Playing Pool with $|\psi\rangle$:
from Bouncing Billiards to Quantum Search

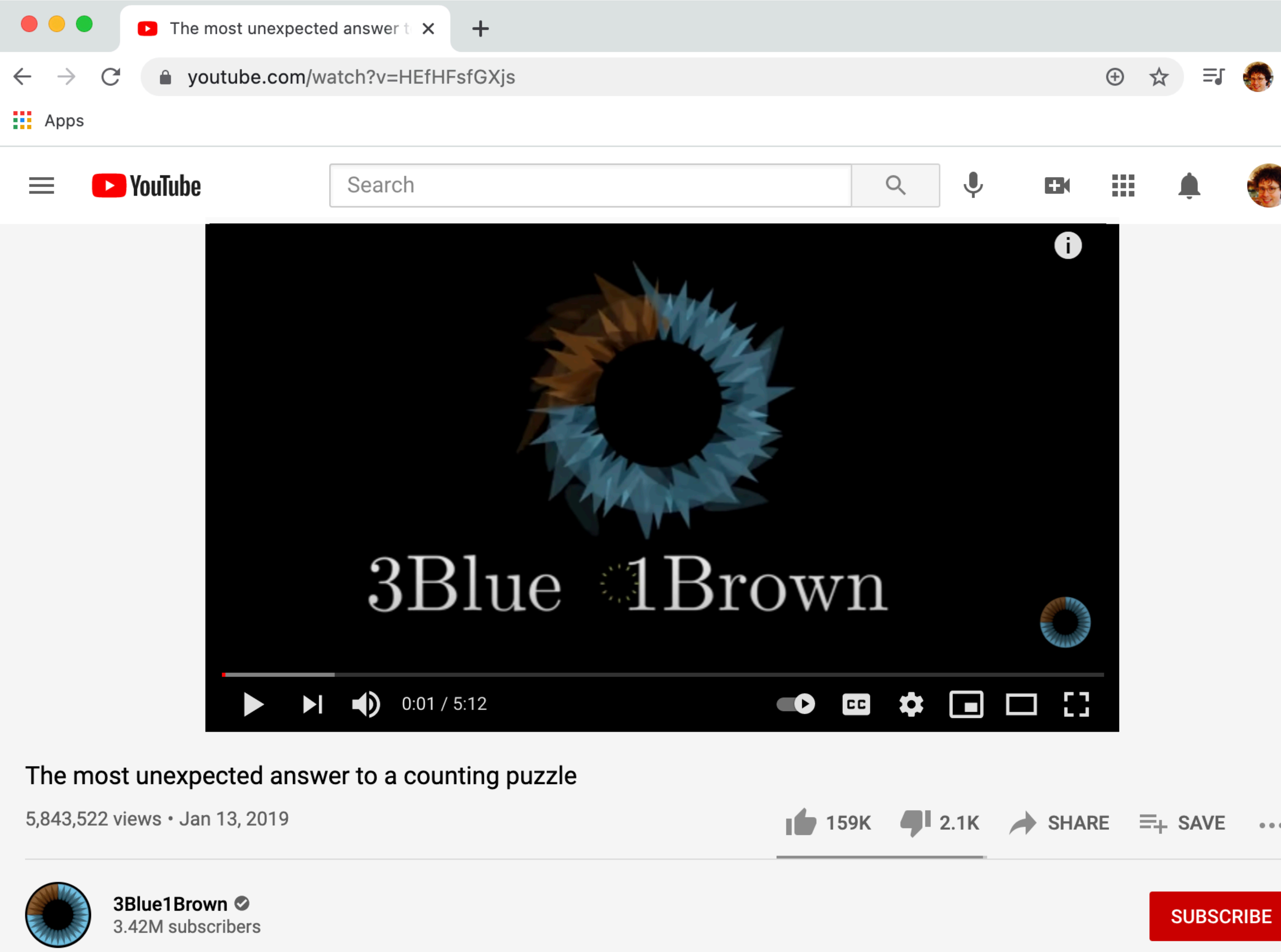
Adam Brown

1912.02207

Quantum Gravity in the Lab:
Teleportation by Size and Traversable Wormholes

Adam Brown, Hrant Gharibyan, Stefan Leichenauer, Henry Lin, Sepehr Nezami, Grant Salton, Leonard Susskind, Brian Swingle, Michael Walter,

1911.06314 & 2102.01064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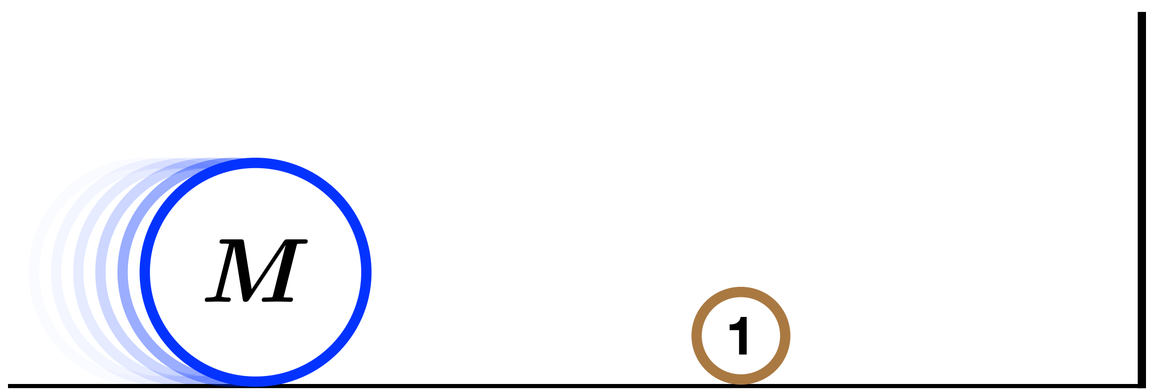

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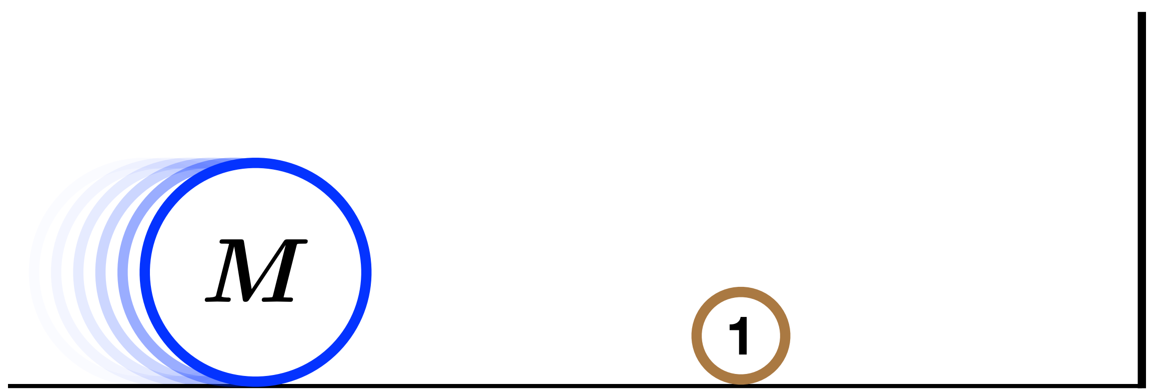
PLAYING POOL WITH π (THE NUMBER π FROM A BILLIARD POINT OF VIEW)

Received December 9, 2003
DOI: 10.1070/RD2003v008n04ABEH000252

Counting collisions in a simple dynamical system with two billiard balls can be used to estimate π to any accuracy.



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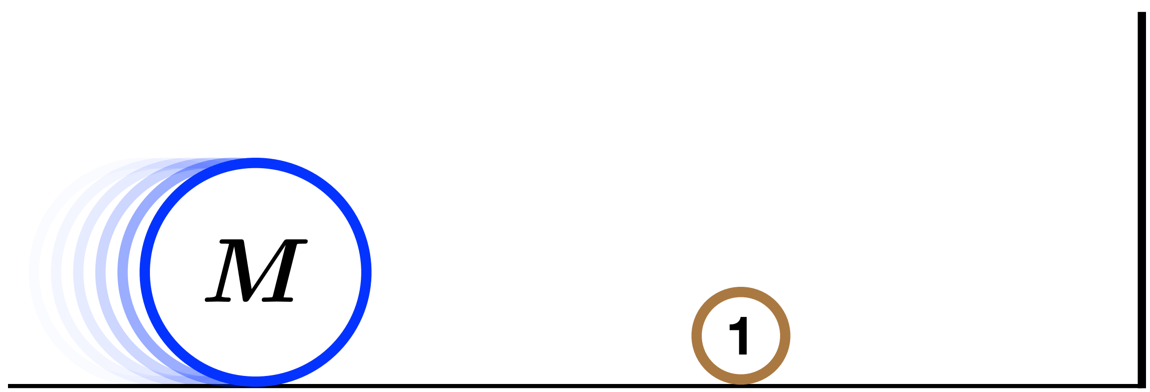
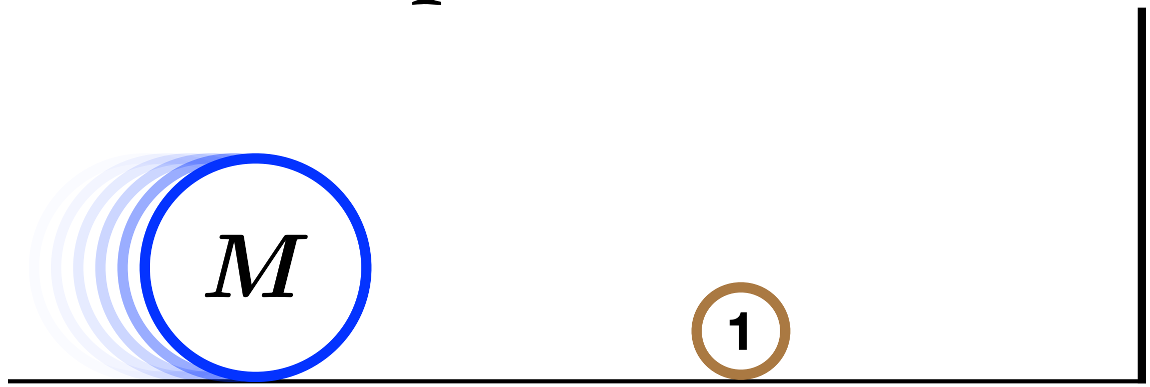


$$M = 100 \quad \rightarrow \quad \#_{\text{collisions}} = 31$$

$$M = 10^6 \quad \rightarrow \quad \#_{\text{collisions}} = 3141$$

$$M = 10^{20} \quad \rightarrow \quad \#_{\text{collisions}} = 31415926535$$

Galperin 1995



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$$\#_{\text{collisions}} = \left\lfloor \pi \sqrt{M} \right\rfloor$$

VOLUME 79, NUMBER 2 PHYSICAL REVIEW LETTERS 14 JULY 1997

Quantum Mechanics Helps in Searching for a Needle in a Haystack

Lov K. Grover*
3C-404A Bell Labs, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 4 December 1996)

Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing N names arranged in completely random order. To find someone's phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database. [S0031-9007(97)03564-3]

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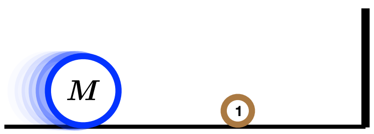

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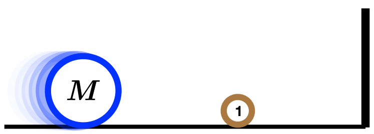
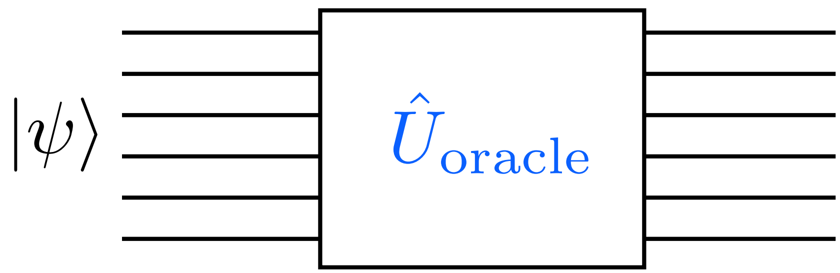


$$\#_{\text{collisions}} = \left\lfloor \pi\sqrt{M} \right\rfloor$$

task: find 'needle in haystack'

find 1 item amongst d



$$\#_{\text{oracle calls}} = \lfloor \frac{1}{4}\pi\sqrt{d-1} \rfloor$$

Galperin 1995

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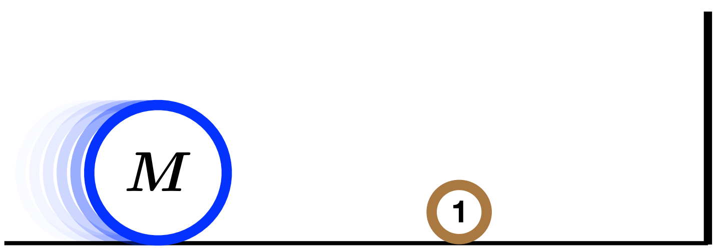

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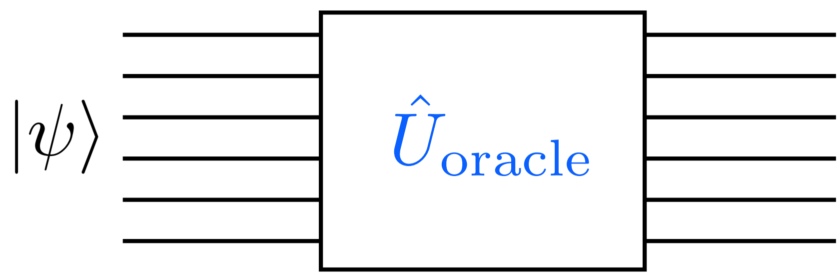

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arXiv.org > quant-ph > arXiv:1912.02207

[Submitted on 4 Dec 2019]

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Galperin 1995 ⟷ 1912.02207 ⟷ Grover 1996

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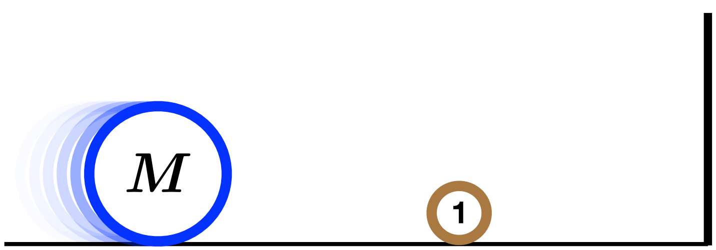

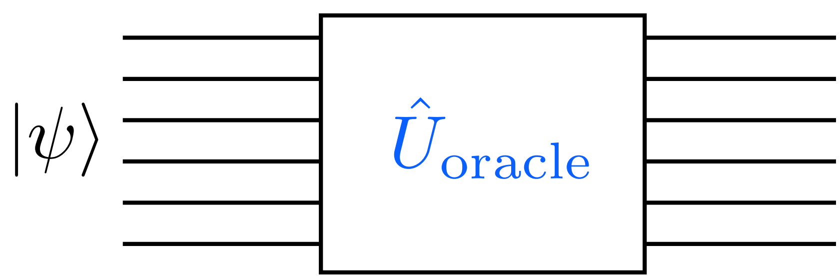

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Quantum Algorithms 101

Shor 1994		Grover 1996

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Shor 1994		Grover 1996
factoring	GOAL	black box search
special purpose (not NP-complete)	STRUCTURE NEEDED?	general purpose (includes NP-complete)
exponential	SPEED UP	quadratic
steal bitcoin	USE CASE	mine bitcoin

Quantum Algorithms 101

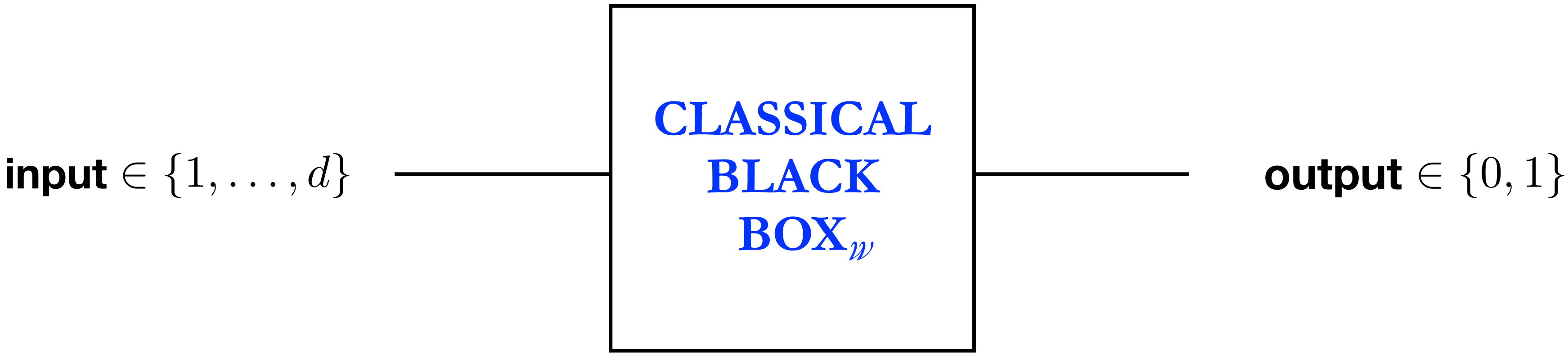
Shor 1994		Grover 1996
factoring	GOAL	black box search
special purpose (not NP-complete)	STRUCTURE NEEDED?	general purpose (includes NP-complete)
exponential $\text{classical} \sim \exp[(\log d)^{\frac{1}{3}}]$ $\text{quantum} \sim (\log d)^3$	SPEED UP	quadratic $\text{classical} \sim d/2$ $\text{quantum} \sim \pi\sqrt{d}/4$
steal bitcoin	USE CASE	mine bitcoin

Grover 1996

black box search

GOAL: find "w"

$$\mathrm{BB}_w[\textbf{input}] = \begin{cases} \textbf{0} & \text{if } \textbf{input} \neq w \\ \textbf{1} & \text{if } \textbf{input} = w \end{cases}$$



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$$\mathrm{BB}_w[\textbf{5}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{4}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{3}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{2}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{1}] = \textbf{0}$$

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$w = 11$ → $\mathrm{BB}_w[\textbf{11}] = \textbf{1}$ ← $w = 11$

$$\mathrm{BB}_w[\textbf{10}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{9}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{8}] = \textbf{0}$$

$$\mathrm{BB}_w[\textbf{7}] = \textbf{0}$$

GOAL: find "w"

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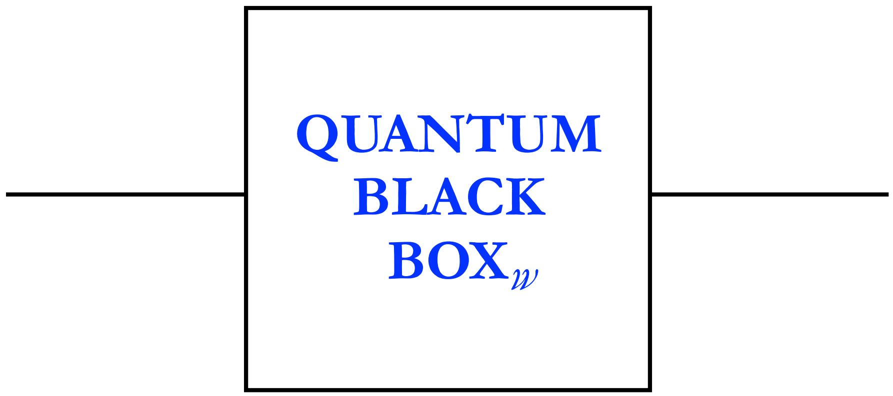
$$w = 11 \longrightarrow \mathrm{BB}_w[\mathbf{11}] = \mathbf{1} \longleftarrow w = 11$$

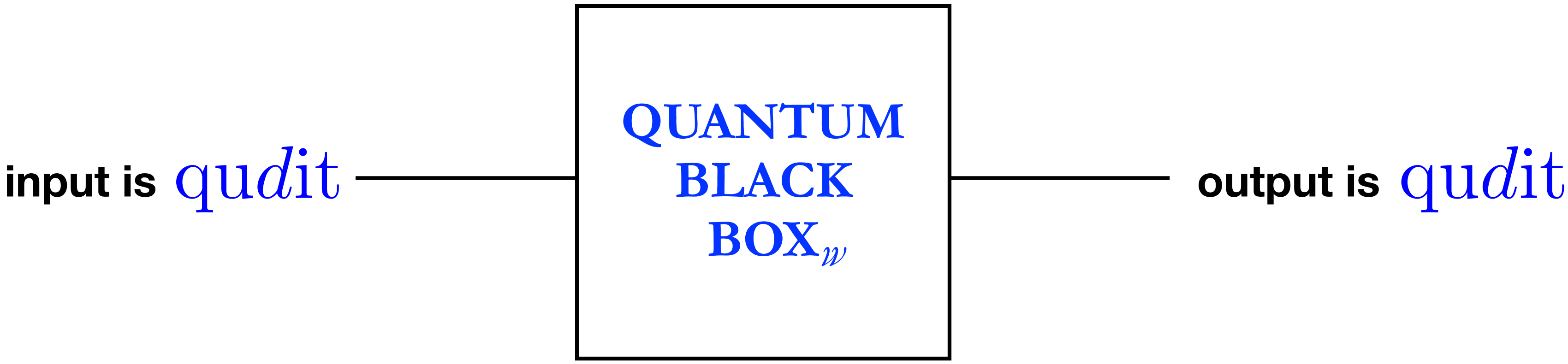
$$\langle \text{queries} \rangle = \frac{1}{2} d$$

GOAL: find "w"



? $$\langle \text{queries} \rangle = \frac{1}{2} d$$?

GOAL: find "w"



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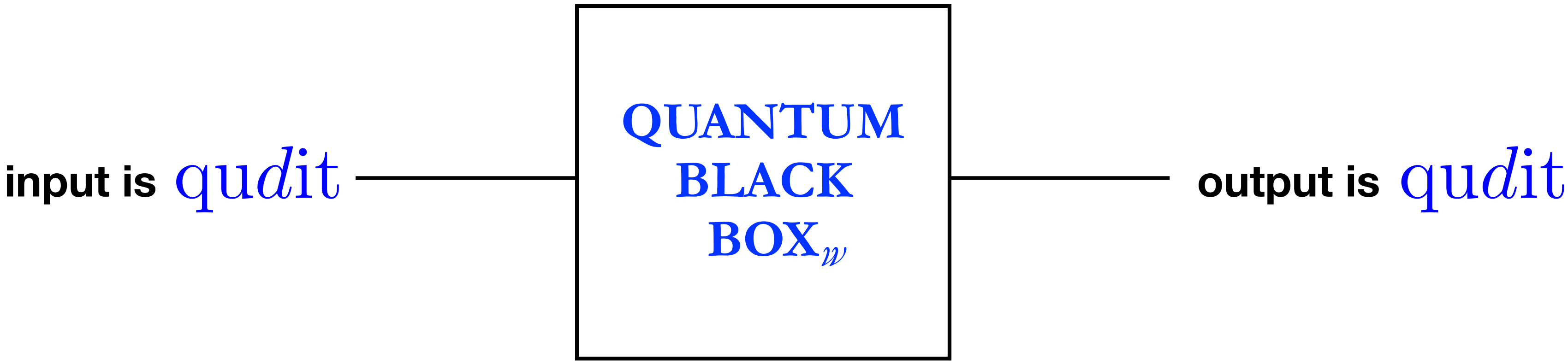
GOAL: find "w"

$$\hat{U}_w |\text{input}\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)|\text{input}\rangle$$



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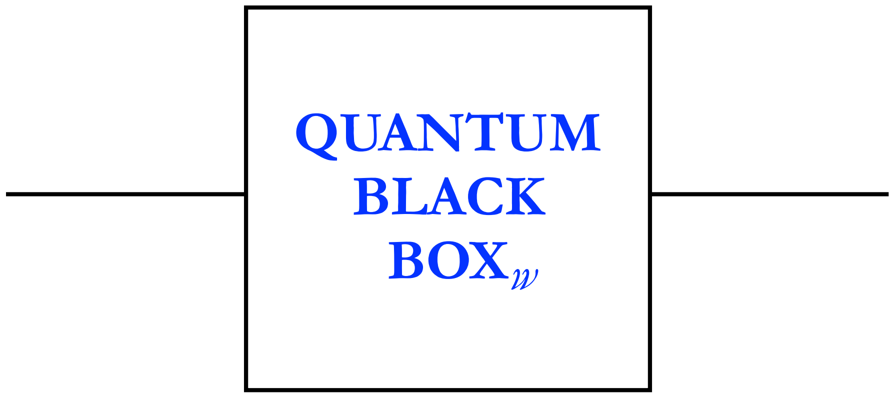
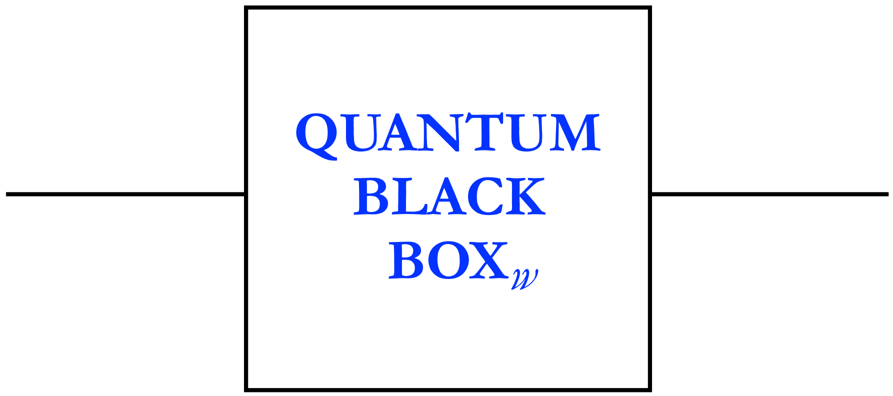


$$\hat{U}_w|\textsf{1}\rangle = +|\textsf{1}\rangle$$

GOAL: find "w"

$$\hat{U}_w|\text{input}\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)|\text{input}\rangle$$

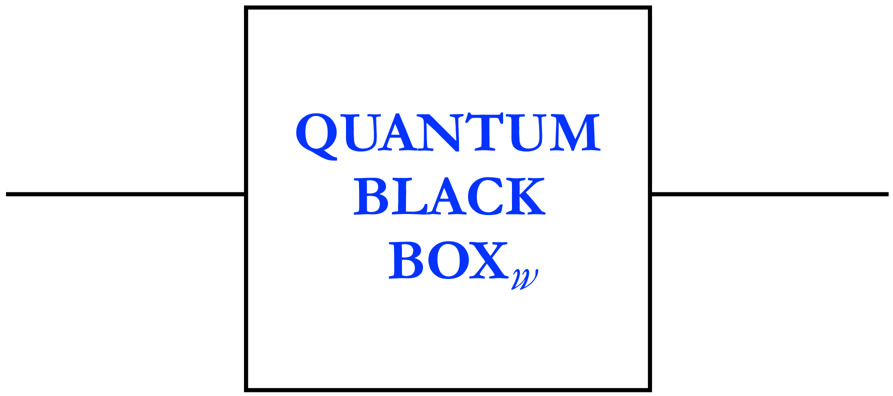


$$\hat{U}_w|1\rangle = +|1\rangle$$

$$\hat{U}_w|w\rangle = -|w\rangle$$

GOAL: find "w"

$$\hat{U}_w |\text{input}\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)|\text{input}\rangle$$



$w = 8$ → $\hat{U}_w|8\rangle = -|8\rangle$ ← $w = 8$

$$\hat{U}_w|7\rangle = +|7\rangle$$

$$\hat{U}_w|6\rangle = +|6\rangle$$

$$\hat{U}_w|5\rangle = +|5\rangle$$

$$\hat{U}_w|4\rangle = +|4\rangle$$

GOAL: find "w"

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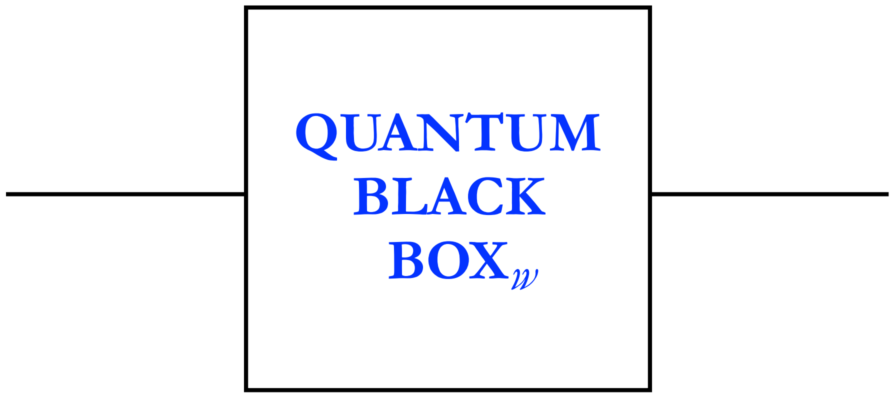

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$$\langle \text{queries} \rangle = \frac{1}{2} d$$

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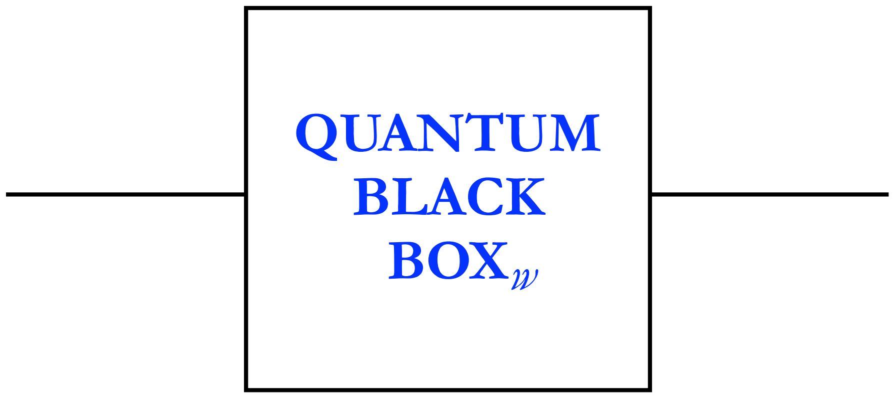

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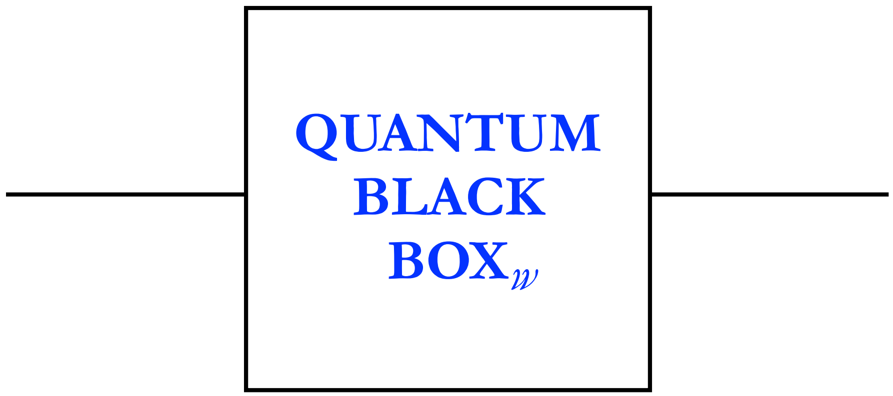
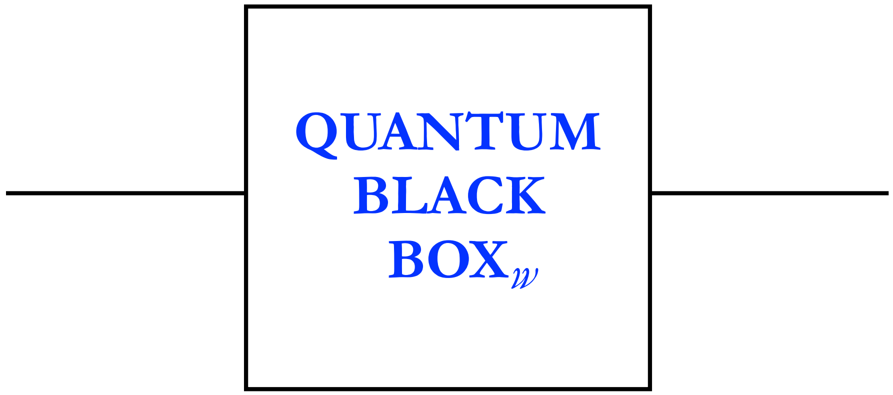


$$v_1|\textbf{1}\rangle + v_2\,|\textbf{2}\rangle + \ldots + v_w|w\rangle + \ldots + v_d|\textbf{d}\rangle$$

GOAL: find "w"

$$\hat{U}_w|\textbf{input}\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)|\textbf{input}\rangle$$

$$\hat{U}_w\left[v_1|\mathbf{1}\rangle + \ldots + v_w|w\rangle + \ldots + v_d|\mathbf{d}\rangle\right] = v_1|\mathbf{1}\rangle + \ldots - v_w|w\rangle + \ldots + v_d|\mathbf{d}\rangle$$



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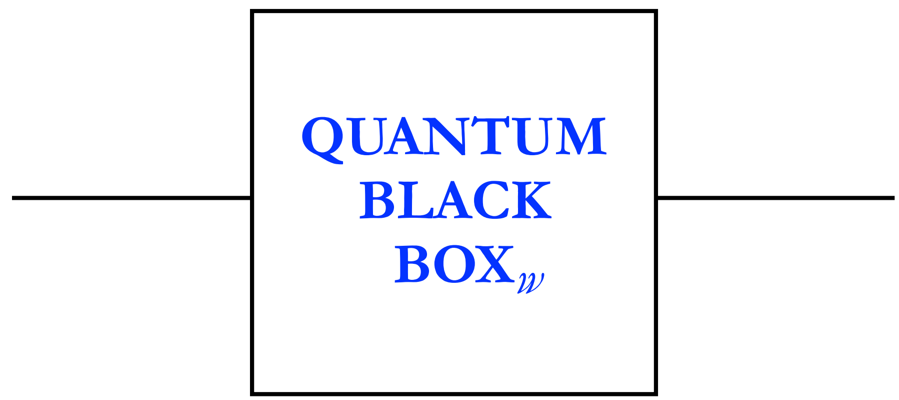

$$|s\rangle \equiv \frac{1}{\sqrt{d}}\Big(|\mathbf{1}\rangle + |\mathbf{2}\rangle + \cdots + |w\rangle + \cdots + |\mathbf{d}\rangle\Big)$$

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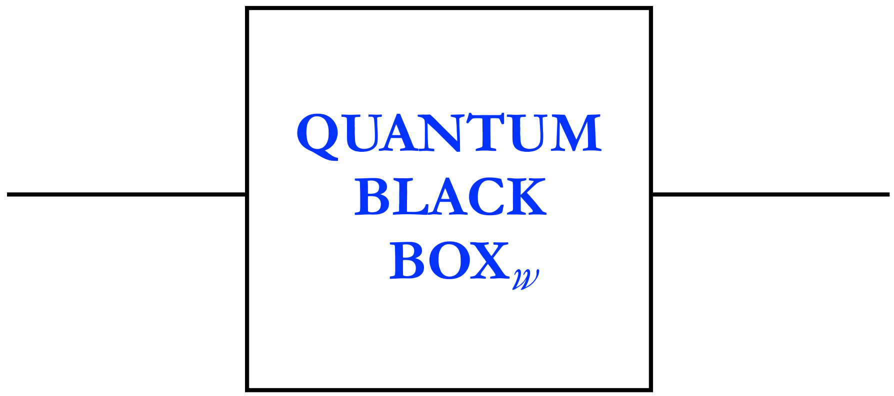

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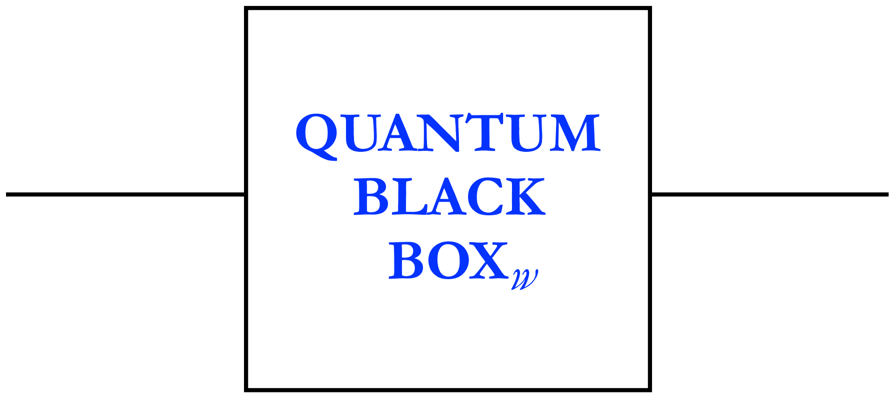

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$$\hat{U}_w|s\rangle = \frac{1}{\sqrt{d}}\Big(|\textsf{1}\rangle + |\textsf{2}\rangle + \cdots - |w\rangle + \cdots + |\textsf{d}\rangle\Big)$$

$$\hat{U}_{w'=2}|s\rangle = \frac{1}{\sqrt{d}}\Big(|\textsf{1}\rangle - |\textsf{2}\rangle + \cdots + |w\rangle + \cdots + |\textsf{d}\rangle\Big)$$

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NOT ORTHOGONAL

$$\hat{U}_{w'=2}|s\rangle = \frac{1}{\sqrt{d}}\left(|\mathbf{1}\rangle - |\mathbf{2}\rangle + \cdots + |w\rangle + \cdots + |\mathbf{d}\rangle\right)$$

GOAL: find "w"

$$\hat{U}_w \big|\textbf{input}\big\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)\big|\textbf{input}\big\rangle$$

$$\hat{U}_w \left[v_1 \big|\mathbf{1}\big\rangle + \ldots + v_w |w\rangle + \ldots + v_d \big|\mathbf{d}\big\rangle \right] = v_1 \big|\mathbf{1}\big\rangle + \ldots - v_w |w\rangle + \ldots + v_d \big|\mathbf{d}\big\rangle$$

$$|s\rangle \equiv \frac{1}{\sqrt{d}} \left(\big|\mathbf{1}\big\rangle + \big|\mathbf{2}\big\rangle + \cdots + |w\rangle + \cdots + \big|\mathbf{d}\big\rangle \right)$$

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NOT ORTHOGONAL

DC
Kr
Kryptonite

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$$\hat{U}_w\left[v_1|\mathbf{1}\rangle + \ldots + v_w|w\rangle + \ldots + v_d|\mathbf{d}\rangle\right] = v_1|\mathbf{1}\rangle + \ldots - v_w|w\rangle + \ldots + v_d|\mathbf{d}\rangle$$

$$|s\rangle \equiv \frac{1}{\sqrt{d}}\Big(|\mathbf{1}\rangle + |\mathbf{2}\rangle + \cdots + |w\rangle + \cdots + |\mathbf{d}\rangle\Big)$$

$$\hat{U}_w|s\rangle = \frac{1}{\sqrt{d}}\Big(|\mathbf{1}\rangle + |\mathbf{2}\rangle + \cdots - |w\rangle + \cdots + |\mathbf{d}\rangle\Big)$$

$$\hat{U}_w|s\rangle$$

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$$\hat{U}_w|s\rangle = \frac{1}{\sqrt{d}}\Big(|\mathbf{1}\rangle + |\mathbf{2}\rangle + \cdots - |w\rangle + \cdots + |\mathbf{d}\rangle\Big)$$

$$\hat{U}_w\hat{U}_w|s\rangle = |s\rangle$$

GOAL: find "w"

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$$|s\rangle \equiv \frac{1}{\sqrt{d}}\Big(|\mathbf{1}\rangle + |\mathbf{2}\rangle + \cdots + |w\rangle + \cdots + |\mathbf{d}\rangle\Big)$$

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$$\hat{U}_w \left|\textbf{input}\right\rangle = \left(\hat{I} - 2|w\rangle\langle w|\right)\left|\textbf{input}\right\rangle$$

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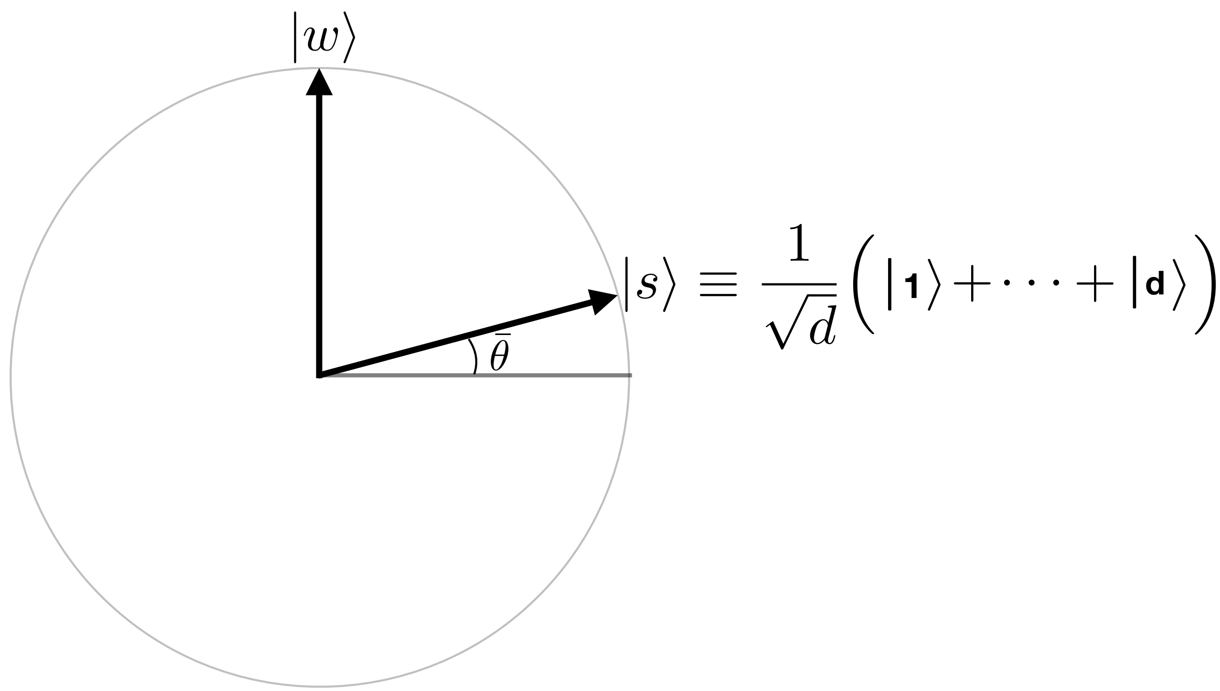

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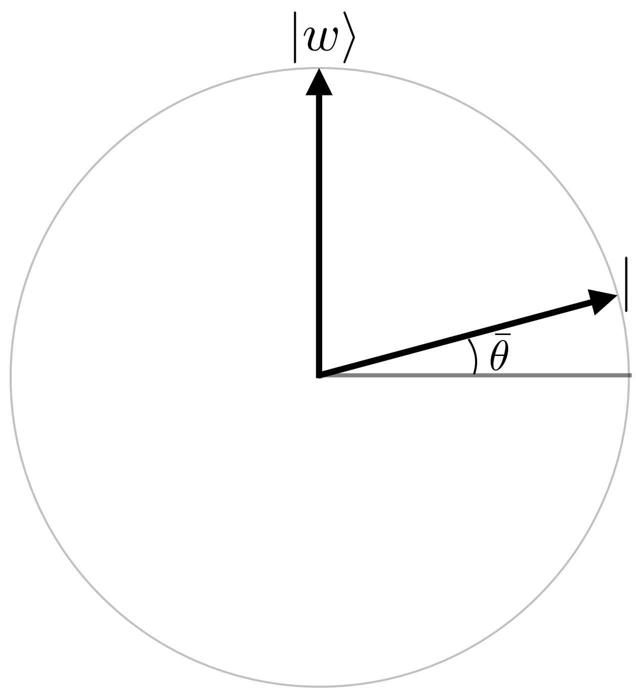

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$$\sin\bar{\theta} \equiv \langle w|s\rangle = \frac{1}{\sqrt{d}}$$

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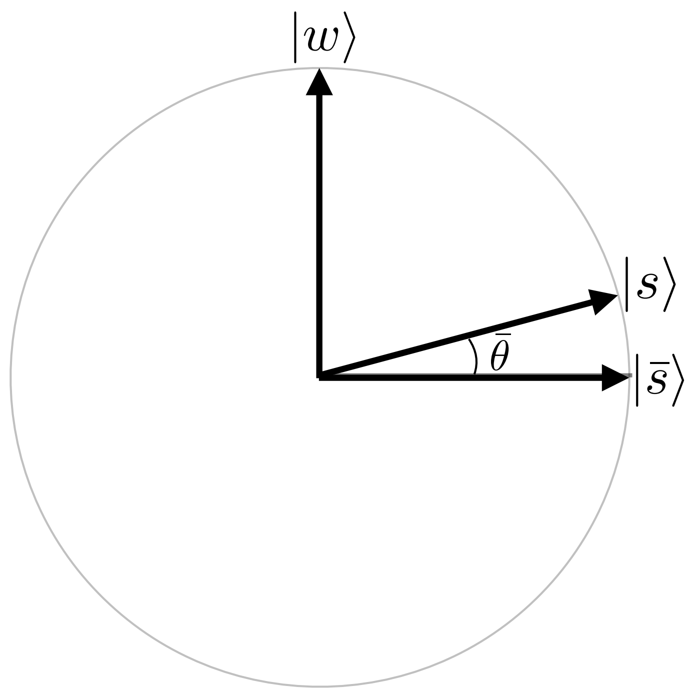

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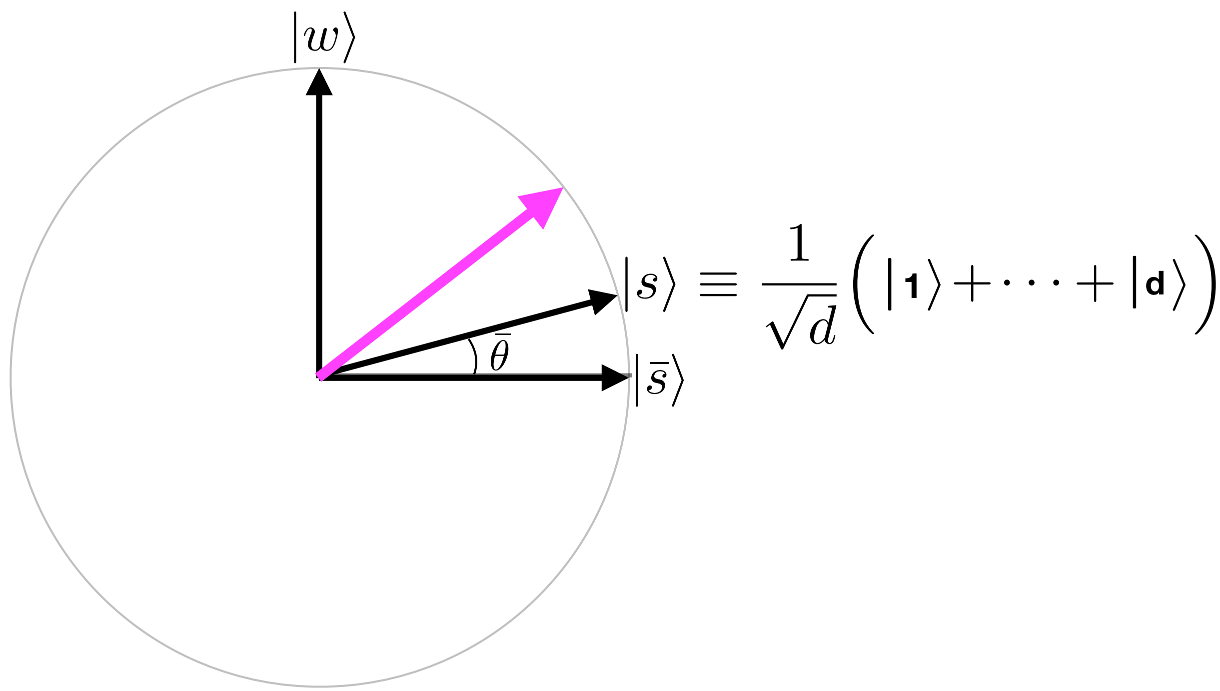

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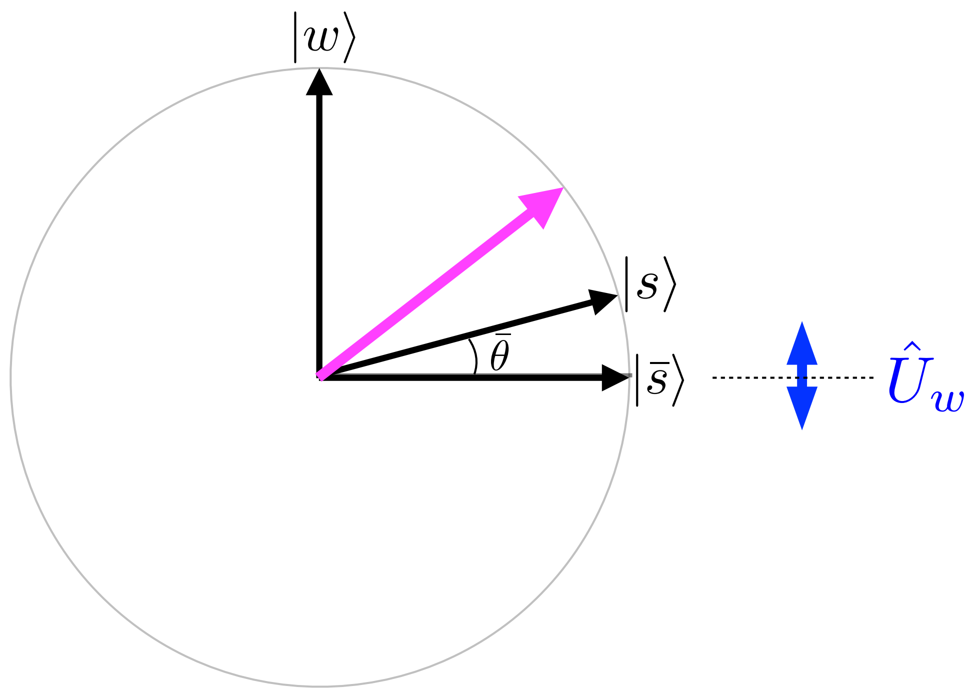

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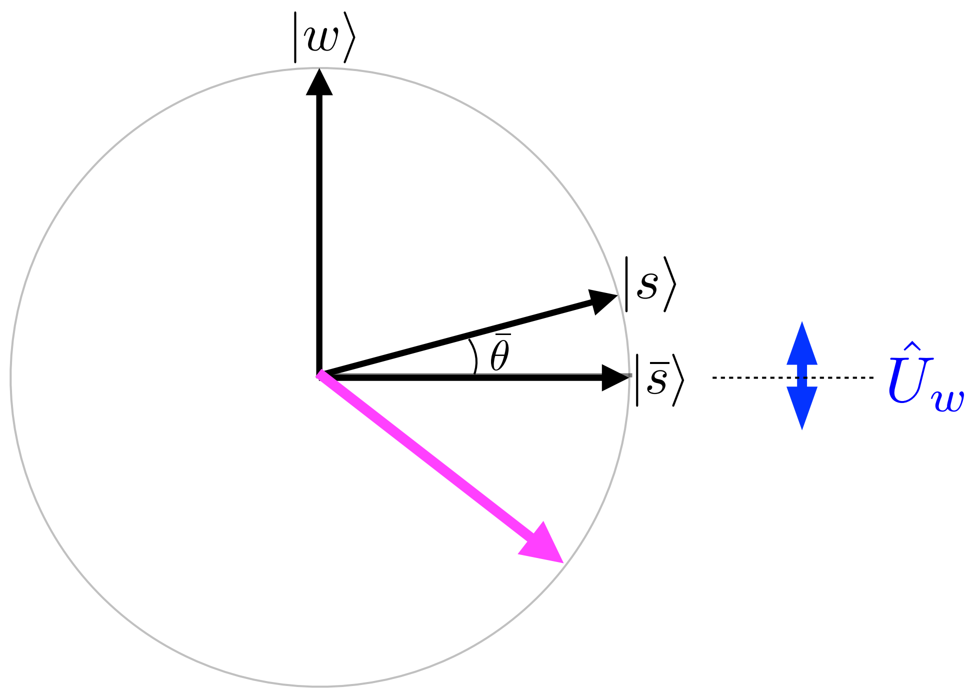

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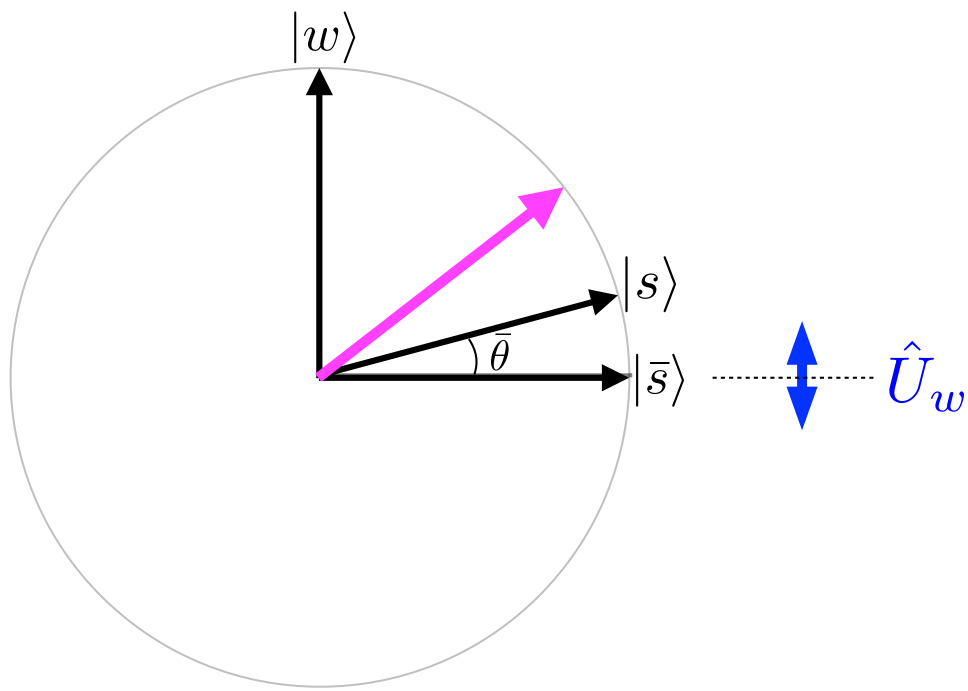

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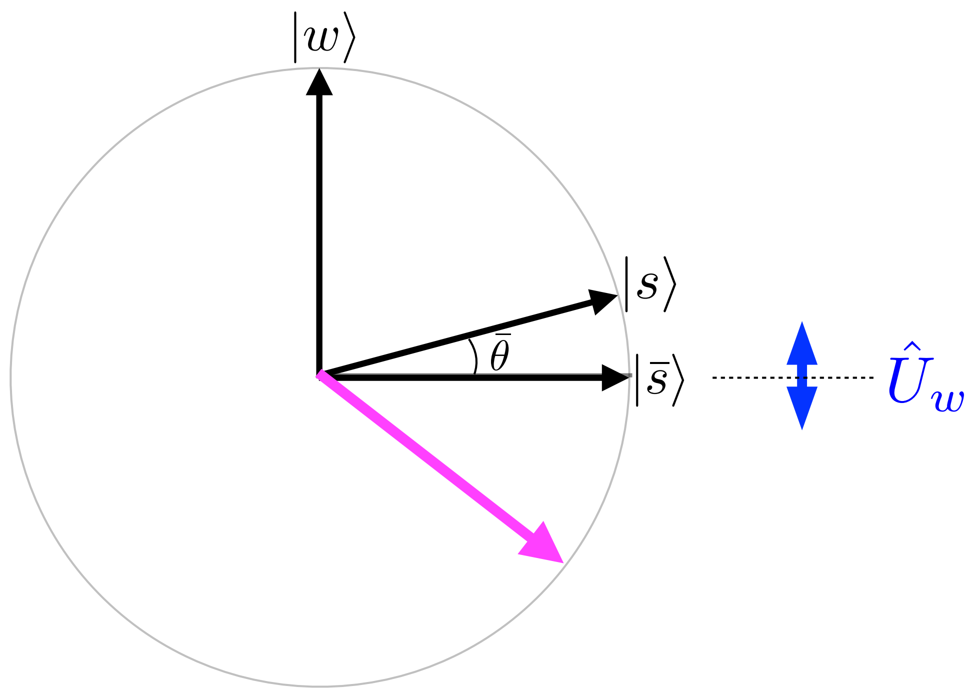

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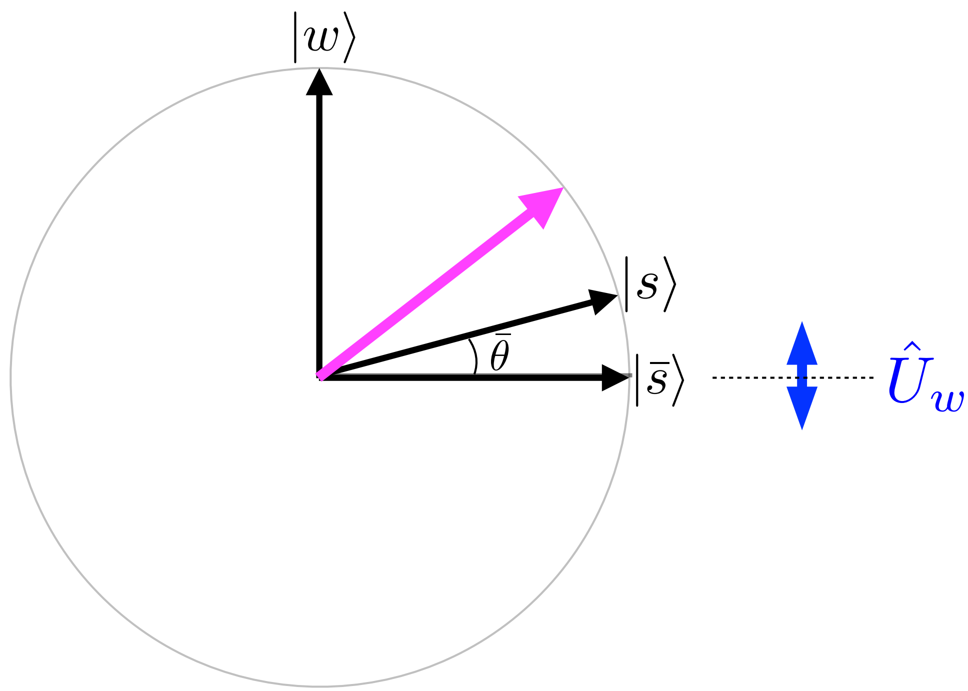

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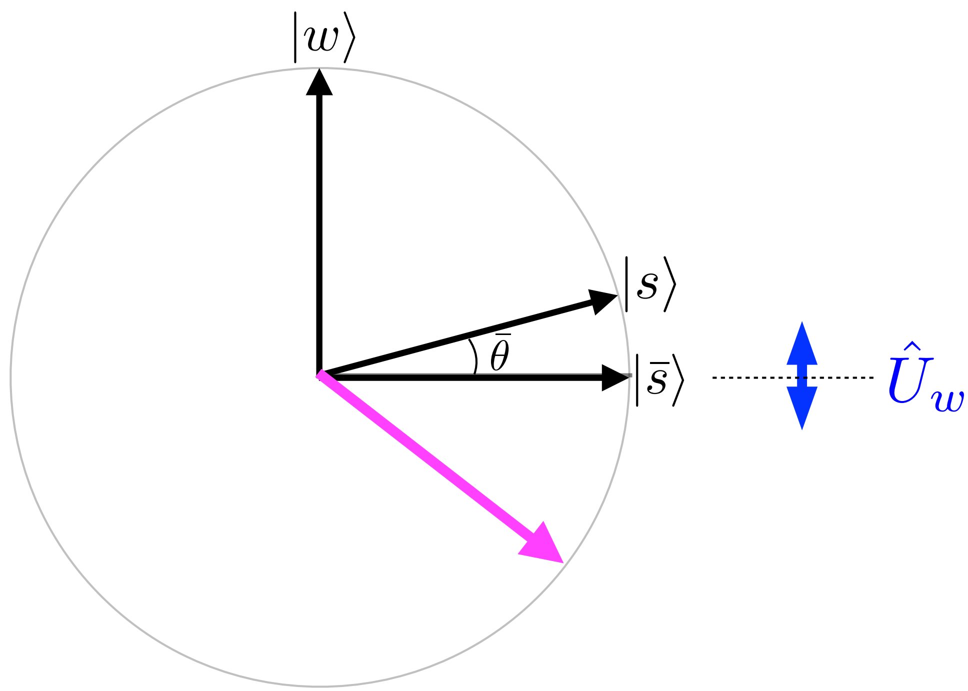

