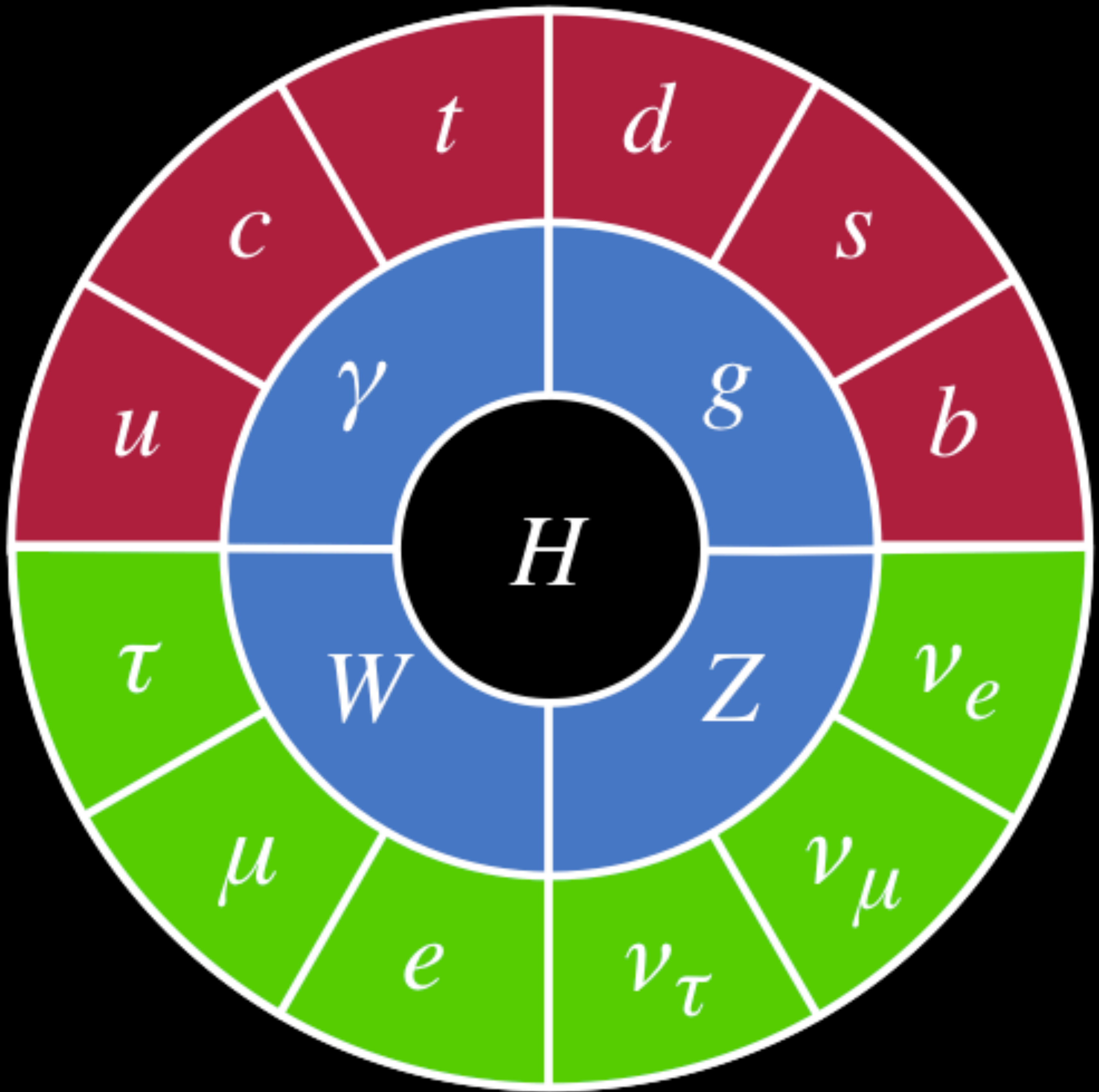
Quantum Computing for Colliders

with Benjamin Nachman, Marat Freytsis

arXiv: 2102.05044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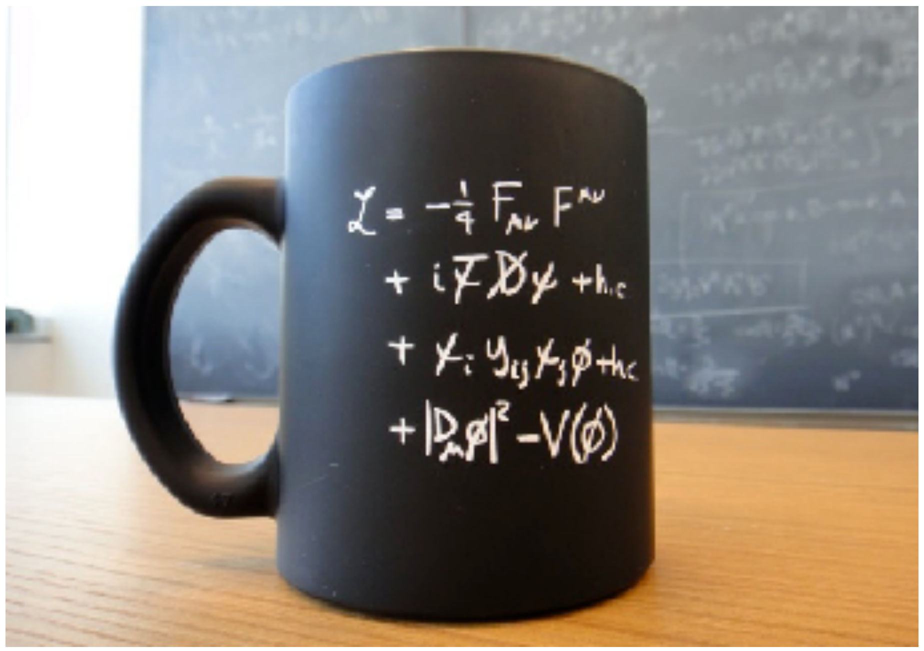
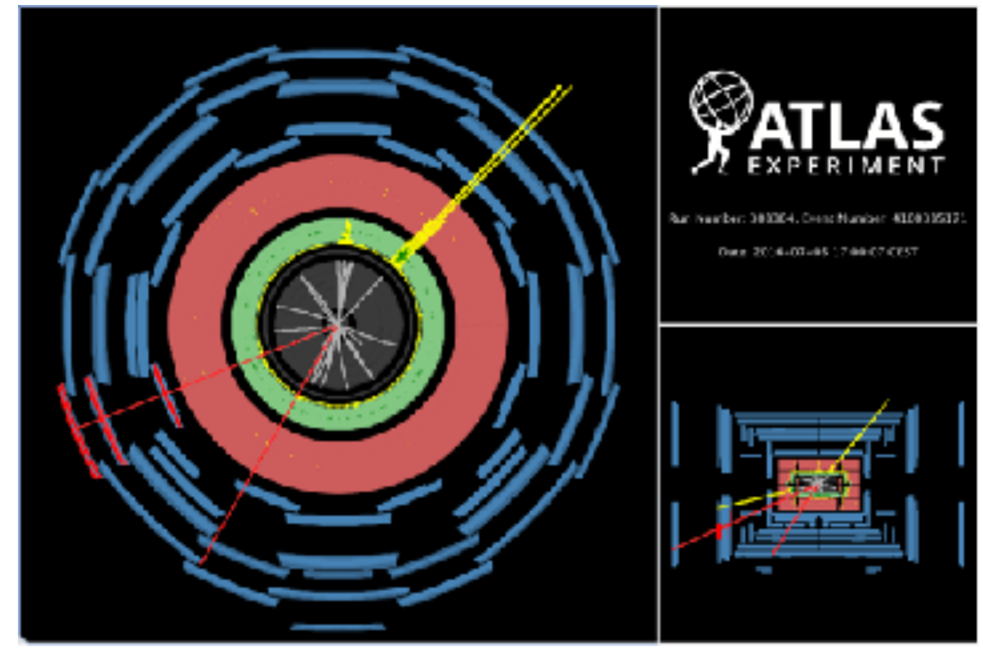
Christian Bauer
Quantum Computing for Colliders

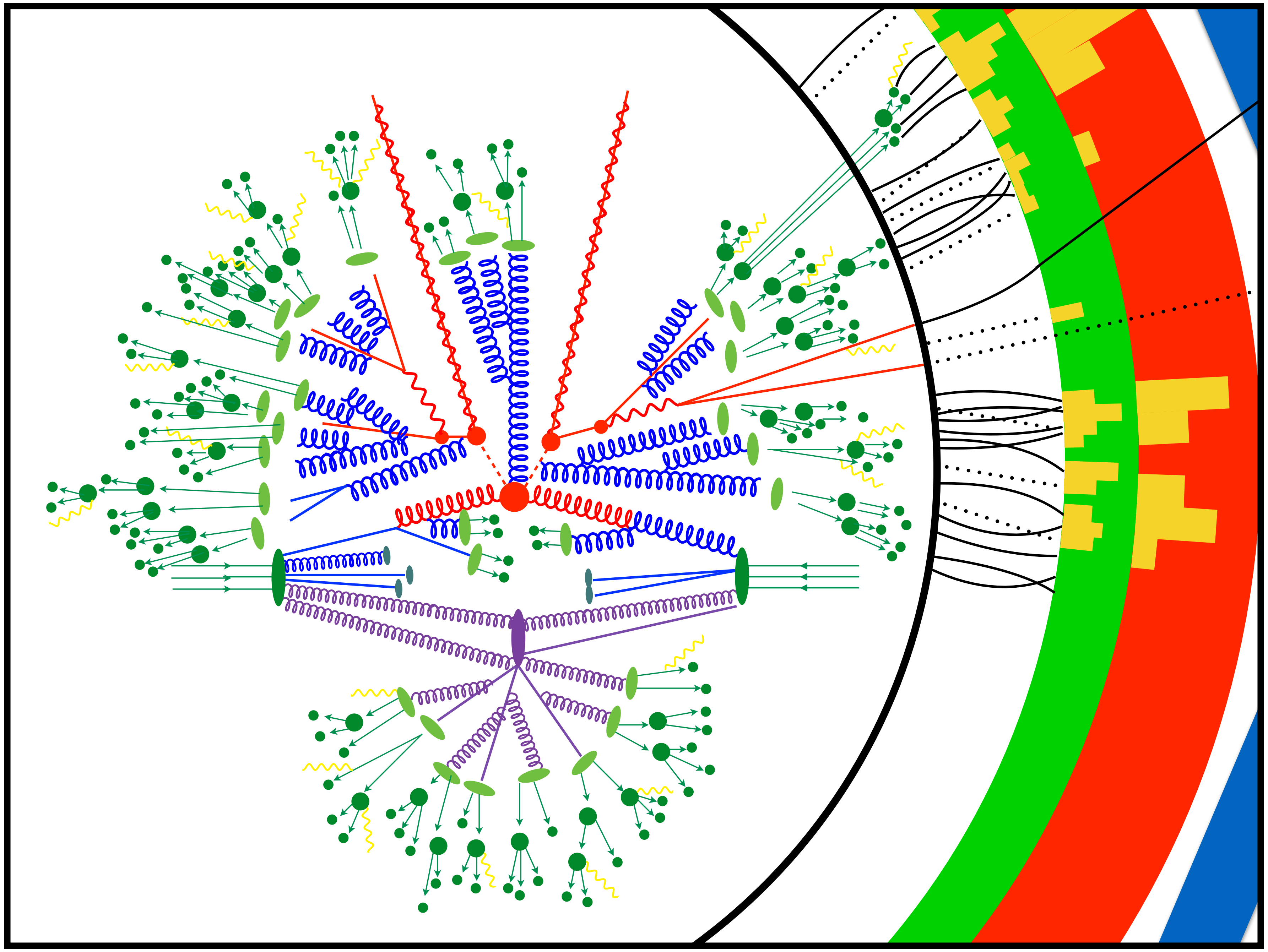
To truly understand if Standard Model describes data observed at LHC, need to connect theory and data





For this, need to be able to go from Lagrangian to fully exclusive events

One of the holy grails of HEP is the full simulation of scattering processes at colliders



One of the holy grails of HEP is the full simulation of scattering processes at colliders

Dream would be to literally compute the full S-matrix

Perform measurement of final state at time T

Create initial state with 2 protons at time -T

$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

Perform time evolution with full SM Hamiltonian from initial time -T to final time T

One of the holy grails of HEP is the full simulation of scattering processes at colliders

Perform measurement of final state at time T

Create initial state with 2 protons at time -T

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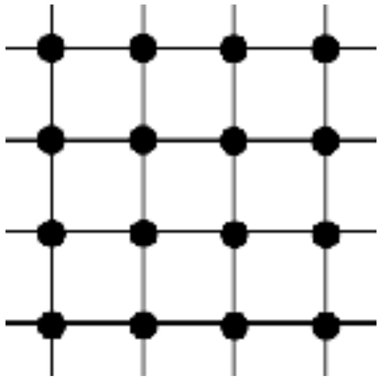
Perform time evolution with full SM Hamiltonian from initial time -T to final time T

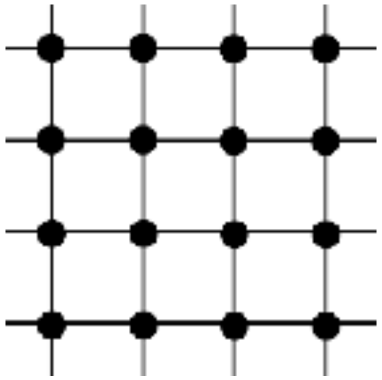
1. This clearly requires Quantum Physics (Quantum Field Theory)
2. This is something that is not even remotely feasible using classical computers
3. Would revolutionize how we can compare experimental collider measurements with theoretical predictions



Calculating an S-Matrix
on a Lattice

Separating high and
low scales

Computations on a
Quantum computer



Calculating an S-Matrix on a Lattice

One can turn the QFT calculation into a QM calculation by discretization / digitization

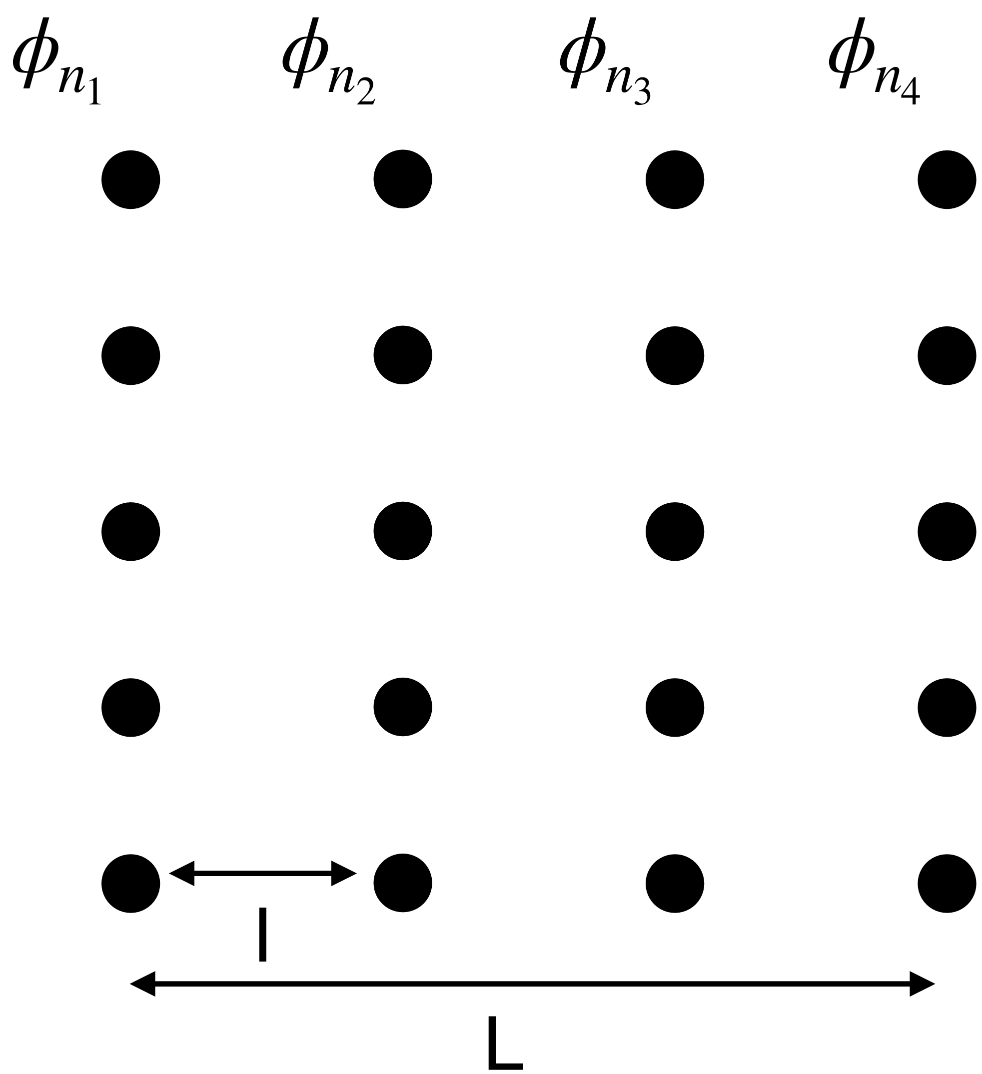
$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

All elements in this expression in terms of fields $\phi(x)$
Both position x and field $\phi(x)$ are continuous

Discretizing position x and digitizing field value $\phi(x)$ turn continuous (QFT) problem into discrete (QM) problem

Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

Instead of having a continuous field ϕ at each position x, we put a digitized field ϕ_n at discrete points x_k arranged on a lattice



$$L = N\, l$$

Hilbert space has dimension

$$\left(n_\phi\right)^{N^d}$$

n_ϕ : # of digitized field values
N : # of lattice points per dim
d : # of dimensions

Problem reduced to matrix multiplication

Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

$$\left| \langle X(T) \,|\, U(T, -T) \,|\, pp(-T) \rangle \right|^2$$

3 basic steps:

1. Create an initial state vector at time (-T) of two proton wave packets
2. Evolve this state forward in time from to time T using the Hamiltonian of the full interacting field theory
3. Perform a measurement of the state

