pQCD with Jets

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Overview

Goal

Precise determination and understanding of QCD parameters α_S and quark masses (top, bottom) to help in the search for new physics.

Tools

- **Jets**
- **Event Shapes:** distributions very sensitive to QCD dynamics $\Rightarrow$ measurements of α_S, m_q
- **Effective Field Theories** (based on QCD):
  - ∗ Soft Collinear Effective Theory $\rightarrow$ SCET
  - ∗ (boosted)-Heavy Quark Effective Theory $\rightarrow$ bHQET

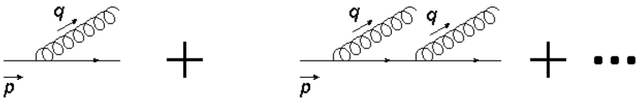
to sum large logs:

Do we have to use EFT?

Reference: QFT and SM. by Schwartz. Chapter 36.

SCET.

Jets



Massless quark ($p^2 = 0$):

$$Prob \sim \sum_n \mathcal{O}\left[\left(\frac{\alpha_s}{(p-n\,q)^2}\right)^n\right]$$

$$Den = (p-n\,q)^2 = -2n\,p\cdot q$$

- $p \cdot q << 1$:
  - $*$ q soft: ($q^\mu \sim \lambda^2$; $\lambda << 1$) $\Rightarrow p \cdot q << 1$
  - $*$ q collinear: ($\vec{q} \parallel \vec{p}$) $\Rightarrow p \cdot q << 1$

Massive quark ($p^2 = m^2$):

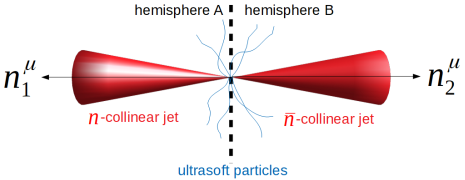
$$Prob \sim \sum_n \mathcal{O}\left[\left(\frac{\alpha_s}{(p-n\,q)^2 - m^2}\right)^n\right]$$

$$Den = (p-n\,q)^2 - m^2 = -2n\,p\cdot q$$

- $p \cdot q << 1$:
  - $*$ q soft: ($q^\mu \sim \lambda^2$; $\lambda << 1$) $\Rightarrow p \cdot q << 1$
  - $*$ q collinear: ($\vec{q} \parallel \vec{p}$) $\Rightarrow p \cdot q \sim \mathcal{O}\left(m/|\vec{p}|\right)$

$$\boxed{Prob \sim \sum \mathcal{O}(1) \Rightarrow Jets}$$

Dijet kinematics



Scales

- CM energy: Q
- Collinear: $p^2_{n,\bar{n}} \sim Q^2\lambda^2$ Primary production $\Rightarrow \hat{m} \sim \lambda$; $\hat{m} \equiv \frac{m}{Q}$
- Ultrasoft: $p_s^2 \sim Q^2\lambda^4$

Light-cone basis:

Two light-like vectors $n_1^2 = n_2^2 = 0$ and a normalization $n_1 \cdot n_2 = N$

- Convention: $n_1^\mu = n^\mu = (1,0,0,1)$, $n_2^\mu = \bar{n}^\mu = (1,0,0,-1)$, $n \cdot \bar{n} = 2$
- Vector decomposition:
  $p^\mu = \bar{n} \cdot p \, \frac{n^\mu}{2} + n \cdot p \, \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv \left(p^+, p^-, p_\perp^\mu\right)$
- Dijet momenta scaling: $p_n \sim Q\left(\lambda^2, 1, \lambda\right)$, $p_{\bar{n}} \sim Q\left(1, \lambda^2, \lambda\right)$, $p_s \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$

Event shapes

Event shapes:

Observables which contain information about the geometric distribution of the final-state particles momenta.

Dijet event shapes:

Reach their minimal value (e_{min}) for two back-to-back narrow jets:

* Thrust ($\tau \equiv 1 - T$)
* C-parameter
* Hemisphere Jet mass
* ...

Dijet limit:

$$e_{\text{dijet}} = e_n + e_{\bar{n}} + e_s$$

$$e_{n,\bar{n},s} = \sum_{i \in n,\bar{n},s} f_{n,\bar{n},s}(p_i)$$

EFTs

Effective Field Theories

EFTs describe the physics of the light degrees of freedom of a system i.e. the physics at a scale much lower than the typical scale of the full system.

Two different ways of use

- **Top-Down:** The EFT is built from a general theory:

  Both theories must agree at low energies but could be different at high energies

- Bottom-Up: The underlying theory is unknown:
  - All possible interactions must be taken into account
  - Fit parameters/couplings to experimental/numerical data

Top-Down

Starting from the full theory lagrangian, high energy degrees of freedom can be integrated out. Using equations of motion and field redefinitions an expansion of the full theory lagrangian can be performed to obtain the EFT.