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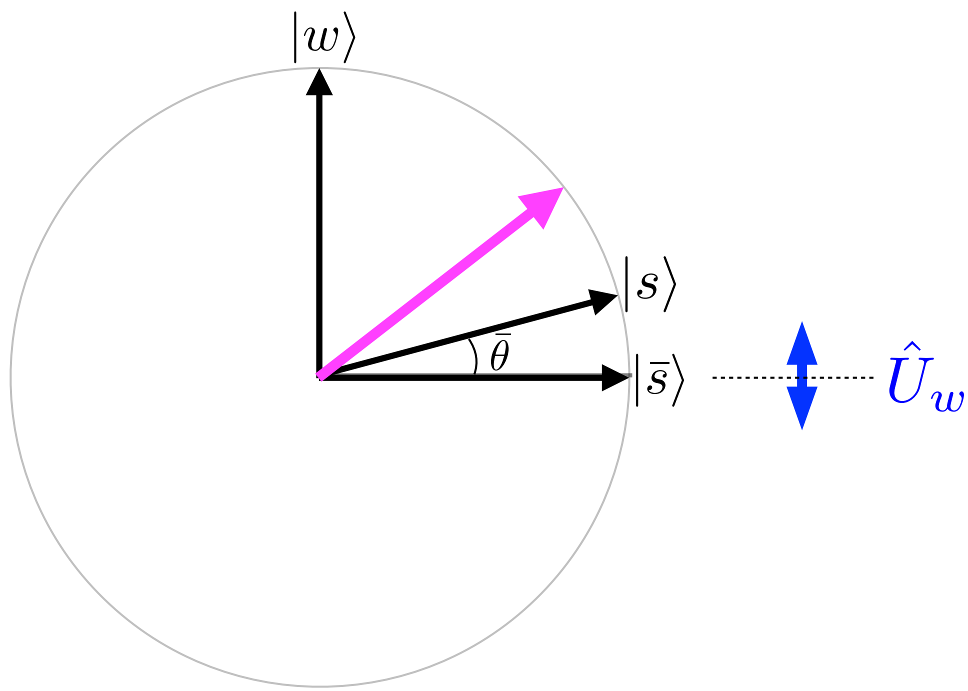

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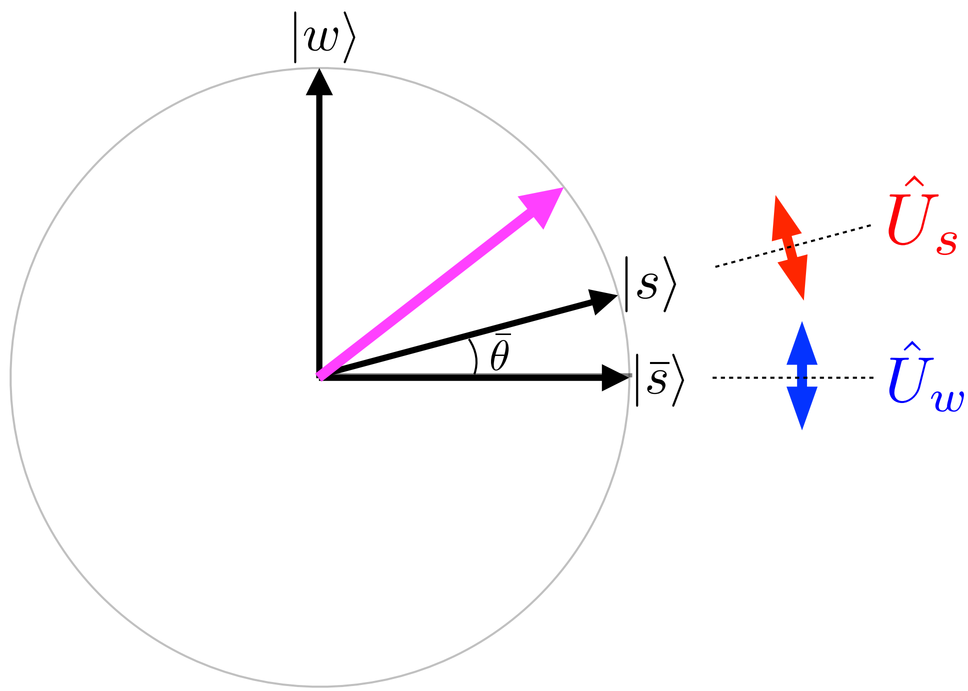

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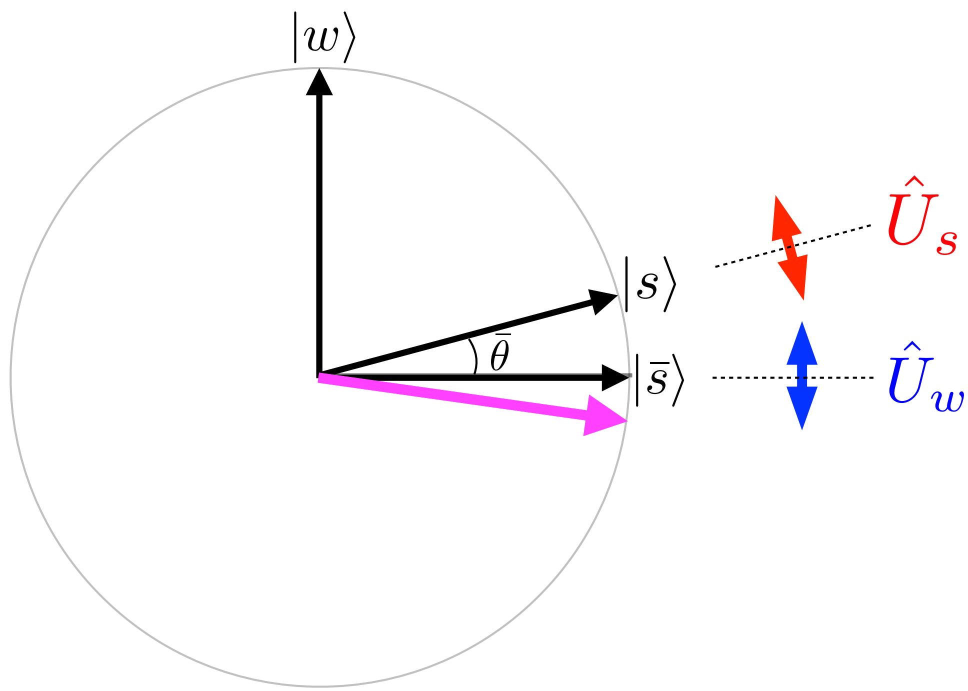

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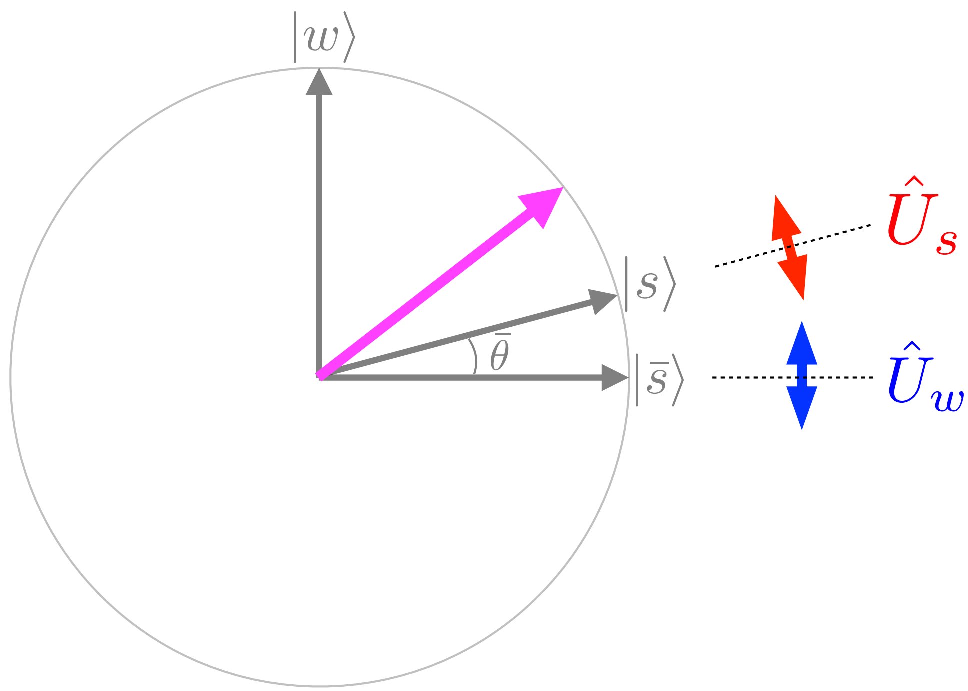

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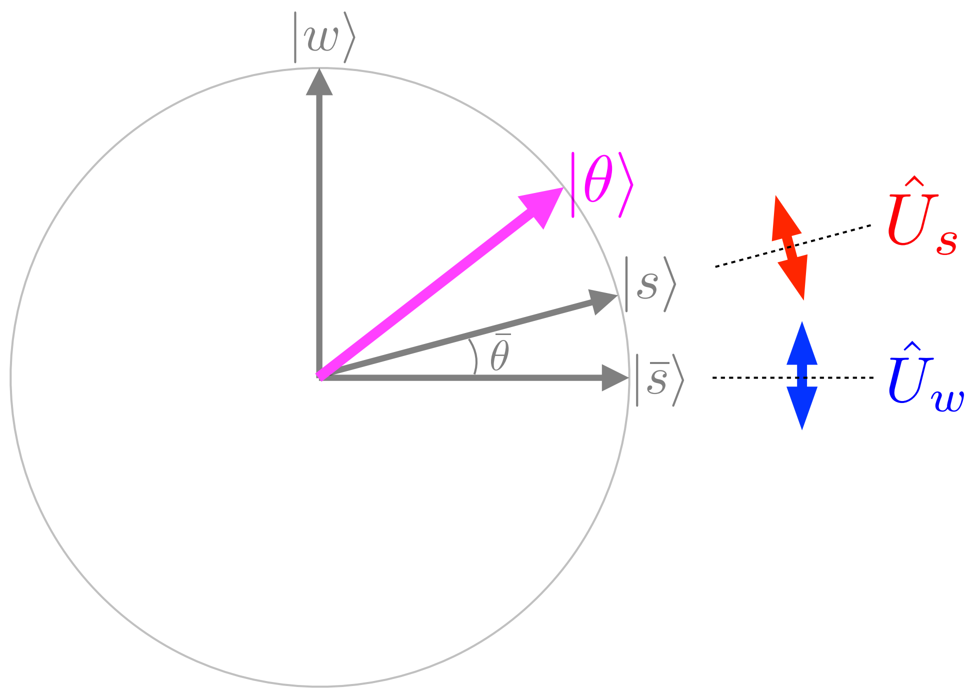

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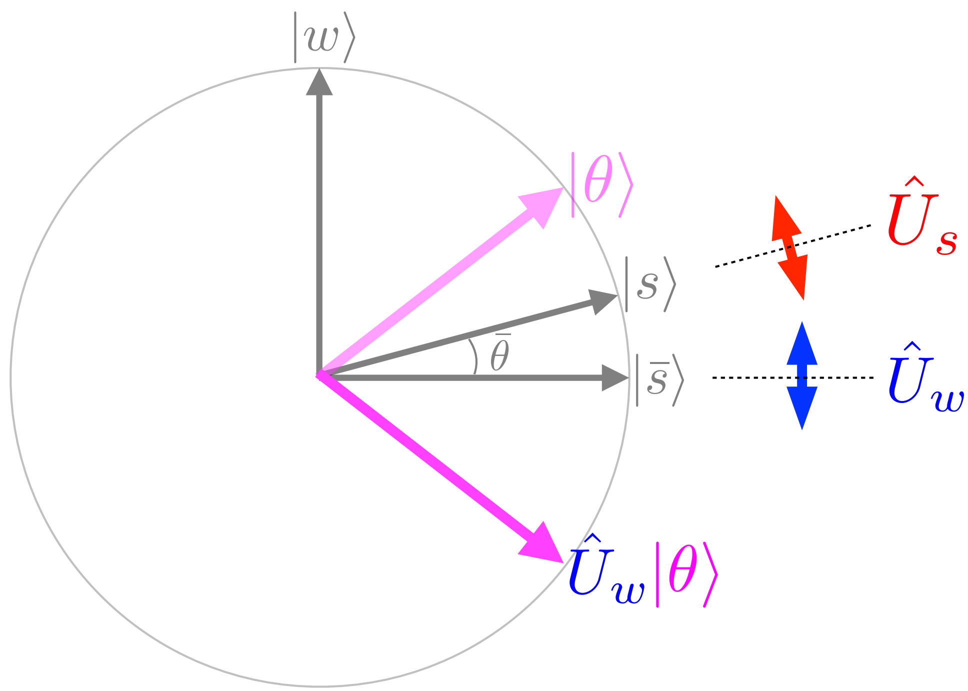

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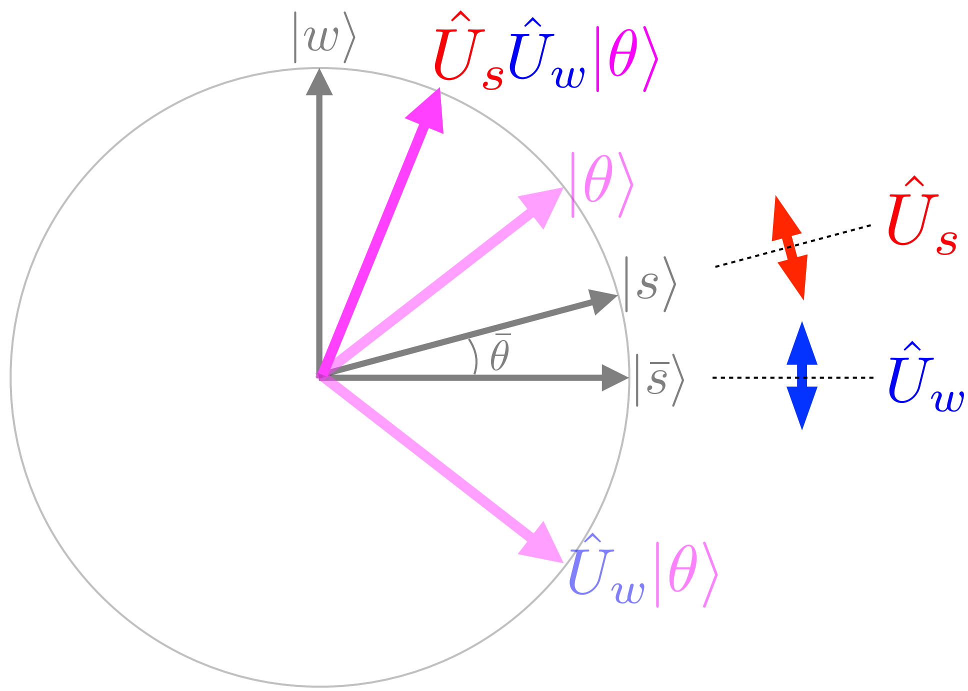

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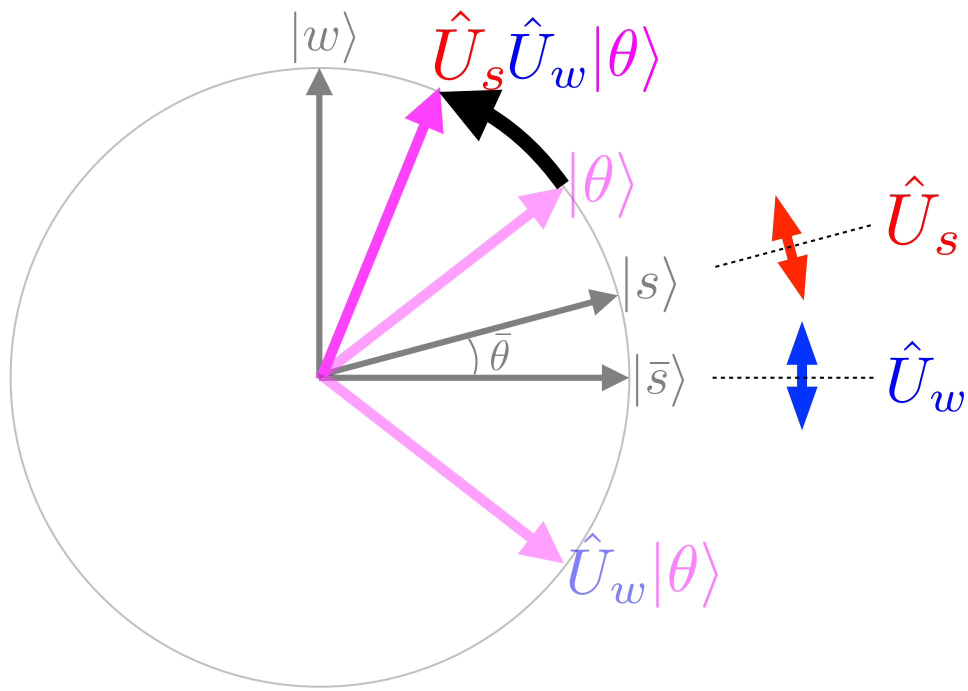

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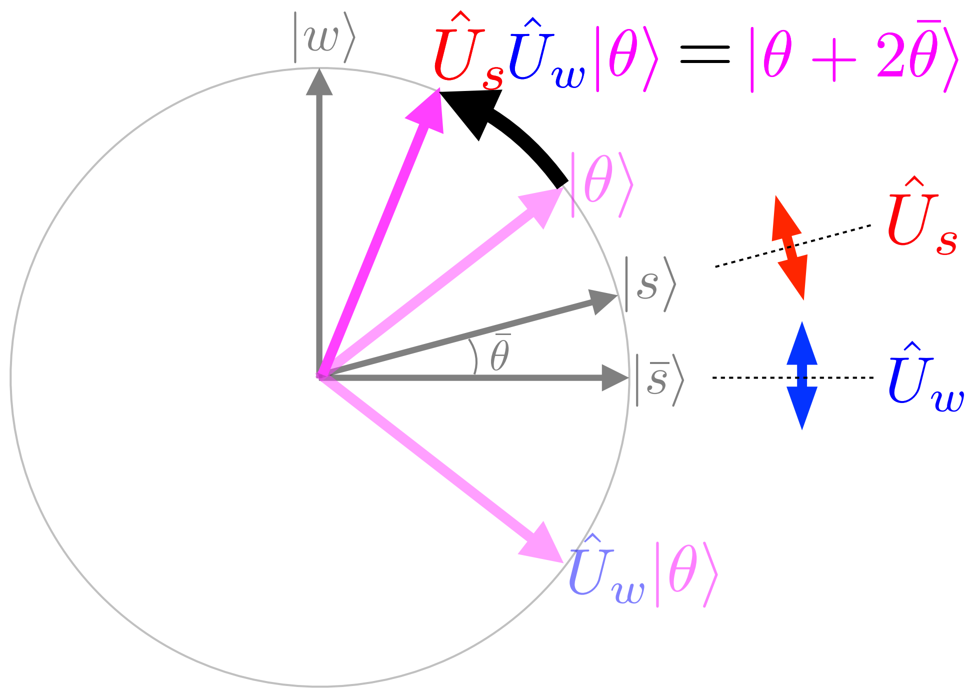
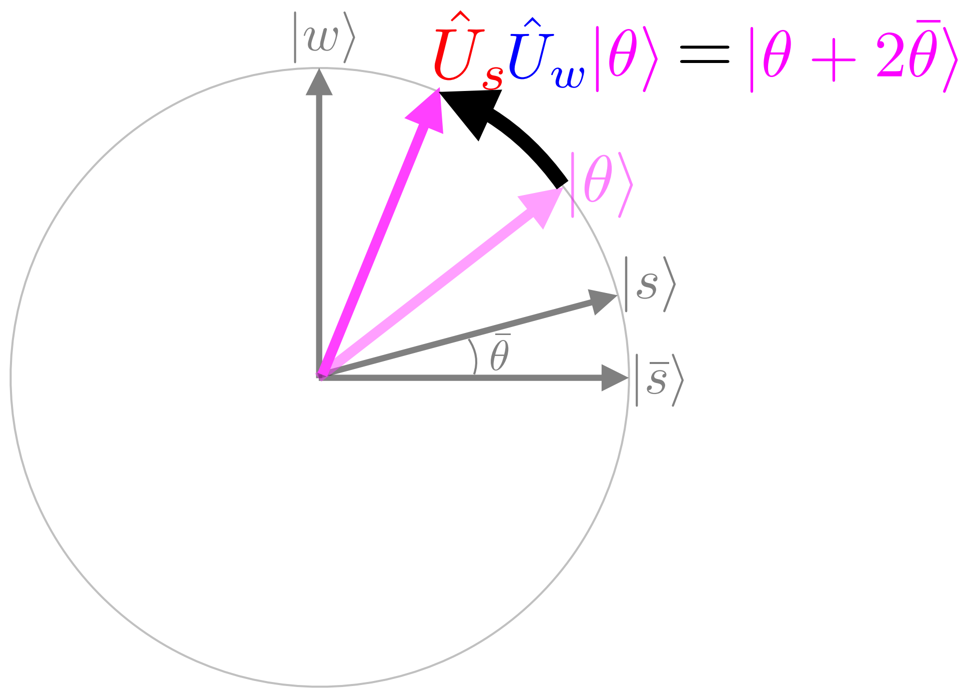


$$\sin\bar{\theta} \equiv \langle w|s\rangle = \frac{1}{\sqrt{d}}$$

$$\hat{U}_s \hat{U}_w |\theta\rangle = |\theta + 2\bar{\theta}\rangle$$



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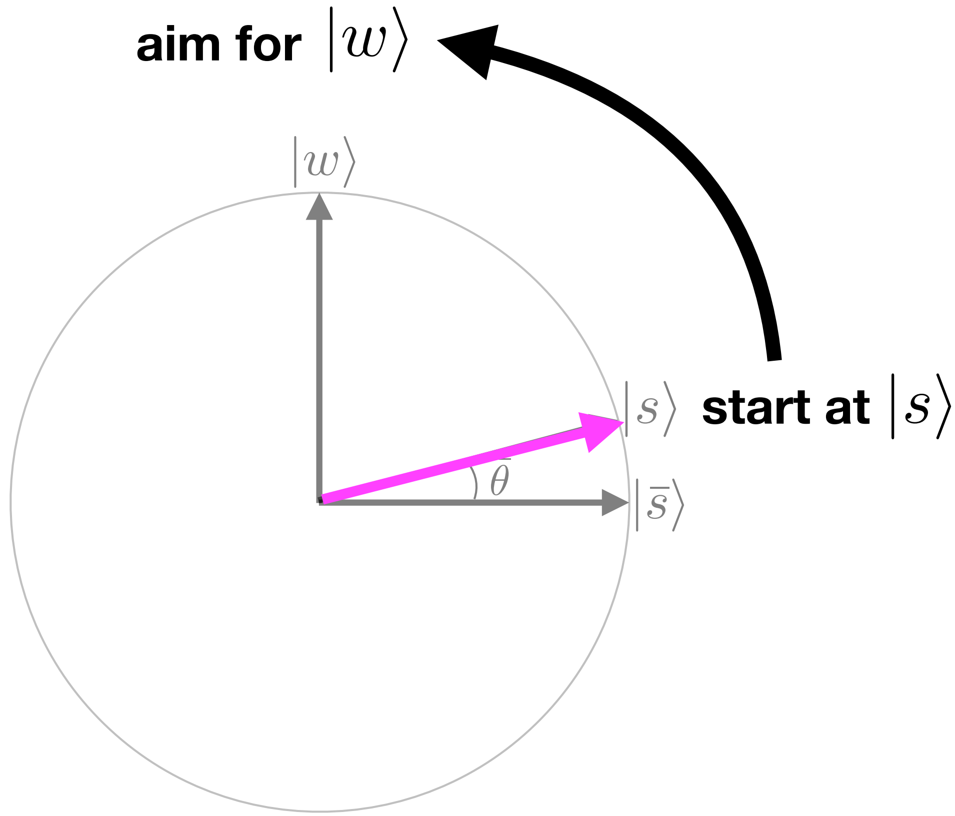

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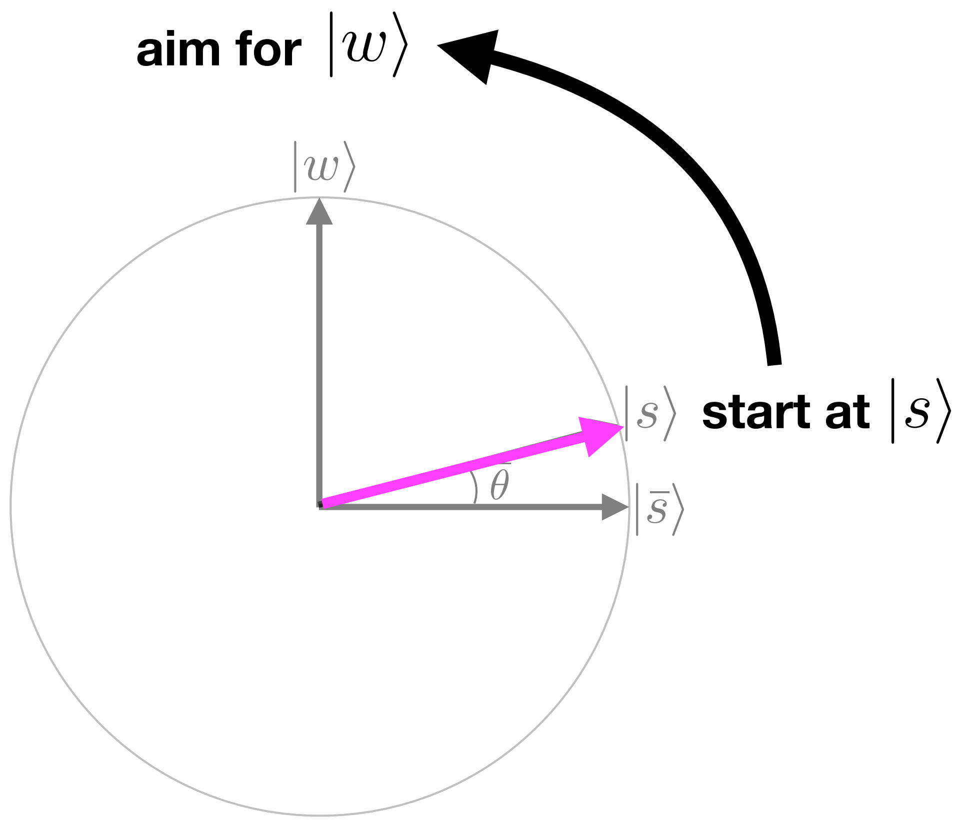
$$|s\rangle$$



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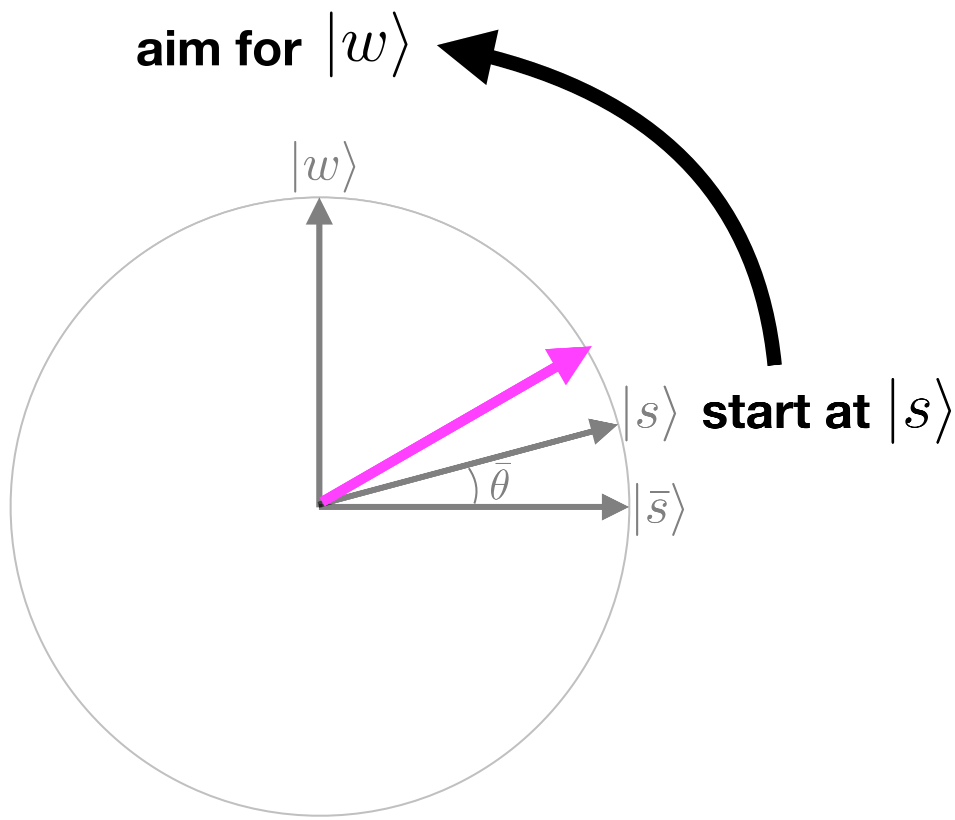

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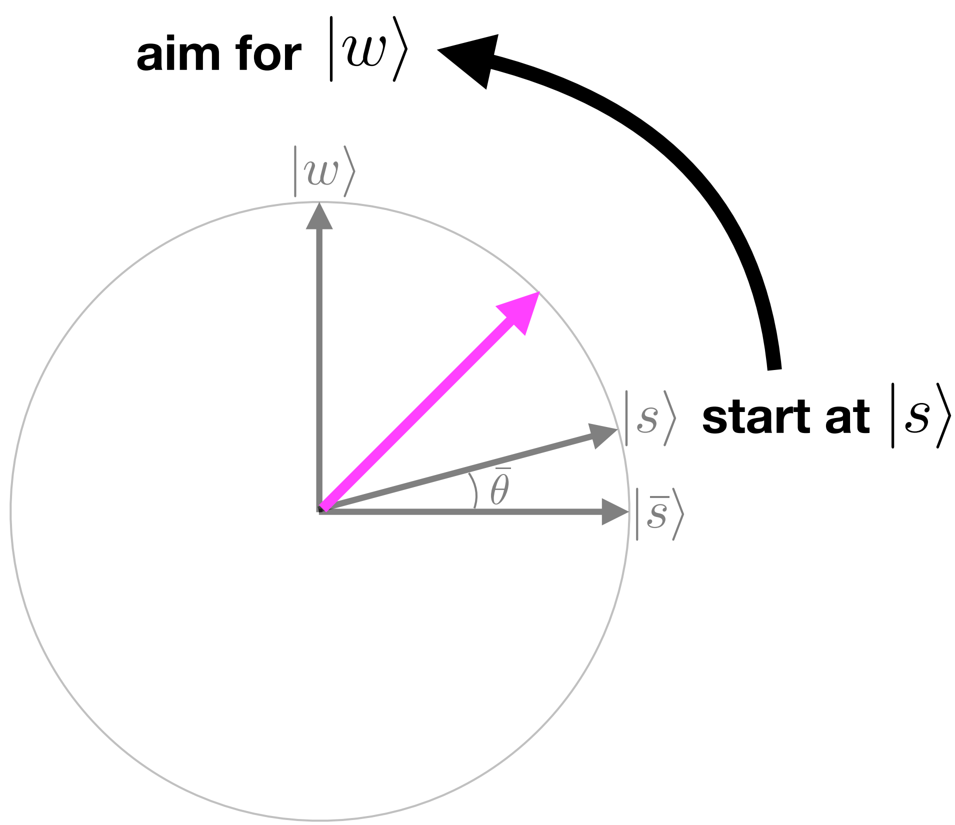

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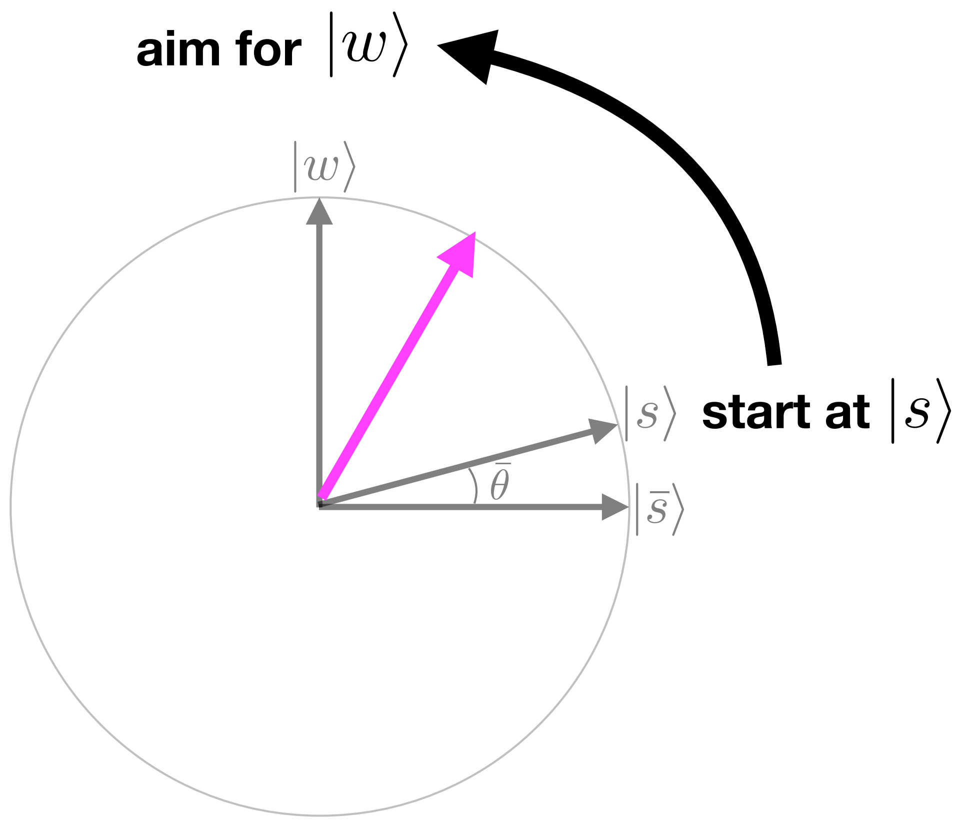

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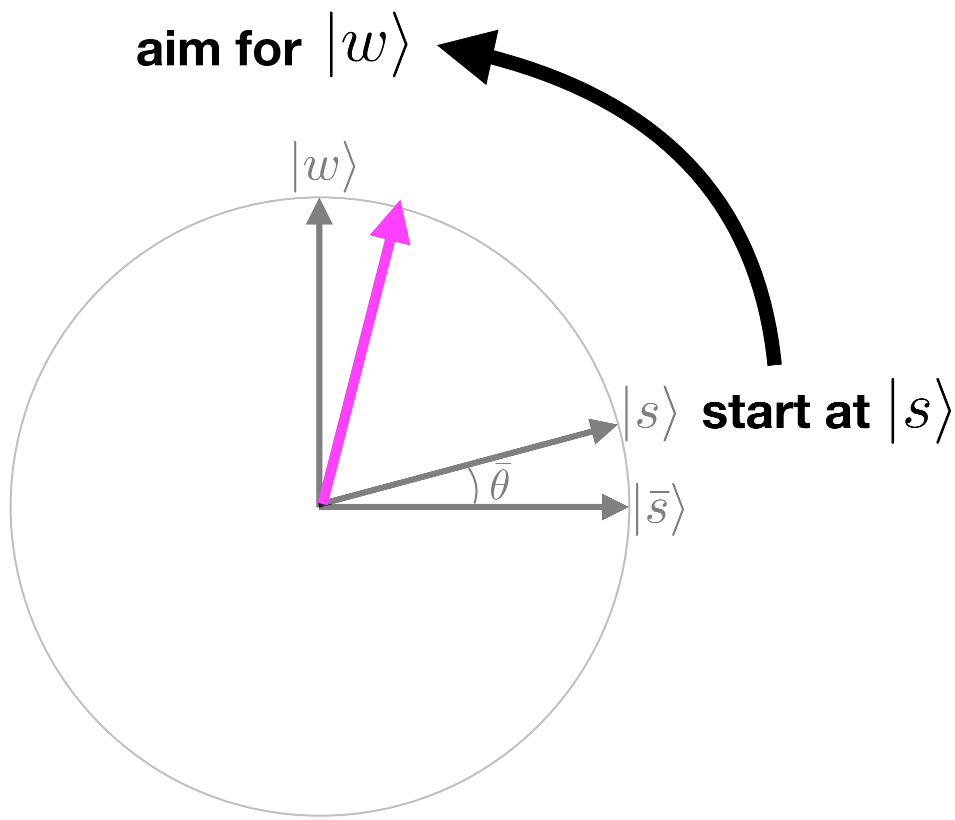

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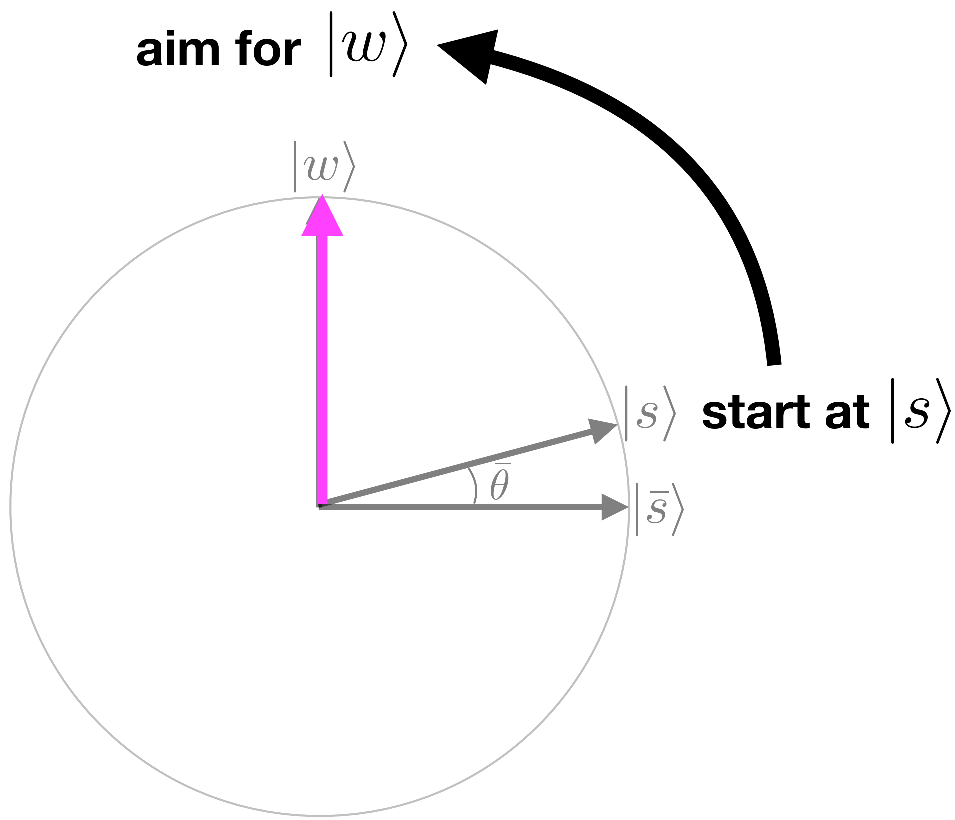

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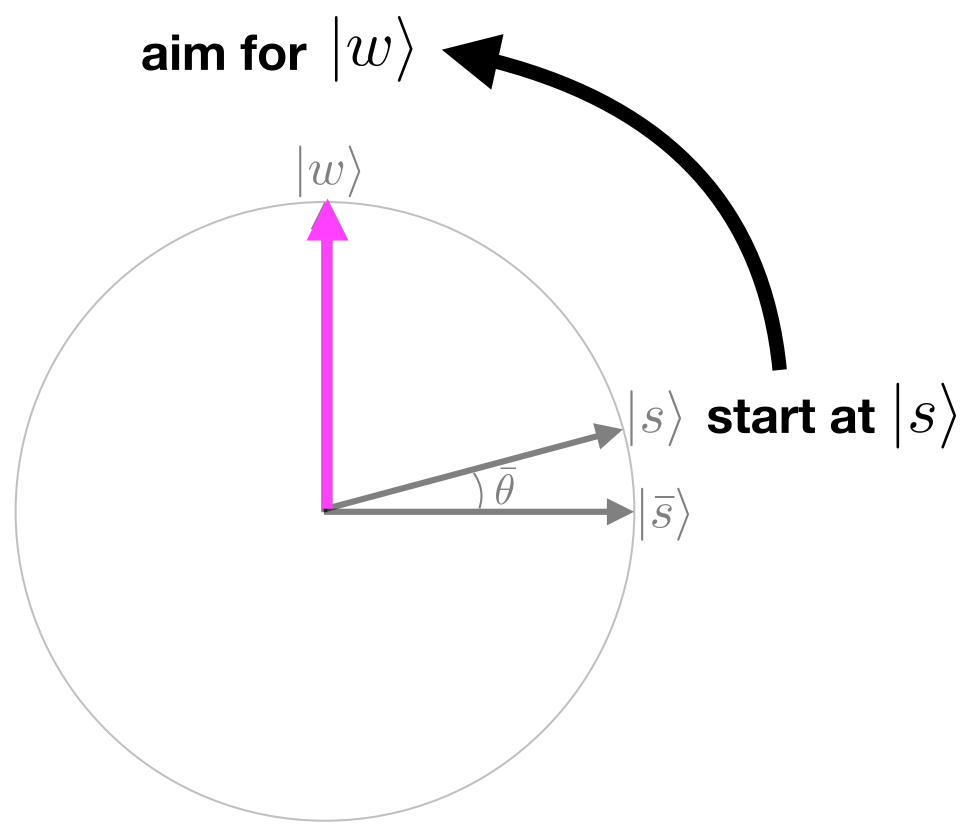

$$\sin\bar{\theta} \equiv \langle w|s\rangle = \frac{1}{\sqrt{d}}$$

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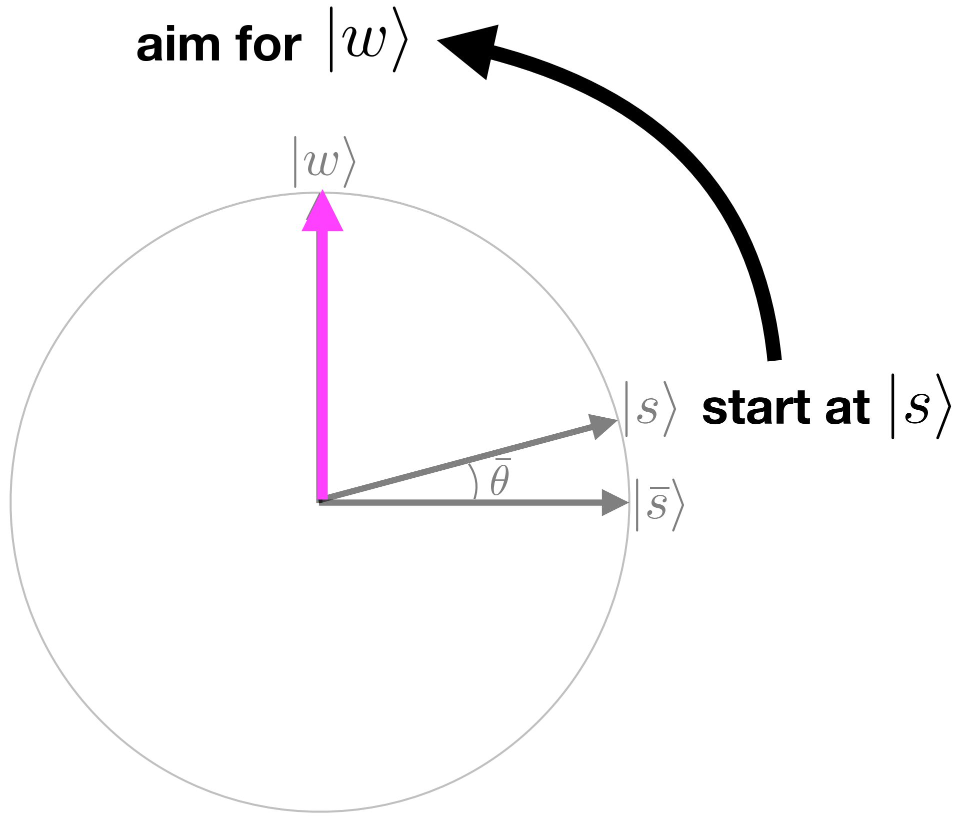
$$\langle\text{queries}\rangle = \left\lfloor \frac{\pi}{4}\sqrt{d-1} \right\rfloor$$



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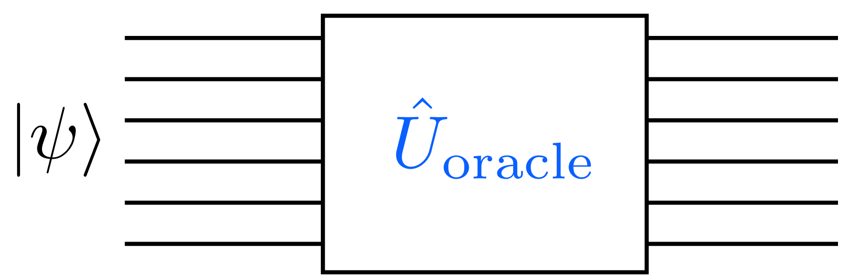
Grover 1996

VOLUME 79, NUMBER 2 PHYSICAL REVIEW LETTERS 14 JULY 1997

Quantum Mechanics Helps in Searching for a Needle in a Haystack

Lov K. Grover*
3C-404A Bell Labs, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 4 December 1996)

Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing N names arranged in completely random order. To find someone's phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database.
[S0031-9007(97)03564-3]



$$\langle \text{queries} \rangle = \left\lfloor \frac{\pi}{4} \sqrt{d-1} \right\rfloor$$

Galperin 1995 ⟷ 1912.02207 ⟷ Grover 1996

G. GALPERIN
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PLAYING POOL WITH π (THE NUMBER π FROM A BILLIARD POINT OF VIEW)

Received December 9, 2003 DOI: 10.1070/RD2003v008n04ABEH000252

Counting collisions in a simple dynamical system with two billiard balls can be used to estimate π to any accuracy.

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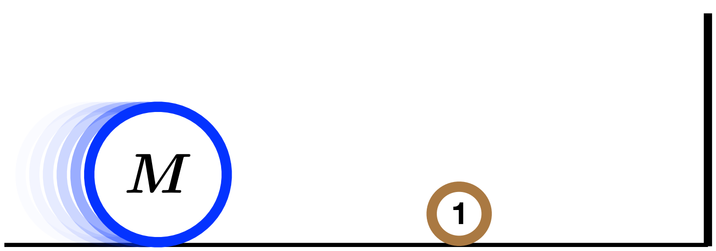
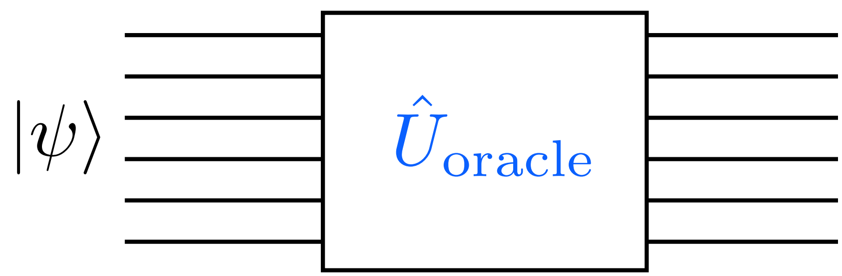
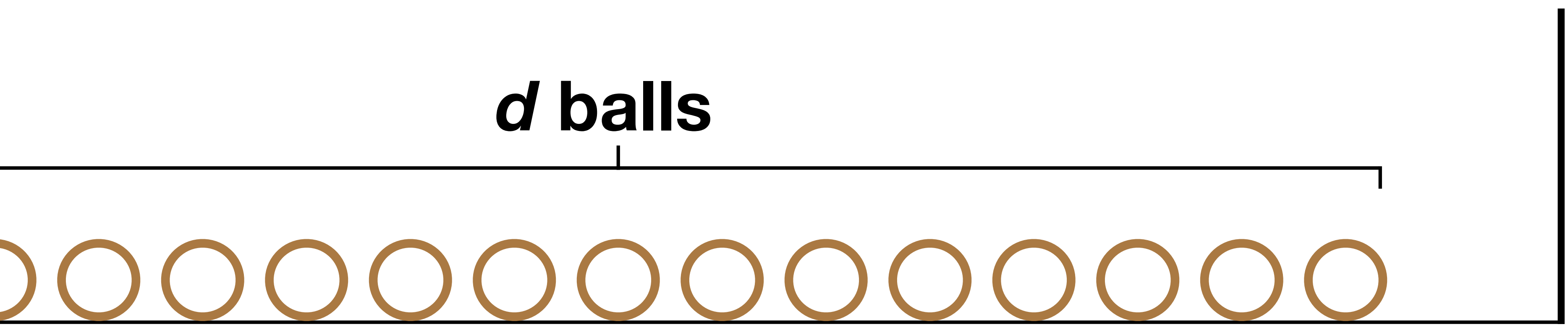


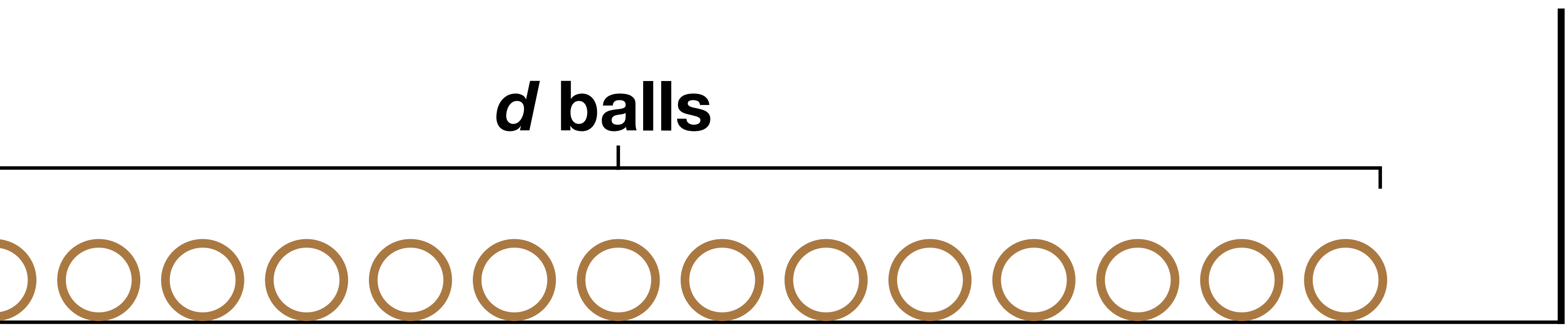


$$\#_{\text{collisions}} = \left\lfloor \pi\sqrt{M} \right\rfloor$$

$$\langle \text{queries} \rangle = \left\lfloor \frac{\pi}{4}\sqrt{d-1} \right\rfloor$$

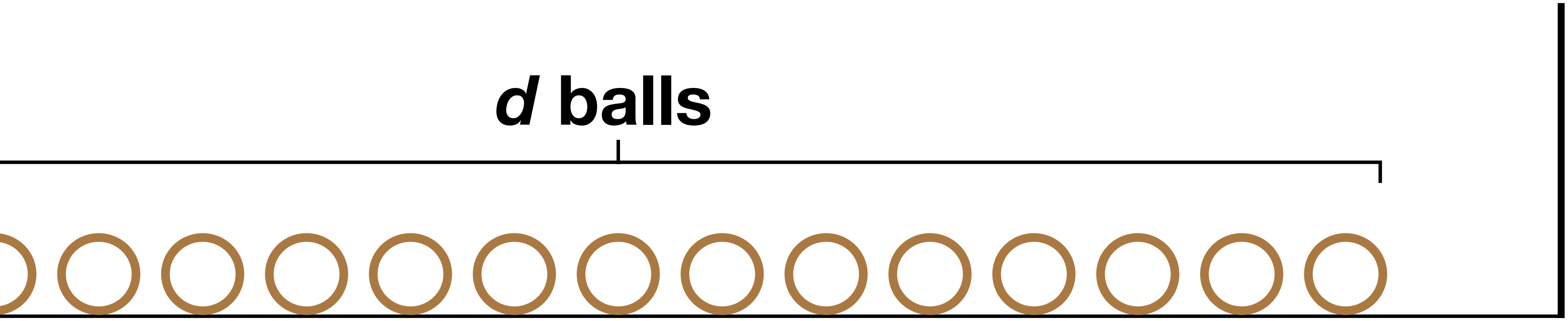

d balls

velocity vector is v^i



velocity vector is v^i

$$|v\rangle = v_1|\mathbf{1}\rangle + v_2\,|\mathbf{2}\rangle + \ldots + v_w\,|w\rangle + \ldots + v_d\,|\mathbf{d}\rangle$$



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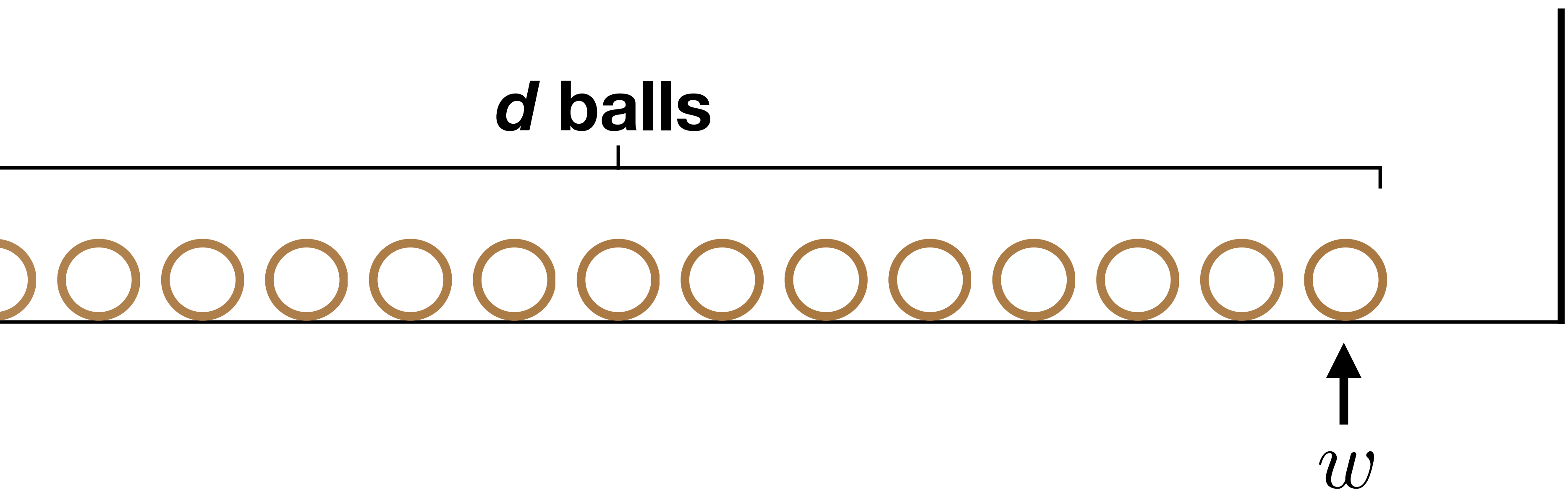

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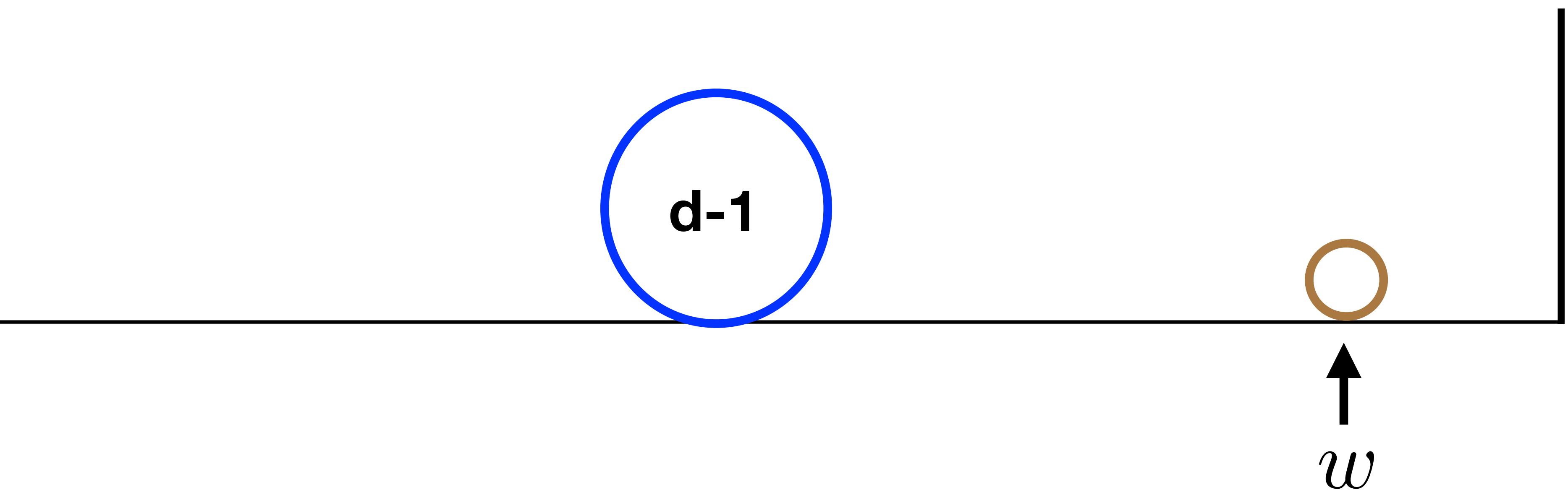

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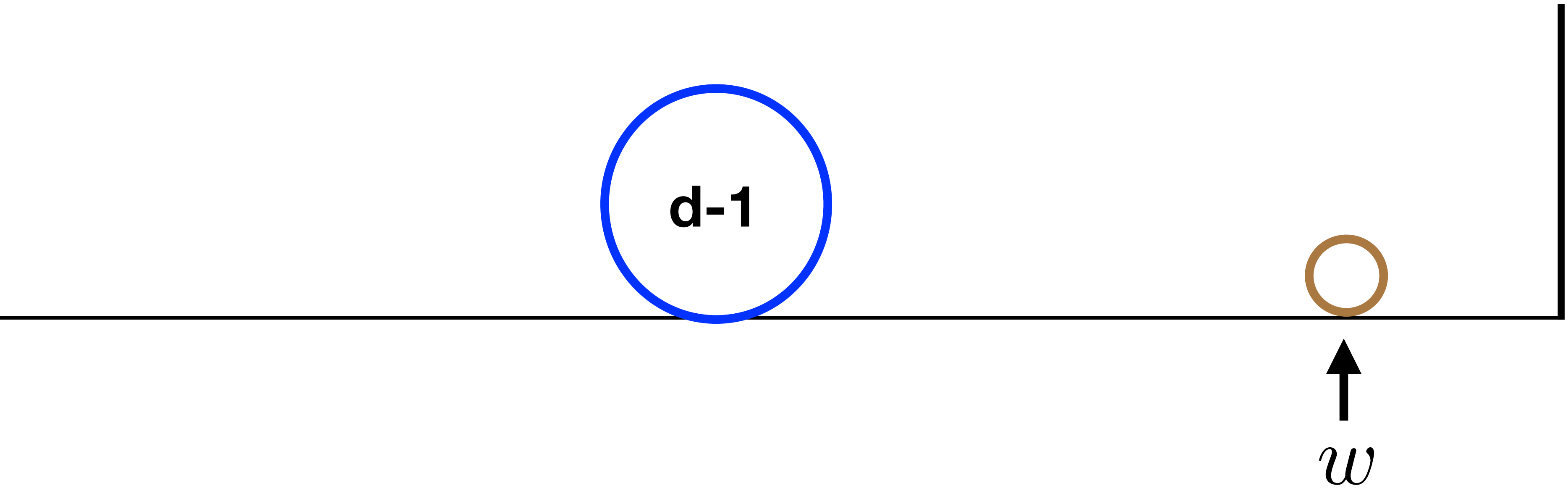

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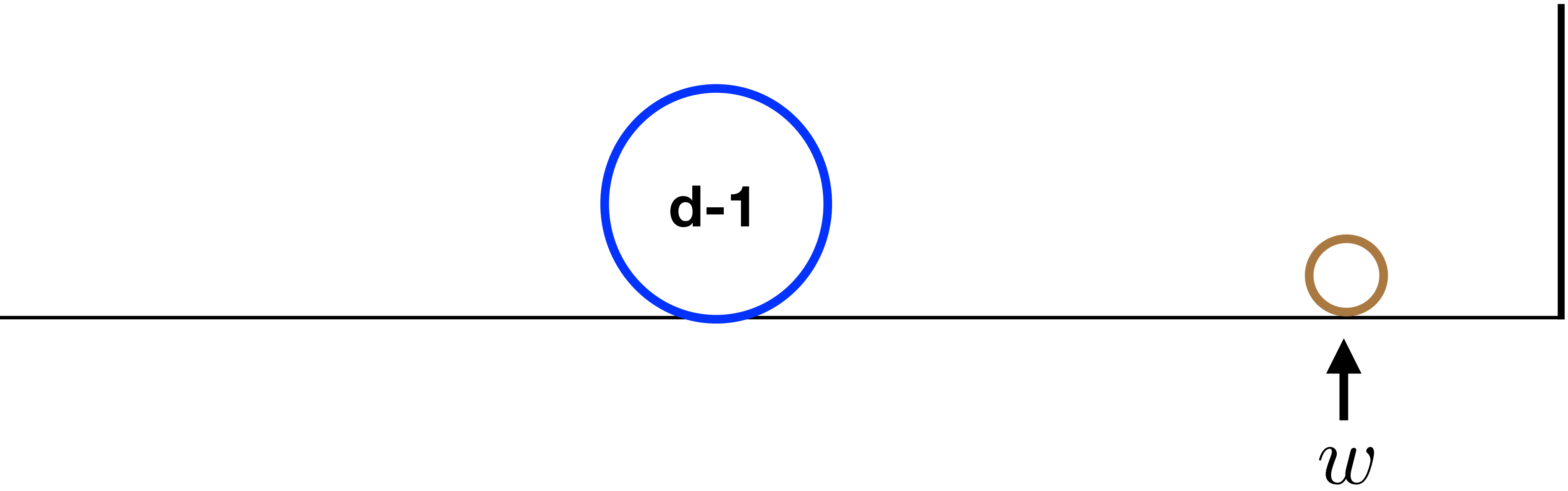