Let's try to estimate the resources we need to simulate physics at the LHC

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

To simulate full energy range of LHC need

$$100\,\mathrm{MeV} \lesssim E \lesssim 7\,\mathrm{TeV}$$

This needs $\mathcal{O}(70{,}000^3) \sim 10^{14}$ lattice sites

Assume I need at least 5 bit digitization $\Rightarrow n_\phi = 2^5 = 32$

Dimension of Hilbert space is

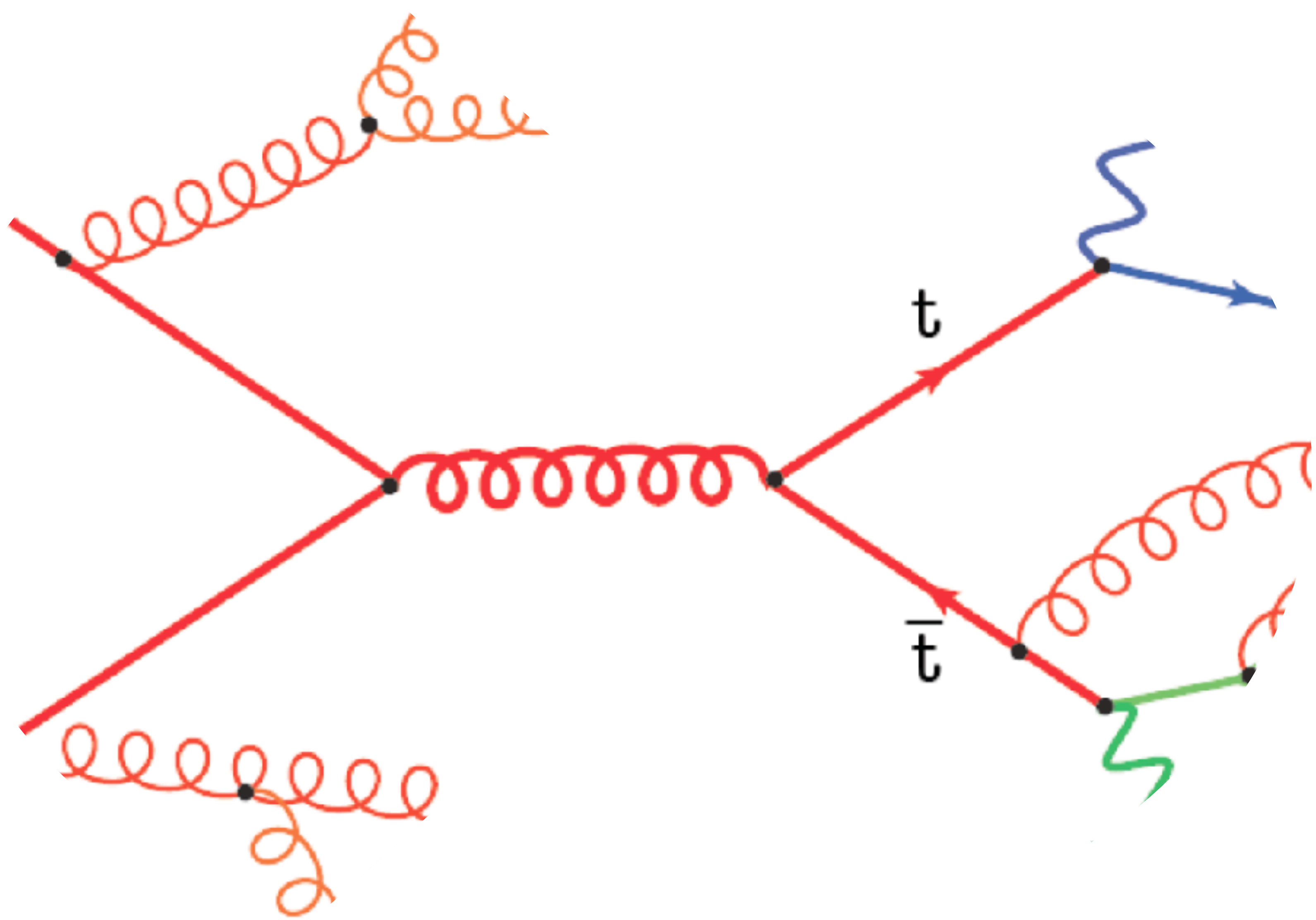
$$32^{10^{14}} \sim \infty$$

Clearly completely impossible to perform such a calculation

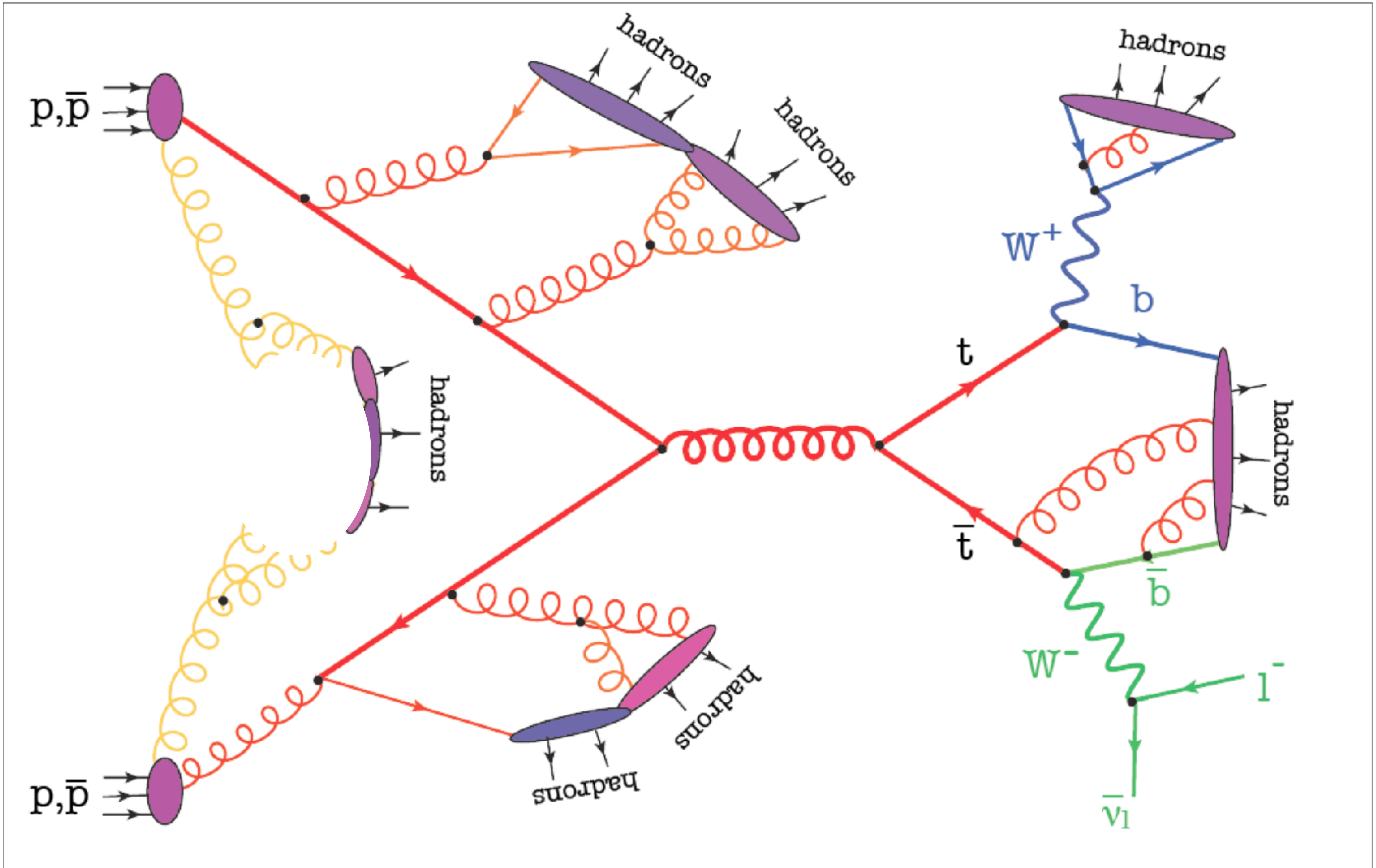
Separating high and low scales

Typical event at LHC involves very different energy scales:
High energy / short distance: Perturbation Theory

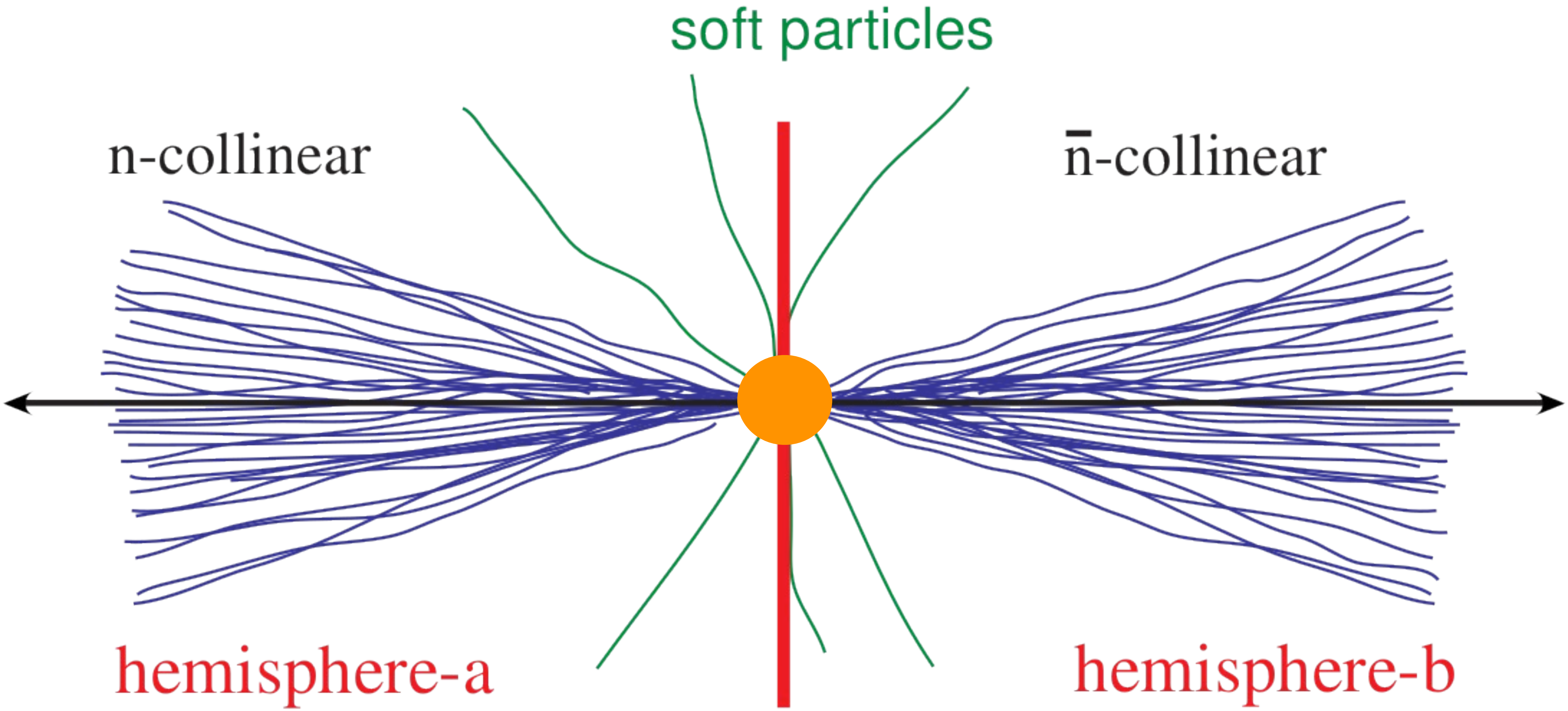
Typical event at LHC involves very different energy scales: Medium energy / medium distance: Parton shower



Typical event at LHC involves very different energy scales: Low energy / long distance: soft radiation / hadronization



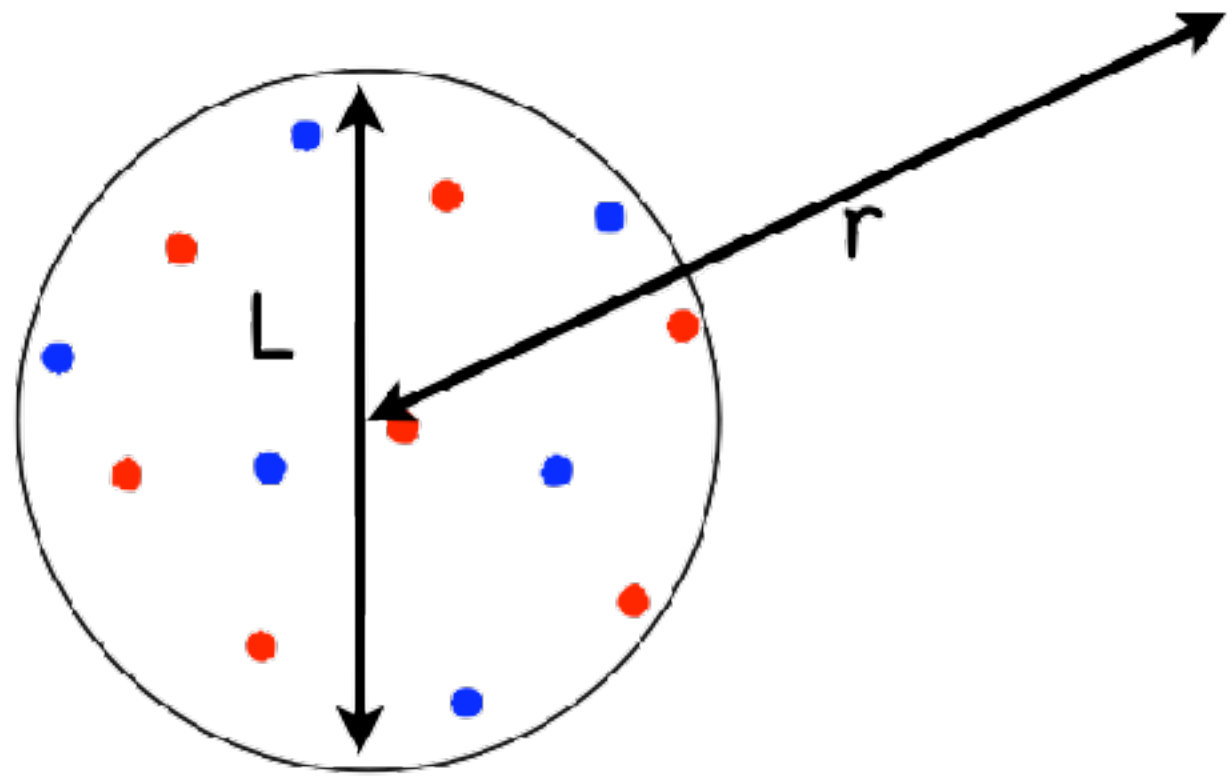
Can separate physics into three main categories: Hard, Collinear, Soft



Hard: Q
Collinear: m_J
Soft: m_J^2/Q

$$m_J^2/Q \ll m_J \ll Q$$

It is well known that scale separation simplifies problems significantly



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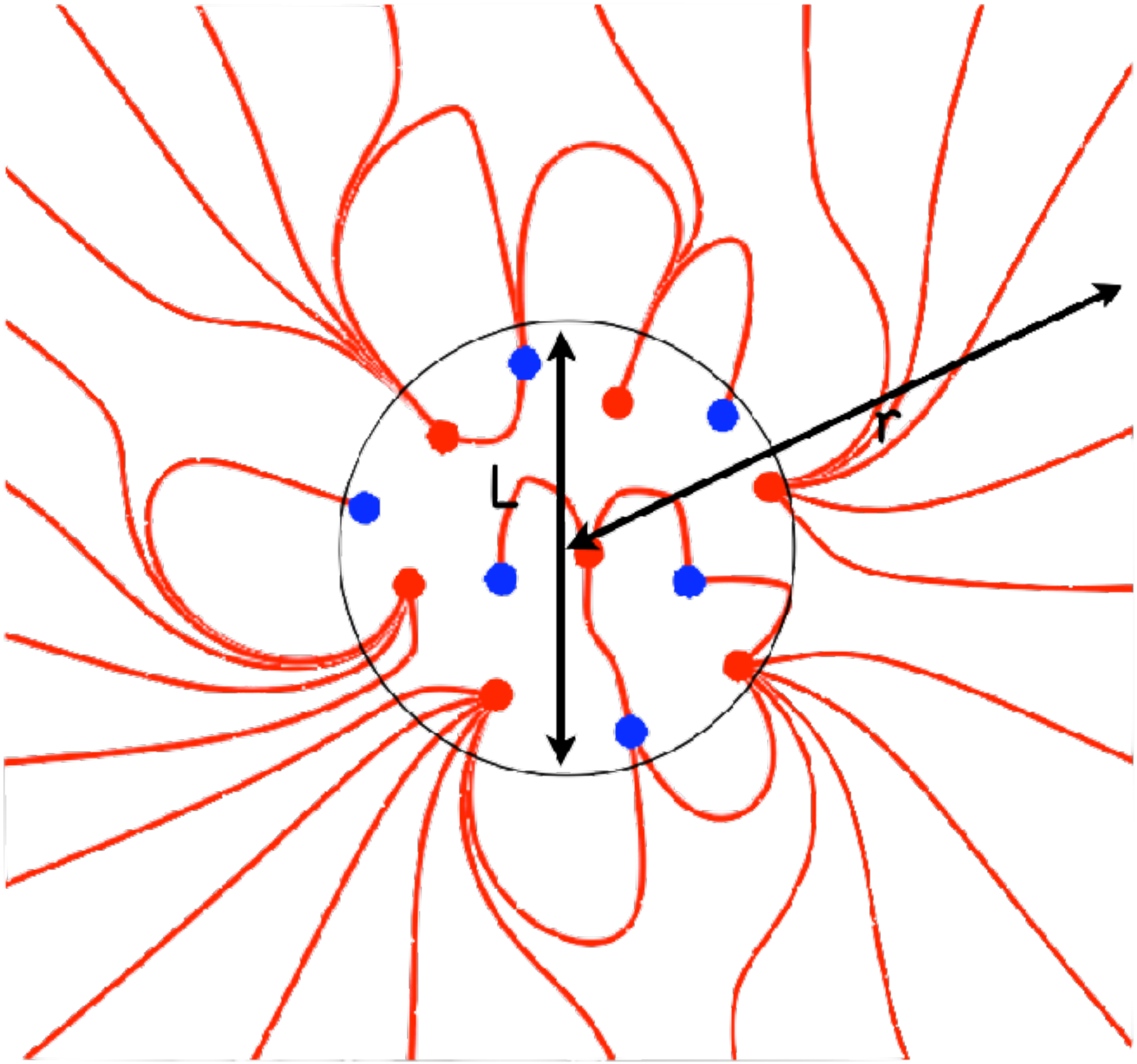

