Playing Pool with $|\psi\rangle$:
from Bouncing Billiards to Quantum Search
Adam Brown
1912.02207

Quantum Gravity in the Lab:
Teleportation by Size and Traversable Wormholes
Adam Brown, Hrant Gharibyan, Stefan Leichenauer, Henry Lin, Sepehr Nezami, Grant Salton, Leonard Susskind, Brian Swingle, Michael Walter,
1911.06314 & 2102.01064
The most unexpected answer to a counting puzzle

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PLAYING POOL WITH $\pi$
(THE NUMBER $\pi$ FROM A BILLIARD POINT OF VIEW)

Received December 9, 2003

DOI: 10.1070/RD2003v008n04ABEH000252

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$M = 10^6 \quad \rightarrow \quad \#\text{collisions} = 3141$

$M = 10^{20} \quad \rightarrow \quad \#\text{collisions} = 31415926535$
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\[ \# \text{collisions} = \left\lfloor \pi \sqrt{M} \right\rfloor \]
Quantum Mechanics Helps in Searching for a Needle in a Haystack

Lov K. Grover*
3C-404A Bell Labs, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 4 December 1996)

Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing $N$ names arranged in completely random order. To find someone’s phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database. [S0031-9007(97)03564-3]

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[S0031-9007(97)03564-3]

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$\text{task: find ‘needle in haystack’}$

$\text{find 1 item amongst } d$

$\#\text{collisions} = \left\lfloor \pi \sqrt{M} \right\rfloor$

$|\psi\rangle \overset{\hat{U}_{\text{oracle}}}{\rightarrow}$

$\#\text{oracle calls} = \left\lfloor \frac{1}{4} \pi \sqrt{d - 1} \right\rfloor$
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Galperin

Department of Mathematics & Computer Science, Eastern Illinois University, Charleston, IL 61920, USA
E-mail: cgg@uis.etsu.edu

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[80031-9007(97)03564-3]
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Department of Mathematics & Computer Sciences, Eastern Illinois University, Charleston, IL 61920, USA
E-mail: cgg@eic.etsu.edu

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COUNTING COLLISIONS IN A SIMPLE DYNAMICAL SYSTEM WITH TWO BILLIARD BAllS CAN BE USED TO ESTIMATE $\pi$ TO ANY ACCURACY.

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[Submitted on 4 Dec 2019]
Quantum Algorithms 101

Shor 1994 | Grover 1996
## Quantum Algorithms 101

<table>
<thead>
<tr>
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<th>Grover 1996</th>
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<tbody>
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</tr>
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</tr>
<tr>
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<td>classical ~ ( d/2 )</td>
</tr>
<tr>
<td>classical ~ ( \exp[(\log d)^{1/3}] )</td>
<td>quantum ~ ( \pi \sqrt{d}/4 )</td>
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Grover 1996

black box search

CLASSICAL BLACK BOX \_w
GOAL: find “$w$”

\[ BB_w[\text{input}] = \begin{cases} 
0 & \text{if } \text{input} \neq w \\
1 & \text{if } \text{input} = w 
\end{cases} \]

input $\in \{1, \ldots, d\}$

\[ BB_w \]

output $\in \{0, 1\}$
GOAL: find “$w$”

$$BB_w[input] = \begin{cases} 
0 & \text{if } input \neq w \\
1 & \text{if } input = w 
\end{cases}$$

$$BB_w[5] = 0$$
$$BB_w[4] = 0$$
$$BB_w[3] = 0$$
$$BB_w[2] = 0$$
$$BB_w[1] = 0$$
GOAL: find “$w$”

$$BB_w[\text{input}] = \begin{cases} 
0 & \text{if } \text{input} \neq w \\
1 & \text{if } \text{input} = w 
\end{cases}$$

CLASSICAL BLACK BOX$_w$

$w = 11 \quad \Rightarrow \quad BB_w[11] = 1 \quad \Rightarrow \quad w = 11$

$BB_w[10] = 0$

$BB_w[9] = 0$

$BB_w[8] = 0$

$BB_w[7] = 0$
GOAL: find “\(w\)"

\[
BB_w[input] = \begin{cases} 
0 & \text{if } \text{input} \neq w \\
1 & \text{if } \text{input} = w
\end{cases}
\]