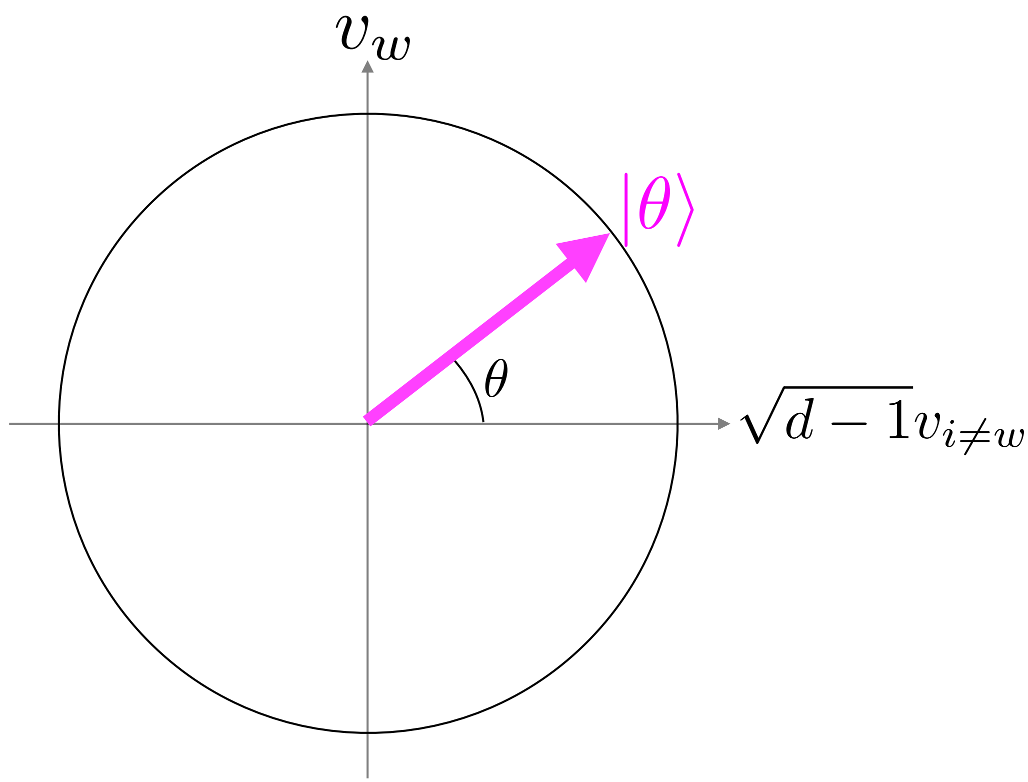

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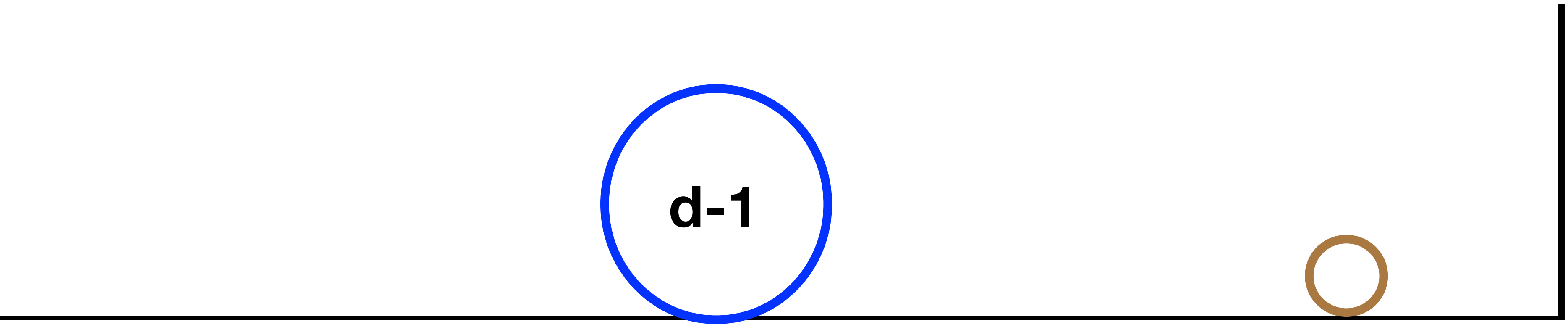

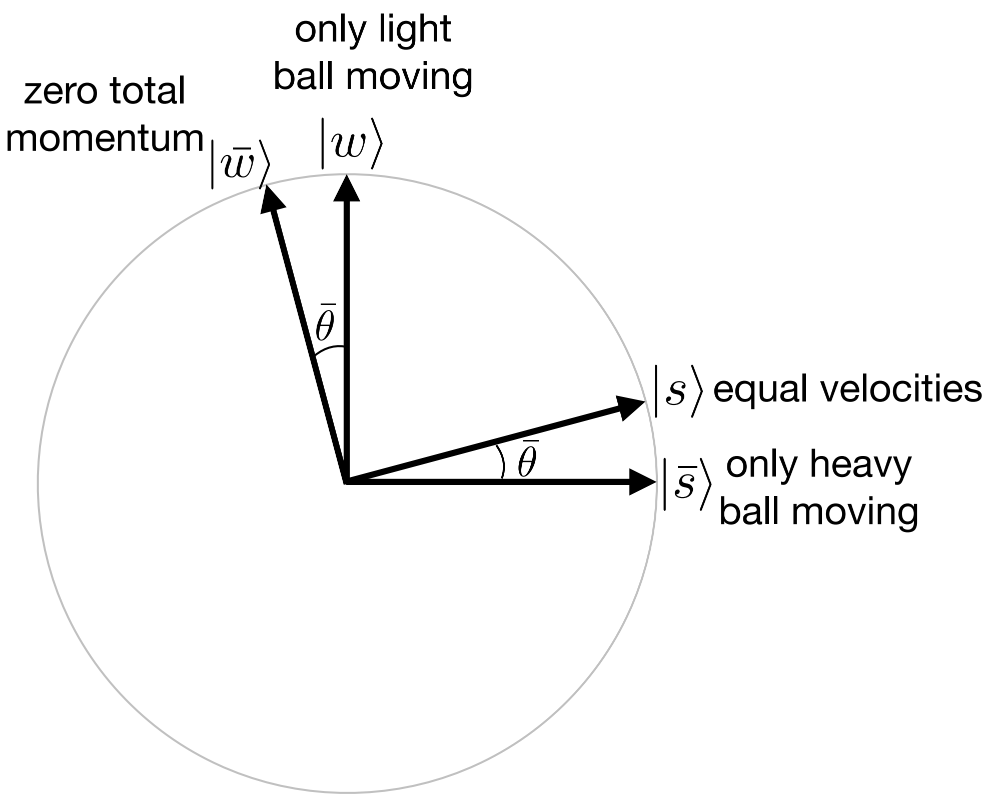

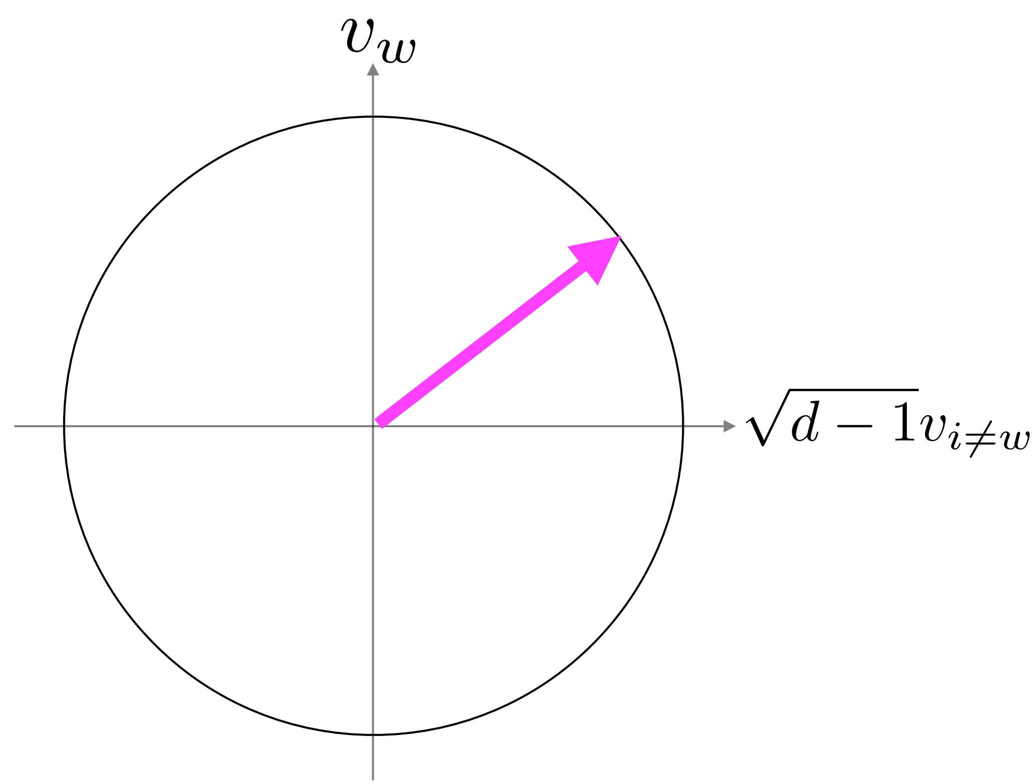

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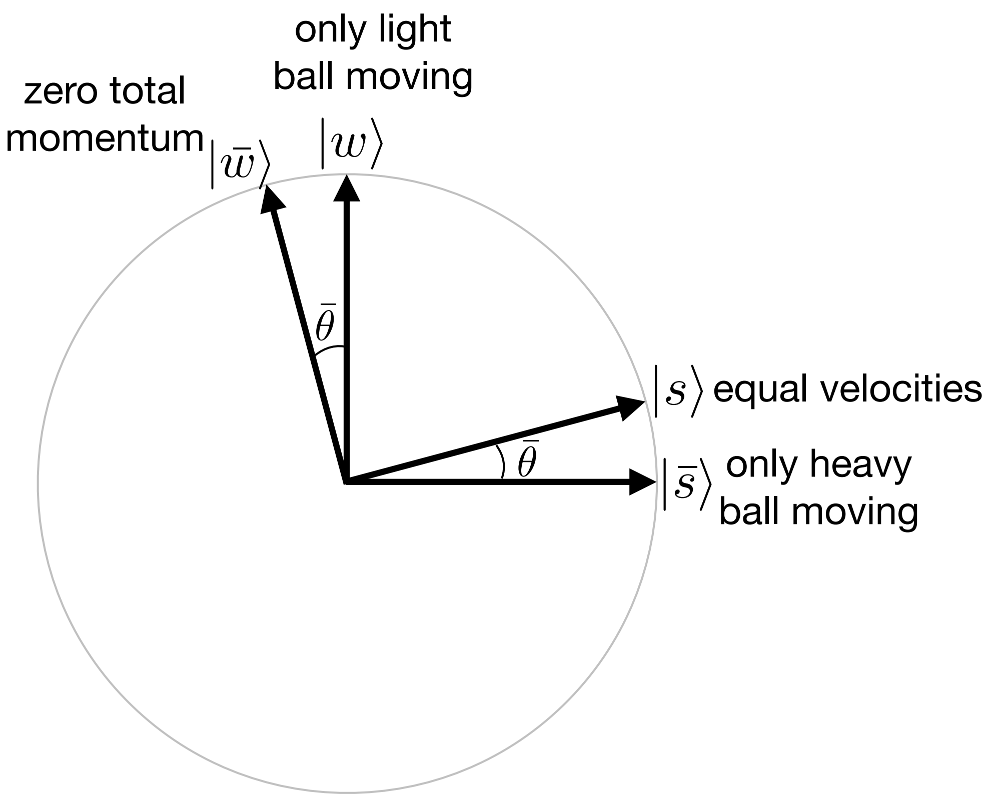

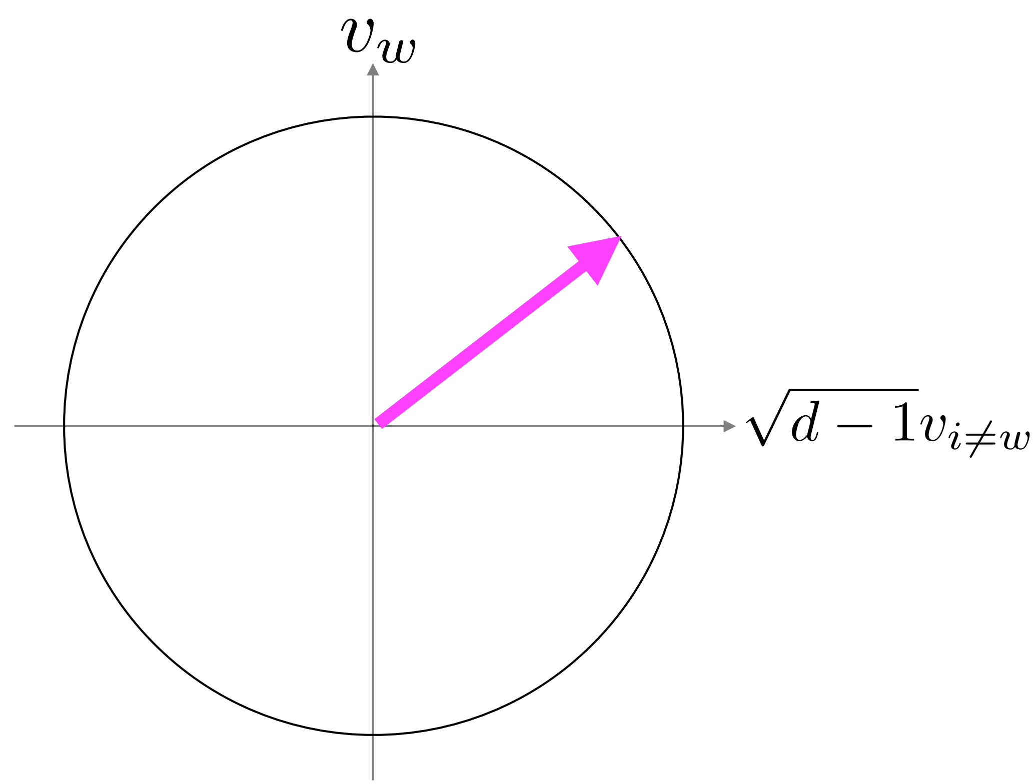

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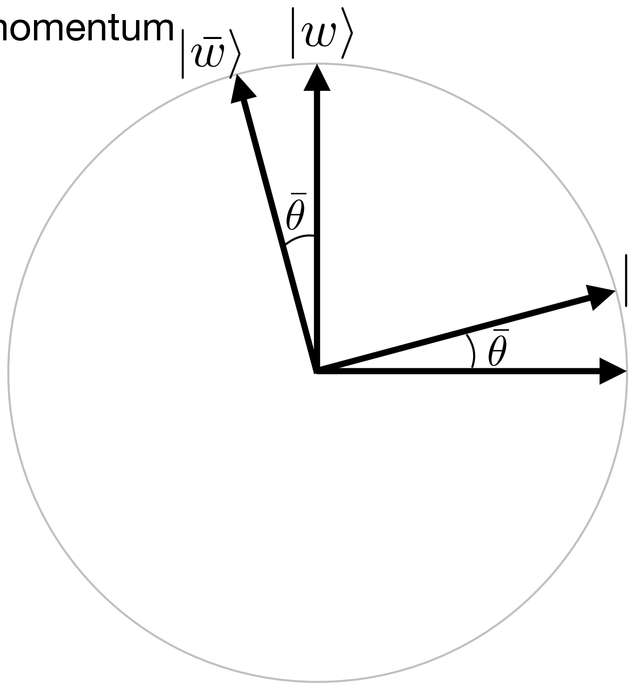

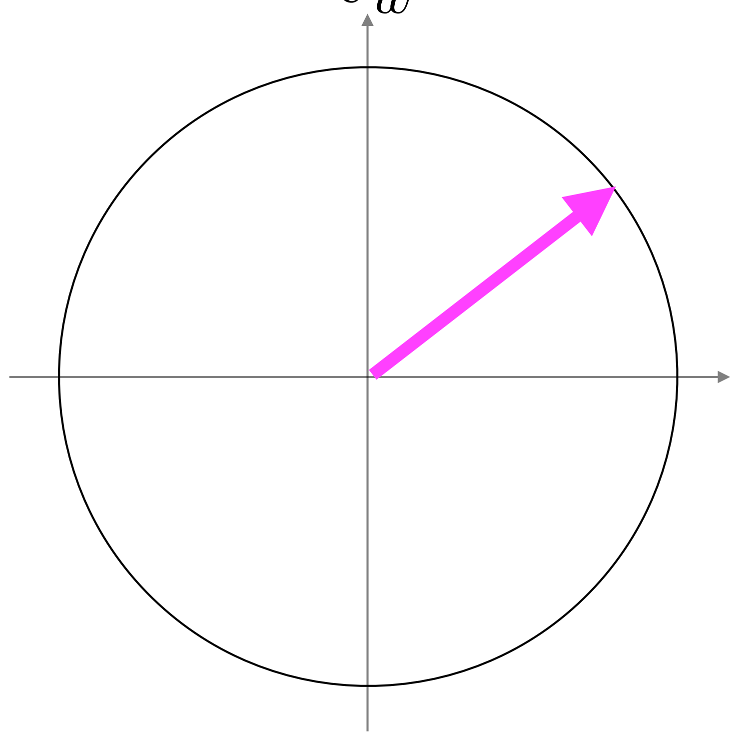

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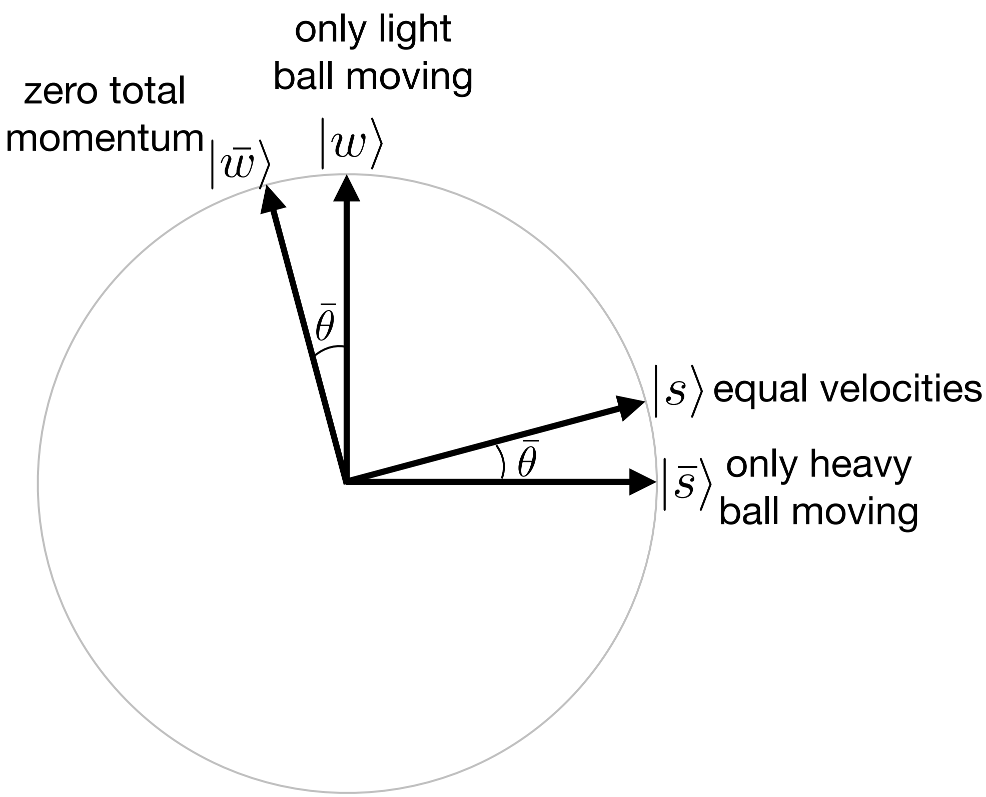

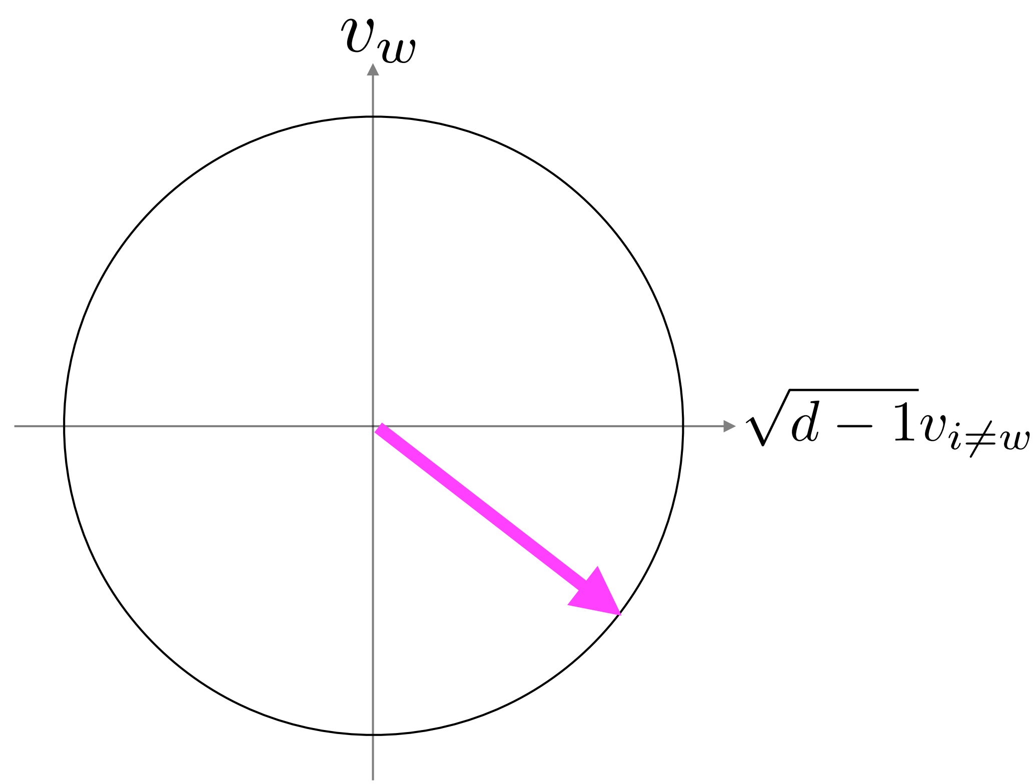

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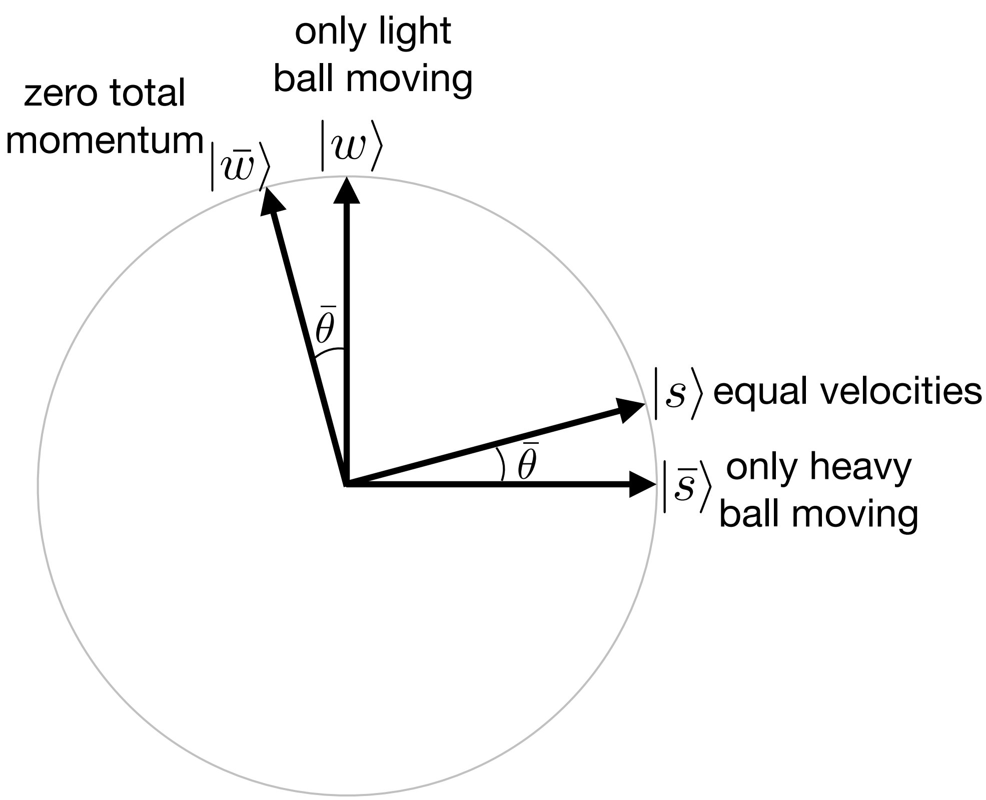

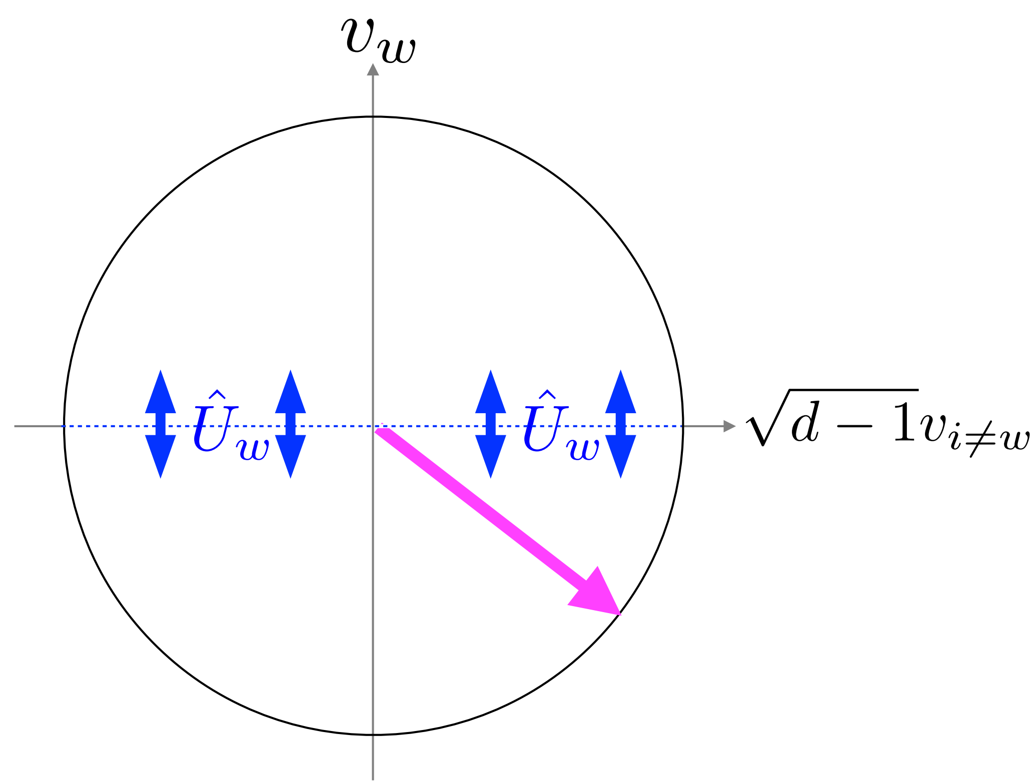

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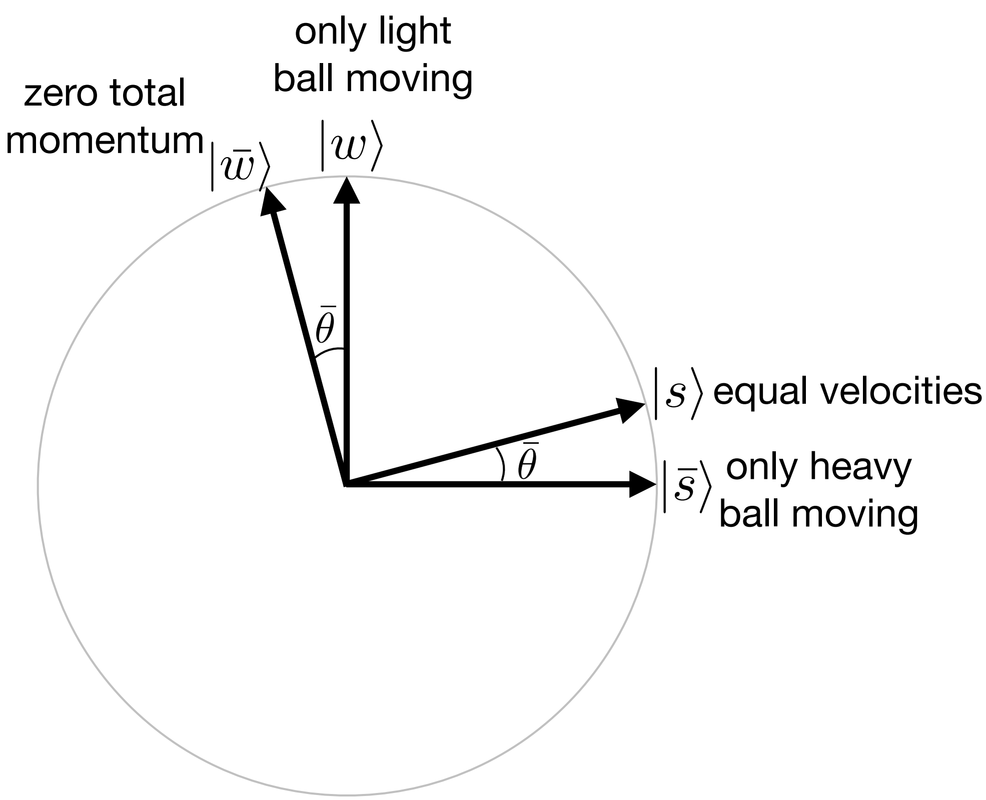

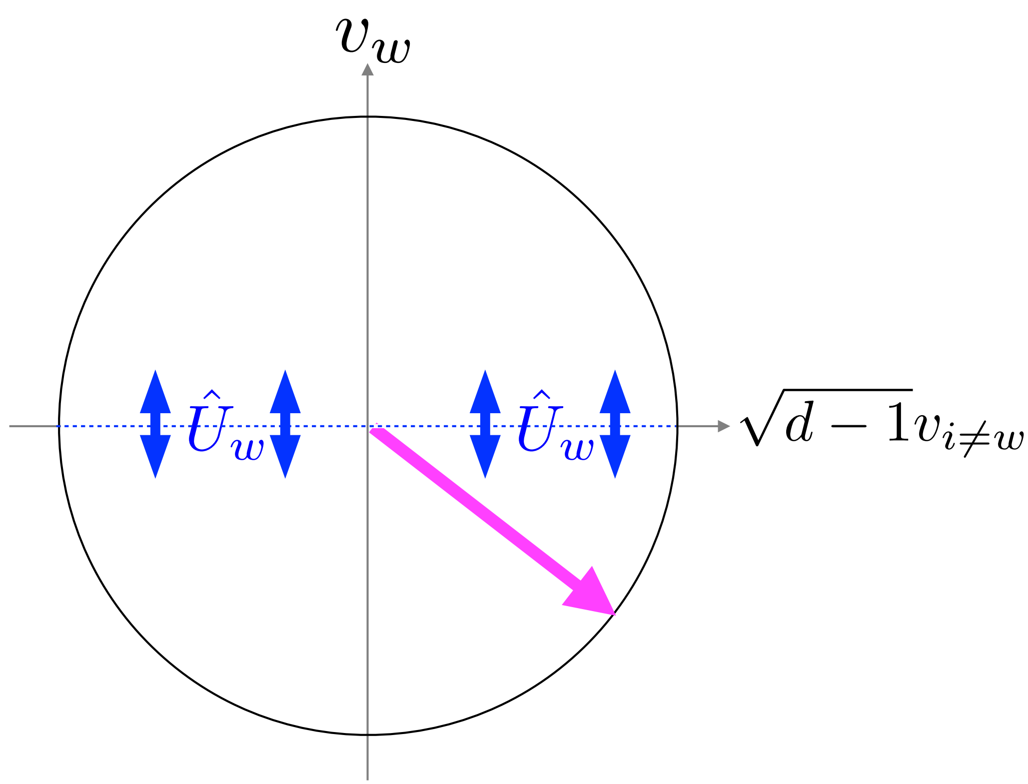

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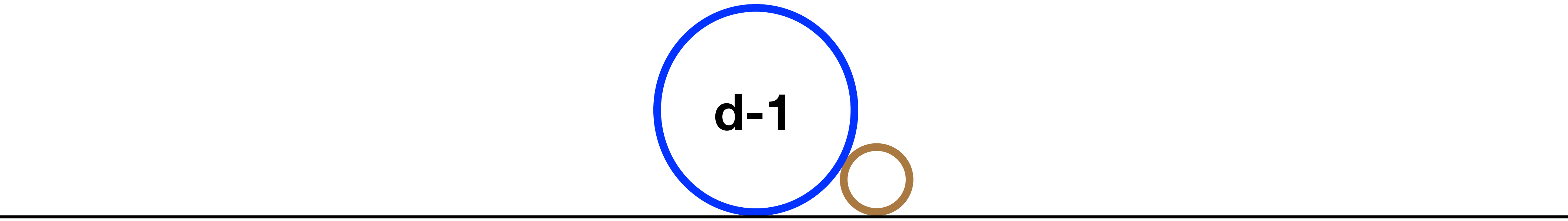
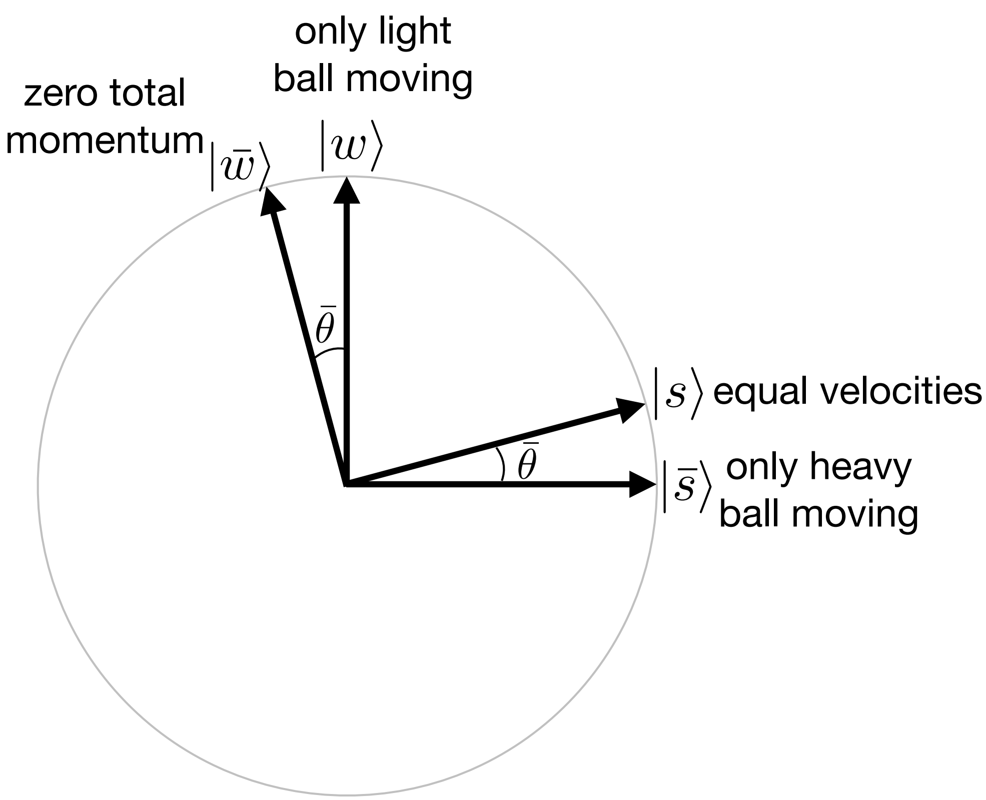
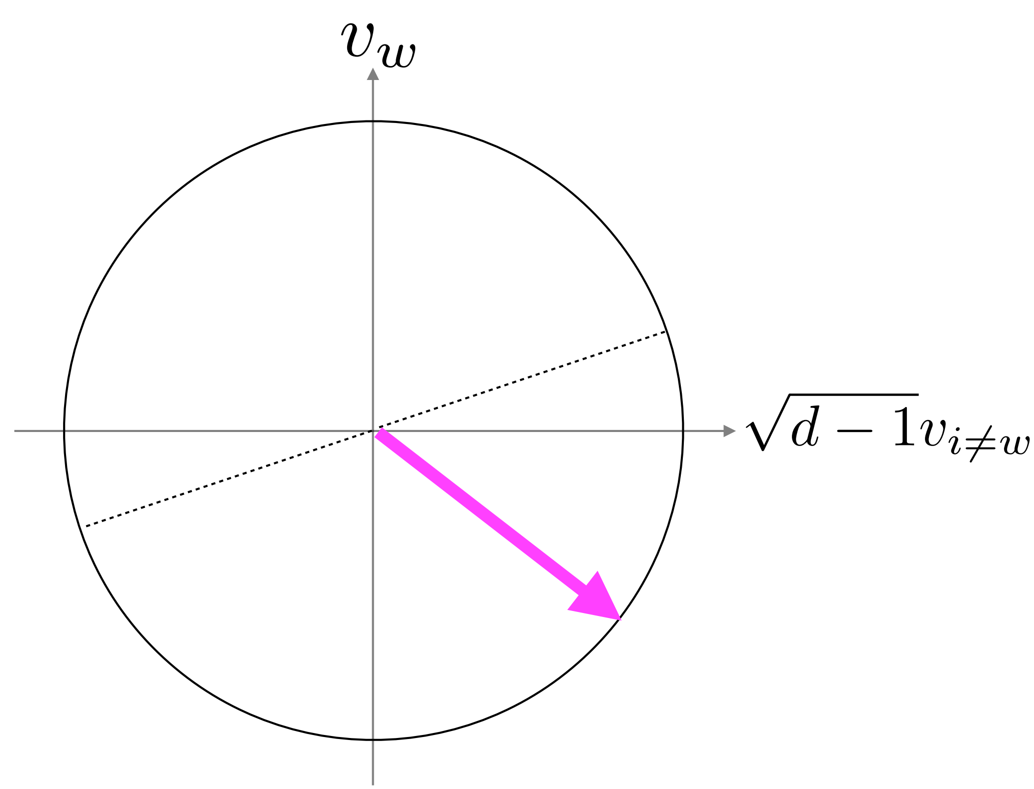


$\hat{U}_s$





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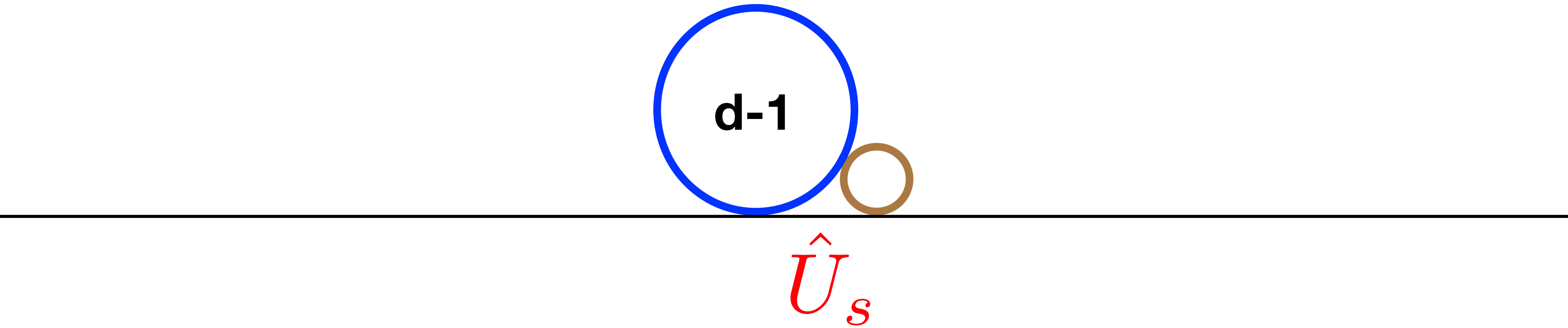

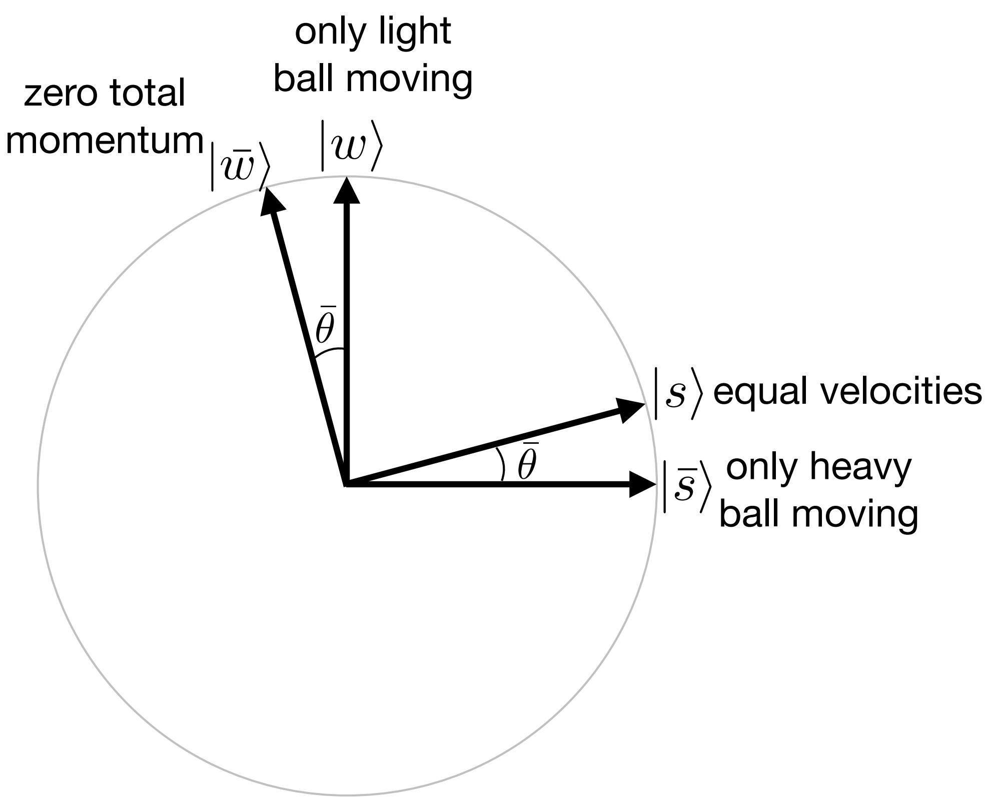

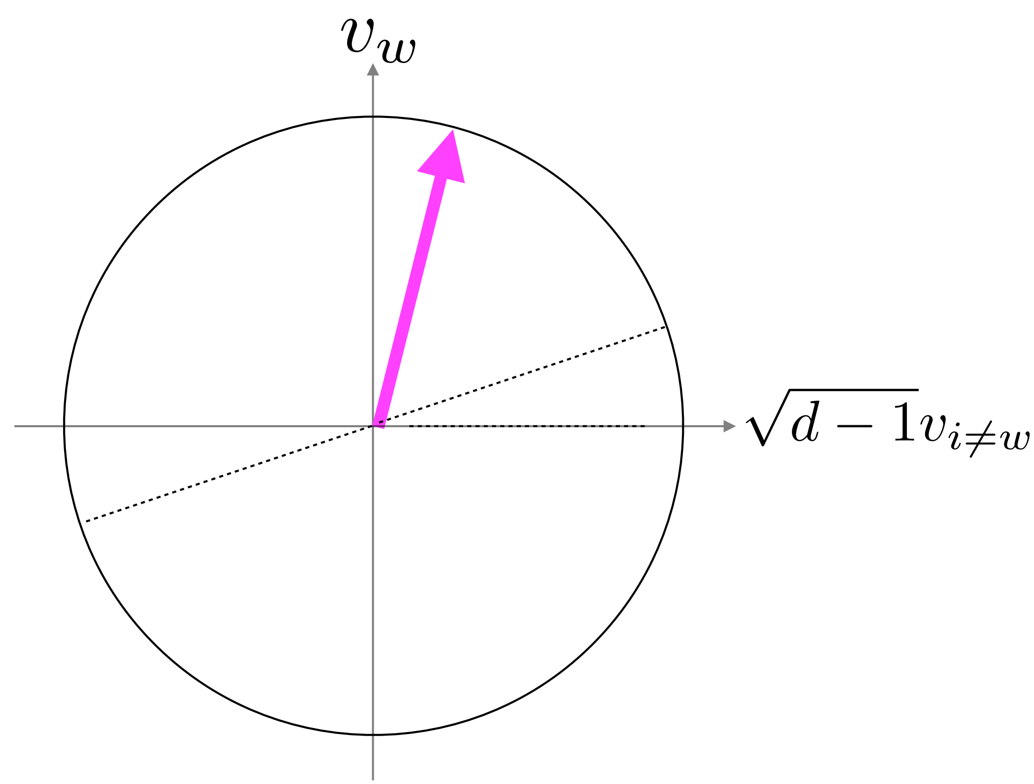

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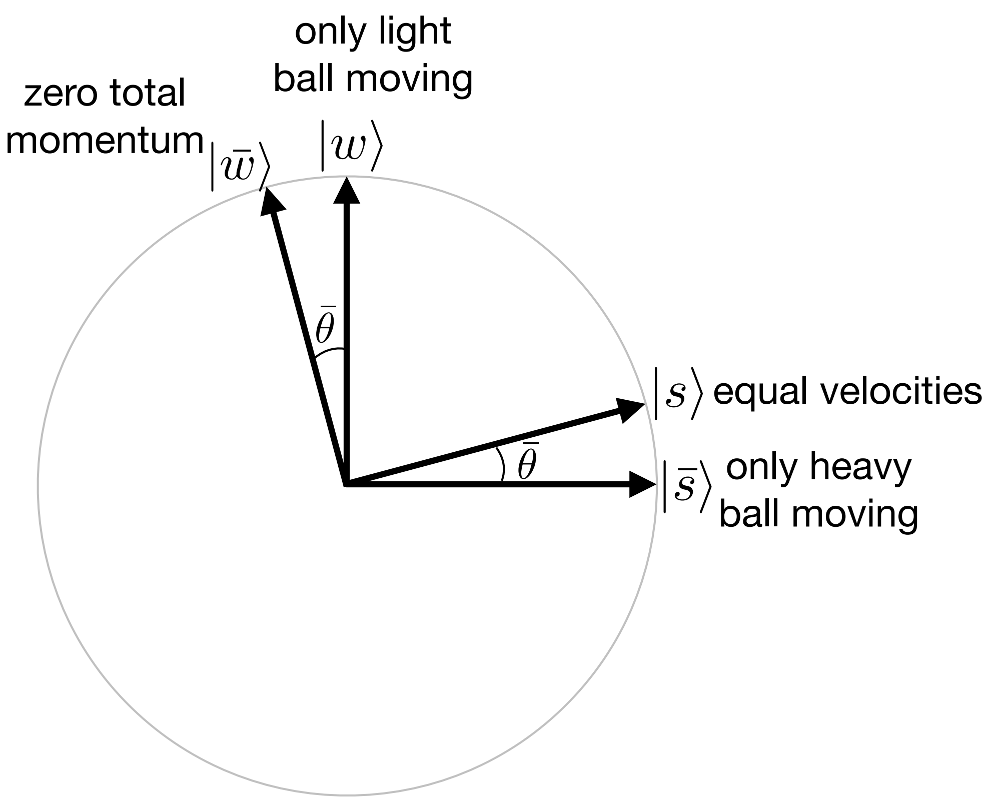

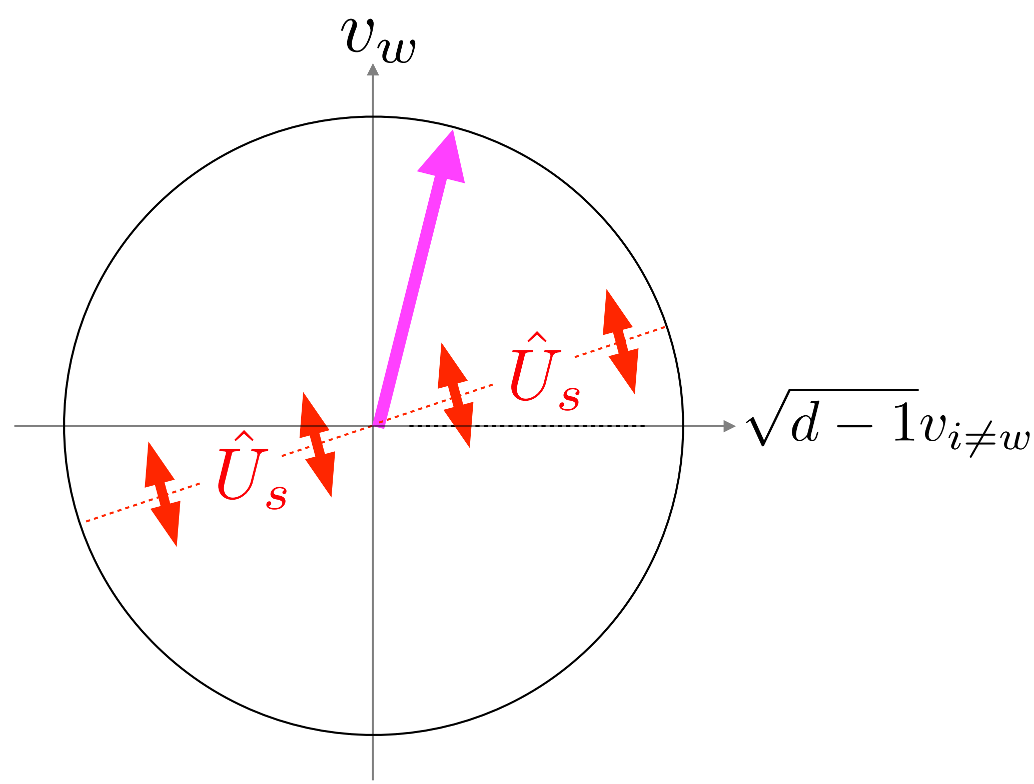

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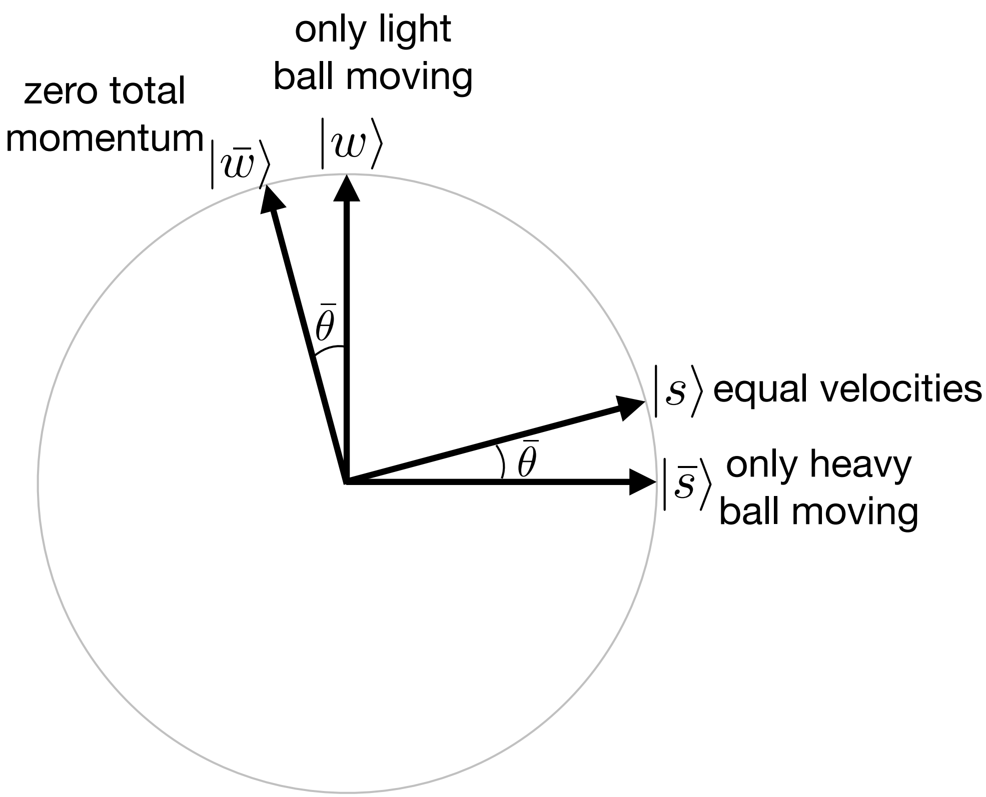

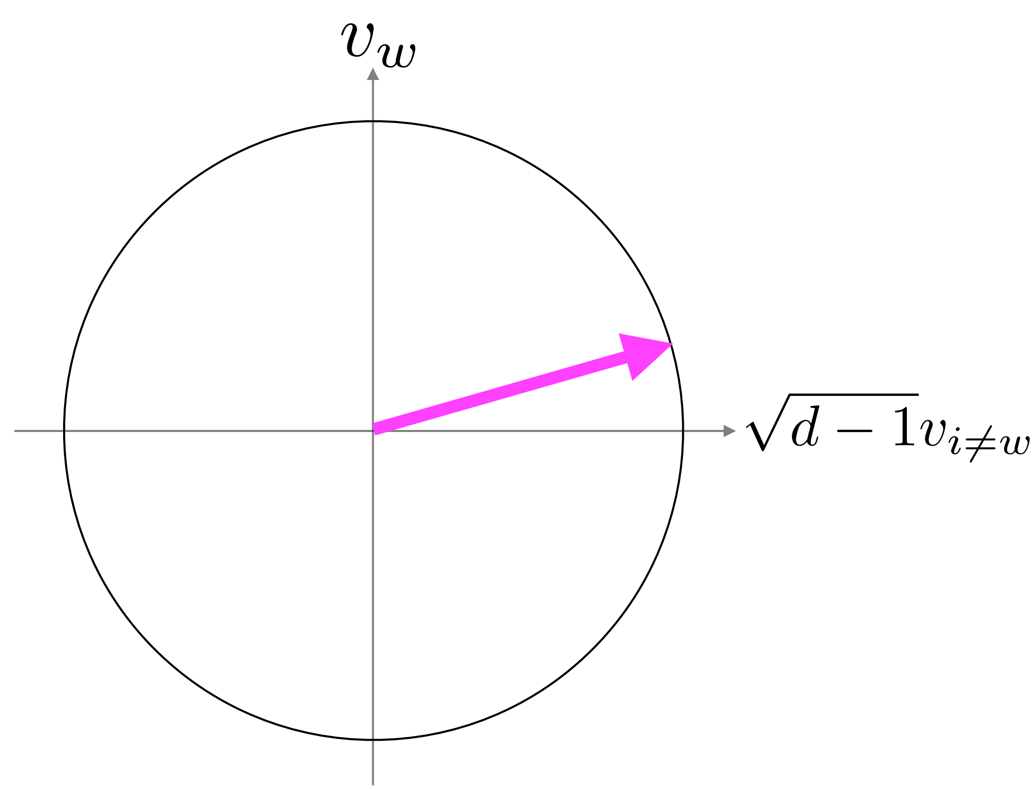

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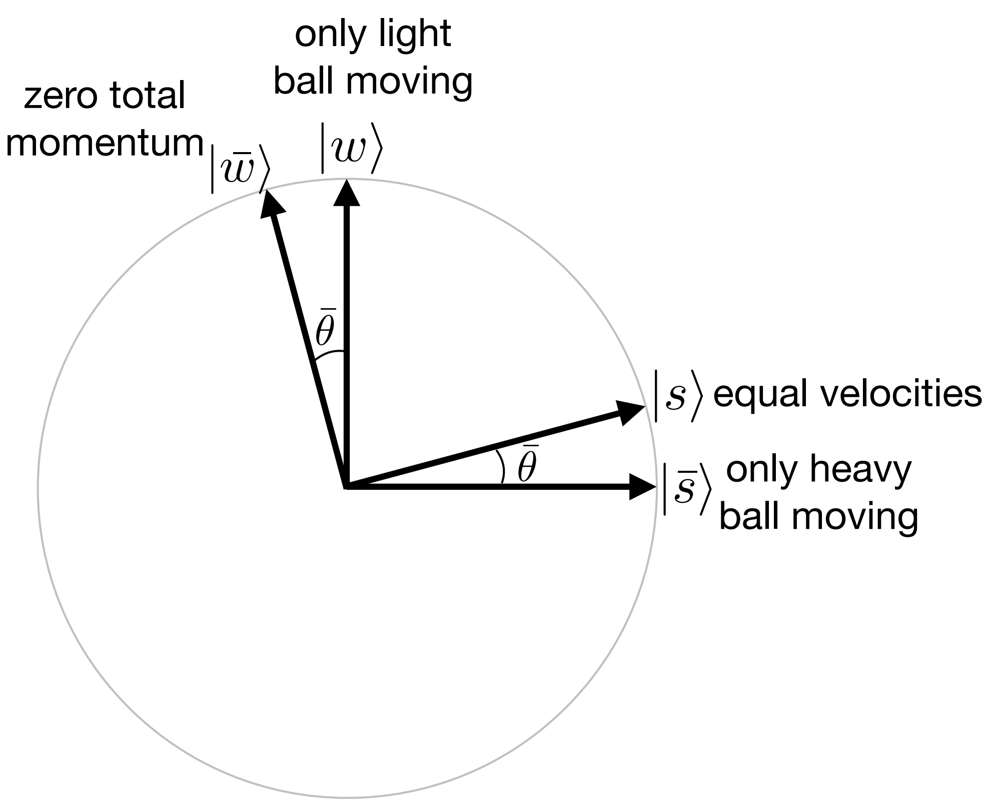

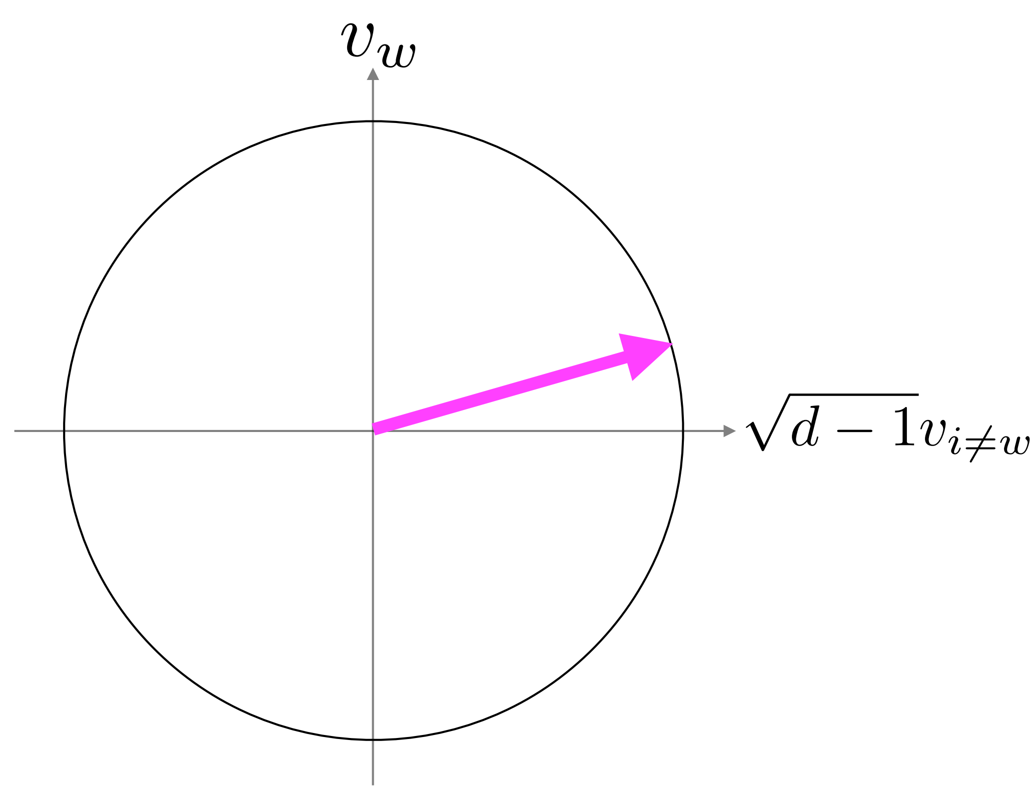

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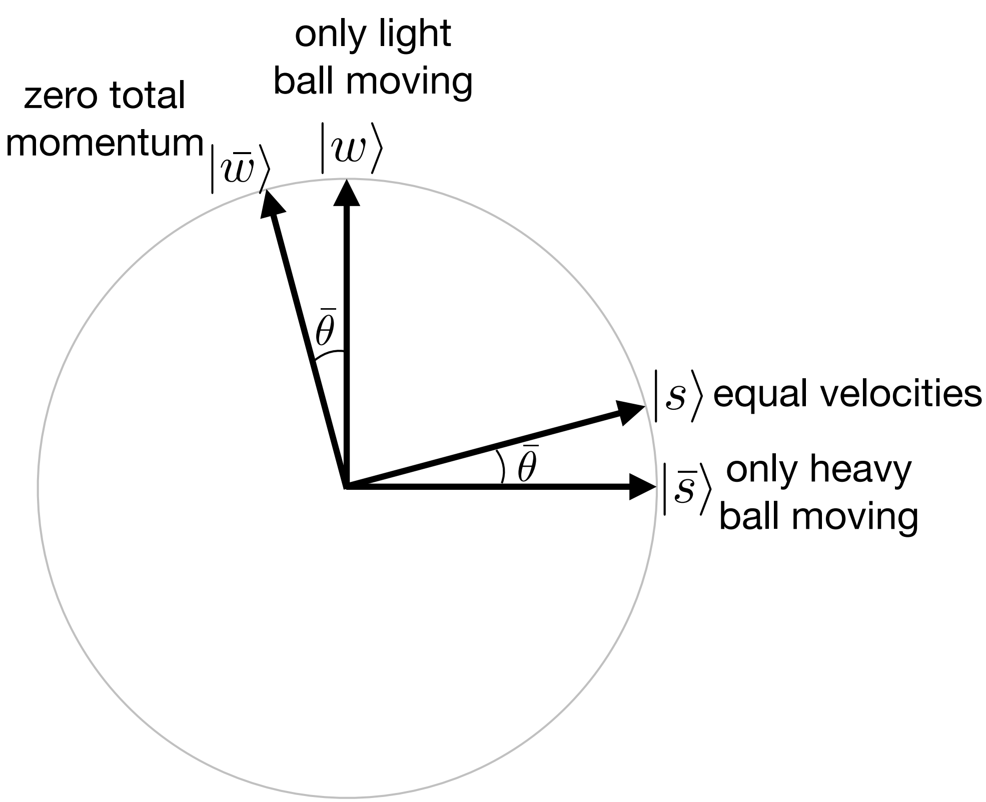

