

$$x = \frac{Q^2}{2(q\cdot p)}, \quad Q^2 = -q^2$$



Coefficient function C:

- Computed from QCD in perturbation theory

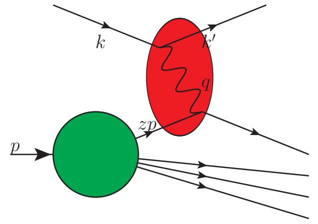
Parton distribution functions f (PDF):

- Q^2 scale dependence is governed by DGLAP equations

$$Q^2 \frac{\mathrm{d}}{\mathrm{d}Q^2} f_i\left(z, Q^2\right) = \int_z^1 \frac{\mathrm{d}w}{w} P_{ij}(w, \alpha_s) f_j\left(\frac{z}{w}, Q^2\right) = P_{ij}(\alpha_s) \otimes f_j\left(Q^2\right),$$

with the splitting functions P being perturbatively determined themselves

Collinear factorisation



$$\sigma\left(x, Q^2\right) = \sum_{i\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, Q^2\right) f_i\left(z, Q^2\right)$$

$$x = \frac{Q^2}{2(q\cdot p)}, \quad Q^2 = -q^2$$

Coefficient function C:

- Computed from QCD in perturbation theory

Parton distribution functions f (PDF):

- Q^2 scale dependence is governed by DGLAP equations

$$Q^2 \frac{\mathrm{d}}{\mathrm{d}Q^2} f_i\left(z, Q^2\right) = \int_z^1 \frac{\mathrm{d}w}{w} P_{ij}(w, \alpha_s) f_j\left(\frac{z}{w}, Q^2\right) = P_{ij}(\alpha_s) \otimes f_j\left(Q^2\right),$$

with the splitting functions P being perturbatively determined themselves

hadronic-cross section σ

- can be predicted given the two above
- must be provided as experimental input in order to determine PDFs
- PDFs are a **large source of theoretical uncertainty** $\rightarrow$ refine theory and include more data to improve results

LHC @ small-x

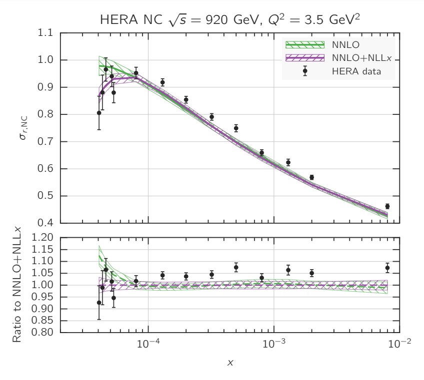


Figure: from Eur.Phys.J.C 78 (2018) 4, 321

- High-energy precision physics from LHC ↔ constrain PDFs to lower regions of x
- Proper description of **small-x** requires accounting for a special class of logarithms

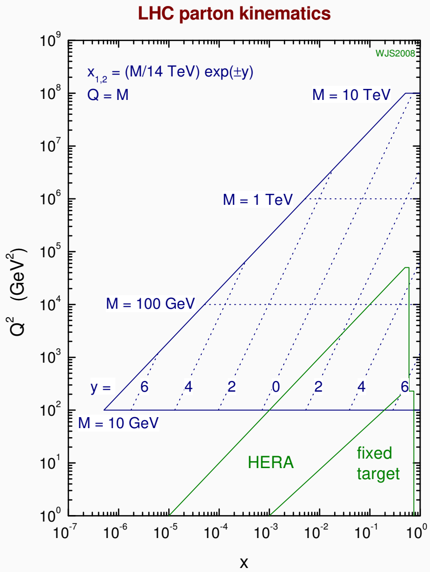


Figure: "W.J. Stirling, private communication"

Features of small-x logarithms

Single logarithm enhancement: one extra power of the logarithm at each successive order of perturbation theory.

- When $\alpha_s \ln(x) = \mathcal{O}(1)$ → breakdown of fixed order pert. theory → resum to all orders in α_s

$$
\begin{aligned}
C = {} & \alpha_s \ln(x) b_{11} && + \alpha_s b_{10} \\
& + \alpha_s^2 \ln^2(x) b_{22} + \alpha_s^2 \ln(x) b_{21} && + \alpha_s^2 b_{20} \\
& + \alpha_s^3 \ln^3(x) b_{33} + \alpha_s^3 \ln^2(x) b_{32} + \alpha_s^3 \ln(x) b_{31} + \alpha_s^3 b_{30} \\
& \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \ddots \\
& \text{(LL)} \qquad \text{(NLL)} \qquad \text{(NNLL)} \qquad \cdots
\end{aligned}
$$

Features of small-x logarithms

Single logarithm enhancement: one extra power of the logarithm at each successive order of perturbation theory.