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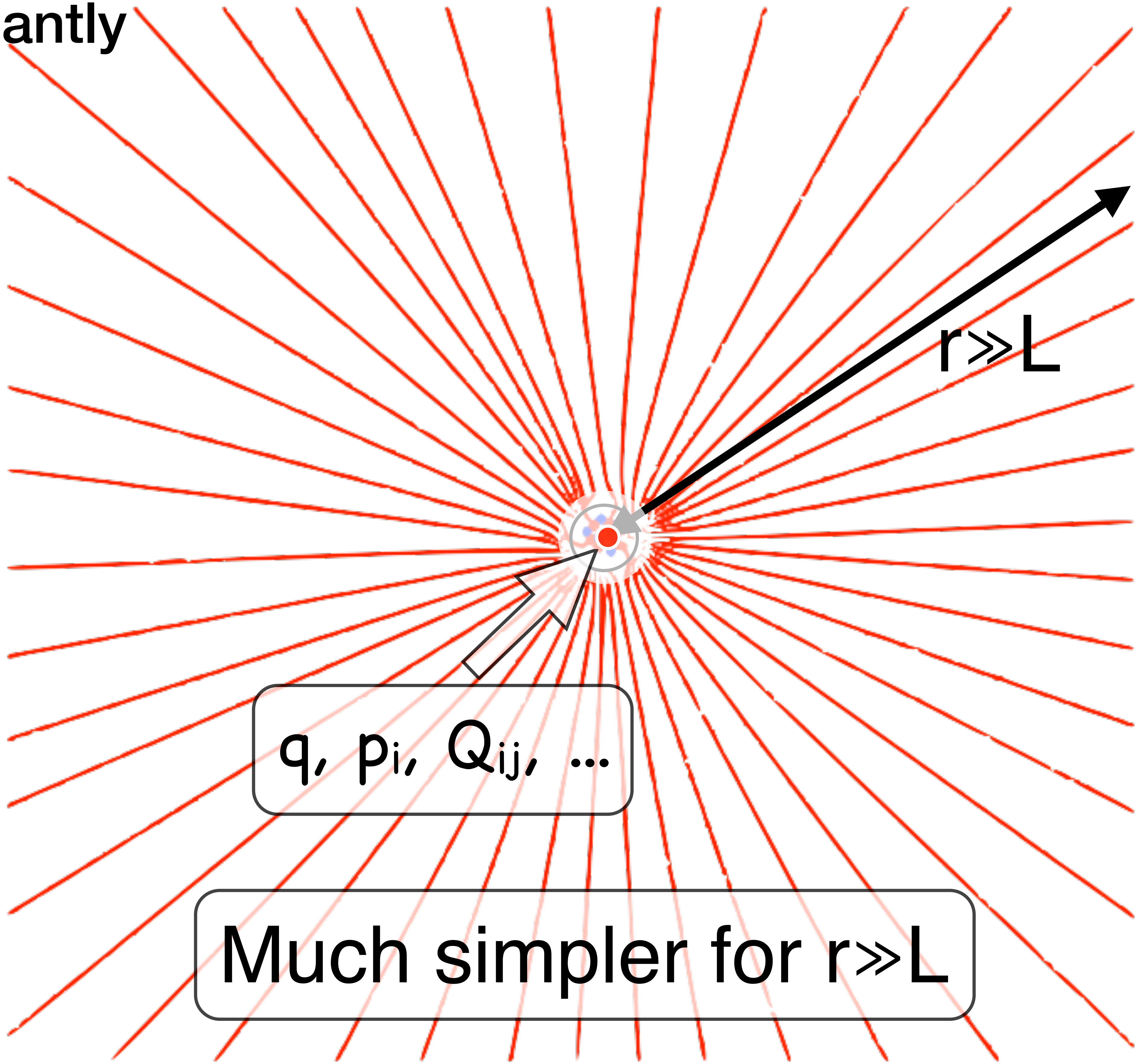


It is well known that scale separation simplifies problems significantly

Potential expanded as

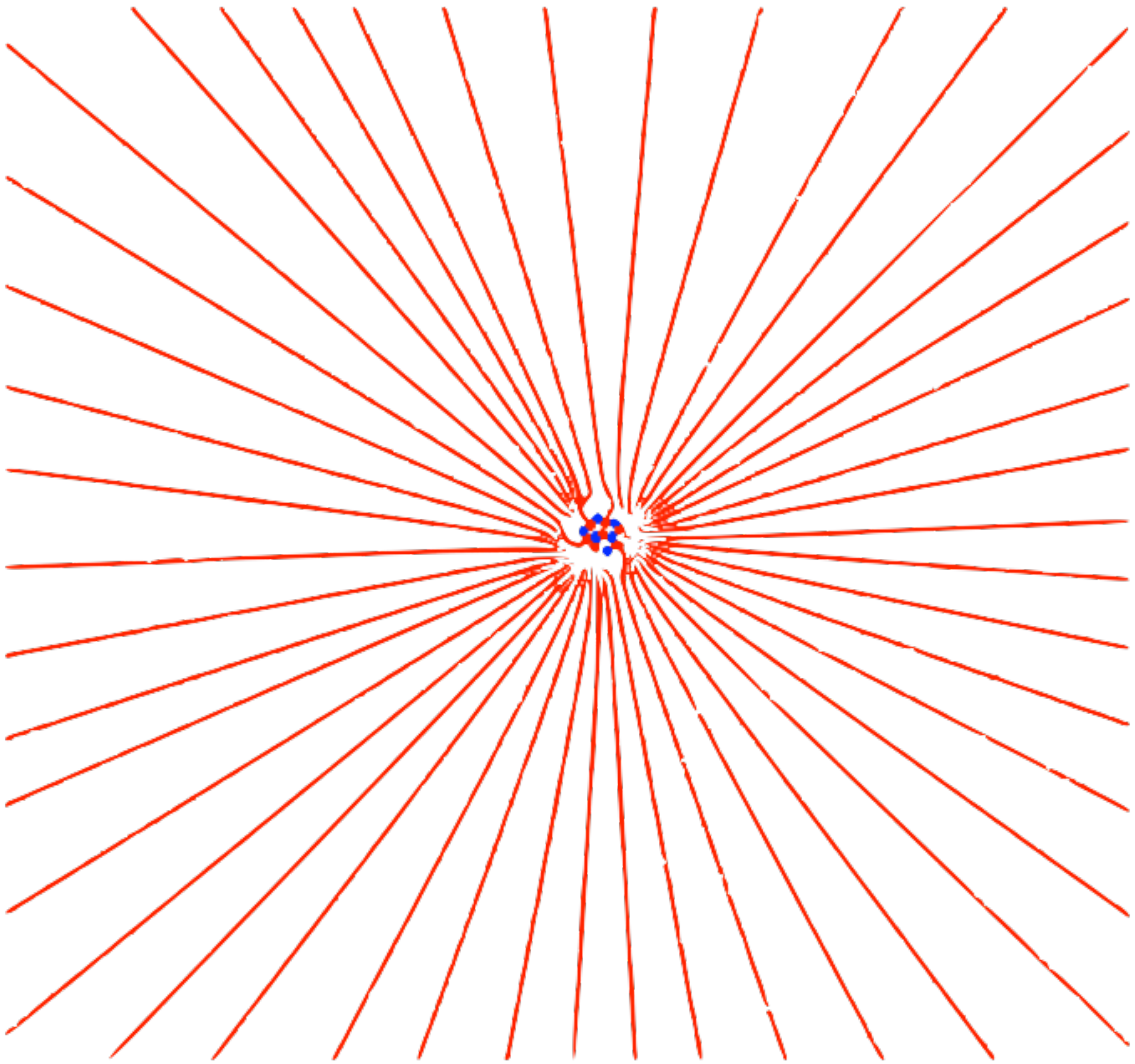
$$V(r) = \frac{q}{r} + \frac{\vec{p} \cdot \vec{x}}{r^3} + \frac{Q_{ij} x_i x_j}{2r^5}$$

$q\,,\quad \vec{p}\,,\quad Q_{ij}$

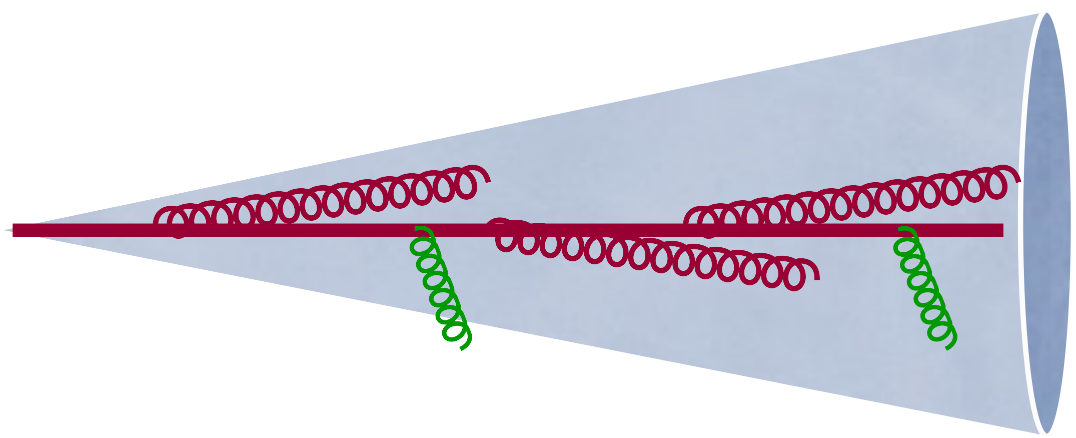
Short distance physics

$\left\langle \frac{1}{r} \right\rangle, \quad \left\langle \frac{\vec{x}}{r^3} \right\rangle, \quad \left\langle \frac{x_i x_j}{r^5} \right\rangle$

Long distance physics



Simple separation of long and short distances



Energetic particles	In same direction as jet	Collinear
Non-energetic particles	Can be anywhere	Soft

Soft-Collinear Effective Theory

SCET

CWB, Fleming, Luke ('00)
CWB, Fleming, Pirjol, Stewart ('00)

Soft-Collinear Effective Theory
SCET

CWB, Fleming, Luke ('00)
CWB, Fleming, Pirjol, Stewart ('00)

For two jets, have two collinear directions

Type	(p ,p ,p)	Fields
collinear 1	$(\lambda^2, 1, \lambda)$	χ_{n1}, A_{n1}
collinear 2	$(1, \lambda^2, \lambda)$	χ_{n2}, A_{n2}
soft	$(\lambda^2, \lambda^2, \lambda^2)$	q_s, A_s

Soft-Collinear Effective Theory
SCET

CWB, Fleming, Luke ('00)
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Formal understanding of QCD	Proofs of factorization	Jet substructure	Event generation
Fixed order calculations	Jet quenching in heavy Ion collisions	Flavor physics	Parton distribution functions
Resummed calculations	Non-global logarithms	Quarkonia physics	Parton showers

Soft-Collinear Effective Theory

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Effective theories allow to separate short and long distance physics from one another

Goal is to separate ingredients that are calculable in perturbation theory from those that really benefit from non-perturbative techniques

Effective Field Theories (SCET)

$$d\sigma = H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

Most interesting object in above equation is the soft function S, which as discussed lives at the lowest energies

For 1TeV jets with 100GeV mass, find

$$\Lambda_S = (100\,\text{GeV})^2/(1000\,\text{GeV}) = 10\,\text{GeV}$$

Other ideas to compute part of a full scattering process have been put forth in slightly different contexts

- Implement parton shower evolution on quantum devices
  - Include classically intractable quantum interference effects

CWB, deJong, Nachman, Provasoli ('18)

- Compute light-front matrix elements (parton distributions) on quantum devices
  - Compute PDFs from first principle

Echevarria, Egusquiza, Rico, Schnell ('21)

Let's try to estimate the resources we need to simulate physics at the LHC

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

As I will argue later, can use effective field theories to limit required range to

$$100\,\mathrm{MeV} \lesssim E \lesssim 10\,\mathrm{GeV}$$

This needs $\mathcal{O}(100^3) \sim 10^6$ lattice sites

Dimension of Hilbert space is

$$32^{10^6} \sim \infty$$

While $32^{10^6} \ll 32^{10^{14}}$,

still completely impossible to perform such a calculation

Computations on a Quantum computer

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1*] Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

Science 336 (2012) 1130

The resources on a quantum computer are much smaller, but still very large

From the discussion before, size of Hilbert space to simulate full LHC given by

$$\dim(H) \sim 32^{10^{14}}$$

This Hilbert space can be encoded in

$$n_Q = \ln_2 \left[\dim(H)\right] \sim 5 \times 10^{14}$$

While this is much, much smaller, still inconceivable to have a system of this size in any of our lifetimes

Crucial thing to realize is that we don't need quantum computer for most of this physics

First, for most observables not interested in the most general high energy process (typically care about events with relatively small number of jets)

Second, perturbation theory works very well for high energy processes with limited number of final state particles

Should use Quantum Computers only for those calculations that are not possible using known techniques

Combine quantum computing with EFTs

Soft function is the expectation value of a "Wilson line" operator between initial and final state

Soft function can be written as

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

$$Y = \mathrm{P}\exp\left[ig \int_0^\infty \mathrm{d}s\, \phi(ns) \right] \qquad ns = (s,0,0,s)$$

Since soft function has much lower characteristic scale, can potentially compute "easily" on quantum device

From the discussion before, size of Hilbert space to simulate soft function

$$\dim(H) \sim 32^{10^6}$$

This Hilbert space can be encoded in

$$n_Q = \ln_2 \left[\dim(H)\right] \sim 5 \times 10^6$$

It seems possible to perform such a calculation on a quantum device in a realistic time scale

A Wilson line is a relatively simple object on a lattice

Wilson line can be easily discretized on the lattice

$$Y_n = \mathrm{P}\exp\left[ig\,\delta x \sum_{i=n_0}^{2n_0} \phi_{x_i}(t = x_i - n_0)\right]$$

$$Y_{\bar{n}}^\dagger = \mathrm{P}\exp\left[-ig\,\delta x \sum_{i=0}^{n_0} \phi_{x_i}(t = n_0 - x_i)\right]$$

Use time evolution to change the time at each lattice point

$$\mathrm{T}[Y_n\, Y_{\bar{n}}^\dagger] = e^{-iH\, n_0 \delta x} \exp\left[ig\,\delta x\left(\phi_{x_{2n_0}} - \phi_{x_0}\right)\right] \times e^{iH\delta x} \exp\left[ig\,\delta x\left(\phi_{x_{2n_0-1}} - \phi_{x_1}\right)\right] \times \cdots \times e^{iH\delta x} \exp\left[ig\,\delta x\left(\phi_{x_{n_0}} - \phi_{x_{n_0}}\right)\right].$$

Alternate between exponential of field operator and Hamiltonian evolution

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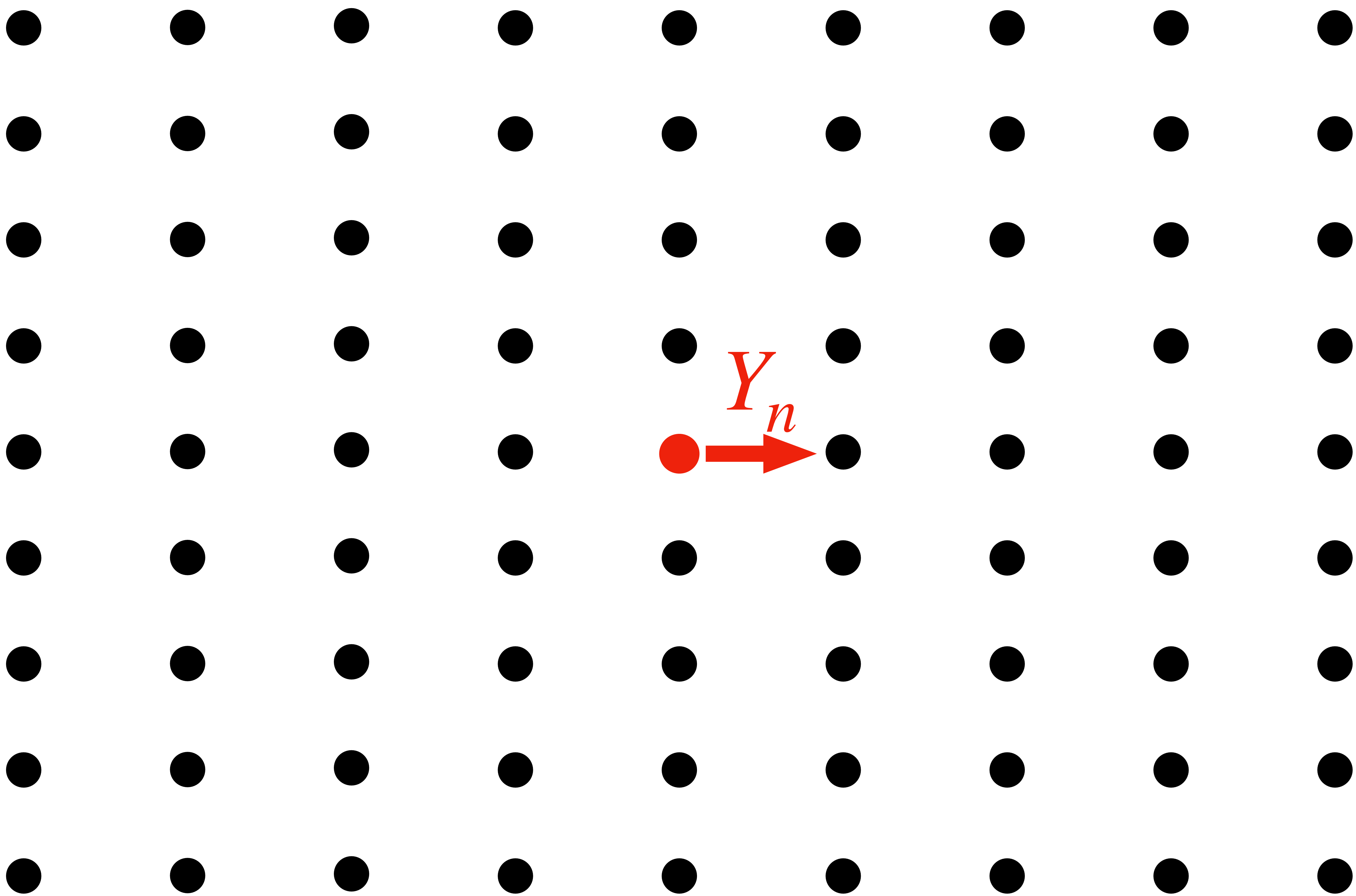