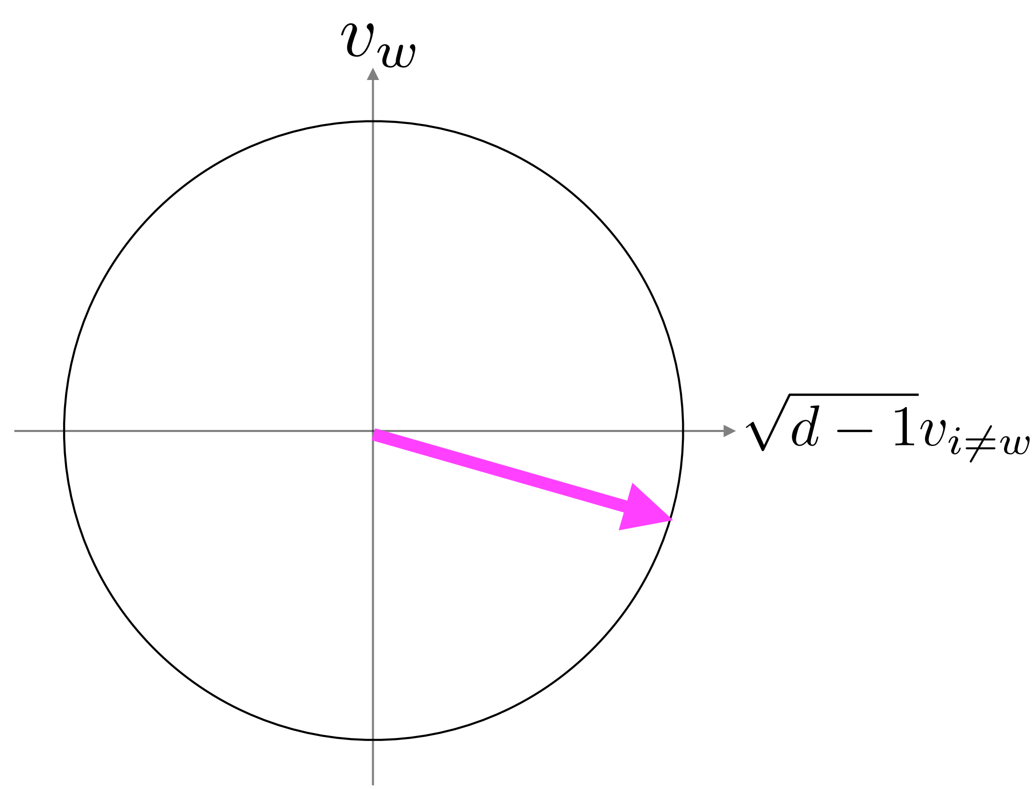

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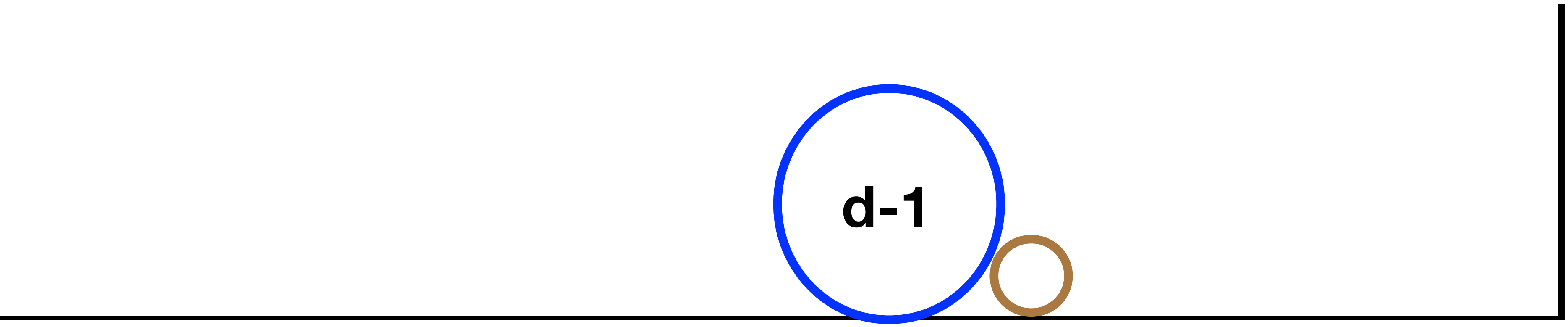

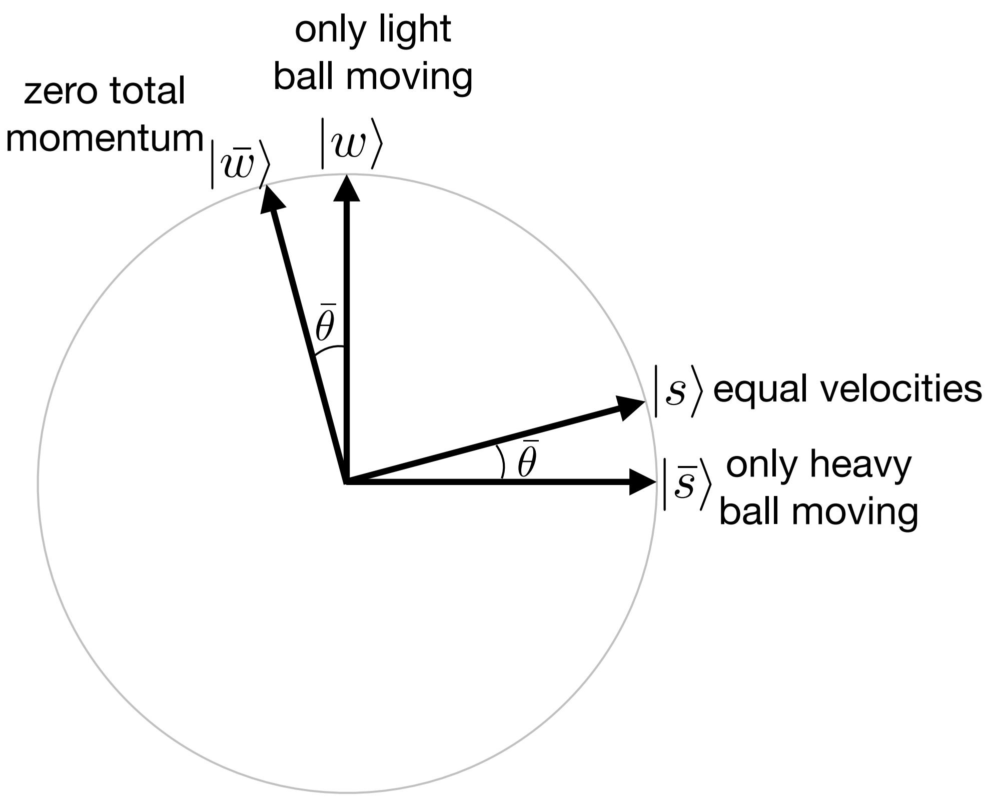

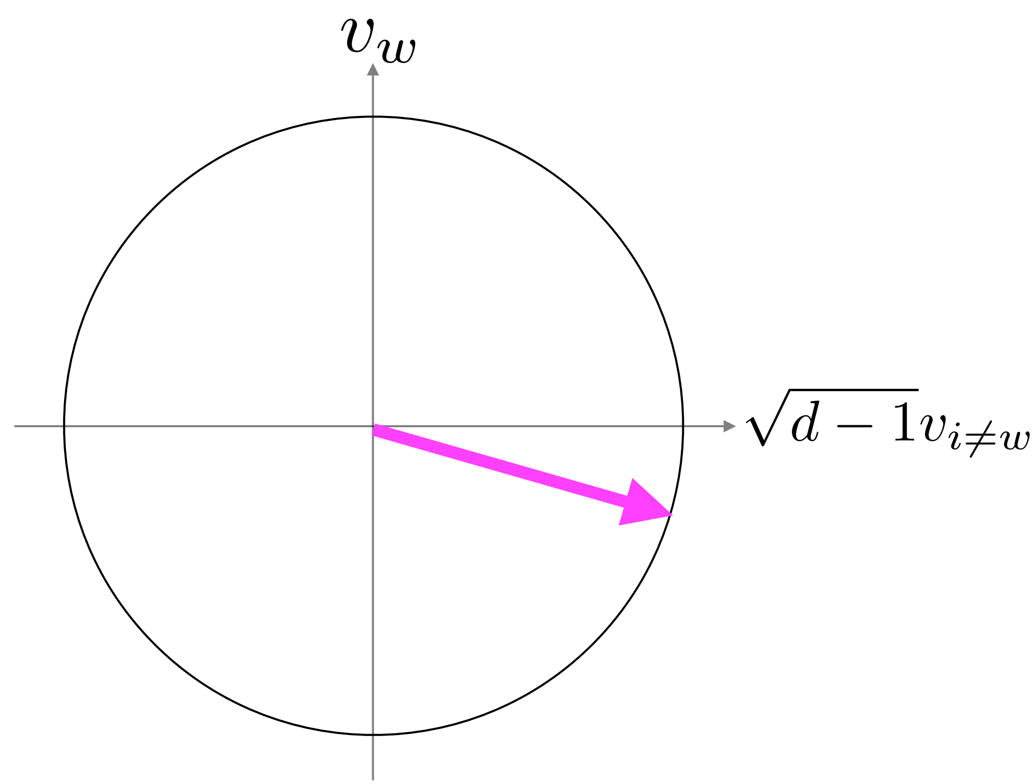

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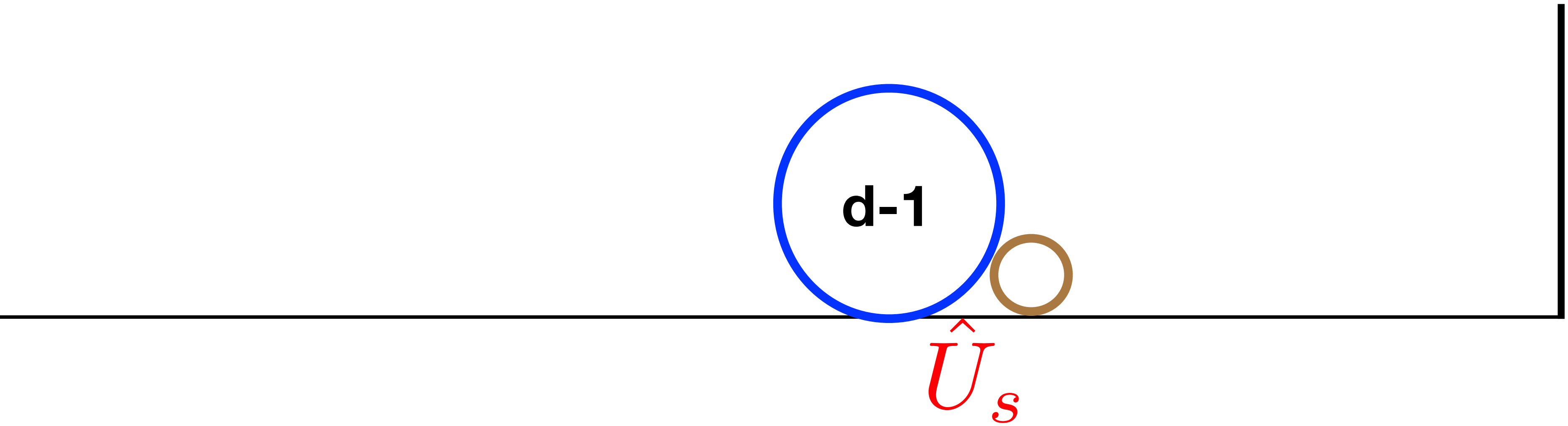

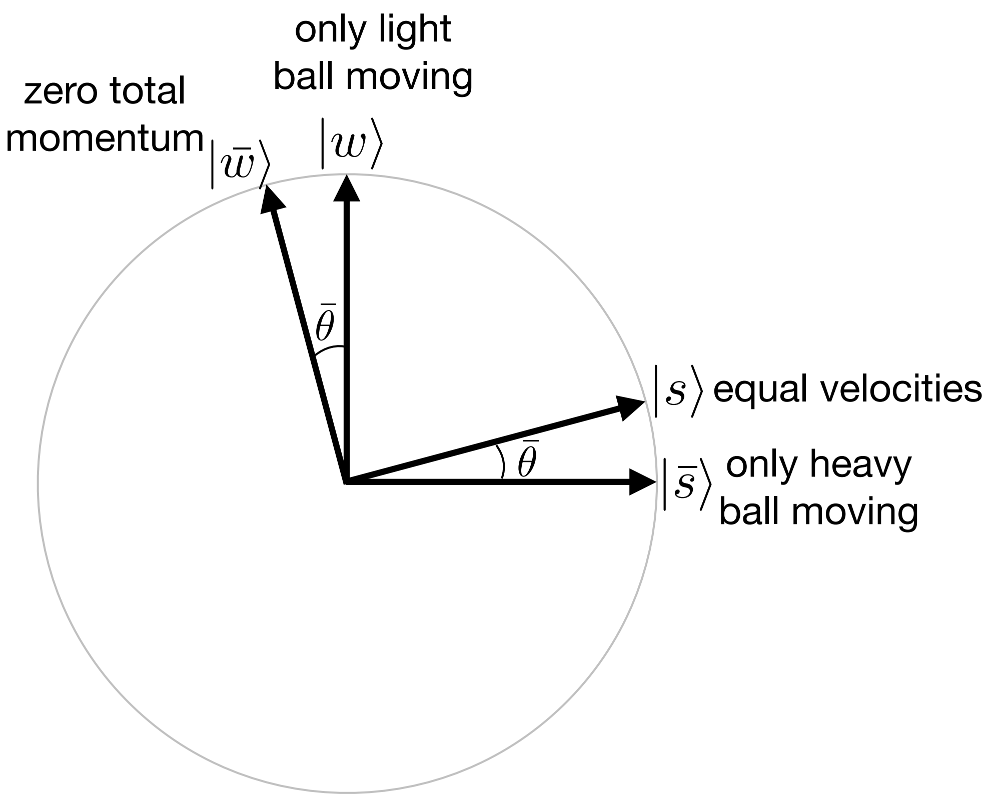

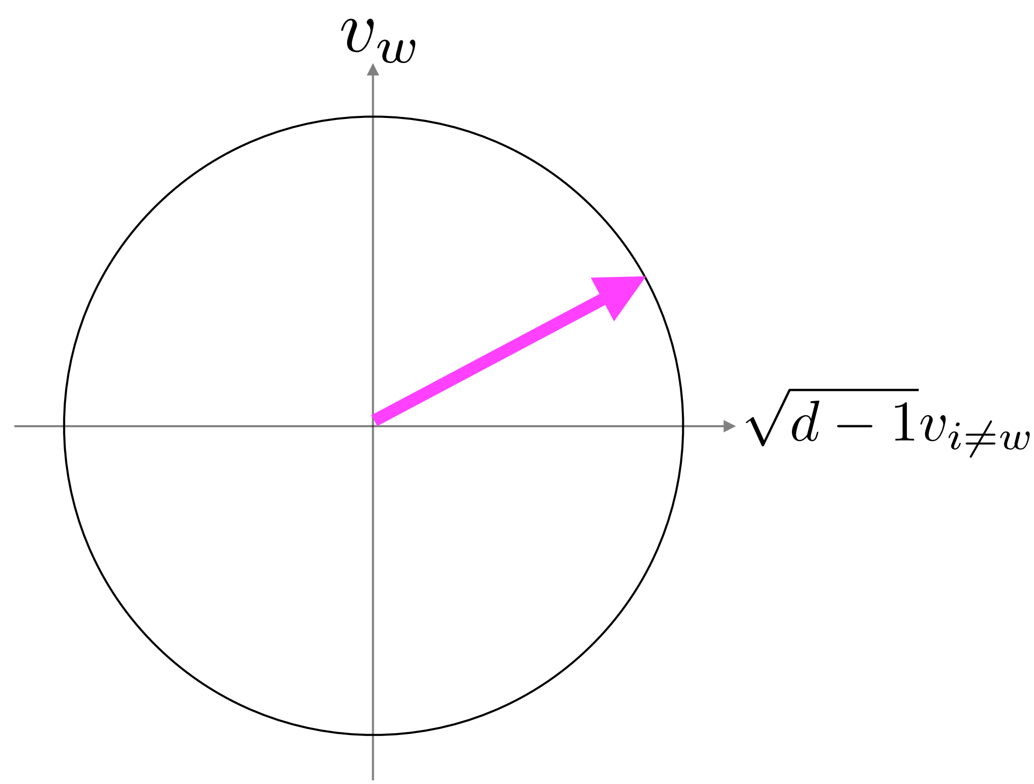

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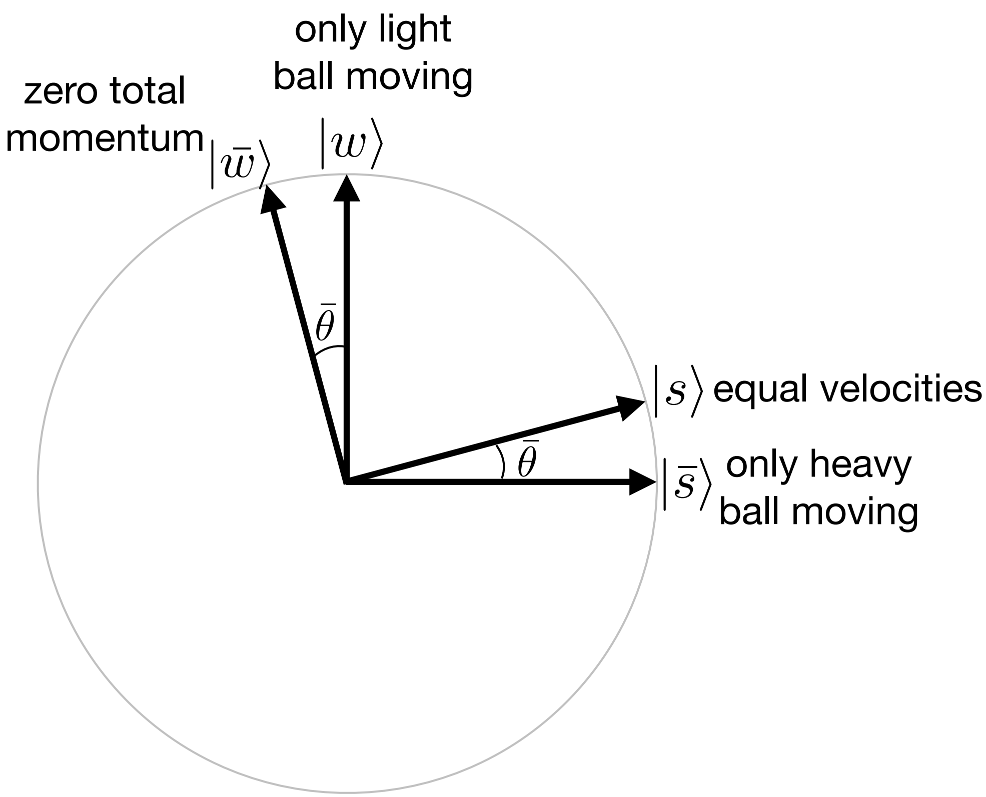

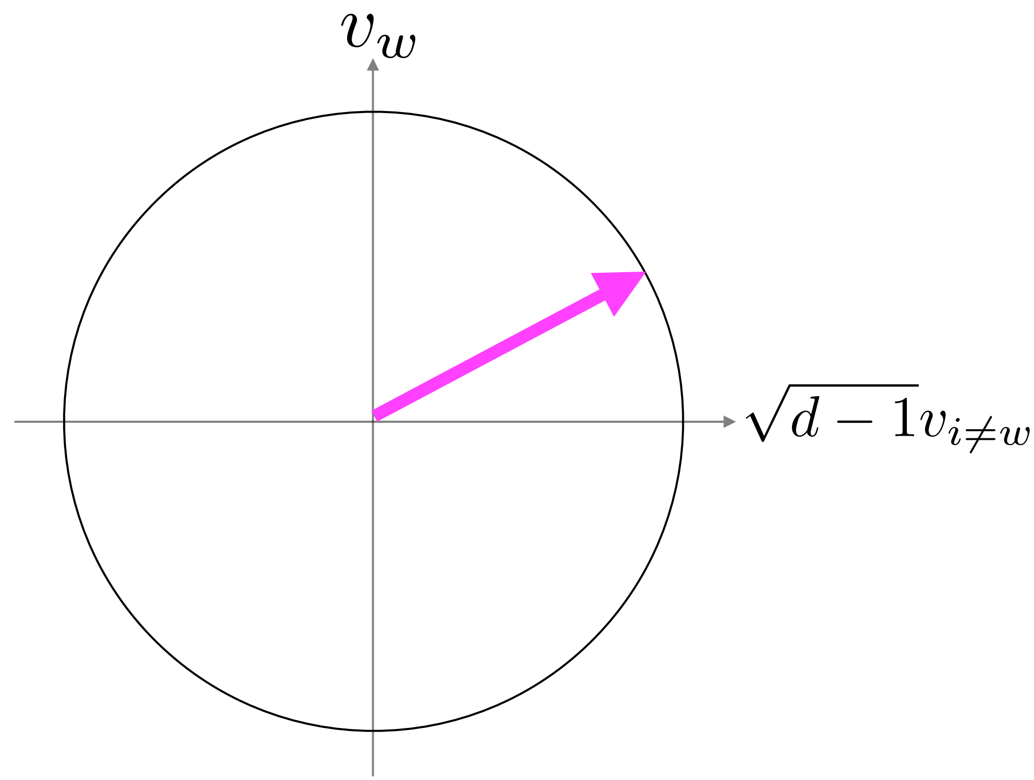

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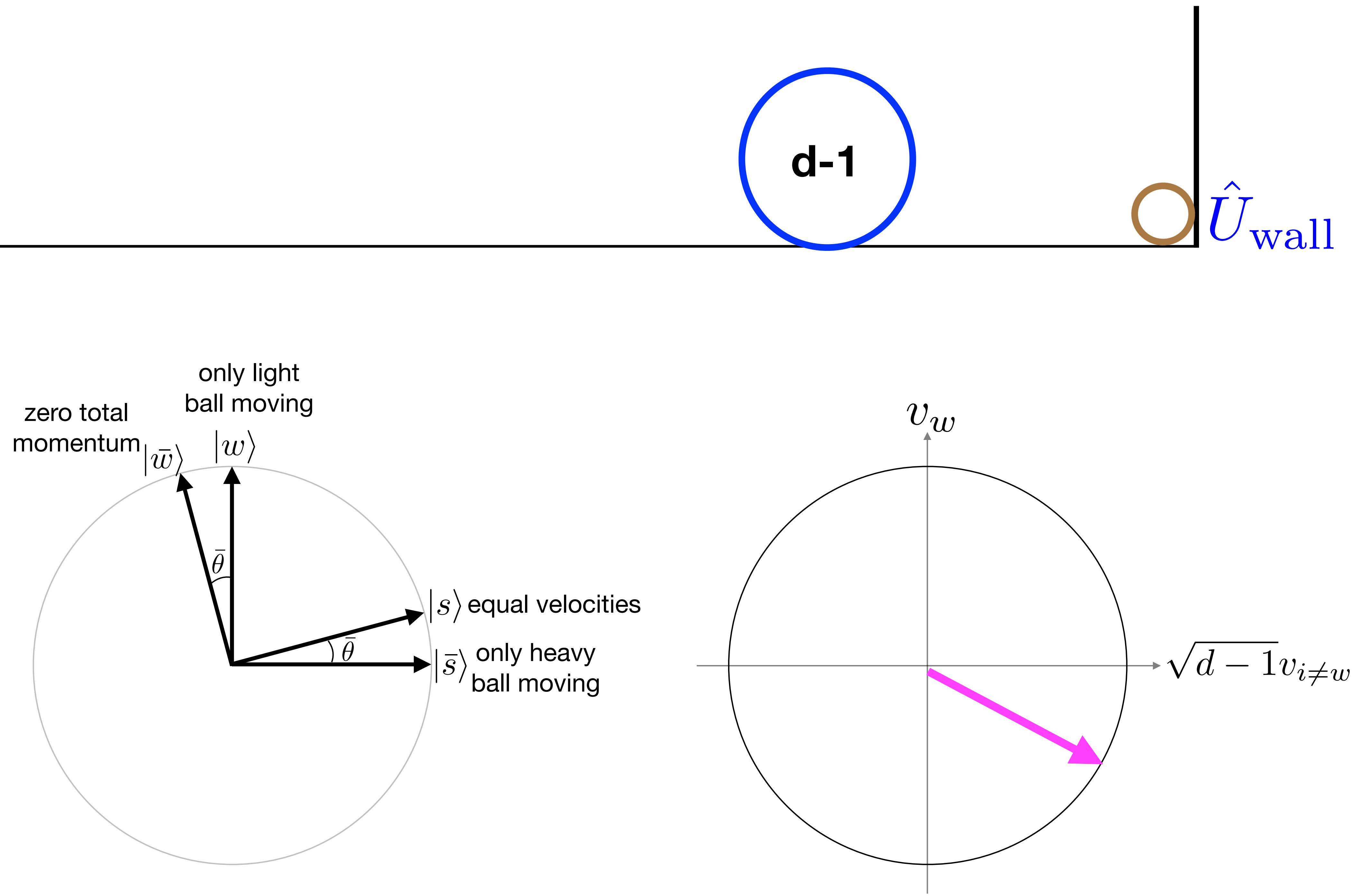

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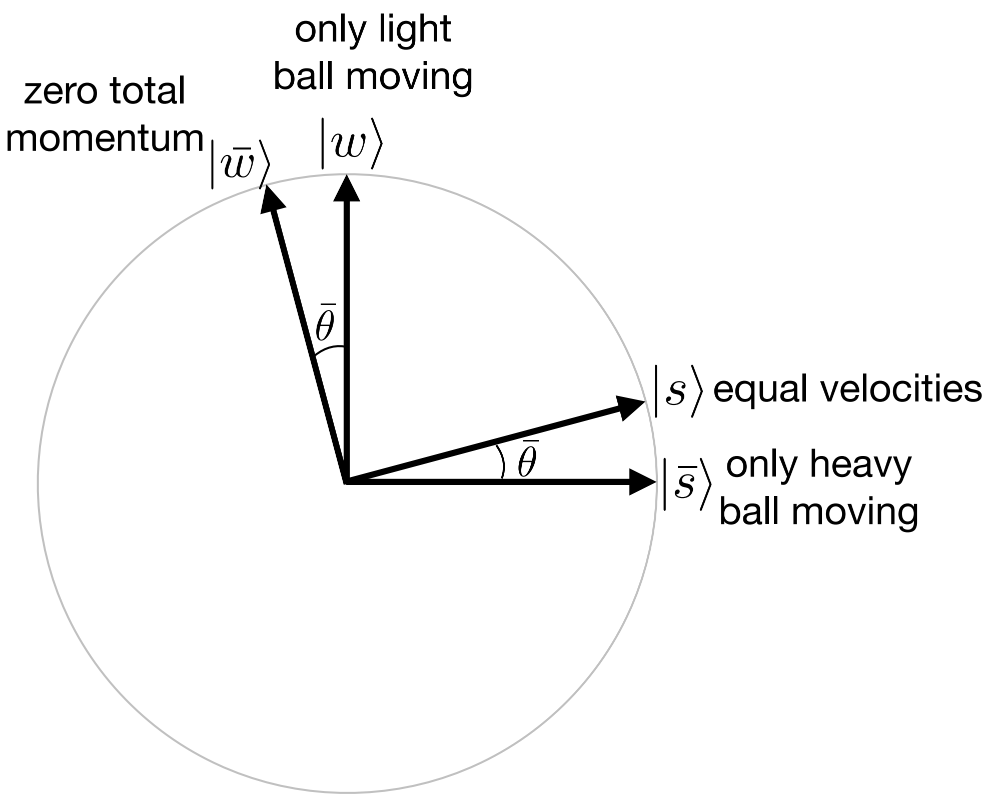

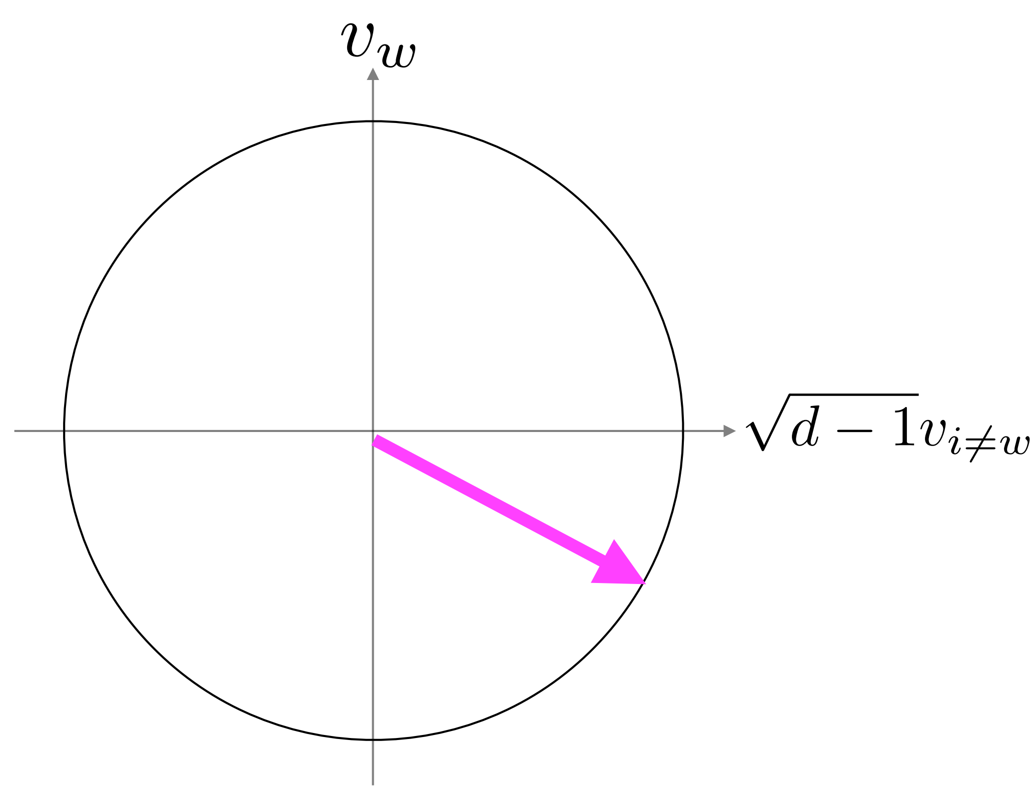

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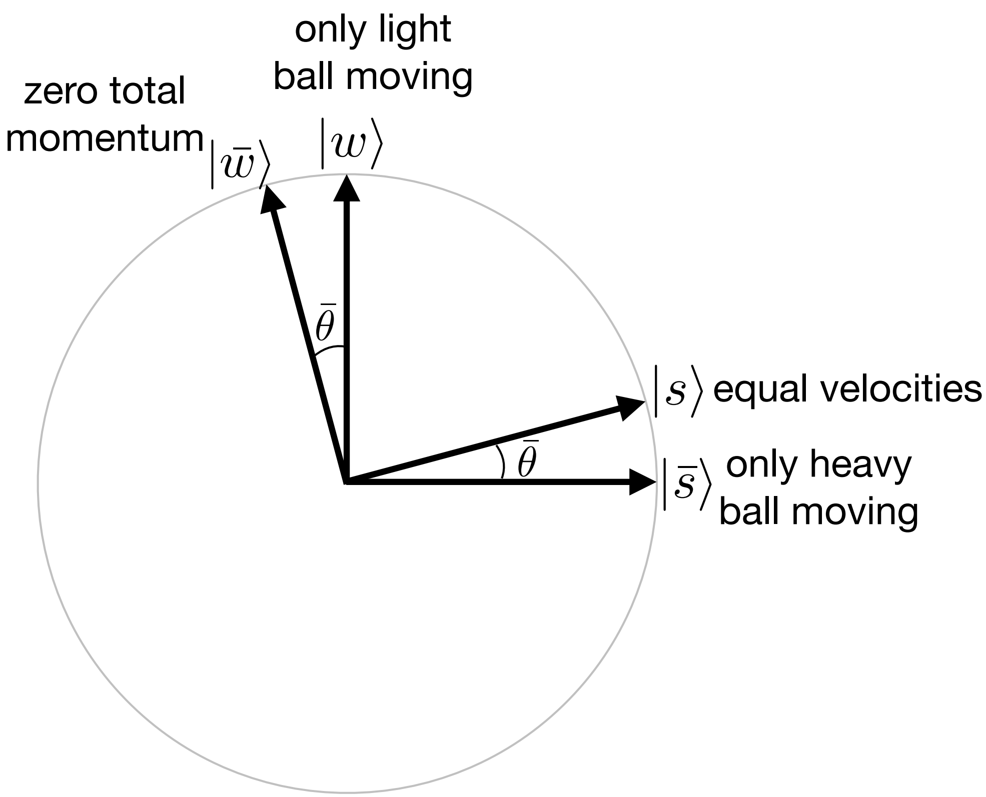

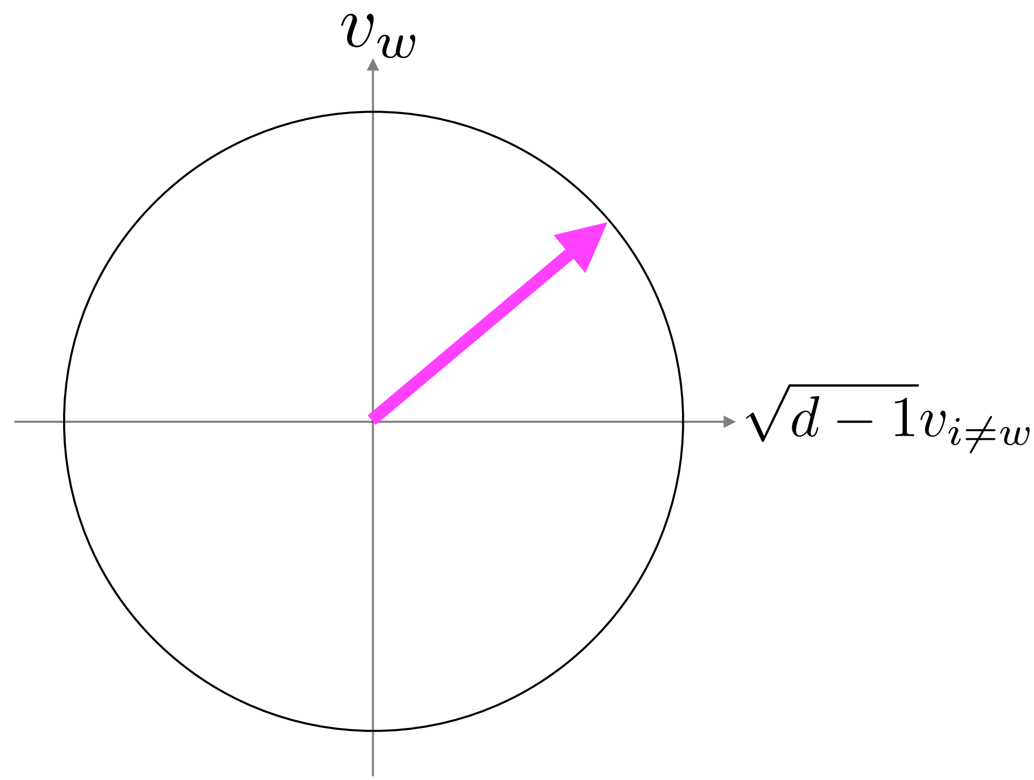

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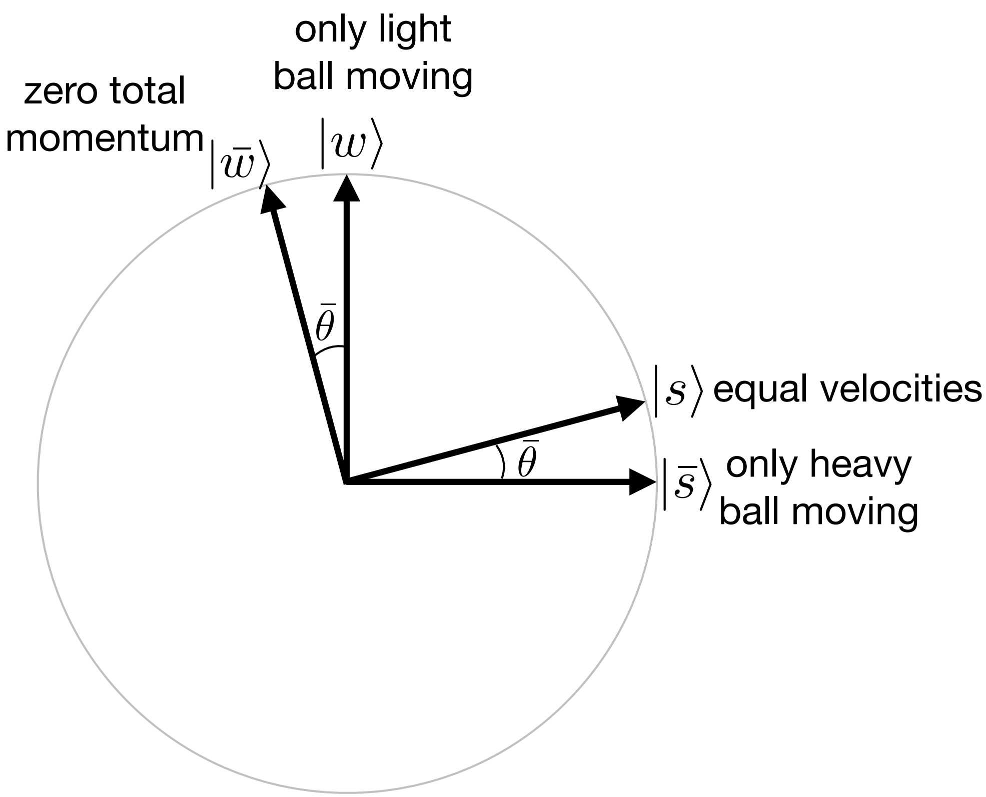

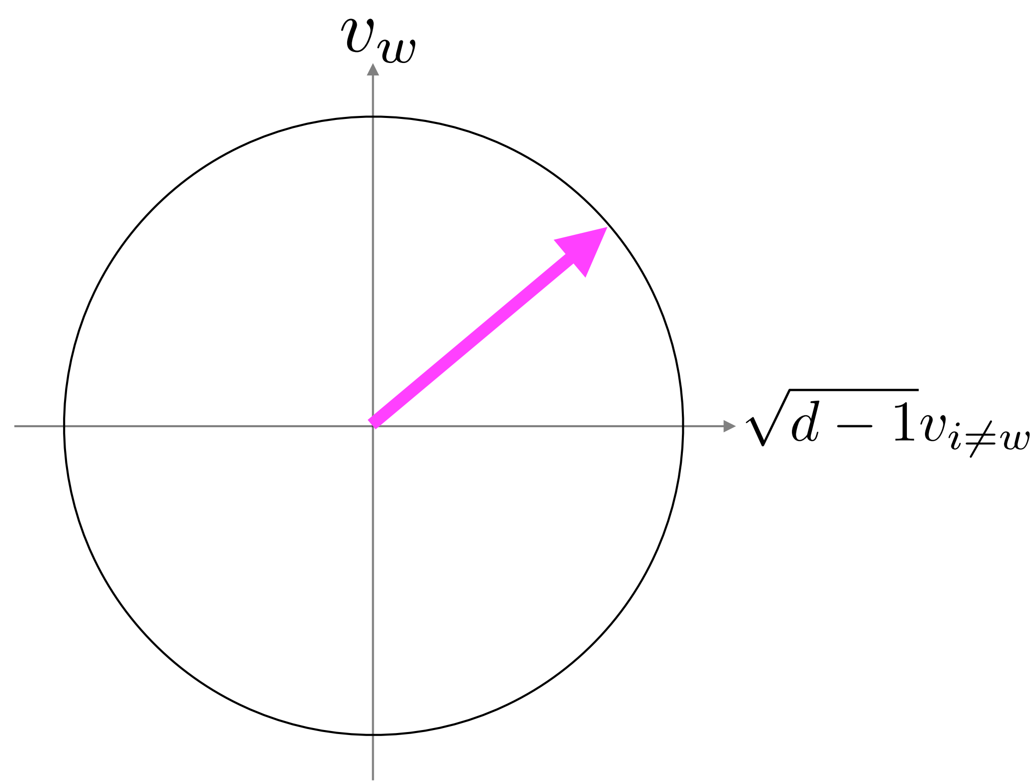

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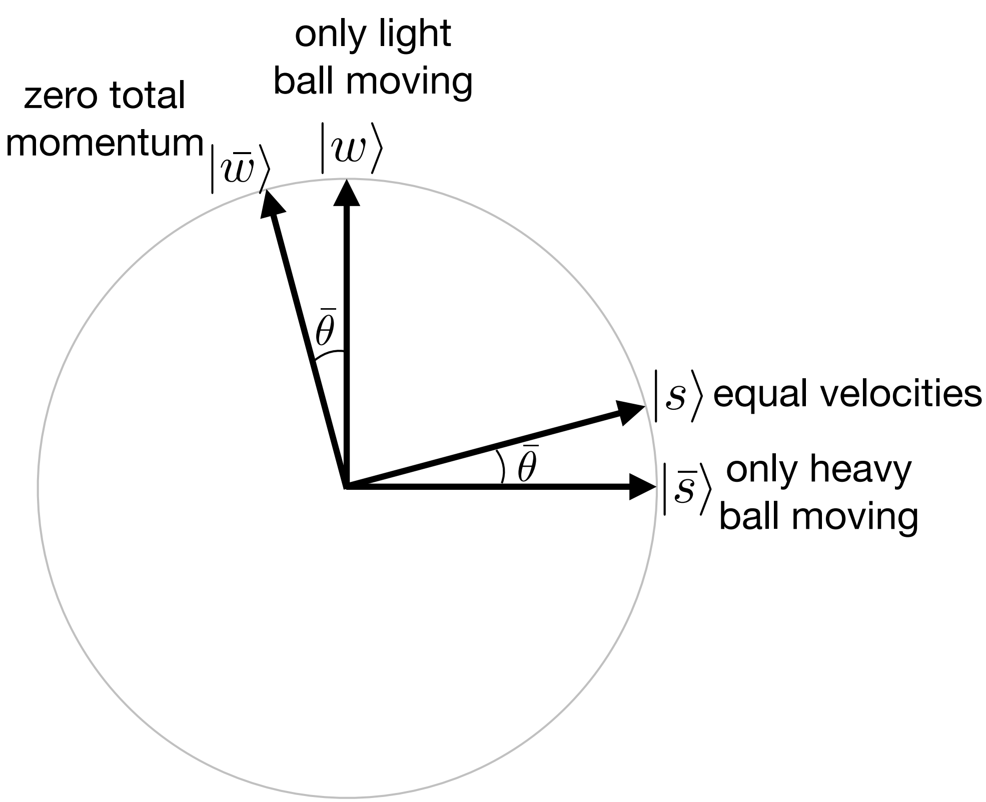

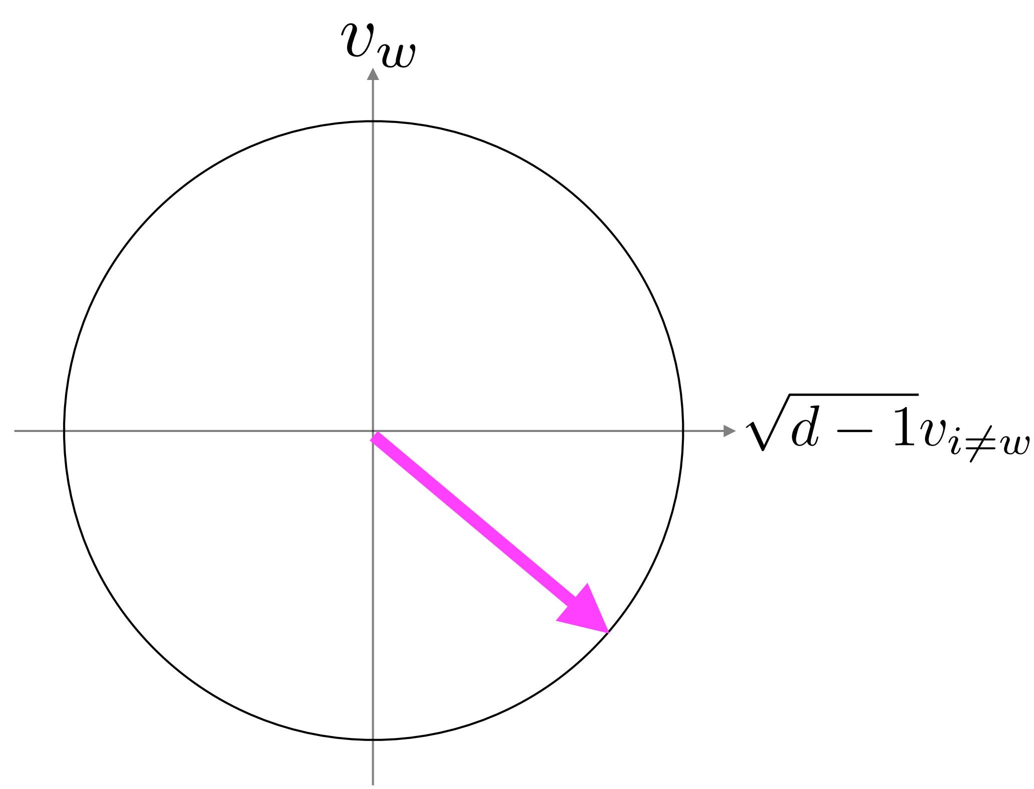

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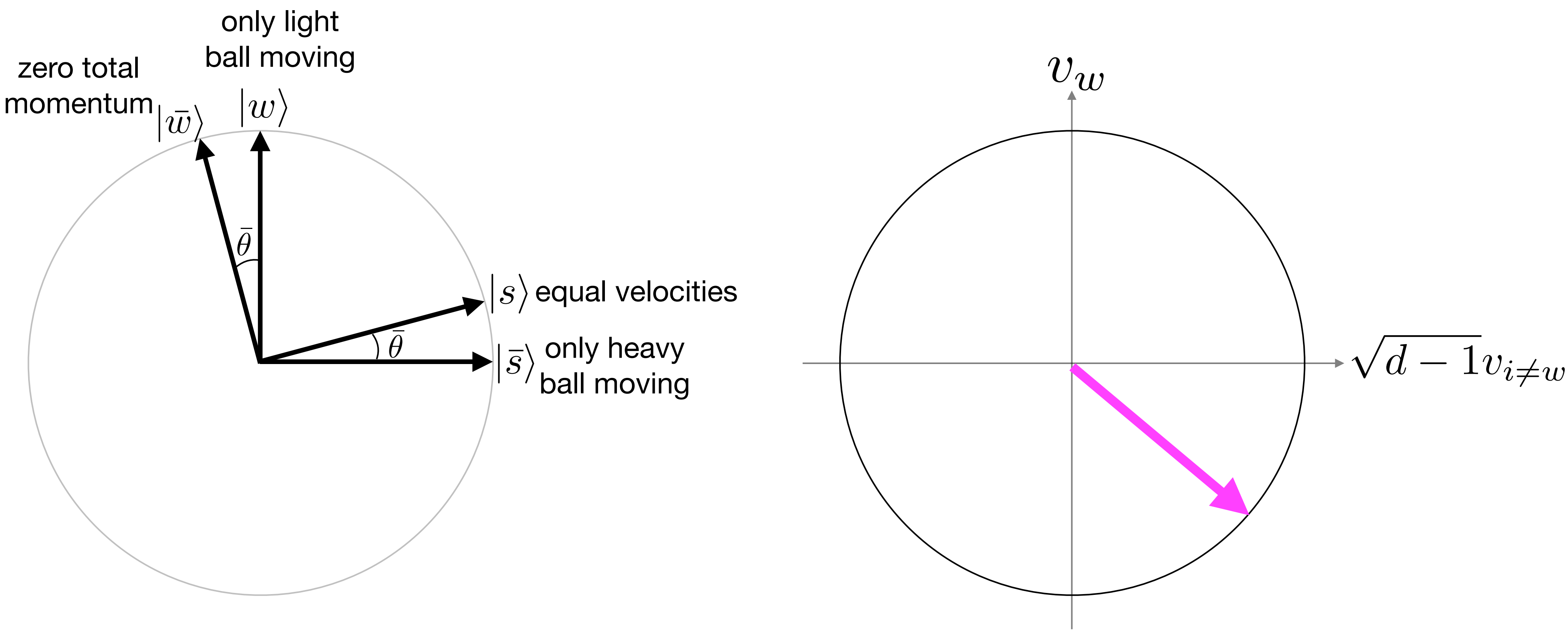

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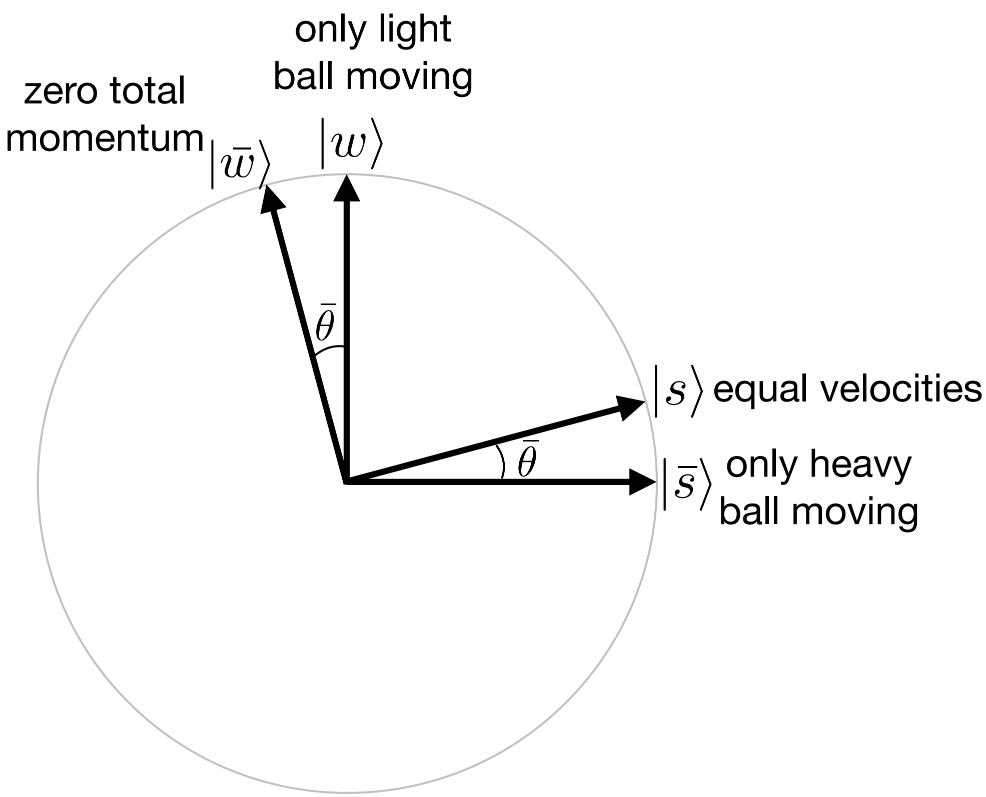

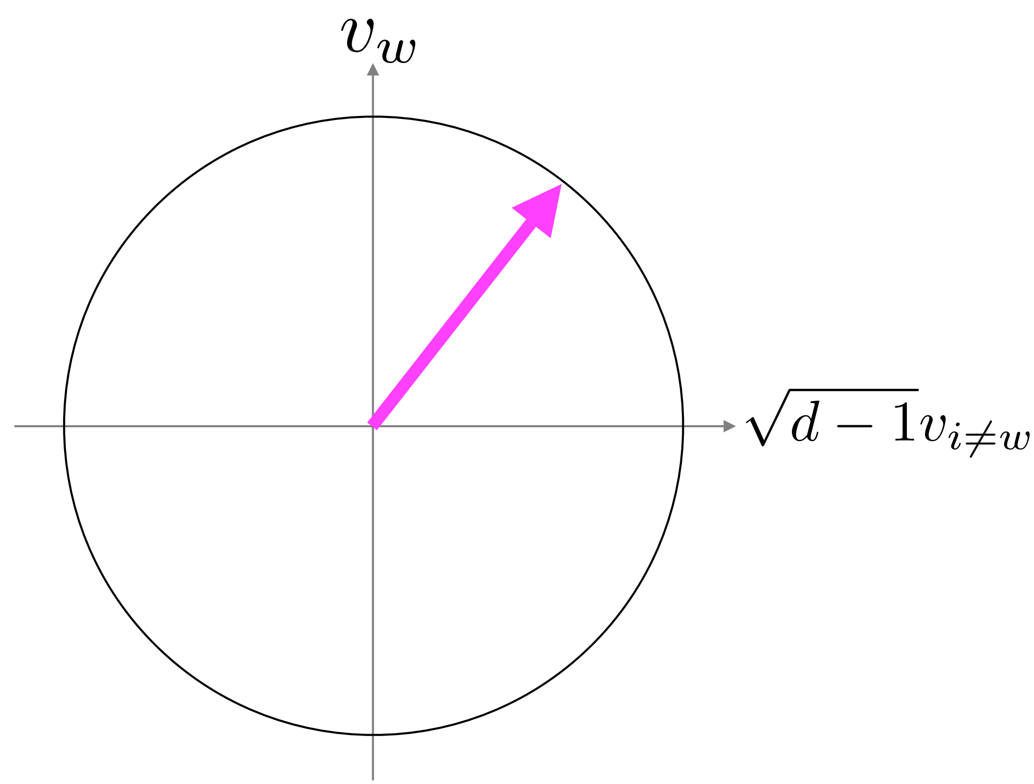

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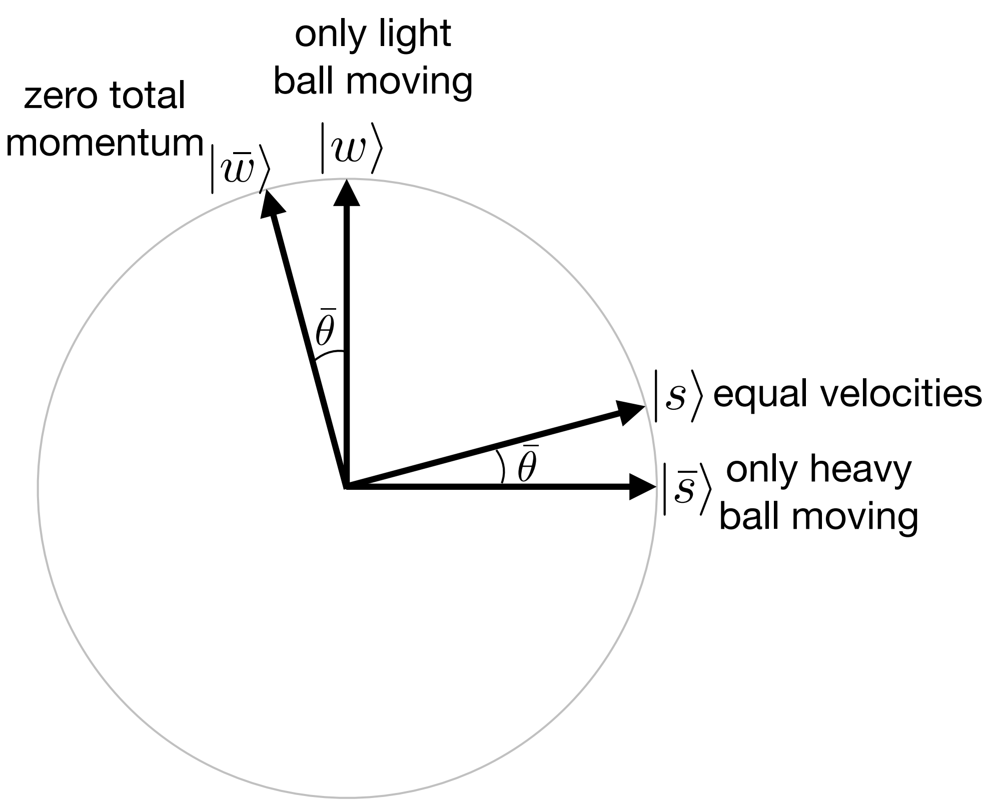

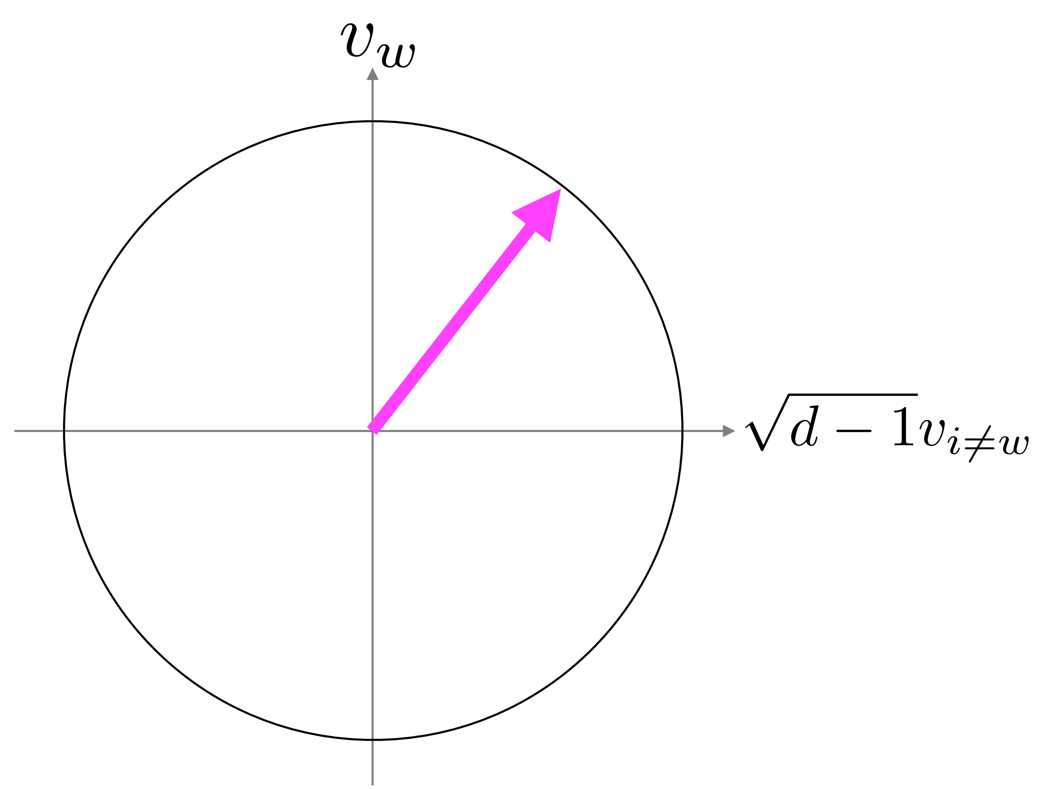

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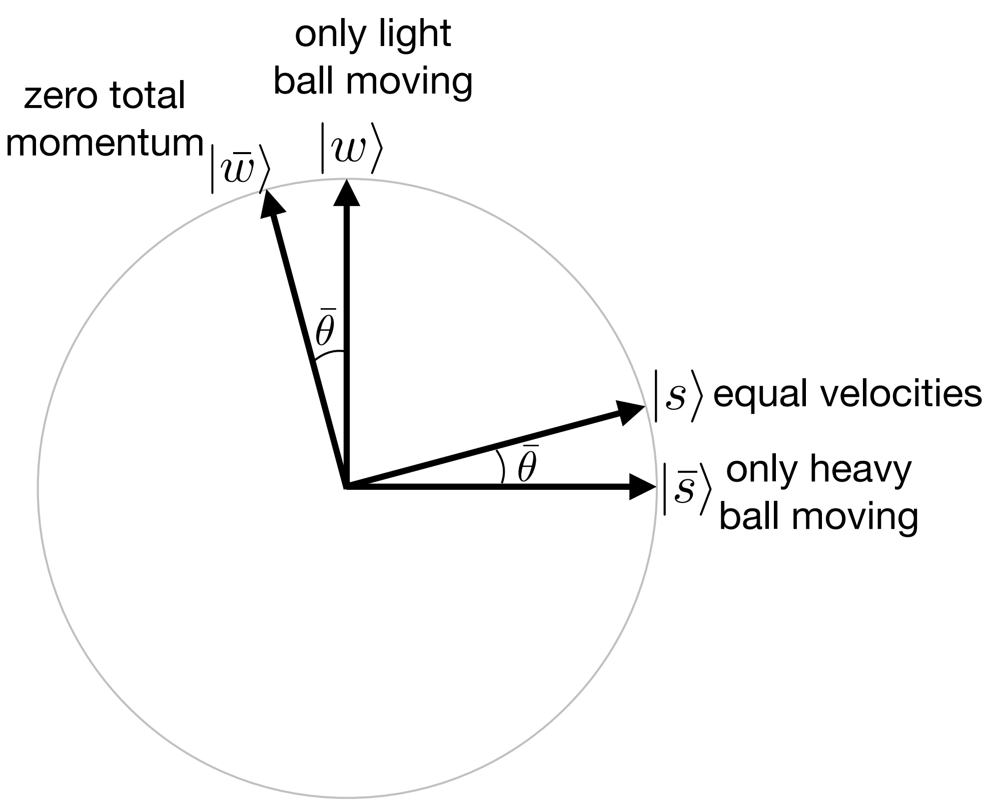

