Effective field theory methods for high temperature/density plasmas

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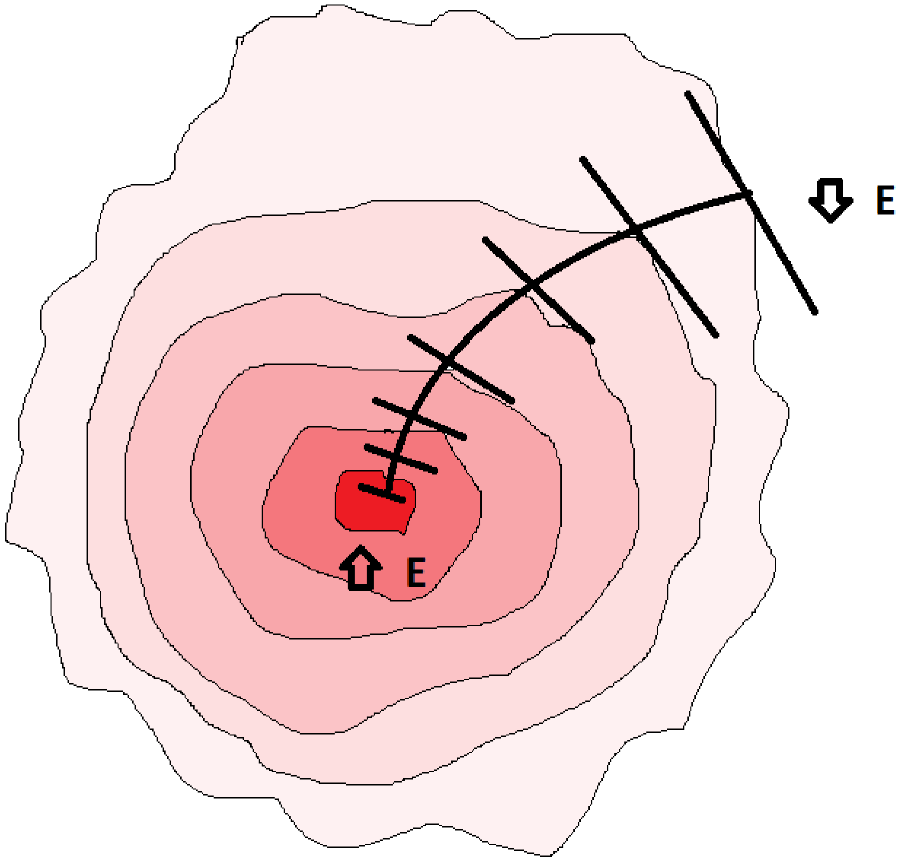
Outline

- 1. The on-shell effective field theory.
- 2. Turning on T (or μ).
- 3. Polarization tensor. Mass corrections to HTL.
- 4. Future work and applications.

1. The on-shell effective field theory (OSEFT)

- The rationale behind OSEFT is the same as any other EFT such as HDET (Deog Ki Hong,1999) or SCET (Bauer, Fleming...,2000).

1. Identify the hard scale in the system (p) and define small fluctuations (k) around that scale.
2. Integrate out the high energy modes, to get a theory for the lower energy scales.
3. The EFT is organized as an expansion of operators of increasing dimension over powers of the high energy scale (k/p).



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On-shell $v^\mu = (1, \boldsymbol{v})$

$v^2 = 0$

Residual momentum $k^\mu \ll p$

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$$\psi_{v,\tilde{v}}(x) = e^{-ipv\cdot x}\left(P_v \chi_v(x) + P_{\tilde{v}} H_{\tilde{v}}^{(1)}(x)\right) + e^{ip\tilde{v}\cdot x}\left(P_{\tilde{v}} \xi_{\tilde{v}}(x) + P_v H_v^{(2)}(x)\right)$$

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$$P_v = \frac{\not{v}\gamma^0}{2} \qquad P_{\tilde{v}} = \frac{\not{\tilde{v}}\gamma^0}{2}.$$

Particle and antiparticle projectors

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Hard momentum

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Residual momentum

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Plug this expression into the QED Lagrangian.

$$\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi$$

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Plug this expression into the QED Lagrangian $\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi$ And find

$$\mathcal{L}_{p,v} = \sum_{p,v}\Big[\overline{\chi}_v i(v\cdot D)\not{\tilde{v}}\chi_v + \overline{H}^{(1)}_{\tilde{v}}(2p + i\tilde{v}\cdot D)\not{v}H^{(1)}_{\tilde{v}}$$

$$+\overline{\chi}_v(i\not{D}_\perp - m)H^{(1)}_{\tilde{v}} + \overline{H}^{(1)}_{\tilde{v}}(i\not{D}_\perp - m)\chi_v\Big]$$

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Using the Lagrange equations of motion

$$H_{\tilde{v}}^{(1)} = -\not{v} \frac{i\not{D}_\perp - m}{2p + i\tilde{v}\cdot D}\chi_v = \frac{i\not{D}_\perp + m}{2p + i\tilde{v}\cdot D}\frac{\not{\tilde{v}}}{2}\chi_v$$

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- OSEFT (m=0) C.Manuel, J.Soto, S.Stetina 2016.
- OSEFT (m≠0) C.Manuel, J.M Torres-Rincon 2021.

$$\mathcal{L}_{p,v} = \bar{\chi}_v(x) \left(i\, v \cdot D \, + i \not{D}_\perp \frac{1}{2p + i\tilde{v} \cdot D} i \not{D}_\perp - m^2 \frac{1}{2p + i\tilde{v} \cdot D} \right) \frac{\tilde{\not{v}}}{2} \chi_v(x)$$
$$- \bar{\chi}_v(x) \left(m \left[\frac{1}{2p + i\tilde{v} \cdot D}, i \not{D}_\perp \right] \right) \frac{\tilde{\not{v}}}{2} \chi_v(x) \; ,$$

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$$- \bar{\chi}_v(x)\left(m\left[\frac{1}{2p + i\tilde{v}\cdot D}, i\not{D}_\perp\right]\right)\frac{\not{\tilde{v}}}{2}\chi_v(x)\ ,$$

- Now it can be expanded using (p) as the hard scale in the system.
- From the OSEFT Lagrangian one can extract the vertices and propagators in order to compute Feynman diagrams.

