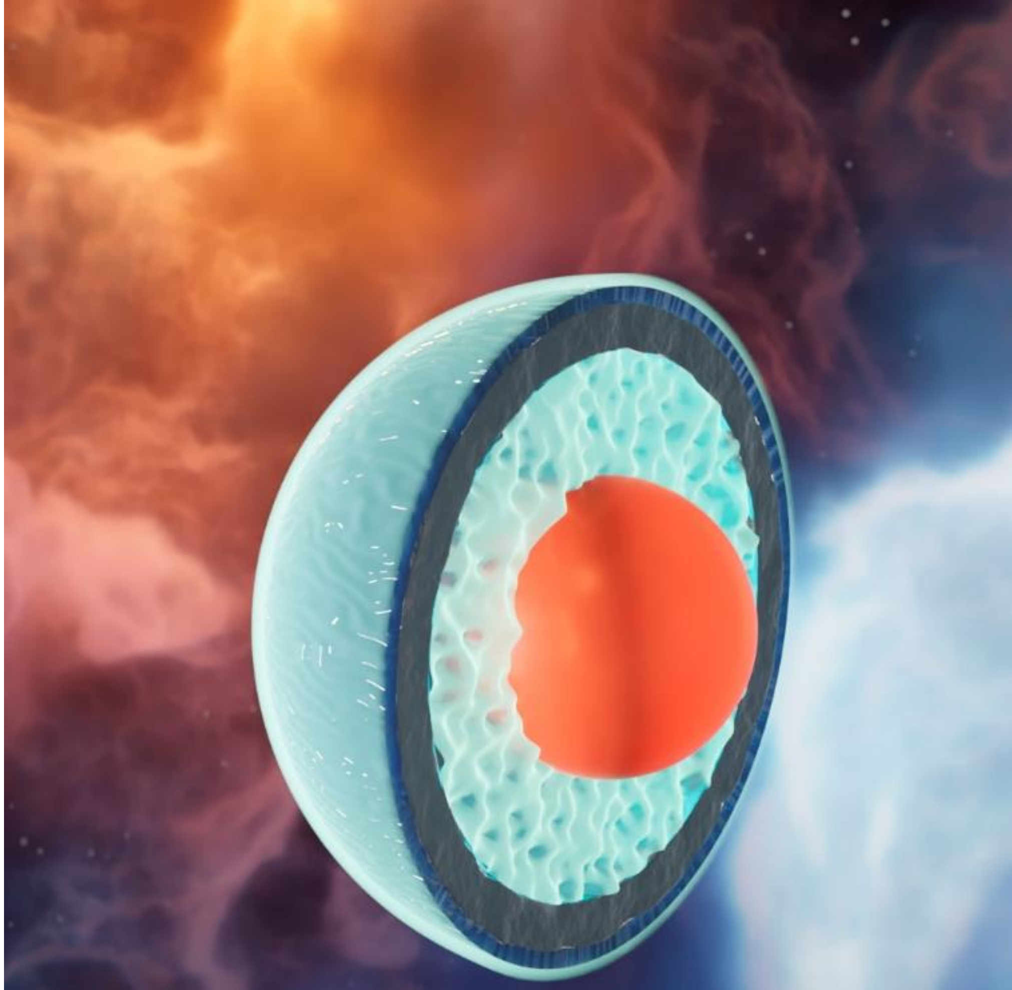
Perturbative Thermal QCD: from Hard Thermal Loops to Neutron Stars

Aleksi Vuorinen

University of Helsinki &
Helsinki Institute of Physics

QCD Master Class 2021

7-9 September 2021



Main references:
1) Laine, AV, Springer Lect. (2016), 1701.01554 [LV]
2) Annala, Gorda, Kurkela, Nättilä, AV, Nature Physics (2020), 1903.09121 [AGKNV]
3) Ghiglieri, Kurkela, Stricland, AV, Phys. Rept. 880 (2020), 2002.10188 [GKSV]
4) Gorda, Kurkela, Paatelainen, Säppi, AV, PRL & PRD (out soon), 2103.05658 & 2103.07427 [GKPSV]

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Plan for this week's lectures:

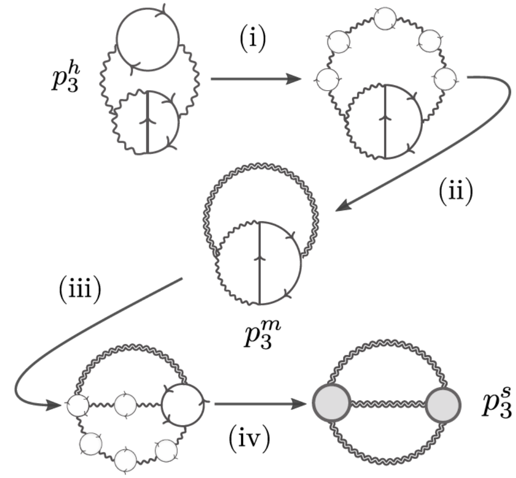
Tue 5.30-6.30pm: *Motivation and outline of lectures*

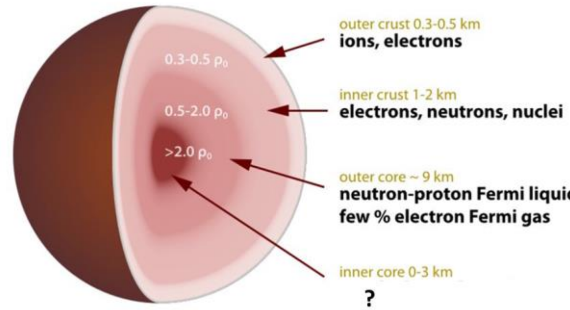
Wed 8-10am: *Equilibrium thermodynamics from pQCD – basic formalism at finite T and μ*

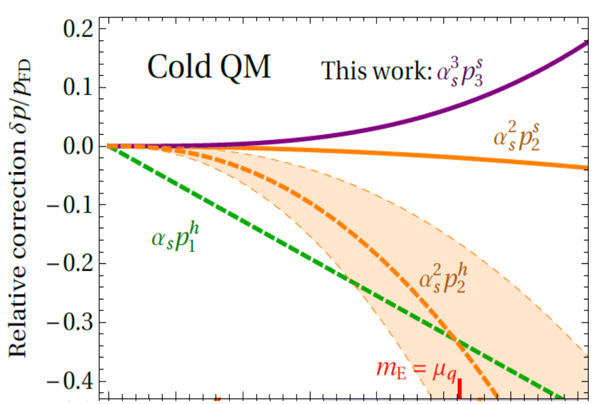
Wed 4.30-6.30pm: *Towards loop calculations at high density – computational tools and IR challenges*

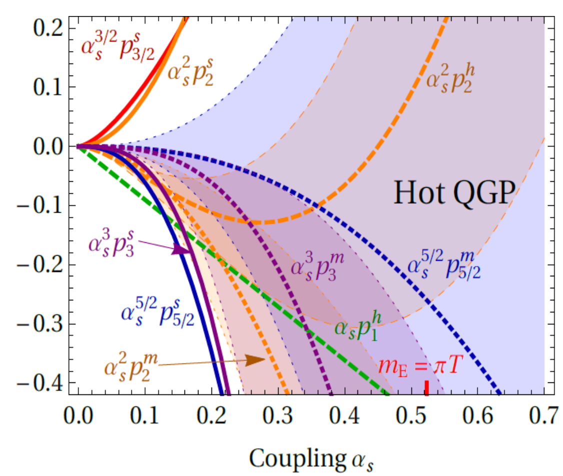
Thu 8-10am: *Neutron stars – introduction to basic properties and observations*

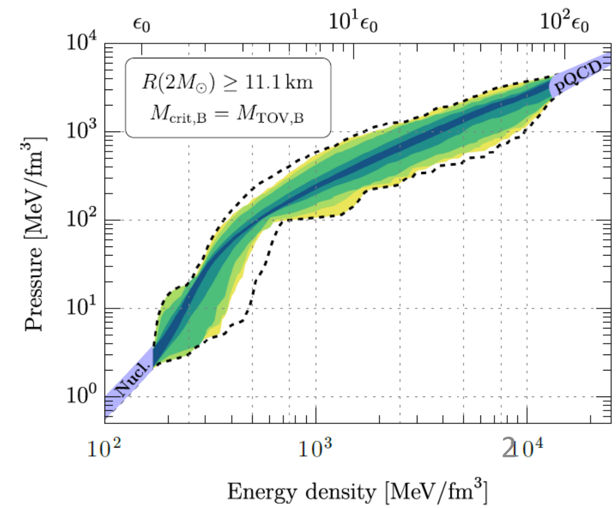
Thu 4.30-5.30pm: *From pQCD to neutron-star physics – where are we now?*





