Lecture 5:
Inclusive dijet and lepton pair productions in pp and AA collisions

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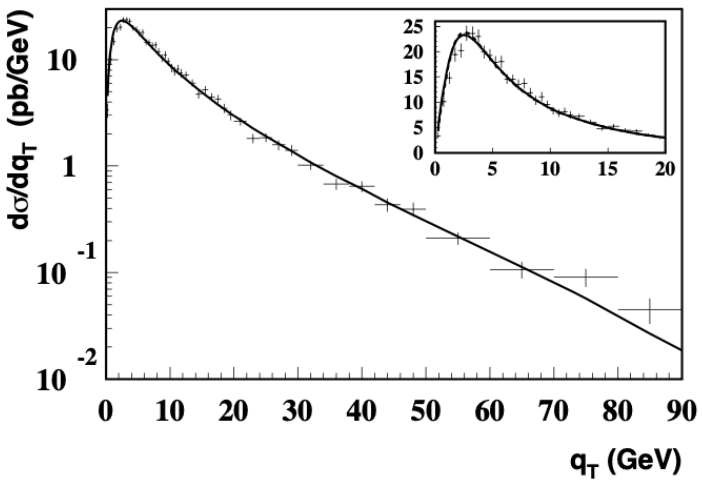
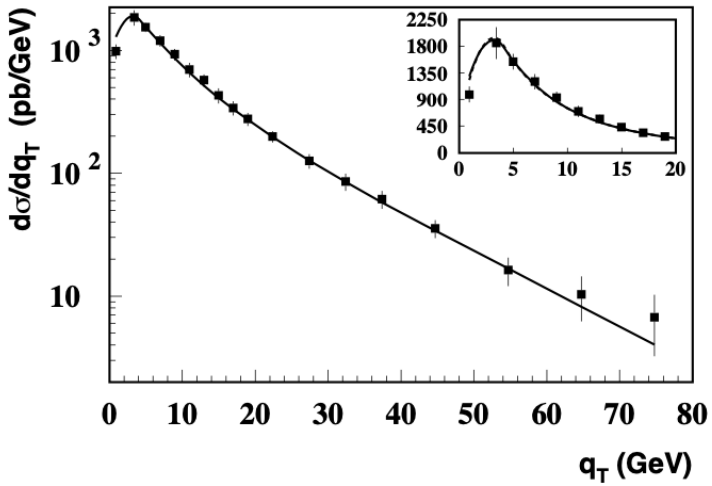
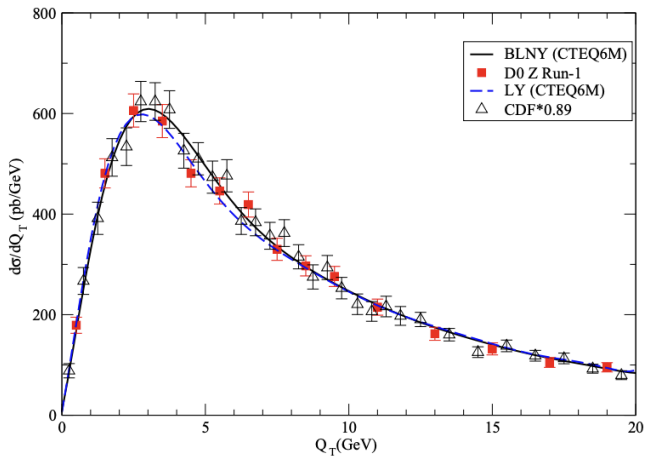
QCD Master Class 2021

QCD Sudakov (CSS) Resummation for Boson Productions





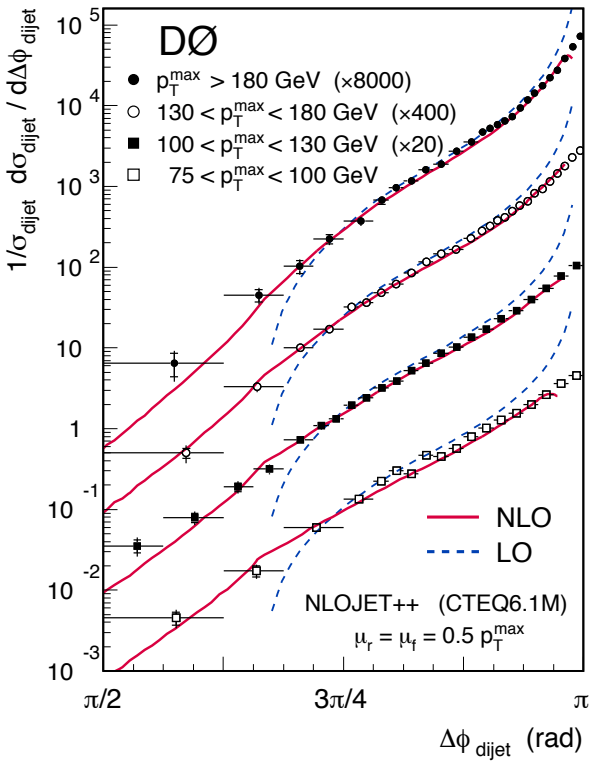
[J.w. Qiu and X. Zhang, 02] DY and W

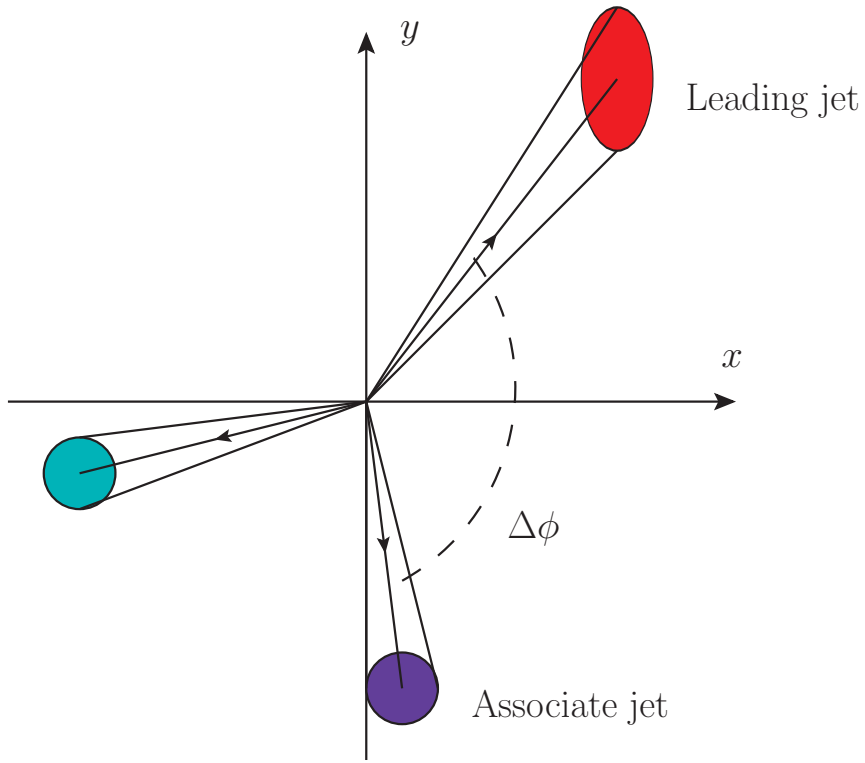


[Landry, Brok, Nadolsky, C. Yuan, 03] Z boson

- CSS Resummation describes DY, W and Z for both small and large Q_T.

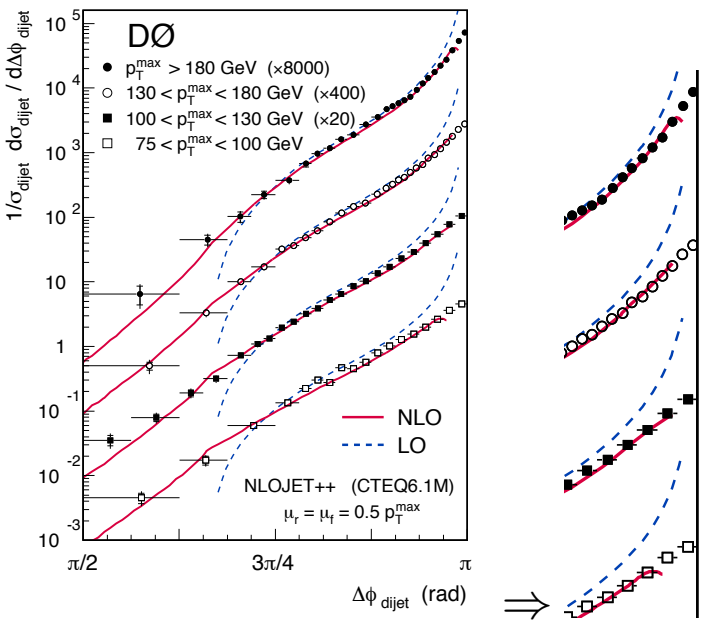
Dijet angular correlation in pp collisions

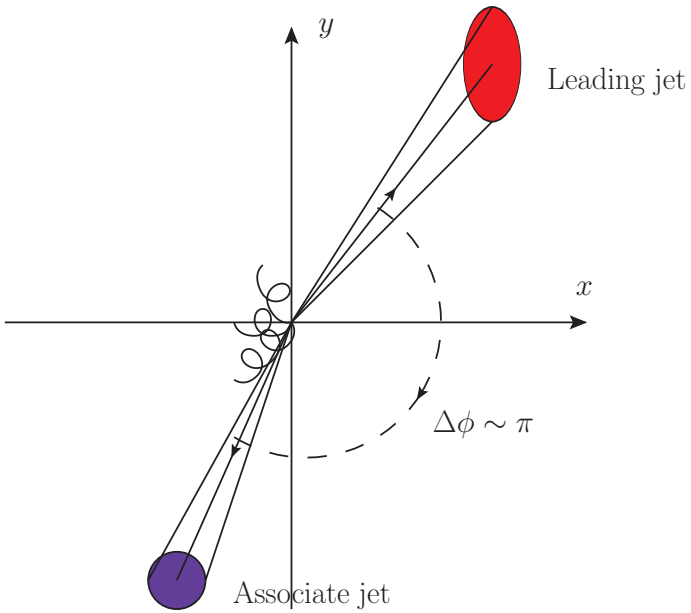




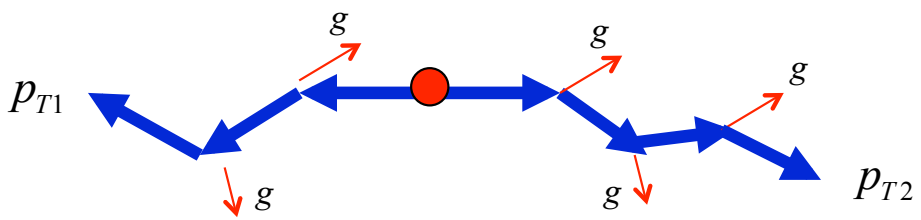
- Dijet azimuthal angular correlation has been measured at Tevatron and the LHC. [DØ collaboration, hep-ex/0409040]
- Well-described by pQCD at NLO [Nagy, 02] except in the back-to-back region.
- Large angle region, deficiency due to kinematical constraints. (LO: $2\pi/3$)