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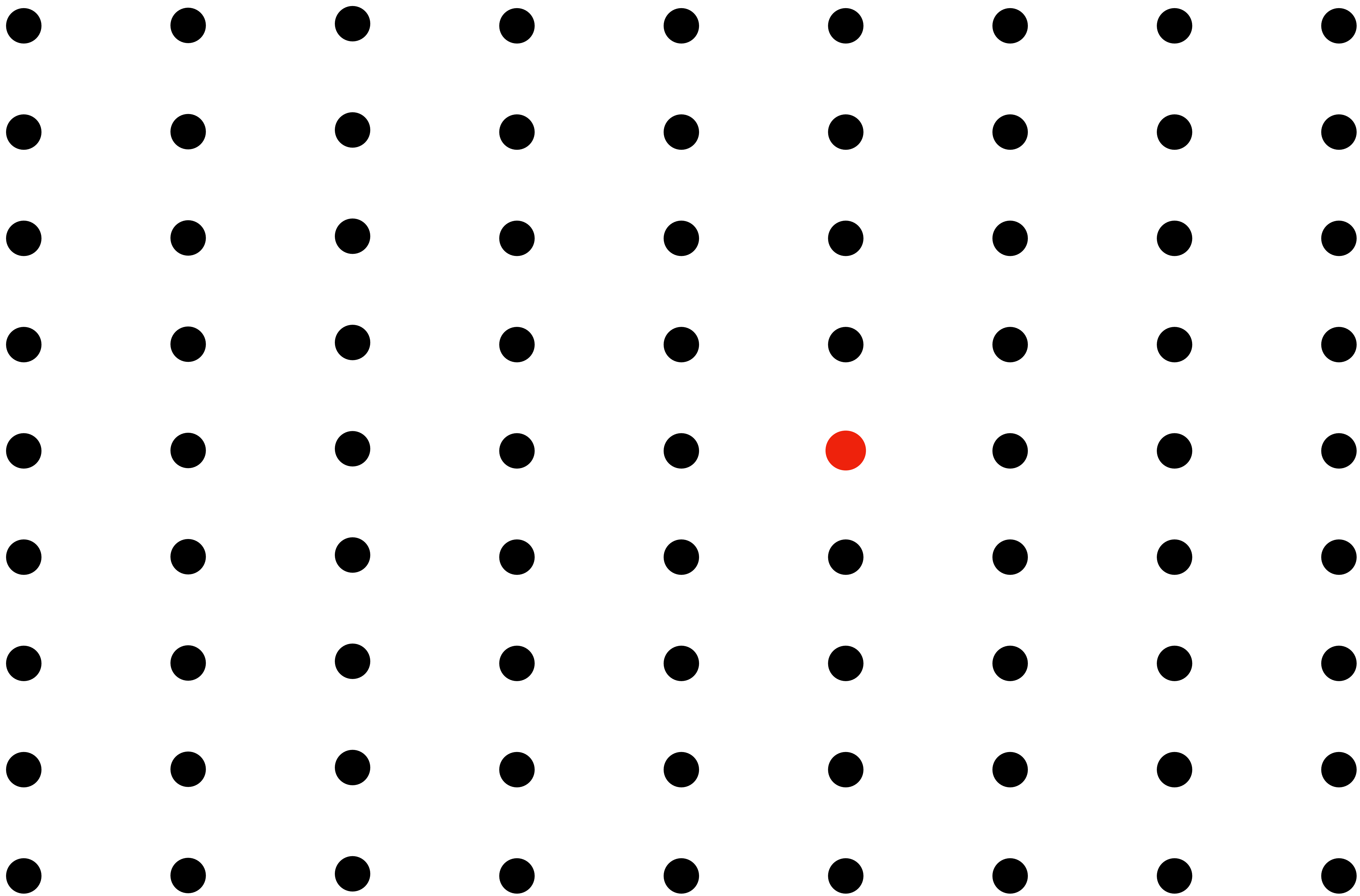

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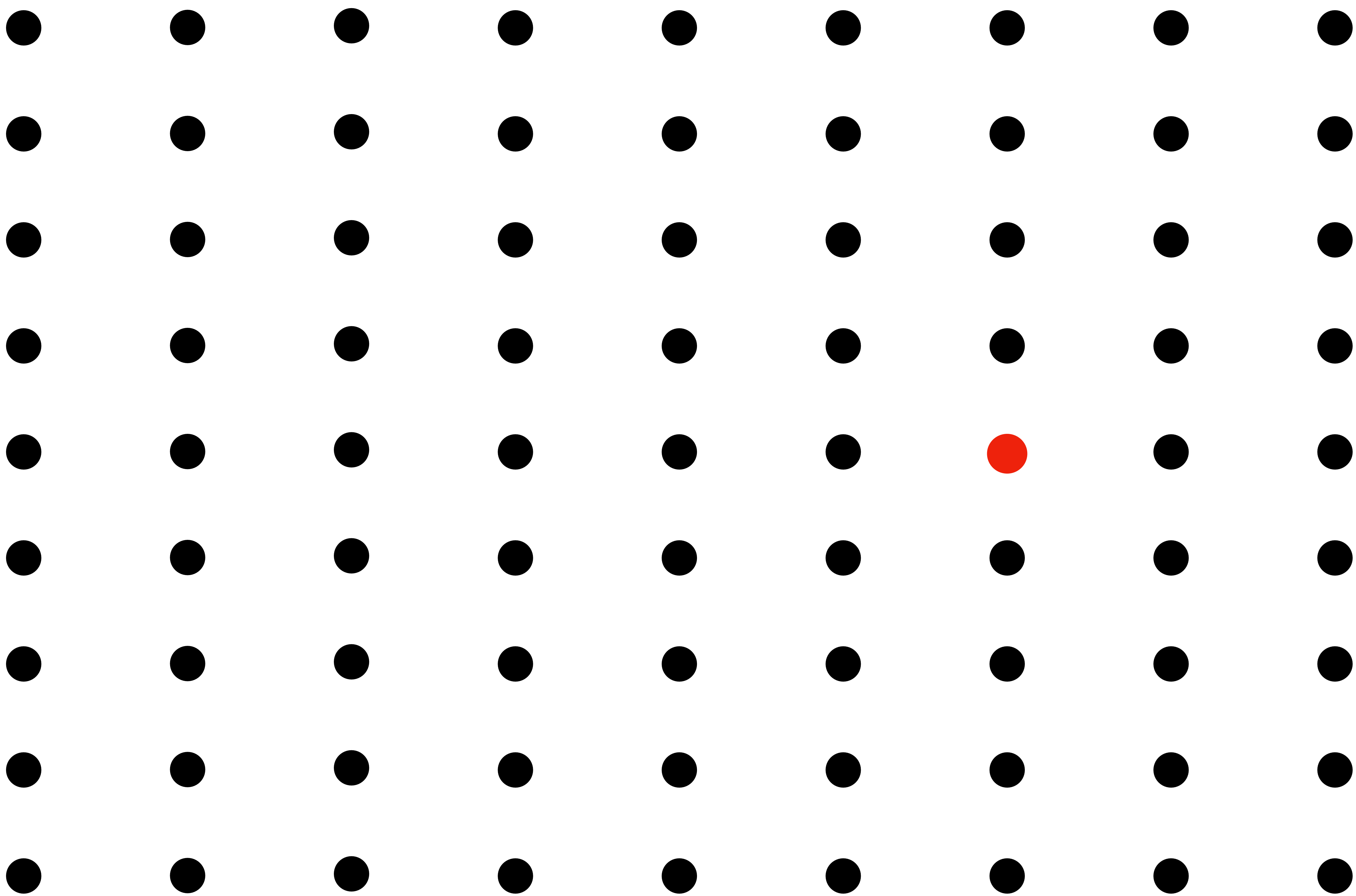

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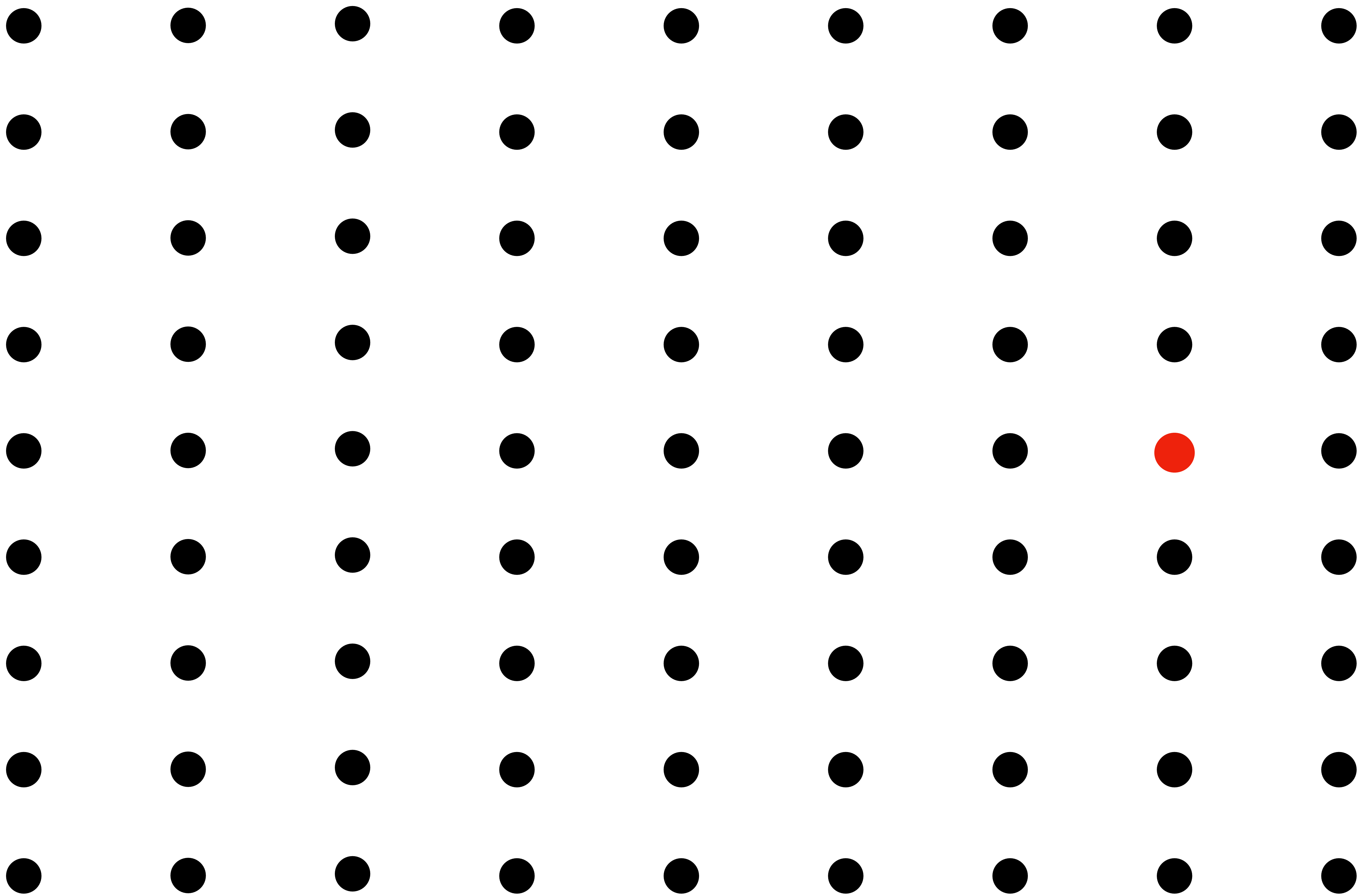

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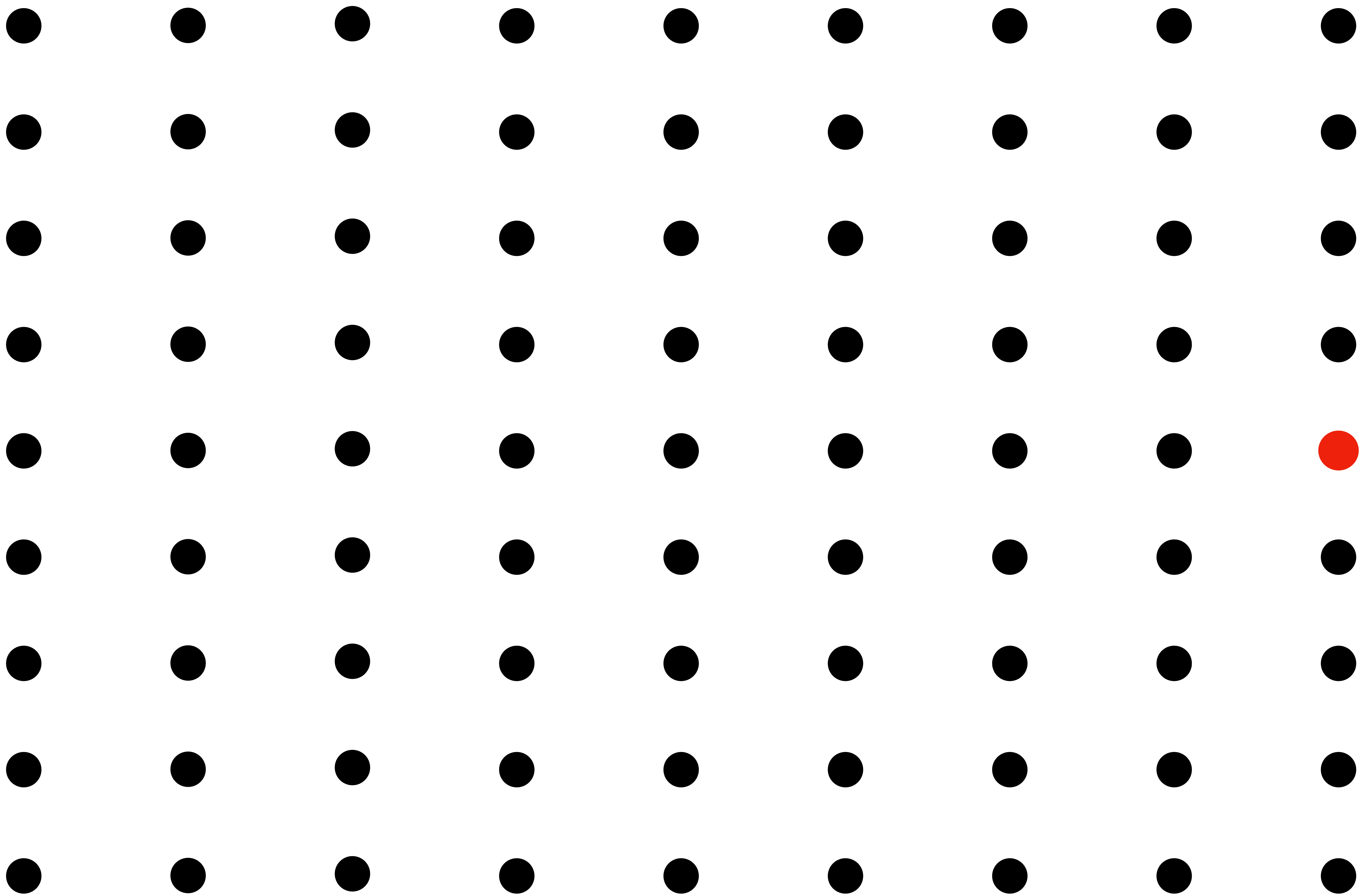

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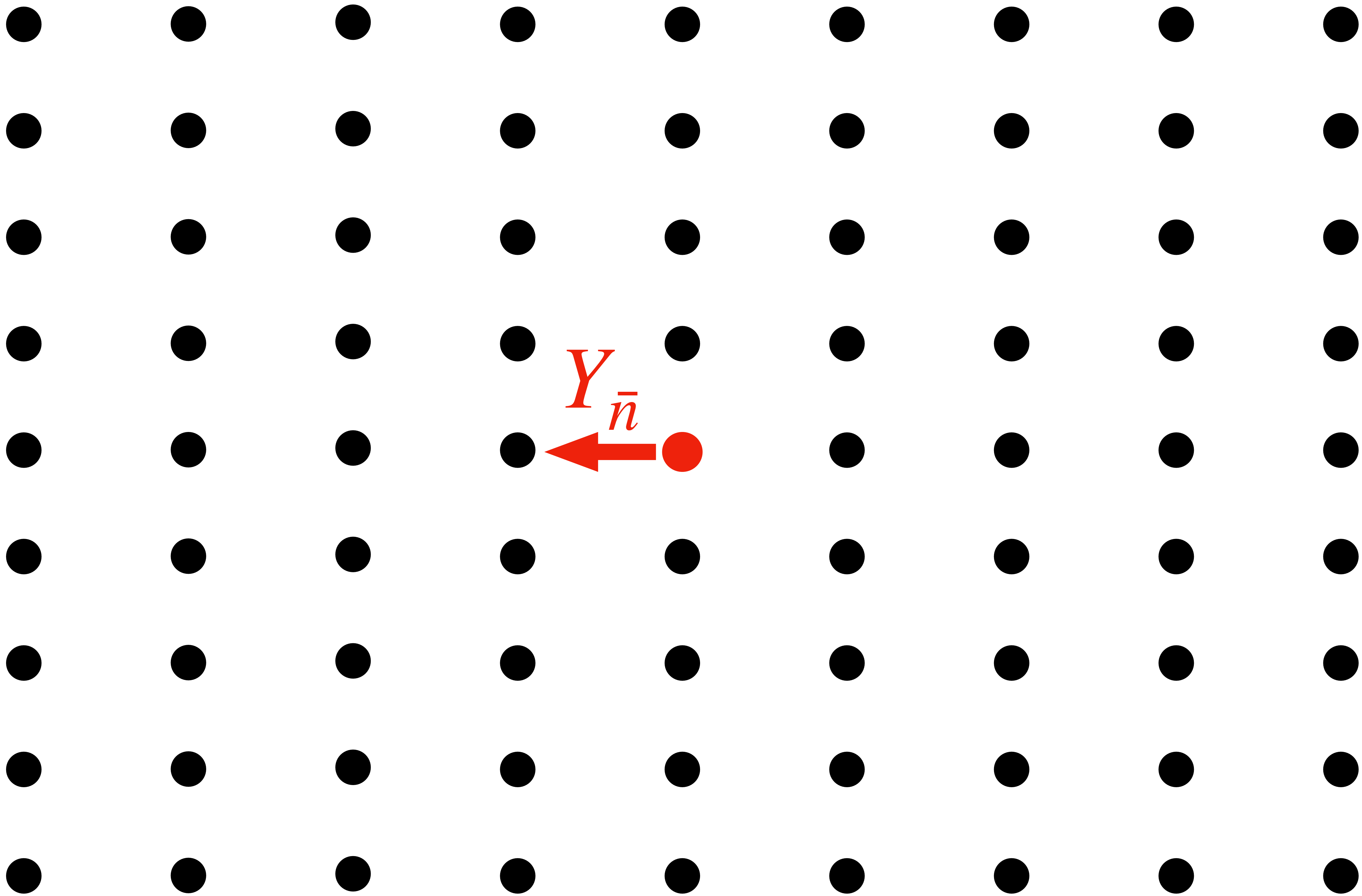