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$$\mathcal{L}^{(2)}_{p,v} = \bar{\chi}'_v \frac{1}{8p^2} \Big(\left[\not{D}_\perp \, , \, \left[i\tilde{v} \cdot D \, , \, \not{D}_\perp \right] \right] - \left\{ \not{D}_\perp^2 + m^2 , \; iv \cdot D - i\tilde{v} \cdot D \right\} + 2iem \, \tilde{v}^\mu F_{\mu\alpha} \gamma_\perp^\alpha \Big) \gamma^0 \chi'_v$$

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Breaks chirality ↑

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$$\begin{aligned} \mathcal{L}^{(3)}_{p,v} = \frac{1}{8p^3} \bar{\chi}''_v \Bigg(& (\not{D}_\perp^2 + m^2)^2 + [i\tilde{v} \cdot D, \not{D}_\perp]^2 - (iv \cdot D - i\tilde{v} \cdot D)(\not{D}_\perp^2 + m^2)(iv \cdot D - i\tilde{v} \cdot D) \\ & + (iv \cdot D - i\tilde{v} \cdot D) \not{D}_\perp [i\tilde{v} \cdot D, \not{D}_\perp] - [i\tilde{v} \cdot D, \not{D}_\perp] \not{D}_\perp (iv \cdot D - i\tilde{v} \cdot D) \\ & + m \left\{ iv \cdot D - i\tilde{v} \cdot D, [i\tilde{v} \cdot D \, , \, i\not{D}_\perp] \right\} \Bigg) \gamma^0 \chi''_v . \end{aligned}$$

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Local field redefinitions at second and third order simplify computations

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- OSEFT propagator for particles (T=0): $$S(k) = \frac{P_v \gamma^0}{k_0 - f(\boldsymbol{k}) + i\epsilon}$$

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Dispersion relation

$$f(\boldsymbol{k}) = k_{\parallel} + \frac{\boldsymbol{k}_{\perp}^2 + m^2}{2p} - \frac{(\boldsymbol{k}_{\perp}^2 + m^2)k_{\parallel}}{2p^2}$$

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- Eventually propagators must also be expanded in powers of (k/p) but it is better to keep them unexpanded (as long as possible).

↓

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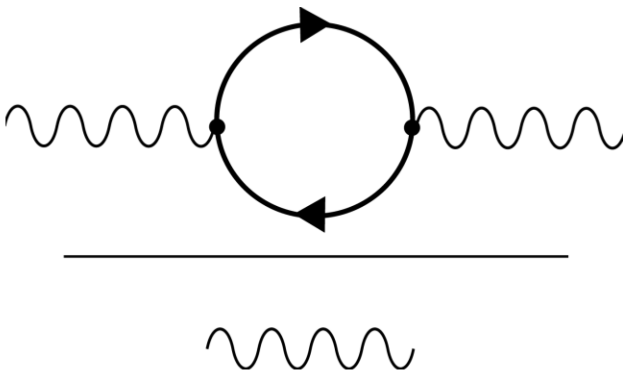
2. Turning on T

A difficulty when turning on the temperature (or μ):



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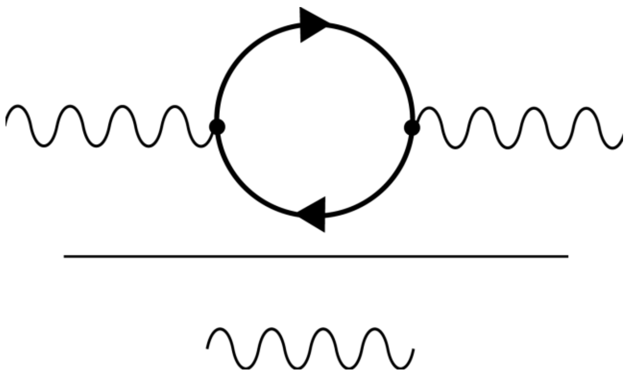

$$\sim \frac{e^2T^2}{l^2}$$

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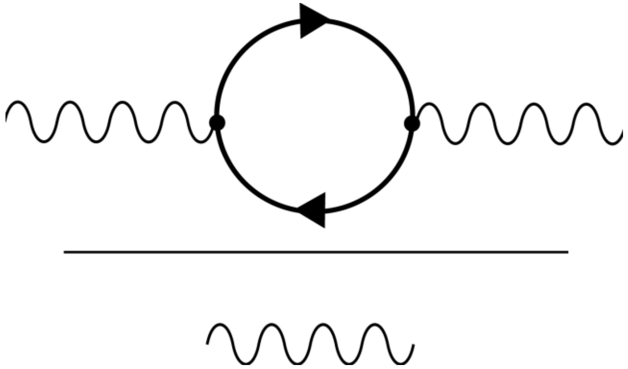

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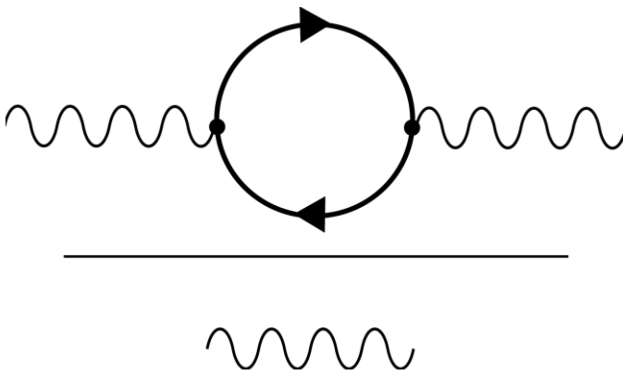

$$\sim \frac{e^2 T^2}{l^2} \sim 1 \quad \text{if} \quad l \sim eT$$

- So that HTL are as relevant as the tree amplitudes in the theory if the photons have soft momenta => Resummation.
- HTL's are important to compute physical quantities!