Dijet angular correlation in pp collisions

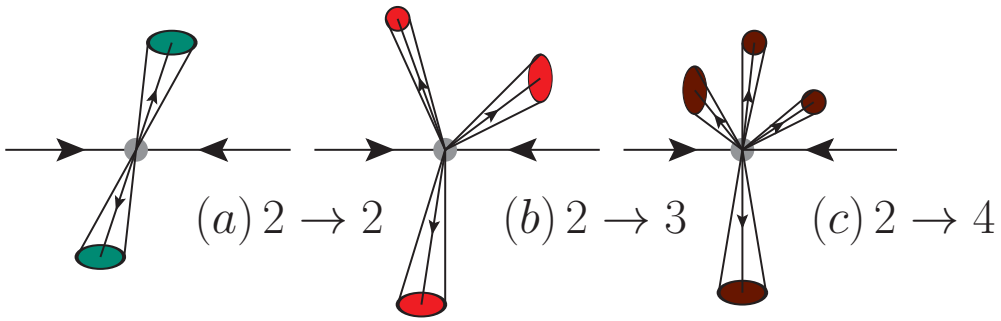




- In the back-to-back region, multiple soft gluon radiation becomes important!
- The goal is to show how to take care of these soft gluon radiations, without relying on MC generators. (Most MC calculation is LO)



Perturbative expansions in dijet productions



Correlations:

$2 \to 2$: 0th order
$2 \to 3$: leading order
$2 \to 4$: next-to-leading order

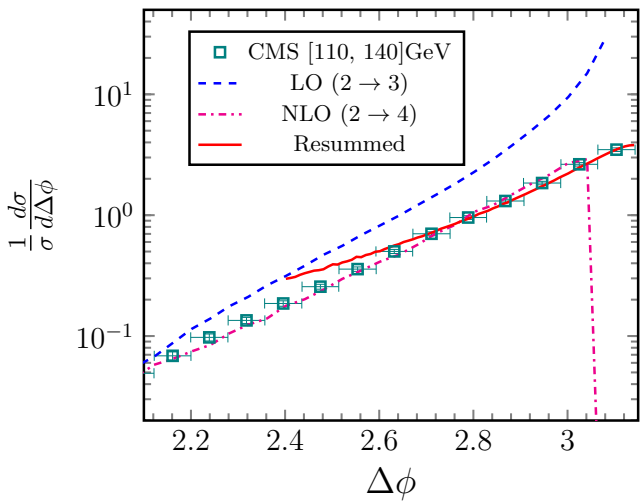
$$\sigma_0 \sum_{i=0}^{\infty} \alpha_s^i \left(L^i + C^{(i)} \right) \quad \text{ideal QCD expansion}$$

$$\sigma_0 \sum_{i=0}^{n-1} \alpha_s^i L^i \;\Big|\; \sigma_0 \sum_{i=0}^{n-1} \alpha_s^i C^{(i)} \quad \Leftarrow \text{pQCD}$$

$$\sigma_0 \sum_{i=n}^{\infty} \alpha_s^i L^i \;\Big|\; \sigma_0 \sum_{i=n}^{\infty} \alpha_s^i C^{(i)}$$

$\Uparrow$ resummation

$\nwarrow$ negligible



- pQCD expansion eventually breaks down in the back-to-back region.
- Appearance of large logarithms such as $L \sim \ln^2 \frac{P_\perp^2}{q_\perp^2}$ with $P_\perp \gg q_\perp$.
- Momentum imbalance $\vec{q}_\perp \equiv \vec{p}_{1\perp} + \vec{p}_{2\perp}$, jet momenta $P_\perp \sim p_{1\perp} \sim p_{2\perp}$.

Sudakov formalism

Dijet productions in the Sudakov formalism (starting from collinear factorization)

$$\begin{aligned}
\frac{d\sigma_{\text{dijet}}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}} &= \sum_{ab} \sigma_0 \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} W(Q, b_\perp), \\
\text{with} \quad W(Q, b_\perp) &= x_1 f_a(x_1, \mu_b) x_2 f_b(x_2, \mu_b) e^{-S(Q, b_\perp)}, \\
S(Q, b_\perp) &= S_{pert}(Q, b_*) + S_{NP}(Q, b_\perp) \\
S_{pert}(Q, b_*) &= \int_{\mu_b^2 = c_0^2/b_*^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B + (D_1 + D_2) \ln \frac{1}{R^2} \right].
\end{aligned}$$

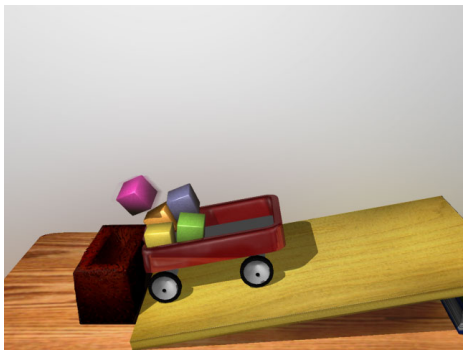
- Soft gluon emissions factorize from the born cross section σ_0.
- Resummation is performed in the $b_\perp$ space. Use
  $\delta^{(2)}(k_\perp - q_\perp) = \int \frac{d^2b_\perp}{(2\pi)^2} e^{i(k_\perp - q_\perp)\cdot b_\perp}$

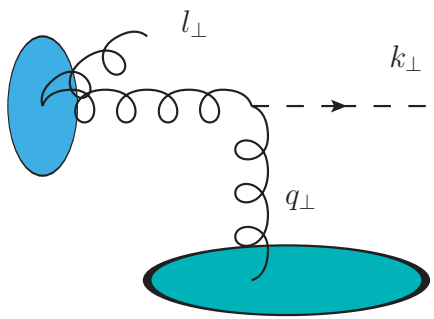
$$\begin{aligned}
\frac{d\sigma}{d^2q_\perp} &= \sigma_0 \sum_n \frac{(-1)^n}{n!} \int d^2k_{1\perp} \cdots d^2k_{n\perp} S(k_{1\perp}) \cdots S(k_{n\perp}) \delta^{(2)}(k_{1\perp} + \cdots k_{n\perp} - q_\perp) \\
&= \sigma_0 \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} e^{-S(b_\perp)}
\end{aligned}$$

- Use $b_* = b/\sqrt{1 + b^2/b_{max}^2}$ prescription to separate perturbative and NP regions.
- All the A, B, C, D coefficients can be computed perturbatively.

Sudakov effects 101

Inertia $\Rightarrow$ Sudakov effects (parton showers in MC generators)





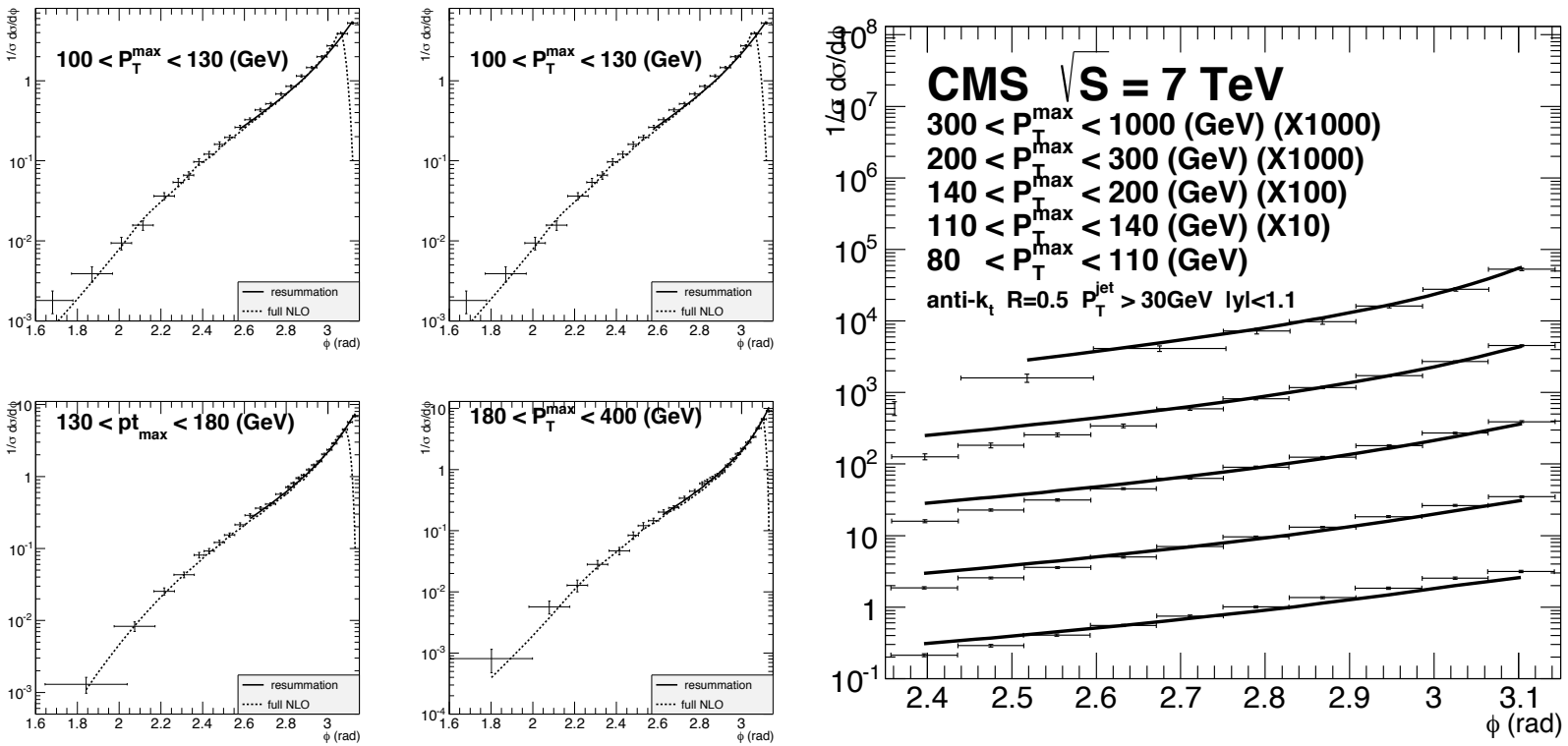
- The Sudakov factor $S \equiv e^{-\mathcal{S}_{\rm sud}} \simeq 1 - \mathcal{S}_{\rm sud}$ is just the probability of those gluons having transverse momentum much less than $k_\perp$.
- Use prob conservation $S + T = 1$ with T the prob of radiation $l_\perp > k_\perp$

$$T = \frac{\alpha_s N_c}{\pi} \int_{k_\perp^2}^{M^2} \frac{dl_\perp^2}{l_\perp^2} \int_{l_\perp^2/M^2}^{1} \frac{d\xi}{\xi} = \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M^2}{k_\perp^2} \simeq \mathcal{S}_{\rm sud}$$

- Dijet production, soft emissions due to hard collisions cause imbalance.

