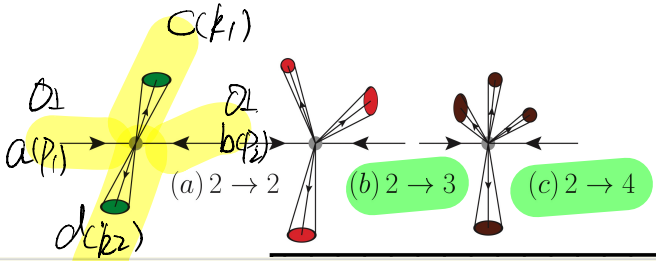
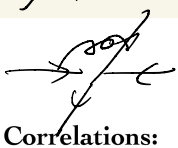
Lecture 5:

Sudakov in dijets in PP, eP

γjet

H+jet, Z/W± +jet





Correlations:

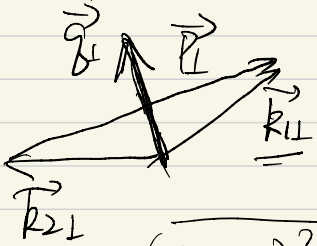
2 → 2: 0th order
2 → 3: leading order
2 → 4: next-to-leading order

collinear factorization

imbalance

$p_1 + p_2 = k_1 + k_2$

$\vec{k}_{1\perp} + \vec{k}_{2\perp} = \vec{q}_\perp$



$P_\perp \equiv \frac{1}{2}(k_{1\perp} - k_{2\perp})$

$|P_\perp| \gg |q_\perp|$

$\ln \frac{P_\perp^2}{q_\perp^2} = L$ in Dijets

$$\frac{d^4\sigma}{dy_1 dy_2 d^2P_\perp^2 d^2q_\perp} = \sum_{ab} \alpha_0^{ab\to cd} x_1 f_a(x_1) x_2 f_b(x_2)$$

$\delta^{(2)}(q_\perp)$

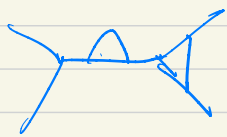
$$\int \frac{d^2b_\perp}{(2\pi)^2} e^{\pm i q_\perp b_\perp}$$

To derive the Sudakov factor.

1. Virtual contribution

Ellis and Sexton 86'

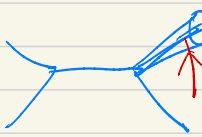
universal



$\delta^{(2)}(\underline{q}_\perp)$, $\frac{1}{\epsilon^2} + \frac{1}{\epsilon}$

2. Real gluon emission.

Inside $\rightarrow$ { I Jet contribution



counted as jet itself.

$\frac{1}{\epsilon}$ $\rightarrow$ II collinear emission to incoming parton.

this is also fixed

jet contribution $\delta^{(1)}(\underline{q}_\perp)$ $\frac{1}{\epsilon^2} + \frac{1}{\epsilon}$

3. Real gluon soft emission outside jet cone.

$\frac{1}{\underline{q}_\perp^2} \ln \frac{Q^2}{\underline{q}_\perp^2}$ $\underline{q}_\perp$ transverse momentum of gluon

$\frac{1}{\epsilon^2} + \cdots$

Arxiv #
1308.2993. 1506.06170. 2106.05307.

Soft contribution

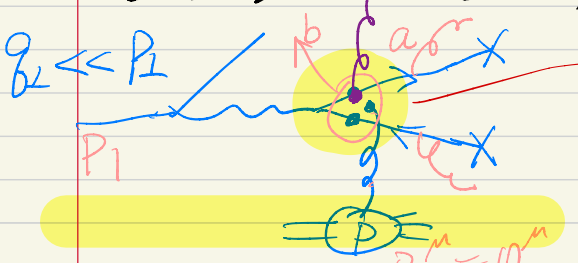
outgoing quark $\frac{2k_1^\mu}{2k_1\cdot k_g} g$

anti quark $-g\frac{2k_1^\mu}{2k_1\cdot k_g}$

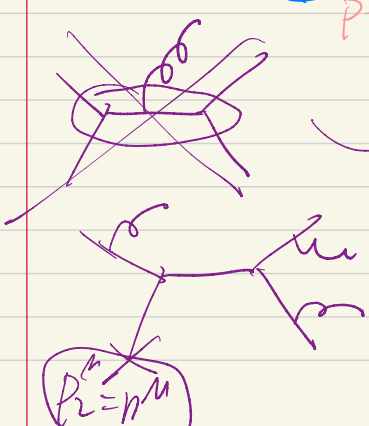
incoming quark $\frac{-2p_1^\mu}{2p_1\cdot k_g} g$

anti- $\frac{2p_1^\mu}{2p_1\cdot k_g} g$

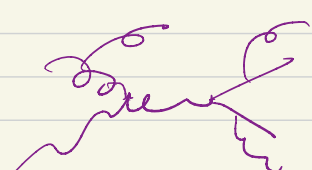
Inclusive dis dijet



→ localized vertex with size $\frac{1}{Q}, \frac{1}{P_\perp}$.