- When $\alpha_s \ln(x) = \mathcal{O}(1)$ $\rightarrow$ breakdown of fixed order pert. theory $\rightarrow$ resum to all orders in α_s

$$\begin{aligned} C = {} & \alpha_s \ln(x) b_{11} + \alpha_s b_{10} \\ & + \alpha_s^2 \ln^2(x) b_{22} + \alpha_s^2 \ln(x) b_{21} + \alpha_s^2 b_{20} \\ & + \alpha_s^3 \ln^3(x) b_{33} + \alpha_s^3 \ln^2(x) b_{32} + \alpha_s^3 \ln(x) b_{31} + \alpha_s^3 b_{30} \\ & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \ddots \\ & \text{(LL)} \qquad \text{(NLL)} \qquad \text{(NNLL)} \qquad \cdots \end{aligned}$$

- In DGLAP splitting functions
  - Controlled via the so called **BFKL equation** $\rightarrow$ Micheal's talk later this week.
  - Known up to NLL

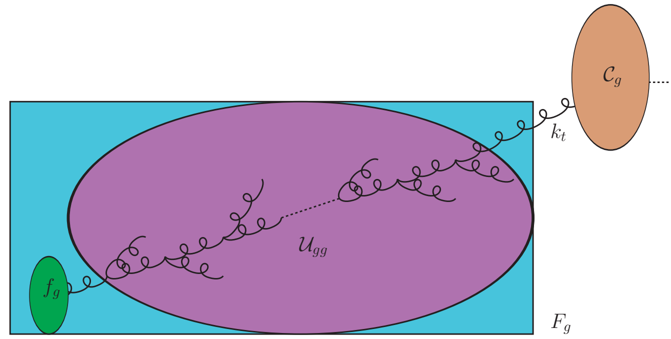
Features of small-x logarithms

Single logarithm enhancement: one extra power of the logarithm at each successive order of perturbation theory.

- When $\alpha_s \ln(x) = \mathcal{O}(1)$ $\to$ breakdown of fixed order pert. theory $\to$ resum to all orders in α_s

$$
\begin{aligned}
C = {} & \alpha_s \ln(x) b_{11} \quad + \alpha_s b_{10} \\
& + \alpha_s^2 \ln^2(x) b_{22} + \alpha_s^2 \ln(x) b_{21} \quad + \alpha_s^2 b_{20} \\
& + \alpha_s^3 \ln^3(x) b_{33} + \alpha_s^3 \ln^2(x) b_{32} + \alpha_s^3 \ln(x) b_{31} + \alpha_s^3 b_{30} \\
& \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \ddots \\
& \quad \text{(LL)} \qquad \text{(NLL)} \qquad \text{(NNLL)} \qquad \cdots
\end{aligned}
$$

- In DGLAP splitting functions
  - Controlled via the so called **BFKL equation** $\to$ Micheal's talk later this week.
  - Known up to NLL
- In coefficient functions
  - The resummation algorithm is based on the k_t-factorization
  - This operation can be carried out exclusively to LL accuracy



- k_t-factorisation[1]: $\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}k_t^2\, \mathfrak{C}_g\left(\frac{x}{z}, \alpha_s, Q^2, k_t^2\right) \mathfrak{F}_g\left(z, Q^2, k_t^2\right),$
- PDF evolutor[2]: $\mathfrak{F}_g\left(x, Q^2, k_t^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} \mathcal{U}_{gg}\left(\frac{x}{z}, k_t^2, Q^2\right) f_g\left(z, Q^2\right)$
- Then $C_g\left(z, Q^2, \alpha_s\right) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}k_t^2\, \mathfrak{C}_g\left(\frac{x}{z}, Q^2, k_t^2, \alpha_s\right) \mathcal{U}_{gg}\left(z, k_t^2, Q^2\right)$

[1] Catani and Hautmann: hep-ph/9405388
[2] Bonvini, Marzani and Peraro: hep-ph/1607.02153