Bottom-Up

Write down all posible lagrangian terms taking into account:

- Relevant degrees of freedom (fields)
- Lorentz invariance and known symmetries
- Equations of motion and integration by parts relate different terms
- Order of the expansion in the power counting parameter

EFTs

Matching & Running

1) Pick a physical process that only has light degrees of freedom as external states
2) Compute this process in both theories (FULL & EFT) and set to zero the difference at some scale $\tilde{\mu}_M$
3) From the previous condition extract the connection (matching) between the couplings of both theories at this scale
4) Using RGEs evolve (running) the EFT Wilson coefficients (couplings) down to a low scale $\tilde{\mu}_L$
5) Compute the RG improved observable using the EFT

SCET

Content and fields-notation

Soft Collinear Effective Theory

EFT built from QCD (Top-Down) which describes collinear and soft degrees of freedom

SCET fields

- $\xi_{n,\bar{n}} \rightarrow n, \bar{n}$-collinear quark
- $A_{n,\bar{n}} \rightarrow n, \bar{n}$-collinear gluon
- $\psi_{us} \rightarrow$ soft quark
- $A_{us} \rightarrow$ soft gluon

Wilson lines

- Collinear $W_{n,\bar{n}}[A_{n,\bar{n}}]$:Interactions between the $\xi_{n,\bar{n}}$ quark and the $A_{n,\bar{n}}$ gluons radiated by the $\xi_{\bar{n},n}$ quark
- Soft $Y_{n,\bar{n}}[A_{us}]$: Interactions between collinear and soft sectors

SCET

Factorization

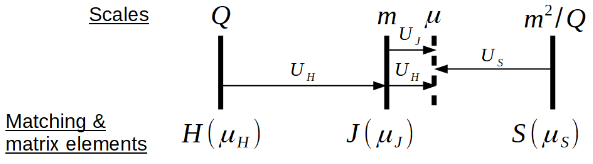
Leading Power SCET Lagrangian $\mathcal{O}(\lambda^0)$

$$\mathcal{L}^{(0)}_{SCET} = \mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{\bar{n}\xi} + \mathcal{L}^{(0)}_{n\,g} + \mathcal{L}^{(0)}_{\bar{n}\,g} + \mathcal{L}^{(0)}_{us}$$

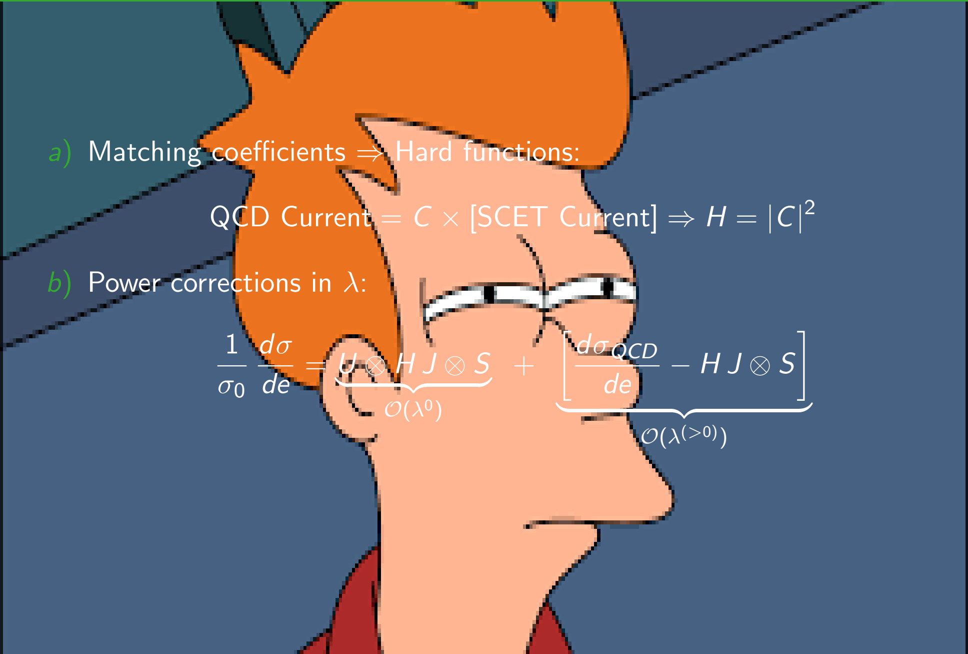
* $\mathcal{L}^{(0)}_{n,\bar{n}\,\xi} \to \xi_{n,\bar{n}}$ interactions with $A_{n,\bar{n}}$ and A_{us}
* $\mathcal{L}^{(0)}_{n,\bar{n}\,g} \to A_{n,\bar{n}}$ interactions with $A_{n,\bar{n}}$ and A_{us}
* $\mathcal{L}^{(0)}_{us} \to$ Interactions between ψ_{us} and A_{us} (QCD type)

Factorization

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H(Q,\tilde{\mu})\,J(m,\tilde{\mu}) \otimes S(\tilde{\mu})$$



Where exactly are the computations in QCD??



a) Matching coefficients ⇒ Hard functions:

$$\text{QCD Current} = C \times [\text{SCET Current}] \Rightarrow H = |C|^2$$

b) Power corrections in λ:

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \underbrace{U \otimes H\, J \otimes S}_{\mathcal{O}(\lambda^0)} + \underbrace{\left[\frac{d\sigma_{QCD}}{de} - H\, J \otimes S\right]}_{\mathcal{O}(\lambda^{(>0)})}$$

bHQET

1) Integrate out the heavy quark and antiquark masses in their corresponding rest frames $\Rightarrow$ $2\times$ HQET's
2) Boost back to c.o.m. frame
3) Match onto SCET in order to account for global soft radiation.