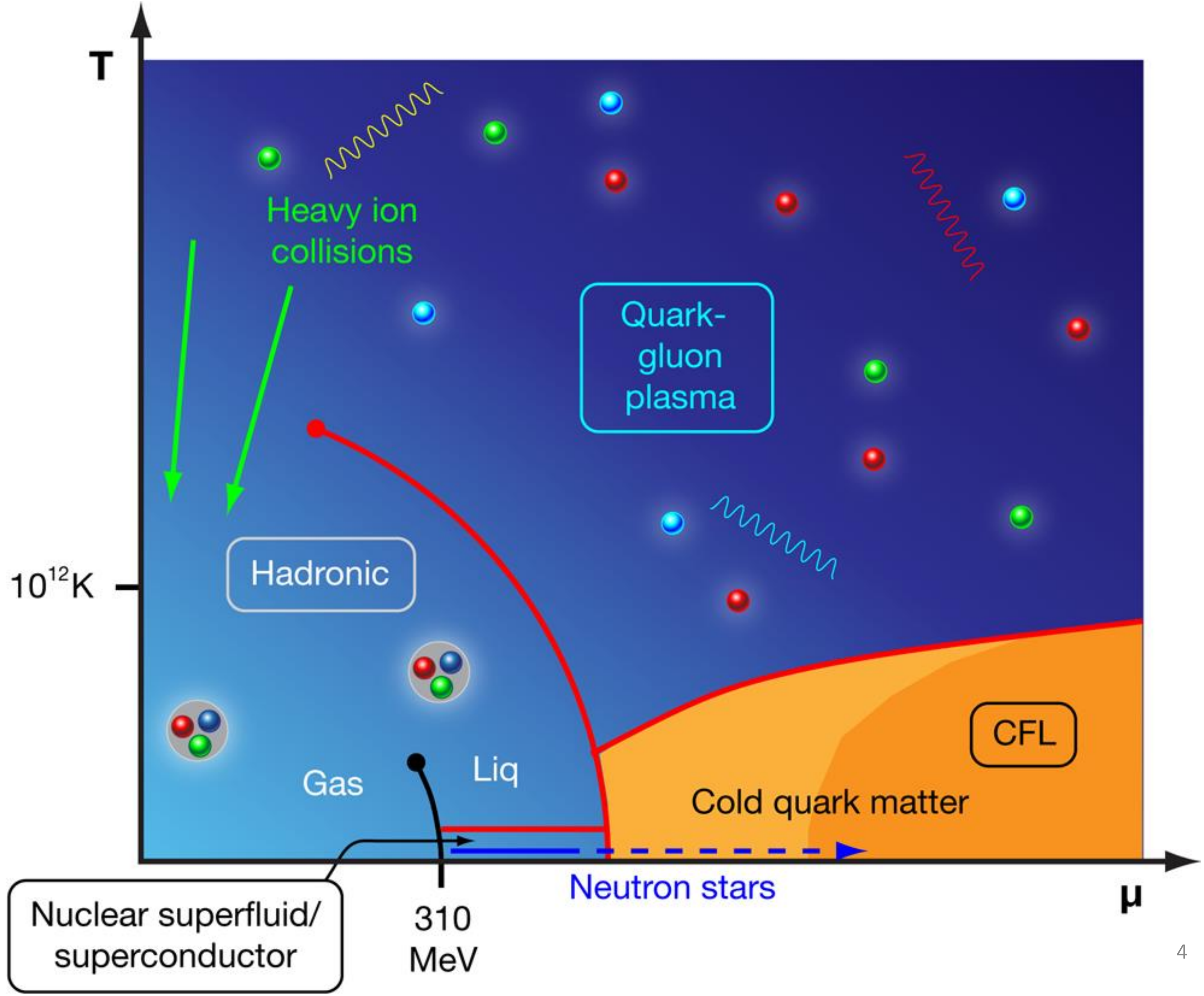


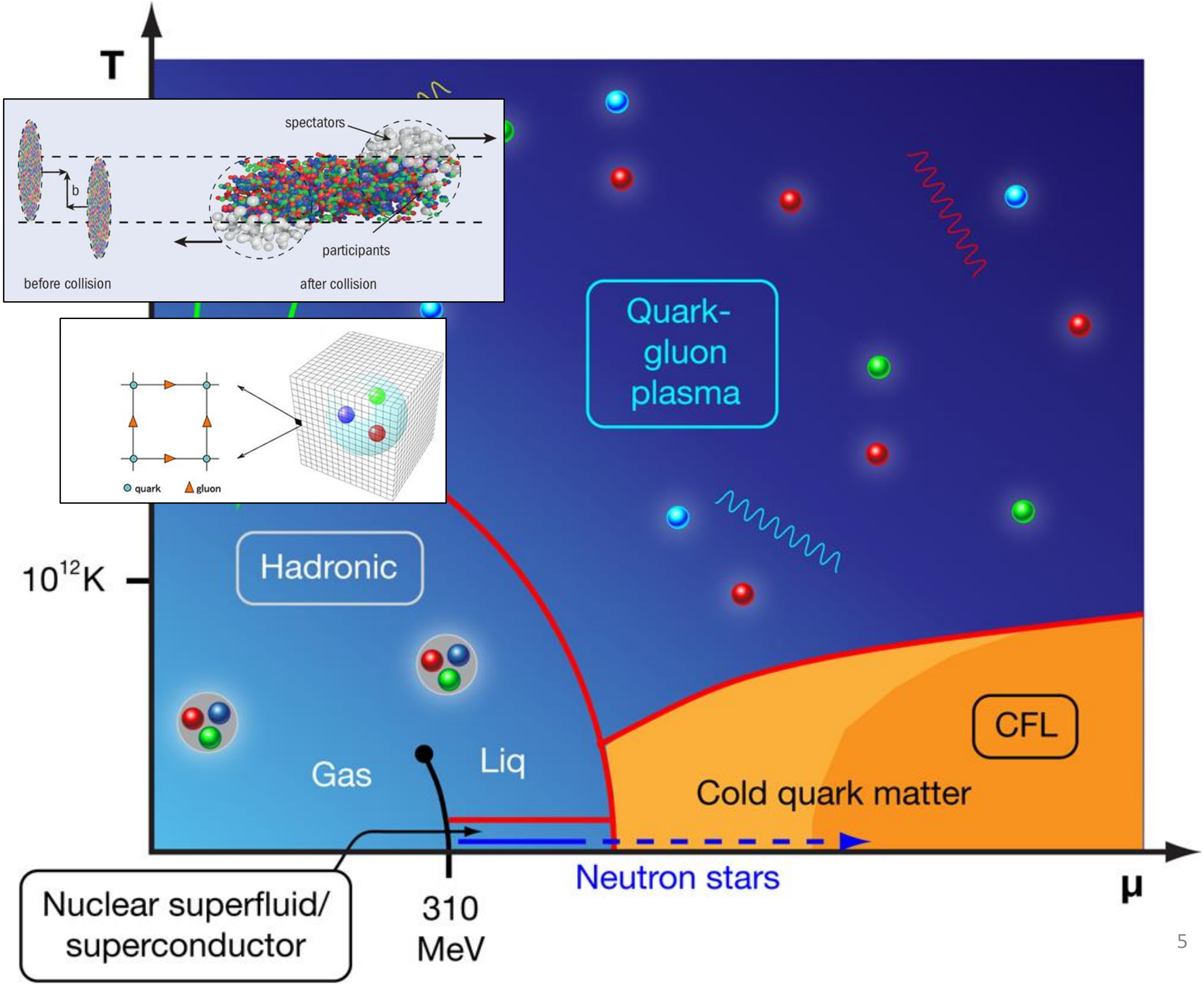




Important: Lots to cover in a relatively short time, so many details omitted.

Many references provided for further study; please ask for more if interested!


T
Heavy ion collisions
Quark-gluon plasma
Hadronic
10^{12}K
CFL
Gas
Liq
Cold quark matter
Neutron stars
μ
Nuclear superfluid/ superconductor
310 MeV



T
spectators
b
participants
before collision
after collision
quark
gluon
Quark-
gluon
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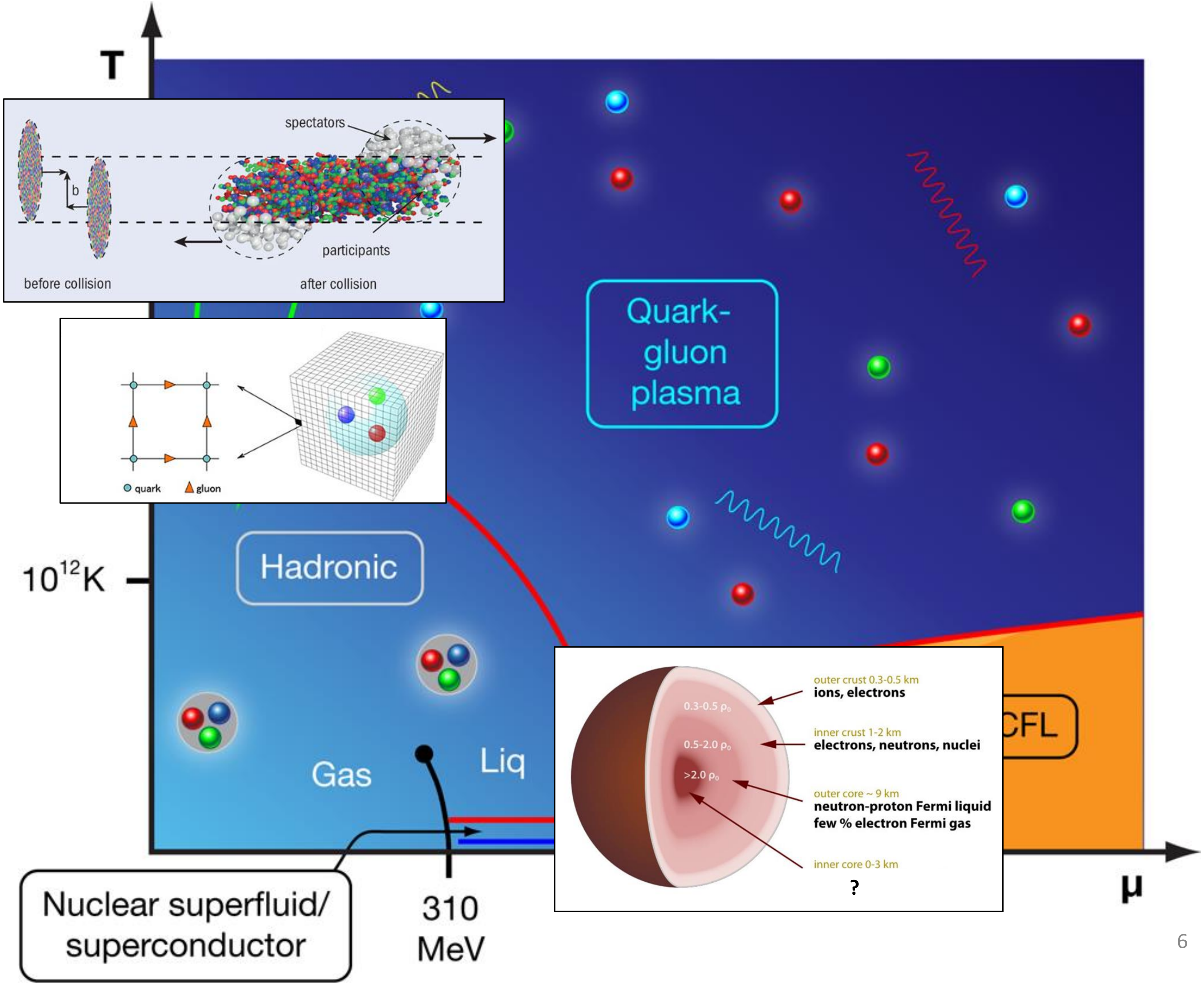

T
spectators
b
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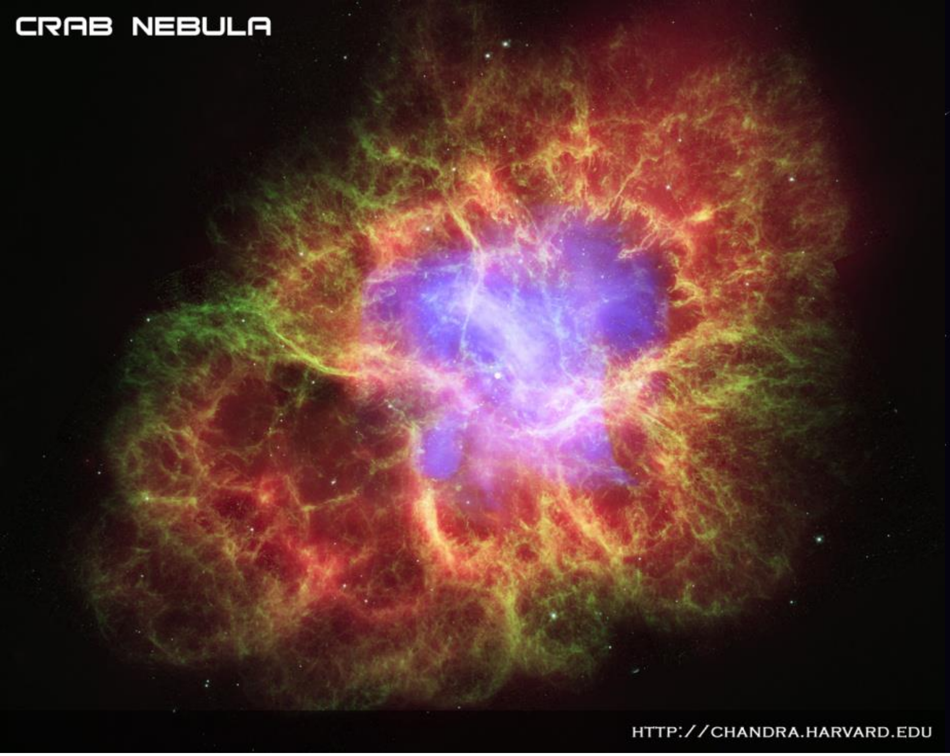
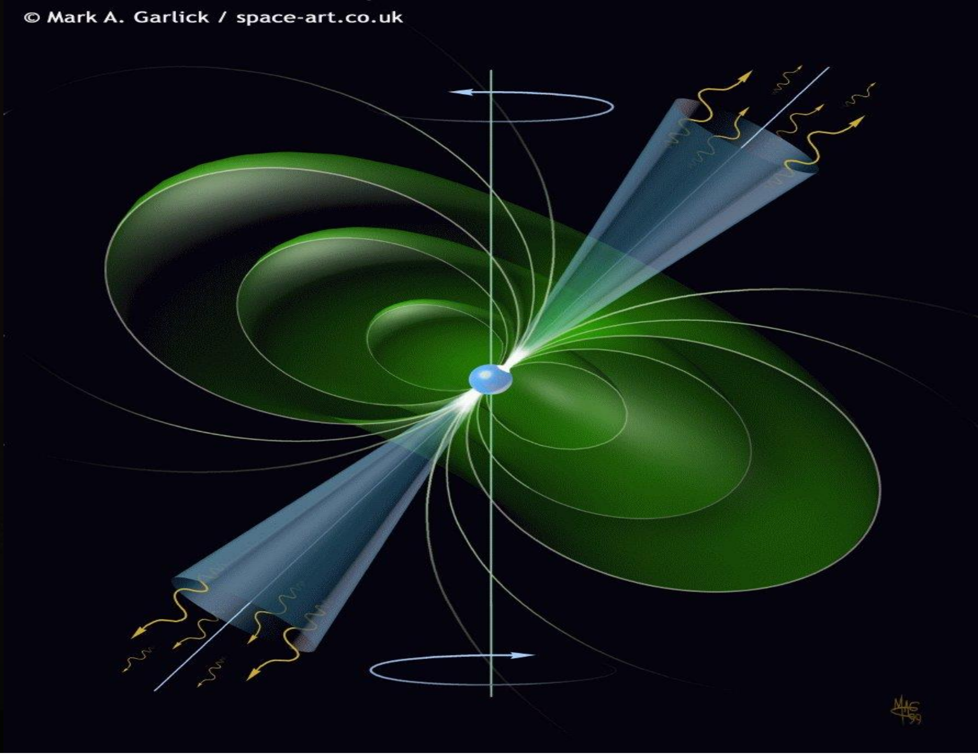
before collision
after collision
quark
gluon
Quark-gluon plasma
Hadronic
10^{12}K
Gas
Liq
CFL
outer crust 0.3-0.5 km
ions, electrons
inner crust 1-2 km
electrons, neutrons, nuclei
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas
inner core 0-3 km
?
0.3-0.5 ρ_0
0.5-2.0 ρ_0
>2.0 ρ_0
Nuclear superfluid/ superconductor
310 MeV
μ