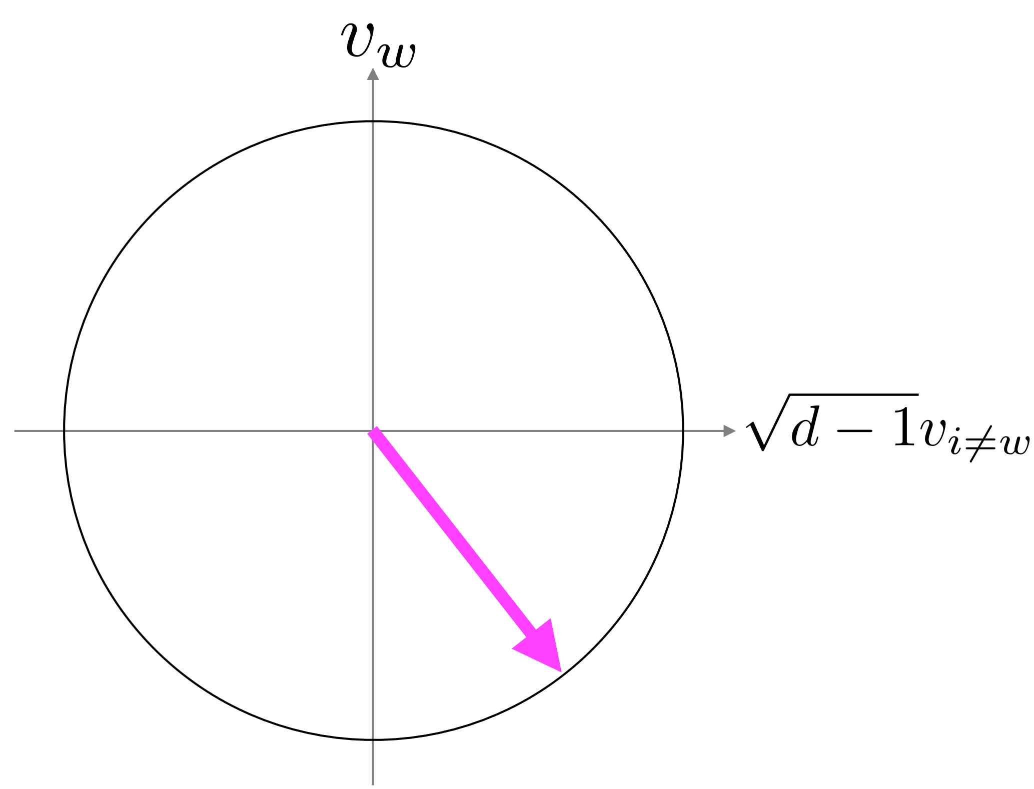

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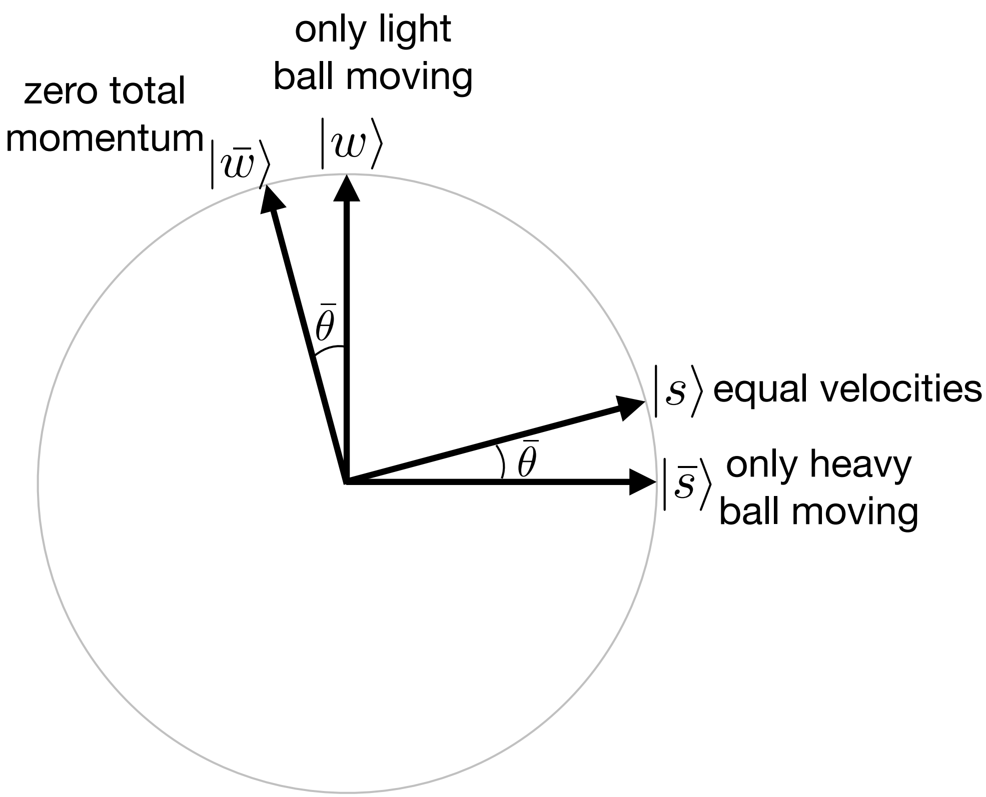

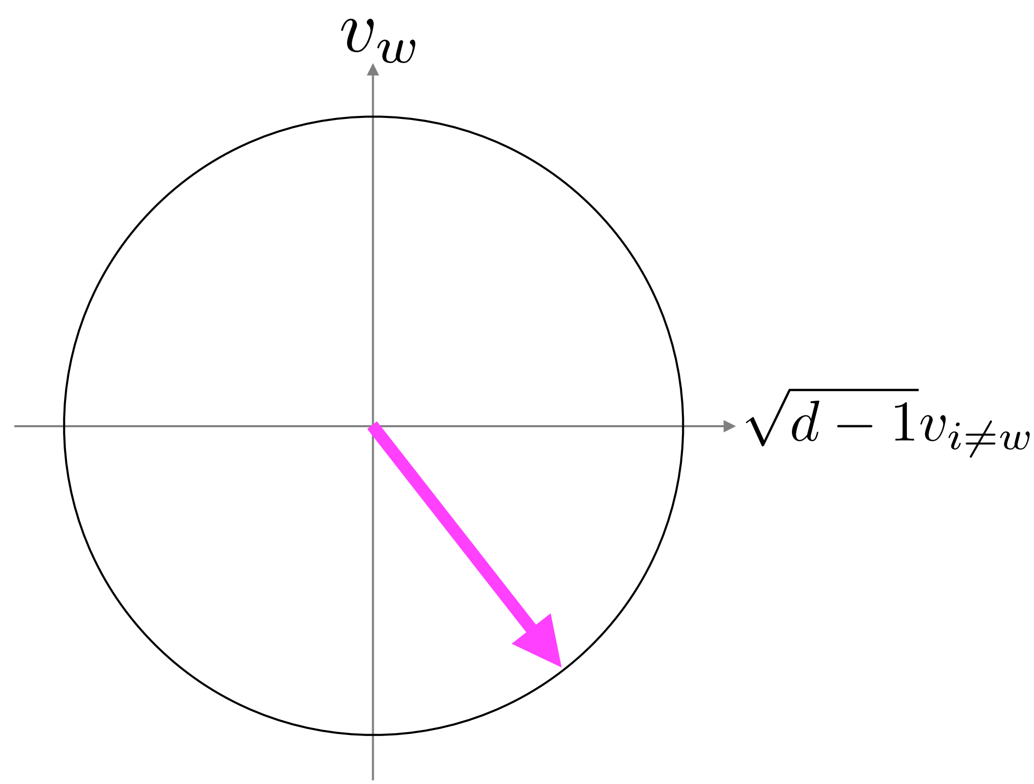

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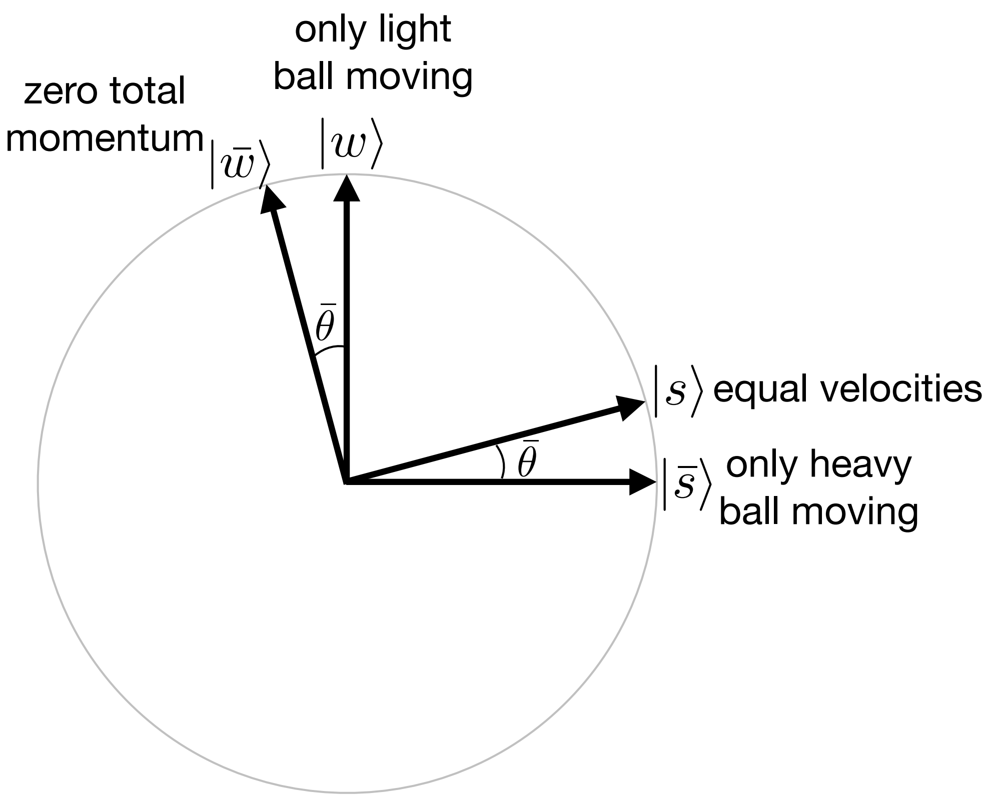

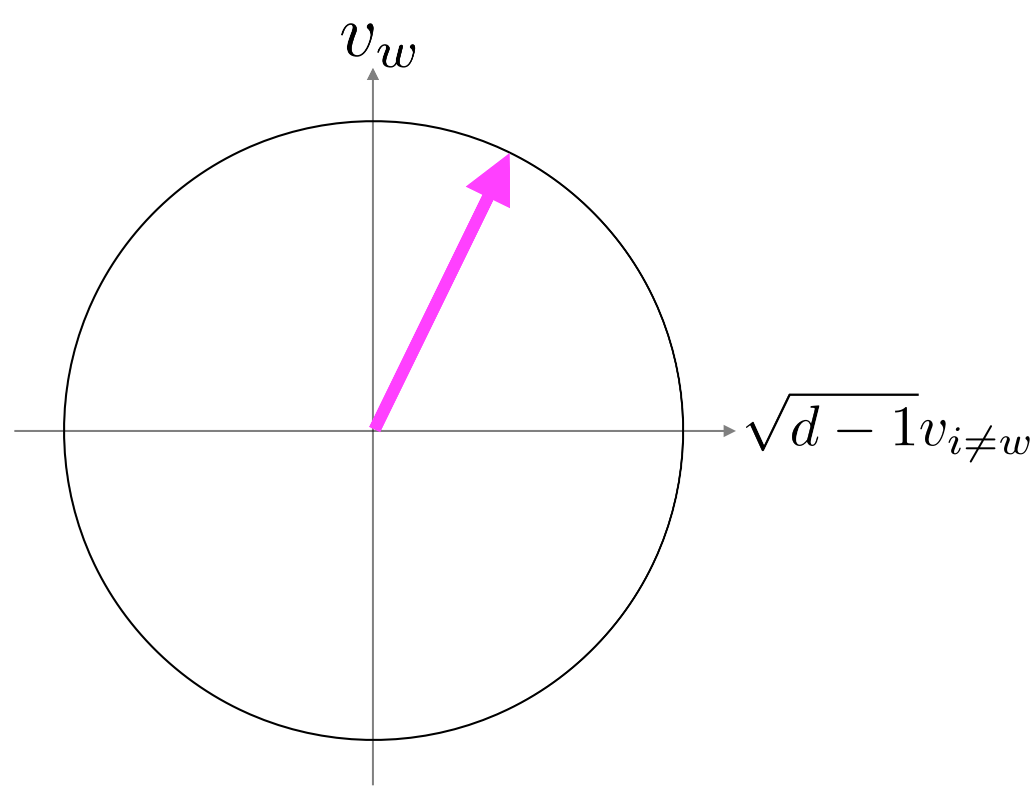

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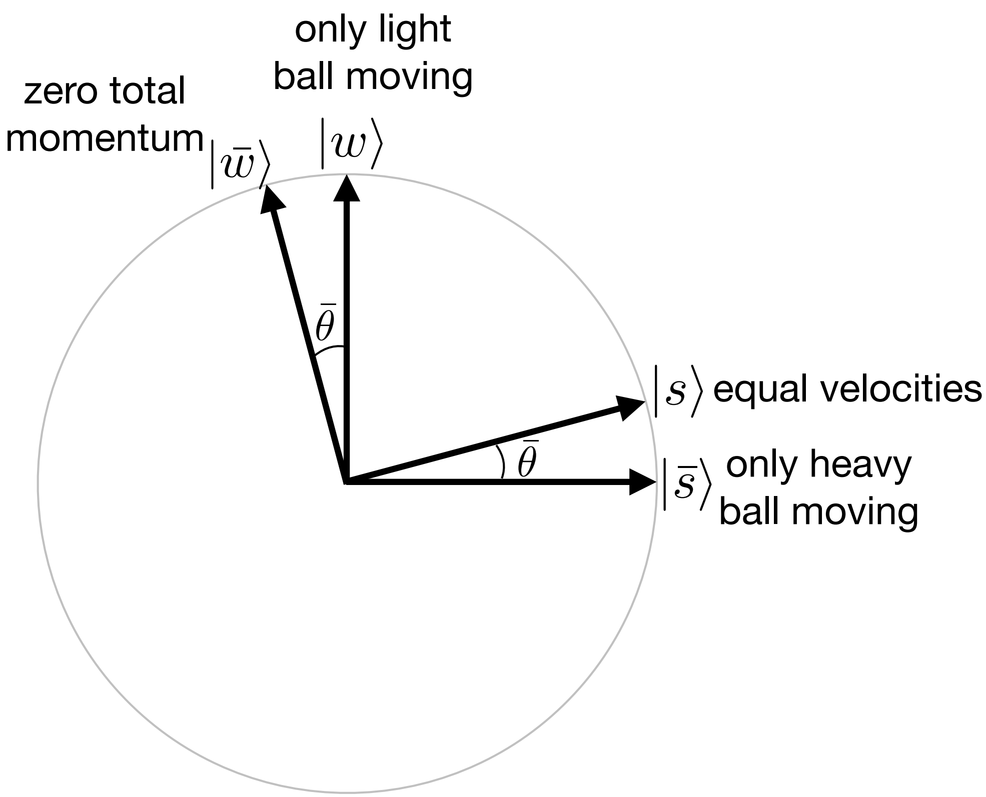

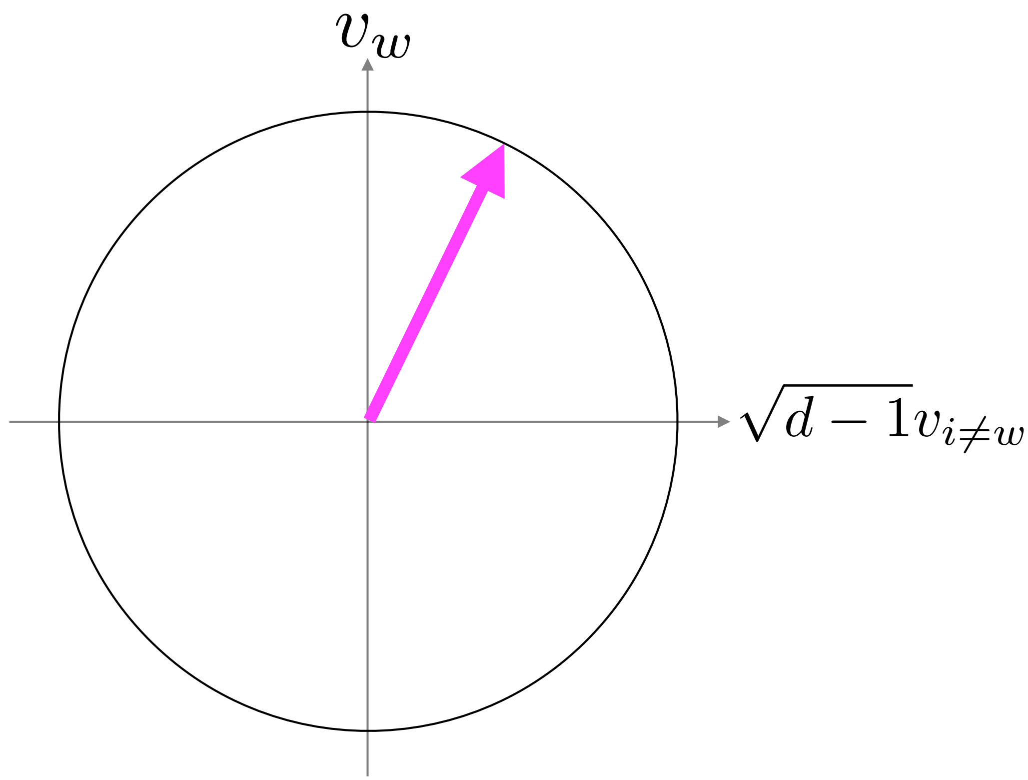

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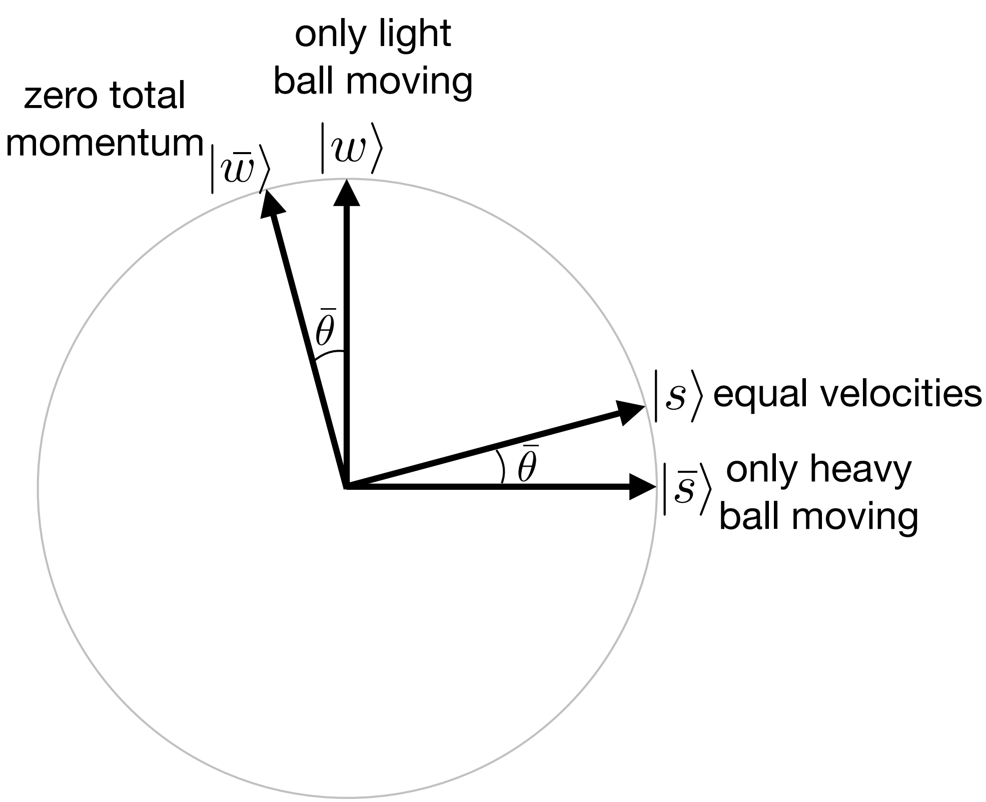

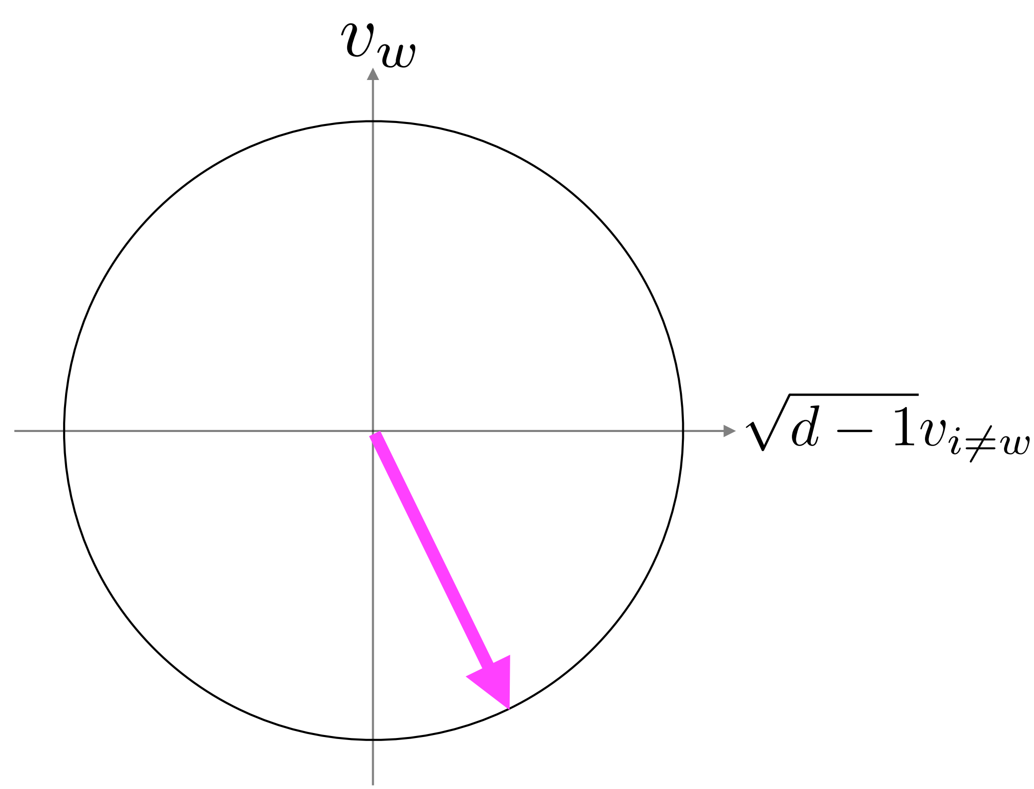

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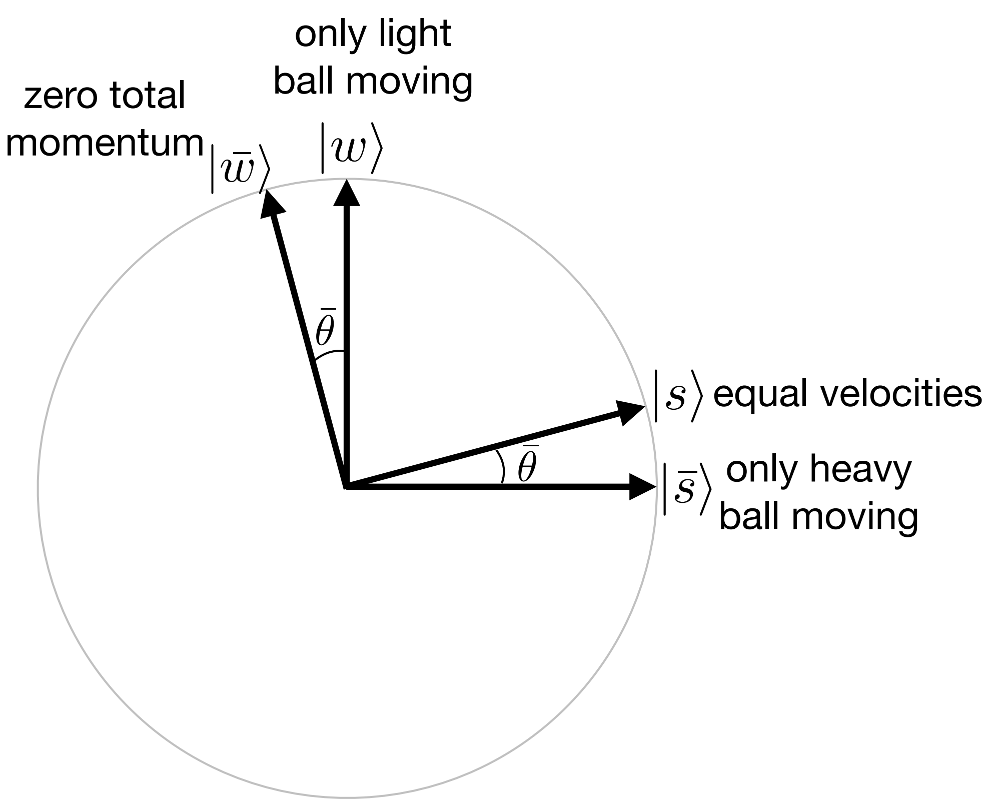

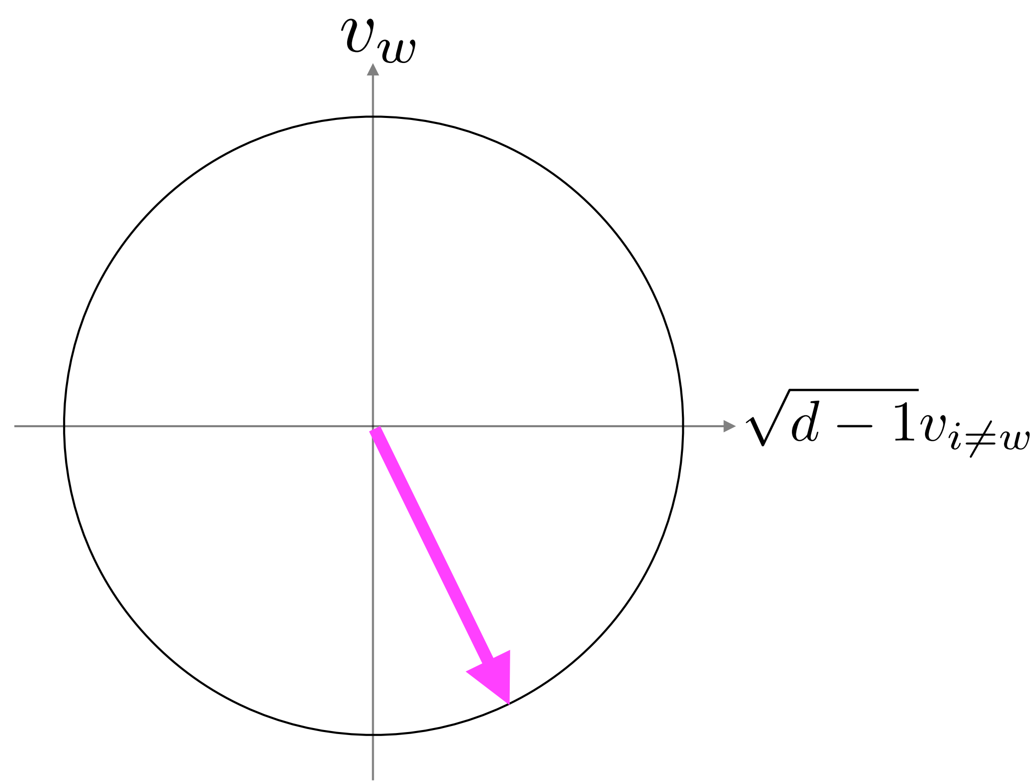

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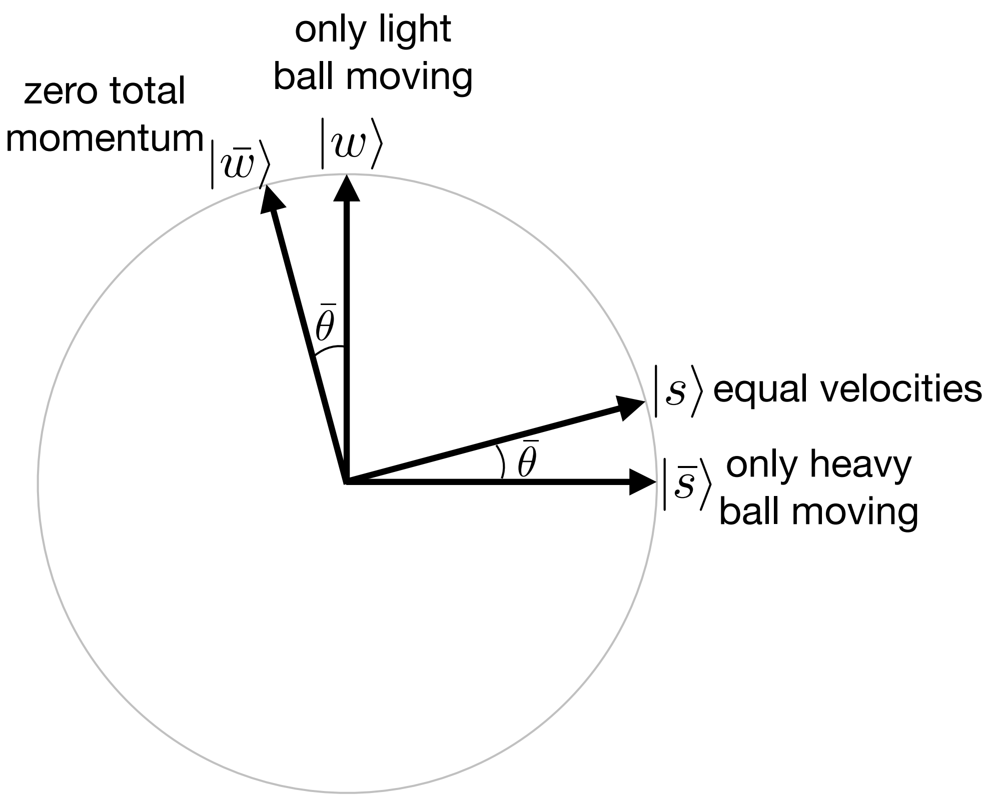

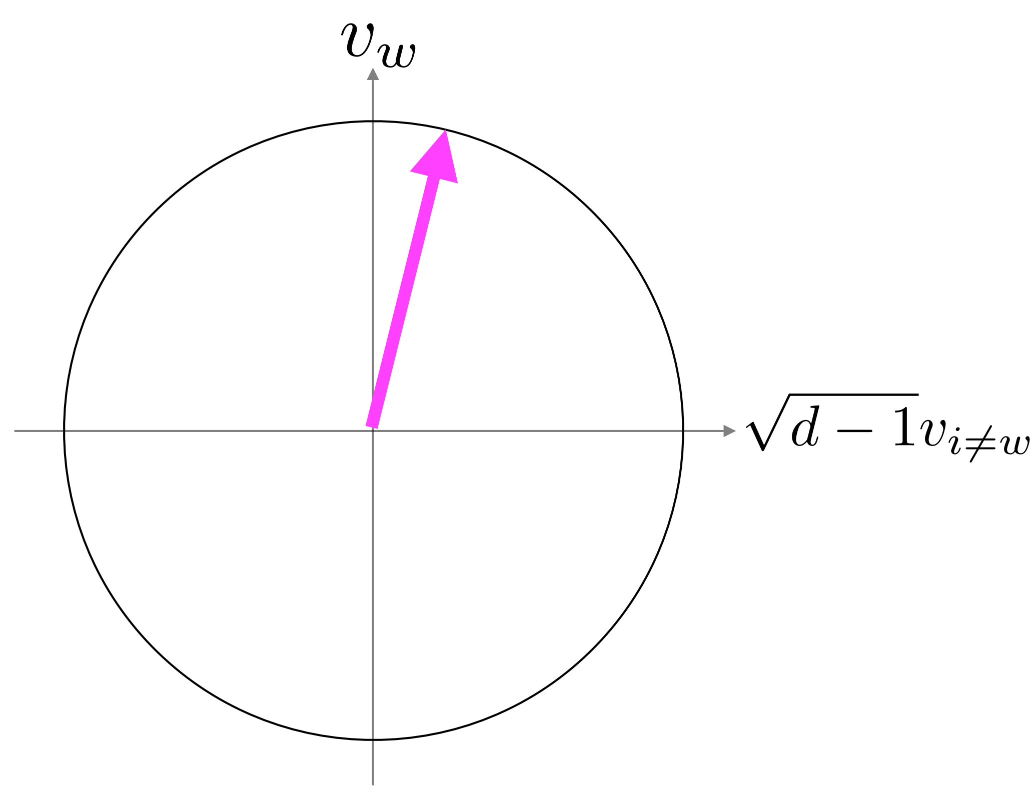

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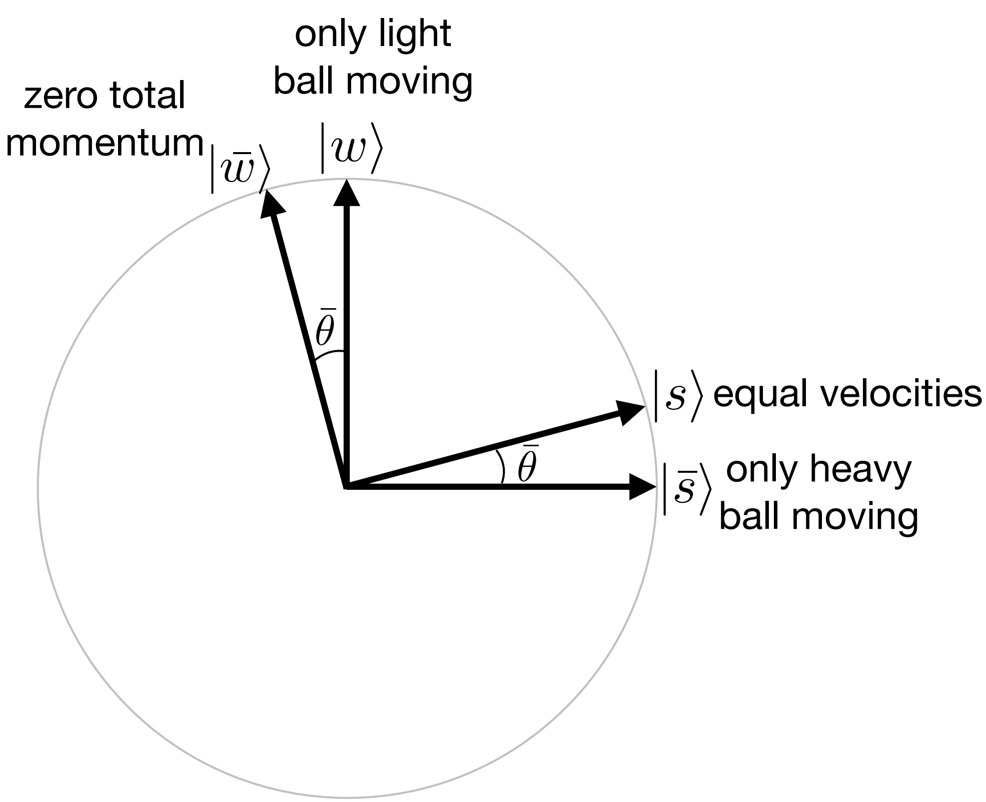

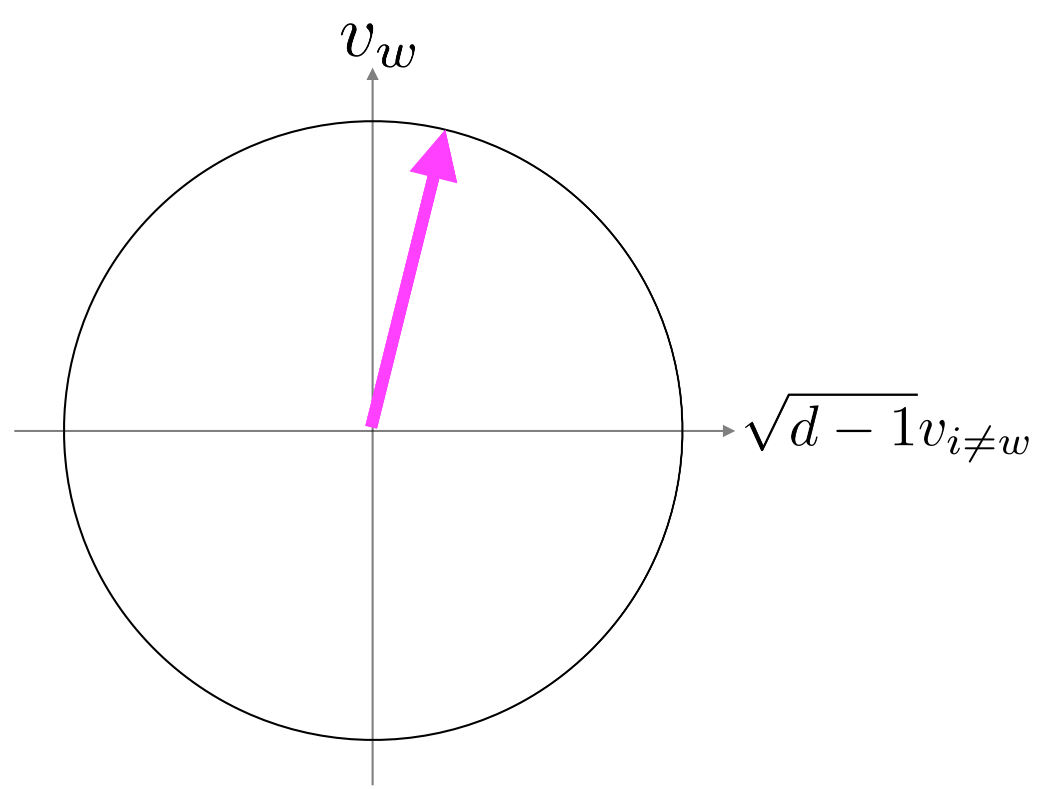

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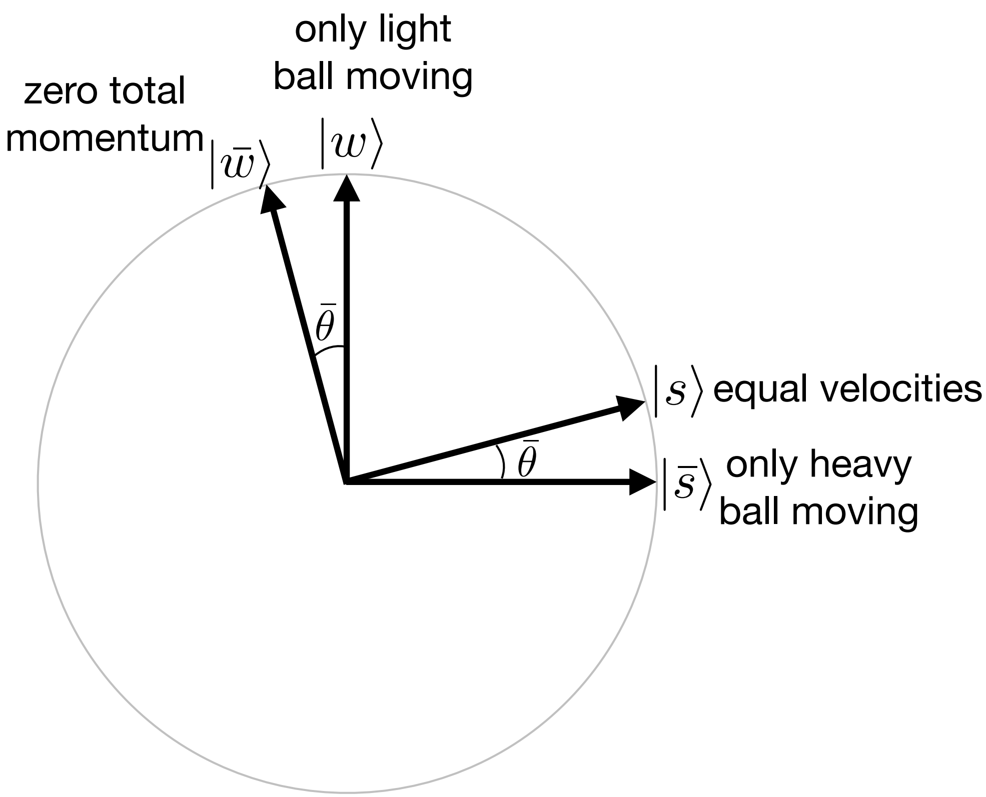

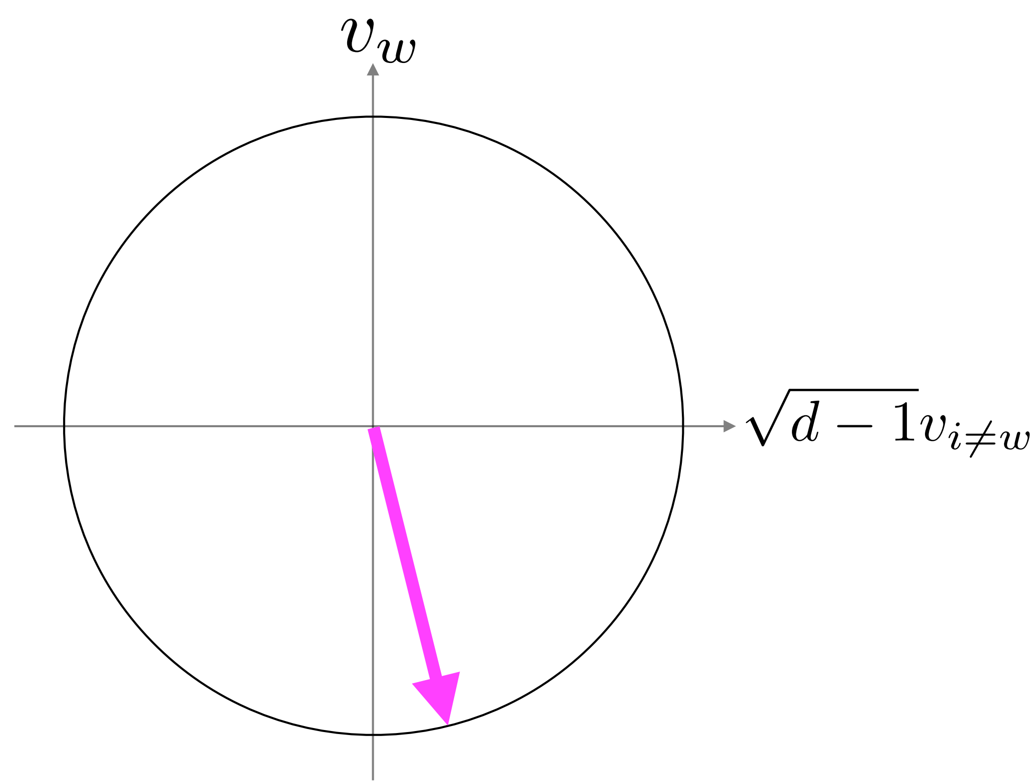

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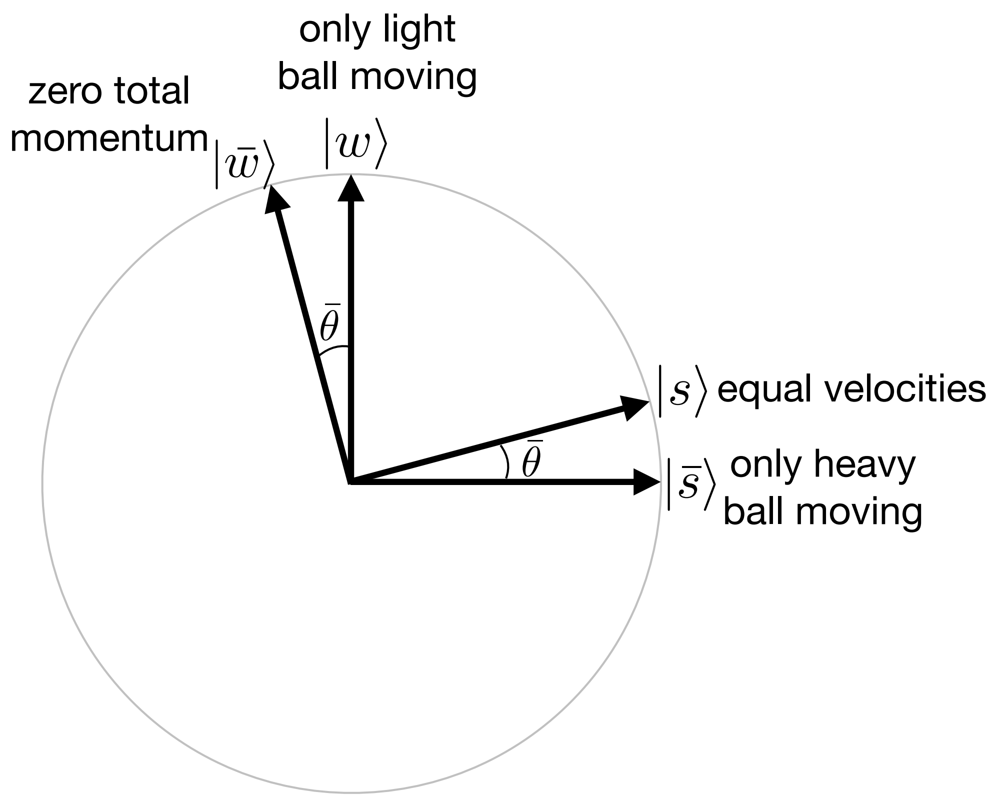

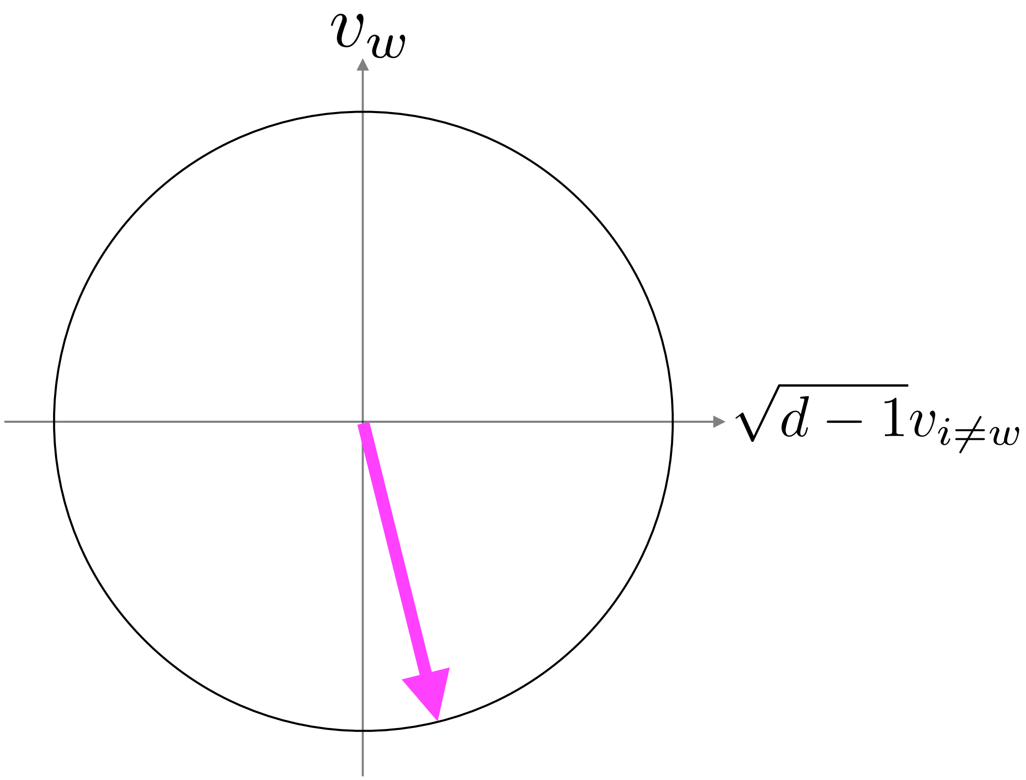

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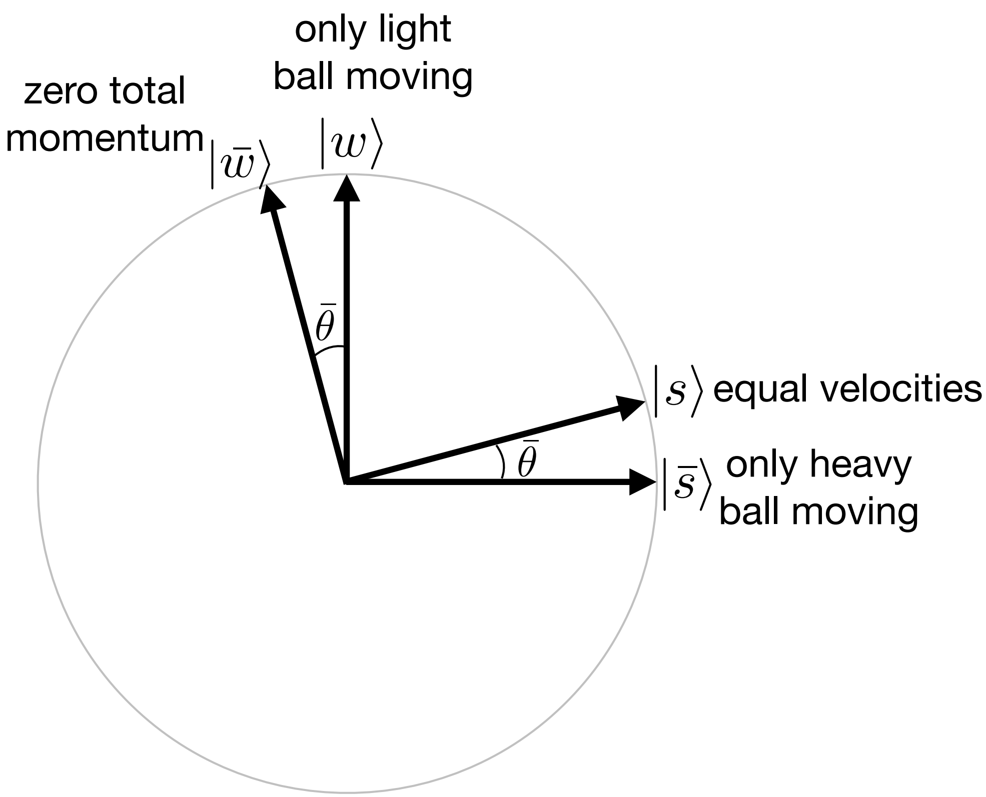

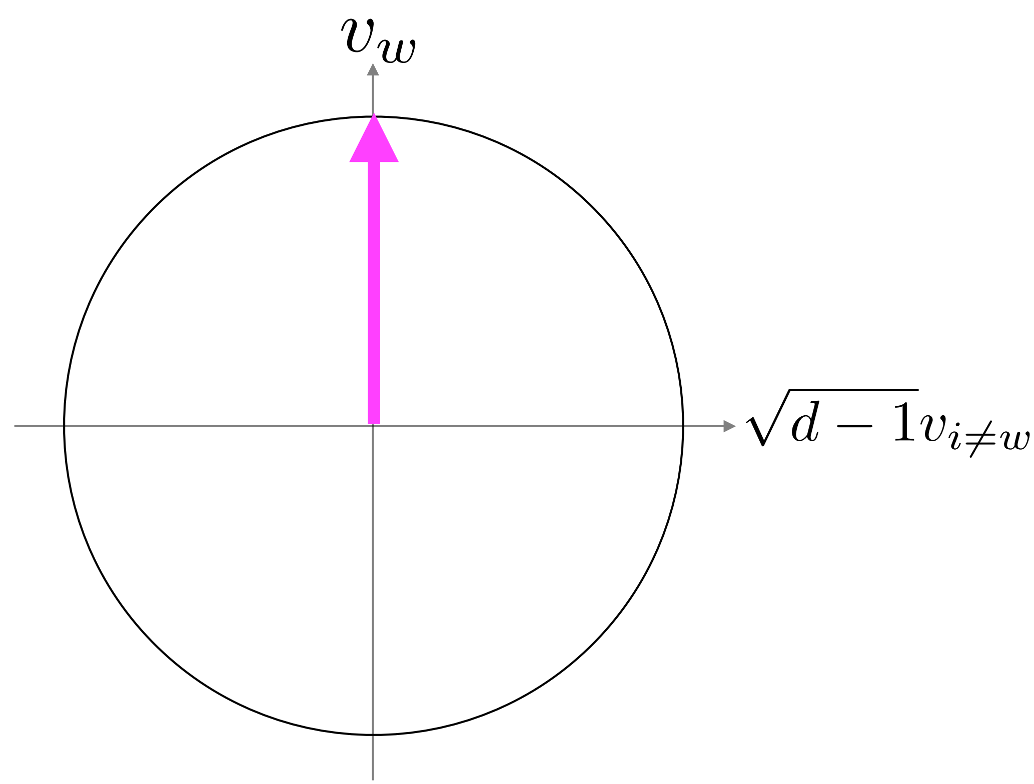

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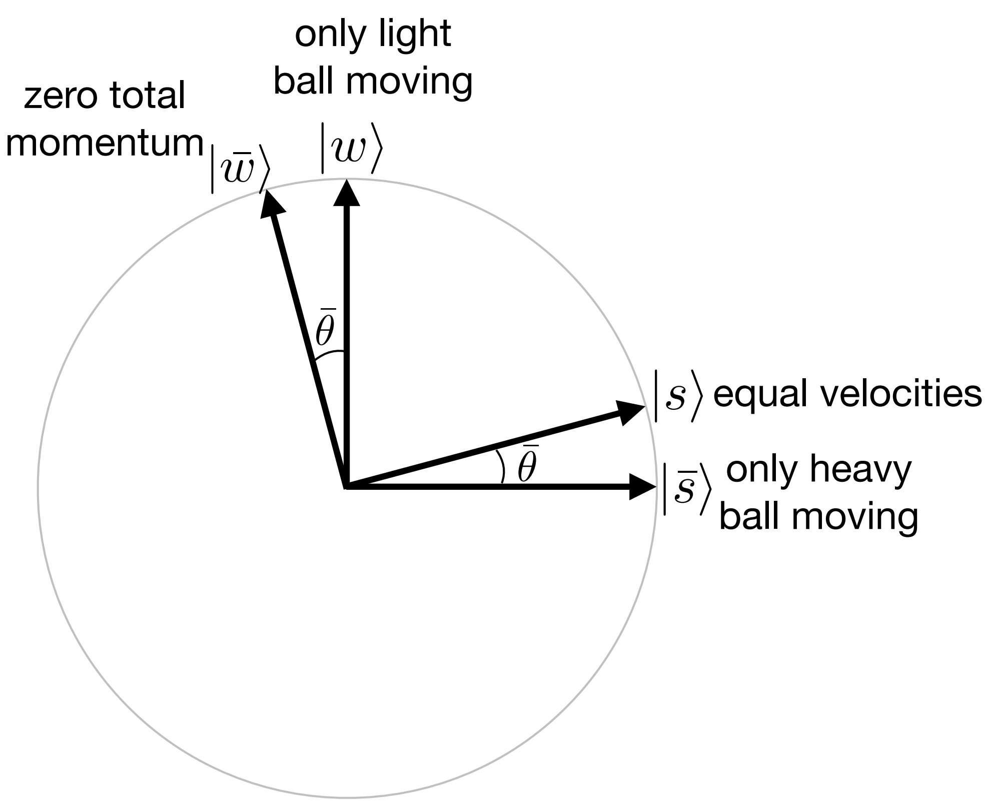

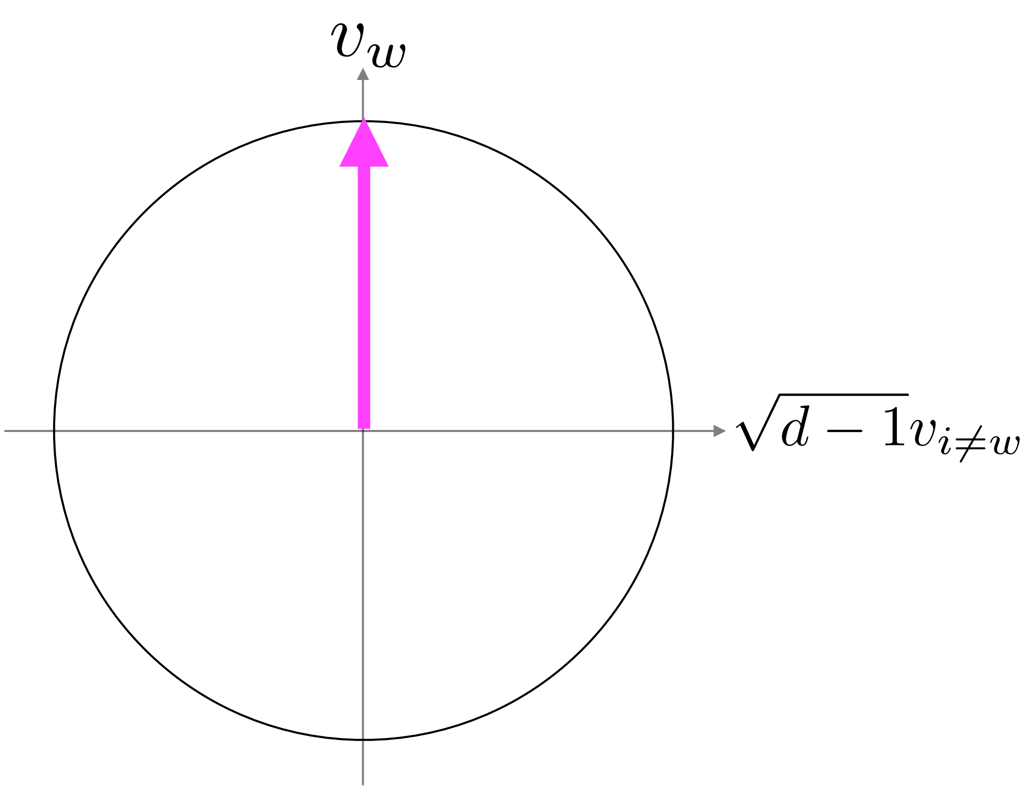

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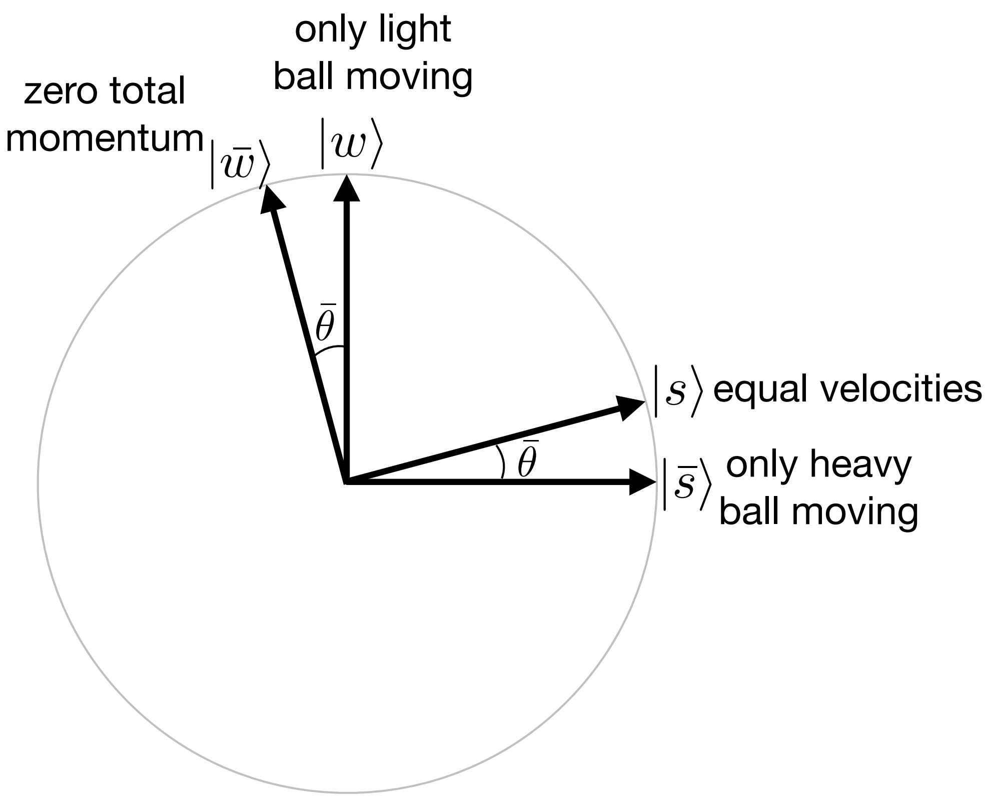