Dijet angular correlation in pp collisions

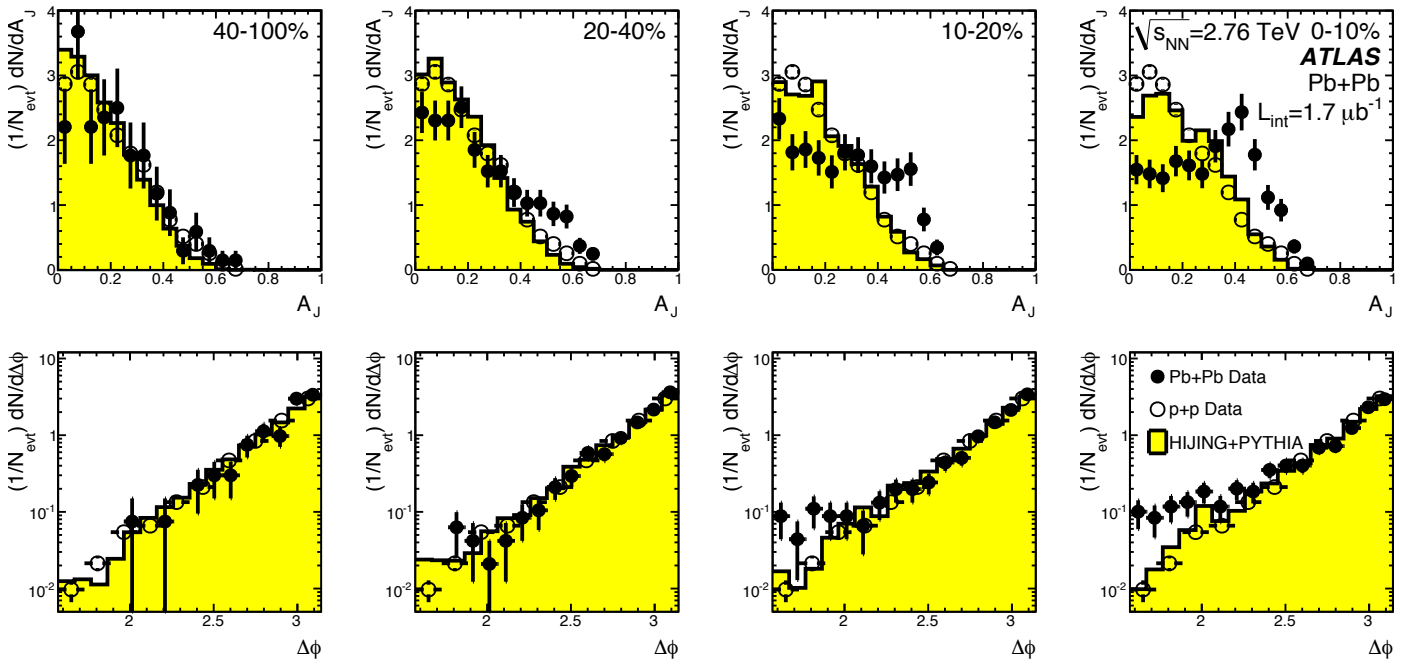
From pQCD to Sudakov resummation: paradigm shift [Sun, Yuan and Yuan, 14, 15]



- pQCD framework can describe the large angle region.
- Sudakov resummation is essential to understand the data near $\phi \sim \pi$.
- QCD theoretical framework to resum soft gluons in dijet processes at NLL accuracy. (*cf.* Pythia is LO.)

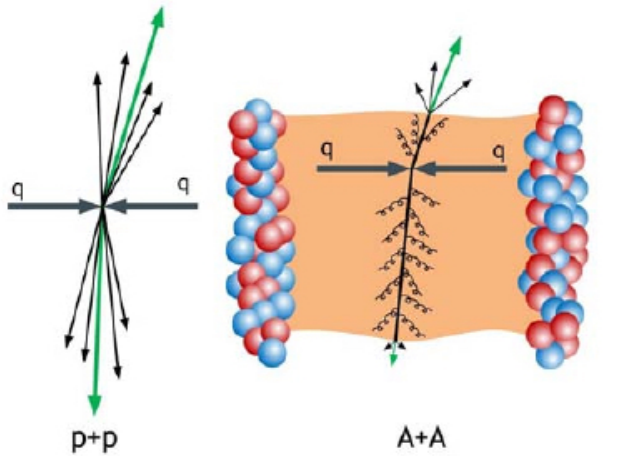
Dijet data in pp and AA collisions at the LHC

ATLAS[2010] and CMS data on dijet asymmetry and angular correlation.



- $A_J \equiv \frac{p_{1\perp}-p_{2\perp}}{p_{1\perp}+p_{2\perp}}$ ($x_J \equiv \frac{p_{2\perp}}{p_{1\perp}}$) reflects the jet energy medium loss in HIC.
  [Qin, Muller, 11; He, Vitev, Zhang, 11;] Photon-jet, Z^0-tagged jet [Vitev, *etc*]
- Dijet angular correlation and transverse energy asymmetry are extremely useful tools to study QGP medium effects in HIC.
- Disappearance of dihadron away side peak at RHIC ⇒ Evidence of QGP.

Dihadron/hadron-jet/dijet angular correlations as a probe to QGP medium

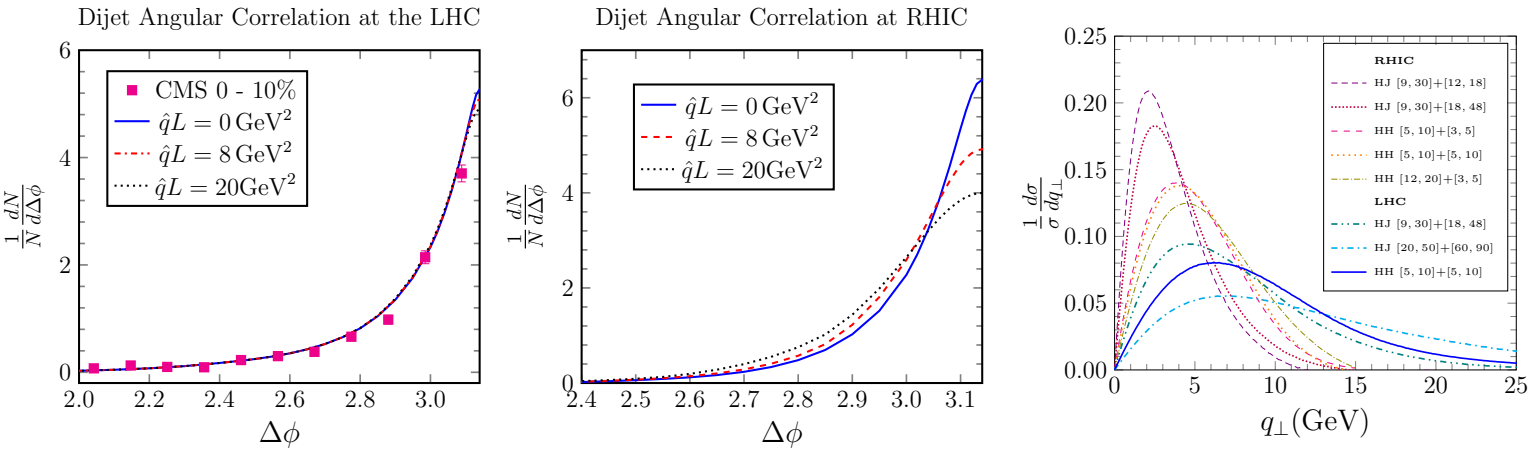


$\hat{q}$ as an important property of QGP medium: [BDMPS formalism]

$$\hat{q} \equiv \frac{\text{Momentum transfer squared}}{\text{Length}} \quad , \quad -\frac{dE}{dx} = \frac{\alpha_s N_c}{4}\hat{q}L.$$

- Transverse momentum broadening $\hat{q}$ (Dijet angular correlation) and energy loss $-\frac{dE}{dx}$ (dijet asymmetry) are two sides of the same coin. Conventional wisdom.
- Reflect the strength of the medium response to traversing jets.
  Pocket formula $q^2_{\perp \mathrm{AA}} \sim q^2_{\perp \mathrm{pp}} + \langle \hat{q}L \rangle$.