The off-shell coefficient function

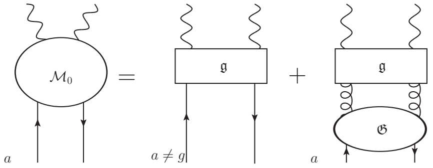


Figure: The only relevant part for x leading logs is the second one on the RHS

$\mathfrak{C}(\cdots)$ is obtained as an off-shell continuation of the Born cross-section.
In practice one requires:

- Off-shellness of the incoming parton (gluon)

$$k_{\text{collinear}} \to k_{\text{in}} = zp_1 + k_t, \qquad \frac{1}{2}\sum_{\lambda} \varepsilon^{\mu}_{\lambda}(k_{\text{in}})\varepsilon^{*\nu}_{\lambda}(k_{\text{in}}) \to -\frac{k_t^{\mu}k_t^{\nu}}{k_t^2} \tag{1}$$

- This is the only process dependent part of the computation.
- Collinear limit may be finite or not depending on the process (i.e. heavy-quark pair production and DIS) $\to$ assuming finiteness for next part.
- By construction it is always at least 2 Gluon Irreducible (2GI) $\implies$ does not contain leading x logarithms (at least in axial gauges).

One gluon emission

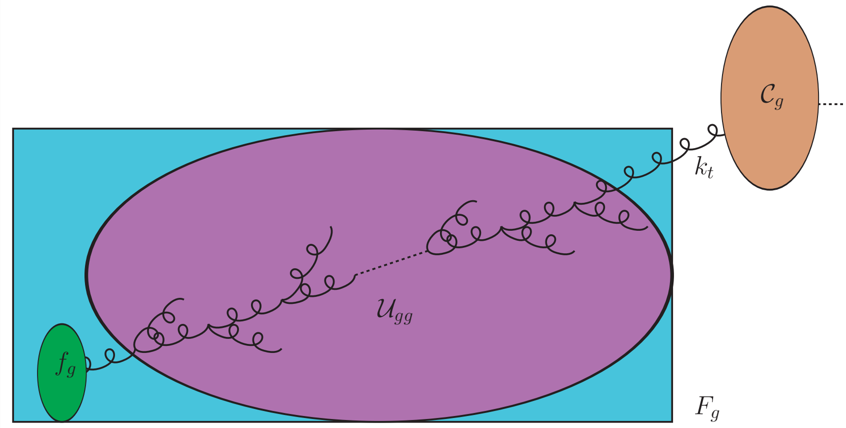


Figure: now we need to pin down the purple part

One gluon emission

Let´s start by considering $\mathfrak{C}(...)$ plus one single emission of a gluon on the incoming leg

$$C_{1,\text{bare}}\left(x, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) = \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}\xi}{\xi^{1+\epsilon}} \mathfrak{C}\left(\frac{x}{z}, \xi, \alpha_s; \epsilon\right) \left[K^1\left(z, \frac{\mu^2}{Q^2\xi}, \alpha_s; \epsilon\right)\right], \quad (2)$$

When taking a Mellin transform

$$f(N) \equiv \int_0^1 dx x^N f(x); \qquad f(x) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} x^{-(N+1)} f(N),$$

logarithms of x are mapped onto poles of N

$$\int_0^1 \mathrm{d}x x^N \frac{\log^{k-1}(x)}{x} = \frac{(-1)^{k-1}(k-1)!}{N^k}.$$

Moreover the convolution reduces to product

$$C_{1,\text{bare}}\left(N, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) = \int_0^\infty \frac{d\xi}{\xi^{1+\epsilon}} \mathfrak{C}(N, \xi, \alpha_s; \epsilon) \left[K^1\left(N, \left(\frac{\mu^2}{Q^2\xi}\right)^\epsilon \alpha_s; \epsilon\right)\right], \quad (3)$$

One gluon emission

Using the expansion

$$\frac{1}{\xi^{1+\epsilon}} = -\frac{\delta(\xi)}{\epsilon} + \sum_{k=0}^{\infty} \left[\frac{\ln^k \xi}{\xi}\right]_+ \frac{(-\epsilon)^k}{k!} \tag{4}$$

in Eq. (3) we get

$$C_{1,\text{bare}}\left(N, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) = -\frac{1}{\epsilon}\mathfrak{C}(N, 0, \alpha_s; \epsilon) \times \left[K^1\left(N, \alpha_s\right)\right] + \text{finite}, \tag{5}$$