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$$\Delta^{T=0}(x,y) = \langle 0|\mathcal{T}\phi(x)\phi(y)|0\rangle \longrightarrow \Delta^{T\neq 0}(x,y) = \langle \mathcal{T}\phi(x)\phi(y)\rangle_{\beta}$$

Vacuum expectation value

Thermal Average

2. Turning on T

Time ordering

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$$\langle A\rangle_\beta = \mathrm{tr}\,(\rho A)$$

Trace

$$\rho = \frac{e^{-\beta H}}{Z}.$$

Density matrix

$$Z = \mathrm{tr}\left(e^{-\beta H}\right).$$

Partition function

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Introduce a solution in terms of creation and annihilation operators

$$\phi(x) = \int \frac{d^3p}{\sqrt{2(2\pi)^3\omega_p}}\left(a(\boldsymbol{p})e^{-ipx} + a^\dagger(\boldsymbol{p})e^{ipx}\right)$$

2. Turning on T

$$i\Delta(k) = \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi n_B(k_0)\delta(k^2 - m^2)$$

Bose-Einstein

$$n_B(k_0) = \frac{1}{e^{\beta|k_0|} - 1}.$$

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Off-Shell

On-Shell

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e.g Markus H. Thoma
arXiv:hep-ph/0010164v1

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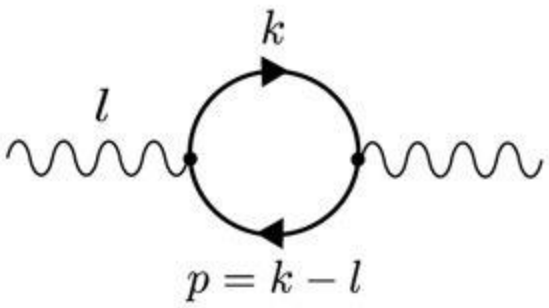

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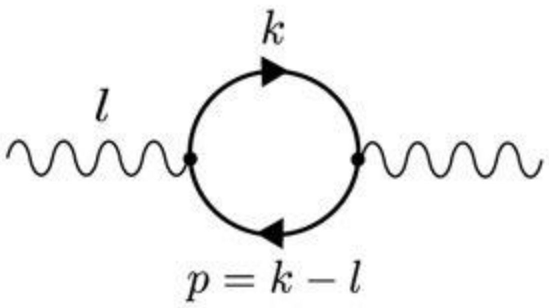

$$\sim [\delta(k^2 - m^2)]^2 \sim \delta(0) \to \infty.$$!

Pinch singularity

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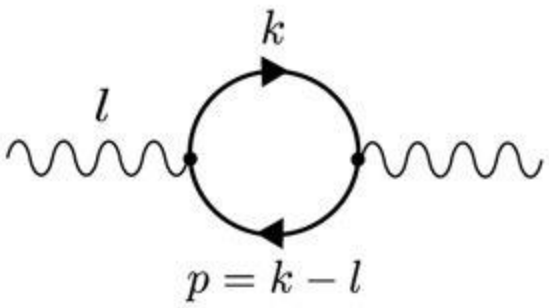

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Pinch singularity

A solution: doubling of degrees of freedom

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The path on the temporal component is parametrized with a certain curve.

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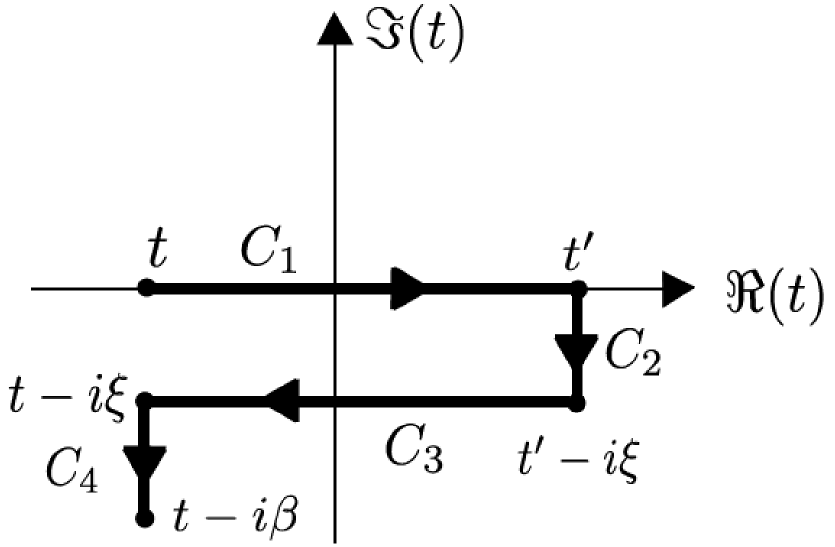
$$C = C_1 + C_2 + C_3 + C_4$$



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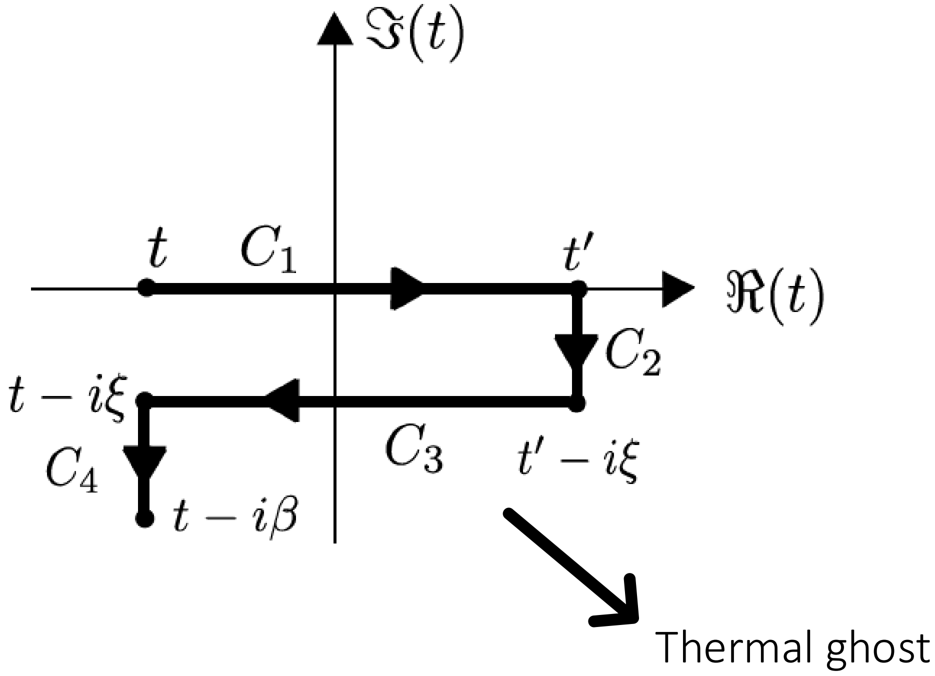