Dijet angular correlation



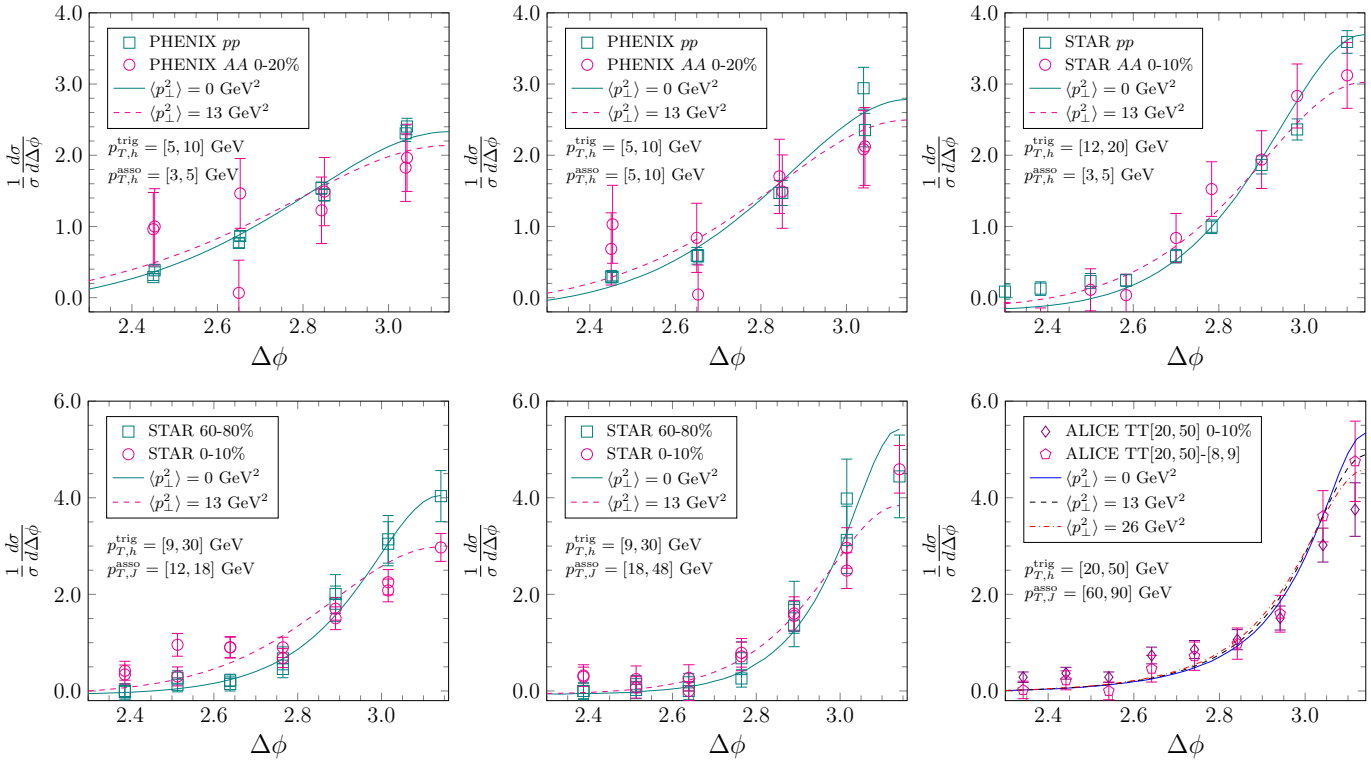
Probing quark gluon plasma medium effects: [Mueller, Wu, Xiao and Yuan, 16]

$$S_{AA}(Q,b) = S_{pp}(Q,b) + \frac{b^2}{4}\langle q_\perp^2\rangle, \quad \text{with} \quad \langle q_\perp^2\rangle \equiv \hat{q}L.$$

- Puzzle of the LHC dijet solved: $\langle q_\perp^2\rangle_{pp}^{LHC} \gg \langle q_\perp^2\rangle_{pp}^{RHIC} \sim \hat{q}L$ (Ratio about 10 : 1) Sudakov becomes so strong that medium broadening is negligible.
- To decrease Sudakov effects: 1. reduce jet energy; 2. decrease collision energy.
- It is possible to measure Jet transport coefficient $\hat{q}$ at RHIC. ($P_T \sim 35$ GeV)

Dihadron and Hadron-Jet correlations as a probe of QGP

[Chen, Qin, Wei, Xiao and Zhang, 16]

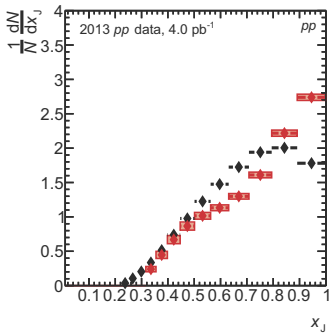


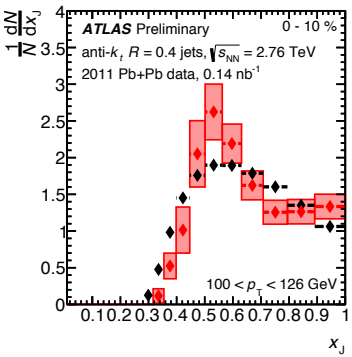
New method to directly probe Jet transport coefficient $\hat{q}$:

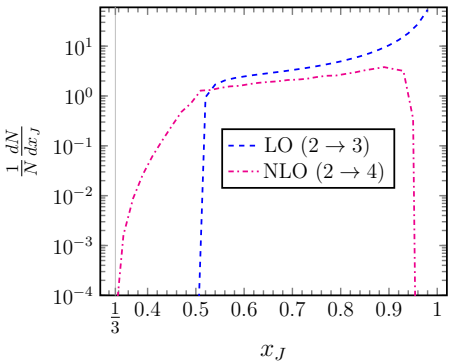
- Global fit $\Rightarrow$ quark jet $\langle p_\perp^2 \rangle_{QGP} = 13^{+5}_{-4}\text{GeV}^2$ at RHIC.
- Use double-log resummed expression for $\hat{q}$. [Liou, Mueller, Wu, 13].
- $\hat{q}_0 = 3.9^{+1.5}_{-1.2}\text{GeV/fm}$ at $T = 0.37\text{GeV}$ with QGP hydro profile.

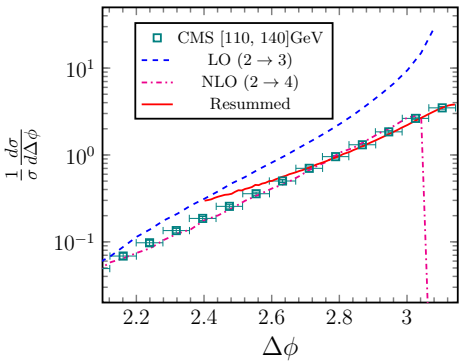
Dijet asymmetry at the LHC

Fully corrected dijet asymmetry $x_J \equiv \frac{p_{2\perp}}{p_{1\perp}} = \frac{1-A_J}{1+A_J}$ data measured by ATLAS.



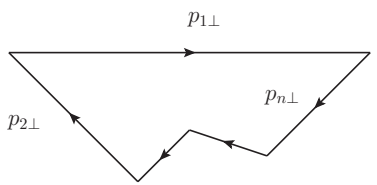






Two interesting features in pQCD calculation for x_J distributions

- pQCD expansion up to $2 \to n$ process is bounded $x_J \geq \frac{1}{n-1}$ (or $A_J \leq \frac{n-2}{n}$).



$\Rightarrow \quad (n-1)p_{2\perp} \geq p_{1\perp}$

- pQCD expansion fails to converge around $x_J \sim 1$ due to back-to-back dijet configurations.

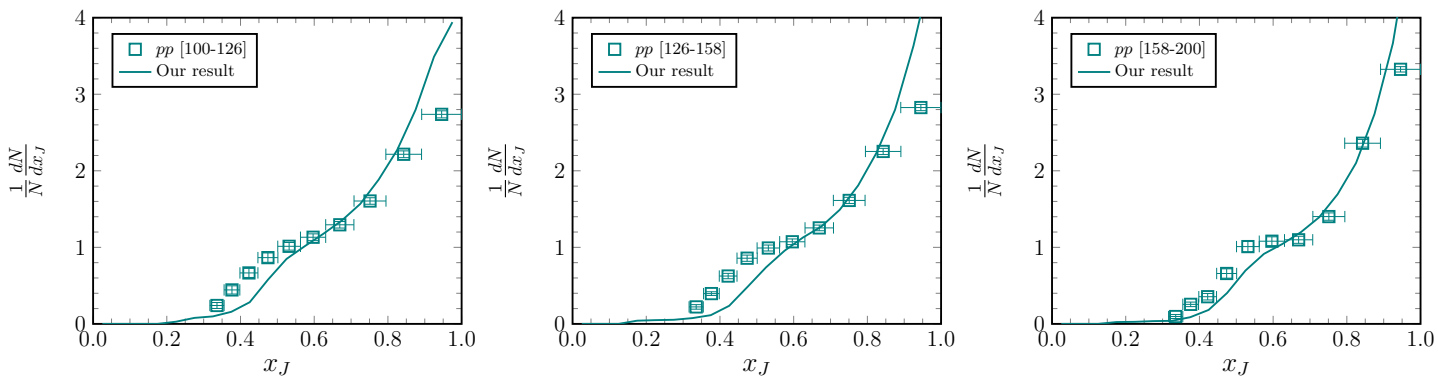
Sudakov resummation improved pQCD approach

Theoretical framework for dijet asymmetries. [Chen, Qin, Wei, Xiao and Zhang, 16]

1. Use resummation for back-to-back configurations;
2. Use pQCD for large angle configurations;
3. Match two formalisms at ϕ_m (where they coincide) in azimuthal angle space.

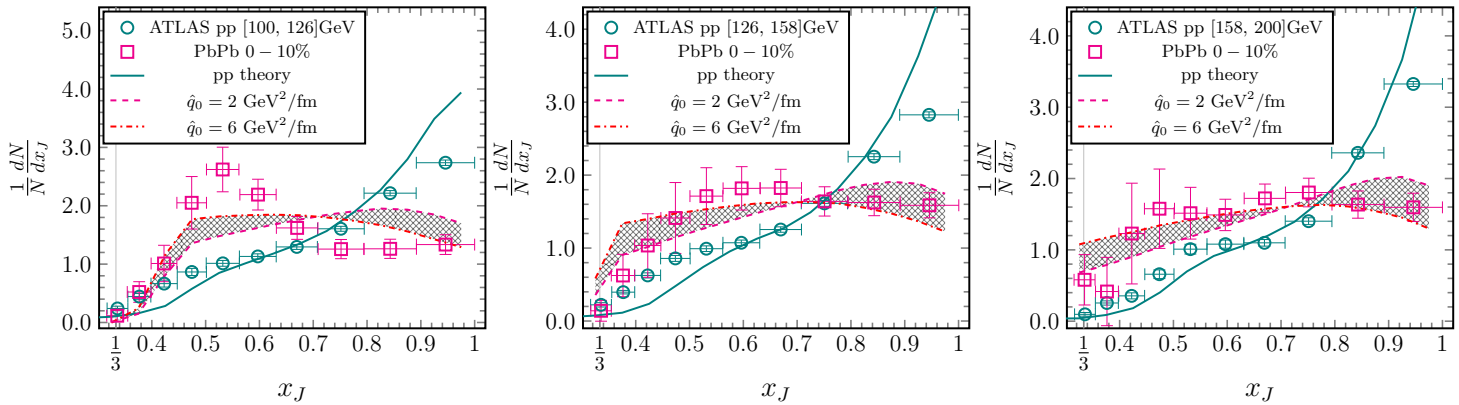
$$\left.\frac{1}{\sigma}\frac{d\sigma}{dx_J}\right|_{\text{improved}} = \left.\frac{1}{\sigma_{\text{NLO}}}\frac{d\sigma_{\text{NLO}}}{dx_J}\right|_{\Delta\phi<\phi_{\text{m}}} + \left.\frac{1}{\sigma_{\text{Sudakov}}}\frac{d\sigma_{\text{Sudakov}}}{dx_J}\right|_{\phi_{\text{m}}<\Delta\phi<\pi},$$

4. More systematic matching[P. Sun, C. P. Yuan and F. Yuan, 14]: Subtracting logarithms up to α_s^2 from NLO calculation. Challenging!