we can convince ourselves that this last expression limit must identify as the leading N pole of the gluon anomalous dimension $\implies \quad K^1(N, \alpha_s) = \frac{\alpha_s C_A}{\pi N} = \alpha_s \gamma_0(N)$. By repeating the same structure n times, we get

$$C_{n,\text{bare}}\left(N, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) = \left[\alpha_s \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \gamma_0(N)\right] \int_0^{\infty} \frac{d\xi_n}{\xi_n^{1+\epsilon}} \mathfrak{C}(N, \xi_n, \alpha_s; \epsilon) \times$$
$$\times \int_0^{\xi_n} \left[\alpha_s \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \gamma_0(N)\right] \frac{d\xi_{n-1}}{\xi_{n-1}^{1+\epsilon}} \times \ldots \times \int_0^{\xi_2} \left[\alpha_s \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \gamma_0(N)\right] \frac{d\xi_1}{\xi_1^{1+\epsilon}}. \tag{6}$$

Collinear subtraction

Collinear singularities can be removed by subtracting the collinear pole before each integration [3]

$$\int_0^{\xi_2}\left[\alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\gamma_0(N)\right]\frac{d\xi_1}{\xi_1^{1+\epsilon}} \to \quad (1-\mathcal{P}_{\overline{\text{MS}}})\int_0^{\xi_2}\left[\alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\gamma_0(N)\right]\frac{d\xi_1}{\xi_1^{1+\epsilon}} =$$

$$= \quad (\alpha_s\gamma_0(N))\left(-\frac{1}{\epsilon}\frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{Q^2\xi_2}\right)^{\epsilon}+\frac{S_\epsilon}{\epsilon}\right);\ (7)$$

$$\text{with}\quad \mathcal{P}_{\overline{\text{MS}}}f(\epsilon)\equiv\sum_{k>0}\lim_{\epsilon\to 0}\left[\epsilon^k f(\epsilon)\right]\frac{S_\epsilon^k}{\epsilon^k}\quad\text{and}\quad S_\epsilon=\left(\frac{\mathrm{e}^{-\gamma_E}}{4\pi}\right)^{\epsilon}$$

The insertion of $n-1$ iterative subtractions looks like

$$C_n\left(N,\frac{\mu^2}{Q^2},\alpha_s;\epsilon\right)=\left[\alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\gamma_0(N)\right]\int_0^{\infty}\frac{d\xi_n}{\xi_n^{1+\epsilon}}\mathfrak{C}(N,\xi_n,\alpha_s;\epsilon)\times(1-\mathcal{P})$$

$$\times\int_0^{\xi_n}\left[\alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\gamma_0(N)\right]\frac{d\xi_{n-1}}{\xi_{n-1}^{1+\epsilon}}\times\ldots\times(1-\mathcal{P})\int_0^{\xi_2}\left[\alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\gamma_0(N)\right]\frac{d\xi_1}{\xi_1^{1+\epsilon}}.(8)$$

[3] G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B 175 (1980) 27

Resumming the emission chain

Now we can safely perform the first $n-1$ integrals [4]

$$C_n\left(N, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) = \left[\alpha_s \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \gamma_0(N)\right] \times$$
$$\times \int_0^\infty \frac{d\xi_n}{\xi_n^{1+\epsilon}} \mathfrak{C}(N, \xi_n, \alpha_s; \epsilon) \frac{1}{(n-1)!} \frac{1}{\epsilon^{n-1}} \left[\alpha_s \gamma_0(N) \left(1 - \left(\frac{\mu^2}{Q^2 \xi_n}\right)^{\epsilon}\right)\right]^{n-1}, \quad (9)$$

and then sum over the number of emissions n

$$C\left(N, \frac{\mu^2}{Q^2}, \alpha_s\right) = \sum_{n=0}^{\infty} C_n\left(N, \frac{\mu^2}{Q^2}, \alpha_s; \epsilon\right) =$$
$$= \left[\alpha_s \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \gamma_0(N)\right] \int_0^\infty \frac{\mathrm{d}\xi}{\xi^{1+\epsilon}} \mathfrak{C}(N, \xi, \alpha_s; \epsilon) \exp\left[\frac{\alpha_s \gamma_0(N)}{\epsilon} \left(1 - \left(\frac{\mu^2}{Q^2 \xi_n}\right)^{\epsilon}\right)\right],$$
$$\to \int_0^\infty \mathrm{d}\xi \, \mathfrak{C}(N, \xi, \alpha_s) \left[\alpha_s \gamma_0(N) \xi^{\alpha_s \gamma_0(N) - 1}\right] \quad \text{when} \quad (\epsilon \to 0, \ \mu = Q) \qquad (10)$$

[4] Since the pole operator is defined in the $\overline{\text{MS}}$ scheme, we omit all finite terms even before the dimensional regularisation is relaxed.