	REST FRAMES		bHQET	
	DOF	Scalings	DOF	Scalings
HQET$_1$	quark: $p = mv + k$ soft: k	$v = (1,1,0)$ $k \sim \Gamma\,(1,1,1)$	quark: $p = mv_+ + k_+$ n-ucollinear: k_+	$v_+ = \left(\frac{m}{Q}, \frac{Q}{m}, 0\right)$ $k_+ \sim \Gamma\left(\frac{m}{Q}, \frac{Q}{m}, 1\right)$
HQET$_2$	anti-quark: $p = mv + k$ soft: k	$v = (1,1,0)$ $k \sim \Gamma\,(1,1,1)$	anti-quark: $p = mv_- + k_-$ $\bar{n}$-ucollinear: k_-	$v_- = \left(\frac{Q}{m}, \frac{m}{Q}, 0\right)$ $k_- \sim \Gamma\left(\frac{Q}{m}, \frac{m}{Q}, 1\right)$
			soft: q_s	$q_s \sim \frac{\Gamma m}{Q}\,(1,1,1)$

Massive Event-Shape Distributions at N^2LL

2006.06383v2

in colaboration with Vicent Mateu and Moritz Preisser

Massive schemes

Generalization introduced in the definition of a given event shape for massive particles:

$$e = f(p_i)$$

E-scheme: $p_i = (p_i^0, \vec{p}_i) \rightarrow p_{i,E} = p_i^0\,(1, \vec{p}_i/|\vec{p}_i|)$
P-scheme: $p_i = (p_i^0, \vec{p}_i) \rightarrow p_{i,P} = |\vec{p}_i|\,(1, \vec{p}_i/|\vec{p}_i|)$
M-scheme: $p_i = (p_i^0, \vec{p}_i) = p_{i,M}$

- For massless particles: $p_E = p_P = p$
- $p_E^2 = p_P^2 = 0$ for massive and massless particles.
- No Lorentz covariant $\Rightarrow$ defined in the c.o.m. frame.
- Change the cross section sensitivity to the quark mass:

	τ	C	ρ
M-scheme	$1-\beta$	$12\hat{m}^2(1-\hat{m}^2)$	$\hat{m}^2$
P- and E- schemes	0	0	0

Table: Threshold position for various event shapes in the case of primary production of a stable quark-antiquark pair in different massive schemes. $\beta = \sqrt{1-4\hat{m}^2}$ is the velocity of the quarks at threshold in natural units.

Collinear limit

- In dijets:

$$e_{\text{dijet}} = \boxed{e_n} + e_{\bar{n}} + e_s \qquad\qquad e_{n,\bar{n},s} = \sum_{i\in n,\bar{n},s} f_{n,\bar{n},s}(p_i)$$

- For any scheme:
  - ◇ The momentum scaling of the collinear and soft particles remains the same.
  - ◇ Light-cone decomposition applies.

n-collinear limit: $n = (1,0,0,1)$; $\quad \bar{n} = (1,0,0,-1)$; $\quad p_n = (p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$

$$\left.\begin{aligned} p^0 &= (p^+ + p^-)/2 \simeq p^-/2 + \mathcal{O}(\lambda^2) \\ |\vec{p}\,| &= \sqrt{(p^0)^2 - m^2} \simeq p^-/2 + \mathcal{O}(\lambda^2) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{\begin{aligned} p^- &= p_E^- = p_P^- \\ p^\perp &= p_E^\perp = p_P^\perp \end{aligned}}$$

$$\left.\begin{aligned} p^+ &= p^0 - p_z = p^0 - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{p_\perp^2 + m^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_P^+ &= |\vec{p}\,| - p_z = |\vec{p}\,| - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{|\vec{p}_\perp|^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_E^+ &= p^0 - \frac{p^0}{|\vec{p}\,|}\, p_z = \frac{p^0}{|\vec{p}\,|}\, p_P^+ \simeq p_P^+ + \mathcal{O}(\lambda^4) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{p_P^+ = p_E^+ = p^+ - \frac{m^2}{p^-}}$$

At Leading Order in λ: $p_{n,P} = p_{n,E} \neq p_n \implies \boxed{e_n^P = e_n^E \neq e_n^M}$

Thrust, Hemisphere Jet Mass, C-parameter

	τ	C	ρ
E	$\frac{1}{Q}\min_{\hat{t}}\sum_i \frac{p_i^0}{\lvert\vec{p}_i\rvert}(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1-\frac{1}{Q^2}\sum_{i,j}\frac{p_i^0 p_j^0(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert^2\lvert\vec{p}_j\rvert^2}\right]$	$\frac{1}{Q^2}\sum_{i,j\in \mathrm{h}}\frac{p_i^0 p_j^0(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert - \vec{p}_i\cdot\vec{p}_j)}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}$
P	$\frac{1}{Q_P}\min_{\hat{t}}\sum_i(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1-\frac{1}{Q_P^2}\sum_{i,j}\frac{(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}\right]$	$\frac{1}{Q_P^2}\sum_{i,j\in\mathrm{h}}(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert - \vec{p}_i\cdot\vec{p}_j)$
M	$\frac{1}{Q}\min_{\hat{t}}\sum_i(p_i^0 - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[2-\frac{1}{Q^2}\sum_{i\neq j}\frac{(p_i\cdot p_j)^2}{p_i^0 p_j^0}\right]$	$\frac{1}{Q^2}\left(\sum_{i\in h} p_i\right)^2$