- No free parameter, since x_J results are insensitive to different choice of ϕ_m.

Dijet asymmetries as a probe of QGP



New methods to probe transport coefficient $\hat{q}$: [Chen, Qin, Wei, Xiao, Zhang, 16]

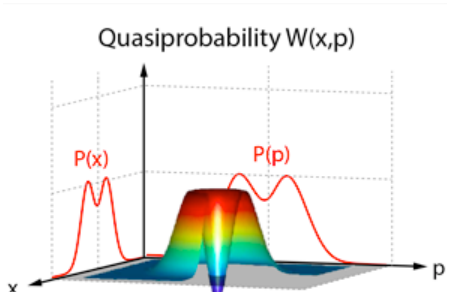
- Use BDMPS energy loss distribution for medium induced soft gluon emissions

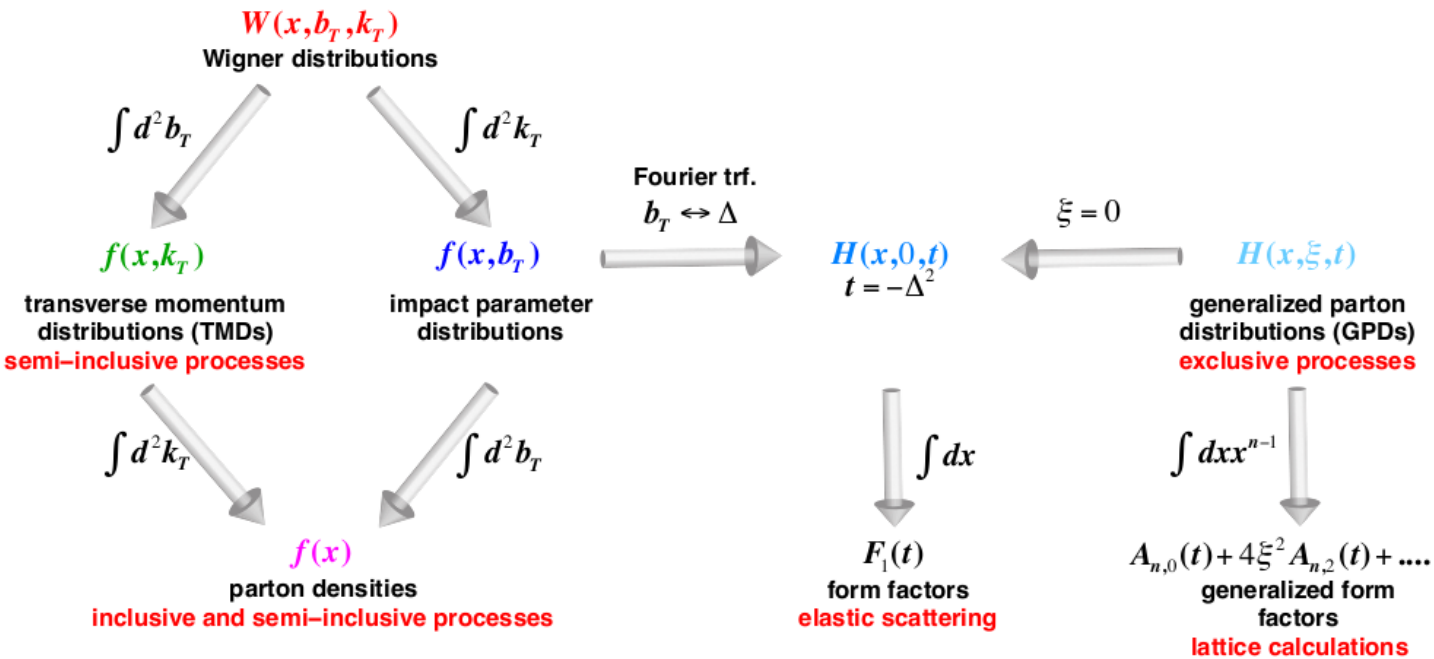
$$D(\epsilon) = \sqrt{\frac{\alpha^2\omega_c}{2\epsilon}}\exp\left(-\frac{\pi\alpha^2\omega_c}{2\epsilon}\right), \text{ with } \quad \omega_c \equiv \int dL\hat{q}L \quad \text{and} \quad \alpha \equiv \frac{2\alpha_s C_R}{\pi}.$$

- Calculation in medium is embedded in OSU $2+1$ d viscous hydro. [Z. Qiu, C. Shen, and U. Hein, 11]
- Dijet asymmetries gives $\hat{q}_0 \sim 2-6\text{GeV}^2/\text{fm}$ at the LHC. ($T = 481\text{MeV}$)
- Assuming T^3 scaling, this roughly agrees with the original BDMPS estimate of $\hat{q}_0 = 0.5\text{GeV}^2/\text{fm}$ at $T = 250\text{MeV}$.
- Global fit from dihadron correlations $\Rightarrow$ quark jet $\hat{q}_0 \sim 4\text{GeV}^2/\text{fm}$ at RHIC.

Embedding small-x gluon in 3D Tomography

Wigner distributions [Belitsky, Ji, Yuan, 04] ingeniously encode all quantum information of how partons are distributed inside hadrons.

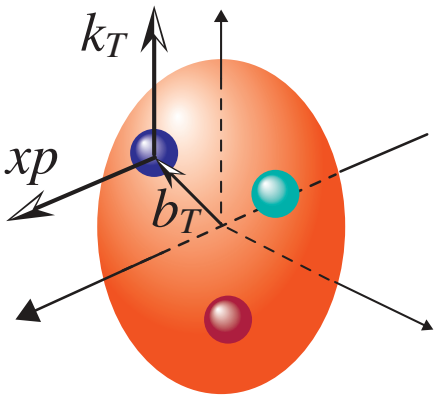


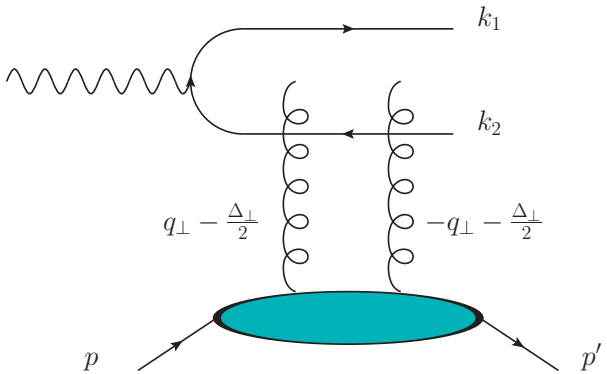


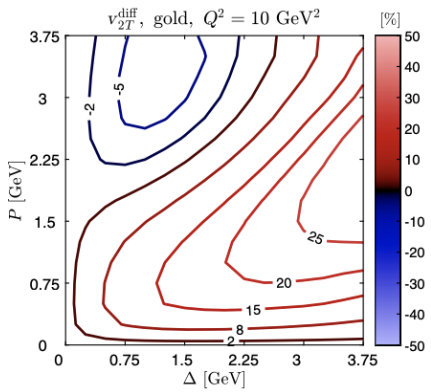
Can we measure Wigner distributions?

- Can we measure Wigner distribution/GTMD? Yes, we can!
- Diffractive back-to-back dijets in *ep*/*eA* collisions. [Hatta, Xiao, Yuan, 16] Link
- Further predictions of asymmetries due to correlations.

$$xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) = x\mathcal{W}_g^T \quad \text{Symmetric part}$$
$$+ \quad 2\cos(2\phi)x\mathcal{W}_g^\epsilon + \cdots \quad \text{Anisotropies}$$



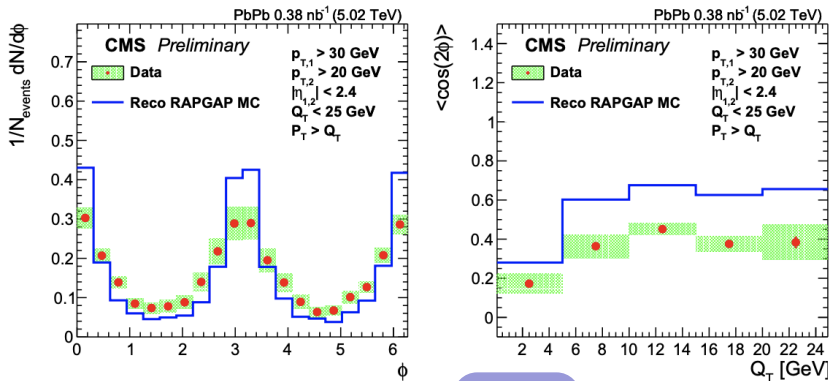


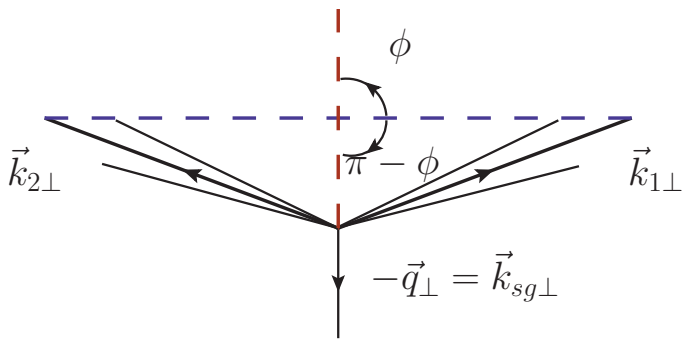


- Study of the elliptic anisotropy.[Mäntysaari, Mueller, Salazar and Schenke, 20] Link