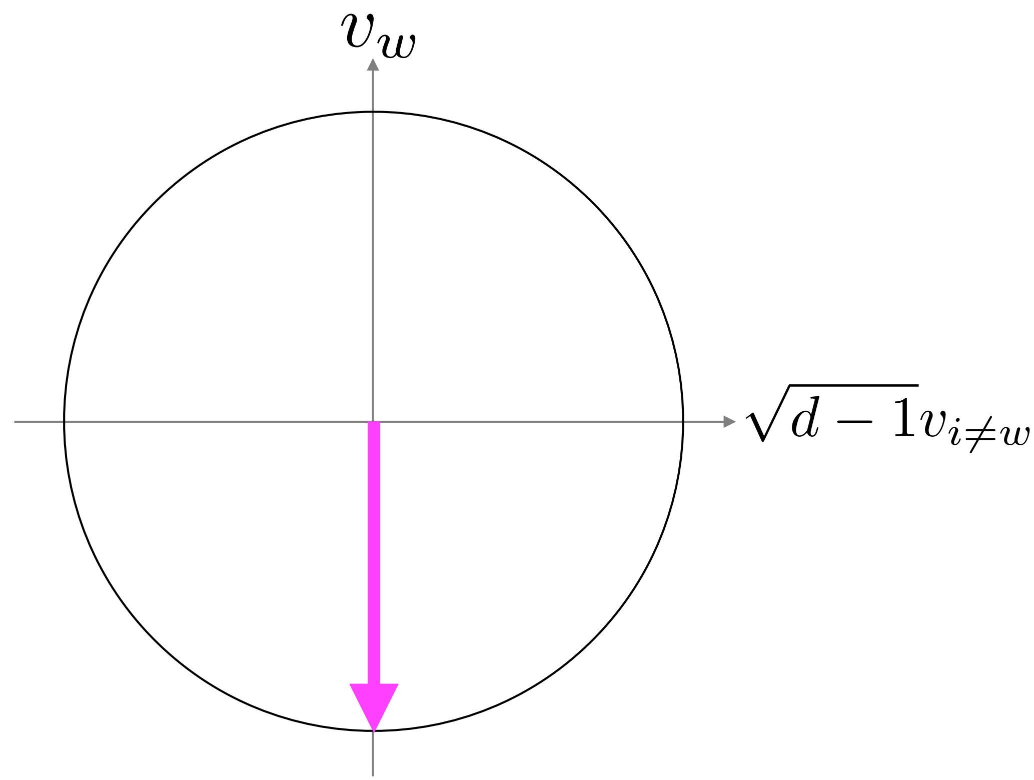

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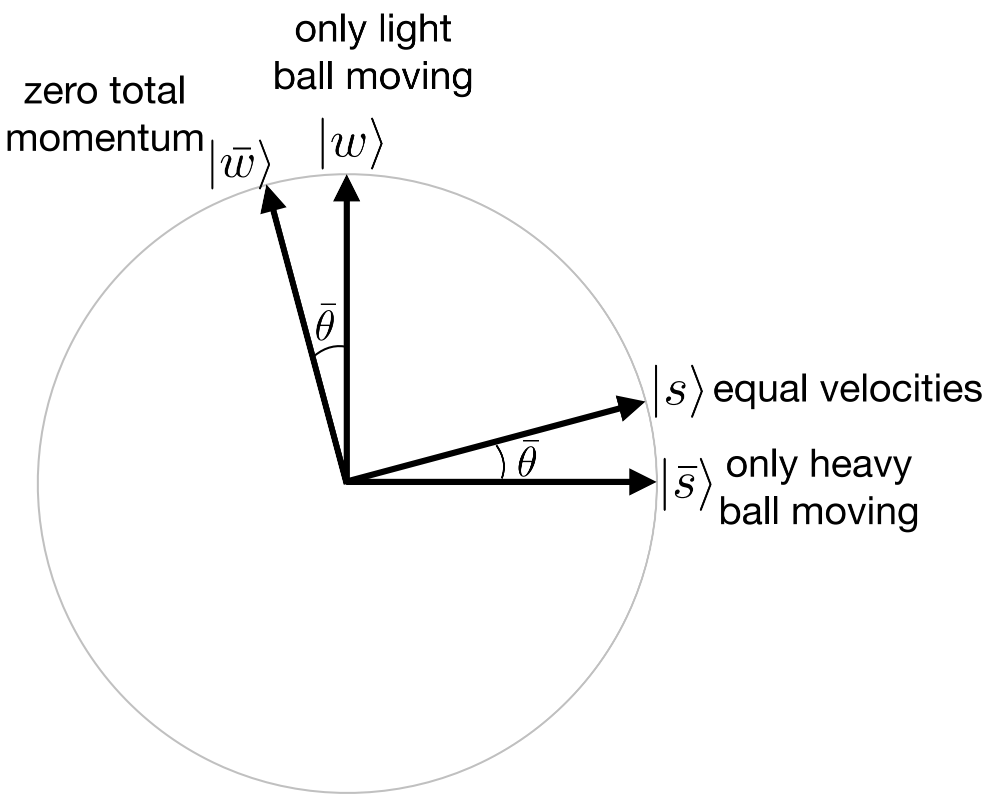

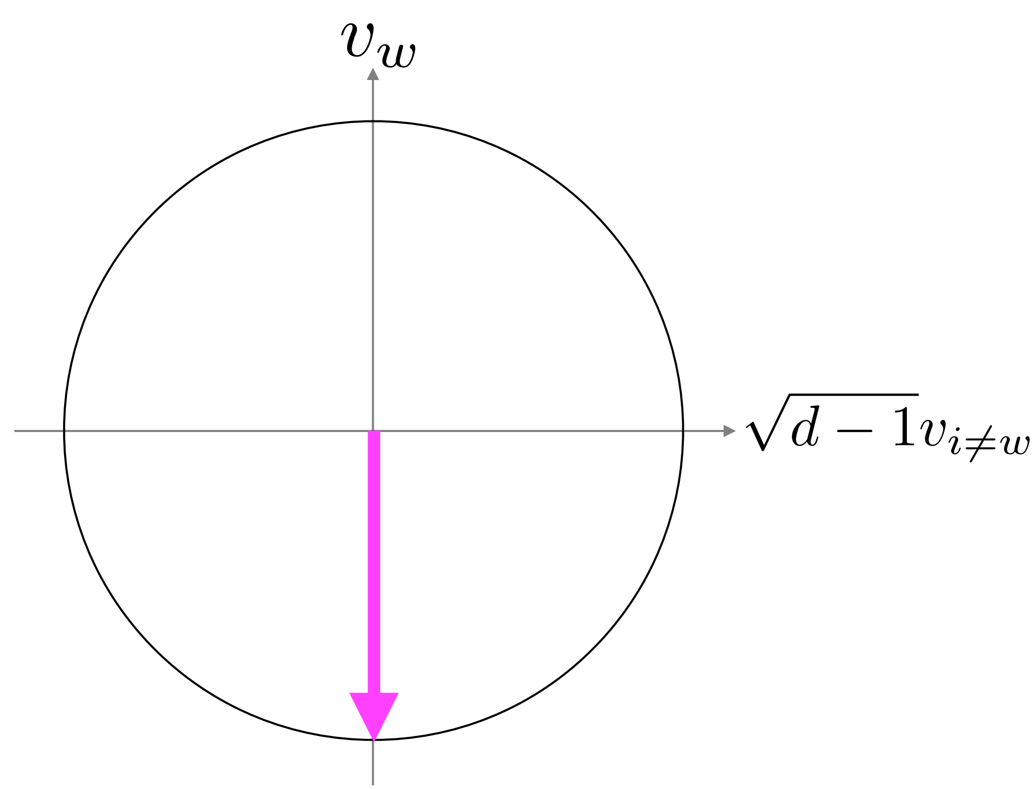

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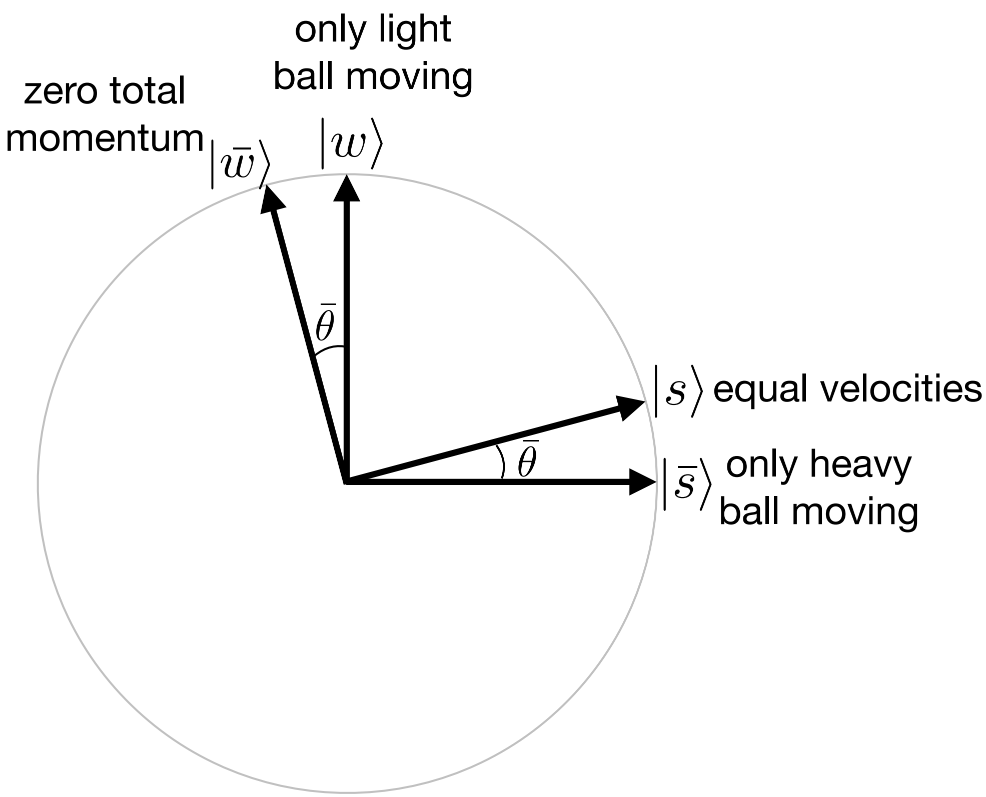

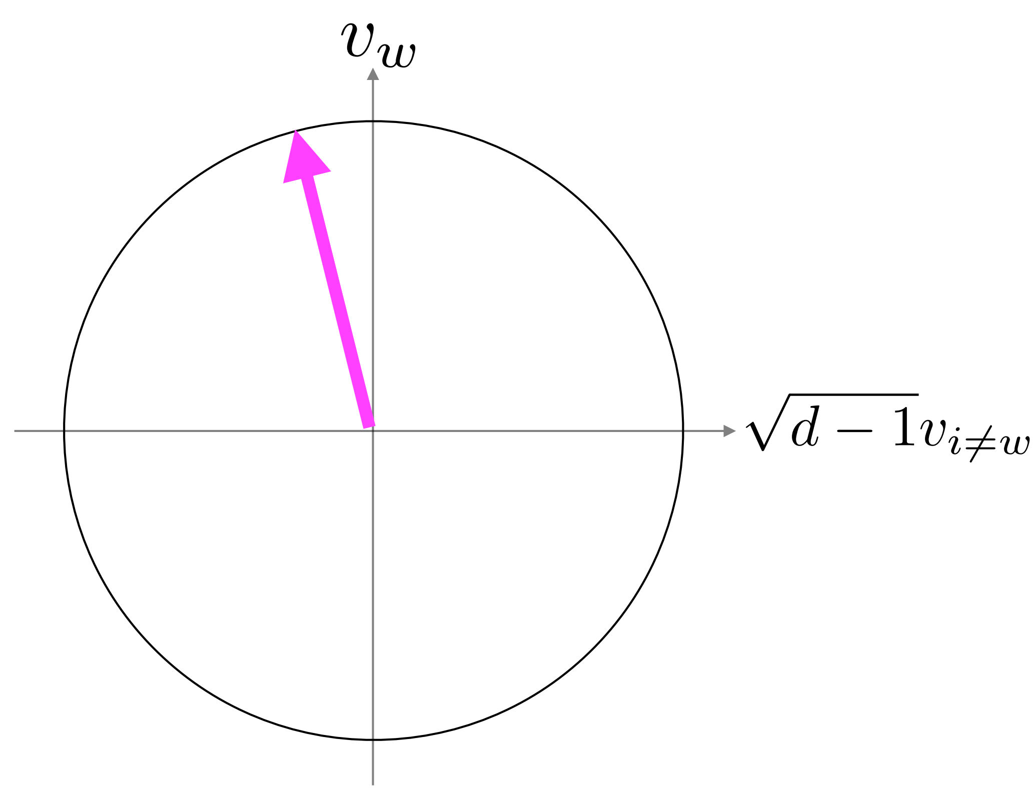

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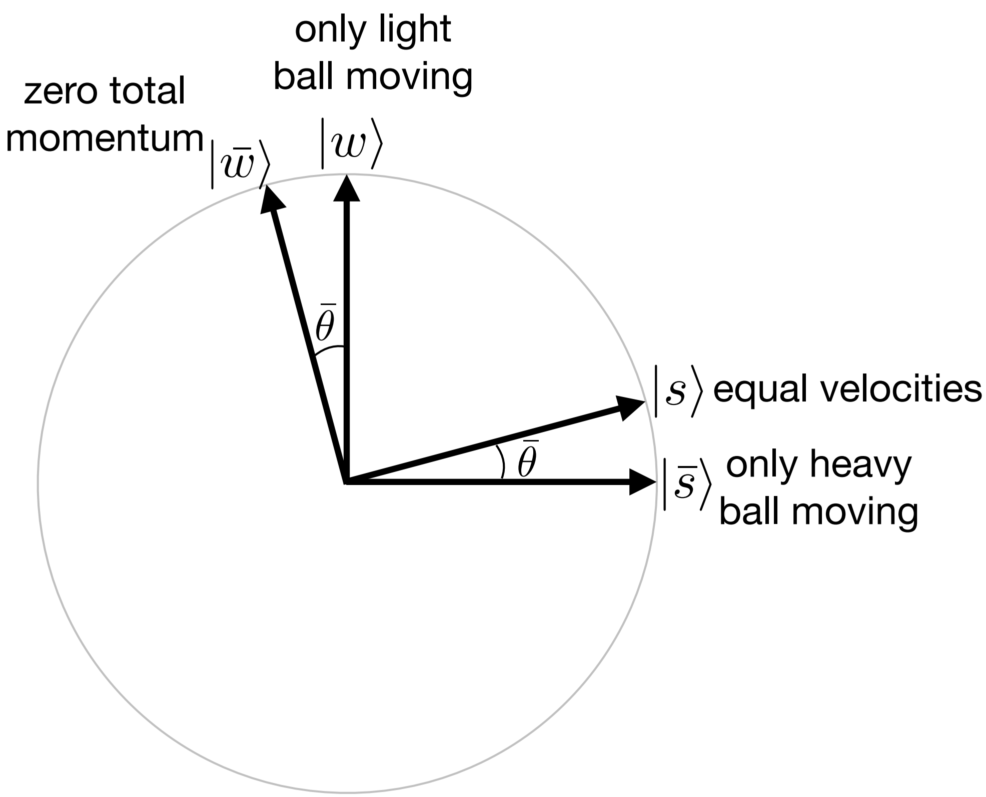

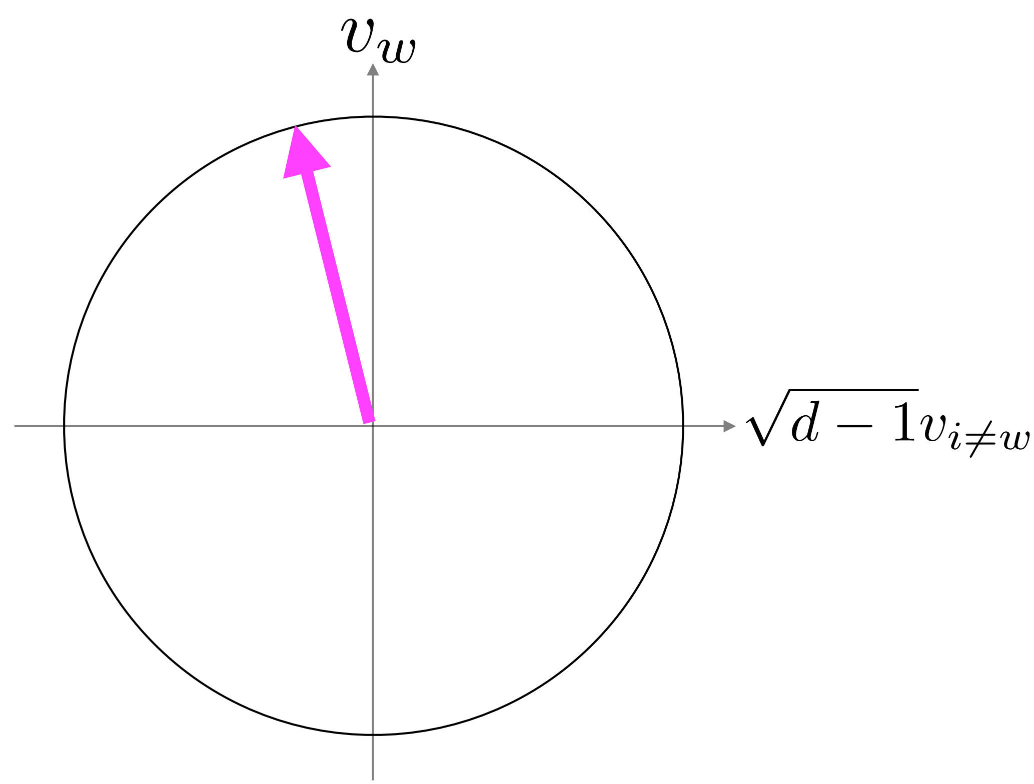

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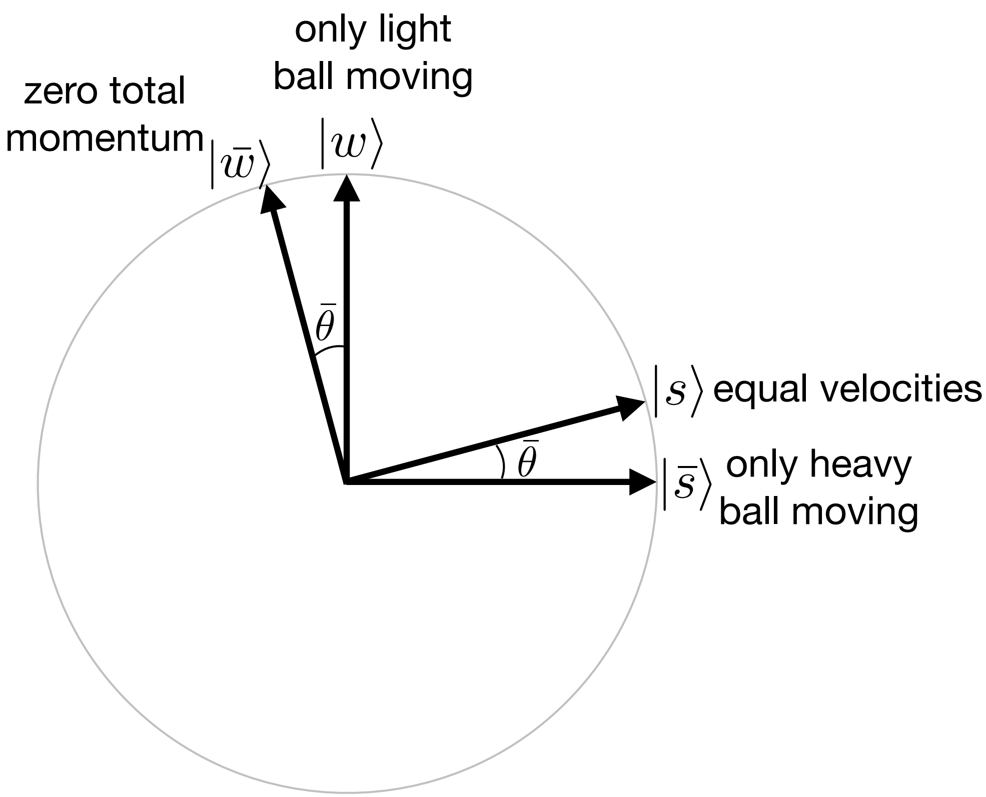

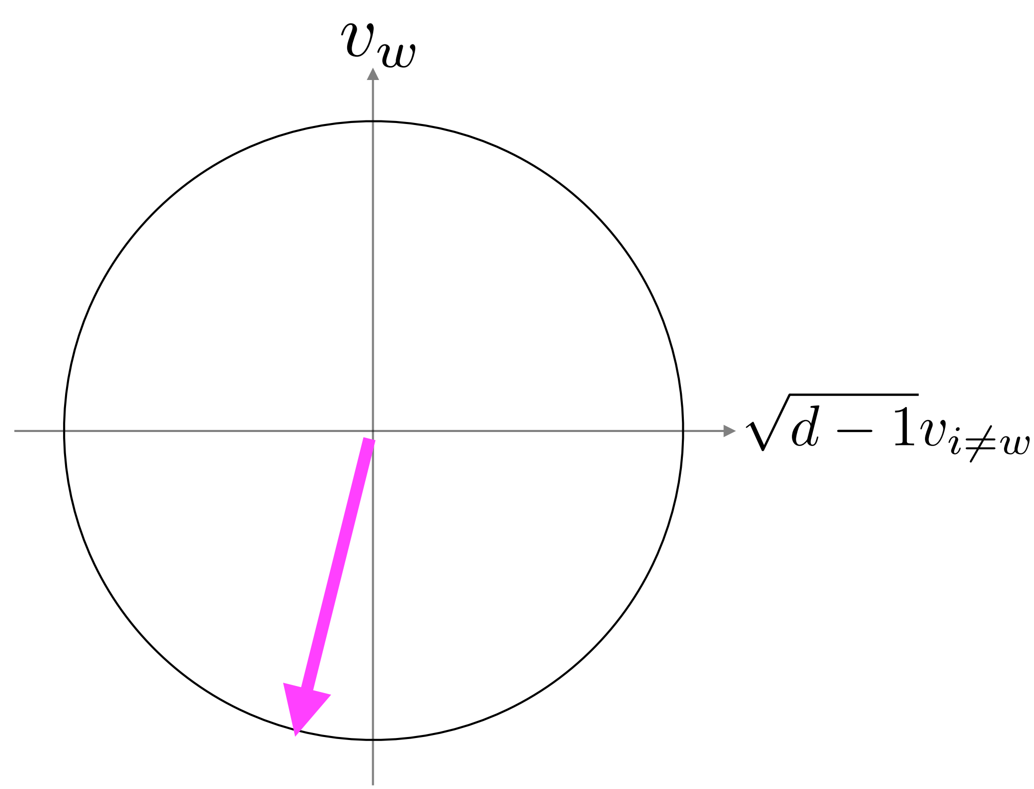

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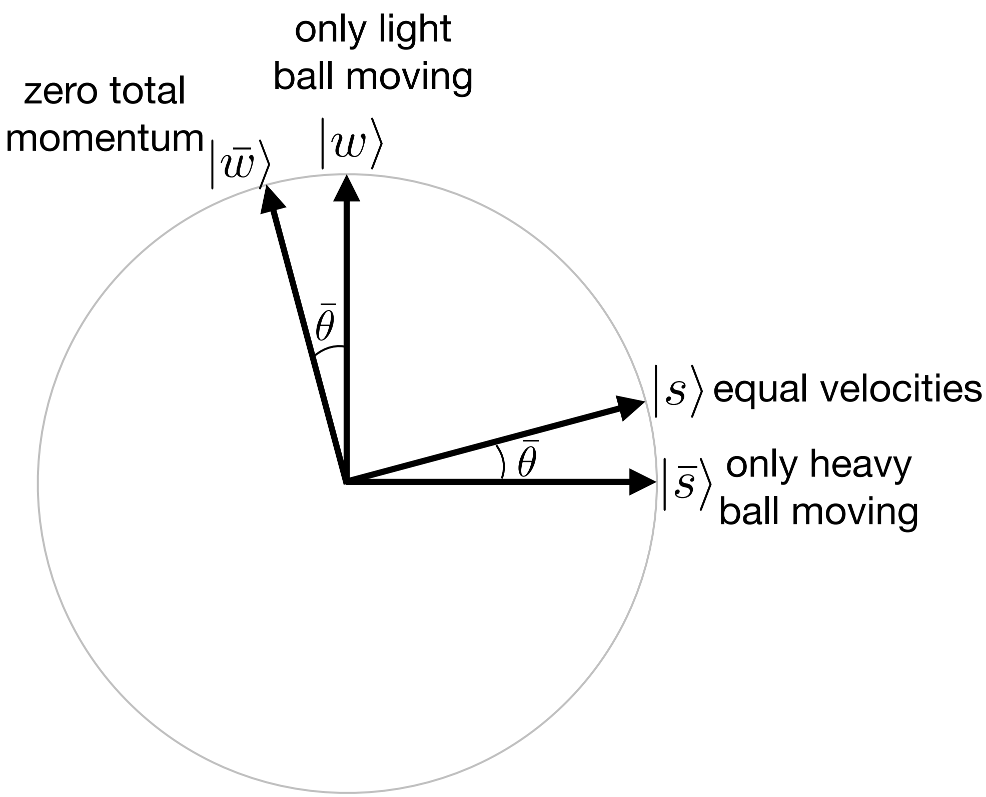

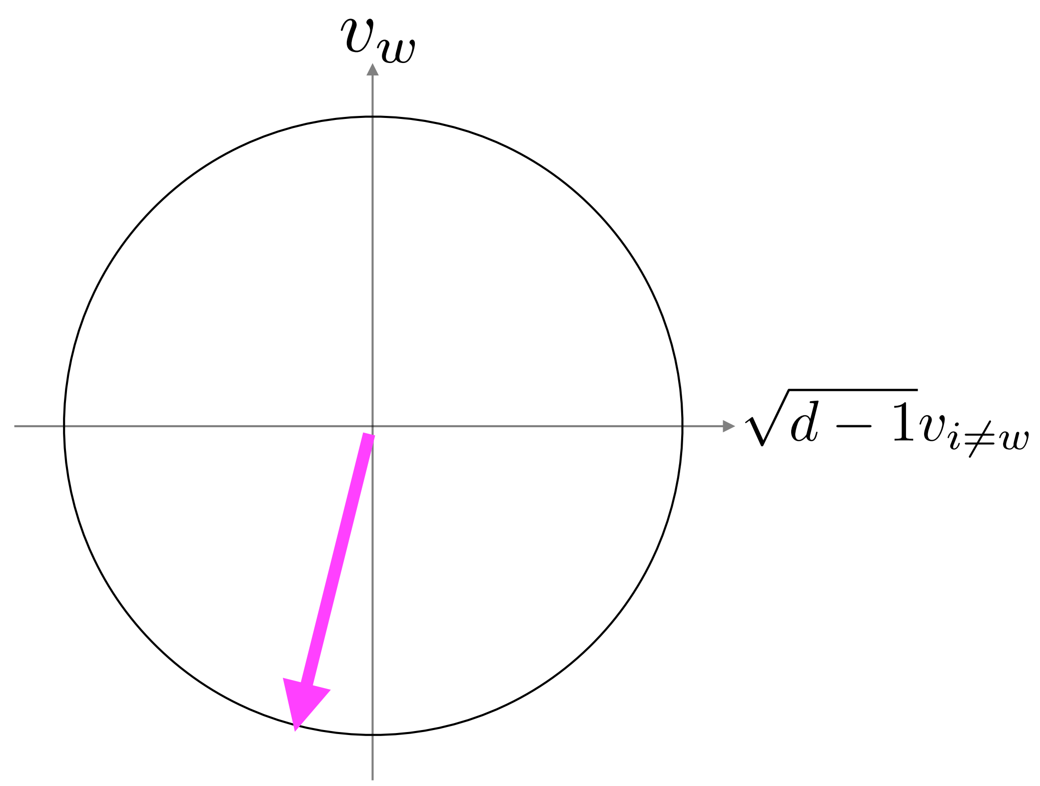

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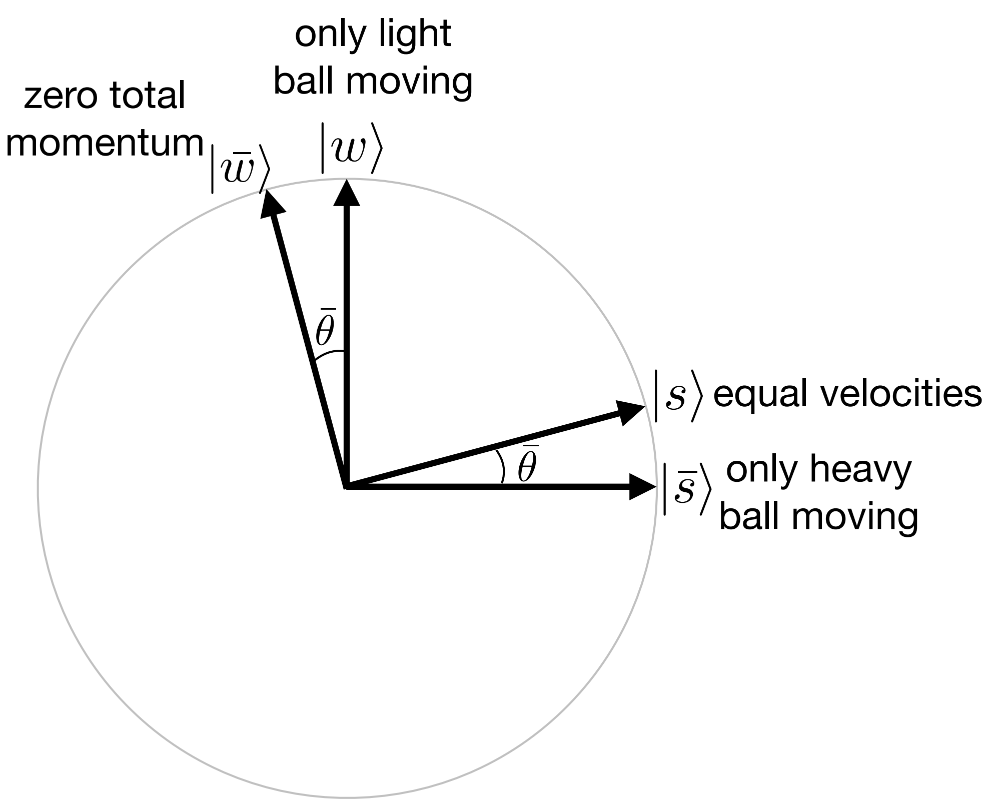

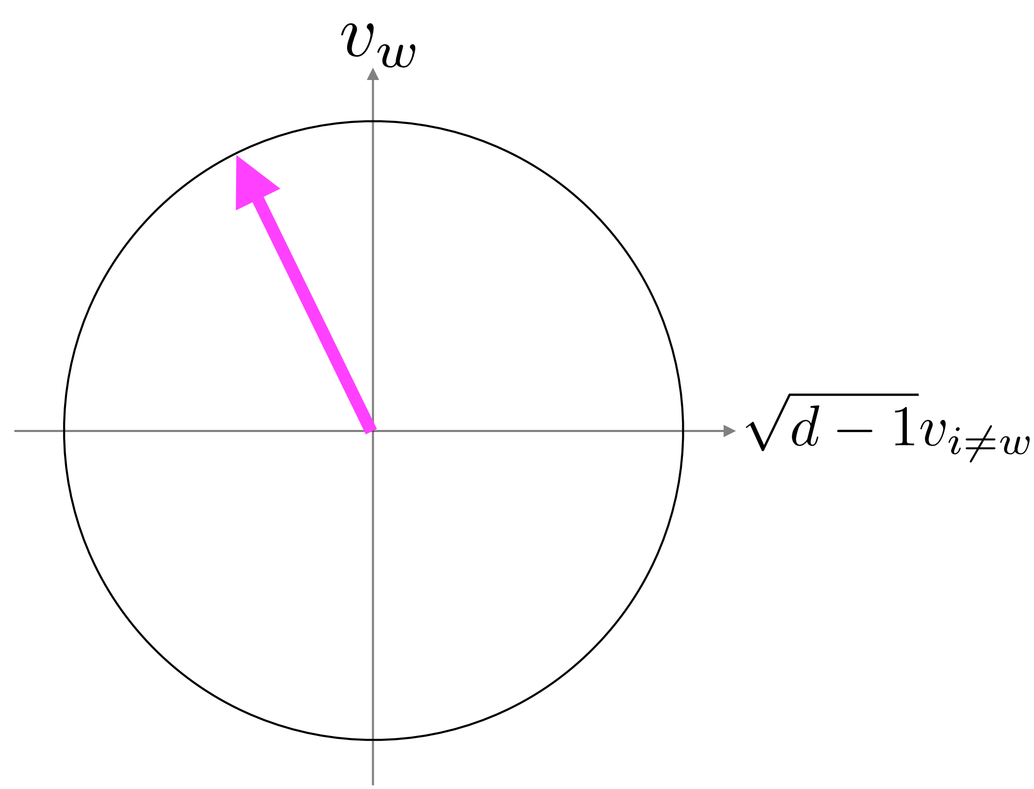

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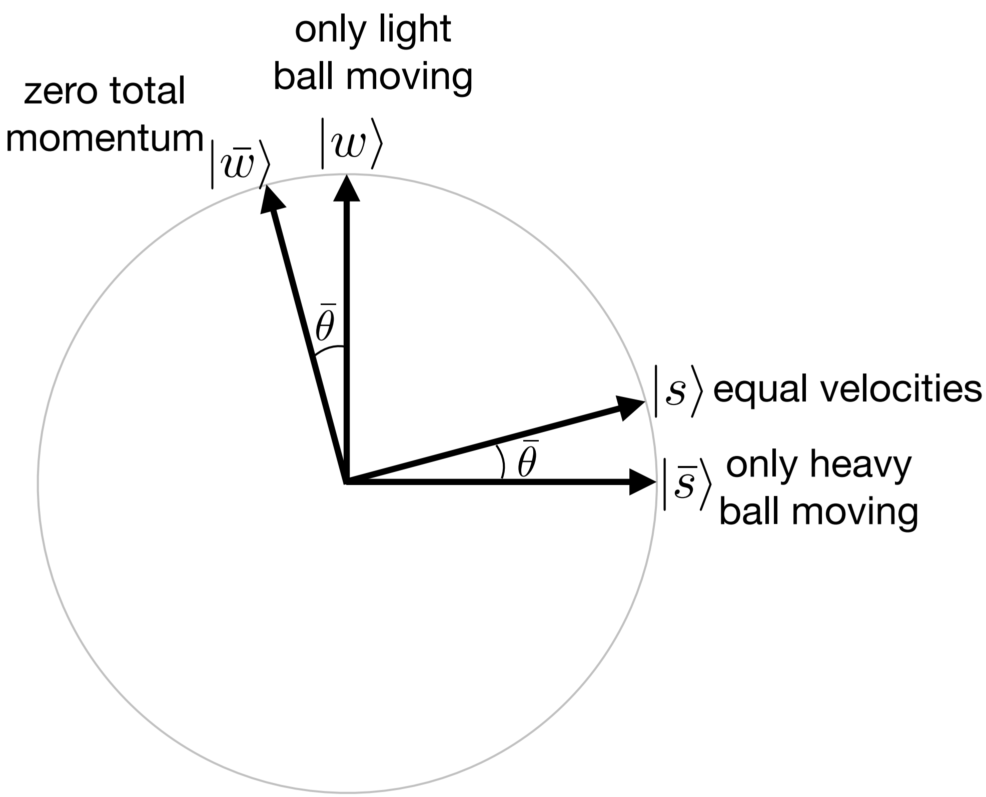

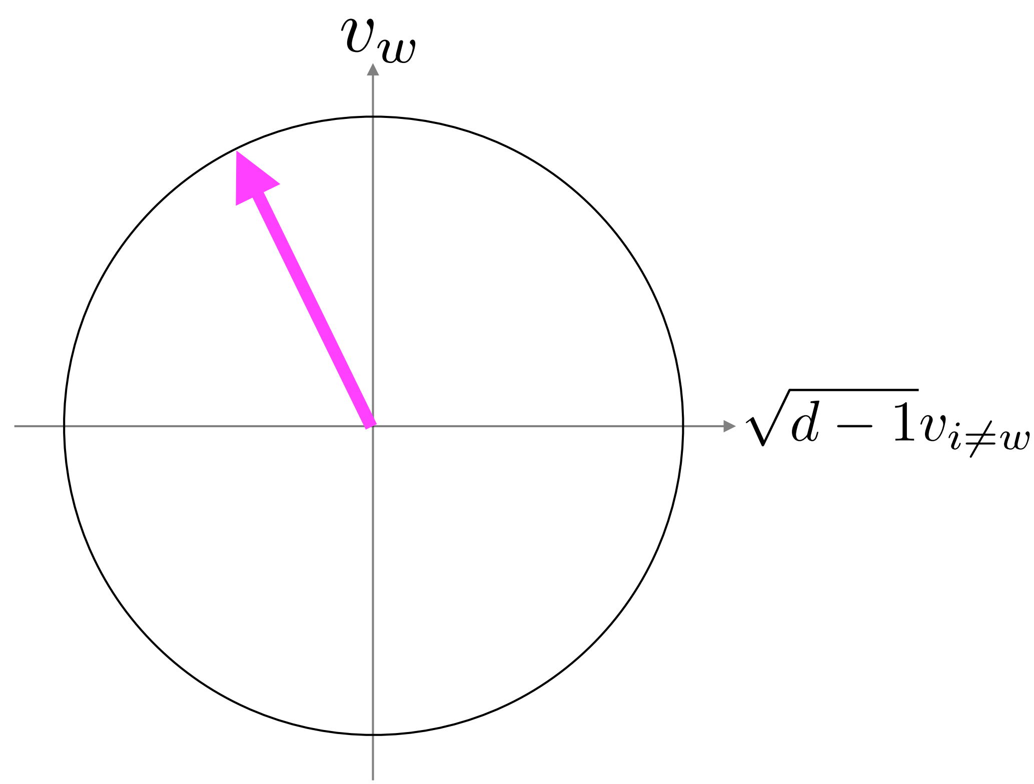

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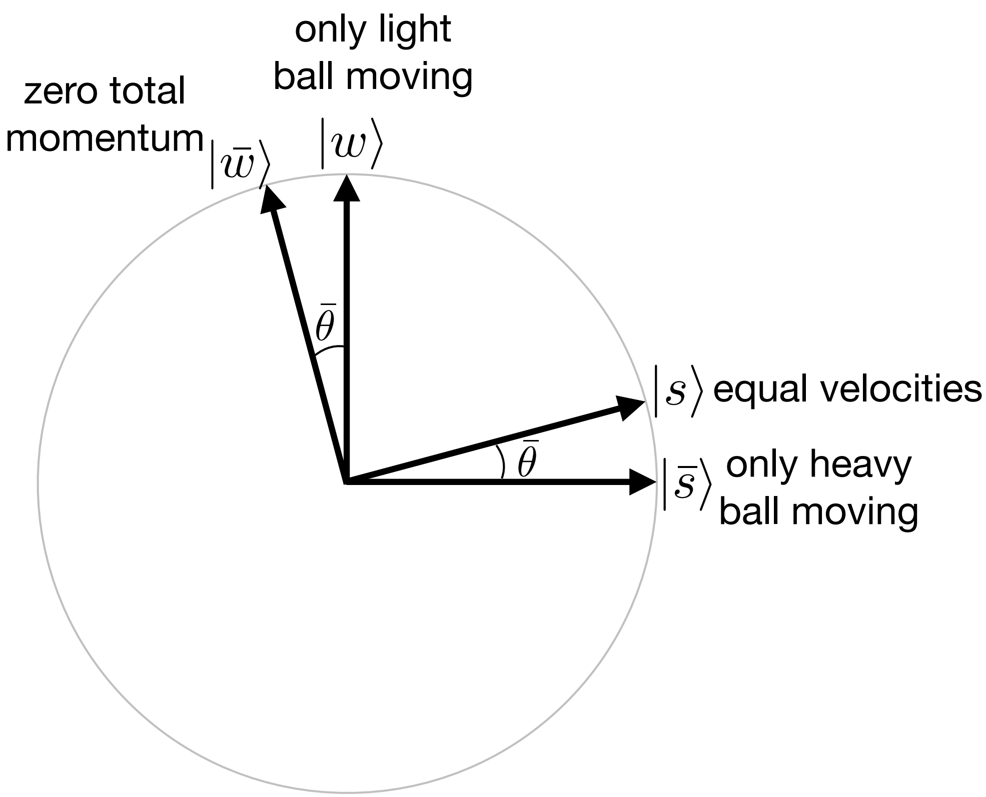

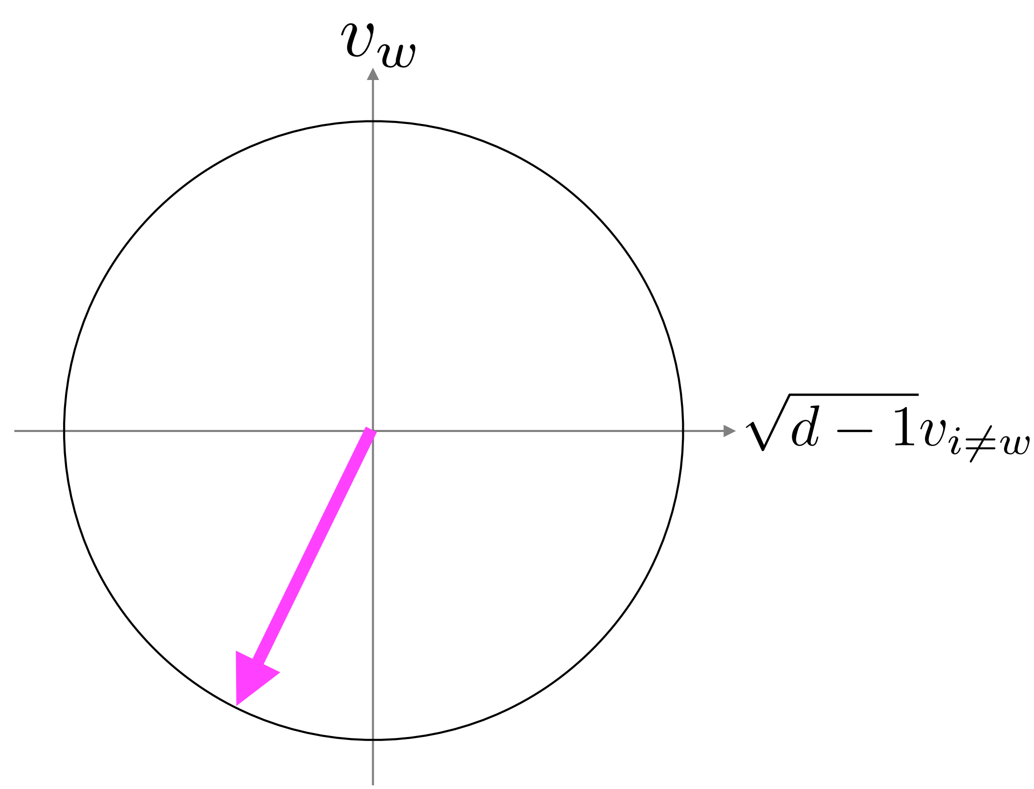

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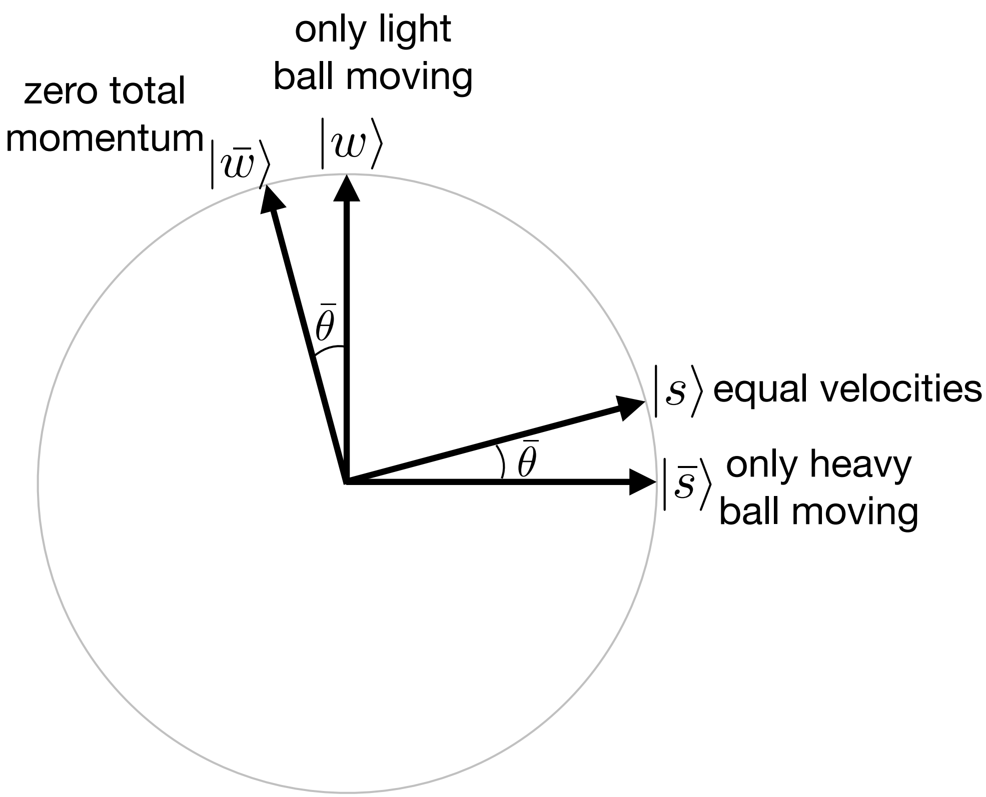

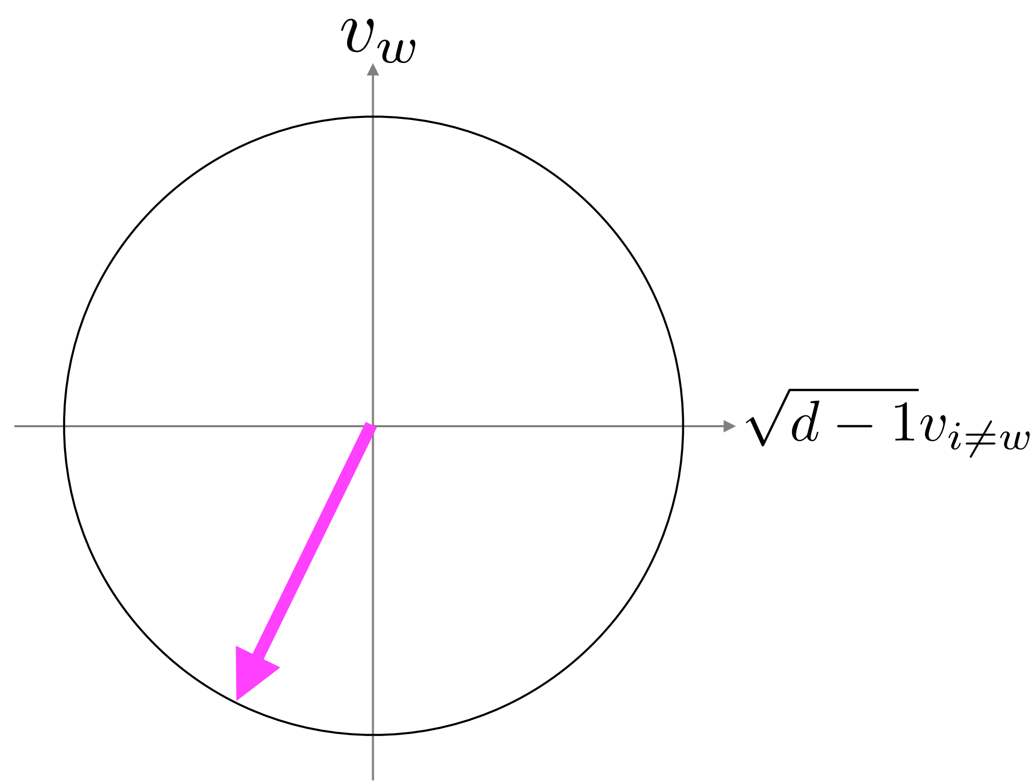

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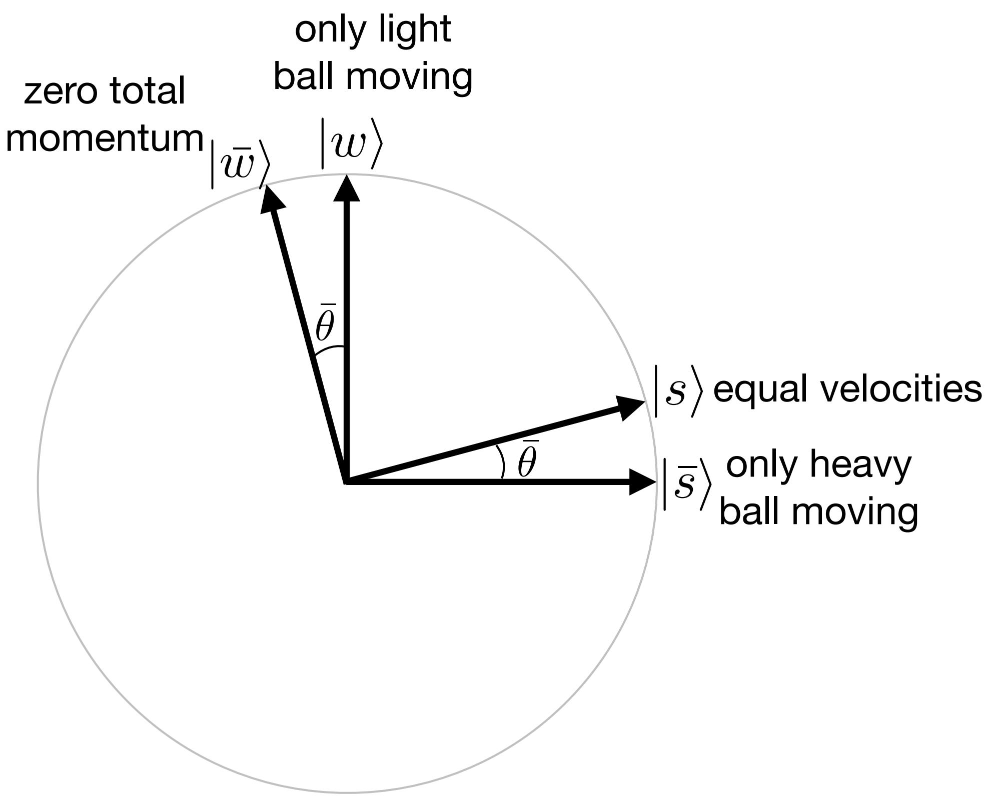

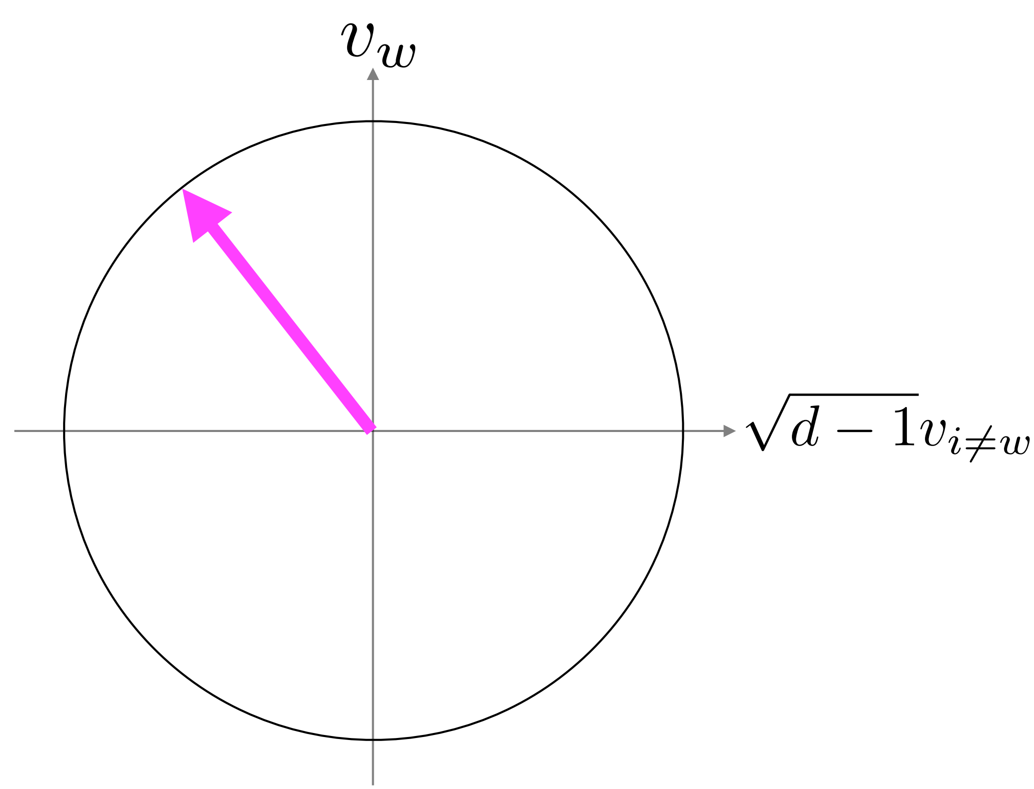

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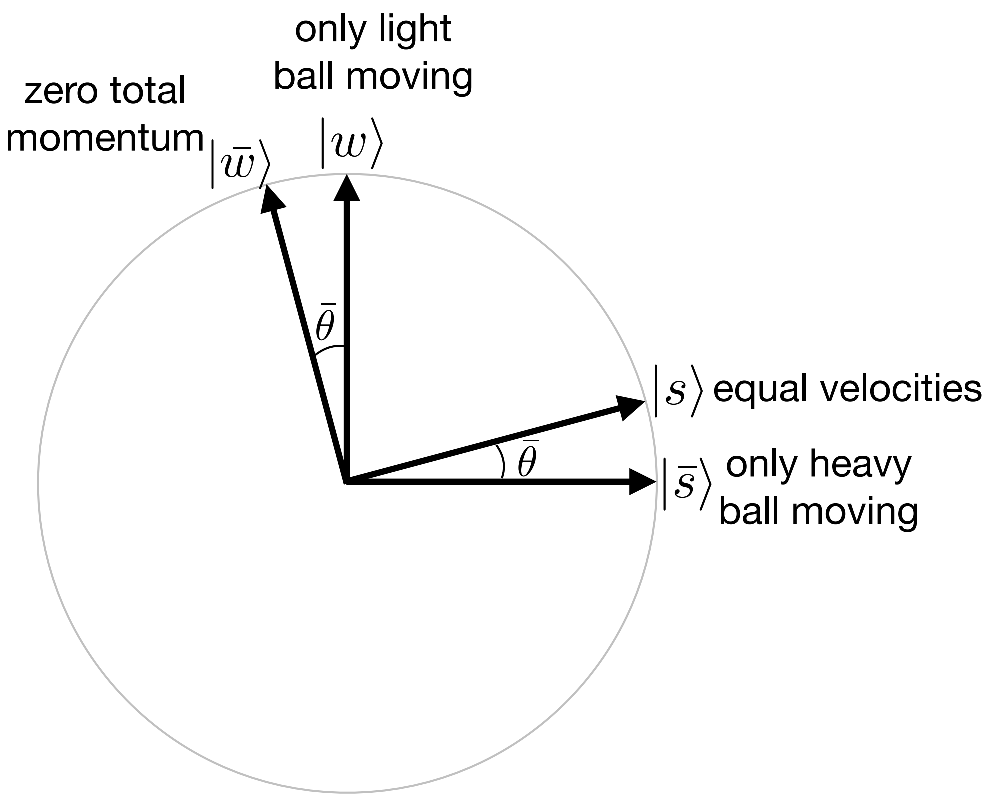

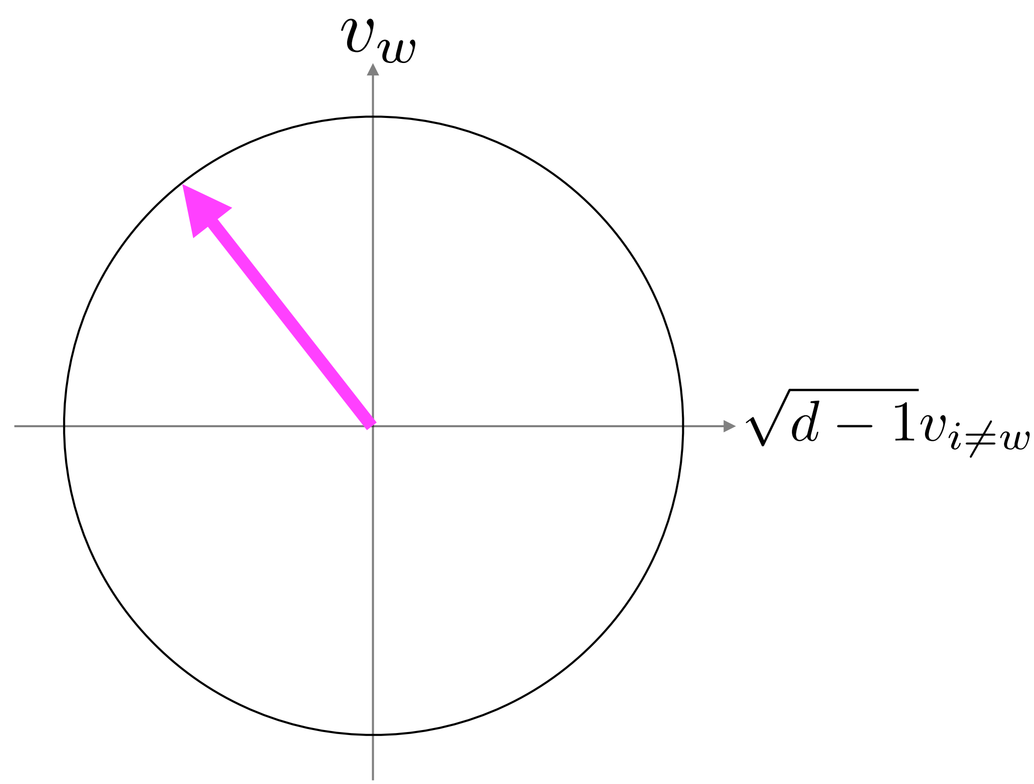

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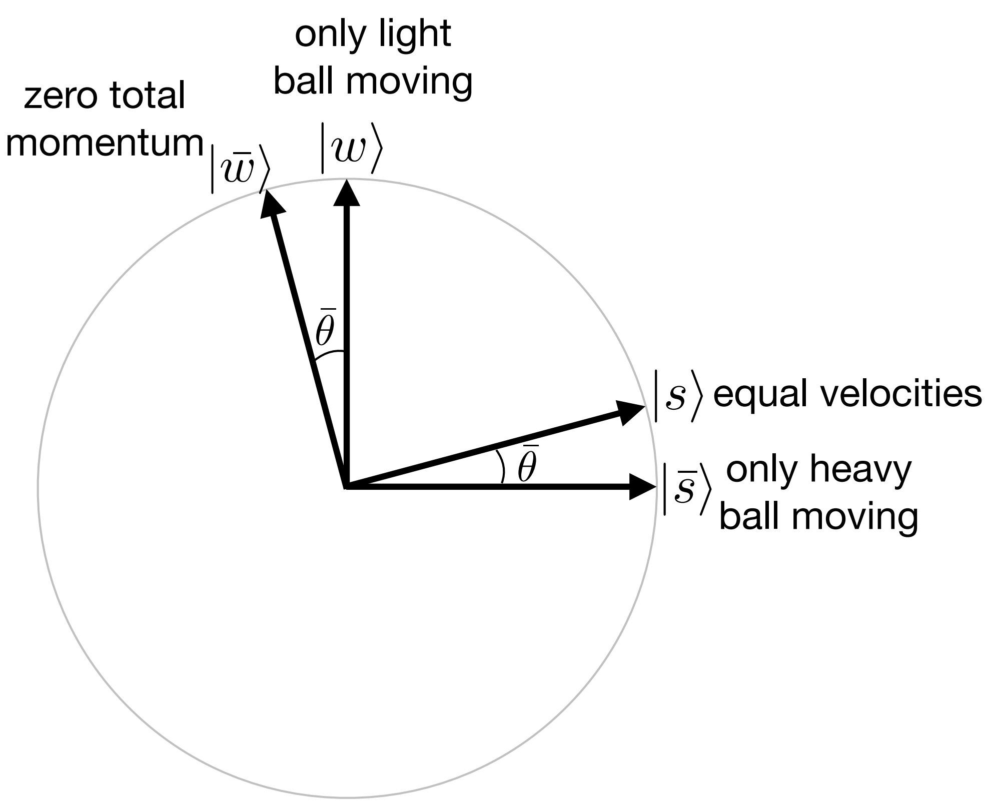