Table: Thrust, C-parameter and hemisphere jet mass in the three massive schemes. In green, the original definitions. $Q_P \equiv \sum_i \lvert\vec{p}_i\rvert$. h is one of the hemispheres delimited by the plane normal to the thrust axis $\hat{t}$

	τ_n	C_n	ρ_n
E, P	$\tau_n^{E,P} = \frac{1}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$C_n^{E,P} = \frac{6}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$\rho_n^{E,P} = \frac{1}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$
M	$\tau_n^J = \frac{1}{Q}\sum_{i\in +} p_i^+$	$C_n^J = \frac{6}{Q}\sum_{i\in +} p_i^+$	$\rho_n = \frac{1}{Q}\sum_{i\in +} p_i^+$

Table: Thrust, C-parameter and hemisphere jet mass collinear limits.

e^+e^- production
SCET

Resummed distribution (position space):

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\rm SCET}}{\mathrm{d}\tau} = \frac{H(Q,\mu_H)\,G(Q,\mu_i)}{[Q(\tau-\tau_{\rm min})]^{1+\tilde{\omega}}}\int\frac{\mathrm{d}x}{2\pi}\,e^{ix}(i\,x)^{\tilde{\omega}}\,\tilde{J}_\tau\Big(\frac{x}{Q^2(\tau-\tau_{\rm min})},\mu_J\Big)\,\tilde{S}_\tau\Big(\frac{x}{Q(\tau-\tau_{\rm min})},\mu_S\Big)$$

Jet function:

$$J_n(s,\mu) = \int\frac{\mathrm{d}\ell^+}{2\pi}\,\left[\frac{1}{4N_c}\mathrm{Tr}\int\mathrm{d}^d x\,e^{i\ell x}\,\langle 0|\,\bar{\not{n}}\chi_n(x)\,\delta(s-Q^2\hat{e}_n)\,\overline{\chi}_{n,Q}(0)|0\rangle\right]$$

- $\chi_{n,Q}$ BARE Jet field $W_n^+\xi_n$ with $p^- = Q$
- $\ell^- = Q$, $\vec{\ell}_\perp = 0$

Factorization theorem:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{bHQET}}}{\mathrm{d}\tau} = Q^2 H(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int \mathrm{d}\ell\, B_\tau\left(\frac{Q^2(\tau-\tau_{\mathrm{min}}) - Q\ell}{m}, \mu\right) S_\tau(\ell,\mu)$$

Jet function:

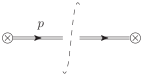
$$B_n(\hat{s}) = \frac{(2\pi)^{d-1} Q}{2m^2 N_C} \mathrm{Tr}\langle 0| W^\dagger_{v_+}(0) h_{v_+}(0) \delta\left[\hat{s} - \frac{Q^2}{m}(\hat{e}_n - e_{\mathrm{min}})\right] \delta^{(d-2)}(\vec{\mathcal{K}}^\perp)\delta(\mathcal{K}^-)\bar{h}_{v_+}(0) W_{v_+}(0)|0\rangle$$

* $\mathcal{K}$ residual momentum operator
* W_{v_+} Wilson lines with u-collinear gluons
* h_{v_+} Heavy quark field

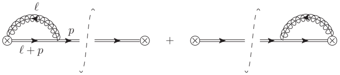
Jet function. Feynman diagrams

Wave-function renormalization:

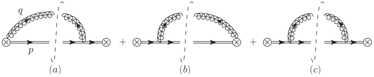
Tree-level:



One loop:



Virtual



Real

SCET Numerical analysis

Differential cross sections

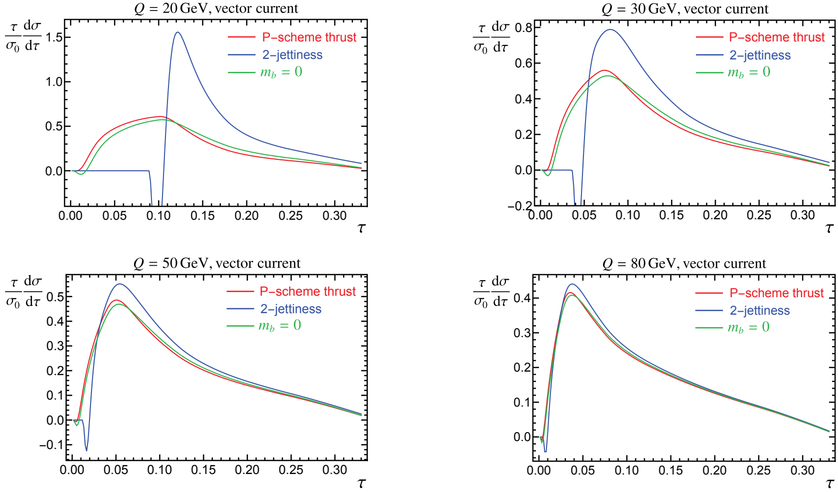
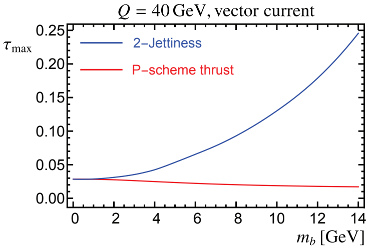


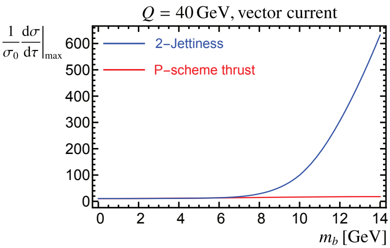
Figure: Differential cross section for massless quarks (green lines), 2-jettiness (blue lines) and P-scheme thrust (red lines) produced through the vector current.