CMS: Dijet photoproduction in UPC (PbPb)

$$\gamma + \mathrm{Pb} \to \mathrm{Jet} + \mathrm{Jet} + \mathrm{Pb}$$

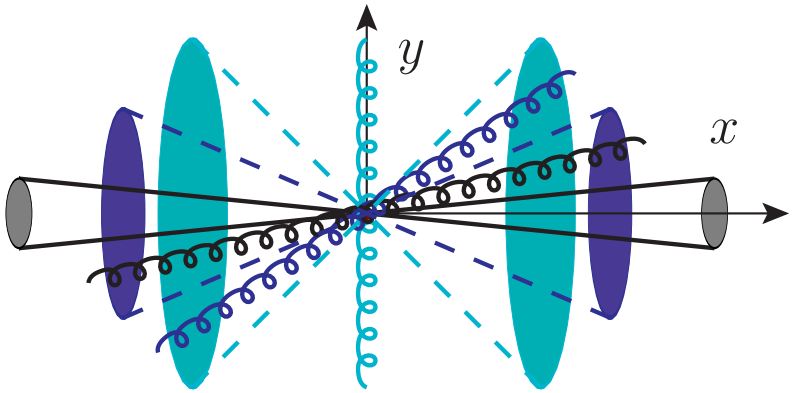


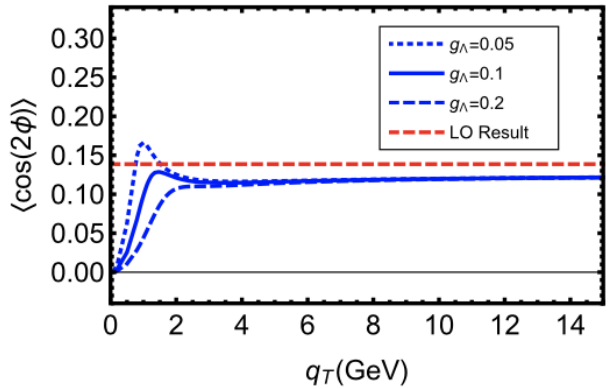


1. Preliminary analysis ▸ Link [CMS-PAS-HIN-18-011]
2. Large asymmetries observed!
3. Indicate additional sources ?

Asymmtries due to final state gluon radiations are important.
[Hatta, Xiao, Yuan, Zhou, 21] ▸ Link

Contributions from final state gluon radiations





Consider soft gluon radiations near jet cone in $\gamma A/p \to q\bar{q} + A/p$

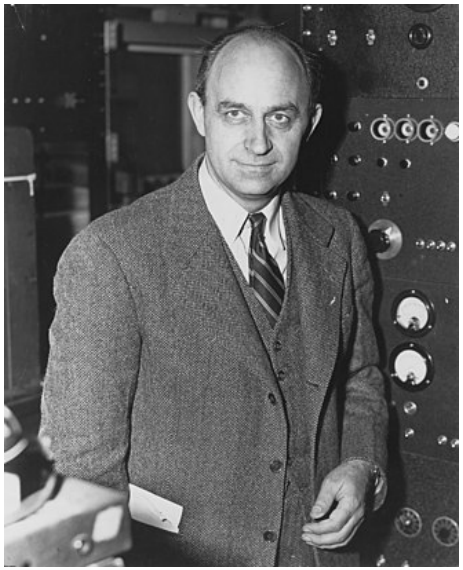
$$g^2 \int \frac{d^3k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F \frac{2k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g}$$

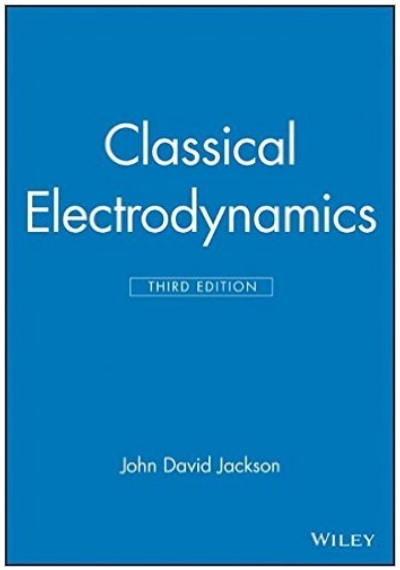
$$= \frac{C_F \alpha_s}{\pi^2 q_\perp^2} \left[c_0^{\text{diff}} + 2\cos(2\phi)\, c_2^{\text{diff}} + \ldots \right].$$

$$c_0^{\text{diff}} = \ln \frac{a_0}{R^2}\,, \qquad c_2^{\text{diff}} = \ln \frac{a_2}{R^2}\,.$$

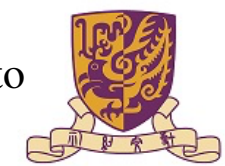
Observed asymmetry should includes initial and final state contributions!

Classical Electrodynamics and Virtual Quanta



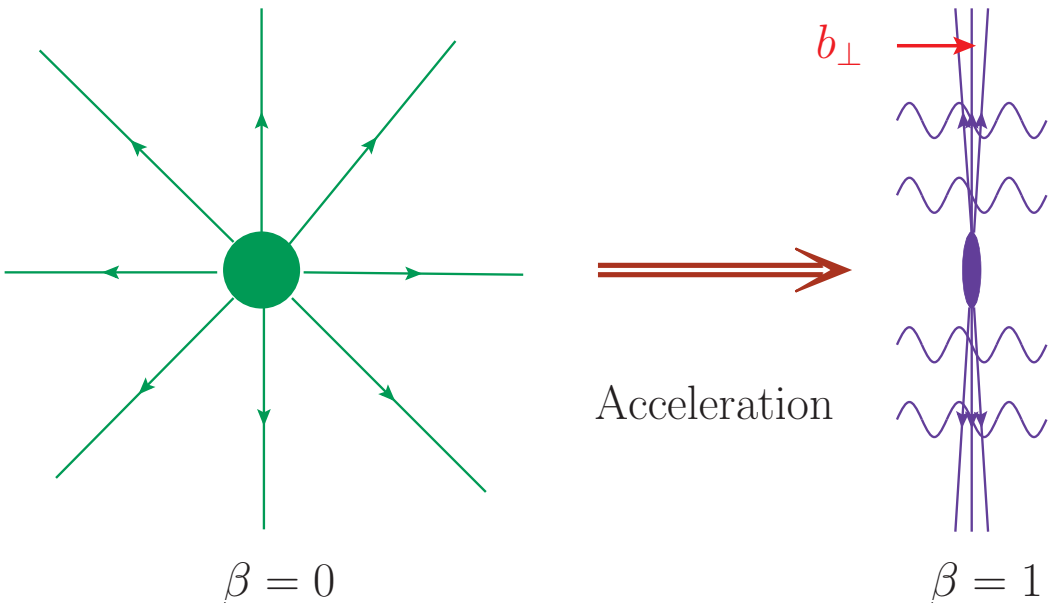


- Following Fermi[24], Weizsäcker [34] and Williams [35] discovered that the EM fields of a relativistically moving charged particle are almost transverse.
- This is equivalent to say that the charged particle carries a cloud of quasi-real photons, which are ready to be radiated if perturbed.
- Weizsäcker-Williams method of virtual quanta (Equivalent Photon Approximation).
- Application in QCD: WW gluon distribution. [McLerran, Venugopalan, 94; Kovchegov, 96; Jalilian-Marian, Kovner, McLerran and Weigert, 97]
- Recently, Weizsäcker-Williams EPA and Wigner distribution are combined together to explain the dilepton productions in HIC.



EPA and Weizsäcker-Williams Photon Distribution

Boost the static potential to the infinite momentum frame ($\gamma \to \infty$):
[Jackiw, Kabat and Ortiz, 92] and HW problem (P11.18) in [Jackson]



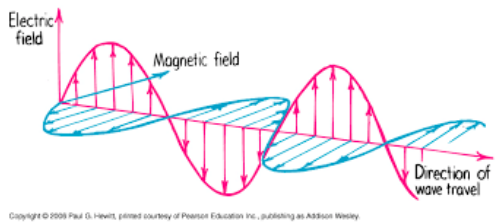
$$A^+_{Cov} = -\frac{q}{\pi}\ln(\lambda b_\perp)\delta(t-z),$$

$$\vec{E} = \frac{q}{2\pi}\frac{\vec{b}_\perp}{b^2_\perp}\delta(t-z),$$

$$\vec{B} = \frac{q}{2\pi}\frac{\hat{v}\times\vec{b}_\perp}{b^2_\perp}\delta(t-z),$$

$$\vec{A}^{LC}_\perp = -\frac{q}{2\pi}[\vec{\nabla}_\perp \ln(\lambda b_\perp)]\theta(t-z).$$

Static electric fields $\Rightarrow$ Electro-Magnetic Wave
$\Rightarrow$ The EM pulses are equivalent to a lot of photons

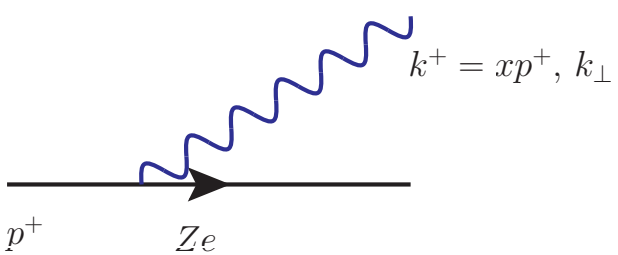


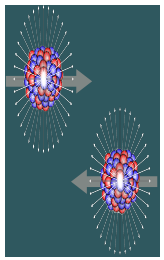
- The gauge potentials A_μ in Covariant gauge and LC gauge are related by a gauge transformation. λ is an irrelevant parameter setting the scale.
- Classical EM: transverse EM fields $\Leftrightarrow$ QM: Co-moving Quasi-real photons.