The resummed coefficient function

This is exactly the expression promised for the collinear coefficient function

$$C(N,\alpha_s) = \int_0^\infty \mathrm{d}\xi\, \mathfrak{C}(N,\xi,\alpha_s)\frac{\mathrm{d}\mathcal{U}(N,\xi)}{\mathrm{d}\xi} \quad \text{with} \quad \mathcal{U}(N,\xi) = \xi^{\alpha_s\gamma_0(N)}\,,$$

Then there are to way to retrieve physical space results

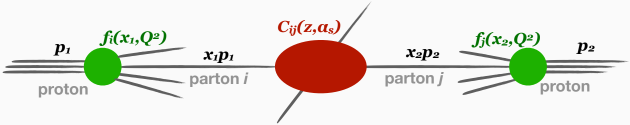
$$C(x,\alpha_s) = \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \frac{\mathrm{d}N}{2\pi\mathrm{i}} x^{-(N+1)} h(N,\alpha_s,\gamma_0)\,, \tag{11}$$

$$\text{with} \quad h(N,\alpha_s,\gamma_0) \equiv -\int_0^\infty \mathrm{d}\xi\, \xi^{\alpha_s\gamma_0(N)} \frac{\mathrm{d}\mathfrak{C}(N,\xi,\alpha_s)}{\mathrm{d}\xi}\,.$$

$$C(x,\alpha_s) = \int_x^1 \frac{\mathrm{d}z}{z}\int_0^\infty \mathrm{d}\xi\mathfrak{C}\left(\frac{x}{z},\xi,\alpha_s\right)\frac{\mathrm{d}\mathcal{U}(z,\xi)}{\mathrm{d}\xi}\,. \tag{12}$$

Differential cross sections in proton-proton collisions

At hadron colliders, two PDFs and suitable variables must be introduced



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}q_t^2}(x) = \sum_{i,j\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}y\, \frac{\mathrm{d}C_{ij}}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_t^2}\left(\frac{x}{z}, y\right) L_{ij}(z, Y-y)$$

$$= \sum_{i,j\in\{q,\bar{q},g\}} \frac{\mathrm{d}C_{ij}}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_t^2} \otimes L_{ij}$$

$$L_{ij}(z,\hat{y}) = f_i\left(\sqrt{z}\mathrm{e}^{-\hat{y}}, Q^2\right) f_j\left(\sqrt{z}\mathrm{e}^{+\hat{y}}, Q^2\right) \vartheta(\mathrm{e}^{-2|\hat{y}|} - x)$$

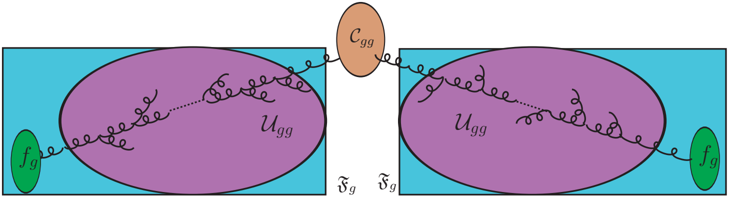
$x = \dfrac{Q^2}{S}$ $\quad$ $Q^2 =$ final state invariant mass $\quad$ $\sqrt{S} =$ collider energy

$$x_{1,2} = \sqrt{z}\mathrm{e}^{\mp\hat{y}} \to z = x_1 x_2\,, \quad \hat{y} = \frac{1}{2}\ln\left(\frac{x_2}{x_1}\right)$$

Resummation for triple differential cross sections

$$\frac{d\sigma}{dQ^2\, dY\, dq_t^2}(x, Y) = \int dk_{1t}^2 \int dk_{2t}^2\, \frac{d\mathfrak{C}_{gg}}{dQ^2\, dy\, dq_t^2}\left(k_{1t}^2, k_{2t}^2\right) \otimes \mathfrak{L}_{gg}\left(k_{1t}^2, k_{2t}^2\right)$$