suppressed by powers of $\frac{q_\perp^2}{P_\perp^2} \sim Q^2$



$\propto N_c C_F$ / color average.

$$\frac{1}{N_c C_F}\left(\frac{k_1^\mu}{k_1\cdot k_g}t^a t^b - \frac{k_2^\mu}{k_2\cdot k_g}t^b t^a\right)$$

$$C_F\left(-g_{\mu\nu} + \frac{p_{2\mu}k_{g\nu} + k_{2\nu}k_{g\mu}}{p_2\cdot k_g}\right)$$

$$\left(\frac{k_1^\nu}{k_1\cdot k_g}t^b t^a - \frac{k_2^\nu}{k_2\cdot k_g}t^a t^b\right)$$

$\mathrm{tr}[t^a t^b t^b t^a] = C_F^2 N_c$

$\mathrm{tr}[t^a t^b t^a t^b] = -\frac{1}{2N_c}C_F N_c$

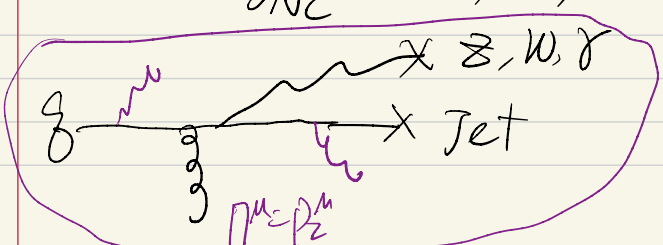
$$= C_F\left[\frac{2k_1\cdot p_2}{k_1\cdot k_g\, p_2\cdot k_g} + \frac{2k_2\cdot p_2}{k_2\cdot k_g\, p_2\cdot k_g}\right]$$

$$-\frac{1}{2N_c}\left[\frac{2k_1\cdot k_2}{k_1\cdot k_g\, k_2\cdot k_g} - \frac{2k_1\cdot p_2}{k_1\cdot k_g\, p_2\cdot k_g} - \frac{2k_2\cdot p_2}{k_2\cdot k_g\, p_2\cdot k_g}\right]$$

$$= \frac{N_c}{2}\left[S(k_1,p_2) + S(k_2,p_2)\right] - \frac{1}{2N_c}S(k_1,k_2)$$

DY

$C_F S(q_1,p_2)$



crossing symmetry

$p_1 \leftrightarrow k_1 / k_2$

$p_1, p_2, k_1, k_2 \quad / C_4^2 = 6$

isotropic

$$S(p_1,p_2) = \frac{2p_1\cdot p_2}{p_1\cdot k_g \; p_2\cdot k_g} \Rightarrow \begin{cases} \hat{s} = (p_1+p_2)^2 \\ \hat{u} \\ \hat{t} \end{cases}$$

$$\int dy_g \Rightarrow \frac{2}{k_{g\perp}^2} \ln \frac{\hat{s}}{k_{g\perp}^2}$$

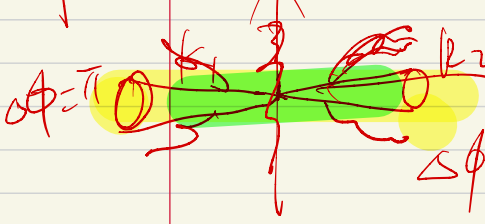
$$S(k_1,p_1) \Rightarrow \frac{1}{k_{g\perp}^2}\left[\ln\frac{\hat{s}}{k_{g\perp}^2} + \ln\frac{\hat{t}}{\hat{u}} + \ln\frac{1}{R^2}\right]$$

$$+2\sum_{n=1}^{f_i} \cos n\phi\,(-1)^n \frac{1}{k_{g\perp}^2}$$

odd, even

$$\int \frac{d\Delta\phi}{2\pi}\int dy_g \; \frac{\theta(\Delta y^2+\Delta\phi^2) - R^2}{} \times S(k_1,p_1) \rightsquigarrow \frac{2k_1\cdot p_1}{k_1\cdot k_g \; p_1\cdot k_g}$$

anisotropy

purely final

$$S(k_1,k_2) = \frac{2}{k_{g\perp}^2}\left[\ln\frac{\hat{s}}{u} + \ln\frac{\hat{s}}{t} + \ln\frac{1}{R^2} + 2\sum_{2n} C_{2n}^{ff}\cos 2n\phi\right]$$