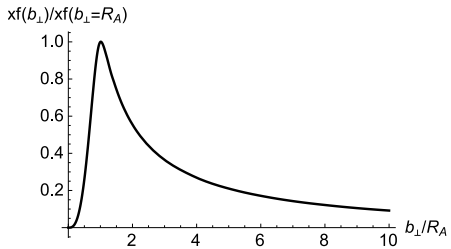
Transverse Momentum Dependent (TMD) Photon Distribution

The photon distribution (flux) for a point particle can be computed from $\vec{A}_\perp^{LC}$

$$xf_\gamma(x,b_\perp) = \frac{Z^2\alpha}{\pi^2}x^2m^2K_1^2(xm\,b_\perp) = \frac{Z^2\alpha}{\pi^2 b_\perp^2}\bigg|_{m\to 0} \quad \text{with} \quad q = Ze.$$





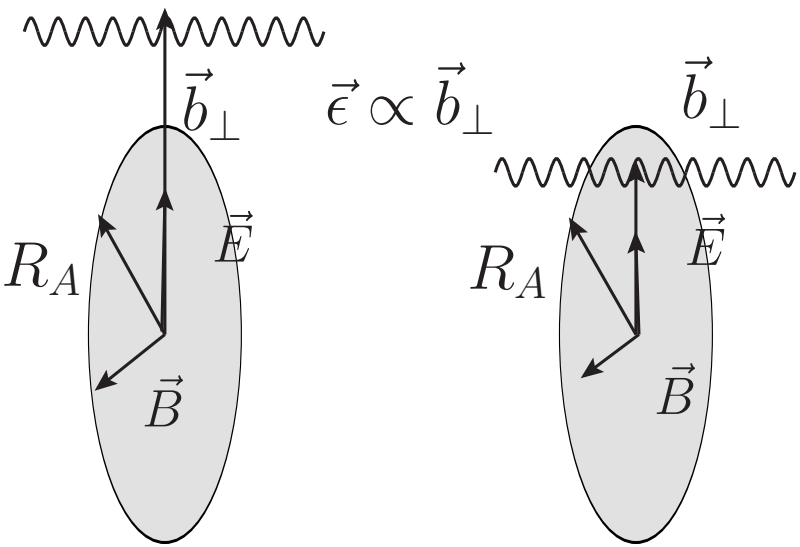


The photon distribution in the transverse momentum space

$$\begin{aligned} xf_\gamma(x,\vec{k}_\perp) &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \left\langle A \left| F^{+i}\left(\frac{\xi}{2}\right) F^{+i}\left(-\frac{\xi}{2}\right)\right| A\right\rangle \\ &\simeq \frac{Z^2\alpha}{\pi^2}\frac{k_\perp^2}{\left(k_\perp^2 + x^2M^2\right)^2}F_A^2(k^2). \end{aligned}$$

- $F_A(k^2)$ is the charge form factor with $k^2 = k_\perp^2 + x^2M^2$. $F_A = 1$ for point charge.
- Wood-Saxon or Gaussian models for realistic nuclei. (*Pb* is very bright!)
- Typical transverse momentum of the photon is $1/R_A$, which is 30MeV for *Pb*.

Linearly Polarized Photon



- E is linearly polarized along the impact parameter $b_\perp$ direction;
- $\vec{B} \perp \vec{E}$;
- The LC gauge potential $A_\perp \propto \vec{b}_\perp$;
- Polarization vector $\vec{\epsilon}_\perp = \vec{b}_\perp / b_\perp$.
- Similar case in momentum space.

- WW photon distribution is maximumly polarized, since $xf_\gamma = xh_\gamma$.

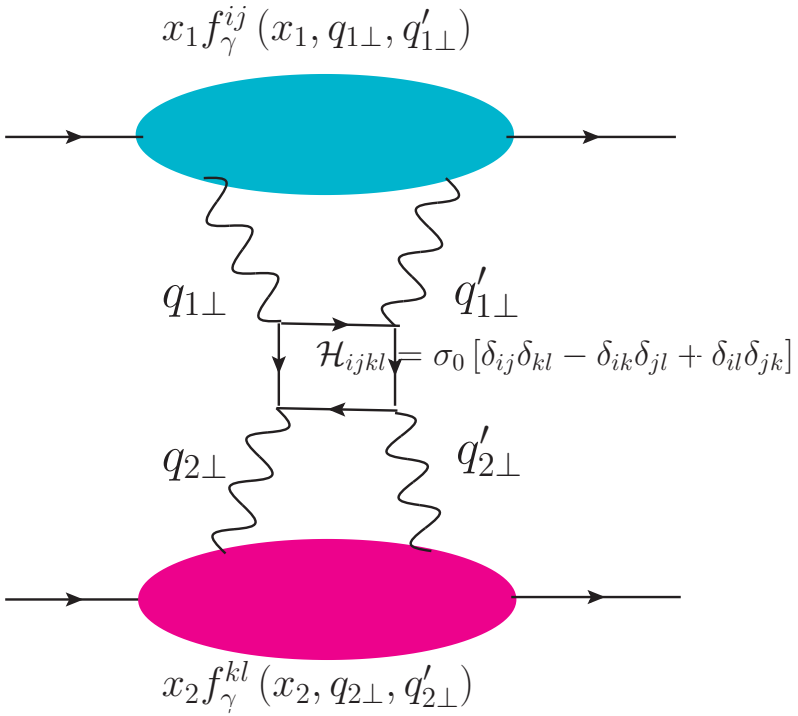
$$xf_\gamma^{ij}(x;b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \langle A, -\frac{\Delta_\perp}{2} | F^{+i} F^{+j} | A, \frac{\Delta_\perp}{2} \rangle ,$$

$$xf_\gamma^{ij}(x;b_\perp) = \frac{\delta^{ij}}{2} xf_\gamma(x;b_\perp) + \left(\frac{b_\perp^i b_\perp^j}{b_\perp^2} - \frac{\delta^{ij}}{2} \right) xh_\gamma(x;b_\perp) = \frac{b_\perp^i b_\perp^j}{b_\perp^2} xf_\gamma ,$$

$$xh_\gamma(x,b_\perp) = xf_\gamma(x,b_\perp) = 4Z^2\alpha \left| \int \frac{d^2k_\perp}{(2\pi)^2} e^{ik_\perp \cdot b_\perp} \frac{\vec{k}_\perp}{k^2} F_A(k^2) \right|^2$$

The application of Wigner distribution in the lepton pair production

Assuming the GEPA factorization at LO, and compute the hard factor $\mathcal{H}$



$$\frac{d\sigma(AB[\gamma\gamma] \to \mu^+\mu^-)}{dy_1 dy_2 d^2p_{1T} d^2p_{2T} d^2b_\perp}$$

$$= \int d^2k_{1T} d^2k_{2T} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp}$$

$$\times x f_\gamma^{ij}(k_{1T}; \Delta_\perp) x_2 f_\gamma^{kl}(k_{2T}; \Delta_\perp) H_{ijkl}$$

$$\times \delta^{(2)}(p_T - k_{1T} - k_{2T})$$

$$\propto \int d^2b_{1\perp} d^2b_{2\perp} \delta^{(2)}(b_\perp - b_{1\perp} + b_{2\perp})$$

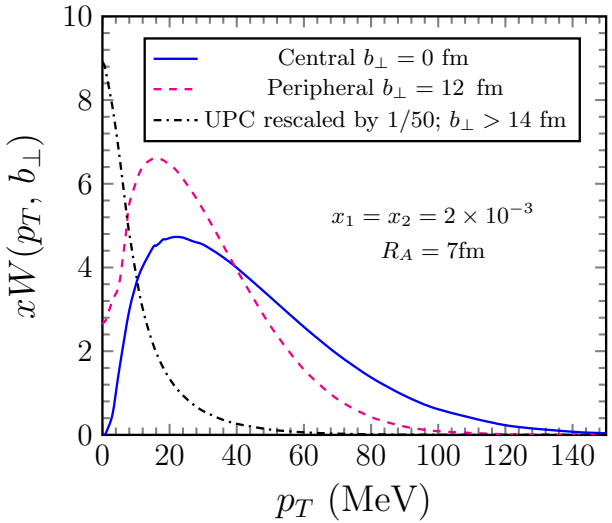
$$\times x f_\gamma^{ij}(k_{1T}; b_{1\perp}) x_2 f_\gamma^{kl}(k_{2T}; b_{2\perp}) H_{ijkl}$$

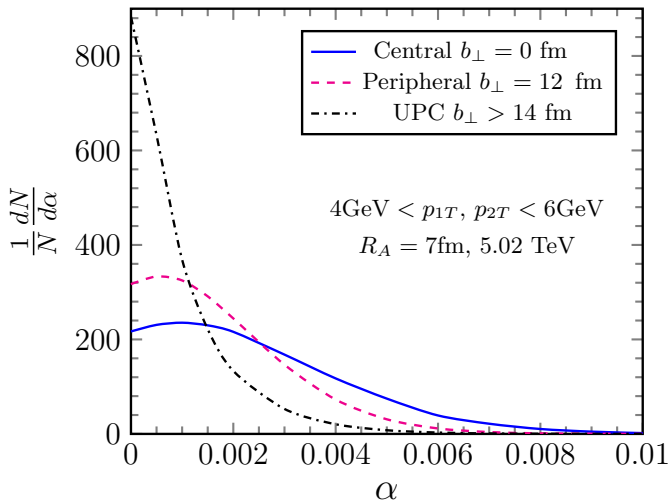
- Notations for the momenta: $q_\perp = k_T - \frac{\Delta_\perp}{2}$, and $q'_\perp = k_T + \frac{\Delta_\perp}{2}$.
- Need the off-diagonal momenta $\Delta_\perp$ to access impact parameter $b_\perp$.
- Will negative region of Wigner Dist $x f_\gamma^{ij}(k_T; b_\perp)$ ever be catastrophic?

Results of GEPA with Wigner Photon

If we define $G^{ik} = \int \frac{d^2k_{1T}}{(2\pi)} e^{ik_{1T}\cdot b_\perp} k_{1T}^i k_{2T}^k \frac{F(k_1^2)}{k_1^2}\frac{F(k_2^2)}{k_2^2}$, and note $k_{1T} + k_{2T} = p_T$

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1T} d^2p_{2T} d^2b_\perp} = \sigma_0 \left[(G^{11} - G^{22})(G^{11*} - G^{22*}) + (G^{12} + G^{21})(G^{12*} + G^{21*})\right] \geq 0$$