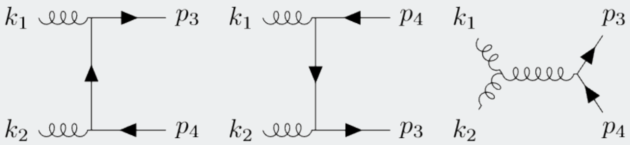
$$\mathfrak{L}_{gg}\left(z, \hat{y}, k_{1t}^2, k_{2t}^2\right) = \mathfrak{F}_g\left(\sqrt{z}e^{-\hat{y}}, k_{1t}^2\right)\mathfrak{F}_g\left(\sqrt{z}e^{+\hat{y}}, k_{2t}^2\right)\vartheta(e^{-2|\hat{y}|} - x)$$



$$\mathfrak{F}_g\left(\sqrt{z}e^{\mp\hat{y}}, Q^2, k_t^2\right) = \mathcal{U}_{gg}\left(k_{1,2t}^2, Q^2\right) \otimes f_g\left(Q^2\right)$$

$$\frac{dC_{gg}}{dQ^2\, dy\, dq_t^2}(z, y) = \int dk_{1t}^2 \int dk_{2t}^2\, \frac{d\mathfrak{C}_{gg}}{dQ^2\, dy\, dq_t^2}\left(k_{1t}^2, k_{2t}^2\right) \otimes \mathcal{U}_{gg}\left(k_{1t}^2, Q^2\right) \otimes \mathcal{U}_{gg}\left(k_{2t}^2, Q^2\right)$$

Application to $Q\overline{Q}$ production [6]



- Why? Recent measurements from LHCb for B/D mesons down to $x \gtrsim 10^{-6}$ + resummed prediction $\rightarrow$ small-x PDF fit [5]
- Final state has been studied as both
  - quark-antiquark pair $\rightarrow$ simplified kinematics
  - single quark $\rightarrow$ useful for phenomenology
- Mostly complete, but some numerical artefacts must still be removed.

[5] see hep-ex/1302.2864

[6] "Differential heavy quark pair production at small x", M. Bonvini and F. Silvetti (upcoming)

small-x NLL in the coefficient function[8]

In principle, k_t-factorization should offer the same handle

$$C(x) = \int d\xi \int_x^1 \frac{dz}{z} \mathfrak{C}\left(\frac{x}{z}, \xi, \alpha_s\right) \mathcal{U}(z, \xi, \alpha_s)$$

- Evolutors
  - definition relies on QCD anomalous dimension $\rightarrow$ already known to NLL
  - numerical implementation available in public code HELL [7]
- Off-shell coefficient function
  NLL extension $\leftrightarrow$ 1-loop off-shell coefficient function
  - Higgs-induced Deep Inelastic Scattering with $n_f = 0$ and $m_t \rightarrow \infty$ is being studied as a test case.
  - Include only 2 gluon irreducible contribution from NLO $\rightarrow$ loss of gauge invariance
  - Formally work in axial gauge (difficult analytical computation) or work in covariant one (simpler but less convenient collinear subtraction and contamination from gauge-dependent terms)

[7] hep-ph/1607.02153

[8] In collaboration with M. Bonvini (INFN), A. Rinaudo, S. Marzani and G. Ridolfi (University of Genoa)

Conclusions and outlook

small-x resummation carry-away

- In some kinematics limit, $\frac{\log^k(x)}{x}$ induce a failure of fixed order perturbation theory.
- Such conditions are interesting for modern hadron colliders.
- The resummation of this single logarithm enhancement in the hard coefficient function can be performed to all orders at leading log accuracy using the k_t-factorisation.

Conclusions and outlook

small-x resummation carry-away

- In some kinematics limit, $\frac{\log^k(x)}{x}$ induce a failure of fixed order perturbation theory.
- Such conditions are interesting for modern hadron colliders.
- The resummation of this single logarithm enhancement in the hard coefficient function can be performed to all orders at leading log accuracy using the k_t-factorisation.

Outlook

- Expressions for the resummed differential coefficient function in any combination of invariant mass, rapidity and transverse momentum was devised
- Direct application of the previous result to heavy-flavor pair production for both single quark and pair final state kinematics
- Extension of resummation strategy to NLL in Scalar-induced DIS is still incomplete

Thank you for your attention!