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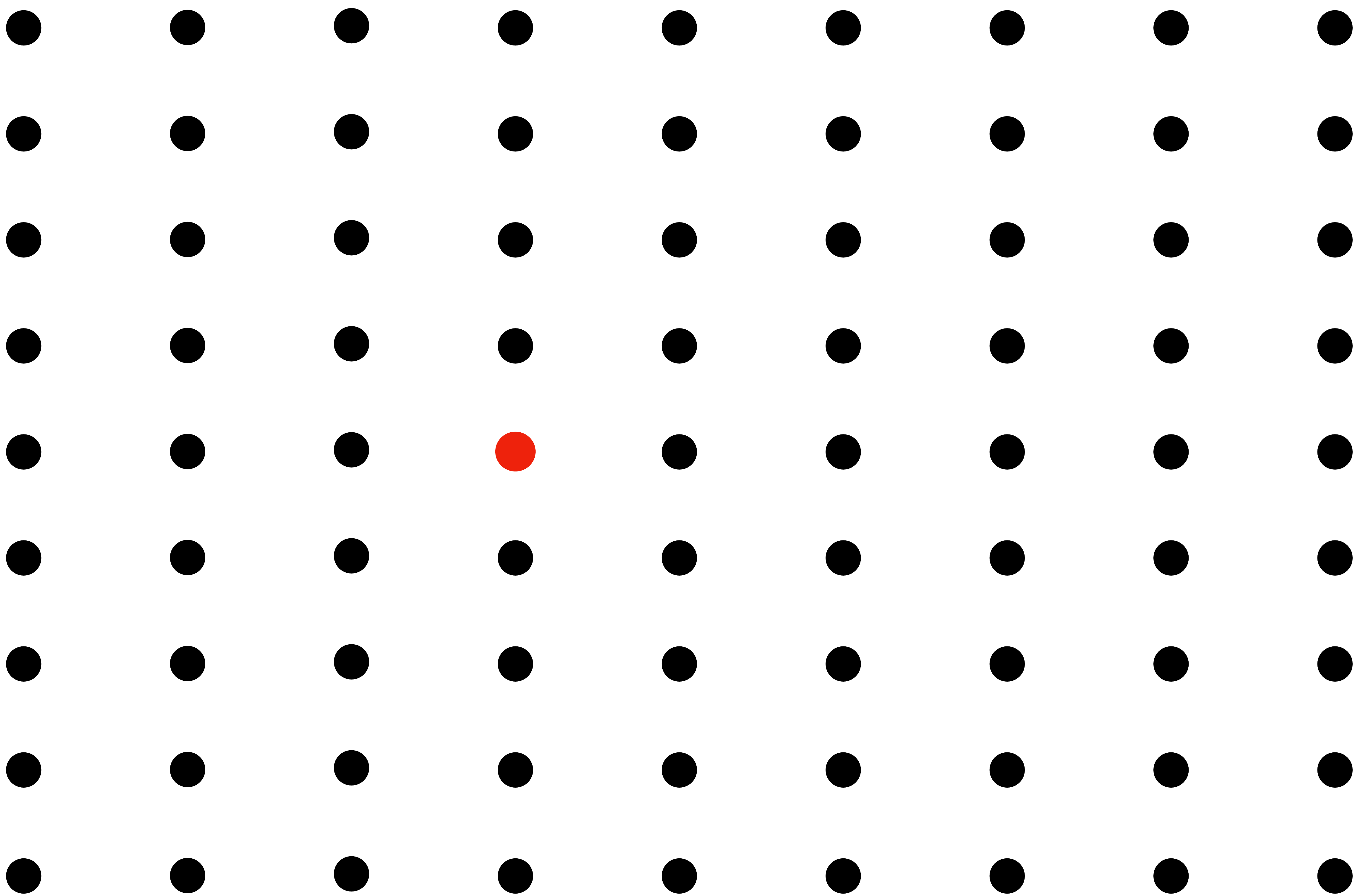

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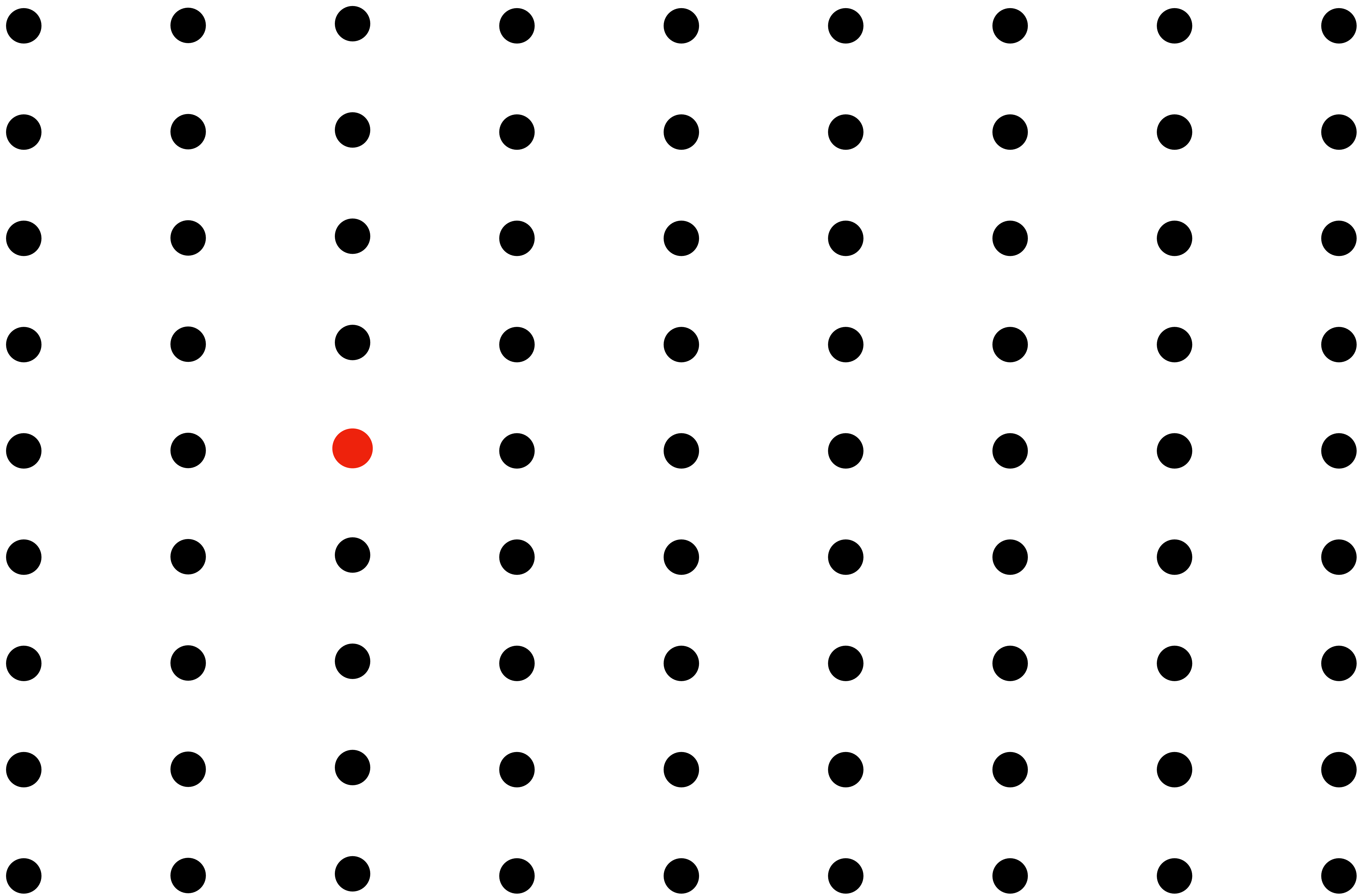

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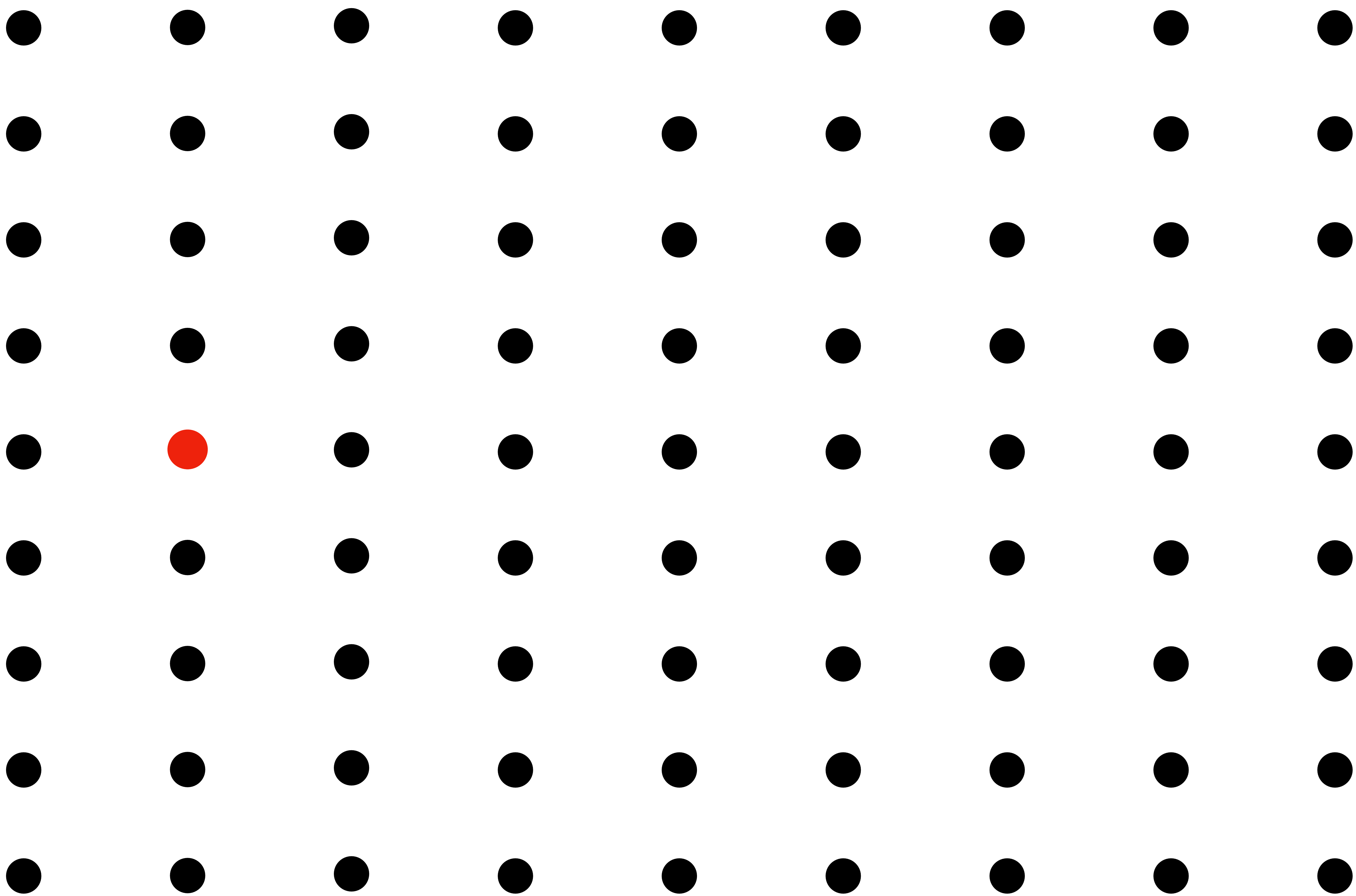

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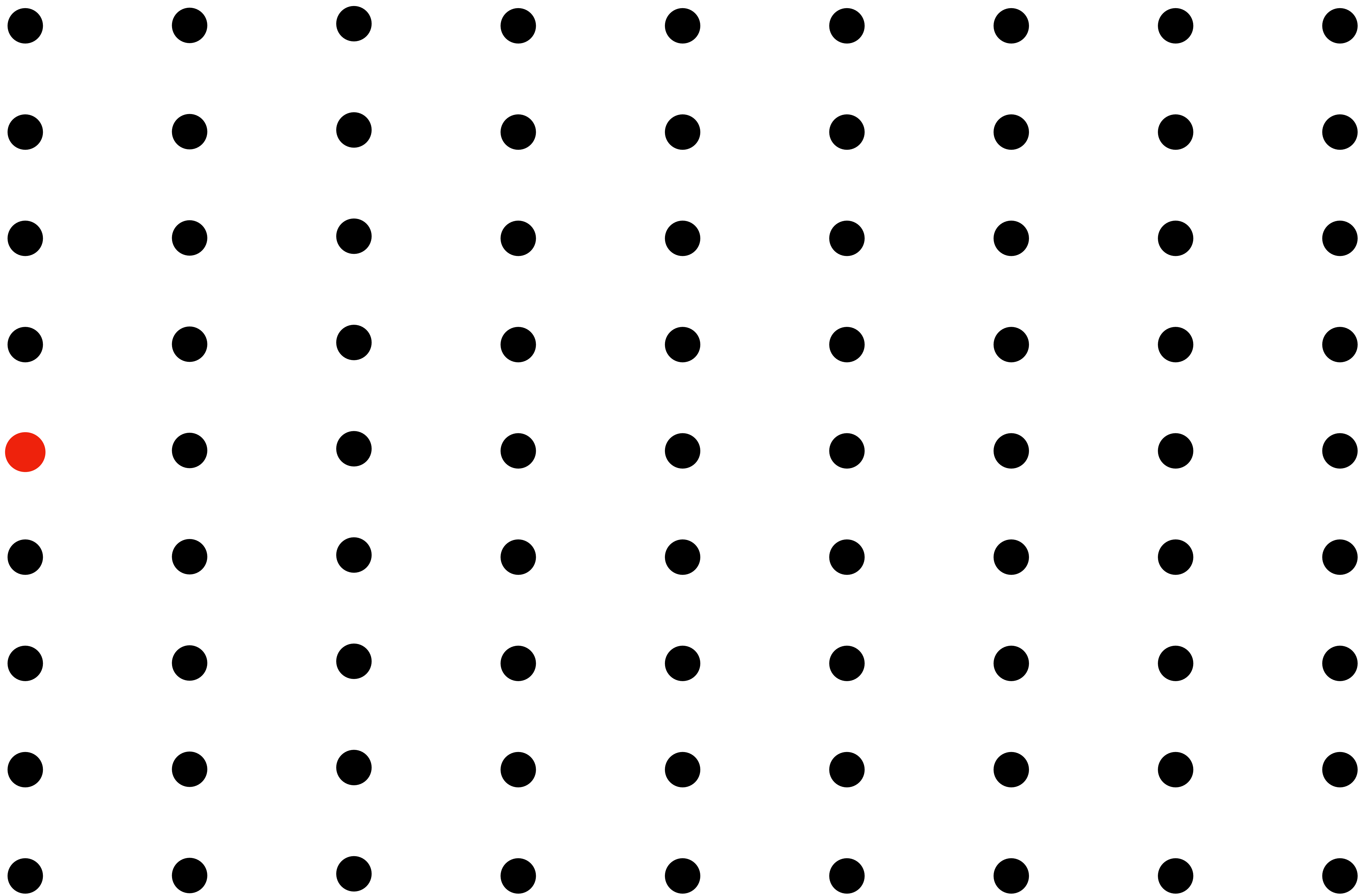

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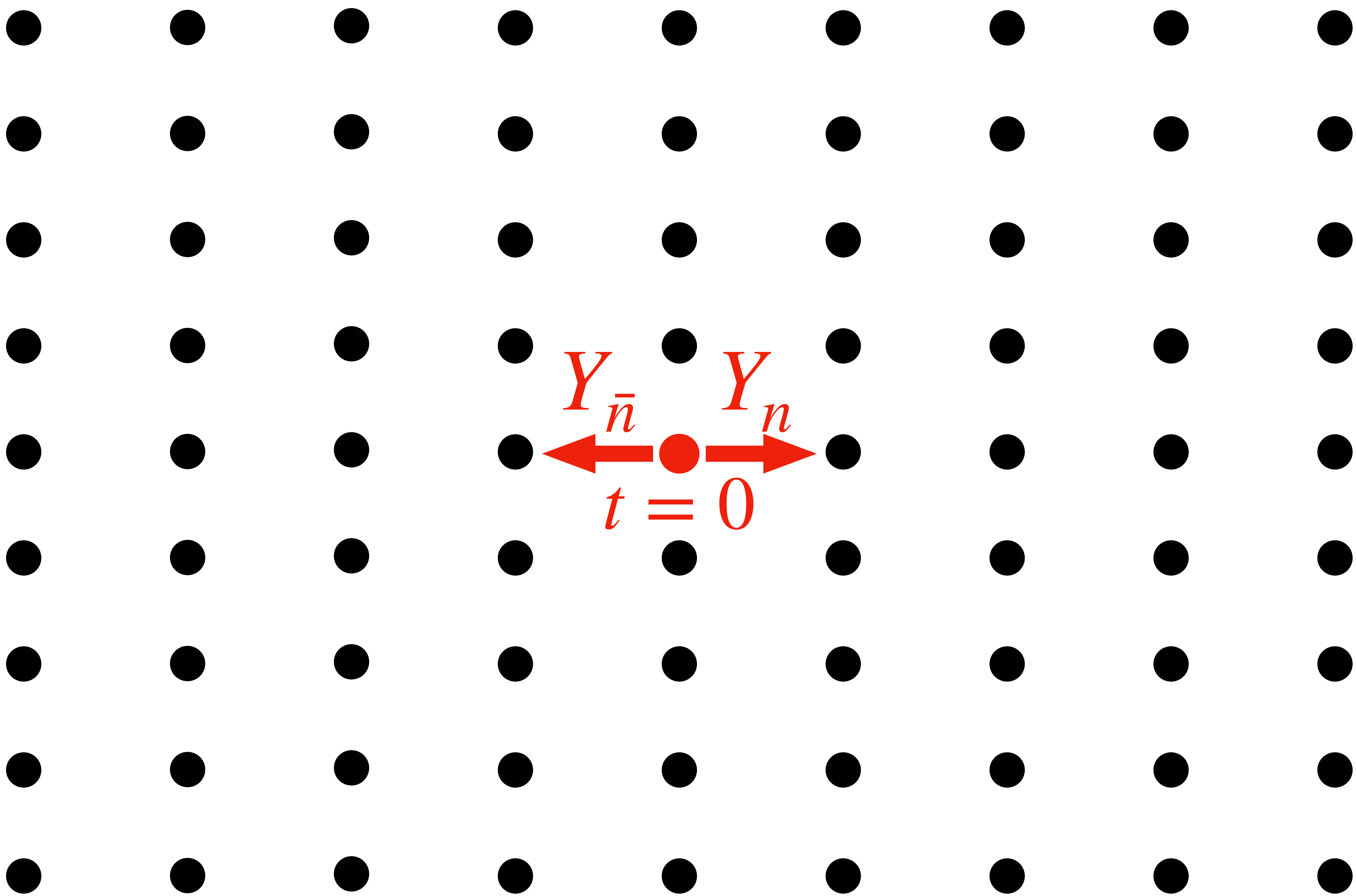