SCET Numerical analysis

Mass dependence of the peak



(a) Peak position



(b) Peak height

Figure: Peak position and peak height for 2-jettiness (blue) and P-scheme thrust (red) massive cross section.

Conclusions

→ Variations on the massive scheme of an event shape allow for different mass sensitivity in the corresponding cross section.

→ P and E schemes are equivalent at leading order in collinear limit but they differ from Q scheme in general.

→ We computed the missing pieces for the SCET and bHQET cross sections in P,E-scheme at $\mathrm{N}^2\mathrm{LL} + \mathcal{O}(\alpha_s)$ accuracy, the jet functions.

→ We carried out RG evolution of non-distributional terms and studied its optimization for numerical implementation.

→ Mass, kinematic and hadronization power corrections were added for a final numerical analysis in SCET of 2-Jettiness and P-scheme thrust distributions.

→ Setup easily adaptable to other observables such as sum of hemisphere masses, heavy-jet-mass, C-parameter, etc.


Backup slides

SCET Jet function

Definition

Definition:

$$J_n(s,\mu) = \int \frac{\mathrm{d}\ell^+}{2\pi} \left[\frac{1}{4N_c} \mathrm{Tr} \int \mathrm{d}^d x \, e^{i\ell x} \, \langle 0 | \, \bar{n}\!\!\!/ \chi_n(x) \, \delta(s - Q^2 \hat{e}_n) \, \overline{\chi}_{n,Q}(0) | 0 \rangle \right]$$

* $\chi_{n,Q}$ BARE Jet field $W_n^+ \xi_n$ with $p^- = Q$
* $\ell^- = Q$, $\vec{\ell}_\perp = 0$

$\Rightarrow$ for inclusive measurement computation through imaginary part of forward-scattering matrix element:

$$J_n(s,\mu) = \frac{-1}{4\pi N_c Q} \, \mathrm{Im} \left[i \int \mathrm{d}^d x \, e^{i\ell x} \, \langle 0 | \, \mathrm{T}\{ \overline{\chi}_{n,Q}(0) \, \bar{n}\!\!\!/ \, \chi_n(x) \} | 0 \rangle \right]$$

* $\ell^+ = s/Q$

ONLY FOR M-SCHEME

SCET Jet function

Definition

Computational form

$$J_n(s,\mu) = \frac{(2\pi)^{d-1}}{N_C} \mathrm{Tr}\left[\frac{\bar{\not{n}}}{2}\langle 0|\chi_n(0)\delta^{(d-2)}(\vec{\mathcal{P}}_X^{\perp})\delta(\bar{\mathcal{P}}-Q)\delta(s-Q\hat{e}_n)\bar{\chi}_n(0)|0\rangle\right]$$

- $\vec{\mathcal{P}}_X^{\perp}$ perpendicular momentum operator
- $\bar{\mathcal{P}}$ operator for momentum in the minus direction

Insert the identity after the measurement delta's in the following way:

$$\sum_X |X\rangle\langle X| = \sum_{n=1} \sum_{\text{spin}} \int \prod_{i=1}^{n} \frac{\mathrm{d}p_i^- \, \mathrm{d}^{d-2}\vec{p}_i^{\perp} \, \theta(p_i^-)}{(2\pi)^{d-1}(2p_i^-)} |X_n\rangle\langle X_n|$$

SCET Jet function

Computational issues in P,E-scheme

* Each real-radiation diagram diverges for $s \to m^2$. This divergence cancels out when summing all real-radiation contributions.
* Real-radiation from Wilson Line (diagrams **a** and **b**):

$$J^{\rm real}_{a,P}(s,\mu) = \frac{C_F \alpha_s\, e^{\varepsilon \gamma_E}}{2\pi m^2 \Gamma(1-\varepsilon)} \left(\frac{s}{\mu^2}\right)^{-\varepsilon} \int_0^1 {\rm d}x\, \frac{x^{2-\varepsilon}(1-x)^{-1-\varepsilon}}{1-x\left(1-\frac{s}{m^2}\right)}$$

In the limit $x \to 1$ the denominator in the integral goes as s/m^2 which combined with the prefactor gives $s^{-1-\varepsilon}$, and it leads to distributions when taking the epsilon expansion:

$$s^{-1+\varepsilon} = \frac{1}{\varepsilon}\delta(s) + \sum_{n=0} \frac{\varepsilon^n}{n!}\left[\frac{\log^n(s)}{s}\right]_+$$

⇒ Method 1:

1) $\Sigma(s_c) \equiv \int_0^{s_c} {\rm d}s\, J^{\rm real}_{a,P}(s,\mu)$
2) Use sector decomposition
3) $J^{\rm real}_{a,P}(s,\mu) = {\rm d}\Sigma(s)/{\rm d}s$ (taking into account: $\frac{\rm d}{{\rm d}x}[\theta(x)\log^n(x)] = n\left[\frac{\log^{n-1}(x)}{x}\right]_+$)

⇒ Method 2:

1) Solve the integral $\longrightarrow {}_2F_1\left(1, 3-\varepsilon, 3-2\varepsilon, 1-\frac{s}{m^2}\right)$
2) Apply Euler's identity $\longrightarrow \left(\frac{s}{m^2}\right)^{-1-\varepsilon} {}_2F_1\left(2-2\varepsilon, -\varepsilon, 3-2\varepsilon, 1-\frac{s}{m^2}\right)$
3) Write new hypergeometric function back as an integral:
$\longrightarrow \int_0^1 {\rm d}x (1-x)^{2-\varepsilon} x^{-1-\varepsilon}\left[1-x\left(1-\frac{s}{m^2}\right)\right]^{-2+2\varepsilon}$

SCET Jet function

Results

Final result for the jet function [A. Bris, V. Mateu and M. Preisser, 2020]

$$J_n^{P,E}(s,\mu) = \delta(s) + \frac{\alpha_s C_F}{4\pi}\left\{\left[2\log\left(\frac{m}{\mu}\right) + 8\log^2\left(\frac{m}{\mu}\right) + 4 + \frac{\pi^2}{3}\right]\delta(s) + \frac{8}{\mu^2}\left[\frac{\log(s/\mu^2)}{s/\mu^2}\right]_+ \right.$$
$$\left. - \frac{4}{\mu^2}\left[1 + 2\log\left(\frac{m}{\mu}\right)\right]\left(\frac{\mu^2}{s}\right)_+ + \frac{s-7m^2}{(s-m^2)^2} - \frac{2s(2s-5m^2)}{(s-m^2)^3}\log\left(\frac{s}{m^2}\right)\right\}$$

Our direct computation of 2-jettiness (M-scheme) jet function agrees with Ref.[Fleming et al.]

$$J_n^J(s+m^2,\mu) = \delta(s) + \frac{\alpha_s C_F}{4\pi}\left\{\left[2\log\left(\frac{m}{\mu}\right) + 8\log^2\left(\frac{m}{\mu}\right) + 8 - \frac{\pi^2}{3}\right]\delta(s) + \frac{8}{\mu^2}\left[\frac{\log(s/\mu^2)}{s/\mu^2}\right]_+ \right.$$
$$\left. - \frac{4}{\mu^2}\left[1 + 2\log\left(\frac{m}{\mu}\right)\right]\left(\frac{\mu^2}{s}\right)_+ + \frac{s}{(m^2+s)^2} - \frac{4}{s}\log\left(1+\frac{s}{m^2}\right)\right\}$$

* The two loop result is also known: [A.H. Hoang, C. Lepenik and M. Stahlhofen, 2019]

SCET Jet function

Massless limit

One loop Jet fuction structure:

$$J_n(\bar{s} + s_{\min}, \mu) = \delta(\bar{s}) + \frac{\alpha_s(\mu)}{4\pi} C_F \left[J_{\rm dist}(\bar{s}, \mu) + \frac{1}{m^2} J_{\rm nd}\left(\frac{\bar{s}}{m^2}\right) \right]$$

$$J_{\rm dist}(\bar{s}, \mu) = \frac{1}{\mu^2} J_{m=0}\left(\frac{\bar{s}}{\mu^2}\right) + \frac{1}{m^2} J_m\left(\frac{\bar{s}}{m^2}\right)$$

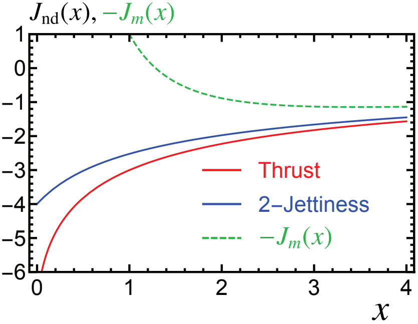


Figure: Massive corrections to the jet function. We show with solid lines the non-distributional functions $J_{\rm nd}$ for P- (red) and M- (blue) schemes. J_m function is shown multiplied -1 as a green dashed line (for $x > 0$ it is common to both schemes).

SCET

P,E-scheme Thrust fixed order cross section

SCET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}^{\mathrm{SCET}}_{\mathrm{FO}}}{\mathrm{d}\tau} = \delta(\tau) + \frac{\alpha_s(\mu)C_F}{4\pi}F_1^{\mathrm{SCET}}(\tau,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$\begin{aligned}F_1^{\mathrm{SCET}}(\tau,\hat{m}) &= \delta(\tau)\left[\frac{10\pi^2}{3} - 8 + 4\log(\hat{m}) + 16\log^2(\hat{m})\right] - 8[1+2\log(\hat{m})]\left(\frac{1}{\tau}\right)_+ \\ &\quad + \frac{2(\tau-7\hat{m}^2)}{(\tau-\hat{m}^2)^2} - \frac{4\tau(2\tau-5\hat{m}^2)}{(\tau-\hat{m}^2)^3}\log\left(\frac{\tau}{\hat{m}^2}\right) \\ &\equiv A^{\mathrm{SCET}}(\hat{m})\delta(\tau) + B^{\mathrm{SCET}}_{\mathrm{plus}}(\hat{m})\left(\frac{1}{\tau}\right)_+ + F^{\mathrm{SCET}}_{\mathrm{NS}}(\tau,\hat{m})\end{aligned}$$

SCET

P,E-scheme Thrust fixed order cross section

QCD: [C. Lepenik and V. Mateu, 2020][See talk by C. Lepenik].