\(w = 11\) \(\Rightarrow\) \(BB_w[11] = 1\) \(\Rightarrow\) \(w = 11\)

\[
\langle \text{queries} \rangle = \frac{1}{2}d
\]
GOAL: find “\( w \)"

\[ \langle \text{queries} \rangle = \frac{1}{2}d \]
GOAL: find "w"

input is qudit

QUANTUM BLACK BOX

output is qudit

\[ \langle \text{queries} \rangle = \frac{1}{2}d \]
GOAL: find “w”

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle \]

input is qudit

\[ \langle \text{queries} \rangle = \frac{1}{2}d \]

output is qudit
GOAL: find “$w$”

$$\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle$$

$$\hat{U}_w |1\rangle = + |1\rangle$$
GOAL: find “$w$”

\[ \hat{U}_w |\text{input}\rangle = \left( \hat{I} - 2 |w\rangle \langle w| \right) |\text{input}\rangle \]

\[ \hat{U}_w |1\rangle = + |1\rangle \]

\[ \hat{U}_w |w\rangle = - |w\rangle \]
GOAL: find "w"

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle \]

\[ w = 8 \]

\[ \hat{U}_w |8\rangle = -|8\rangle \]

\[ \hat{U}_w |7\rangle = +|7\rangle \]

\[ \hat{U}_w |6\rangle = +|6\rangle \]

\[ \hat{U}_w |5\rangle = +|5\rangle \]

\[ \hat{U}_w |4\rangle = +|4\rangle \]

\[ \hat{U}_w |3\rangle = +|3\rangle \]

\[ \hat{U}_w |2\rangle = +|2\rangle \]

\[ \hat{U}_w |1\rangle = +|1\rangle \]

\[ \hat{U}_w |0\rangle = +|0\rangle \]
GOAL: find “w”

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle \]

\[ w = 8 \quad \hat{U}_w |8\rangle = -|8\rangle \quad w = 8 \]

\[ \langle \text{queries} \rangle = \frac{1}{2d} \]
GOAL: find “$w$”

$$\hat{U}_w |\text{input}\rangle = \left( \hat{I} - 2 |w\rangle \langle w| \right) |\text{input}\rangle$$
GOAL: find “$w$”

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle \]

\[ v_1 |1\rangle + v_2 |2\rangle + \ldots + v_w |w\rangle + \ldots + v_d |d\rangle \]
GOAL: find “\( w \)"

\[
\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle
\]

\[
\hat{U}_w [v_1 1 + \ldots + v_w w + \ldots + v_d d] = v_1 1 + \ldots - v_w w + \ldots + v_d d
\]
GOAL: find “w”

\[ \hat{U}_w |\text{input}\rangle = \left( \hat{I} - 2|w\rangle\langle w| \right) |\text{input}\rangle \]

\[ \hat{U}_w [v_1|1\rangle + \ldots + v_w|w\rangle + \ldots + v_d|d\rangle] = v_1|1\rangle + \ldots - v_w|w\rangle + \ldots + v_d|d\rangle \]

\[ |s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right) \]
GOAL: find “*w*”

\[ \hat{U}_w \ket{\text{input}} = (\hat{I} - 2|w\rangle\langle w|)\ket{\text{input}} \]

\[ \hat{U}_w [v_1|1\rangle + \ldots + v_w|w\rangle + \ldots + v_d|d\rangle] = v_1|1\rangle + \ldots - v_w|w\rangle + \ldots + v_d|d\rangle \]

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\[ \hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots - |w\rangle + \cdots + |d\rangle \right) \]
**GOAL: find “w”**

\[ \hat{U}_w \ket{\text{input}} = (\hat{I} - 2 \ket{w} \bra{w}) \ket{\text{input}} \]

\[ \hat{U}_w [v_1 \ket{1} + \ldots + v_w \ket{w} + \ldots + v_d \ket{d}] = v_1 \ket{1} + \ldots - v_w \ket{w} + \ldots + v_d \ket{d} \]