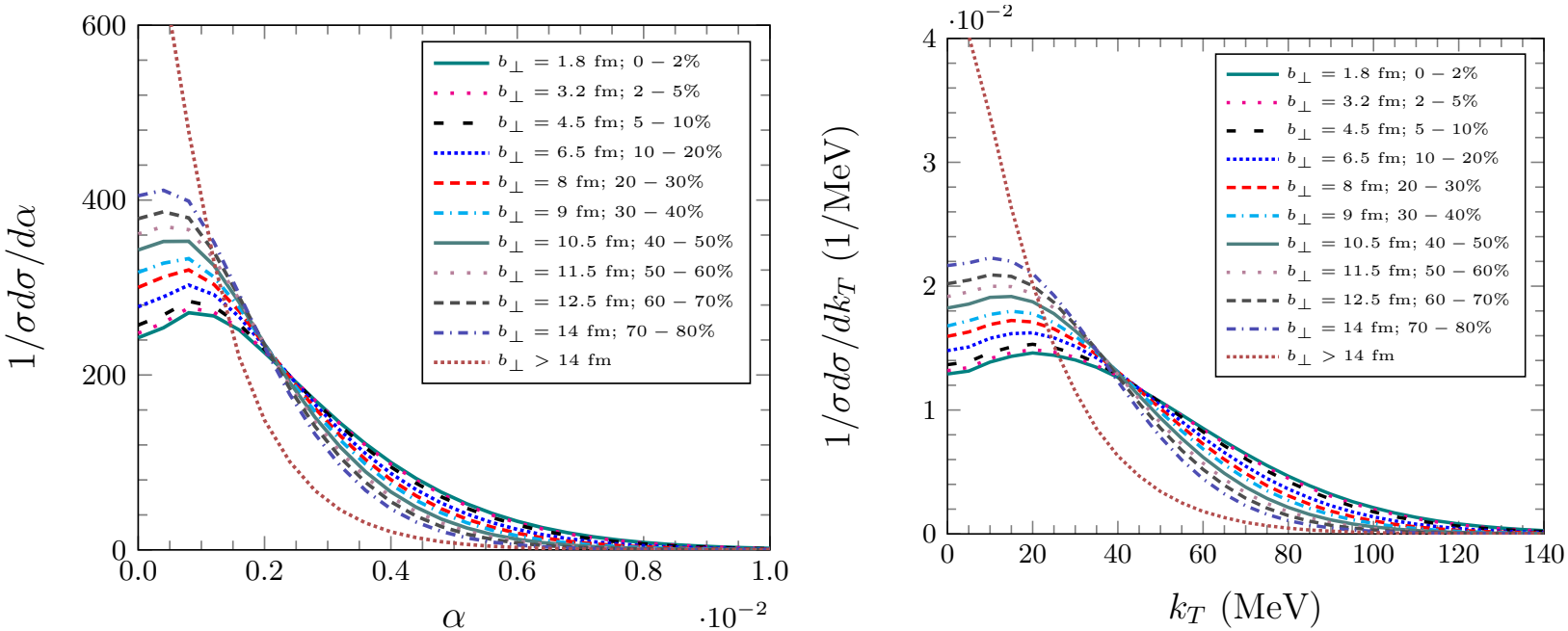




- The cross section $= 0$ when $b_\perp = 0$ and $p_T = 0$ ($G11 = G_{22}$ and $G_{12} = -G_{21}$)
- Explains the dip (displaced peak, ATLAS) in central AA. $\sigma \sim xW(p_T,\ b_\perp)$
- However, the dip becomes much less significant after averaging over momenta.
- Qualitatively explains recent ATLAS CONF-2019-51 data. Still a bit puzzling.

Predictions for upcoming ATLAS measurements

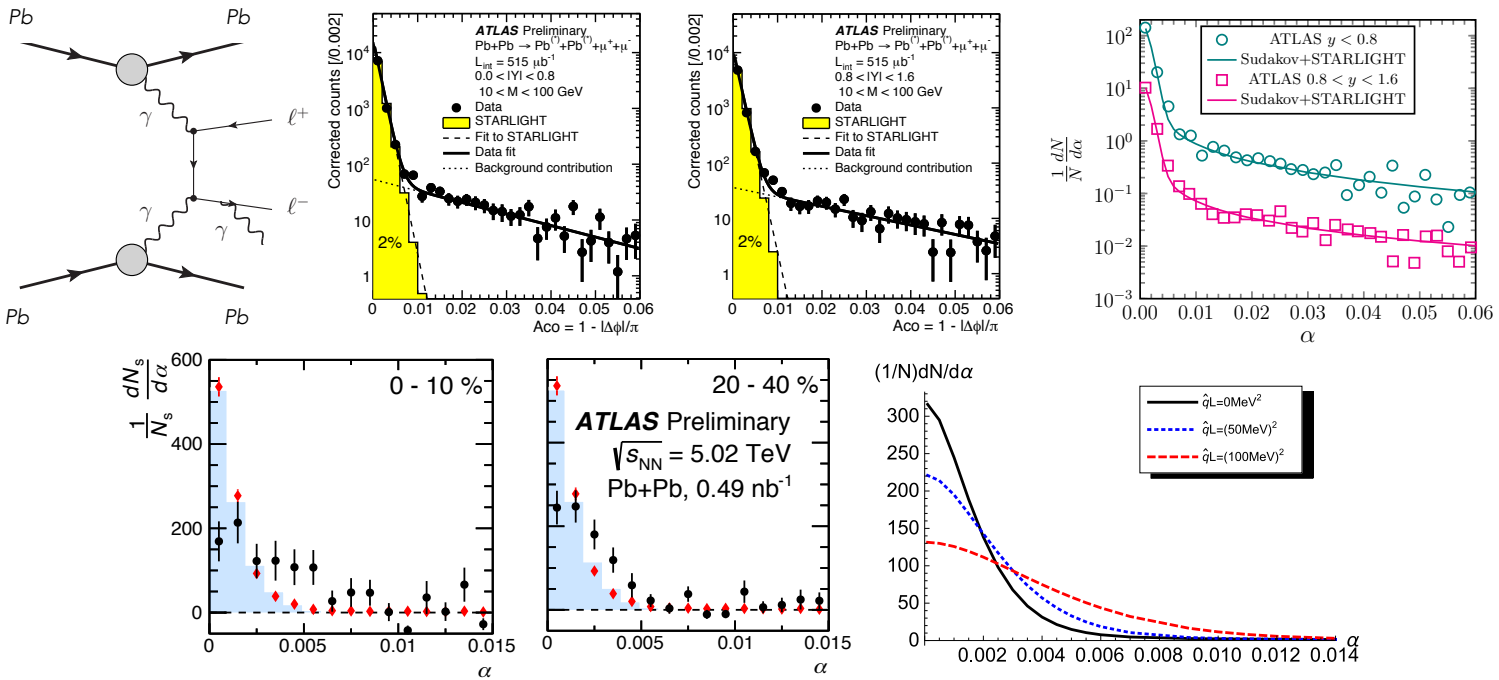
$\sqrt{s} = 5.02\text{TeV}$, $|y| < 2.5$ $p_T > 4\text{GeV}$, $4 < m_{l^+l^-} < 45\text{GeV}$, $|A| \equiv \frac{|p_{T1}-p_{T2}|}{p_{T1}+p_{T2}} < 0.06$ (sent in Nov. 2020)



- Detailed predictions are provided for ALTAS [S.Y. Wei, et al in preparation]

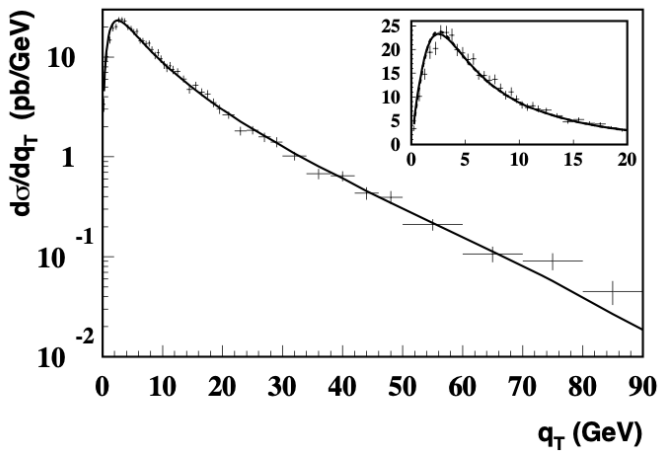
Dilepton productions in AA collisions

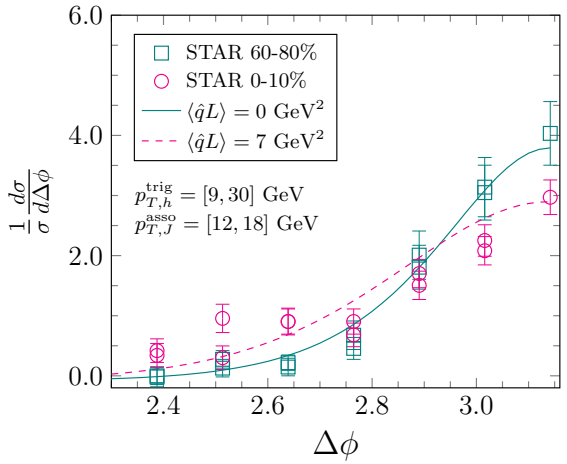
[ATLAS, 1806.08708; 2019 Conf-51]; [Klein, Mueller, Xiao, Yuan, 18; 20]

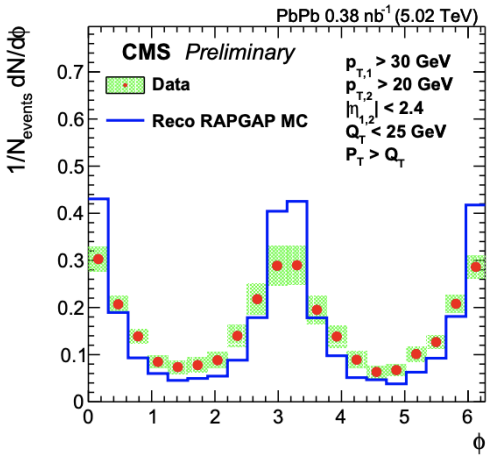


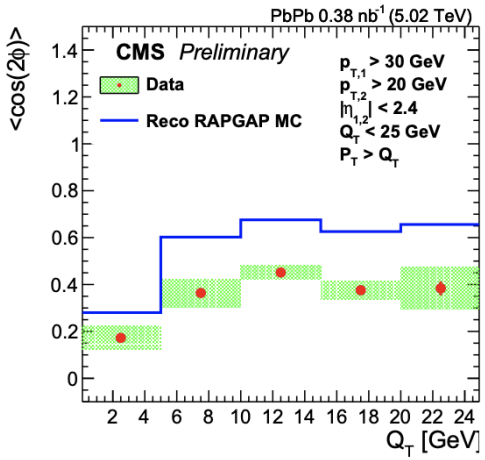
- Acoplanarity $\alpha \equiv 1 - \Delta\phi/\pi$ and asymmetry $A \equiv ||p_T^+| - |p_T^-||/||p_T^+| + |p_T^-||$.
- Initial photon $k_\perp$ distribution plus photon radiation describes the background.
- Sudakov resummation of QED radiative corrections. $\ln 1/R^2 \Rightarrow \ln Q^2/m^2$.

Summary









- Sudakov resummation has tremendous applications in QCD and QED.
- Essential to describe the heavy vector boson Q_T distribution, back-to-back dijets and dilepton angular correlations.
- Useful tool in HIC to extract medium properties.
- Final state gluon radiation generates anisotropy.