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{FO}}^C}{\mathrm{d}\tau} = R_0^C(\hat{m})\,\delta(\tau) + C_F\,\frac{\alpha_s}{\pi}\,F_C^{\mathrm{QCD}}(\tau,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$F_C^{\mathrm{QCD}}(\tau,\hat{m}) = A^C(\hat{m})\delta(\tau) + B_{\mathrm{plus}}^C(\hat{m})\left(\frac{1}{\tau}\right)_+ + F_{\mathrm{NS}}^C(\tau,\hat{m})$$

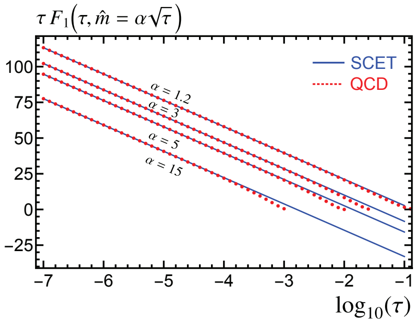


Figure: Comparison of the $\mathcal{O}(\alpha_s)$ correction to the differential cross sections in QCD $[\,F_{\mathrm{V}}^{\mathrm{QCD}}(\tau,\hat{m})\,]$ and SCET $[\,F_1^{\mathrm{SCET}}(\tau,\hat{m})\,]$.

SCET

RG evolution

- The running of the terms involving just distributions can be easily done in position space.
- Non-distributional cross section at N^{2}LL:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\rm nd}}{\mathrm{d}\tau} = \frac{C_F\alpha_s(\mu_J)}{2\pi Q^{1-\tilde{\omega}}}\frac{G(Q,\mu_i)}{\hat{m}^2\Gamma(-\tilde{\omega})}\int_{2s_{\rm min}}^{Q^2\tau}\mathrm{d}s\, J_{\rm nd}\Big(\frac{s-2s_{\rm min}}{m^2}\Big)(Q^2\tau - s)^{-1-\tilde{\omega}}$$

$$= \frac{C_F\alpha_s(\mu_J)}{2\pi}\frac{G(Q,\mu_i)}{Q\hat{m}^2}\left[\frac{1}{Q(\tau-\tau_{\rm min})}\right]^{\tilde{\omega}} I_{\rm nd}\Big(\tilde{\omega},\frac{\tau-\tau_{\rm min}}{\hat{m}^2}\Big)$$

- The running of the non-distributional term reduces to solving:

$$I_{\rm nd}(\tilde{\omega},y) = \frac{1}{\Gamma(-\tilde{\omega})}\int_0^1 \mathrm{d}z\,(1-z)^{-1-\tilde{\omega}}J_{\rm nd}(zy)$$

$$I^{J}_{\rm nd}(\tilde\omega, y) = \frac{1}{\Gamma(-\tilde\omega)}\int_0^1 {\rm d}z\,(1-z)^{-1-\tilde\omega}\left[\frac{zy}{(1+zy)^2} - \frac{4\log(1+zy)}{zy}\right]$$

$$I^{P,E}_{\rm nd}(\tilde\omega, y) = \frac{1}{\Gamma(-\tilde\omega)}\int_0^1 {\rm d}z(1-z)^{-1-\tilde\omega}\left[\frac{zy-7}{(1-yz)^2} + \frac{2zy(2zy-5)}{(1-yz)^3}\log(zy)\right]$$

1) **Analytical results:** Bring the previous expressions into the form of integral representations of hypergeometric functions:

$$\frac{\log(1+zy)}{zy} = \int_0^1 {\rm d}x\,\frac{1}{1+xzy}\ ; \qquad \log(zy) = \left[\log(y) + \frac{\rm d}{{\rm d}\varepsilon}z^{\varepsilon}\right]_{\varepsilon\to 0}$$

⇒ Despite the continuity, the cancellation between different terms divergent at $y \to 1$ ($s \to m^2$) in the P,E-scheme leads to troublesome expressions for numerical implementation.

⇒ Using identities of hypergeometric functions different expressions can be found, each of them beeing more efficient for a given range of values. But they all have cancellations between divergent terms around $y \to 1$.

⇒ Evaluation of hypergemetric functions is needed in both schemes which may be numerically expensive.

2) **Expansion series:** Expand the integrand or use the Mellin-Barnes representation and the converse mapping theorem:

$$\frac{1}{(1+X)^{\nu}} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}t\,(X)^{-t}\,\frac{\Gamma(t)\Gamma(\nu-t)}{\Gamma(\nu)}$$

The expansion around $X \gg 1$ ($X \ll 1$) is obtained integrating by residues the poles of the Mellin transform in the right (left) side of the fundamental strip $(0,\nu)$.

Different quickly convergent series for the regions:

- ◇ Jettiness: $y \to 0$ and $y \to \infty$
- ◇ Thrust P,E-scheme: $y \to 0$, $y \to 1$ and $y \to \infty$

which overlap with each other.