When a hydrogen burning star runs out of fuel, several possible scenarios depending on the initial mass of the star:

- $\mathrm{M} \lesssim 9M_{\mathrm{sun}}$ ⇒ White dwarf
- $\mathrm{M} \gtrsim 9M_{\mathrm{sun}}$ ⇒ Supernova explosion
  - $\mathrm{M} \gtrsim 20M_{\mathrm{sun}}$ ⇒ Gravitational collapse into BH
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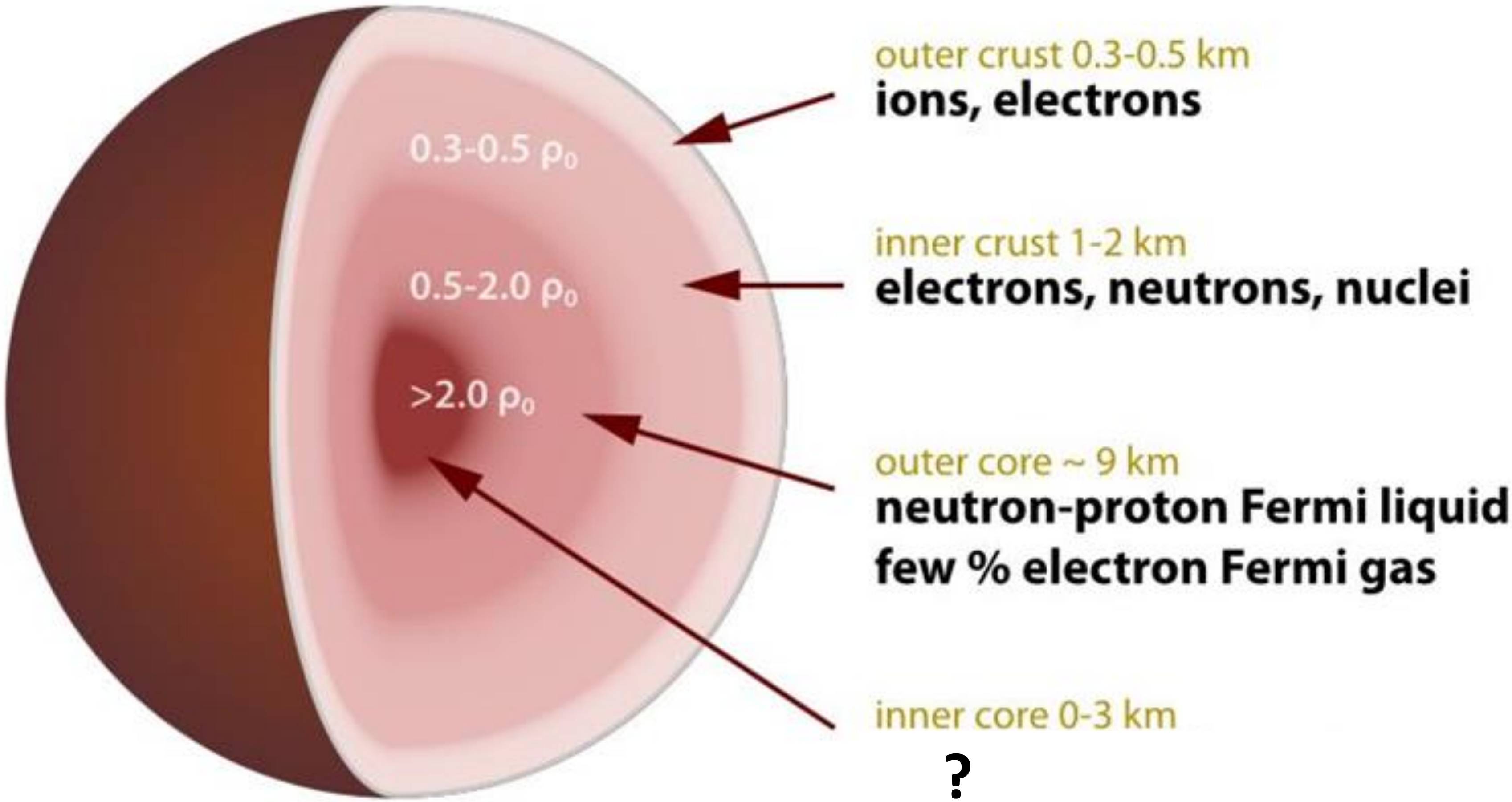
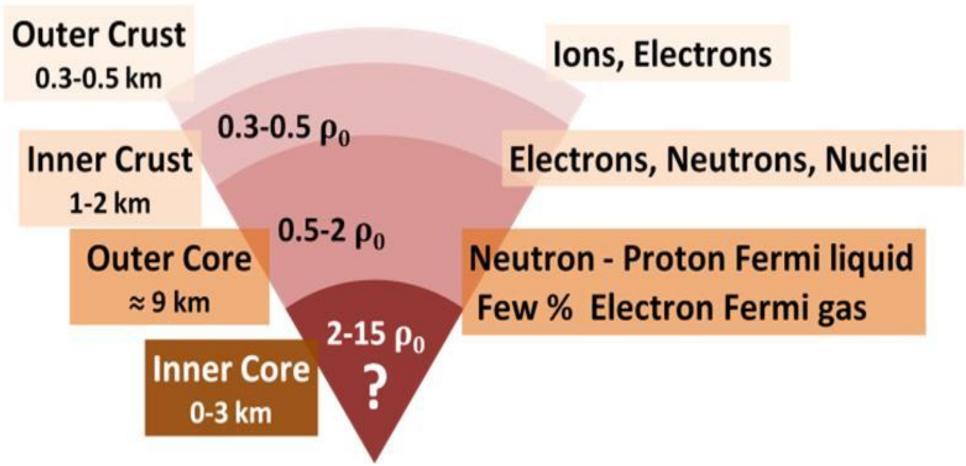
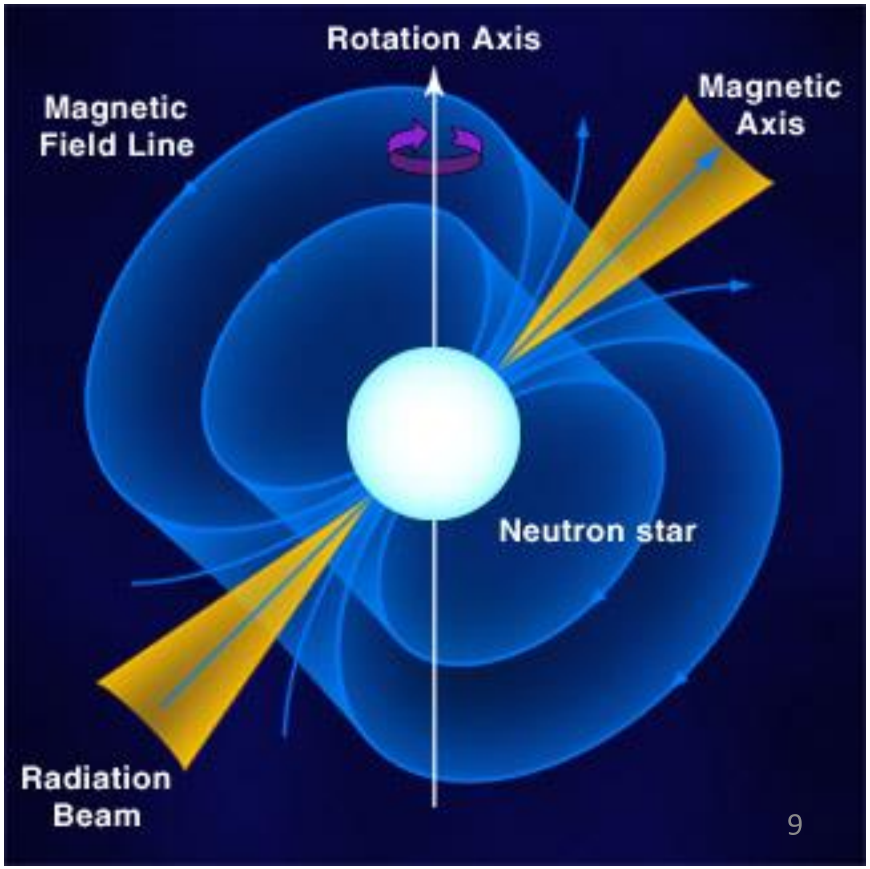

0.3-0.5 ρ_0
0.5-2.0 ρ_0
>2.0 ρ_0
outer crust 0.3-0.5 km
ions, electrons
inner crust 1-2 km
electrons, neutrons, nuclei
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas
inner core 0-3 km
?

