A novel background field approach to the confinement-deconfinement transition

DvE, U. Reinosa, J. Serreau, M. Tissier (arXiv:2104.08974 [hep-ph])

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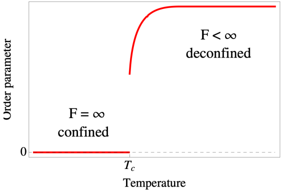
- From this, we know that:
  - At some very high temperature T_c, hadrons become free quarks and gluons $\rightarrow$ quark-gluon plasma.
  - This transition is related to the breaking of the center symmetry of a gauge group when using pure Yang-Mills theory (infinitely heavy quarks):

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2$$

Center symmetry

- The order parameter for the confinement/deconfinement transition is the Polyakov loop:

$$\mathcal{P} \propto e^{-F/T}$$



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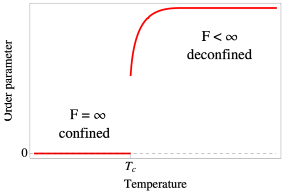

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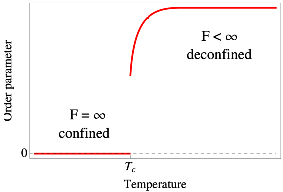

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- Under center symmetry $\mathcal{P} \rightarrow Z_N \mathcal{P}$, with Z_N the center elements of the gauge group. So, breaking of the center symmetry signals deconfinement.

Analytical results

- At high energies, gluon dynamics are well described by an $SU(3)$ pure Yang-Mills action with a Faddeev-Popov gauge fixing:

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \overline{c}^a (D_\mu c)^a + ib^a \partial_\mu A^a_\mu$$

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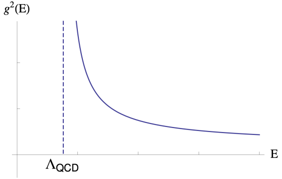

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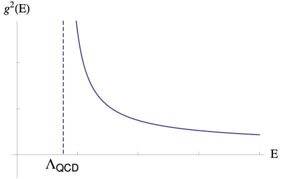

- Does this mean we have an infinite coupling at low energies? Probably not!

Gribov problem

- Gribov: for high values of the coupling constant, the FP gauge fixing does not uniquely fix the gauge field

$$A'^a_\mu = A^a_\mu - D^{ab}_\mu \alpha^b \qquad \partial_\mu A^a_\mu = \partial_\mu {A^{a\prime}}_\mu = 0$$

so that

$$\partial_\mu D^{ab}_\mu \alpha^b = (\partial^2 \delta^{ab} - g f^{abc} \partial_\mu A^c_\mu)\alpha^b = 0$$

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$$+ \bar{\varphi}^{ae}_\nu \partial_\mu D^{ab}_\mu \varphi^{be}_\nu - g\gamma^{1/2} f^{abc} A^a_\mu \left(\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu\right) - \gamma d d_G$$

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  - Curci-Ferrari model: $\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$

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- Gluons do not have a mass perturbatively
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- However, for $p \gg m$, $m \approx 0$
- Confined gluons do not have a physical interpretation, so BRST symmetry might be broken non-perturbatively

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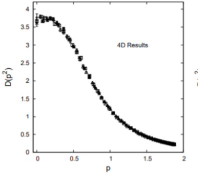

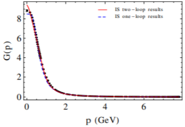

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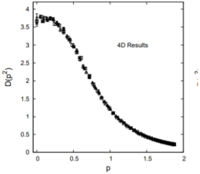

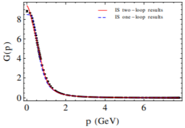

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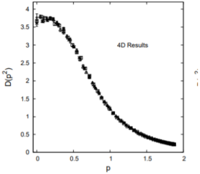

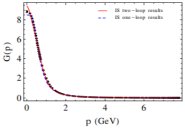

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- Can we use the CF (or GZ) model to describe the confinement/deconfinement transition? In principle: Yes. In practice: Maybe.

In principle...

... one would look for the state A_{min} that minimizes quantum action $\Gamma[A]$ for finite temperature

$$\Gamma[A_{min}] \leq \Gamma[A], \forall A.$$

Then, since for $SU(N)$

$$\mathcal{P} \propto exp[A_0],$$

we can use A_{min} as an order parameter for confinement by checking if it is center-symmetric. The temperature at which we go from a center-symmetric state to a non center-symmetric state is the transition temperature T_c.