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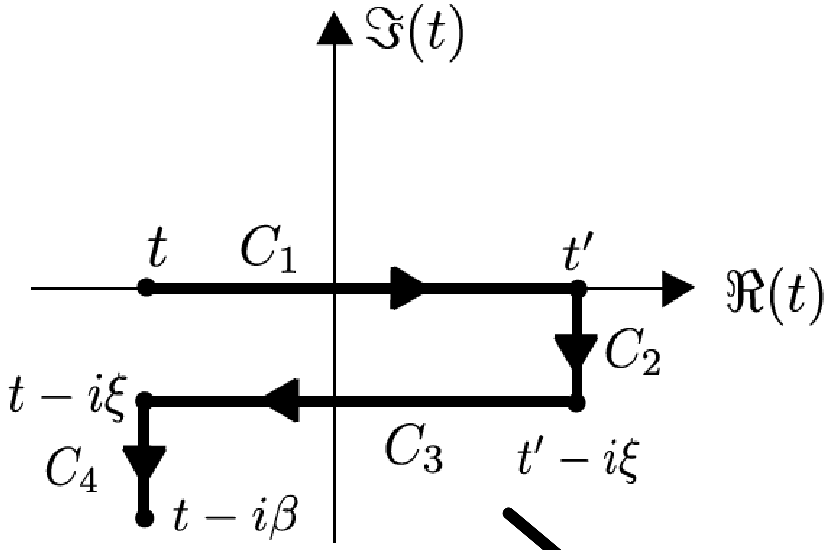


Thermal ghost

Le Bellac:

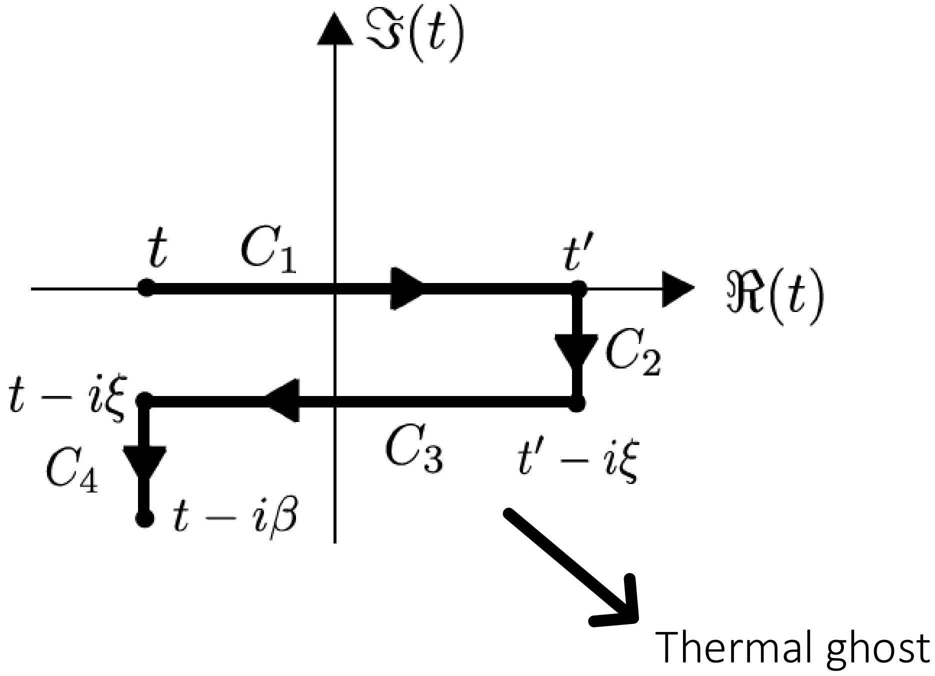
$$\Delta_C(x, x') = -i \int_C \frac{d^4k}{(2\pi)^4} [\theta_C(t - t') + f(k_0)]\rho(k) e^{-ik(x-x')}$$