NS characteristics:

- Masses from $\sim M_{\odot}$ up to more than $2M_{\odot}$
- Radii $\approx 11 - 13$ km
- Spin frequencies $\lesssim$ kHz
- Temperatures $\lesssim$ keV
- Strong magnetic fields up to 10^{15}G

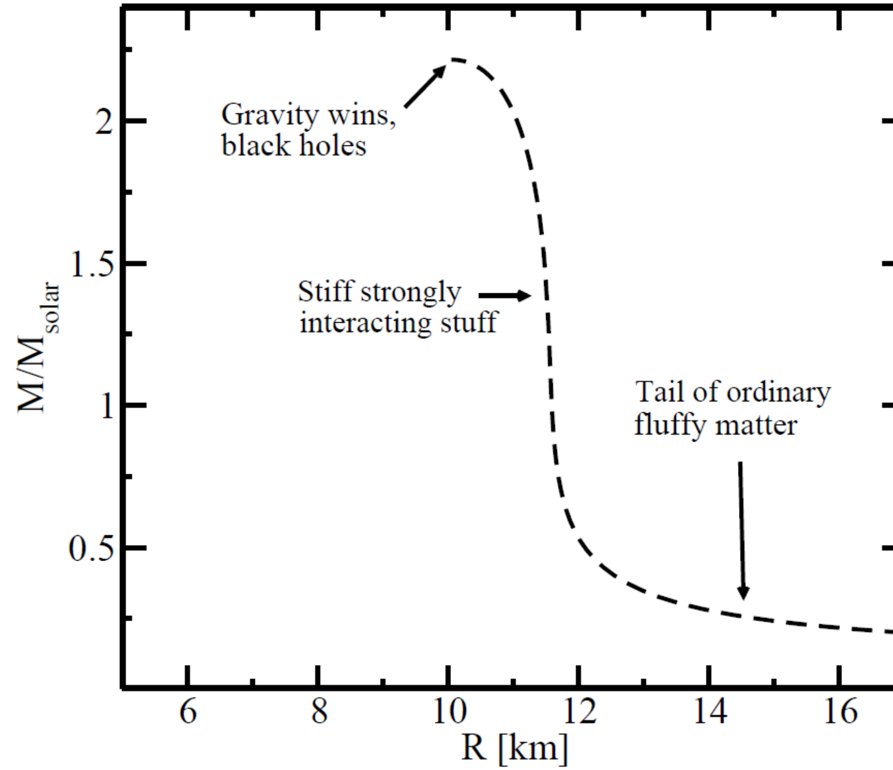
Unique laboratory for strong interaction physics: Density in NS cores high enough to probe nuclear matter well beyond saturation density – and possibly host deconfined matter!





Physics picture: Hydrostatic equilibrium resulting from fierce competition between two forces: gravity and the (Fermi) pressure of QCD matter

General relativistic description via famous Tolman-Oppenheimer-Volkov equations:



$$\begin{aligned}\frac{dM(r)}{dr} &= 4\pi r^2 \varepsilon(r), \\ \frac{dp(r)}{dr} &= -\frac{G\varepsilon(r)M(r)}{r^2}\frac{(1+p(r)/\varepsilon(r))\left(1+4\pi r^3 p(r)/M(r)\right)}{1-2GM(r)/r}\end{aligned}$$

$$\varepsilon(p) \Rightarrow M(R)$$

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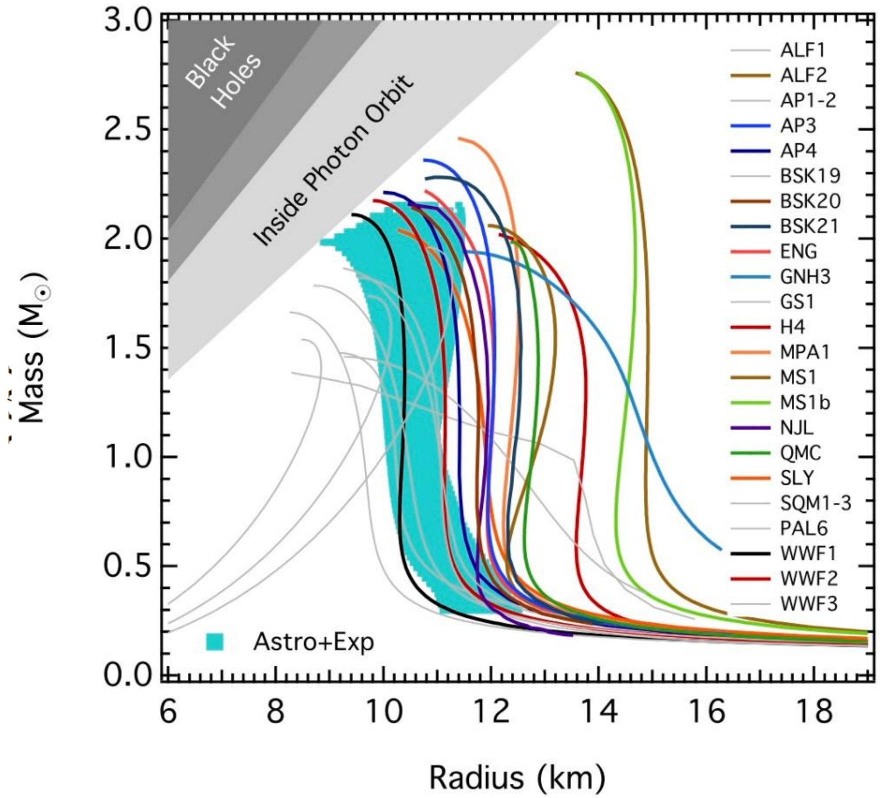

Ozel et al., ApJ 820 (2016)

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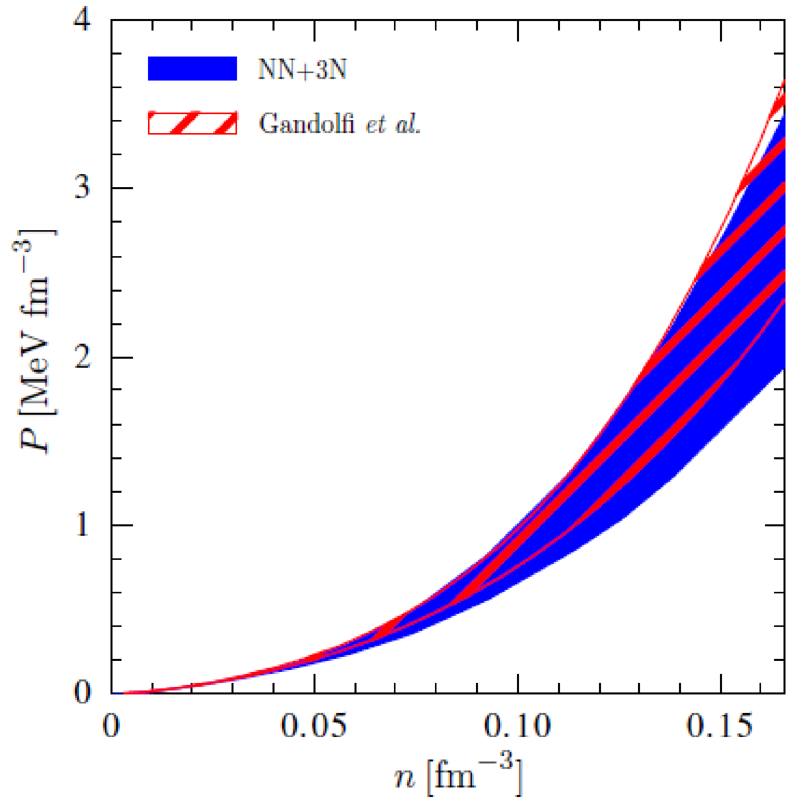
Particle/nuclear theory challenge: Find Equation of State of strongly interacting matter.

Relevant limits for the system:

- Cold and dense: $\mu \gg T$
- Electrical neutrality:
$$\frac{2n_u}{3} - \frac{n_d}{3} - \frac{n_s}{3} + n_e = 0$$
- Beta equilibrium:
$$\frac{\mu_B}{3} = \mu_d = \mu_s = \mu_u + \mu_e$$



Hebeler et al., ApJ 773 (2013)

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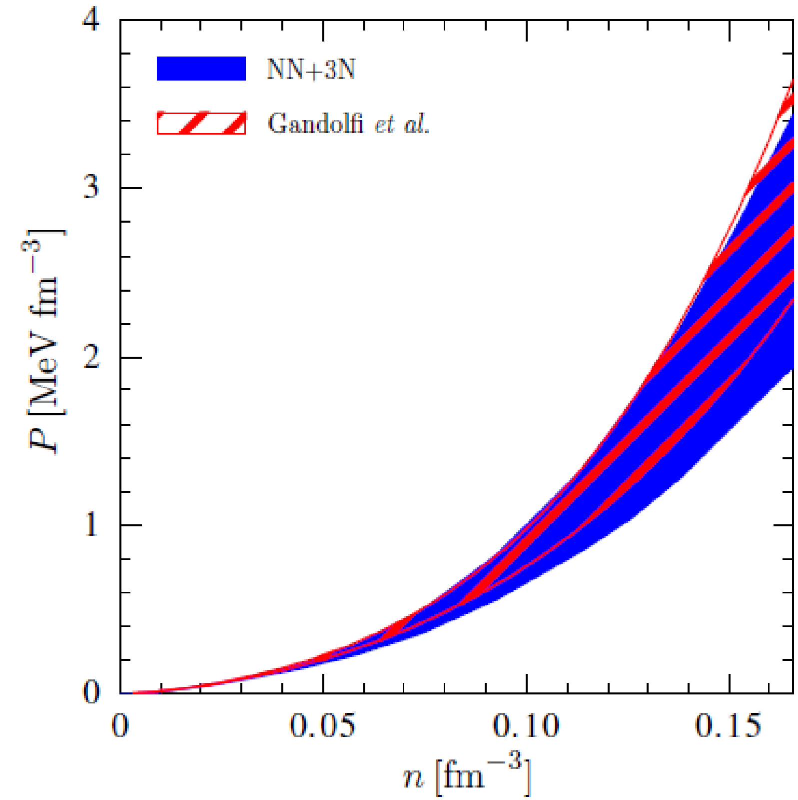

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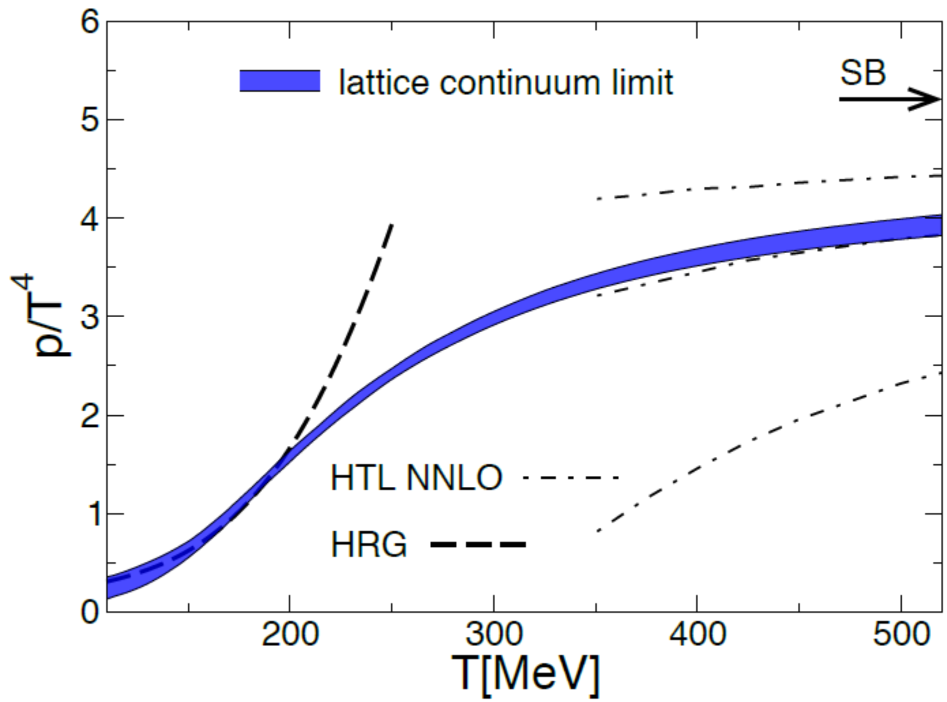
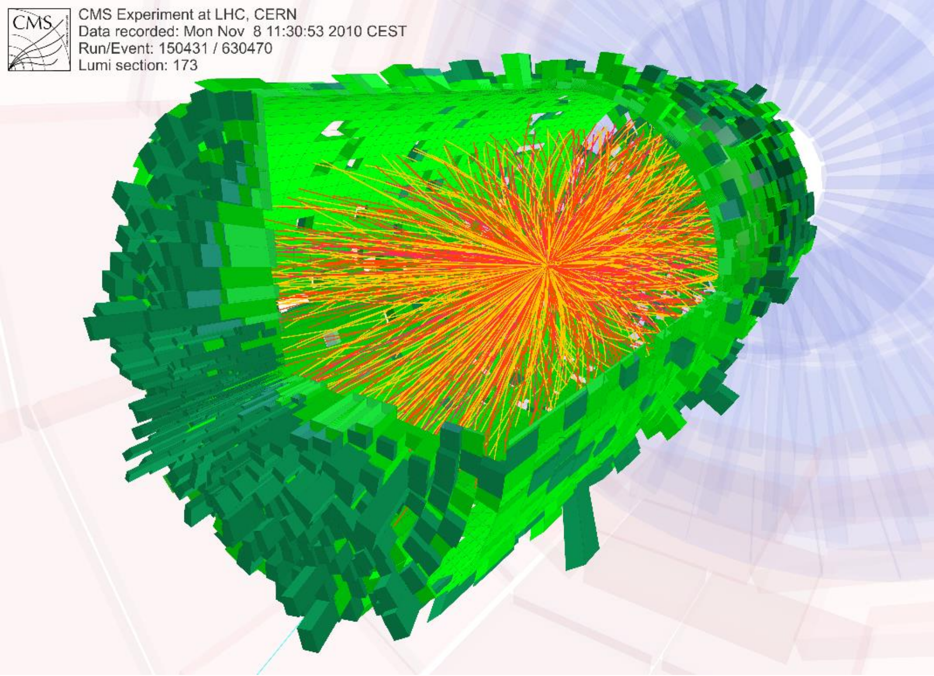
Big open questions:

- Can QCD theorists predict neutron star measurements?
- Can we infer the QCD matter EoS from observations?
- Can deconfined matter be found inside the stars?