Gauge-fixed lattice results: for $SU(2)$ the transition is second order, with $T_c = 295 MeV$. Therefore, at T_c we should find

$$\frac{\delta\Gamma(A)}{\delta A_\mu(p)\delta A_\nu(-p)}|_{A=A_{min}} = 0 \rightarrow \frac{1}{\langle A_0(p)A_0(-p)\rangle}|_{p=0} = \infty.$$

Gauge-fixed lattice vs continuum

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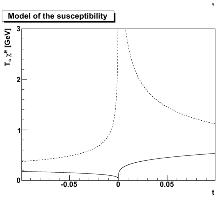
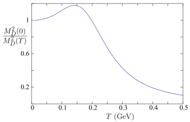




The problem is that by fixing the gauge, we broke the center symmetry Z_N of the $SU(N)$ gauge symmetry:

$$\Gamma[A] \neq \Gamma[A^U].$$

This should not alter the physical results in principle, but it can lead to problems when approximations (loop calculations) are involved.

Background field methods

To retain the center symmetry, we introduce the the Landau-DeWitt gauge

$$\overline{D}_\mu(A^b_\mu - \overline{A}^b_\mu) = 0,$$

with $\overline{A}^a_\mu$ an arbitrary background field and $\overline{D}^{ab}_\mu = \partial_\mu \delta^{ab} - gf^{abc}\overline{A}^c_\mu$. We consider the action

$$S[A, \overline{A}] = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{D}_\mu \overline{c}^a D_\mu c^a + ib^a \overline{D}_\mu (A^a_\mu - \overline{A}^a_\mu) + m^2 (A^a_\mu - \overline{A}^a_\mu) \right\},$$

which is invariant under the simultaneous $SU(N)$ transformation of the fields A^a_μ and $\overline{A}^a_\mu$

$$S[A^U, \overline{A}^U] = S[A, \overline{A}],$$

and center symmetry is preserved.

Background field methods

$$\begin{aligned} S[A^U, \bar{A}^U] &= S[A, \bar{A}] \\ \Gamma[A^U, \bar{A}^U] &= \Gamma[A, \bar{A}] \end{aligned}$$

Checking the invariance under center transformations of $A_{\mathrm{min}}[\bar{A}]$ is ambiguous:

$$\Gamma[A_{min}(\bar{A}), \bar{A}] = \Gamma[A^U_{min}(\bar{A}), \bar{A}^U].$$

Transforming the minimizing state transforms it into the minimizing function of another potential, with another gauge fixing. Therefore, we cannot identify the center-symmetric states.

Background effective action

In the method of the BG effective action[1], we define a new object

$$\tilde{\Gamma}[\overline{A}] \equiv \Gamma[A = \overline{A}, \overline{A}],$$

and we can show that $\overline{A}_{min}$ such that

$$\tilde{\Gamma}[A_{min}(\overline{A})] \leq \tilde{\Gamma}[\overline{A}], \qquad \forall \overline{A}$$

are alternative order parameters for center symmetry. There is no ambiguity anymore since:

$$\tilde{\Gamma}[\overline{A}] = \tilde{\Gamma}[\overline{A}^U].$$

The drawback of the BG method is that $\tilde{\Gamma}[\overline{A}]$ is a formal object an does not relate directly to gauge-fixed quantities such as propagators and vertices.

[1] J. Braun, H. Gies and J.M. Pawlowski, Phys. Lett. B 684,262-267 (2010)

Center-symmetric effective action

In the solution of the CS effective action , we define the center symmetric state A_c and make the gauge choice $\overline{A} = A_c$ and define

$$\Gamma_c[A] = \Gamma[A, \overline{A} = A_c].$$

Now

$$\Gamma_c[A] \neq \Gamma_c[A^U], \quad \forall U$$

but

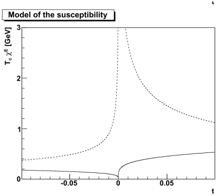
$$\Gamma_c[A] = \Gamma_c[A^{U^c}], \quad \forall U^c$$

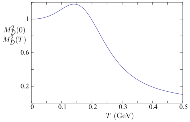
with U^c the center transformations.

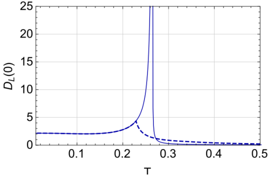
There is no ambiguity since the background field $\overline{A}$ is fixed.

Since $\Gamma_c[A]$ is nothing but a particular gauge choice of $\Gamma[A, \overline{A}]$, we can directly reach gauge-fixed quantities such as propagators and vertices.

Results - electric susceptibility







Results - Transition temperature $T_c(MeV)$

	Lattice	FRG [2]	CF-BG at 1-lp [3]	CF-BG at 2-lp [4]	CF-CS at 1-lp
SU(2)	295	230	238	284	265
SU(3)	270	275	185	254	267

[2] L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010
[3] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.
[4] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

Conclusion and Outlook

- In this work, we have found a transition temperature for the confinement-deconfinement transition through the CS effective action, which is closer to the lattice results than the BG effective action.
- We have shown that the CS effective action gives direct access to gauge-fixed quantities. This gives the opportunity to compare with lattice data.
- For a future work, we hope to extend our work to higher loop orders and also determine the gluon propagator.