⇒ dramatic performance improvement in numerical implementation.

bHQET Jet function

Computational form:

$$B_n(\hat{s}) = \frac{(2\pi)^{d-1}Q}{2m^2 N_C} \mathrm{Tr}\langle 0| W_{v_+}^\dagger(0) h_{v_+}(0) \delta\Big[\hat{s} - \frac{Q^2}{m}(\hat{e}_n - e_{\min})\Big] \delta^{(d-2)}(\vec{\mathcal{K}}^\perp)\delta(\mathcal{K}^-)\bar{h}_{v_+}(0) W_{v_+}(0)|0\rangle$$

* $\mathcal{K}$ residual momentum operator
* W_{v_+} Wilson lines with u-collinear gluons
* h_{v_+} Heavy quark field

Heavy quark Leading Order: $p = mv + k = (m^2/Q, Q, 0) + k$

→ On-shell condition:

$$v \cdot k = 0$$

→ Measurements:

$$Q(\tau_n^J - \hat{m}^2) = p^+ - \frac{m^2}{Q} = k^+\,, \qquad Q\tau_n^{P,E} = p^+ - \frac{m^2}{p^-} = k^+ + \hat{m}^2 k^- = 0$$

→ Phase space:

$$\frac{\mathrm{d}p^+ \mathrm{d}p^- \mathrm{d}^{d-2}\vec{p}_\perp}{2(2\pi)^{d-1}}\,\delta\big[p^- p^+ - |\vec{p}_\perp|^2 - m^2\big]\theta(p^- + p^+) = \frac{\mathrm{d}k^- \mathrm{d}^{d-2}\vec{k}_\perp}{2Q(2\pi)^{d-1}}$$

bHQET Jet function

Diagrams:

- ⇒ Same diagrams as SCET but replacing $p \to k$ for heavy quark momenta.
- ⇒ Virtual diagrams are scaleless ⇒ Vanish in dim.reg.

Thrust P,E-scheme

$$mB_n^{P,E}(\hat{s},\mu) = -\frac{\alpha_s \Gamma(2+\varepsilon) C_F\, e^{\varepsilon\gamma_E}}{\pi\mu\varepsilon}\left(\frac{\hat{s}}{\mu}\right)^{-1-2\varepsilon}$$

$$= \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}+\frac{\pi^2}{6}\right)\delta(\hat{s}) - \frac{4}{\mu}\left(\frac{1}{\varepsilon}+1\right)\left(\frac{\mu}{\hat{s}}\right)_+ + \frac{8}{\mu}\left(\frac{\mu\log(\hat{s}/\mu)}{\hat{s}}\right)_+\right]$$

2-Jettiness: [Fleming et al.]

$$mB_n^{J}(\hat{s},\mu) = -\frac{\alpha_s \Gamma(2-\varepsilon) C_F\, e^{\varepsilon\gamma_E}}{\pi\mu\varepsilon\Gamma(2-2\varepsilon)}\left(\frac{\hat{s}}{\mu}\right)^{-1-2\varepsilon}$$

$$= \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}+4-\frac{\pi^2}{2}\right)\delta(\hat{s}) - \frac{4}{\mu}\left(\frac{1}{\varepsilon}+1\right)\left(\frac{\mu}{\hat{s}}\right)_+ + \frac{8}{\mu}\left(\frac{\mu\log(\hat{s}/\mu)}{\hat{s}}\right)_+\right]$$

- ∗ The two loop result is also known: [A. Jain, I. Scimemi and I.W. Stewart, 2008]

SCET Numerical analysis

Power corrections

- Mass and kinematic:

Mass power corrections are still kinematically singular:

$$\frac{\mathrm{d}\hat{\sigma}^{FO}_{\rm QCD}}{\mathrm{d}\tau} = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\rm QCD}}{\mathrm{d}\tau}(\tau,\hat{m}) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\rm QCD}}{\mathrm{d}\tau}(\tau,\hat{m})$$

$$\frac{\mathrm{d}\hat{\sigma}^{FO}_{\rm SCET}}{\mathrm{d}\tau} = H\,J\otimes S = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\rm SCET}}{\mathrm{d}\tau}(\tau,\hat{m}) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\rm SCET}}{\mathrm{d}\tau}(\tau,\hat{m}) = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\rm QCD}}{\mathrm{d}\tau}(\tau,\hat{m}\to 0) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\rm SCET}}{\mathrm{d}\tau}(\tau,\hat{m})$$

$\Rightarrow$ can absorb into SCET: (same approach as followed in Ref.[arXiv:1608.01318v1])

1) Modify hard and jet functions: $H \to \tilde{H} = H + \Delta H(\hat{m}),\;\; J \to \tilde{J} = J + \Delta J(\hat{m})$
2) Impose: $\tilde{H}\,\tilde{J}\otimes S|_{\rm dist} \equiv \frac{\mathrm{d}\hat{\sigma}^{dist}_{\rm QCD}}{\mathrm{d}\tau}(\tau,\hat{m})$.
3) One relation for two unknowns $\Rightarrow$ The uncertainty is encoded in one parameter variation.
4) Mass,kinematic-corrected cross section: $\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \underbrace{U\otimes}_{\rm resum.}\tilde{H}\,\tilde{J}\otimes S + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\rm QCD}}{\mathrm{d}\tau} - \frac{\mathrm{d}\hat{\sigma}^{nd}_{\rm SCET}}{\mathrm{d}\tau}}$

- Hadronization: convolution with model function $F(p)$

$$\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = \int_0^{Q\tau}\mathrm{d}p\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\left(\tau - \frac{p}{Q}\right)F(p)\,, \qquad F(p) = \frac{128\,p^3}{3\lambda^4}\,e^{-\frac{4p}{\lambda}}$$