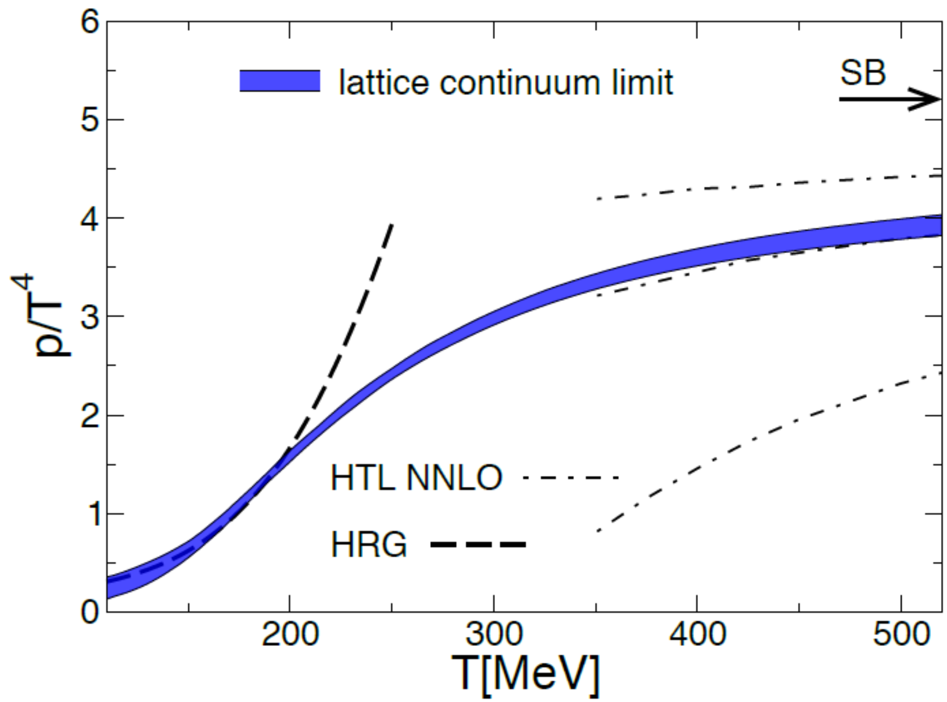
Classic reference for material covered so far: *N. Glendenning, Compact Stars, Springer*

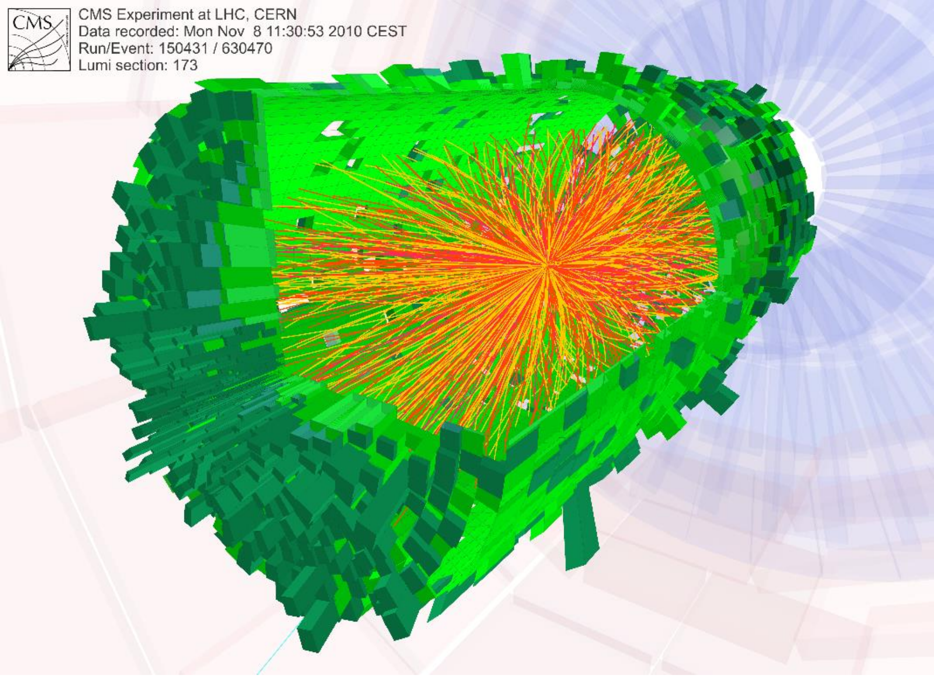
Heavy-ion physics: Been there, done that – what have we learned?





Some lessons from heavy ion experiments & lattice studies of QGP:





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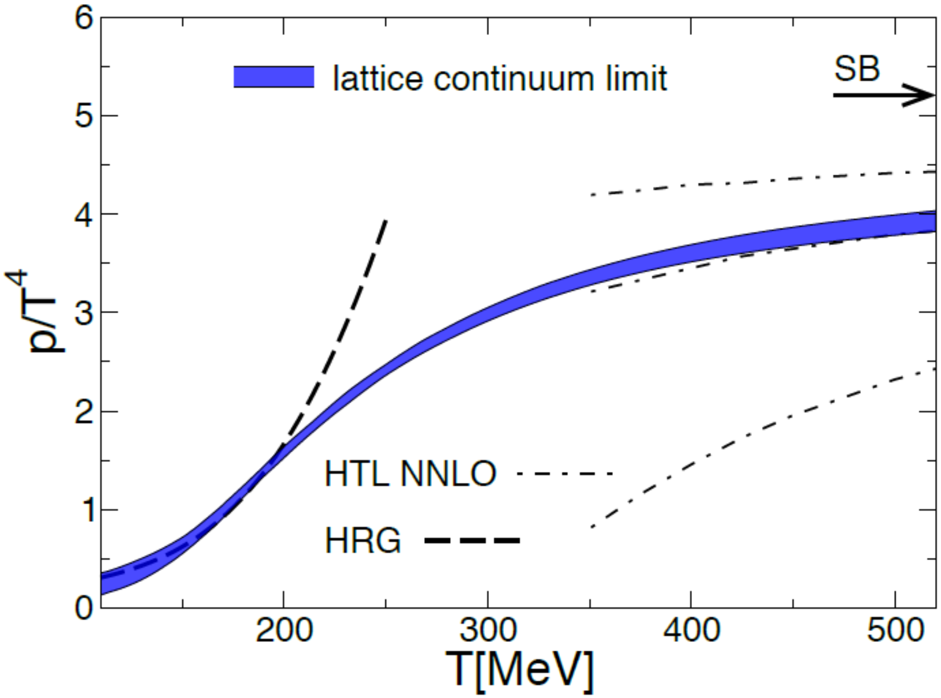
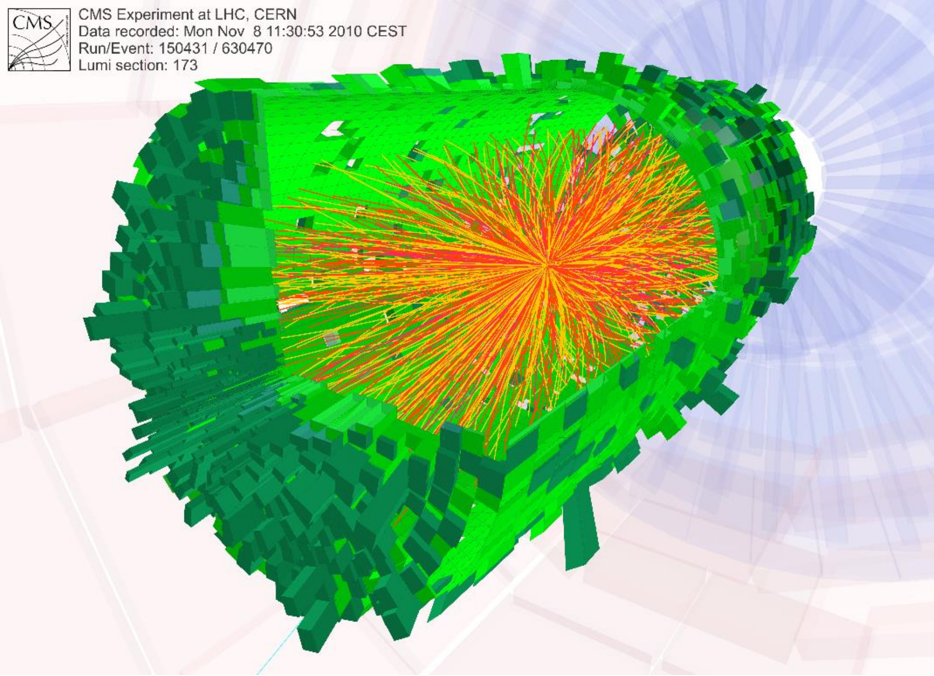
1) Crossover deconf. transition at $T \sim 150$ MeV, $\epsilon \sim 400\ \mathrm{MeV/fm^3}$





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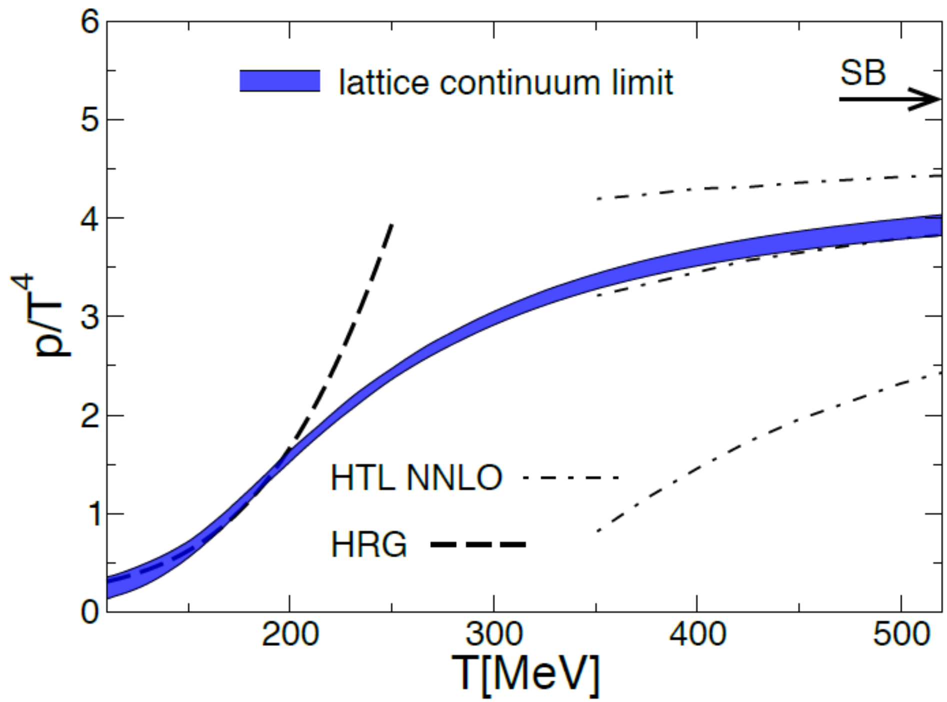