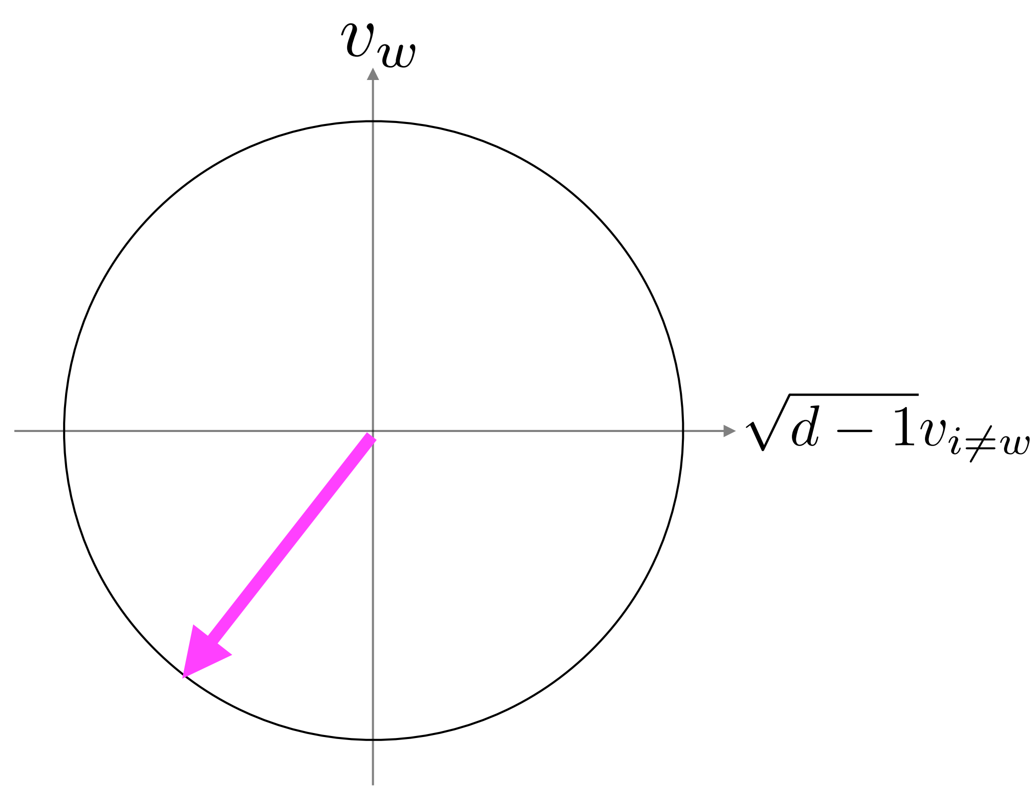

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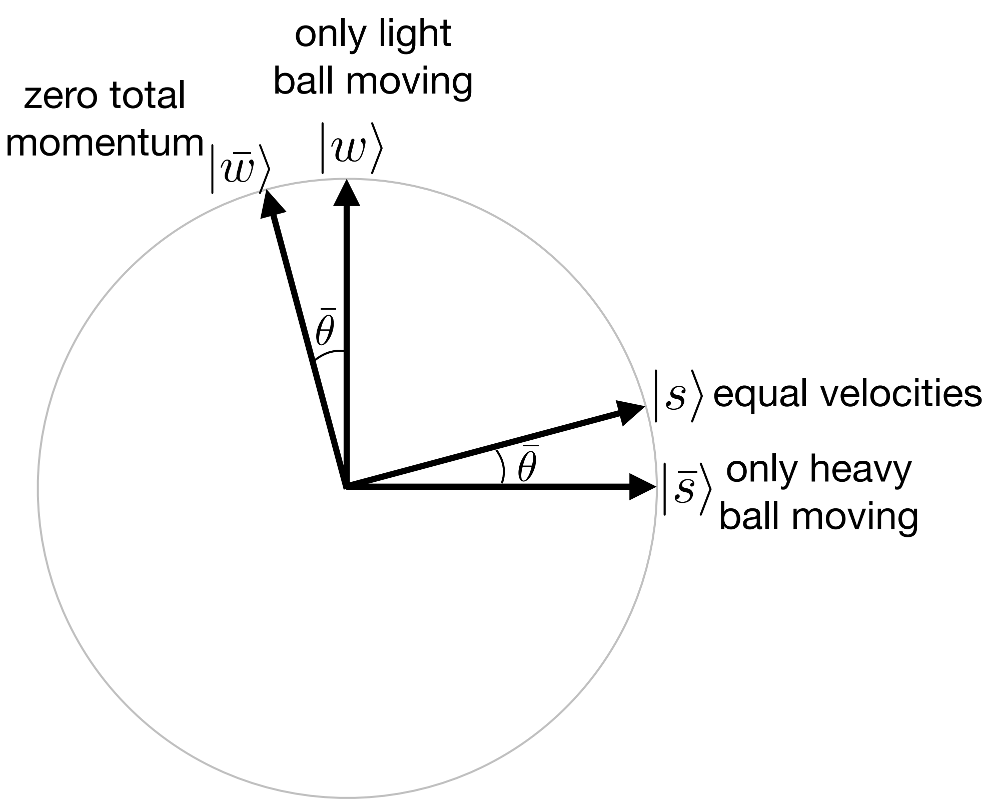

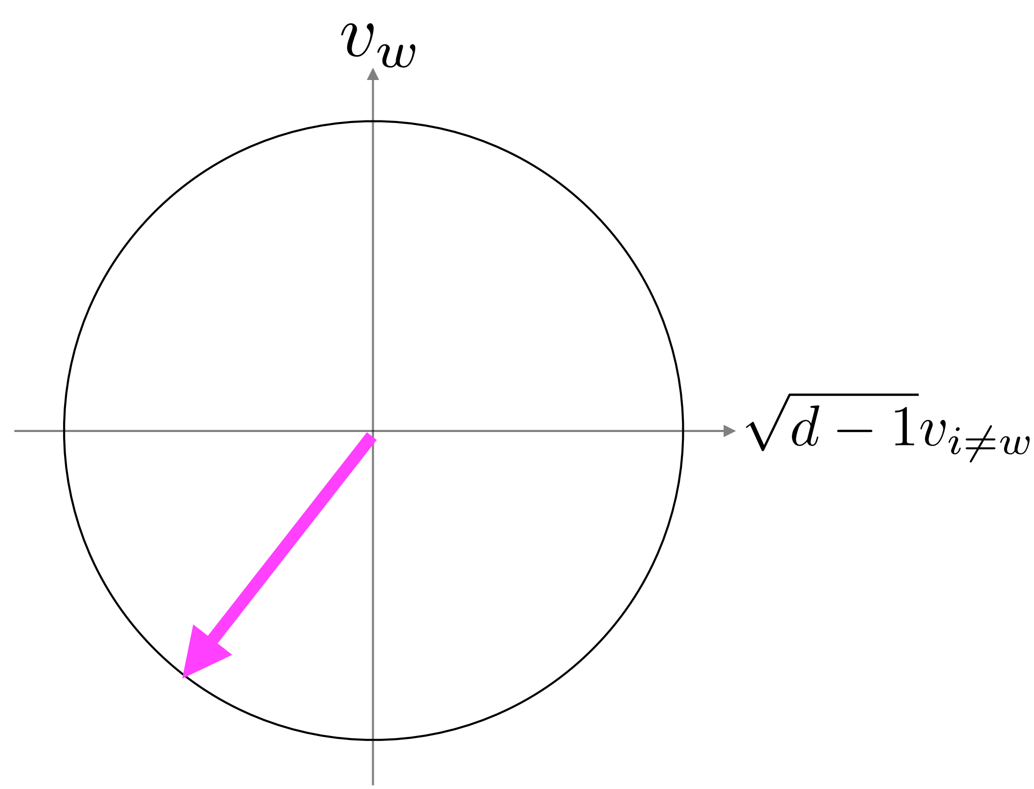

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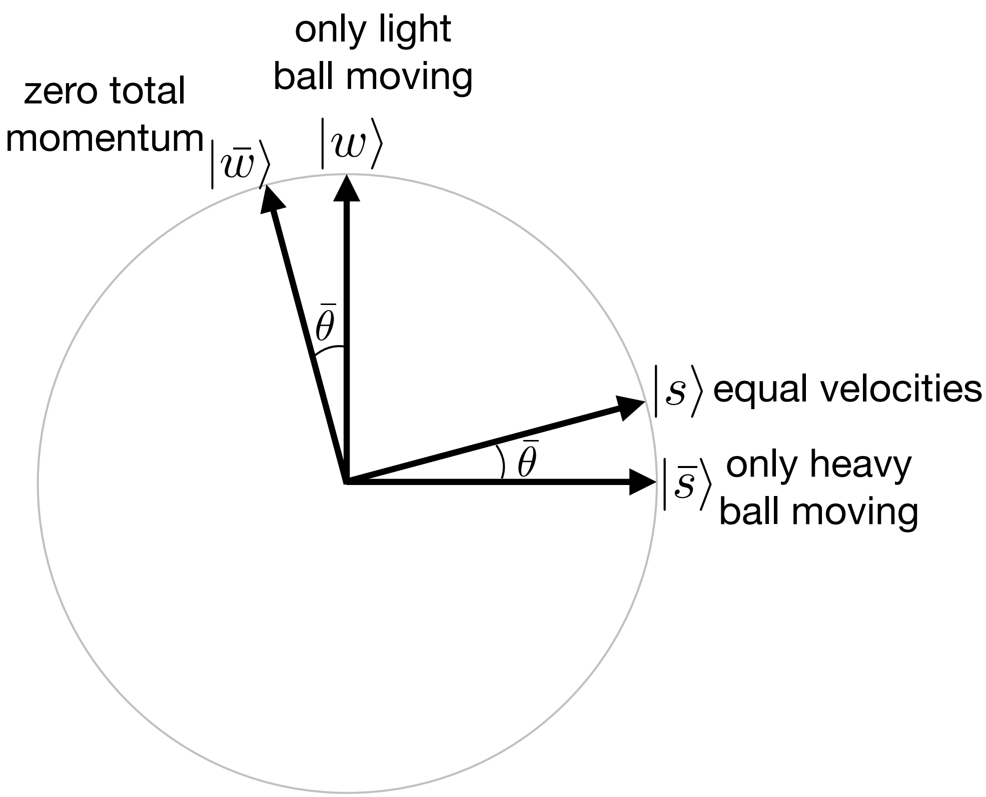

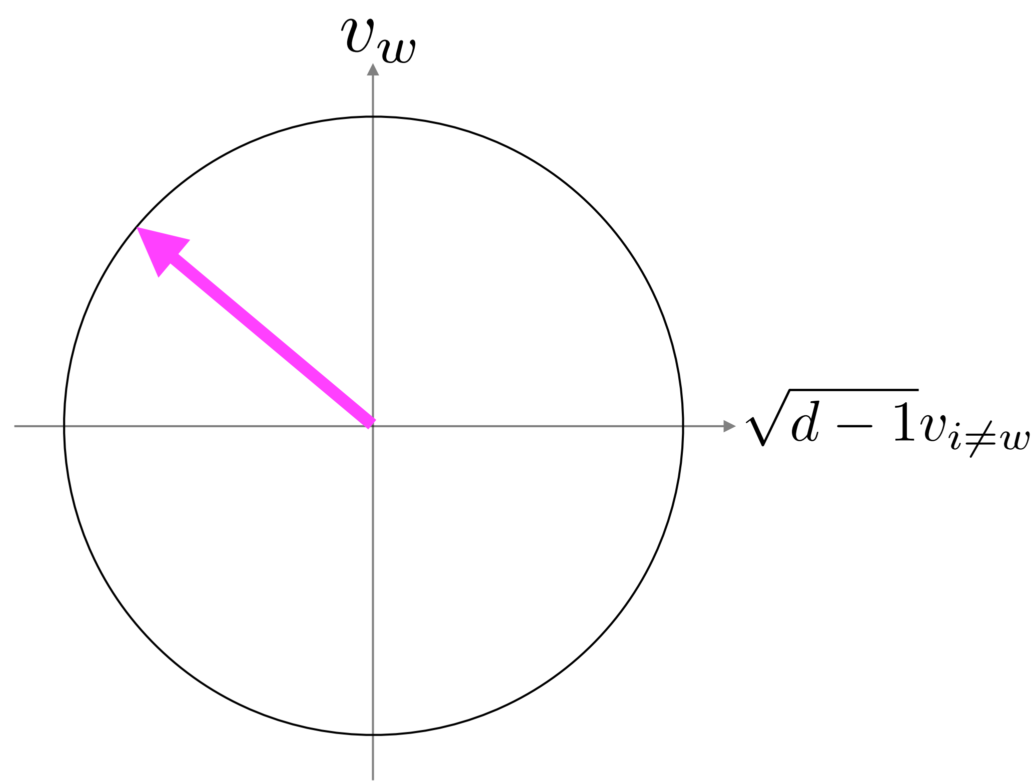

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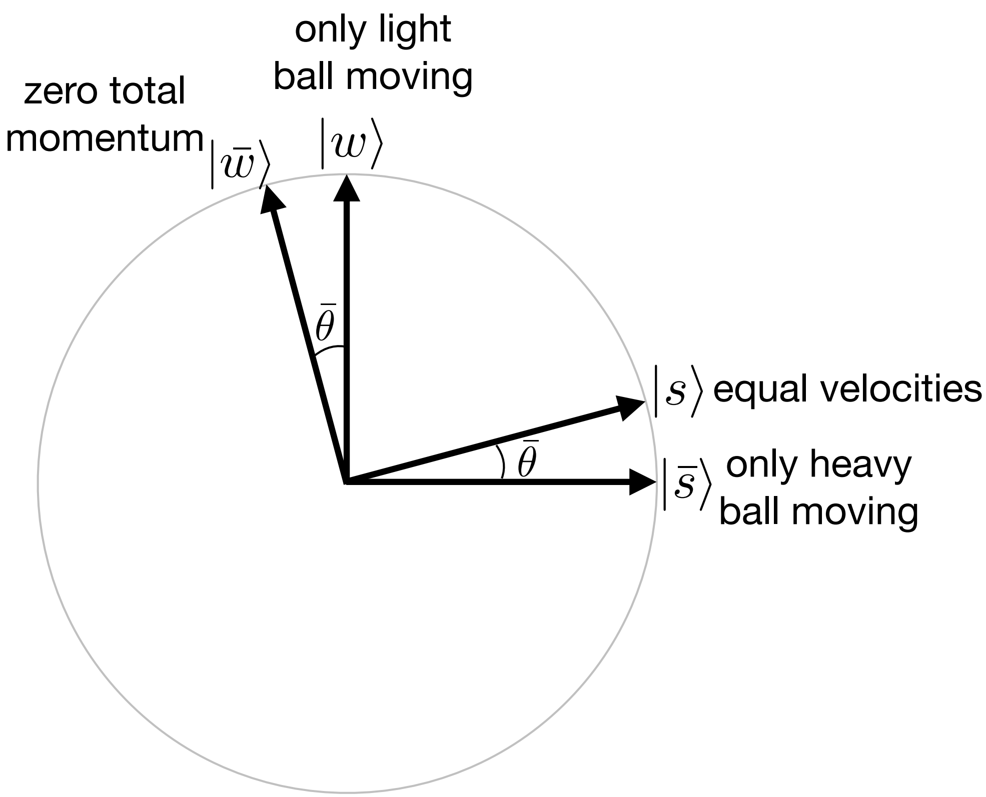

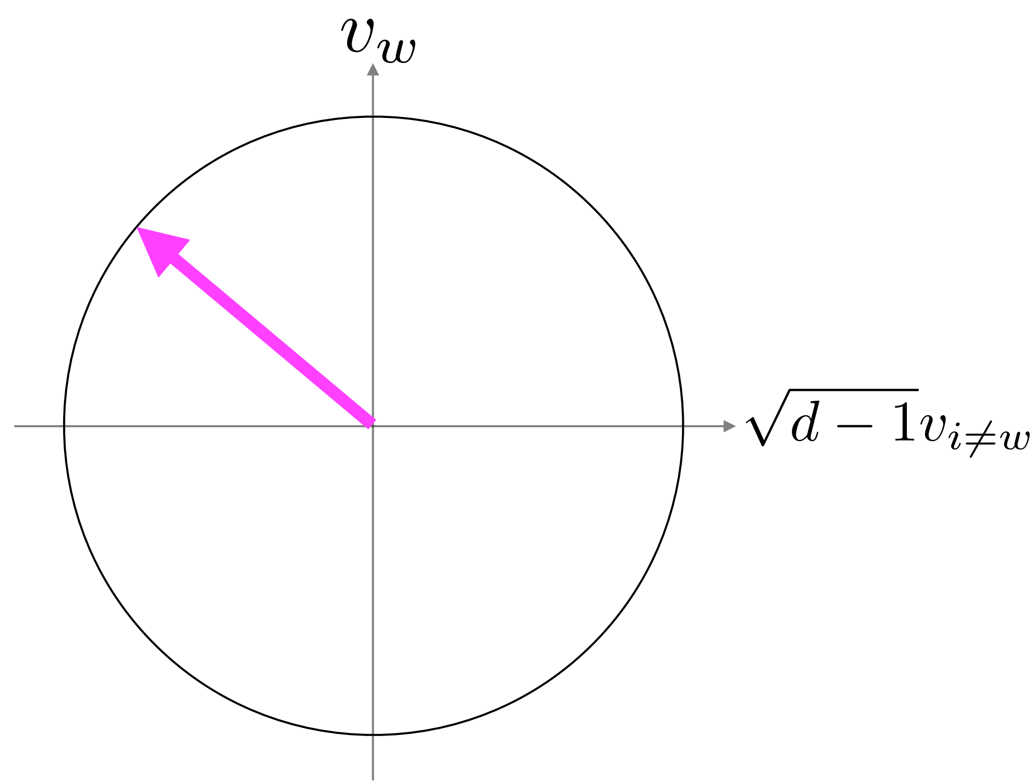

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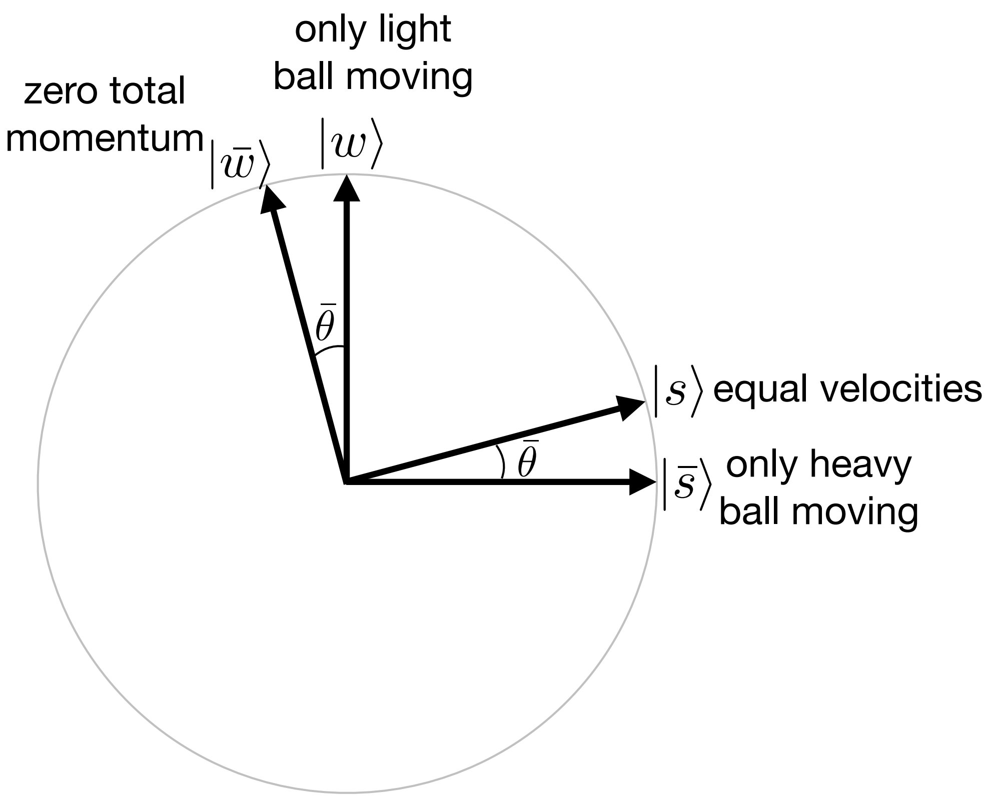

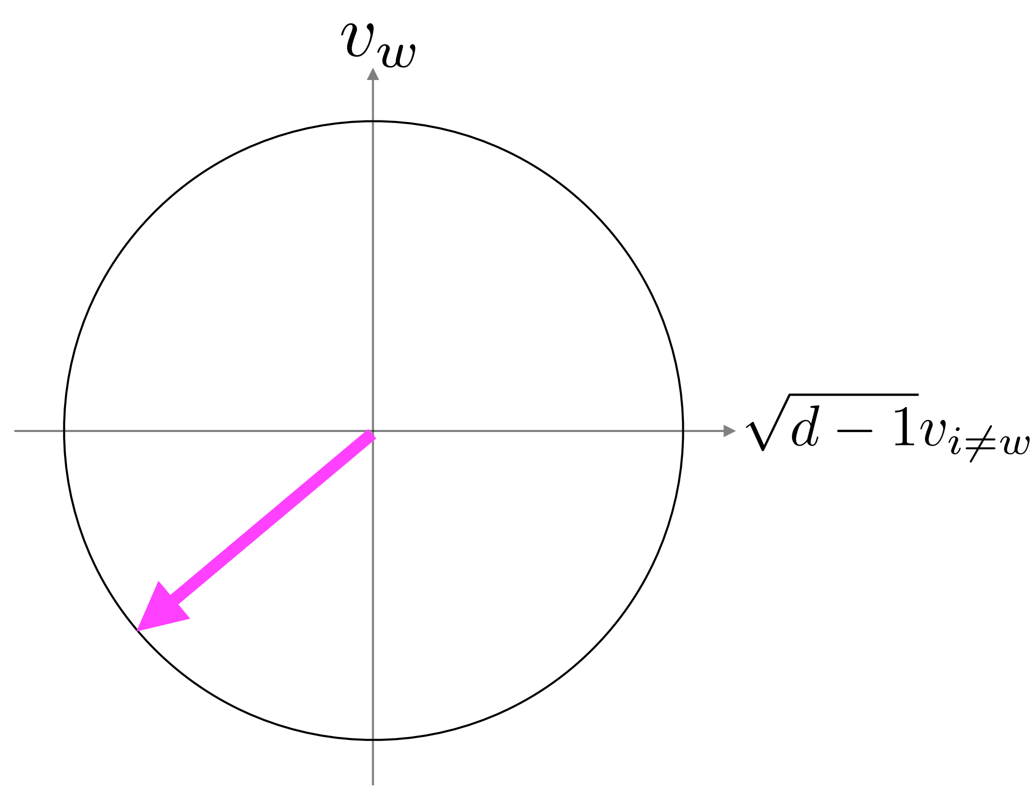

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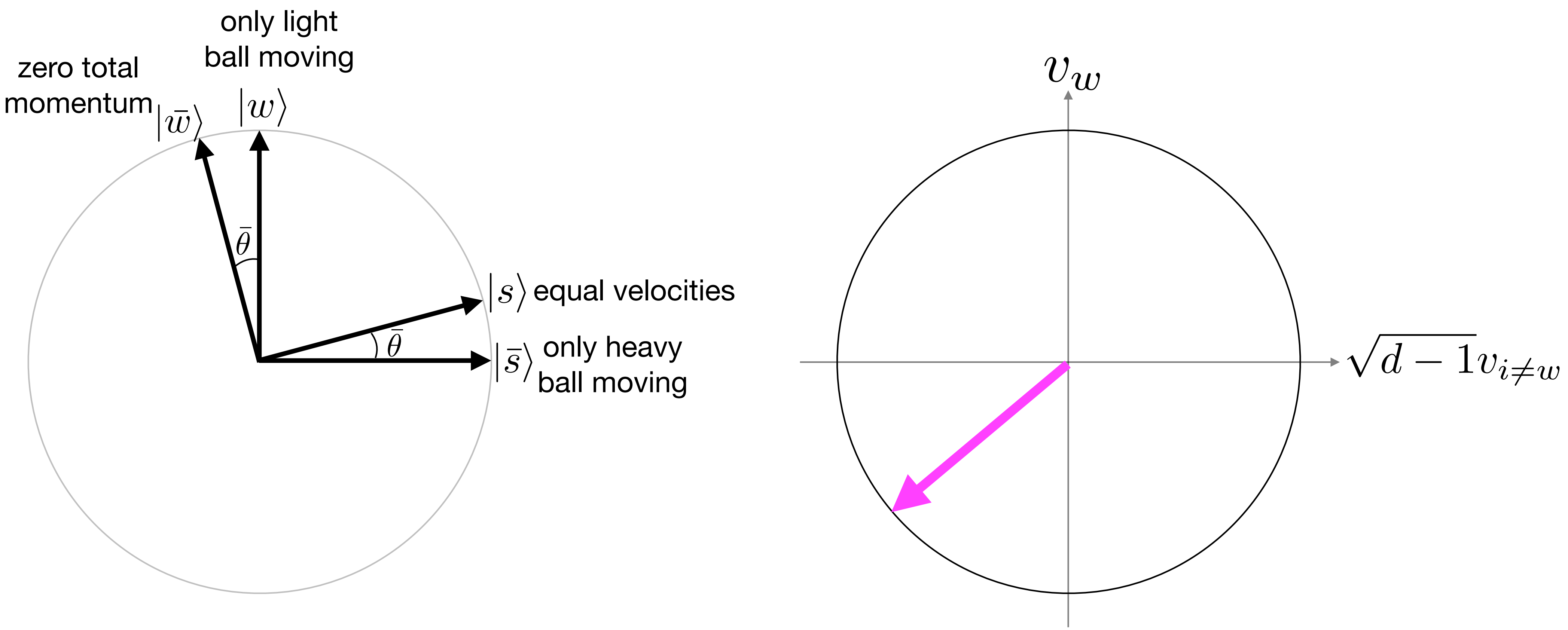

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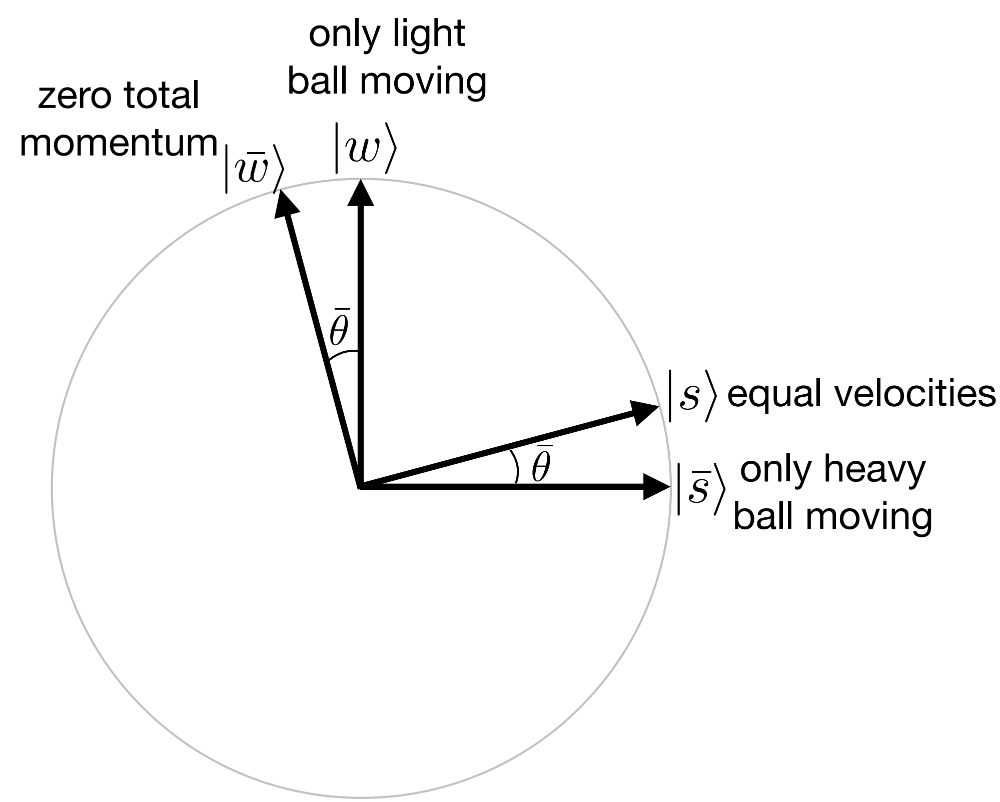

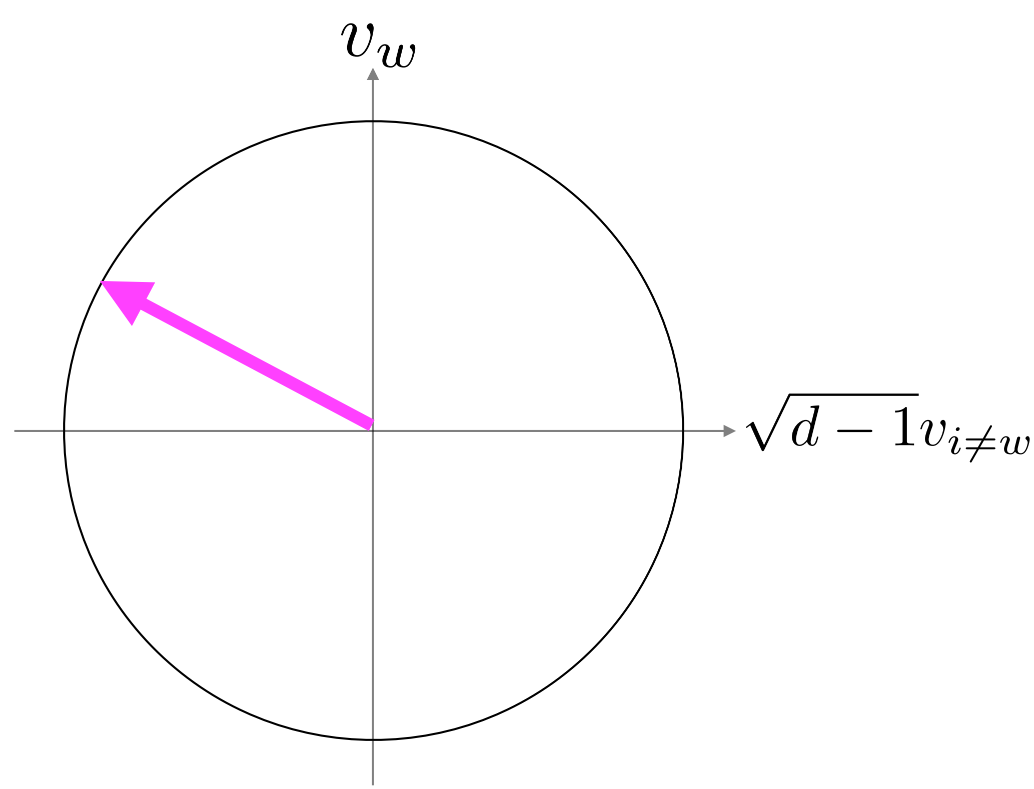

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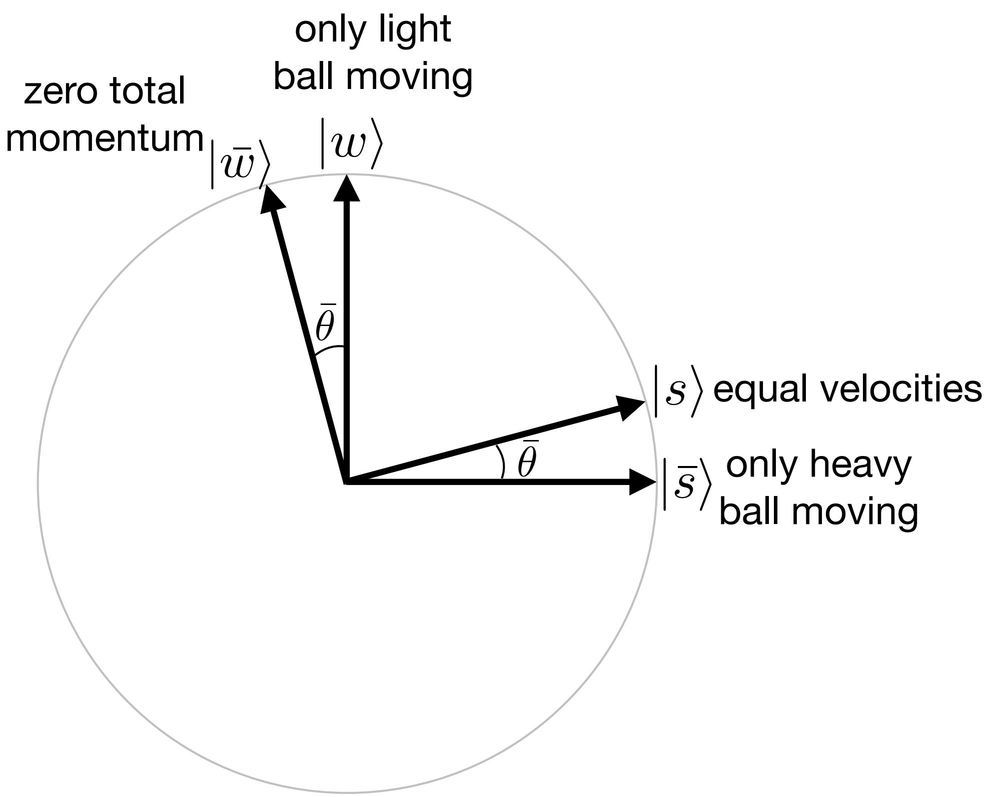

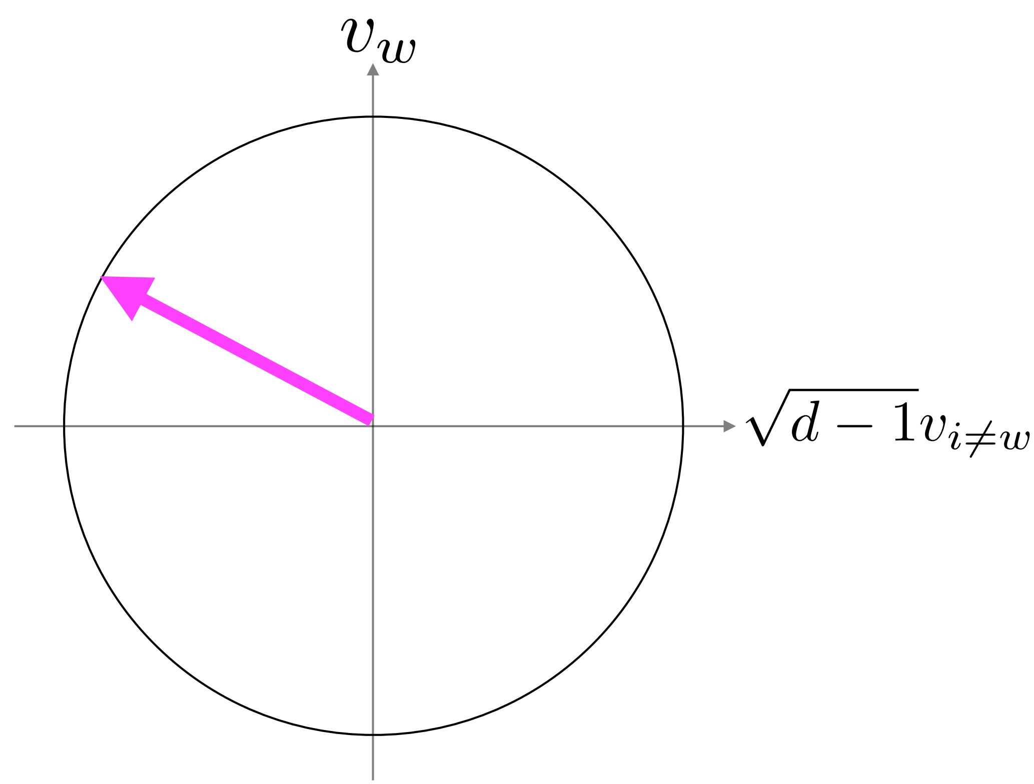

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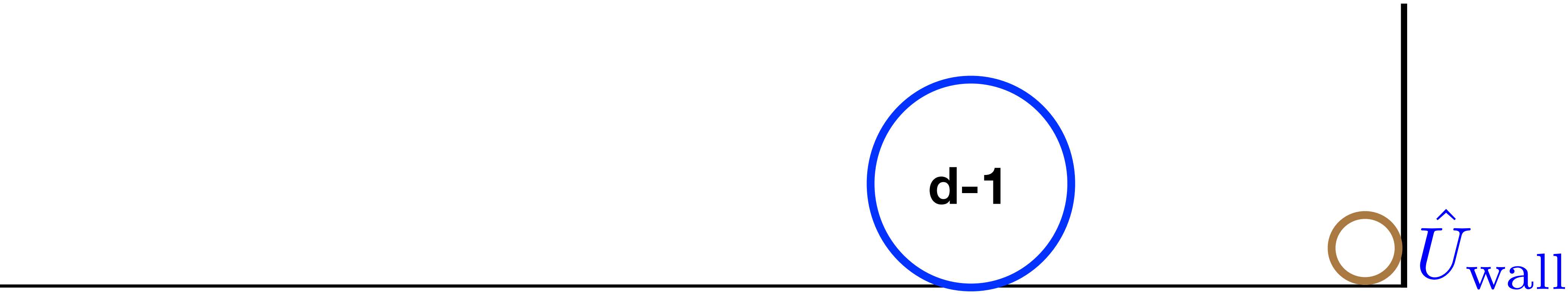

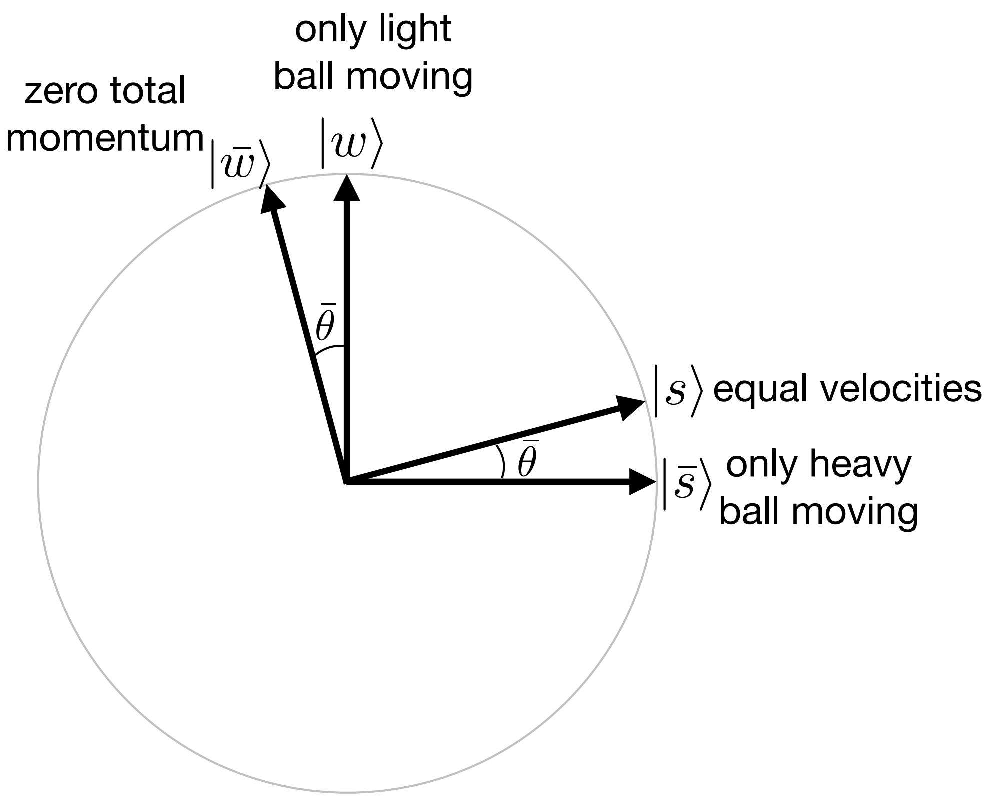

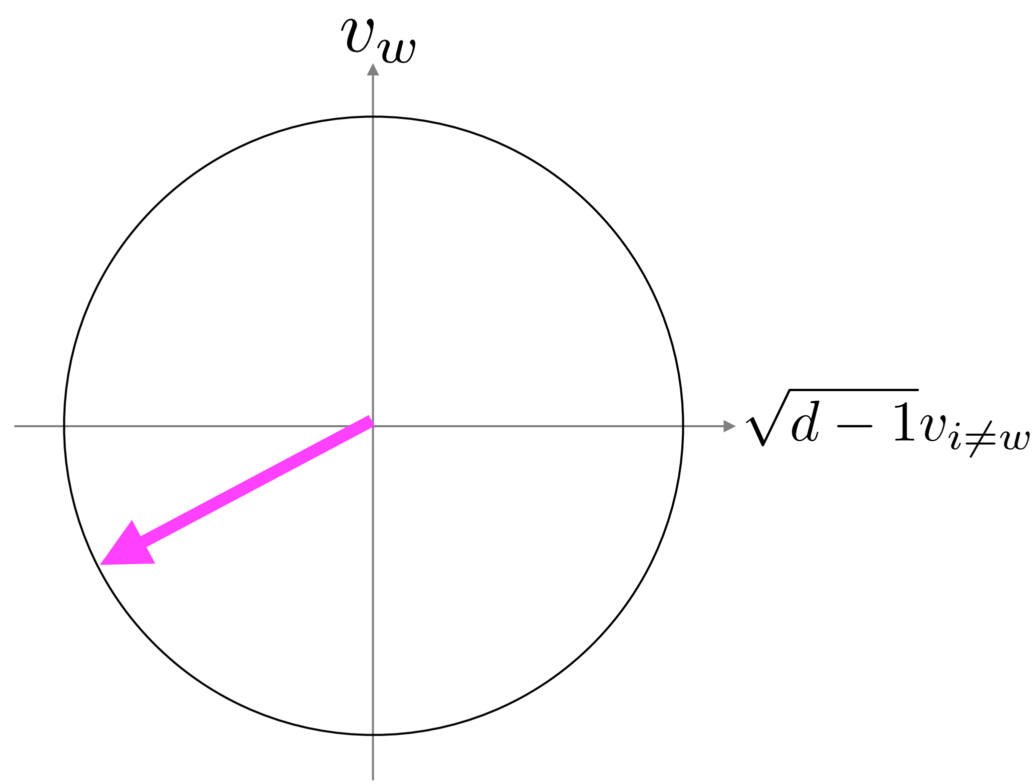

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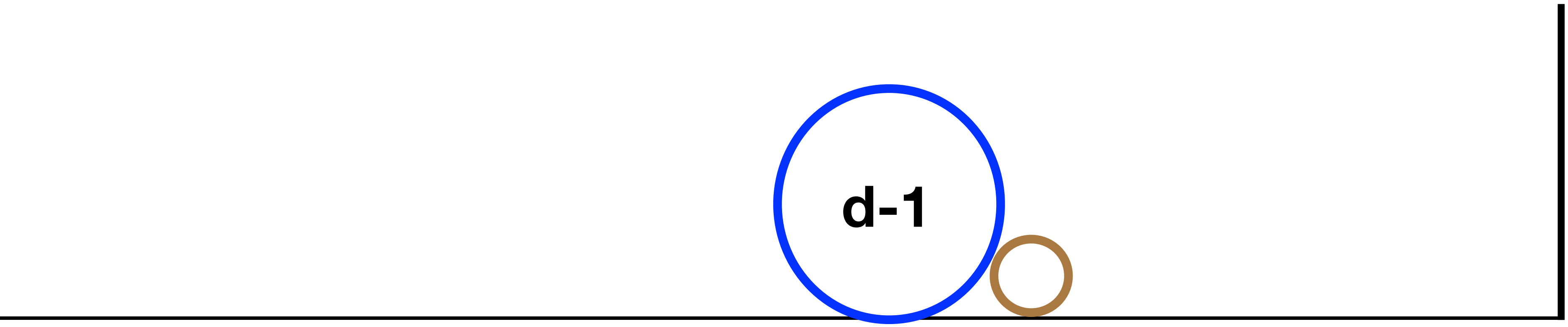

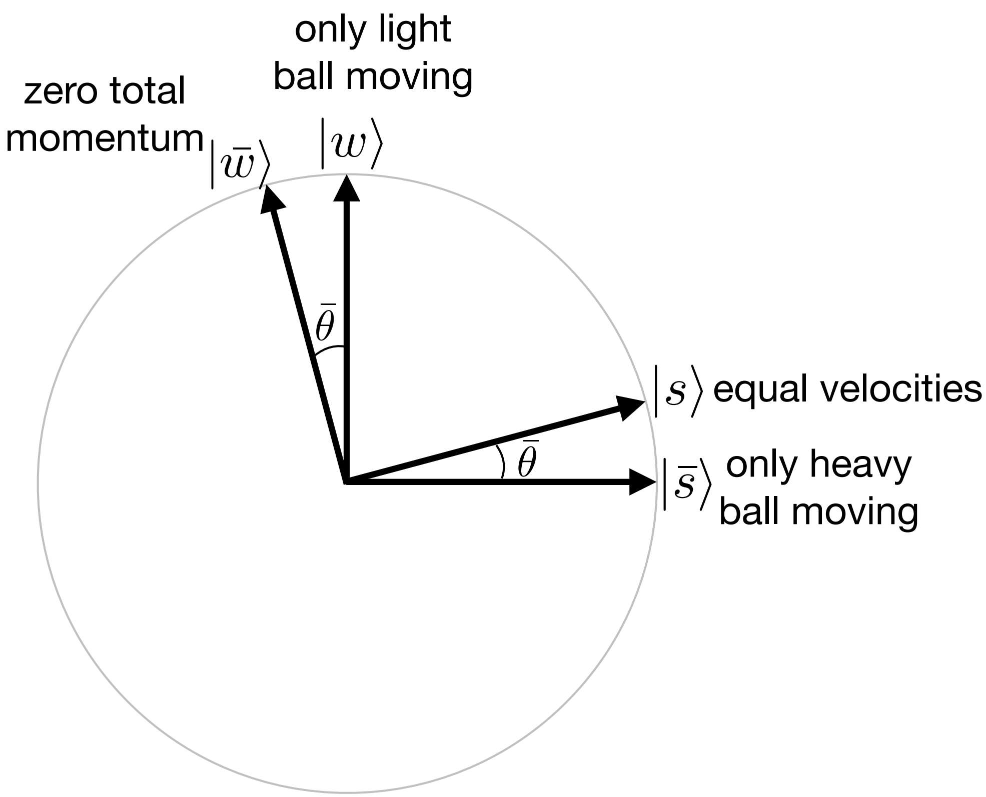

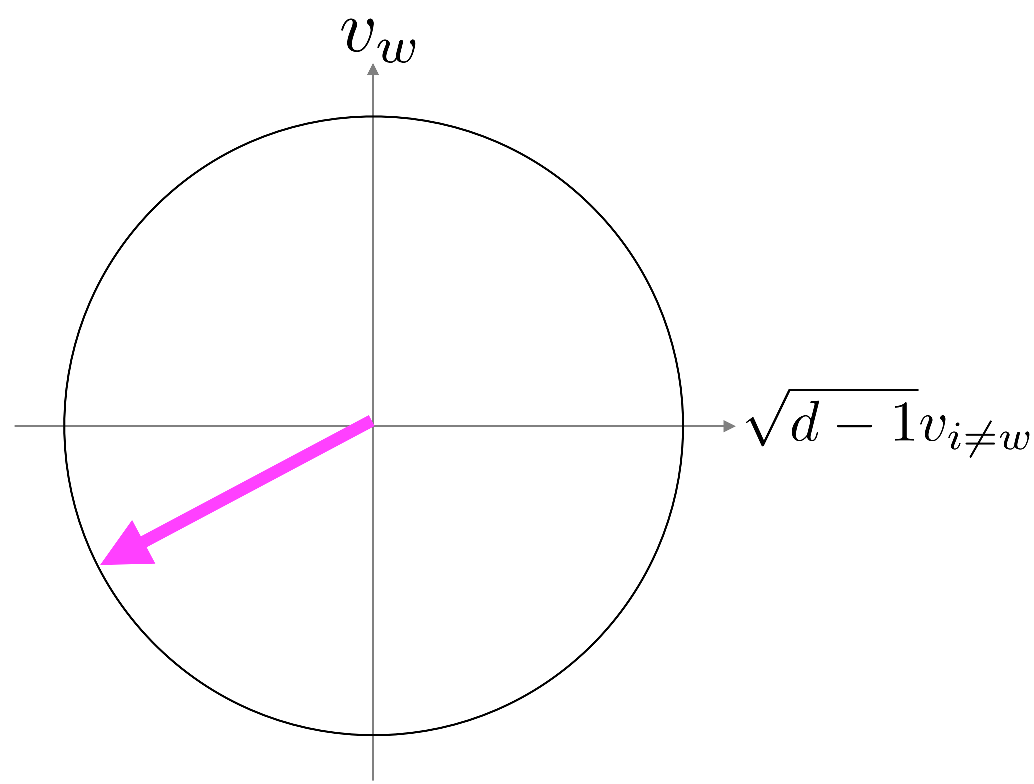

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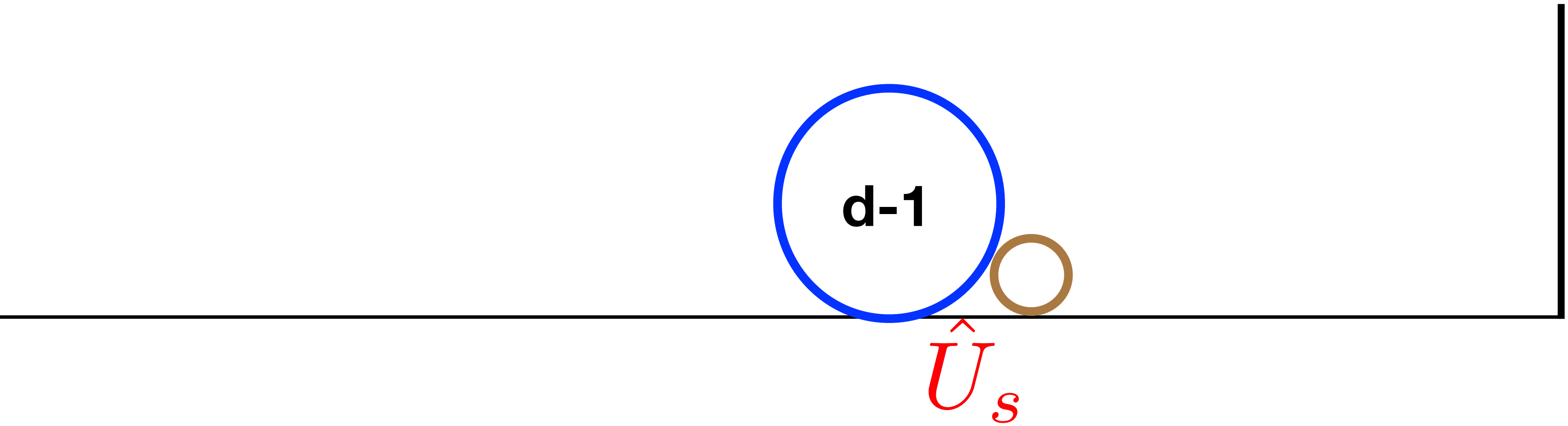

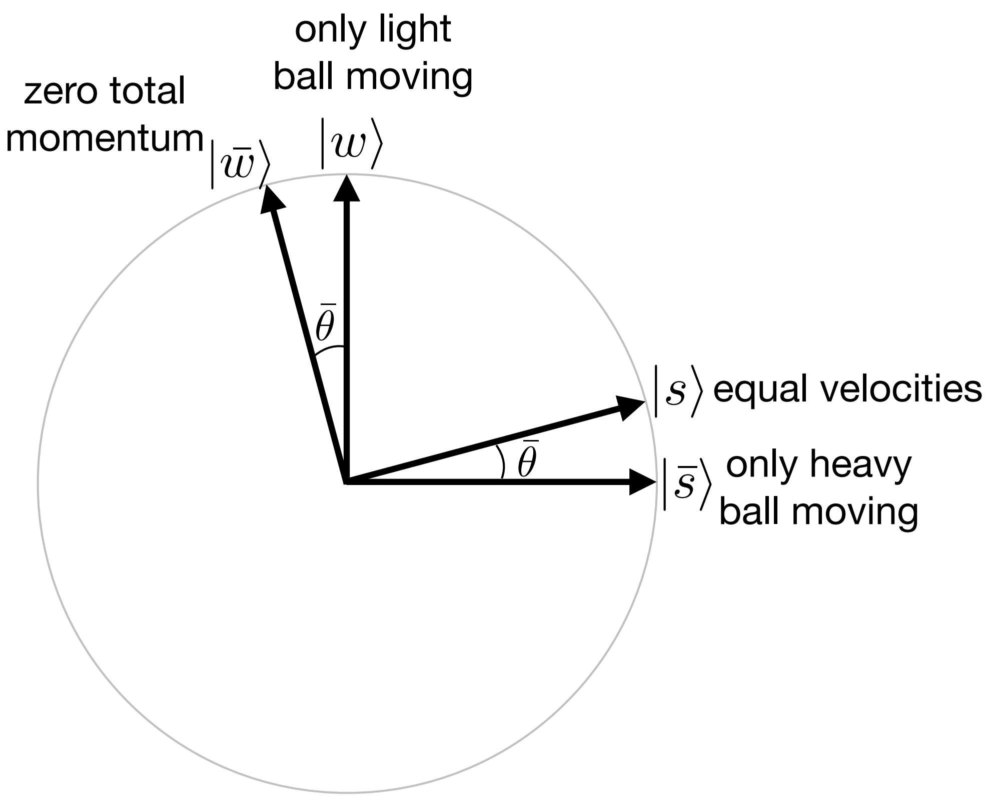

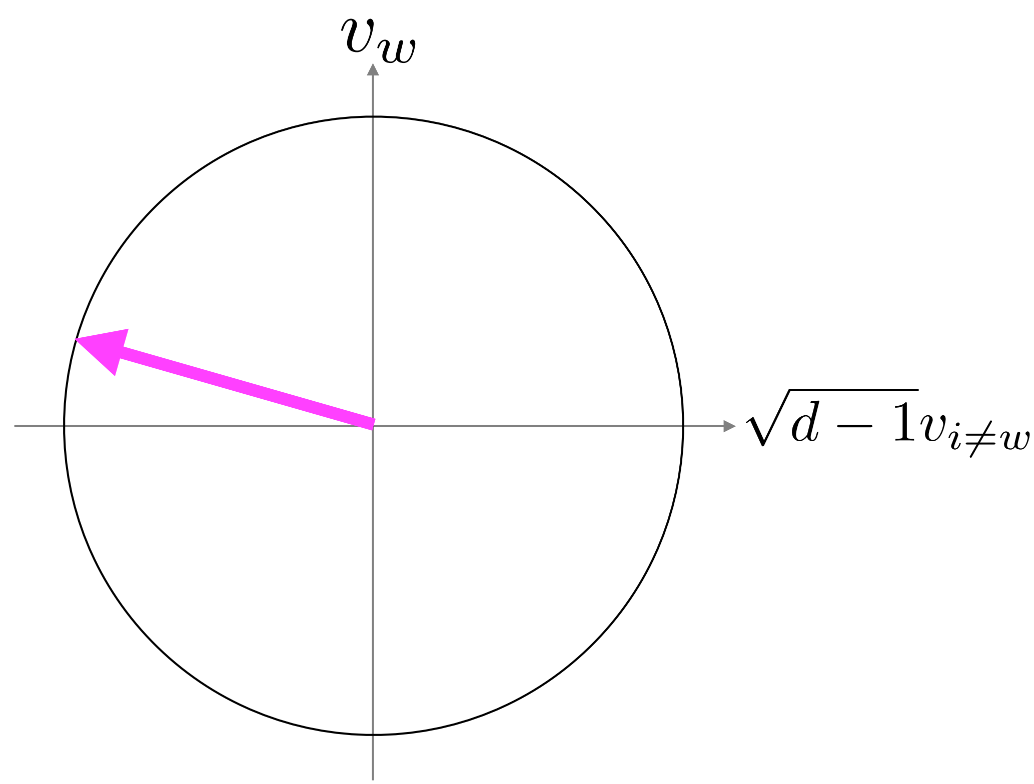

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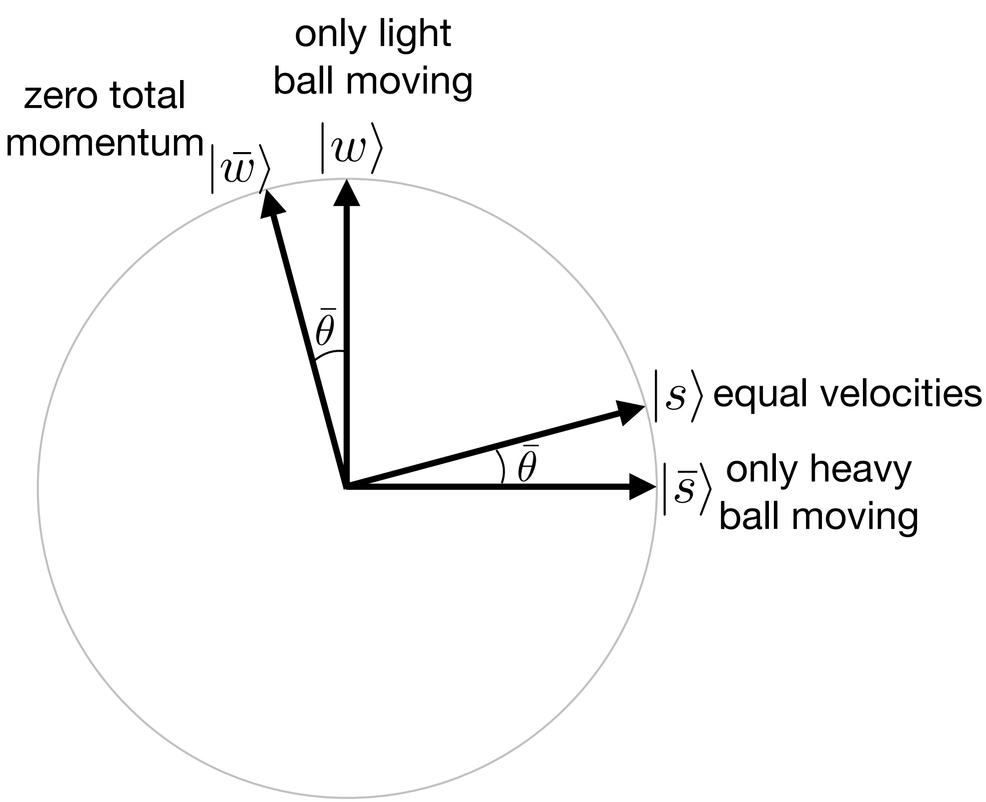

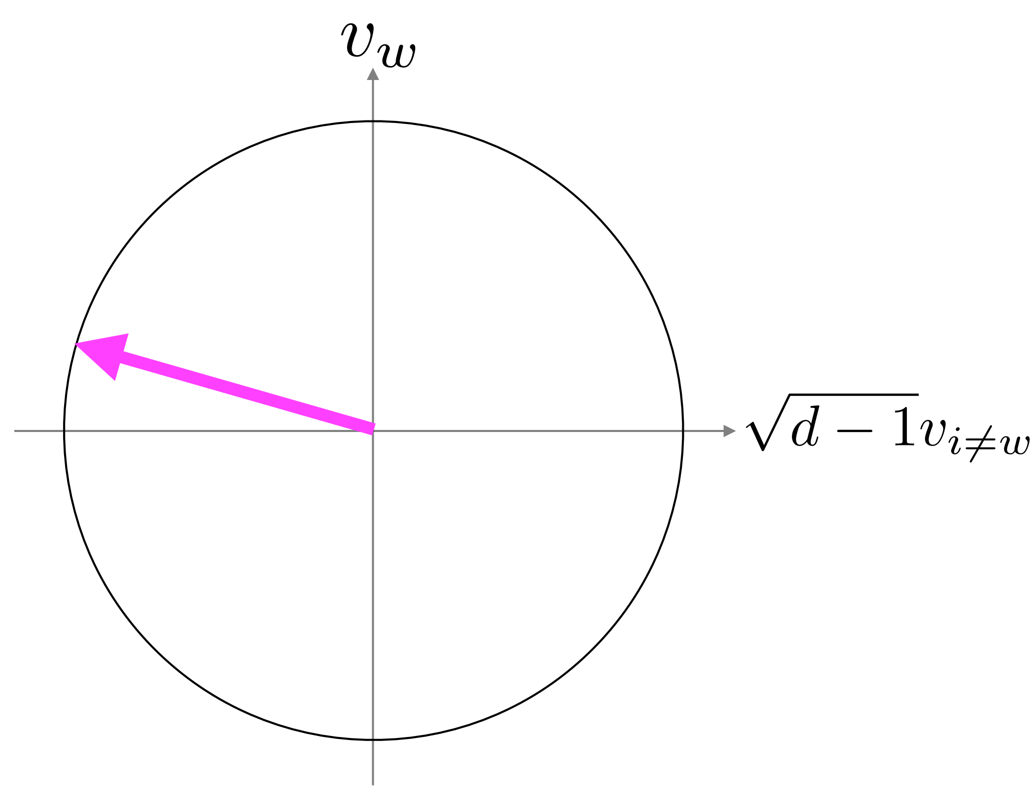

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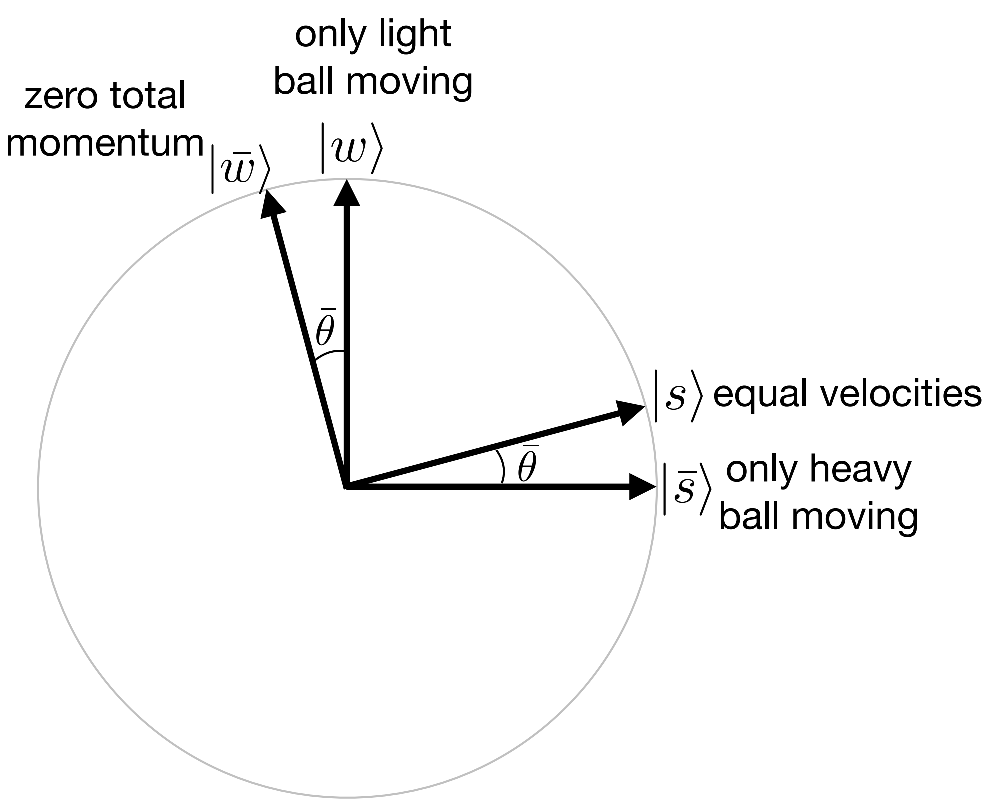

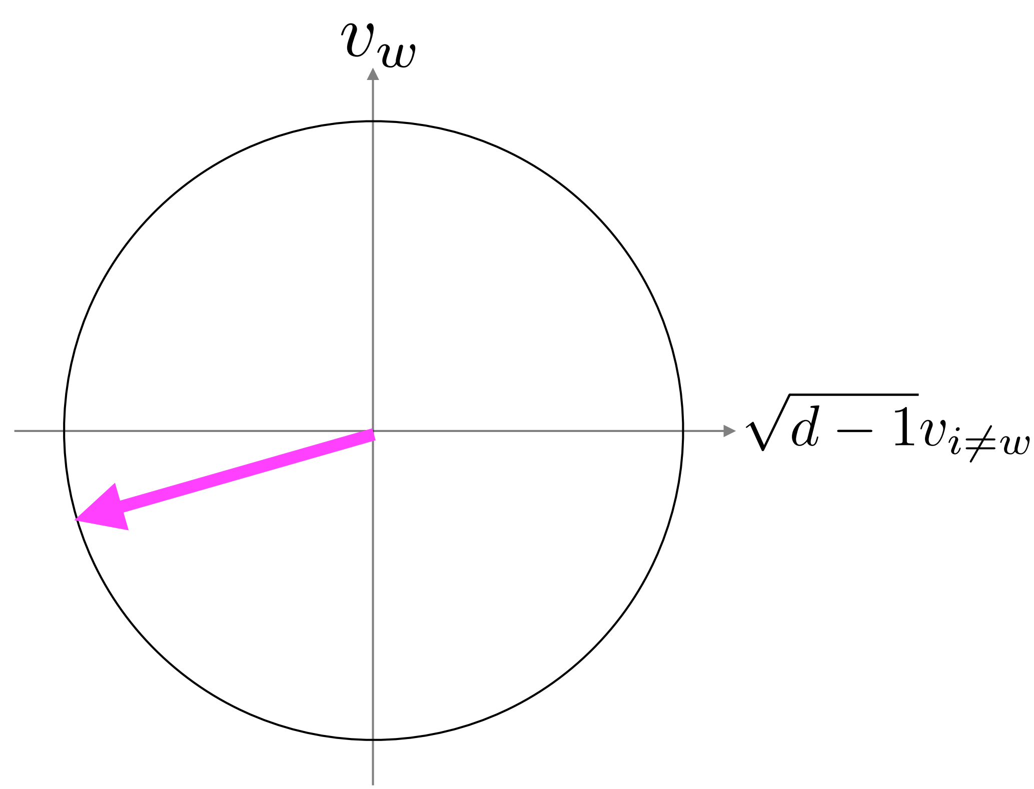

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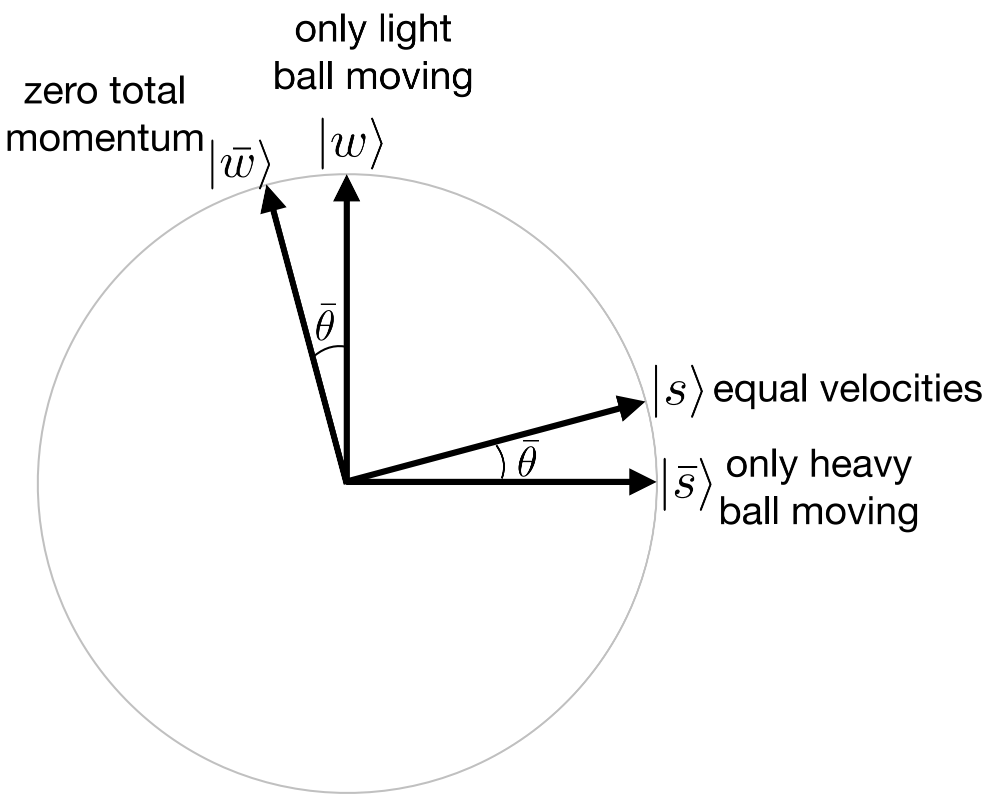

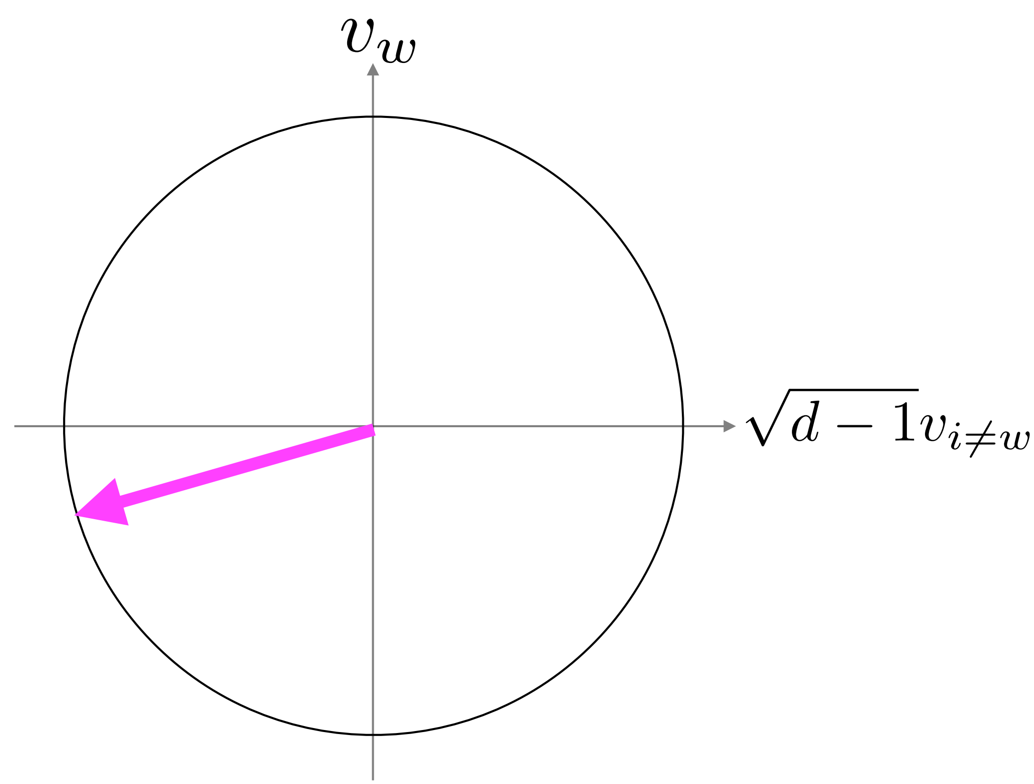

arXiv.org > quant-ph > arXiv:1912.02207

[Submitted on 4 Dec 2019]

Playing Pool with |ψ⟩: from Bouncing Billiards to Quantum Search

Adam R. Brown

Galperin 1995 ⟷ 1912.02207 ⟷ Grover 1996

G. GALPERIN
Department of Mathematics & Computer Sciences, Eastern Illinois University, Charleston, IL 61920, USA
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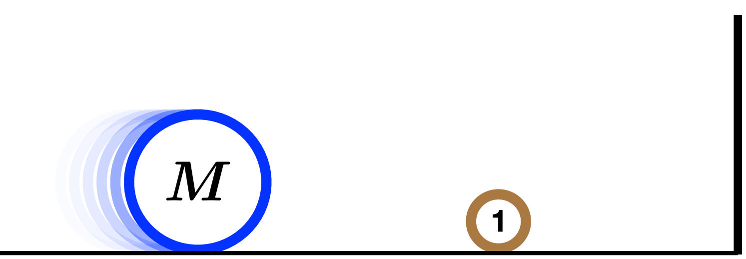
PLAYING POOL WITH π (THE NUMBER π FROM A BILLIARD POINT OF VIEW)

Received December 9, 2003 DOI: 10.1070/RD2003v008n04ABEH000252

Counting collisions in a simple dynamical system with two billiard balls can be used to estimate π to any accuracy.