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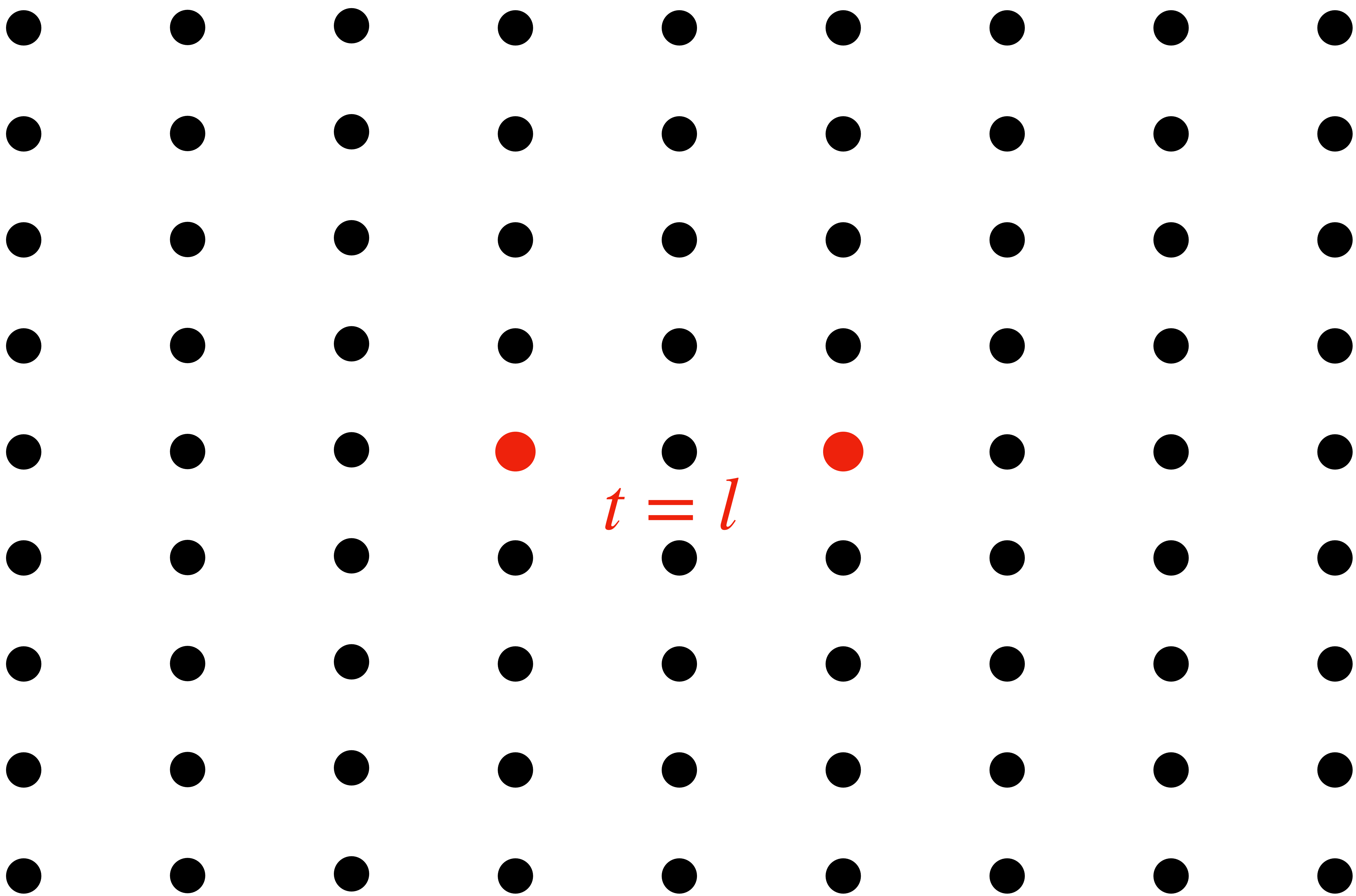

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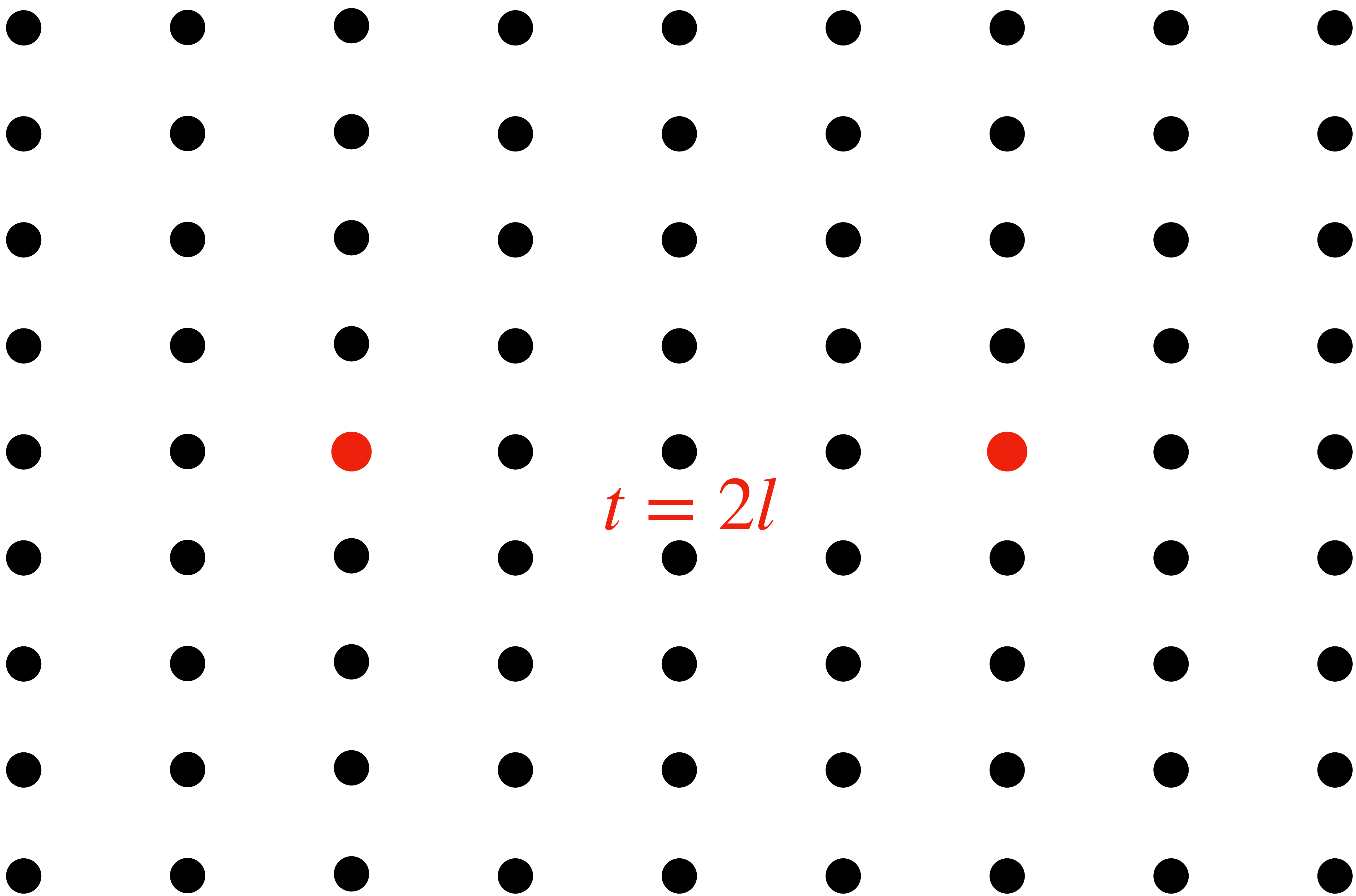

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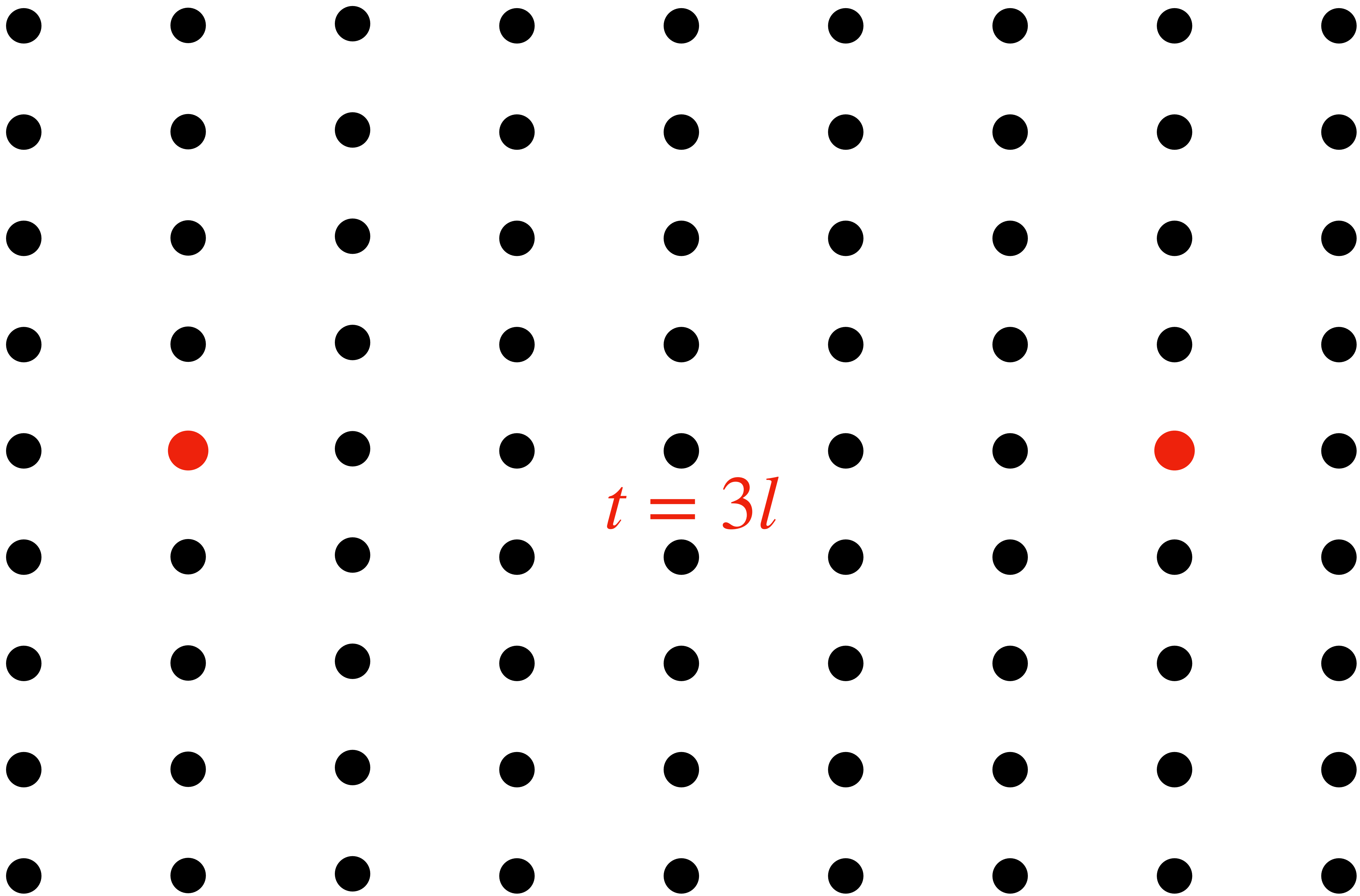

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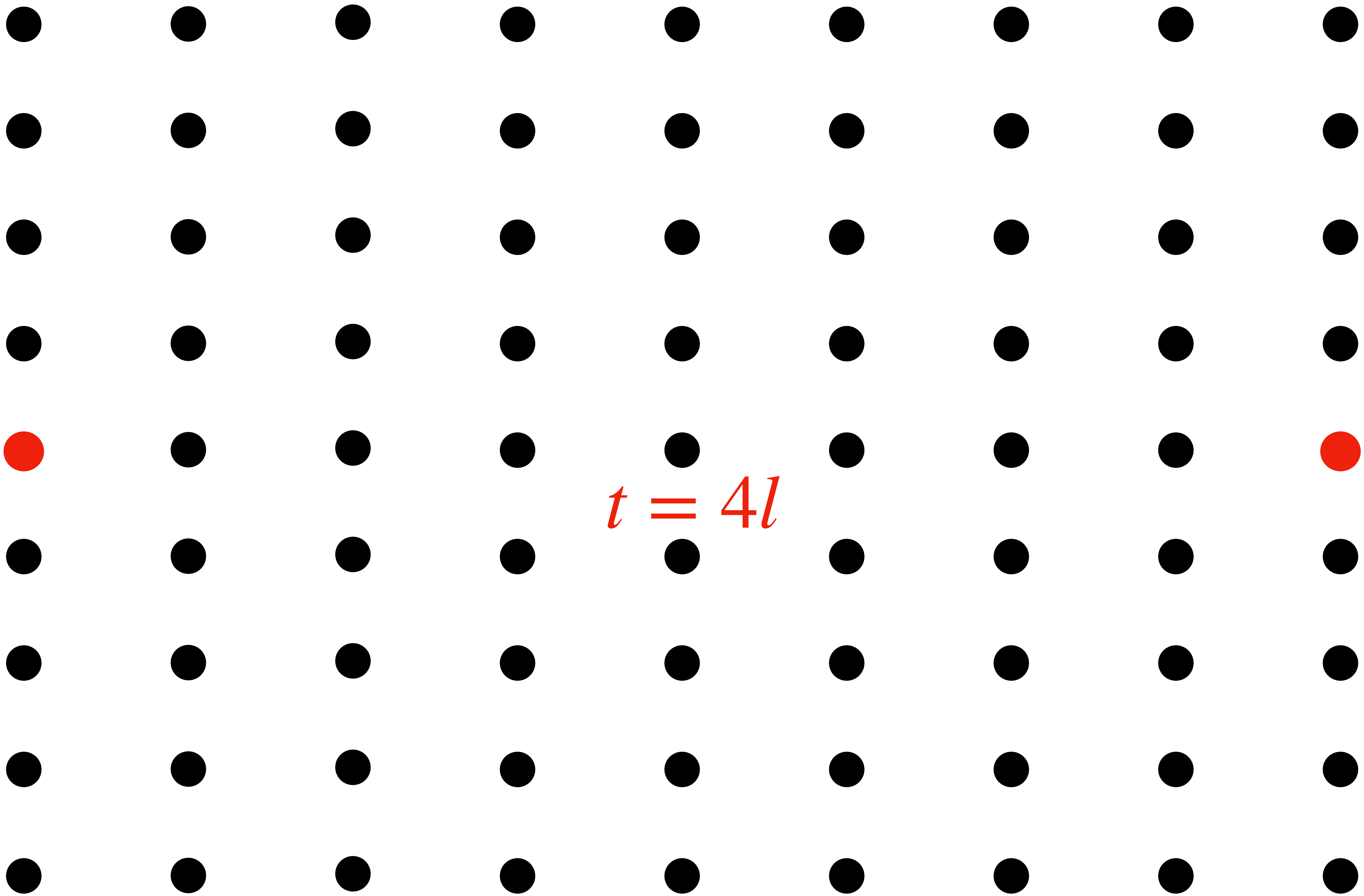
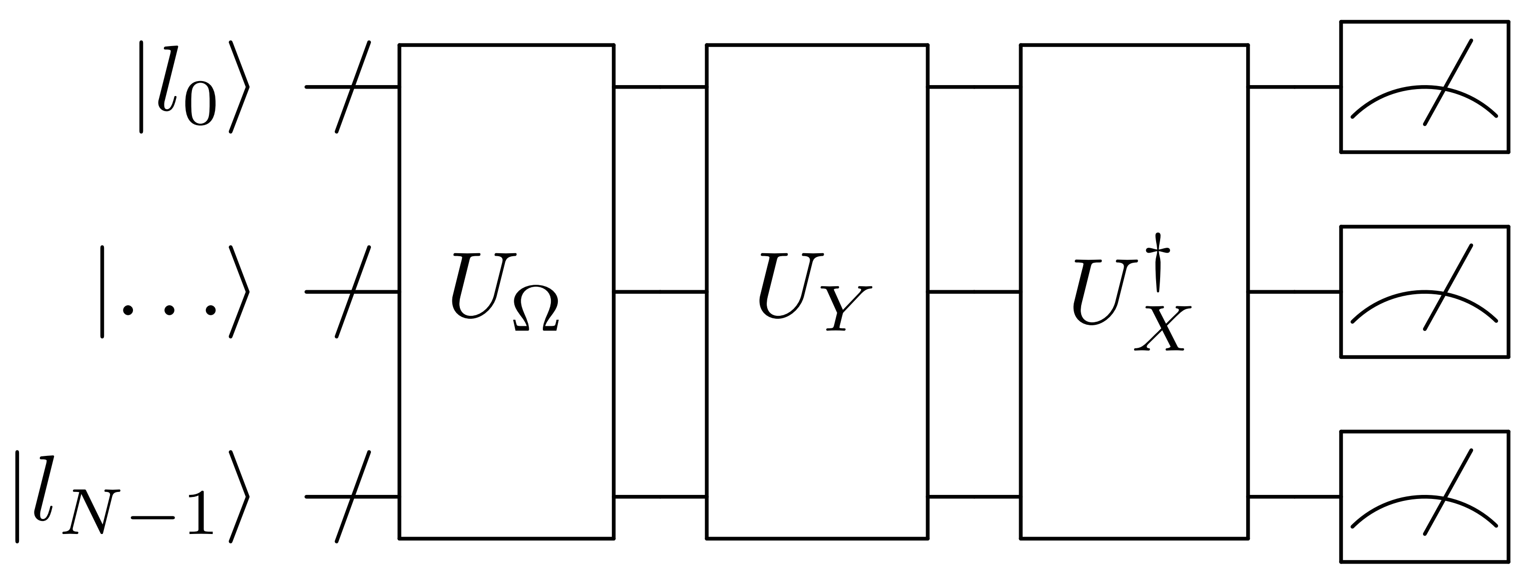


Soft function is the expectation value of a "Wilson line" operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

Have worked out quantum circuit to create vacuum state $|\Omega\rangle$, circuit for $T[Y_n Y_{\bar{n}}^\dagger]$ and circuit to measure final state $|X\rangle$



Constructing the relevant circuit is relatively straightforward

Hamiltonian Evolution

Jordan, Lee, Preskill ('12)
Somma ('16)
Macridin et al ('18)
Savage, Klco ('19)

Crucial simplification: this problem only requires Hamiltonian of free field theory

$$H = H_\phi + H_\pi \qquad H_\phi = \hat{\phi}^2/2\,, \quad H_\pi = \hat{\pi}^2/2$$

Can move between ϕ and π basis via QFT

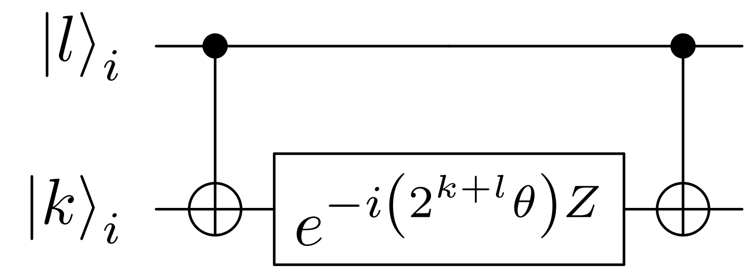
$$e^{iH_\pi t} = \mathrm{QFT}^{-1}\, e^{i\delta x\, t\phi_i^2}\, \mathrm{QFT}$$

and express ϕ operator through Z operators

$$\hat{\phi}_i = \sum_{j=0}^{n_Q-1} 2^j \hat{\sigma}_{z,i}^{(j)}$$

Entire Hamiltonian therefore determined in terms of

$$\exp\left[i\theta\hat{\phi}_i\hat{\phi}_j\right] = \prod_{l=0}^{n_Q-1}\prod_{k=0}^{n_Q-1} \exp\left[i\, 2^{(l+k)}\theta\, \sigma_{z,i}^{(l)}\sigma_{z,j}^{(k)}\right] =$$