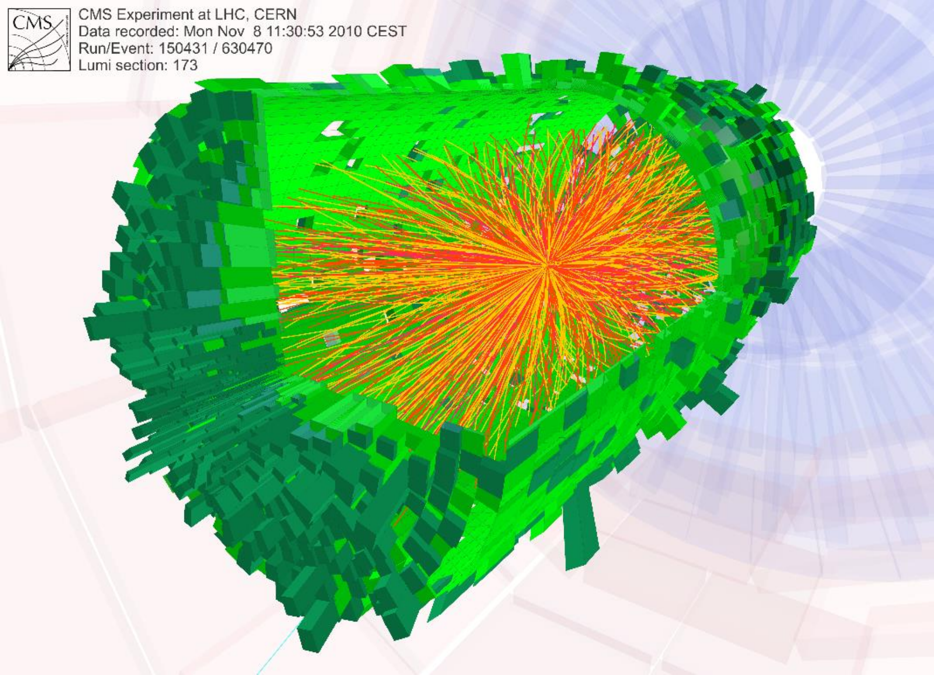

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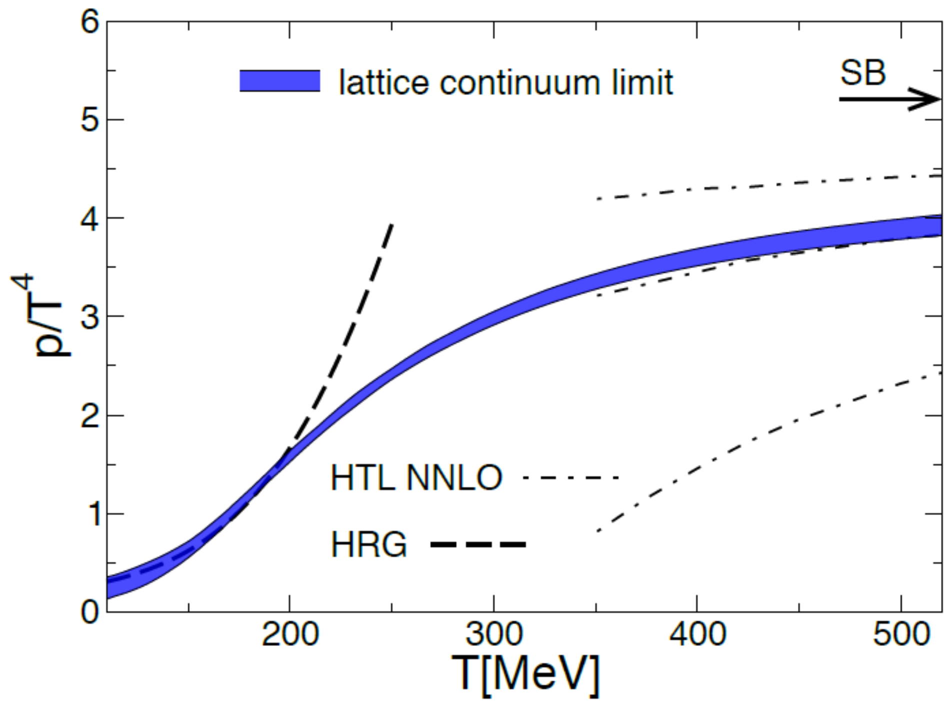

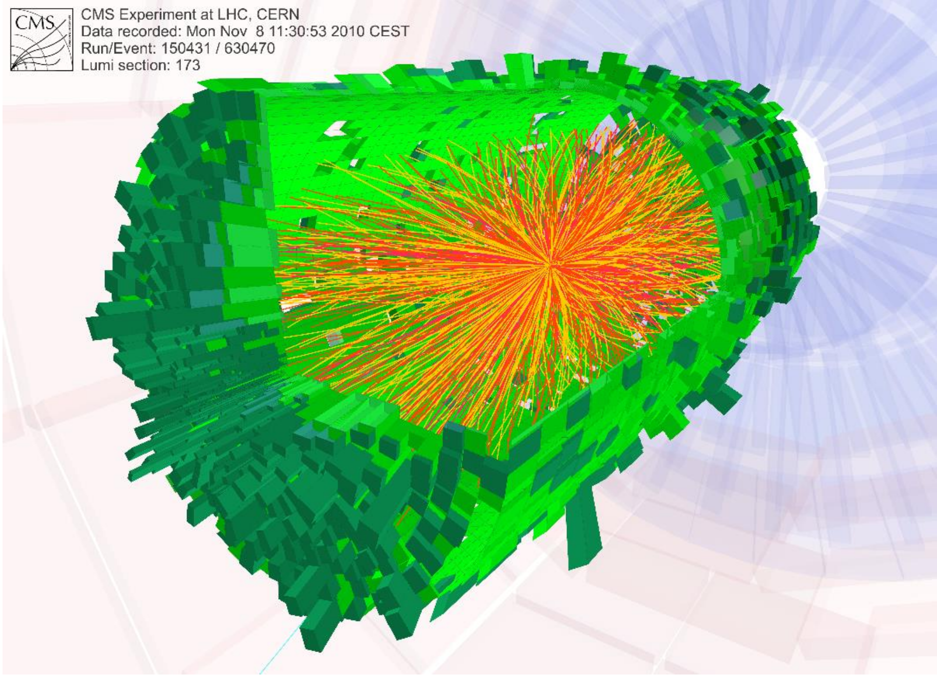

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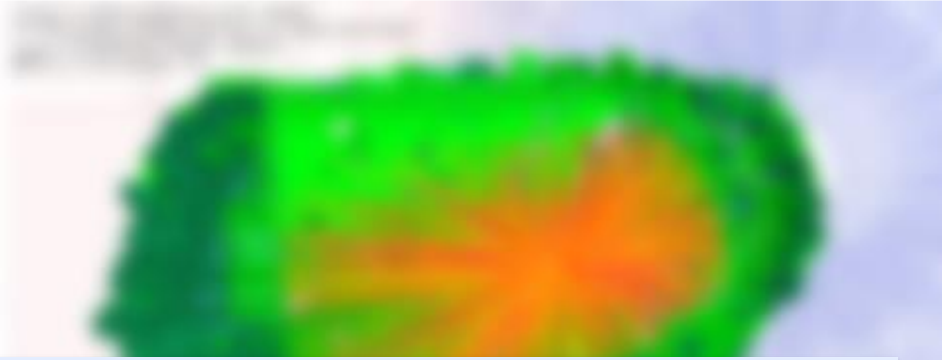
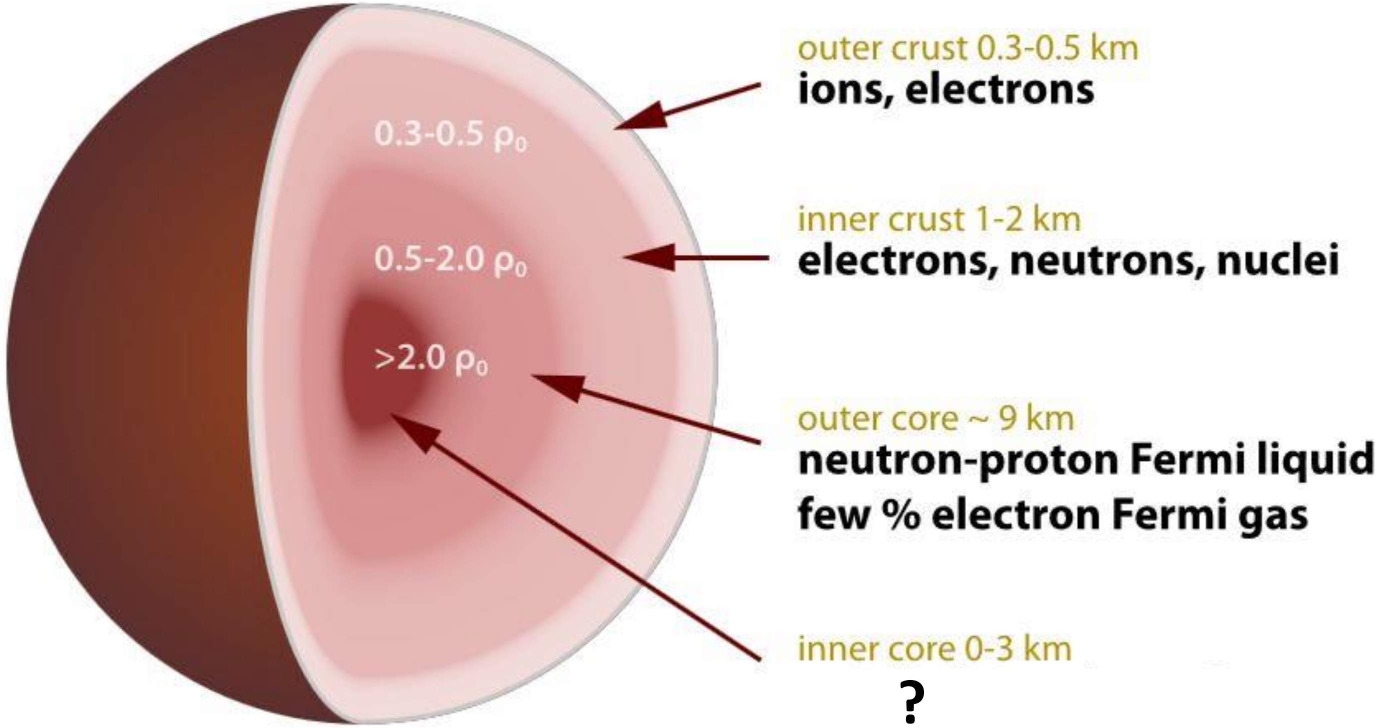
4) Presence of deconfined matter can be confirmed indirectly



Main question for these lectures: how does this generalize to neutron stars?