$$f(k_0) = (e^{\beta k_0} - 1)^{-1}$$

$$\rho(k) = 2\pi \mathrm{sgn}(k_0)\delta(k^2 - m^2)$$

Spectral function

2. Turning on T



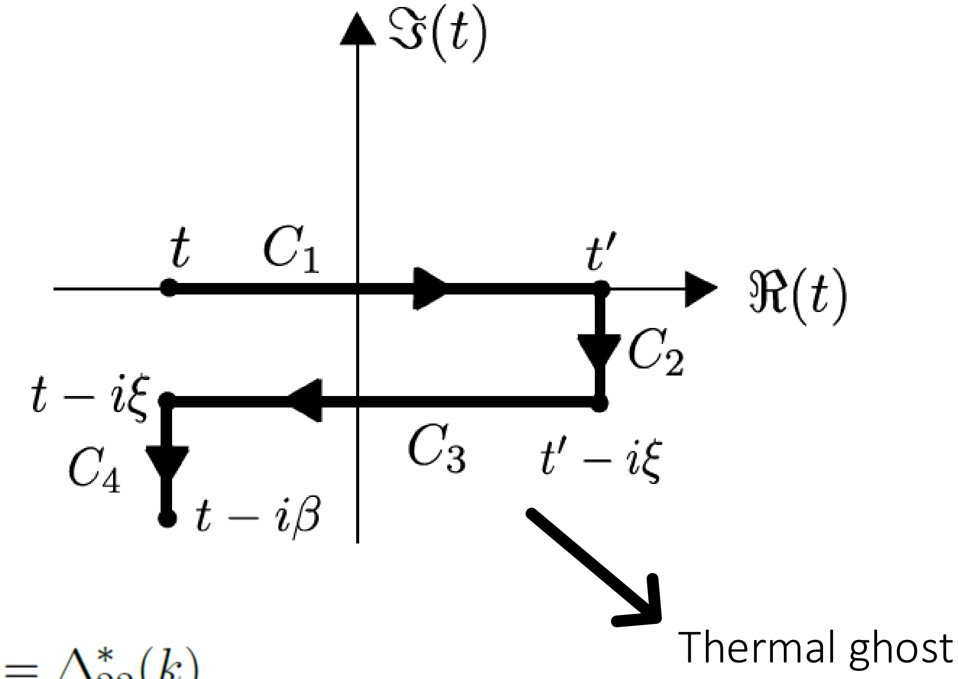
$$\Delta_{C_2}(t', t' - i\xi) = \Delta_{C_2}(0, -i\xi)$$

$$\Delta_{C_3}(t' - i\xi, t - i\xi) = \Delta_{C_3}(t', t) = \Delta^*_{C_3}(t, t')$$

$$\Delta_{C_4}(t - i\xi, t - i\beta) = \Delta_{C_4}(-i\xi, -i\beta) = \Delta_{C_4}(-i\xi, 0)$$

2. Turning on T

$$\Delta_{ij} = \begin{pmatrix} \Delta_{C_1} & \Delta_{C_2} \\ \Delta_{C_3} & \Delta_{C_4} \end{pmatrix} \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}$$



$$\Delta_{11}(k) = \frac{1}{k^2 - m^2 + i\epsilon} - i2\pi n_B(k_0)\delta(k^2 - m^2) = \Delta_{22}^*(k)$$

$$\Delta_{12}(k) = -i2\pi\delta(k^2 - m^2)[\theta(-k_0) + n_B(k_0)]$$

$$\Delta_{21}(k) = -i2\pi\delta(k^2 - m^2)[n_B(k_0) + \theta(k_0)].$$

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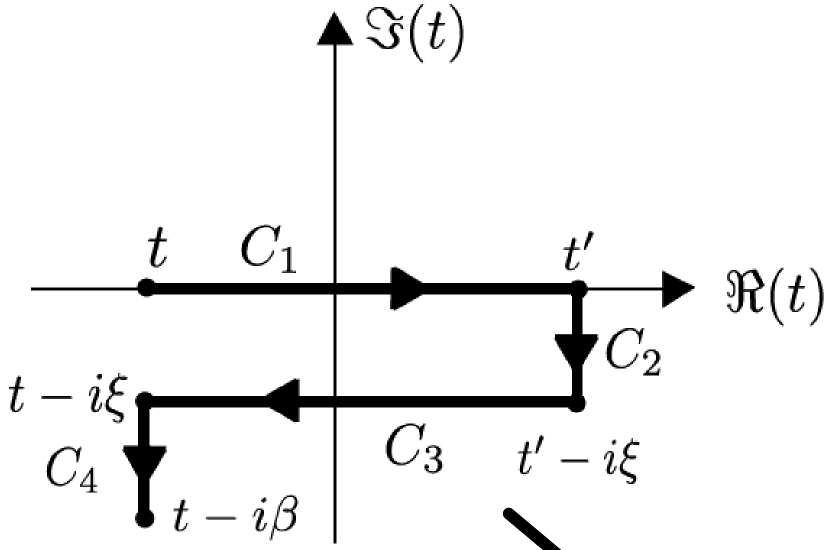

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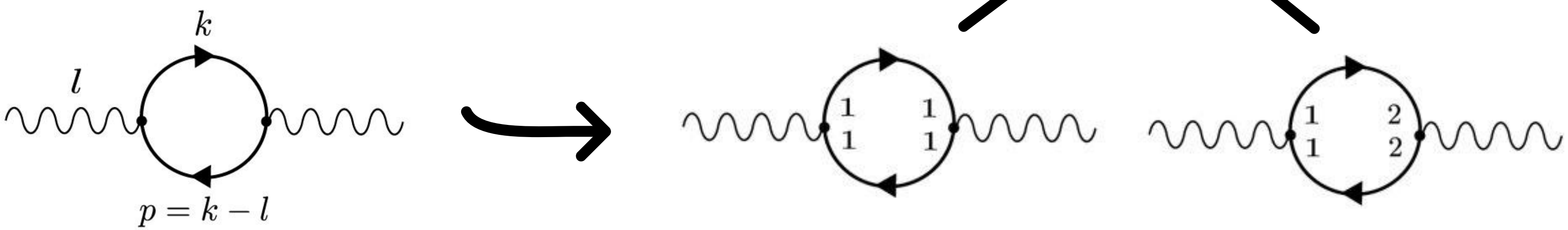
$$\Delta_{ij} = \begin{pmatrix} \Delta_{C_1} & \Delta_{C_2} \\ \Delta_{C_3} & \Delta_{C_4} \end{pmatrix} \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}$$

$$\Delta_{11}(k) = \frac{1}{k^2 - m^2 + i\epsilon} - i2\pi n_B(k_0)\delta(k^2 - m^2) = \Delta_{22}^*(k)$$

$$\Delta_{12}(k) = -i2\pi\delta(k^2 - m^2)[\theta(-k_0) + n_B(k_0)]$$

$$\Delta_{21}(k) = -i2\pi\delta(k^2 - m^2)[n_B(k_0) + \theta(k_0)].$$

Free of pinch singularities



$$\Delta_{11} - \Delta_{12} - \Delta_{21} + \Delta_{22} = 0$$

2. Turning on T

In order to perform computations is better if we move to the Keldysh representation.