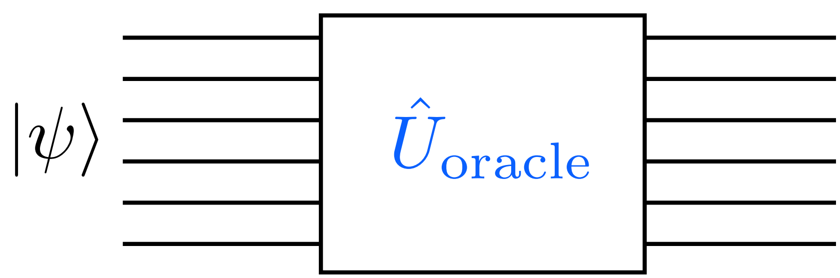
$$\#_{\text{collisions}} = \lfloor \pi\sqrt{M} \rfloor$$

VOLUME 79, NUMBER 2 PHYSICAL REVIEW LETTERS 14 JULY 1997

Quantum Mechanics Helps in Searching for a Needle in a Haystack

Lov K. Grover*
3C-404A Bell Labs, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 4 December 1996)

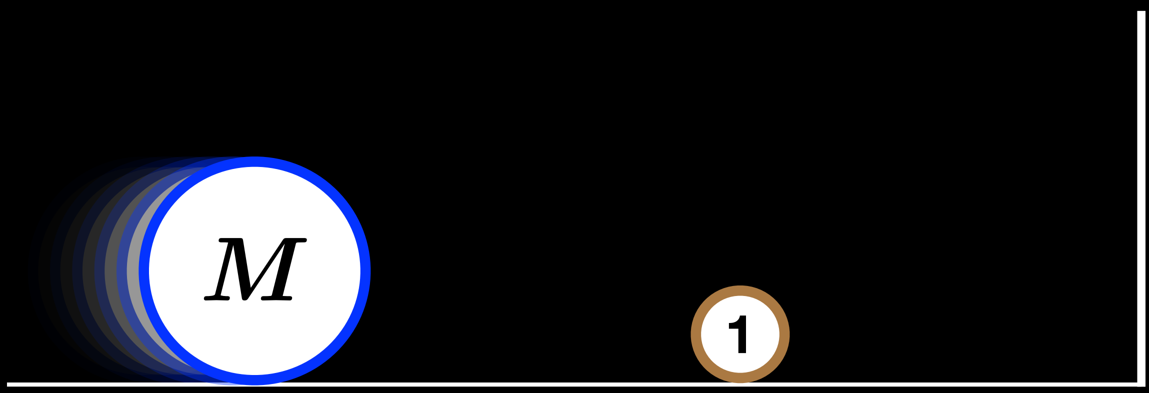
Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing N names arranged in completely random order. To find someone's phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database.
[S0031-9007(97)03564-3]



$$\#_{\text{oracle calls}} = \lfloor \frac{1}{4}\pi\sqrt{d-1} \rfloor$$

BOUNCING BILLIARDS	GROVER SEARCH
all kinetic energy in light ball	$\lvert w\rangle$
both balls equal velocity	$\lvert s\rangle = \frac{1}{\sqrt{d}}\sum_i \lvert i\rangle$
all kinetic energy in heavy ball	$\lvert \bar{s}\rangle = \frac{1}{\sqrt{d-1}}\sum_{i\neq w} \lvert i\rangle$
total momentum zero	$\lvert \bar{w}\rangle = \sqrt{\frac{d-1}{d}}\lvert w\rangle - \frac{1}{\sqrt{d(d-1)}}\sum_{i\neq w} \lvert i\rangle$
the $M = d-1$ billiards in big ball	the $d-1$ wrong answers
$\hat{O}_{\text{ball}}$ = 	$\hat{U}_s = 2\lvert s\rangle\langle s\rvert - \mathbb{1}$
$\hat{O}_{\text{wall}}$ = 	$\hat{U}_w = \mathbb{1} - 2\lvert w\rangle\langle w\rvert$
small ball bounces back and forth	alternate $\hat{U}_s$ and $\hat{U}_w$
velocity v_i of ith billiard	amplitude v_i of ith eigenstate
$2\times$ kinetic energy of ith billiard	probability $\lvert v_i\rvert^2$ of ith eigenstate
conservation of kinetic energy	conservation of probability
conservation of phase space	unitarity
motion purely horizontal	wavefunction purely real
collision order matters	operators don't commute
$\hat{O}_{\text{ball}}$ conserves total momentum	$\left[\lvert s\rangle\langle s\rvert, \hat{U}_s\right] = 0$
$\hat{O}_{\text{wall}}$ conserves big-ball momentum	$\left[\lvert \bar{s}\rangle\langle \bar{s}\rvert, \hat{U}_w\right] = 0$

“Galperin’s π-calculating plan displays a wanton disregard for engineering practicalities. It requires that we overcome friction, overcome inelasticities, overcome the blurring effects of quantum mechanics, and then having overcome all these things it requires exceptional patience, because even pedestrian initial velocities provoke catastrophic corrections from special relativity. Nevertheless, whatever the shortcomings of billiard balls as tools for calculating π, the results of this paper suggest a tool that is even worse. We might start with a qu$(100^N + 1)$it, and then step-by-step enact the quantum mirror of Galperin’s method, mirroring each velocity with an amplitude, mirroring each billiard collision with a unitary, before ending with a painstaking tomographic reconstruction of the final state and ushering in a new-if-pointless era of quantum arithmetic. It would not be easy, it would not be useful, but it would be a picturesquely quixotic way to seek π in the $|\psi\rangle$.”



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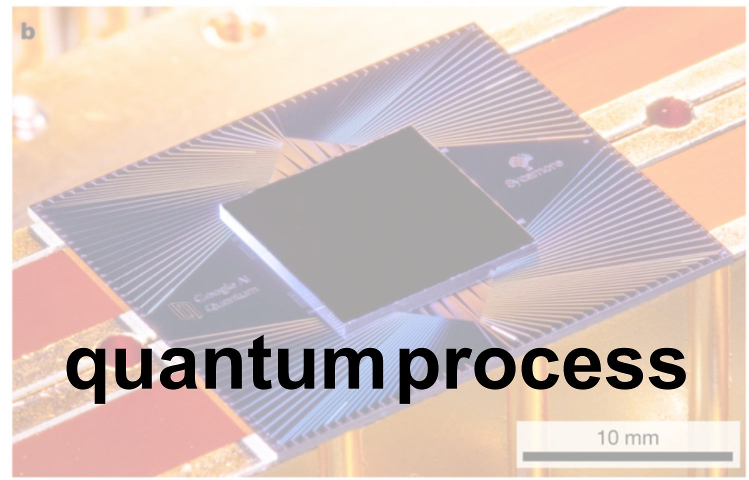
Quantum Gravity in the Lab

Quantum Gravity in the Lab



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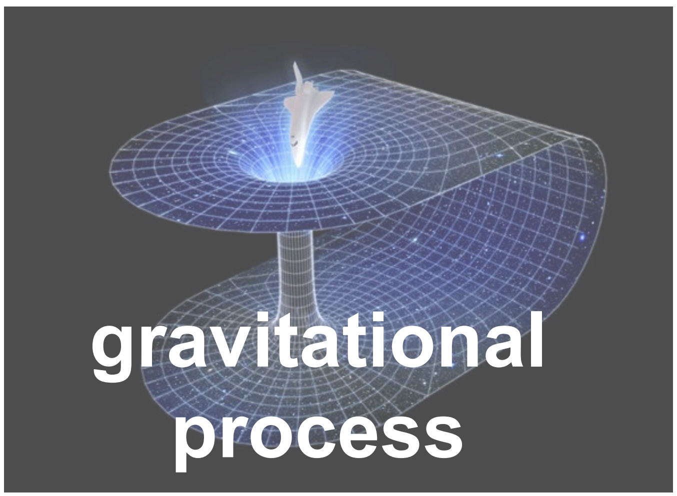

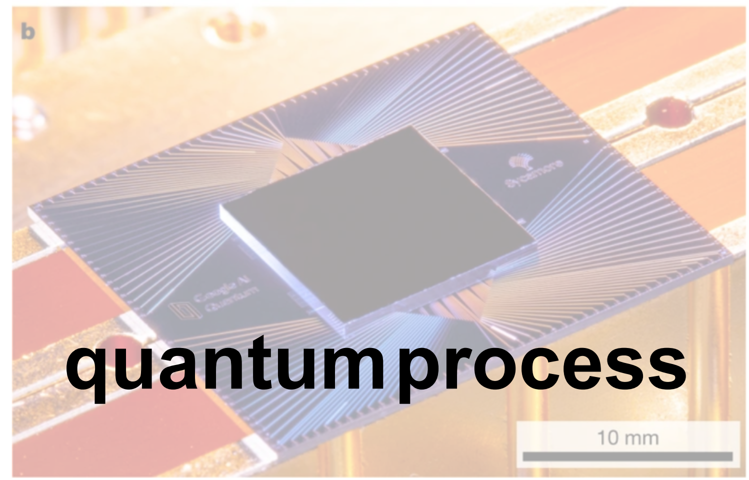

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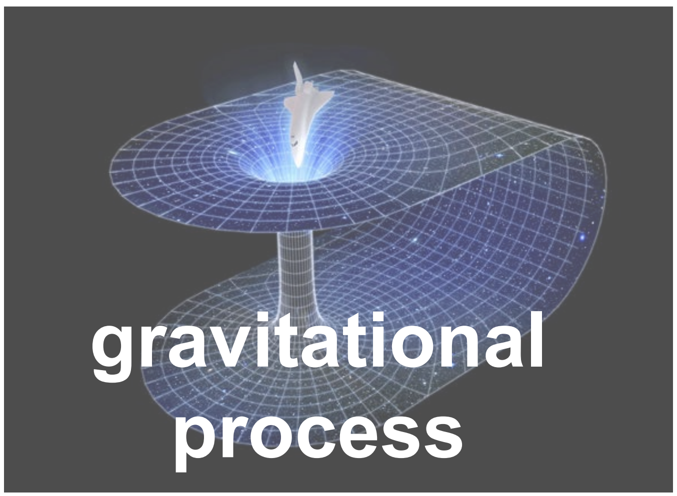
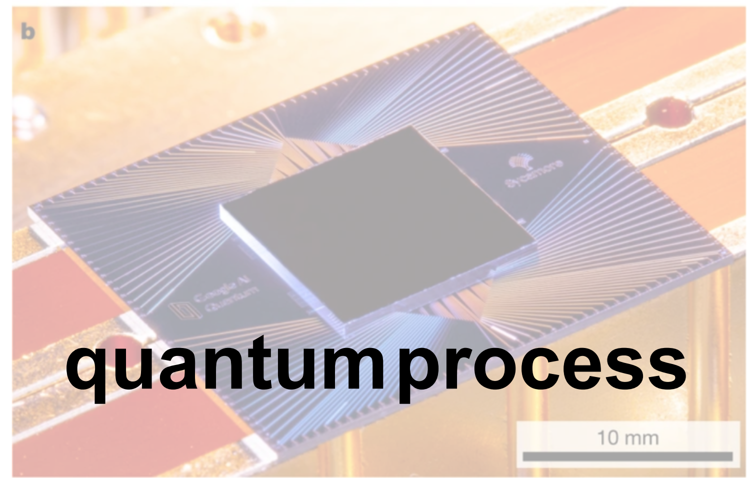


holographic ⟷ dual



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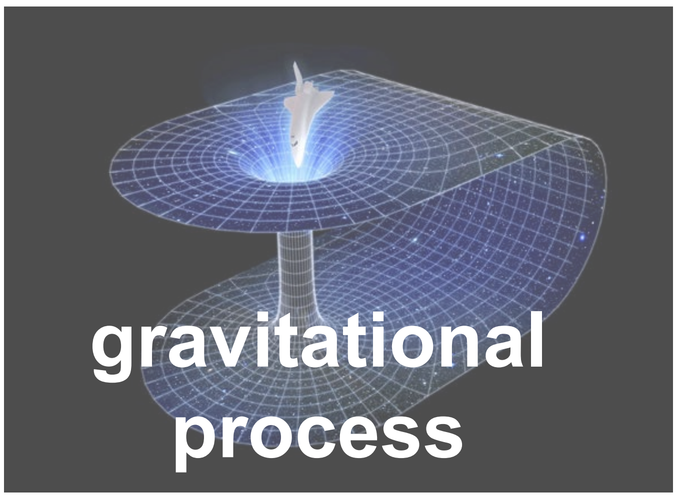

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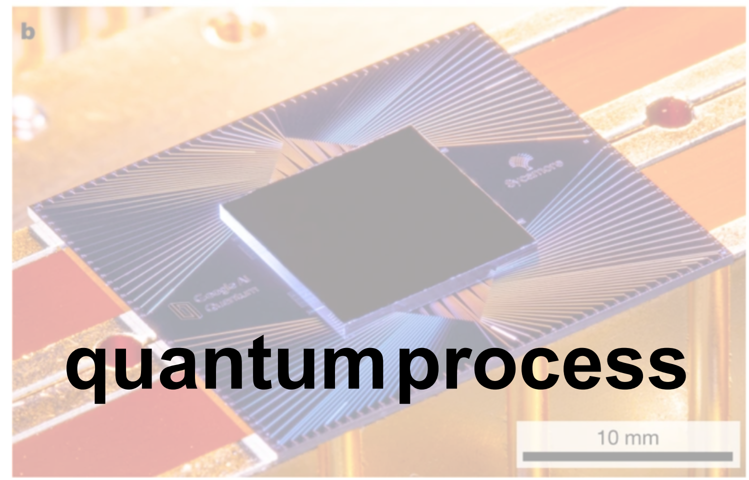

theorists like

experimentalists like

large "N"

few qubits

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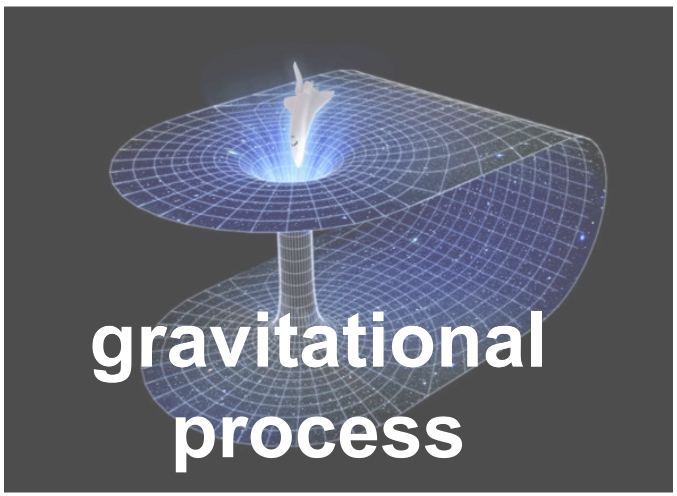

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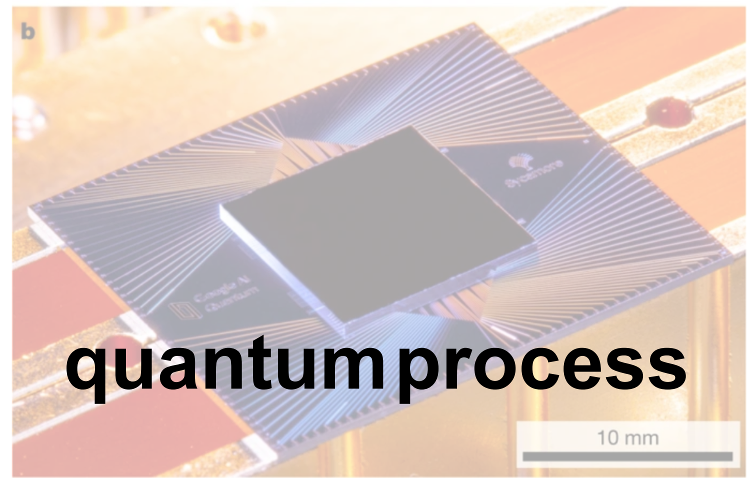

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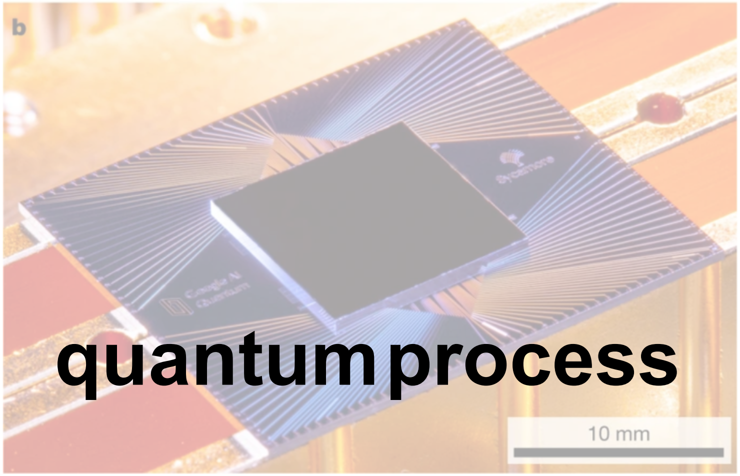
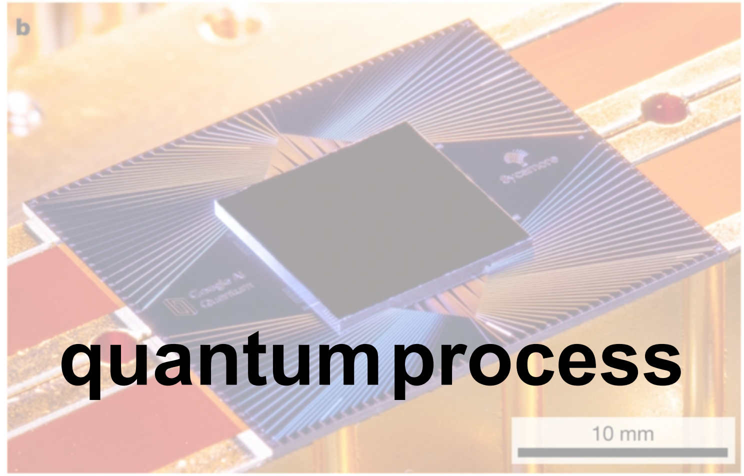


quantum process

experimentalists like few qubits



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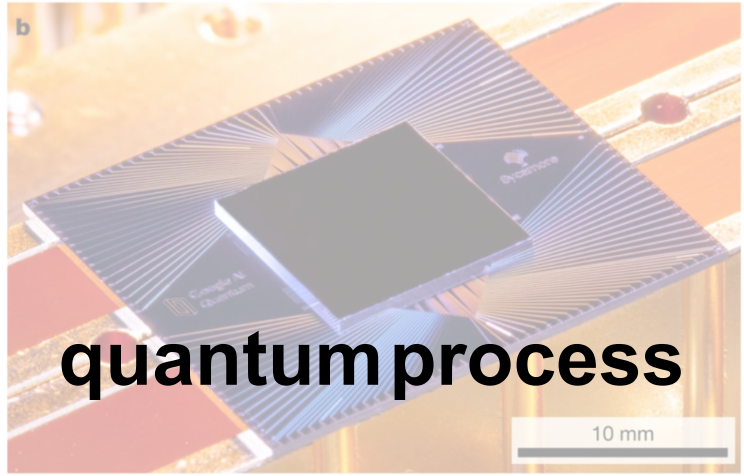
now

time



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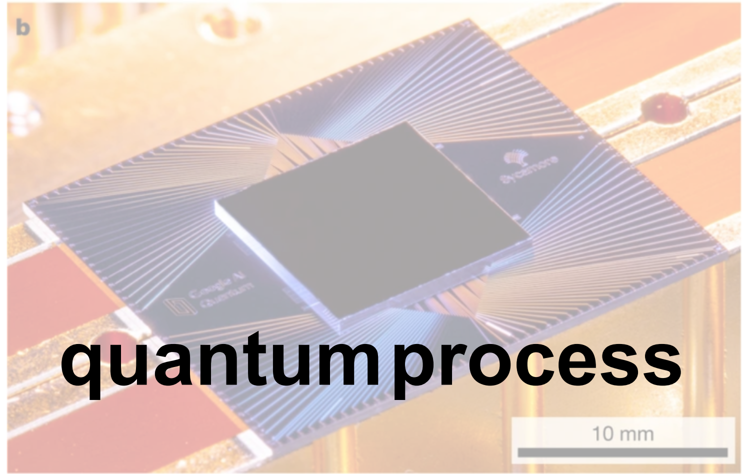

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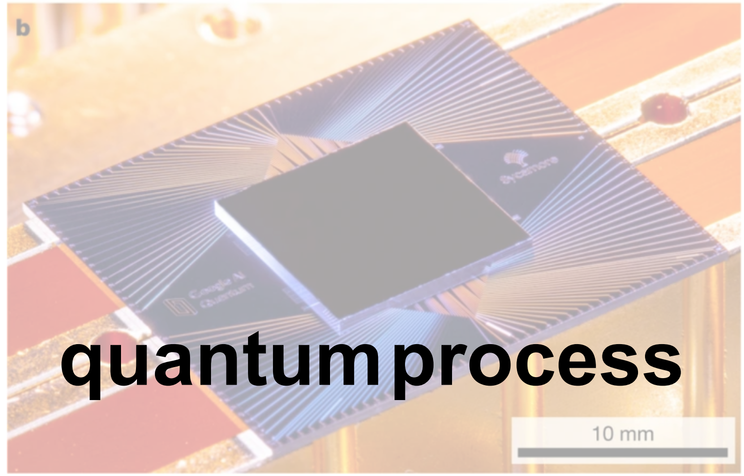
“do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”



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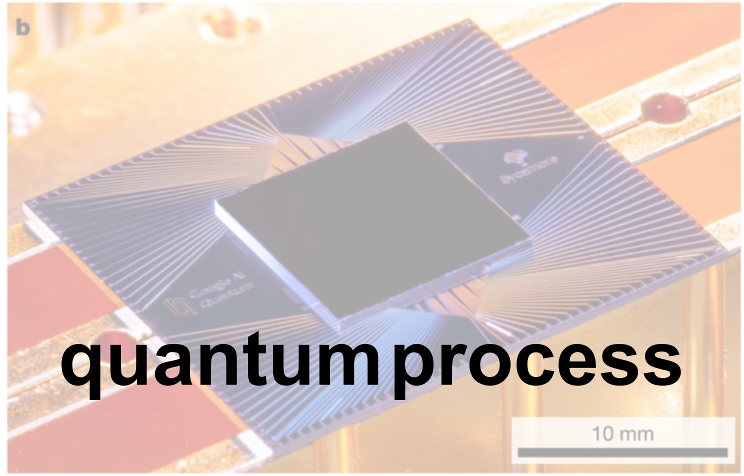

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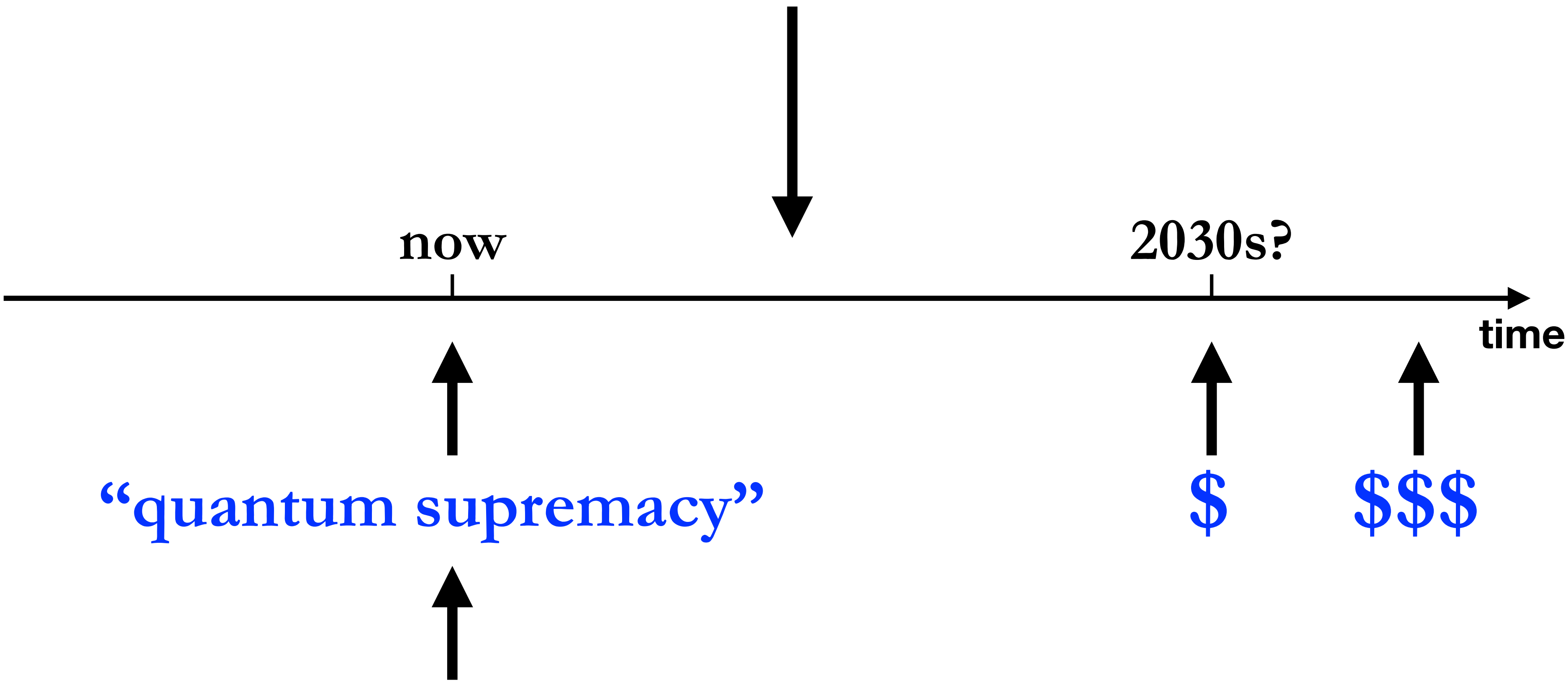

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Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks

Quantum Gravity in the Lab

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2. "Quantum gravitational supremacy"?

Quantum Gravity in the Lab

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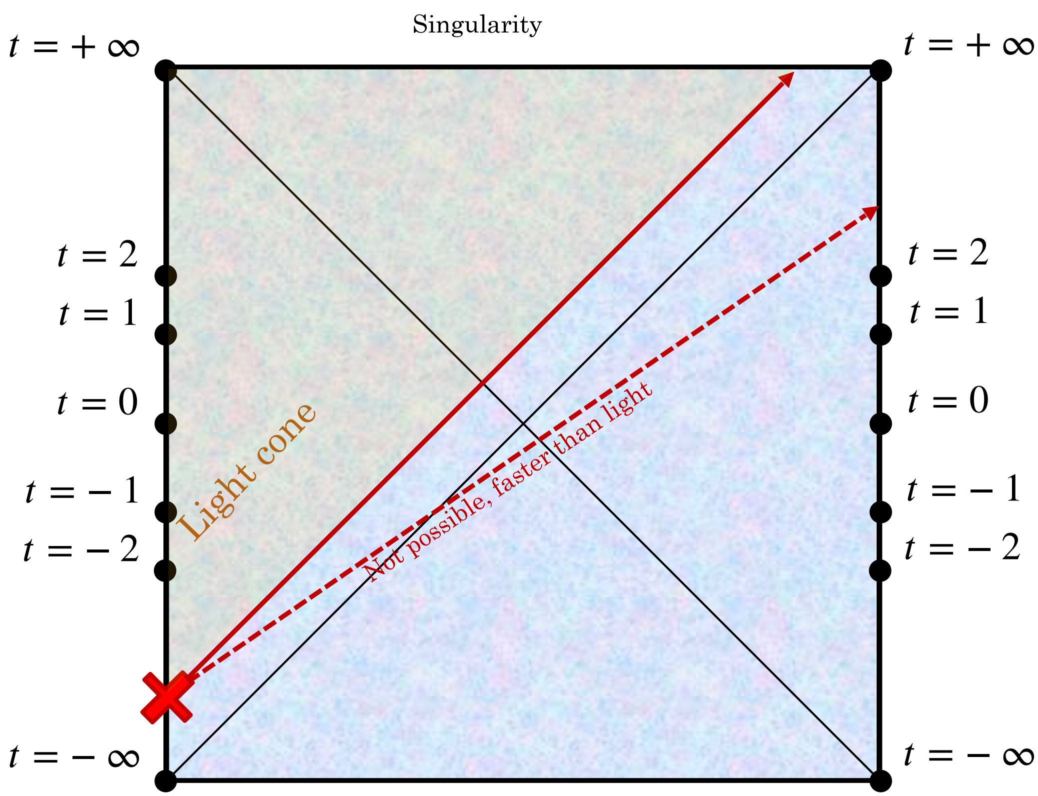

IMAGE CREDIT: SEPEHR

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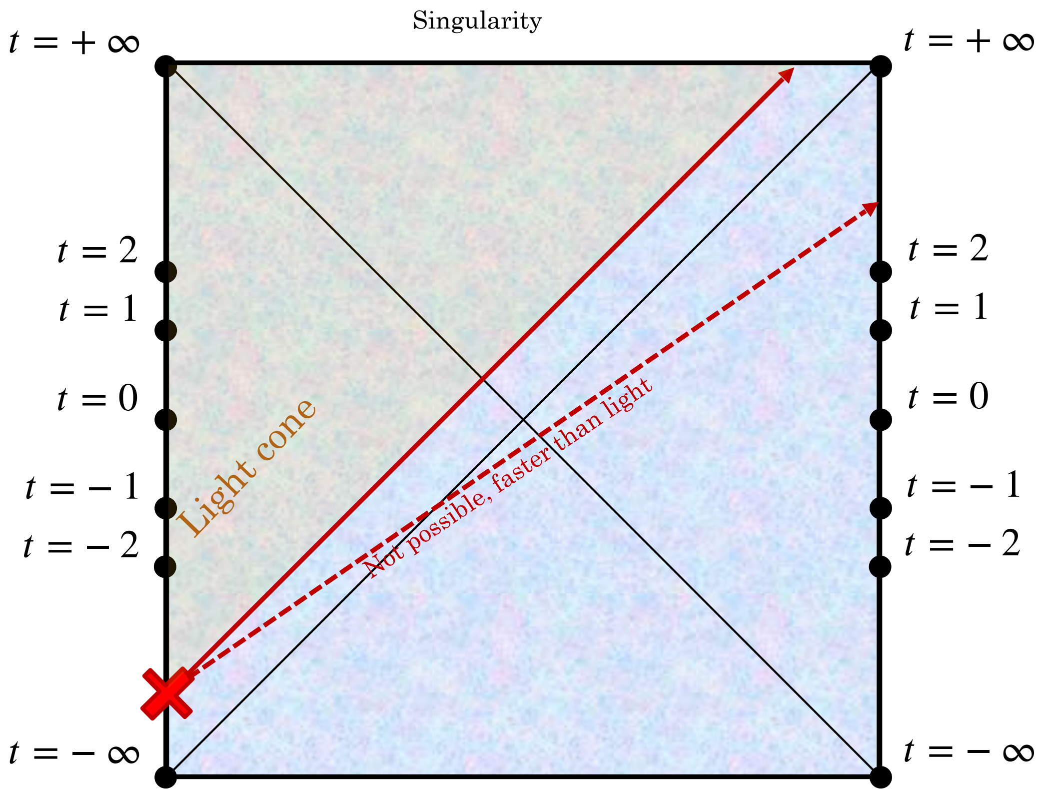

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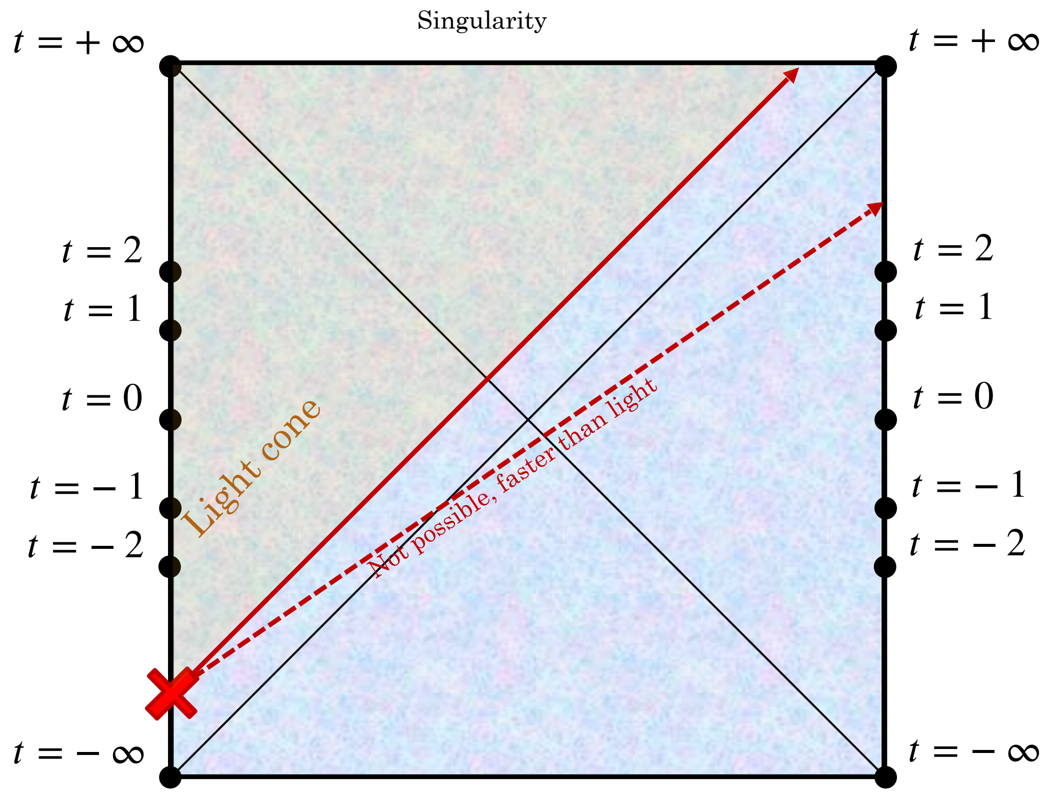

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wormhole not traversable

“ER = EPR”

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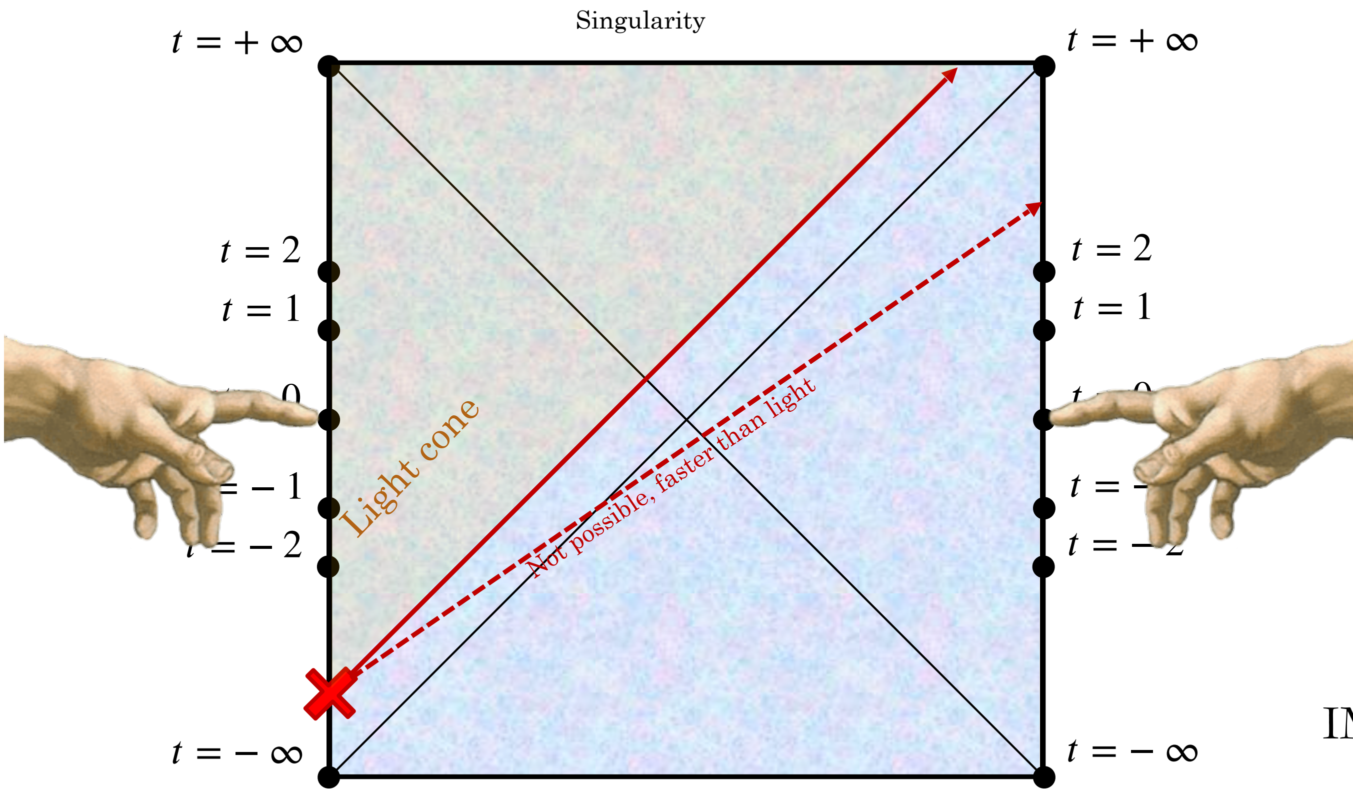

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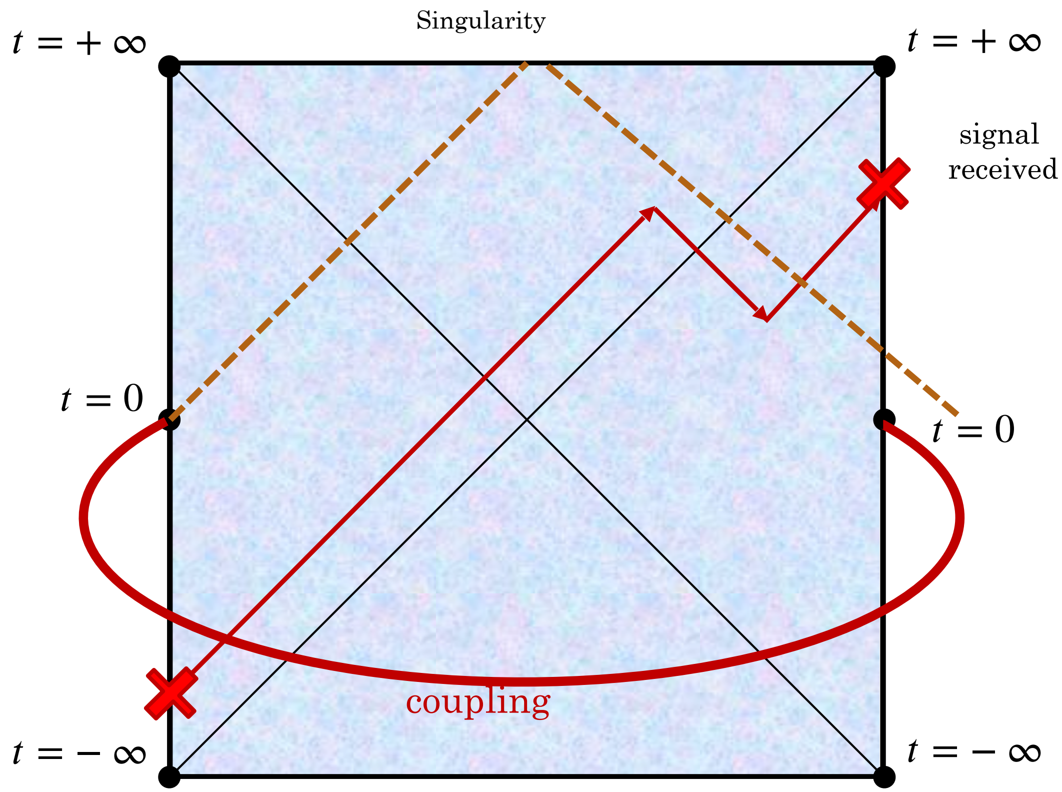

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wormhole traversable

Gao, Jafferis, Wall 1608.05687
Maldacena, Stanford, Yang 1704.05333

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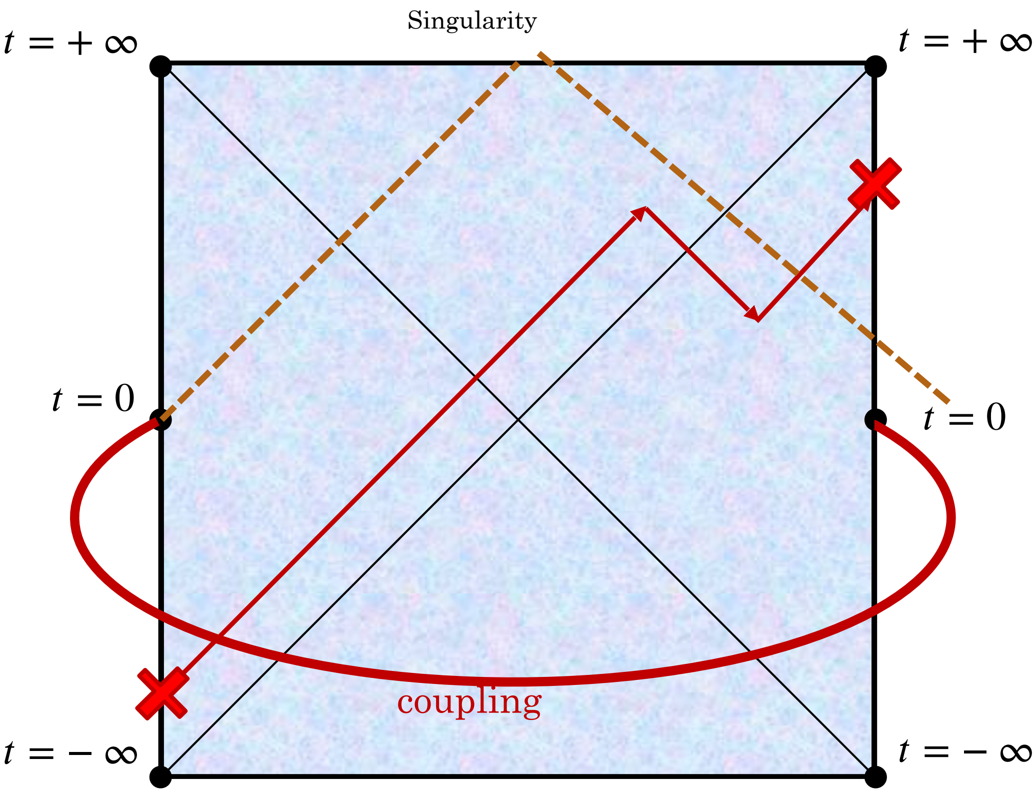

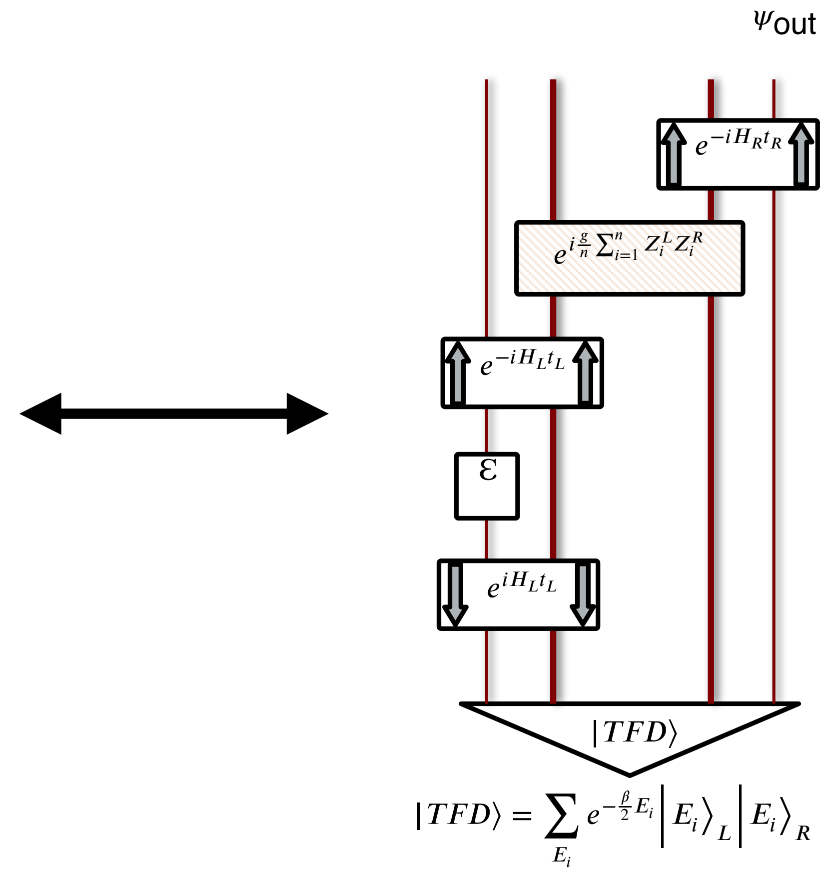

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Maldacena, Stanford, Yang 1704.05333

AB, HG, SL, HL, SN, GS, LS, BS, MW 1911.06314

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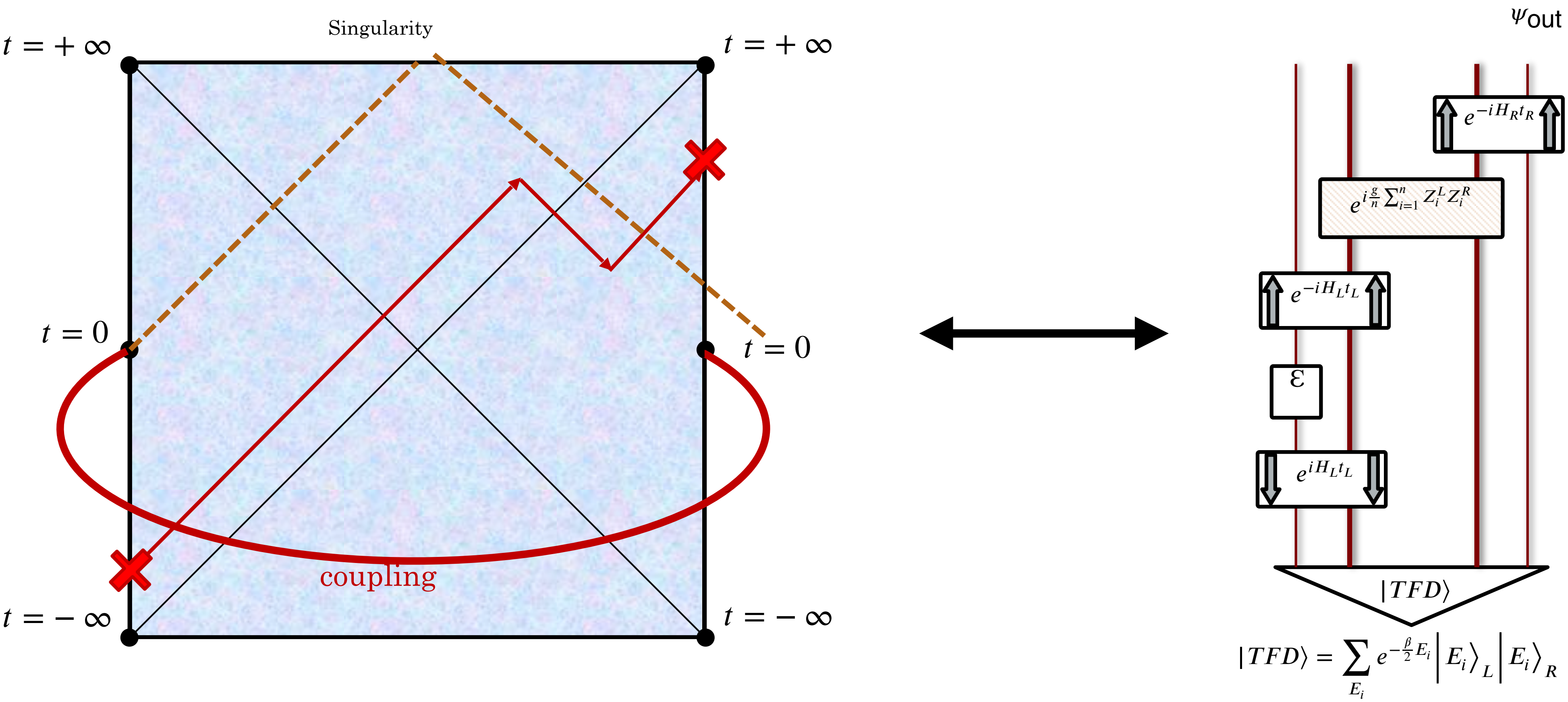

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= “do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”

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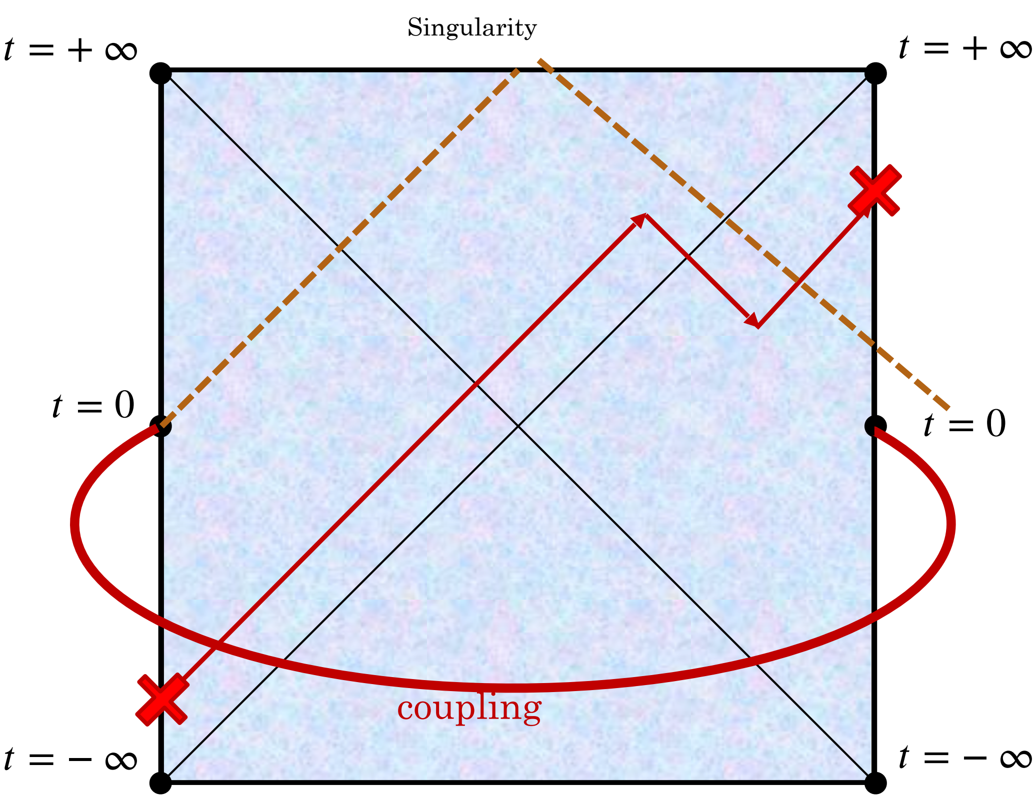
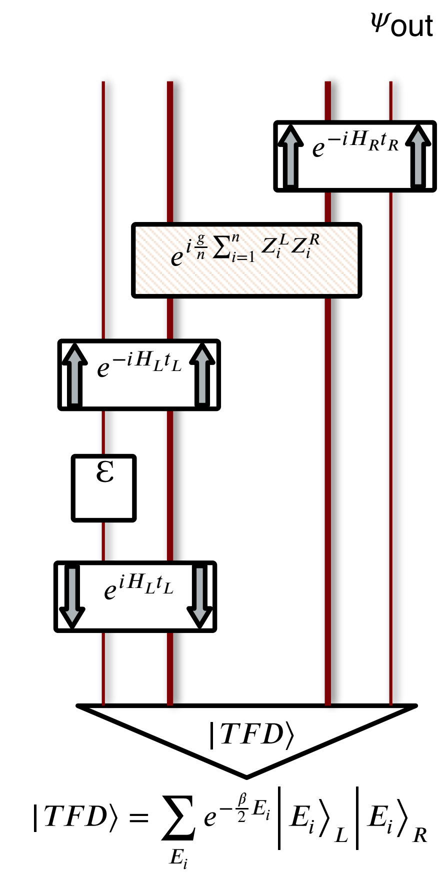
= "exhibit a phenomenon in quantum hardware that, while explainable just in terms of the boundary, is more easily explained by holographic gravity"





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Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks

2. “Quantum gravitational supremacy”?

3. Learn something new about quantum gravity?

theorists like ⟷ experimentalists like

large “N” ⟷ few qubits