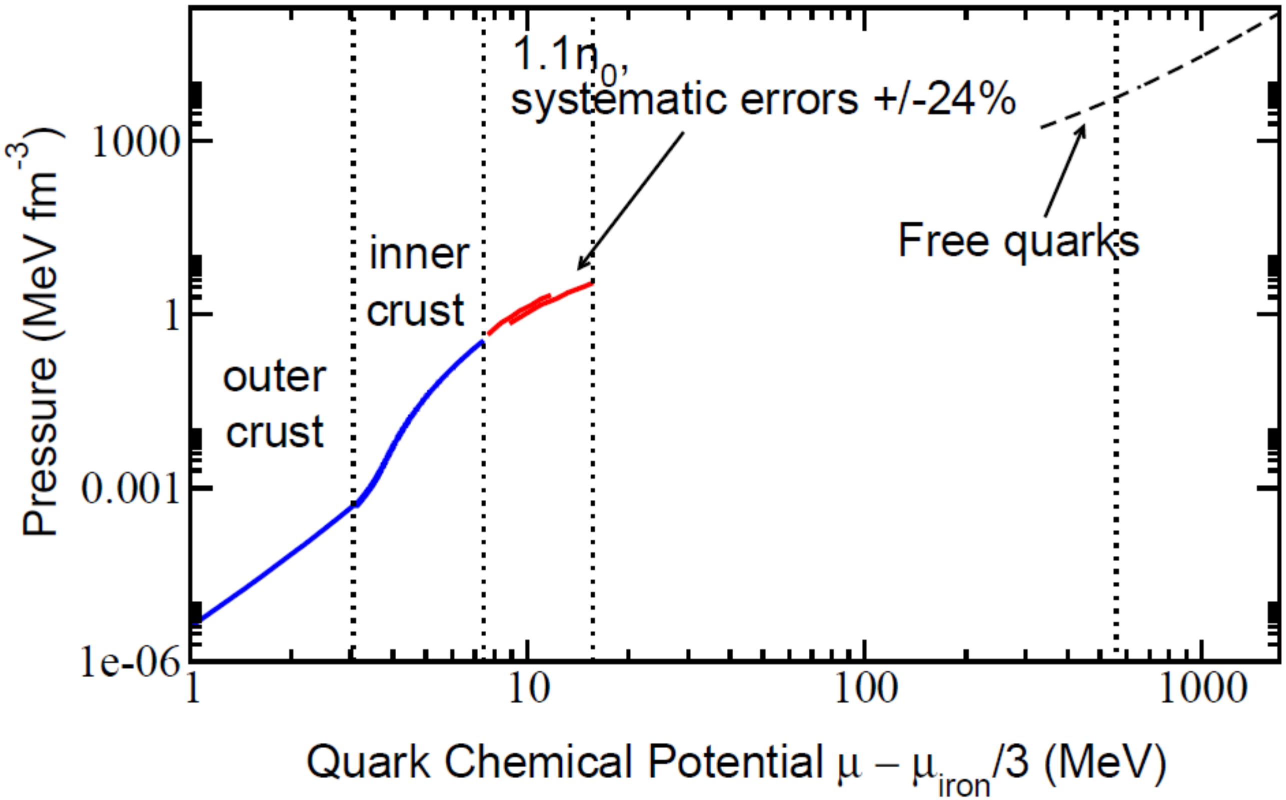
- How can we remedy for the absence of lattice methods (Sign Problem) at high density?
- How should we optimally exploit the observational information available for NSs?
- How can particle theory tools – in particular resummed perturbative QCD – help in the process?

Laying out the problem: what do we need from nuclear and particle theory?



Proceeding inwards from the crust:

- μ_B increases gradually, starting from μ_{Fe}
- Baryon/mass density increase from 0 to beyond $n_s \equiv \rho_0 \approx 0.16/\mathrm{fm}^3 \approx 2 \times 10^{14}\mathrm{g/cm}^3$
- Composition changes from nuclei to neutrons/quarks



Known limits of the EoS (more details on Thursday):

- Low density: phenomenological nuclear theory and systematic Chiral Effective Theory → EoS known to approx. saturation density
- Asymptotic freedom: limiting behavior at very large densities
- In between tricky: no nonperturbative first principles methods available, convergence of naive weak-coupling expansions unclear

$$\begin{aligned} \Omega(T, \mu_u, \mu_d, \mu_s, m_s) &= -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\mathrm{QCD}}}, \\ \mathcal{L}_{\mathrm{QCD}} &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i \gamma_0)\psi_i \end{aligned}$$

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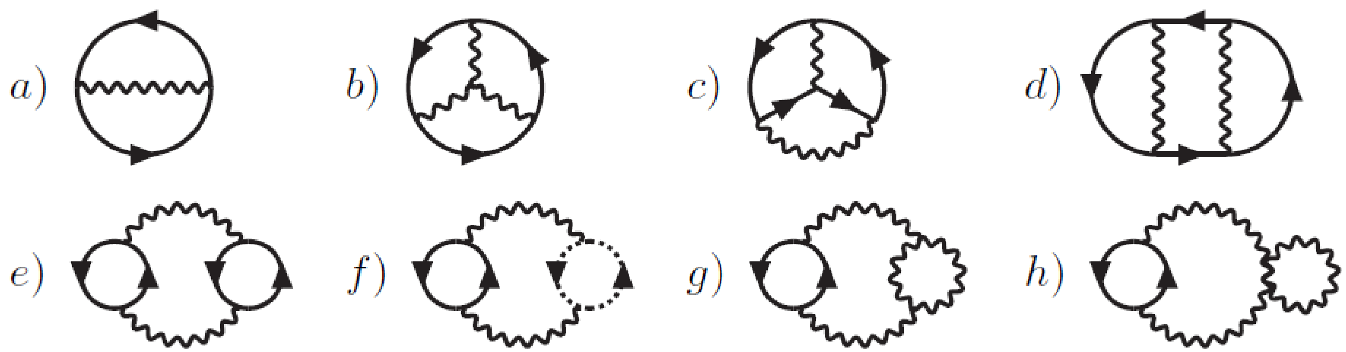
pQCD in unpaired phase: Expand functional integral in a weak-coupling expansion → Vacuum graphs at $\mu \neq 0, T = 0$



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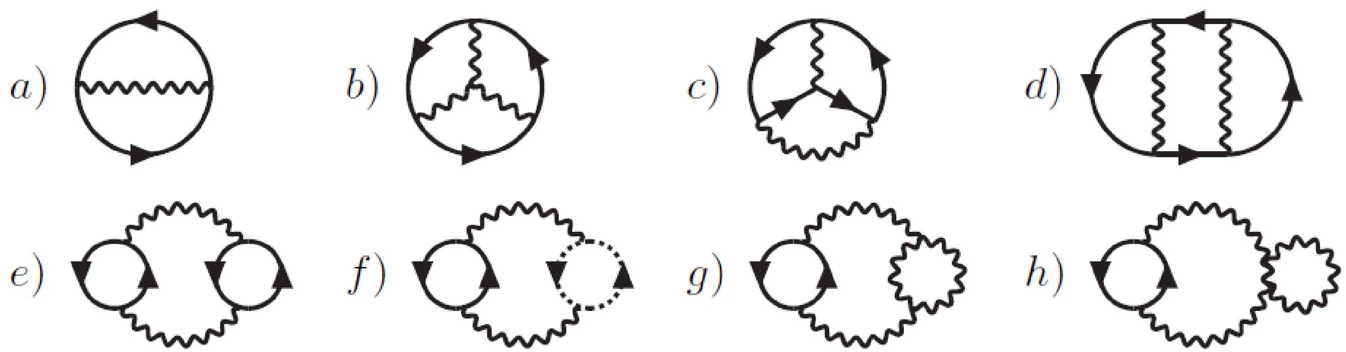

Problem at 3+ loops: Sensitivity to dynamical screening of long-wavelength fields, requiring EFT machinery (topic of tomorrow)

Up to $O(\alpha_s^3)$, three types of contributions to the pressure:

- Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- Soft modes (scale $m_E \sim g\mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- Mixing of soft and hard modes

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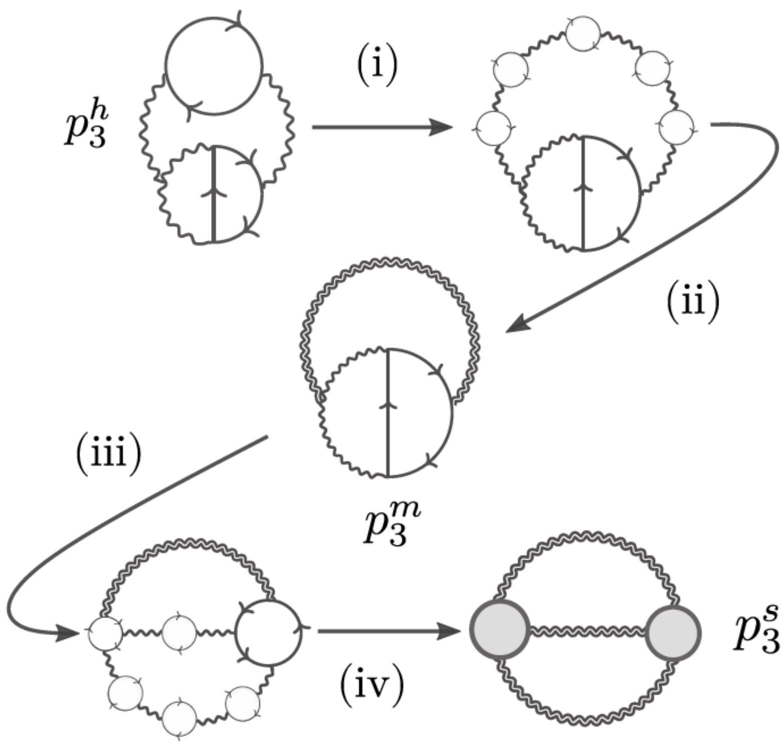

$$p = p_{\rm FD} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3 + p_2^s \alpha_s^2 + p_3^s \alpha_s^3 + p_3^m \alpha_s^3$$