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We can also perform similar definitions for the self-energy.

$$\Pi_A = \Pi_{11} + \Pi_{21} \qquad \Pi_R = \Pi_{11} + \Pi_{12} \qquad \Pi_S = \Pi_{11} + \Pi_{22}$$

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The physical is the retarded self-energy.

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Finally:

$$\Delta_{A,R}(k) = \frac{1}{k^2 - m^2 \mp i\mathrm{sgn}(k_0)\epsilon}$$

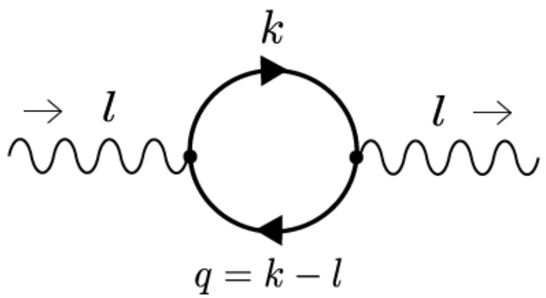
$$\Delta_S(k) = -i2\pi\delta(k^2 - m^2)[1 + 2n_B(k_0)].$$

Marc Comadran, Cristina Manuel. arXiv:2106.08904
Soon in Physics Review D.

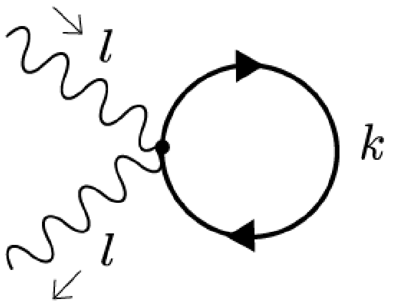
3. Polarization tensor. Mass corrections to HTL.

There are two diagrams that contribute to the photon self-energy in the OSEFT:

Bubble:



Tadpole:



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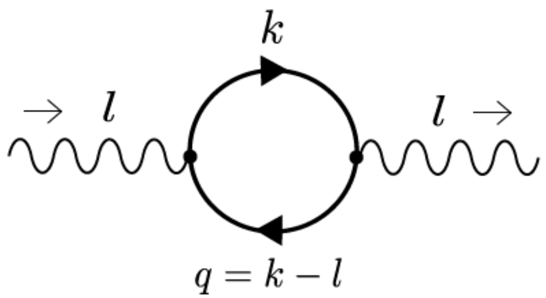

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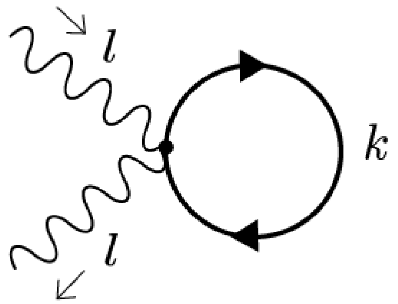

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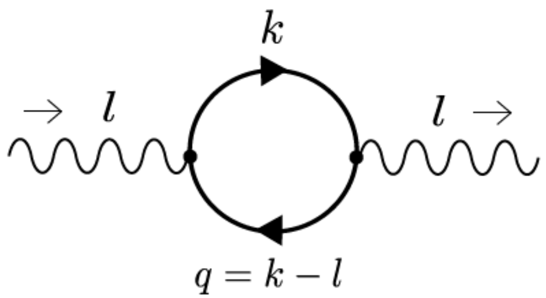
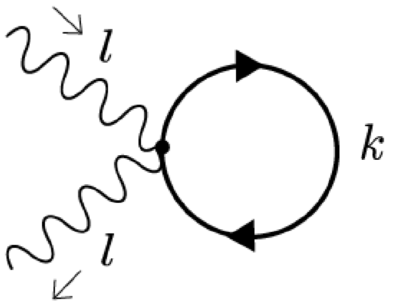


$$\Pi^{\mu\nu}_{b,\chi}(l) = \frac{i}{2}\sum_{p,\mathbf{v}}\int\frac{d^4k}{(2\pi)^4}\left\{\mathrm{Tr}\left[V^\mu S^\chi_S(k-l)V^\nu S^\chi_R(k)\right] + \mathrm{Tr}\left[V^\mu S^\chi_A(k-l)V^\nu S^\chi_S(k)\right]\right\}$$

Tadpole:



$$\Pi^{\mu\nu}_{t,\chi}(l) = -\frac{i}{2}\sum_{p,\mathbf{v}}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[W^{\mu\nu}S^\chi_S(k)\right]$$

3. Polarization tensor. Mass corrections to HTL.

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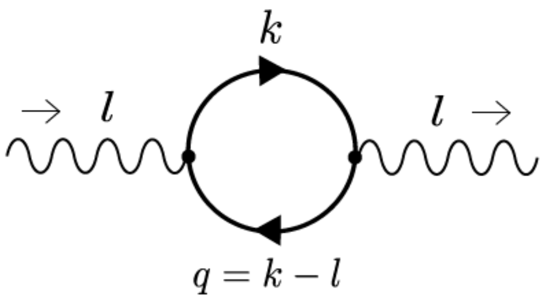

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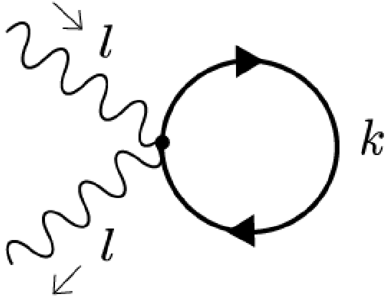

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- Real time formalism
- Keldysh representation

3. Polarization tensor. Mass corrections to HTL.

After inserting the definitions of the (thermal) OSEFT propagators and performing the energy integrals, one can reach the general expressions:

$$\Pi^{\mu\nu}_{b,\chi}(l) = -\sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathrm{Tr}\left[V^\mu P_\chi P_v \gamma^0 V^\nu P_v \gamma^0\right] \frac{n_F(p+f(\mathbf{k}-\boldsymbol{l},m)) - n_F(p+f(\mathbf{k},m))}{l_0 + i0^+ + f(\mathbf{k}-\boldsymbol{l},m) - f(\mathbf{k},m)}$$

And:

$$\Pi^{\mu\nu}_{t,\chi}(l) = -\frac{1}{2}\sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathrm{Tr}\left[W^{\mu\nu} P_\chi P_v \gamma^0\right] \left(1 - 2n_F(p+f(\mathbf{k},m))\right)$$

3. Polarization tensor. Mass corrections to HTL.

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Now this expressions must be expanded in inverse powers of the high energy scale (1/p)^n.