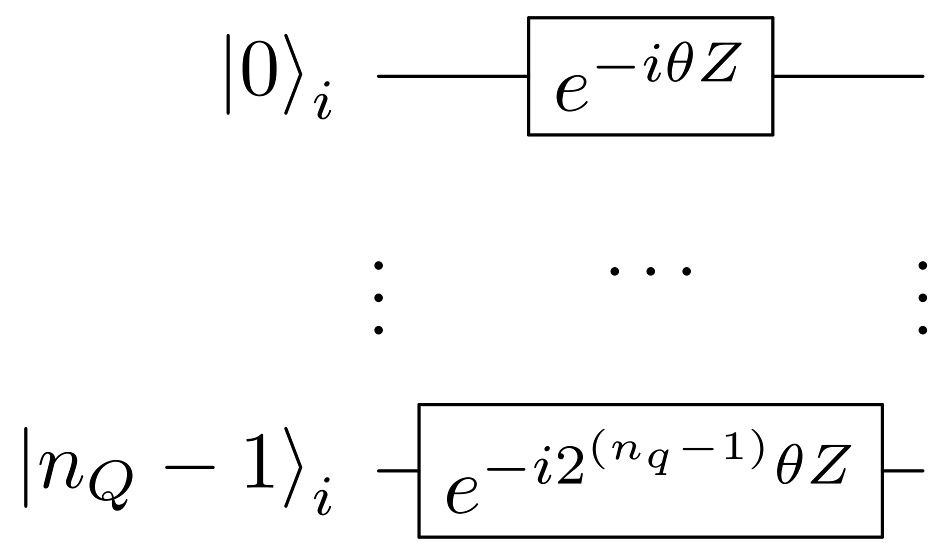
Constructing the relevant circuit is relatively straightforward

Exponential of field operator

CWB, Freytsis, Nachman ('21)

Much simpler to implement, using similar technique as for Hamiltonian

$$\exp[i\theta\hat{\phi}_i] = \prod_{j=0}^{n_Q-1} \exp\left[i2^j\theta\sigma_{z,i}^{(j)}\right] =$$



Put together, allows to implement the whole Wilson line operator

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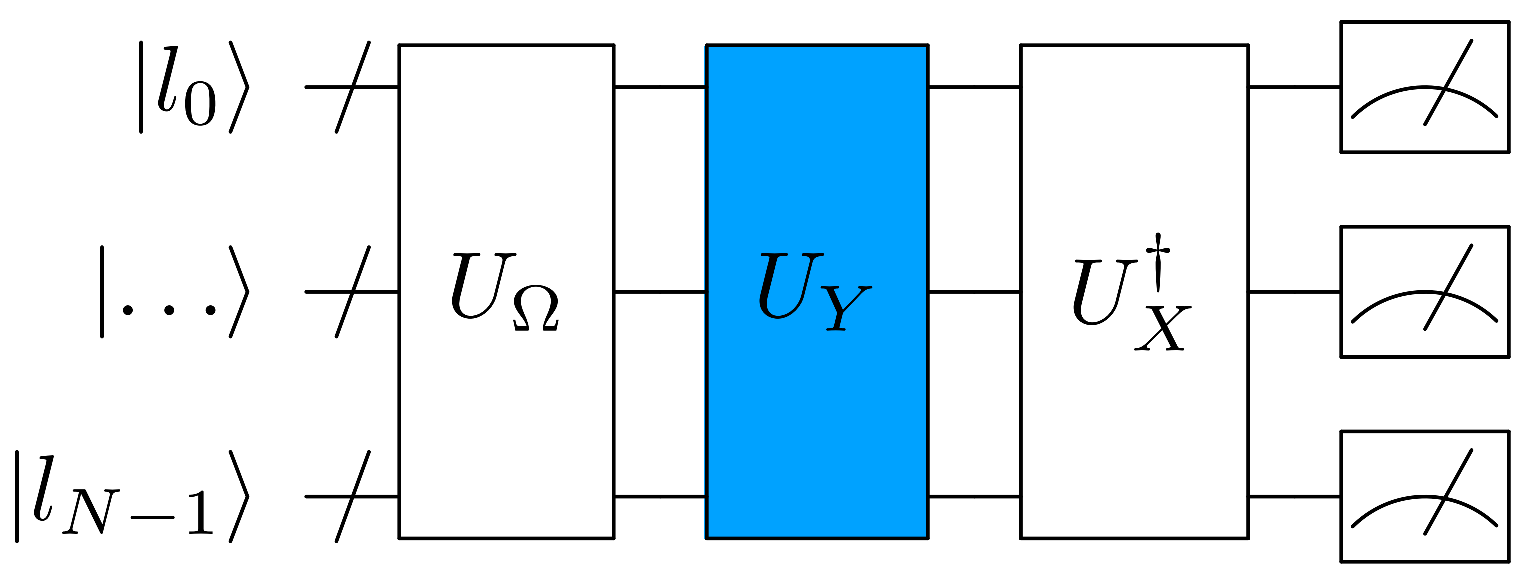


Constructing the relevant circuit is relatively straightforward

Ground state preparation

Kitaev, Webb ('08)
CWB, Deliyannis, Freytsis, Nachman (in preparation)

Ground state of scalar field theory given by multivariate Gaussian

$$|\Psi\rangle = \exp\left[-\frac{1}{2}\hat{\phi}_i G_{ij} \hat{\phi}_j\right] |k_0\rangle \cdots |k\rangle_n$$

The covariance matrix G_{ij} can be diagonalized
$G = MDM^T$, where D is diagonal and M upper triangle matrix

General process is therefore to proceed in two steps

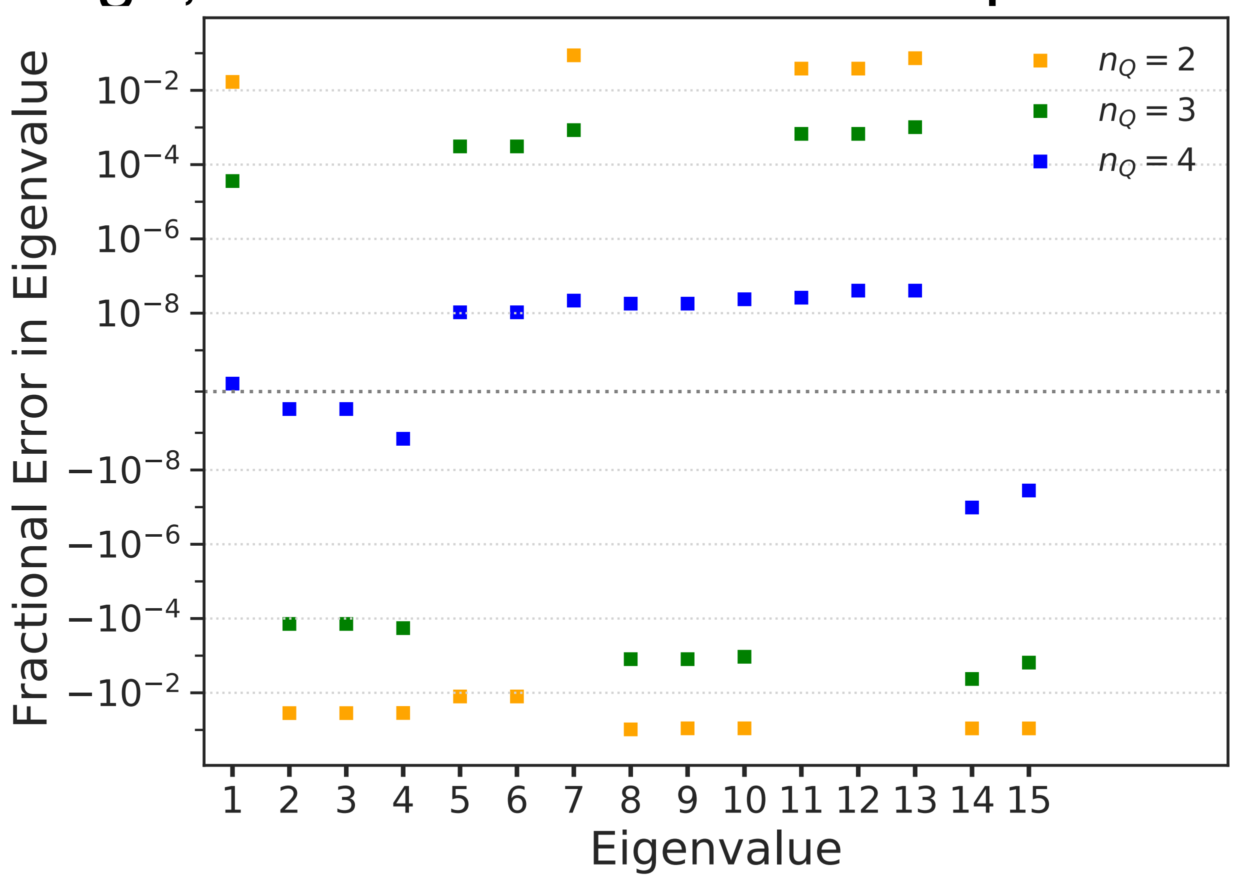
1. Prepare set of uncorrelated Gaussians with widths determined by D
2. Switch basis by applying M (a shearing operation)

Constructing the relevant circuit is relatively straightforward

Ground state preparation

Kitaev, Webb ('08)
CWB, Deliyannis, Freytsis, Nachman (in preparation)

1. Prepare set of uncorrelated Gaussians with widths determined by D
   - Classical complexity scales as $N\exp(n_\phi)$
   - Quantum algorithm exists that has polynomial scaling $Np(n_\phi)$
   - Requires to perform relatively complicated quantum arithmetic
   - Since n_ϕ not large, most efficient to use exponential algorithm?



Constructing the relevant circuit is relatively straightforward

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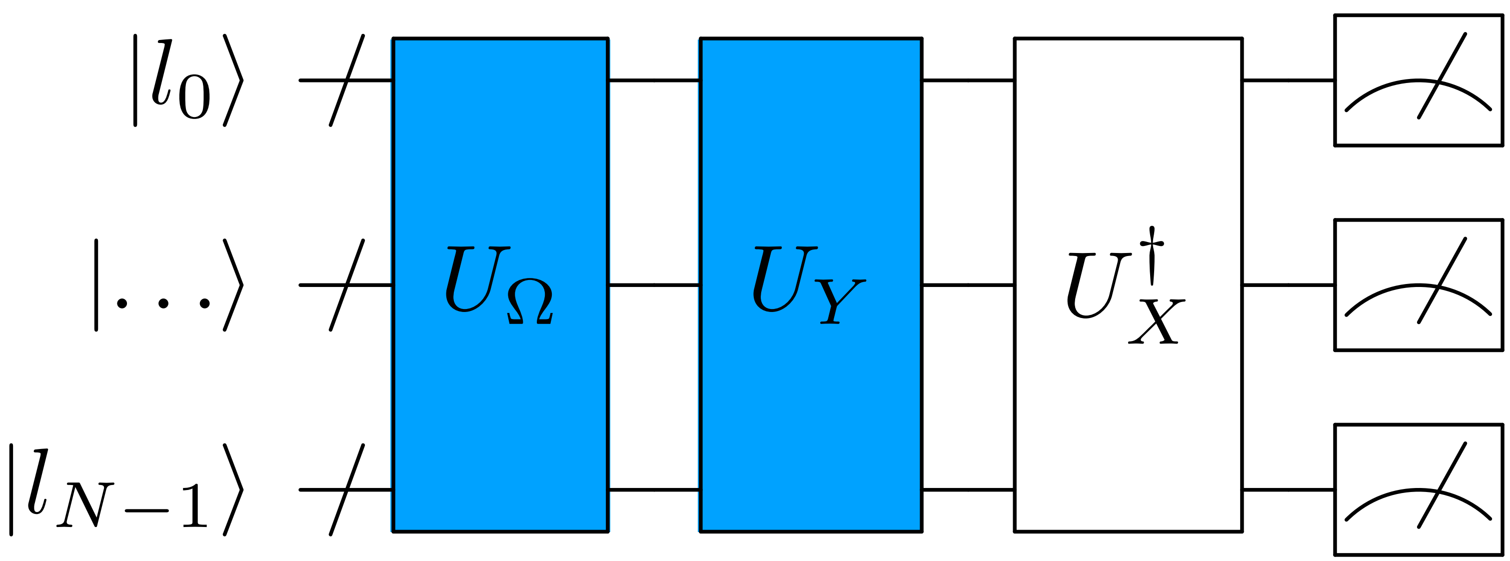
1. Prepare set of uncorrelated Gaussians with widths determined by D
   - Classical complexity scales as $N \exp(n_\phi)$
   - Quantum algorithm exists that has polynomial scaling $Np(n_\phi)$
   - Requires to perform relatively complicated quantum arithmetic
   - Since n_ϕ typically not very large, might be most efficient to simply create classically computed state

2. Switch basis by applying M (a shearing operation)
   - Classical complexity scales as $\exp(Nn_\phi)$
   - Quantum algorithm exists that has polynomial scaling $p(Nn_\phi)$
   - Since N typically large, imperative to use much more efficient quantum algorithm

Soft function is the expectation value of a "Wilson line" operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

Have worked out quantum circuit to create vacuum state $|\Omega\rangle$, circuit for $T[Y_n Y_{\bar{n}}^\dagger]$ and circuit to measure final state $|X\rangle$



Constructing the relevant circuit is relatively straightforward

Excited state preparation

Jordan, Lee, Preskill ('12)

1. Given the ground state of the theory, can obtain excited state by acting with creation operator.
2. Not a unitary operation, but can be implemented using ancillary quit
3. Complexity scales as $p(Nn_\phi)$

Soft function is the expectation value of a "Wilson line" operator between initial and final state

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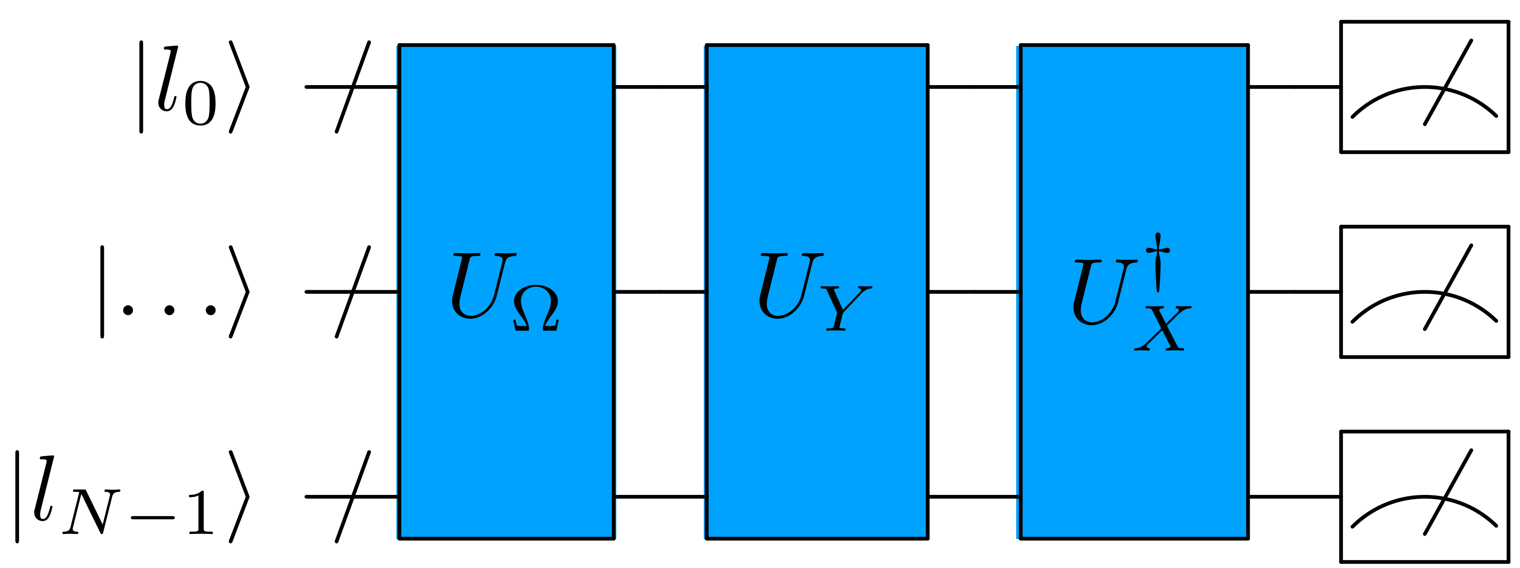