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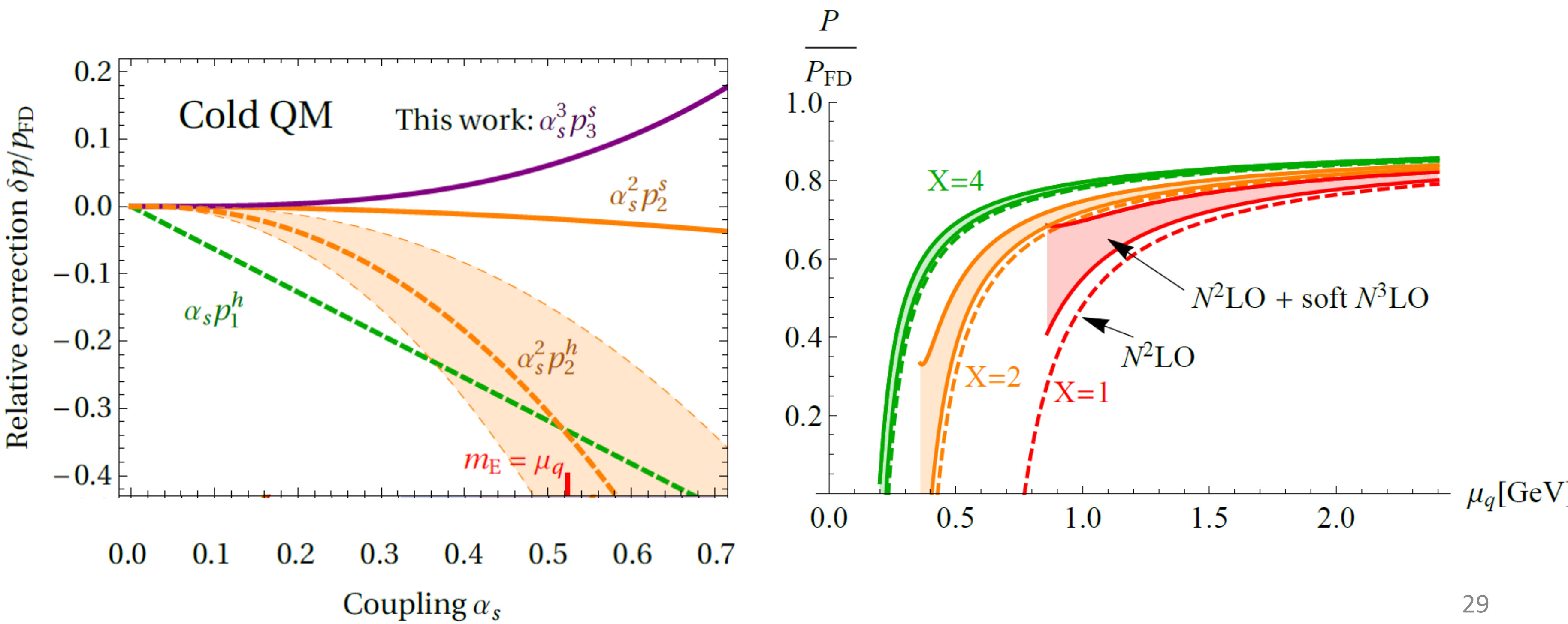
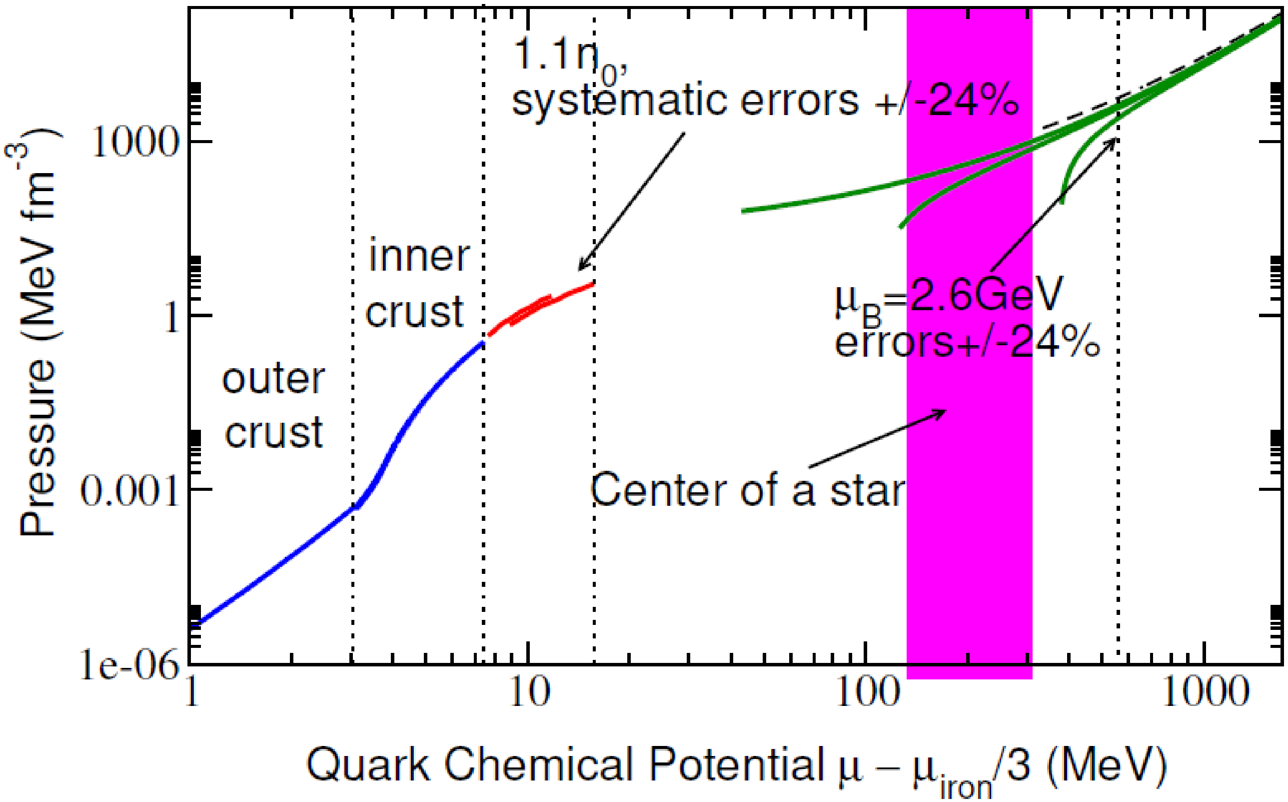
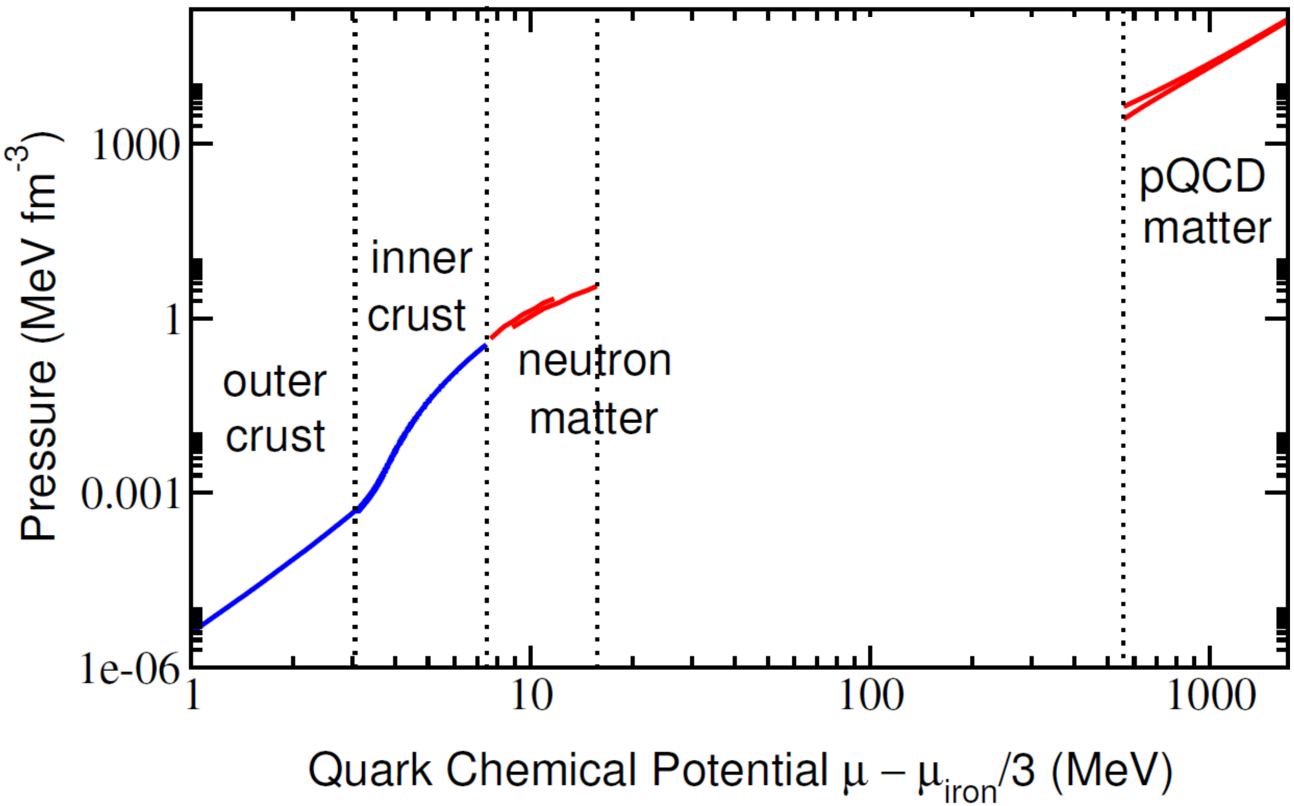




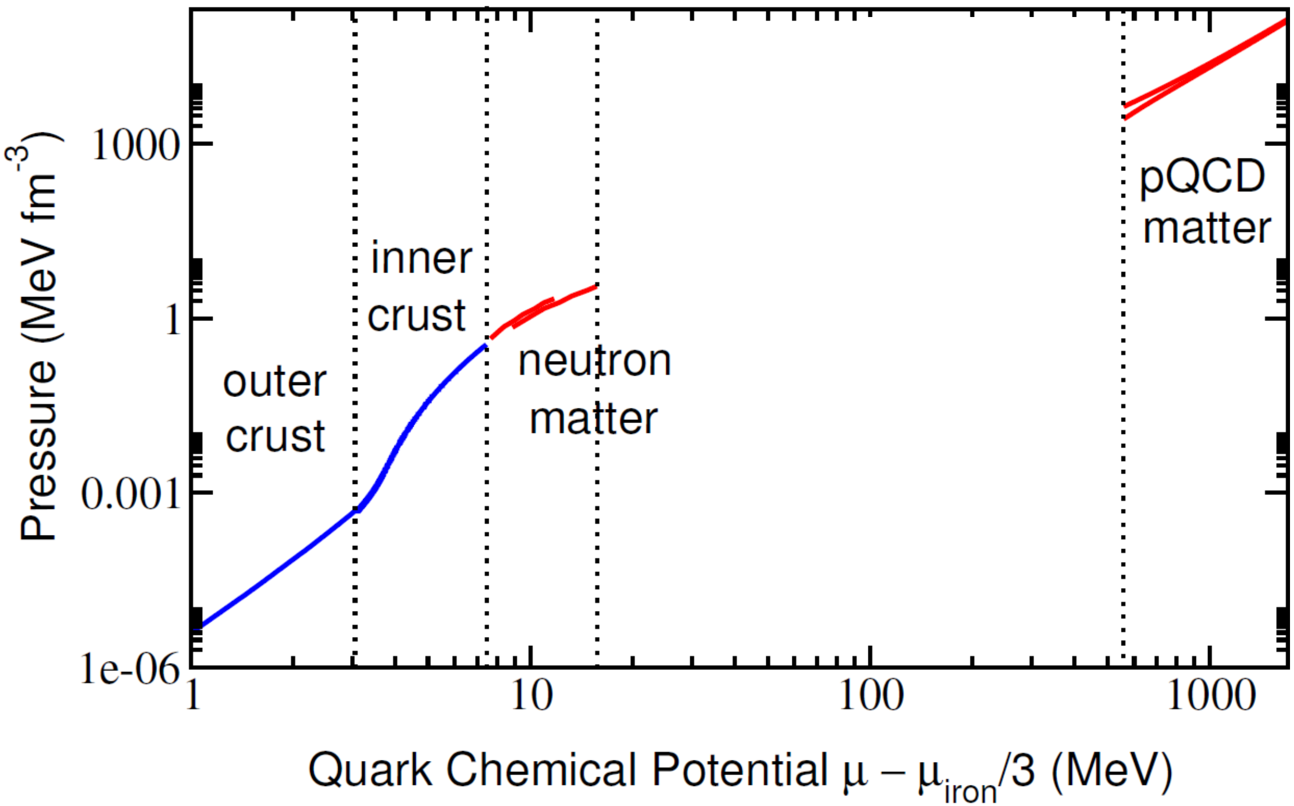
Three-loop EoS with nonzero quark masses [Kurkela, Romatschke, AV, PRD 81 (2009)]

- Uncertainty of result at $\pm 24\%$ level around $40n_s$
- Main uncertainty from renormalization scale dependence
- Pairing contributions to EoS subdominant at $n > 40n_s$, but may become relevant if pQCD result significantly improved



Conclusion: Sizable no man's land extending from NSs' outer core to densities not realized inside physical neutron stars.

Best way forward: try to improve both low- and high-density limits. In between, interpolate & constrain using observational data.



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Tomorrow: pQCD at high density; Thursday: back to NSs.