3. Polarization tensor. Mass corrections to HTL.

For instance at n=1 we find:

$$\Pi^{\mu\nu}(l) = -e^2 \sum_{\chi=\pm} \sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{dn_F}{dp} \left(\frac{P_\perp^{\mu\nu}}{2} + v^\mu v^\nu - l_0 \frac{v^\mu v^\nu}{v \cdot l} \right)$$

Then one goes back to the variables of the full theory:

$$\sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3}$$

And

$$p = q - k_{\parallel,\hat{\mathbf{q}}} + \frac{\mathbf{k}_{\perp,\hat{\mathbf{q}}}^2}{2q} + \mathcal{O}\left(\frac{1}{q^2}\right) ,$$

$$\mathbf{v} = \hat{\mathbf{q}} - \frac{\mathbf{k}_{\perp,\hat{\mathbf{q}}}}{q} - \frac{\hat{\mathbf{q}}\mathbf{k}_{\perp,\hat{\mathbf{q}}}^2 + 2k_{\parallel,\hat{\mathbf{q}}}\mathbf{k}_{\perp,\hat{\mathbf{q}}}}{2q^2} + \mathcal{O}\left(\frac{1}{q^3}\right) ,$$

$$n_F(p) = n_F(q) + \frac{dn_f}{dq}\left(-k_{\parallel,\hat{\mathbf{q}}} + \frac{\mathbf{k}_{\perp,\hat{\mathbf{q}}}^2}{2q}\right) + \frac{1}{2}\frac{d^2 n_F}{dq^2} k_{\parallel,\hat{\mathbf{q}}}^2 + \mathcal{O}\left(\frac{1}{q^3}\right)$$

3. Polarization tensor. Mass corrections to HTL.

The HTL contribution:

$$\Pi^{\mu\nu}_{\rm htl}(l) = -4e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{dn_F}{dq}\left(\delta^\mu_0\delta^\nu_0 - l_0\frac{v^\mu_{\hat{\mathbf{q}}}v^\nu_{\hat{\mathbf{q}}}}{v_{\hat{\mathbf{q}}}\cdot l}\right)$$

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$$\Pi^{00}_R(l) = \Pi^L_R(l) \qquad \Pi^{0i}_R(l) = \frac{l_0 l^i}{|\boldsymbol{l}|^2} \Pi^L_R(l)$$

$$\Pi^{ij}_R(l) = \left(\delta^{ij} - \frac{l^i l^j}{|\boldsymbol{l}|^2} \right) \Pi^T_R(l) + \frac{l^i l^j}{|\boldsymbol{l}|^2} \frac{l_0^2}{|\boldsymbol{l}|^2} \Pi^L_R(l)$$

$$\Pi^L_R(l) = -m_D^2 \left[1 - \frac{l_0}{2|\boldsymbol{l}|} \left(\ln \left| \frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|} \right| - i\pi\theta(\boldsymbol{l}^2 - l_0^2) \right) \right]$$

$$\Pi^T_R(l) = m_D^2 \frac{l_0^2}{2|\boldsymbol{l}|^2} \left[1 - \left(1 - \frac{|\boldsymbol{l}|^2}{l_0^2} \right) \frac{l_0}{2|\boldsymbol{l}|} \left(\ln \left| \frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|} \right| - i\pi\theta(\boldsymbol{l}^2 - l_0^2) \right) \right]$$

3. Polarization tensor. Mass corrections to HTL.

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Debye mass

$$m_D^2 = \frac{e^2 T^2}{3}$$

Landau damping

$$\Pi^L_R(l) = -m_D^2 \left[1 - \frac{l_0}{2|\boldsymbol{l}|} \left(\ln\left|\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right| - i\pi\theta(\boldsymbol{l}^2 - l_0^2)\right)\right]$$

$$\Pi^T_R(l) = m_D^2 \frac{l_0^2}{2|\boldsymbol{l}|^2} \left[1 - \left(1 - \frac{|\boldsymbol{l}|^2}{l_0^2}\right) \frac{l_0}{2|\boldsymbol{l}|} \left(\ln\left|\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right| - i\pi\theta(\boldsymbol{l}^2 - l_0^2)\right)\right]$$

3. Polarization tensor. Mass corrections to HTL.

At $n=2$ there are no contributions. (If there is a chiral chemical potential, one finds the CME)

3. Polarization tensor. Mass corrections to HTL.

At n=3 we find the mass dependent contributions.

$$\Pi^{\mu\nu}_{\mathrm{m}}(l) = -4m^2e^2\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left\{\frac{n_F}{2q^3}\left[P^{\mu\nu}_{\perp,\hat{\mathbf{q}}} + \frac{(v^\mu_{\hat{\mathbf{q}}}-\tilde{v}^\mu_{\hat{\mathbf{q}}})(v^\nu_{\hat{\mathbf{q}}}-\tilde{v}^\nu_{\hat{\mathbf{q}}})}{2}\right]\right.$$

$$\left.+\frac{1}{q^2}\frac{dn_F}{dq}\left[-\frac{P^{\mu\nu}_{\perp,\hat{\mathbf{q}}}}{2} + l_{\parallel,\hat{\mathbf{q}}}\left(\frac{v^\mu_{\hat{\mathbf{q}}}v^\nu_{\hat{\mathbf{q}}}}{v_{\hat{\mathbf{q}}}\cdot l} + \frac{l_{\parallel,\hat{\mathbf{q}}}}{2}\frac{v^\mu_{\hat{\mathbf{q}}}v^\nu_{\hat{\mathbf{q}}}}{(v_{\hat{\mathbf{q}}}\cdot l)^2} + \frac{1}{4}\frac{v^\mu_{\hat{\mathbf{q}}}(v^\nu_{\hat{\mathbf{q}}}-\tilde{v}^\nu_{\hat{\mathbf{q}}}) + v^\nu_{\hat{\mathbf{q}}}(v^\mu_{\hat{\mathbf{q}}}-\tilde{v}^\mu_{\hat{\mathbf{q}}})}{v_{\hat{\mathbf{q}}}\cdot l}\right)\right]\right\}$$

3. Polarization tensor. Mass corrections to HTL.

At n=3 we find the mass dependent contributions.