Backup

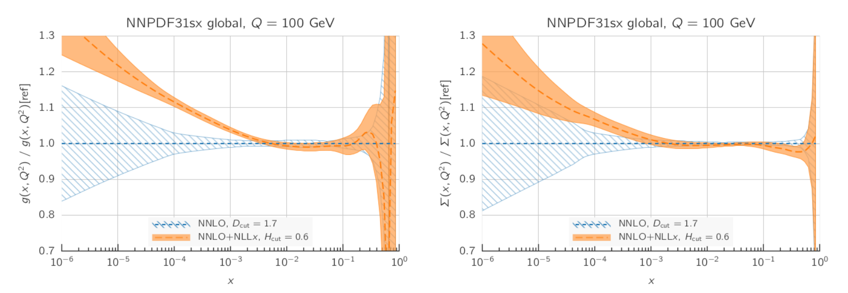


Figure: comparison of gluon and quark singlet pdfs with and without the inclusion of small-x resummation, see 1710.05935

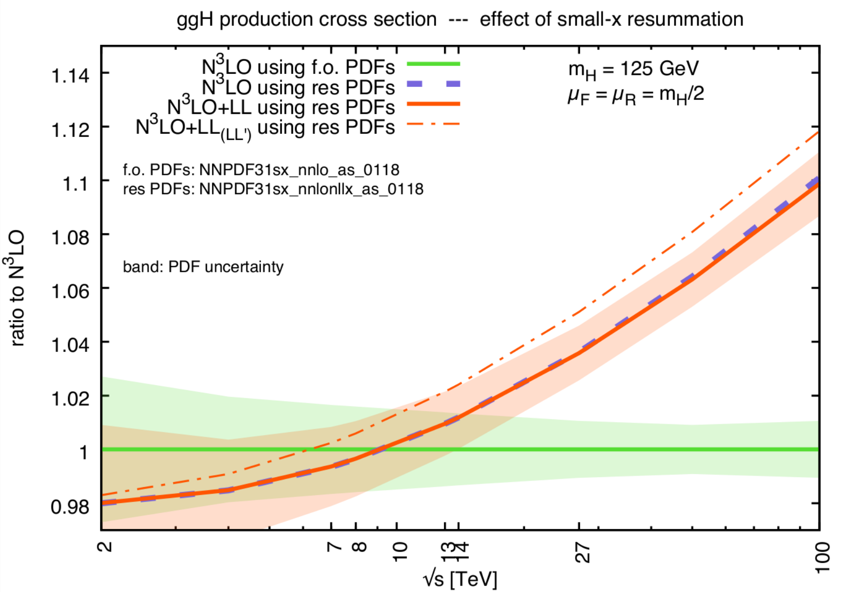


Figure: onset of small-*x* resummation effects in Higgs production via gluon fusion along increasing collider centre-of-mass energy, see 1805.08785

Recursive collinear subtraction[9]

$$\sigma = \bar{M} \otimes \bar{\Gamma}, \tag{13}$$

$\otimes$ denotes the standard convolution product in x space. Assume $d = 4 - 2\epsilon$ dimensions in order to regularize $\bar{M}$ and $\bar{\Gamma}$. Write the coefficient function $\bar{M}$ as the product of a hard part H and the iteration of a kernel K:

$$\bar{M} = H \otimes_{x,k_T} (1 + K + K \otimes_{x,k_T} K + ...) \equiv M(1 + K + K^2 + K^3 + ...) = H\frac{1}{1-K}, \tag{14}$$

where $\otimes_{x,k_T}$ stands both for convolution in x space and k_T integration.

$$1 - K = 1 - \mathcal{P}K - (1-\mathcal{P})K = \left[1 - \mathcal{P}K(1-(1-\mathcal{P})K)^{-1}\right][1-(1-\mathcal{P})K], \tag{15}$$

which leads to (note the reverse order):

$$\frac{1}{1-K} = \left[\frac{1}{1-(1-\mathcal{P})K}\right]\left[\frac{1}{1-\mathcal{P}K(1-(1-\mathcal{P})K)^{-1}}\right]. \tag{16}$$

Then finally

$$\sigma = \bar{M}\otimes\bar{\Gamma} = \left[H\frac{1}{1-(1-\mathcal{P})K}\right]\left[\frac{1}{1-\mathcal{P}K(1-(1-\mathcal{P})K)^{-1}}\bar{\Gamma}\right] \to M\otimes\Gamma \quad \text{when} \quad \epsilon \to 0$$

[9] again lifted from 1010.2743