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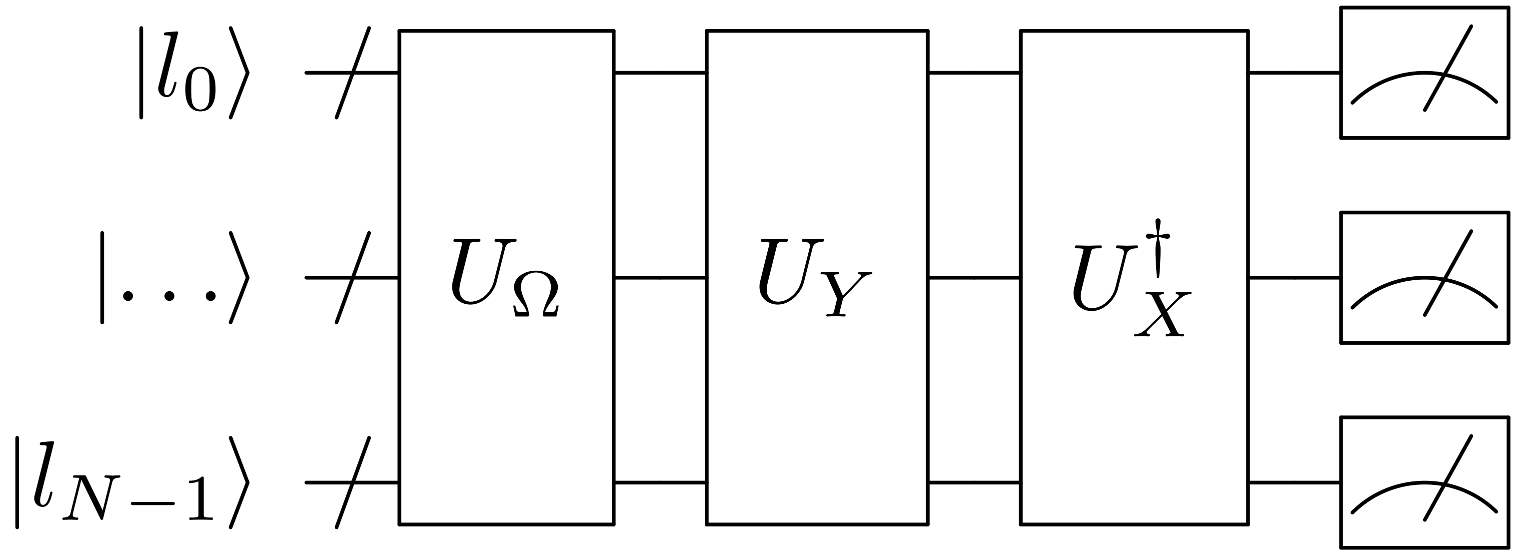
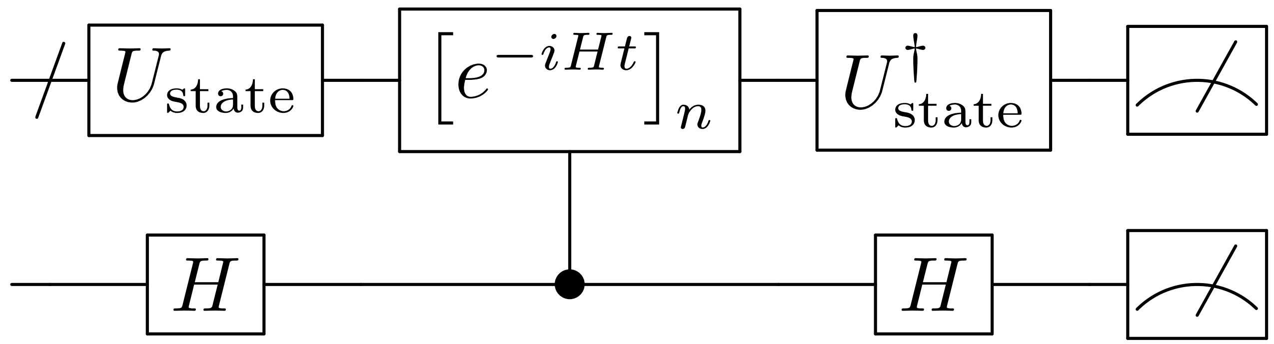


Steps to simulate the soft function S:

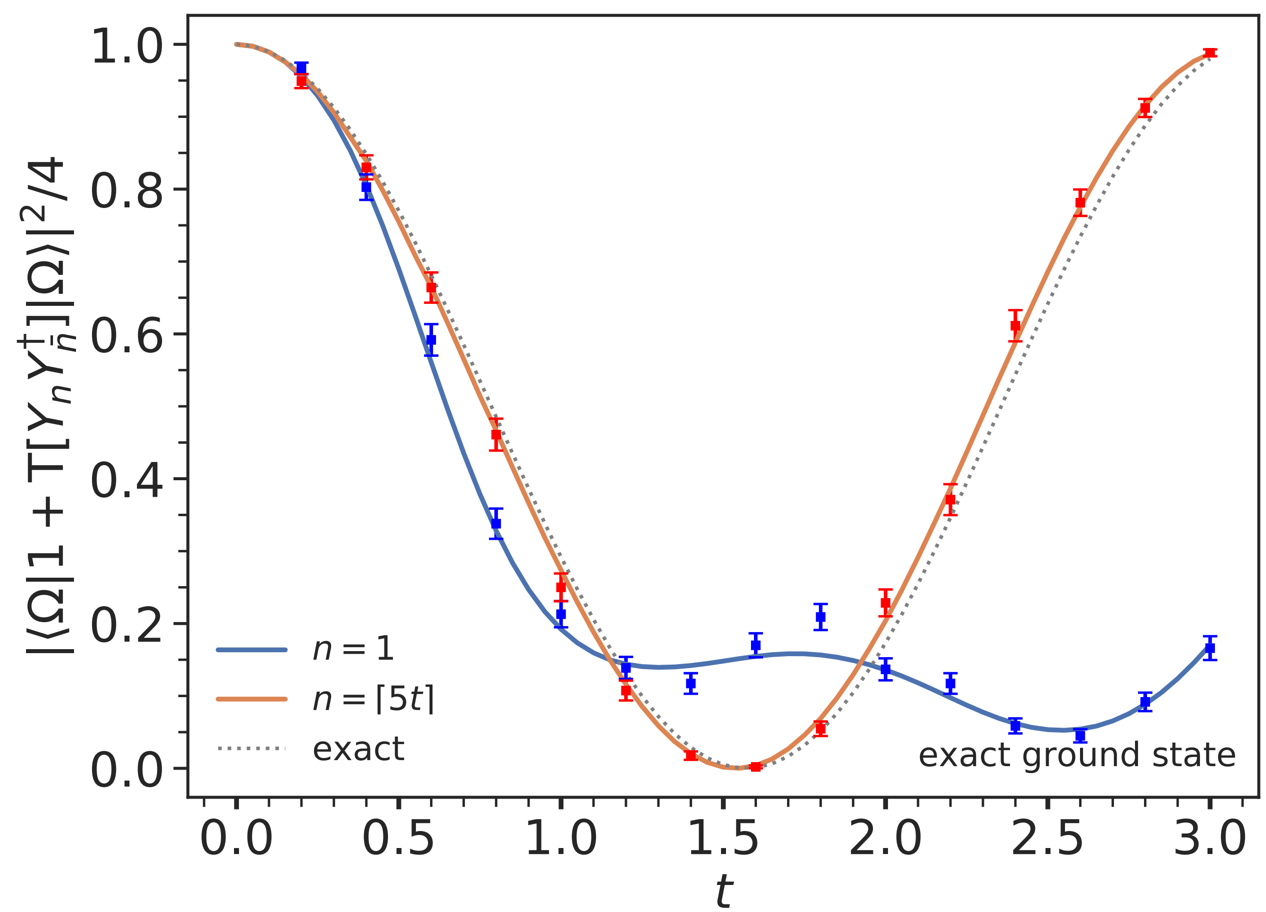
1. Start with all qubits in $|0\rangle$ state
2. Apply operator U_Ω creating ground state $|\Omega\rangle$ of the field QFT
3. Apply the operator $U_Y = T[Y_{\bar{n}}^\dagger Y_n]$
4. Perform the inverse of U_X creating state $|X\rangle$
5. Measure and count number of times all qubits are in $|0\rangle$ state

To know that we implement things right, need to be able to cross check our results

State preparation and Hamiltonian evolution can be checked directly against known result of free scalar field theory



$$f_{\text{ctr}}(t) = \frac{1}{4}\left|1 + \langle\Omega|\left[e^{-iHt}\right]_n|\Omega\rangle\right|^2$$



Soft function is the expectation value of a "Wilson line" operator between initial and final state

For a latticed scalar field theory, can in fact compute the required matrix elements analytically

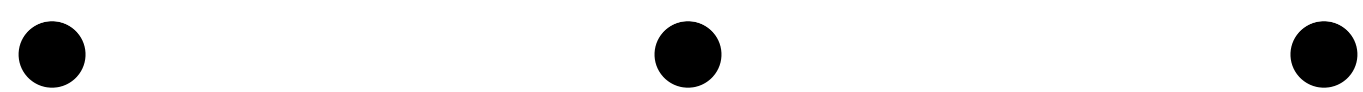
- Exponential of field operator related to coherent states
- Coherent states satisfy relation
  $D_{\mathbf{p}}(\alpha_{\mathbf{p}})D_{\mathbf{p}}(\beta_{\mathbf{p}}) = (2\pi)^d\, D_{\mathbf{p}}(\alpha_{\mathbf{p}} + \beta_{\mathbf{p}})\, e^{i\,\mathrm{Im}(\alpha_{\mathbf{p}}\beta^*_{\mathbf{p}})\left(\frac{2\pi}{\delta p}\right)^d}$
- Can use these results to obtain for example the ground state overlap

$$\left|\langle\Omega|\mathrm{T}[Y_n\, Y^\dagger_{\bar{n}}|\Omega\rangle\right|^2 = \exp\left[-8\frac{g^2}{(2\pi)^d}\sum_{\mathbf{p}}\frac{1}{2\omega_{\mathbf{p}}}\sum_{x\geq y}\cos(\omega_{\mathbf{p}}(x-y))\sin(\mathbf{n}\cdot\mathbf{p}\,x)\sin(\mathbf{n}\cdot\mathbf{p}\,y)\right]$$

- Several interesting effects, that I don't have time to describe
  - Mixed UV-IR divergences that only cancel in physical observables
  - Absence of non-trivial IRC safe observables in 1+1 D

In order to implement this on actual hardware, we need to make the system very small

Use only three lattice sites



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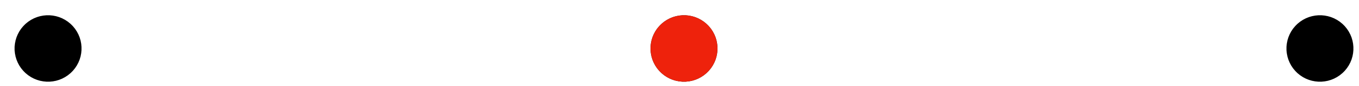

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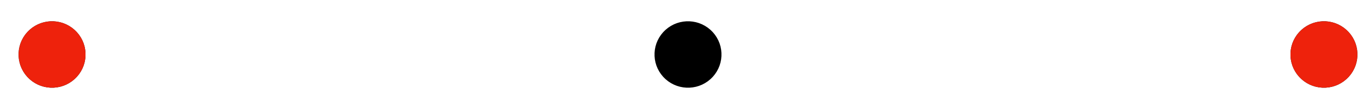


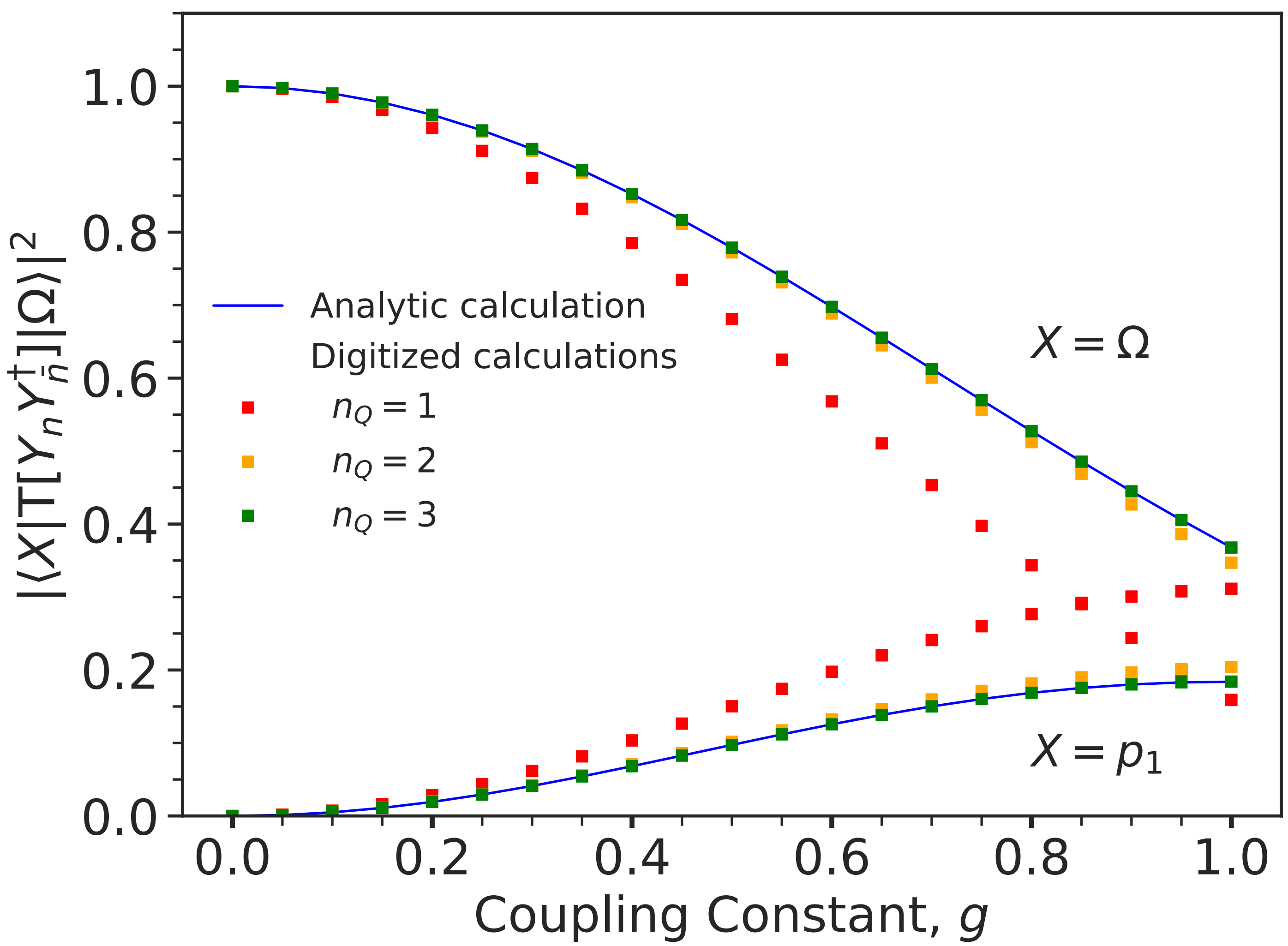
- Hamiltonian evolution produces only an overall phase, since it always acts on initial or final state

Furthermore, use only 2 qubits per lattice site

- Shearing matrix M required for state preparation is trivial (identity), such that covariance matrix is diagonal

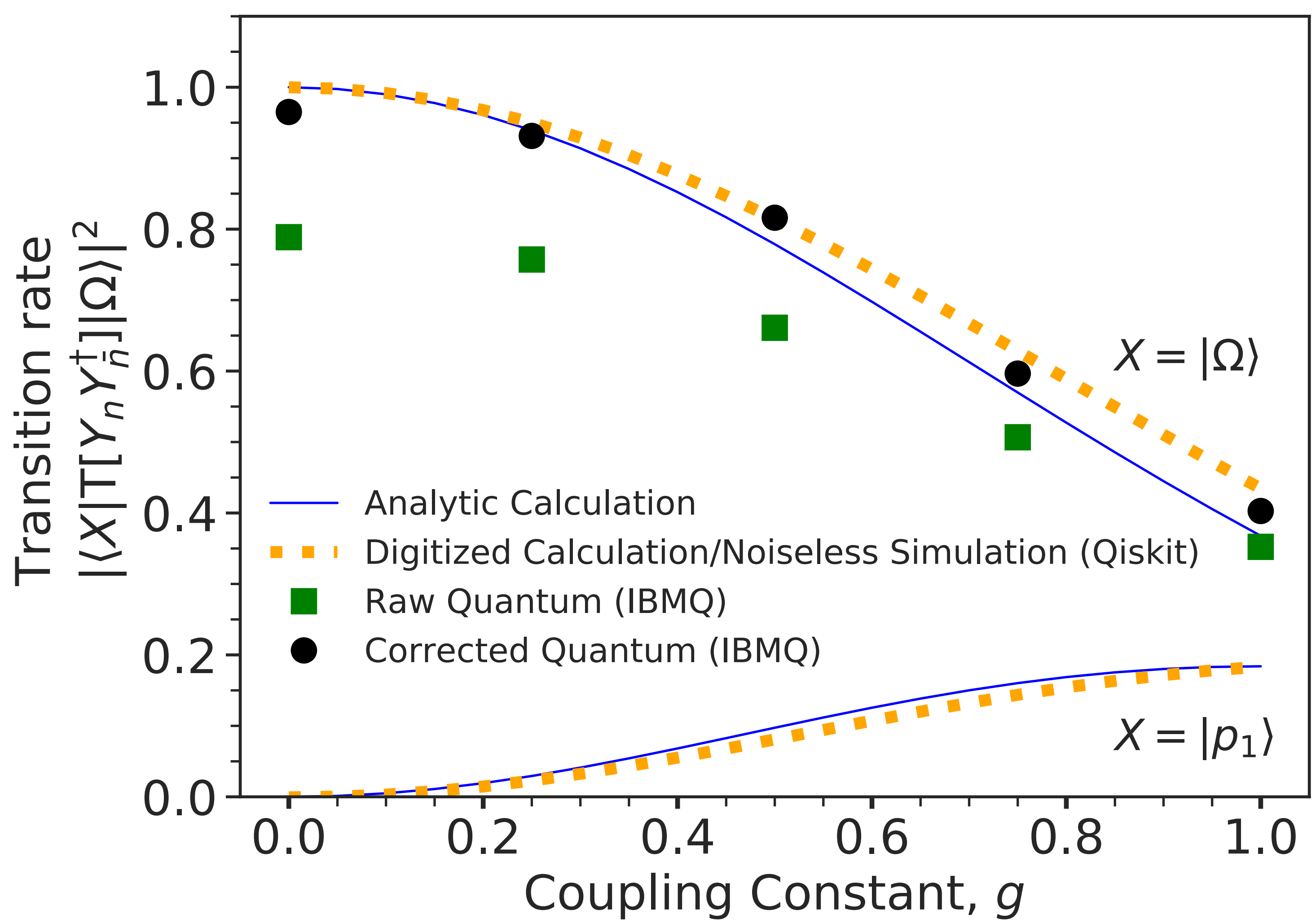
Only 6 qubits required for simulation

Soft function is the expectation value of a “Wilson line” operator between initial and final state



As expected, already with 2-3 qubits per lattice site get answers that are very close to the analytical result

Soft function is the expectation value of a "Wilson line" operator between initial and final state



Quantum computer gives a good description of the analytical result

In summary, by combining effective field theories with quantum algorithms, have the possibility to compute long distance effects in collider physics from first principles.

1. Full simulation of scattering processes can be described by matrix evolution by discretizing space
2. Energy range that needs to be described determines number of lattice points
3. Performing this matrix evolution is completely intractable using classical algorithms, due to exponential scaling
4. Even using most efficient quantum algorithms with polynomial scaling requires completely unrealistic resources
5. Using effective theories can limit problem to the energy range that is not accessible using known techniques
6. Requires much smaller energy range and therefore much smaller quantum resources
7. Have shown that the most novel ingredient in EFT framework can indeed be computed using quantum algorithms

In summary, by combining effective field theories with quantum algorithms, have the possibility to compute long distance effects in collider physics from first principles.

While this has shown that the relevant EFT calculations are possible, much more work required for real world applications

1. Calculation done for scalar field theory
   Implementation for gauge theories

2. Only computed simplest overlaps
   Work on more general state preparation

3. Calculation done in bare theory:
   Think carefully about renormalization in EFT

QUESTIONS?

Items 1. and 2. are already in progress, starting to think about 3.