IR divergent. We have to regularize!

$$\Pi^{\mu\nu}_{\mathrm{m}}(l) = -4m^2e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3}\left\{ \frac{n_F}{2q^3}\left[P^{\mu\nu}_{\perp,\hat{\mathbf{q}}} + \frac{(v^\mu_{\hat{\mathbf{q}}} - \tilde{v}^\mu_{\hat{\mathbf{q}}})(v^\nu_{\hat{\mathbf{q}}} - \tilde{v}^\nu_{\hat{\mathbf{q}}})}{2}\right] \right.$$
$$\left. + \frac{1}{q^2}\frac{dn_F}{dq}\left[-\frac{P^{\mu\nu}_{\perp,\hat{\mathbf{q}}}}{2} + l_{\parallel,\hat{\mathbf{q}}}\left(\frac{v^\mu_{\hat{\mathbf{q}}}v^\nu_{\hat{\mathbf{q}}}}{v_{\hat{\mathbf{q}}}\cdot l} + \frac{l_{\parallel,\hat{\mathbf{q}}}}{2}\frac{v^\mu_{\hat{\mathbf{q}}}v^\nu_{\hat{\mathbf{q}}}}{(v_{\hat{\mathbf{q}}}\cdot l)^2} + \frac{1}{4}\frac{v^\mu_{\hat{\mathbf{q}}}(v^\nu_{\hat{\mathbf{q}}} - \tilde{v}^\nu_{\hat{\mathbf{q}}}) + v^\nu_{\hat{\mathbf{q}}}(v^\mu_{\hat{\mathbf{q}}} - \tilde{v}^\mu_{\hat{\mathbf{q}}})}{v_{\hat{\mathbf{q}}}\cdot l}\right)\right]\right\}$$

Finite pieces

3. Polarization tensor. Mass corrections to HTL.

Radial integral: $$\nu^{-2\epsilon}\int_0^\infty dq q^{-1+2\epsilon} n_F(q) = \frac{1}{4\epsilon} + \frac{1}{2}\ln\left(\frac{\pi T e^{-\gamma_E}}{2\nu}\right) + \mathcal{O}(\epsilon)$$

Angular integral: $$S_{3+2\epsilon}^{-1}\int d\Omega_{3+2\epsilon}\left(-\delta^{ij} + 3\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j\right) = -\frac{2}{3}\epsilon + \mathcal{O}(\epsilon^2)$$

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Angular integral: $$S_{3+2\epsilon}^{-1}\int d\Omega_{3+2\epsilon}\left(-\delta^{ij} + 3\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j\right) = -\frac{2}{3}\epsilon + \mathcal{O}(\epsilon^2)$$

- There is a subtle cancellation of the infrared divergency between the radial and angular integral!
- With a cut-off the infrared divergence still cancels, but one would not get the finite piece needed to preserve the Ward identity.

$$\Pi_{\mathrm{m}}^{\mathrm{L}}(l_0, \boldsymbol{l}) = \frac{e^2 m^2}{2\pi^2}\frac{\boldsymbol{l}^2}{l_0^2 - \boldsymbol{l}^2},$$

$$\Pi_{\mathrm{m}}^{\mathrm{T}}(l_0, \boldsymbol{l}) = \frac{e^2 m^2}{2\pi^2}\frac{l_0}{2|\boldsymbol{l}|}\ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)$$

3. Polarization tensor. Mass corrections to HTL.

$$\Pi_L^{\rm htl}(l_0,\boldsymbol{l}) = \frac{e^2T^2}{3}\left(1-\frac{l_0}{2|\boldsymbol{l}|}\ln\left(\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right)\right) \ ,$$

$$\Pi_L^{\rm pow\cdot corr}(l_0,\boldsymbol{l}) = -\frac{e^2}{4\pi^2}\left(\boldsymbol{l}^2-\frac{l_0^2}{3}\right)\left(1-\frac{l_0}{2|\boldsymbol{l}|}\ln\left(\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right)\right) \ ,$$

$$\Pi_L^{\rm 2loop}(l_0,\boldsymbol{l}) = \frac{e^4T^2L^2}{8\pi^2\boldsymbol{l}^2} \ ,$$

$$\Pi_T^{\rm htl}(l_0,\boldsymbol{l}) = \frac{e^2T^2}{3}\frac{l_0}{4\boldsymbol{l}^3}\left(2|\boldsymbol{l}|l_0 - L^2\ln\left(\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right)\right) \ ,$$

$$\Pi_T^{\rm pow\cdot corr}(l_0,\boldsymbol{l}) = \frac{e^2}{4\pi^2}\left(\frac{l_0^2}{2}+\frac{l_0^4}{6\boldsymbol{l}^2}-\frac{2\boldsymbol{l}^2}{3}-\frac{l_0^3}{12\boldsymbol{l}^3}\left(2\boldsymbol{l}^2+l_0^2-\frac{3\boldsymbol{l}^4}{l_0^2}\right)\ln\left(\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right)\right)$$

$$\Pi_T^{\rm 2loop}(l_0,\boldsymbol{l}) = -\frac{e^4T^2}{16\pi^2}\frac{l_0}{|\boldsymbol{l}|}\ln\left(\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right) \ .$$

3. Polarization tensor. Mass corrections to HTL.

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$m, l \sim eT$ → Same order

$eT < m \ll T$ → Dominant

Future work and applications:

- Energy loss
- Compute corrections for Fermion self-energy
- Apply formalism to QCD
- Plasma oscillations
- Secret (or a Russian guy will do it before you) :P

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