QUANTUM BLACK BOX

\[ |s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right) \]

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\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle
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\]

\[
\hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots - |w\rangle + \cdots + |d\rangle \right)
\]

\[
\hat{U}_{w'} = 2 |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle - |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right)
\]
GOAL: find “w”

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle \]

\[ \hat{U}_w [v_1|1\rangle + \ldots + v_w|w\rangle + \ldots + v_d|d\rangle] = v_1|1\rangle + \ldots - v_w|w\rangle + \ldots + v_d|d\rangle \]

\[ |s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right) \]

\[ \hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots - |w\rangle + \cdots + |d\rangle \right) \]

NOT ORTHOGONAL

\[ \hat{U}_{w'=2} |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle - |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right) \]
GOAL: find “w”

\[
\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle
\]

\[
\hat{U}_w [v_1|1\rangle + \ldots + v_w|w\rangle + \ldots + v_d|d\rangle] = v_1|1\rangle + \ldots - v_w|w\rangle + \ldots + v_d|d\rangle
\]

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|s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right)
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NOT ORTHOGONAL

\[
\hat{U}_{w'=2} |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle - |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right)
\]
**GOAL: find “w”**

\[
\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle \langle w|) |\text{input}\rangle
\]

\[
\hat{U}_w [v_1 |1\rangle + \ldots + v_w |w\rangle + \ldots + v_d |d\rangle] = v_1 |1\rangle + \ldots - v_w |w\rangle + \ldots + v_d |d\rangle
\]

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|s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right)
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\hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots - |w\rangle + \cdots + |d\rangle \right)
\]

\[
\hat{U}_w |s\rangle
\]
GOAL: find "w"

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\[ |s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right) \]

\[ \hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots - |w\rangle + \cdots + |d\rangle \right) \]

\[ \hat{U}_w \hat{U}_w |s\rangle = |s\rangle \]
GOAL: find “w”

\[
\hat{U}_w \ket{\text{input}} = \left( \hat{I} - 2|w\rangle\langle w| \right) \ket{\text{input}}
\]

\[
\hat{U}_s \ket{\text{input}} = \left( 2|s\rangle\langle s| - \hat{I} \right) \ket{\text{input}}
\]

\[
|s\rangle \equiv \frac{1}{\sqrt{d}} \left( \ket{1} + \ket{2} + \cdots + |w\rangle + \cdots + |d\rangle \right)
\]

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\hat{U}_w |s\rangle = \frac{1}{\sqrt{d}} \left( \ket{1} + \ket{2} + \cdots - |w\rangle + \cdots + |d\rangle \right)
\]

\[
\hat{U}_s \hat{U}_w |s\rangle
\]
GOAL: find “$w$”

\[
\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle
\]

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\]

\[
\hat{U}_w \hat{U}_s \hat{U}_w |s\rangle
\]
GOAL: find "w"

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle \]

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\[ \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w |s\rangle \]
GOAL: find “$w$”

$$\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|) |\text{input}\rangle$$

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$$\hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w |s\rangle$$
GOAL: find "w"

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\[ \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w |s\rangle \]
**GOAL: find “w”**

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\hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle
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\hat{U}_s |\text{input}\rangle = (2|s\rangle\langle s| - \hat{I})|\text{input}\rangle
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|s\rangle \equiv \frac{1}{\sqrt{d}} \left( |1\rangle + |2\rangle + \cdots + |w\rangle + \cdots + |d\rangle \right)
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\hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w \hat{U}_s \hat{U}_w |s\rangle
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GOAL: find “w”

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\[ (\hat{U}_s \hat{U}_w)^{\left[ \frac{\pi}{4} \sqrt{d-1} \right]} |s\rangle \sim |w\rangle \]
GOAL: find “w”

\[ \hat{U}_w |\text{input}\rangle = (\hat{I} - 2|w\rangle\langle w|)|\text{input}\rangle \]

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\[ |s\rangle \]

aim for \( |w\rangle \)

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\[ \hat{U}_s \hat{U}_w |s\rangle \]

aim for \[ |w\rangle \]

\[ \text{start at } |s\rangle \]

\[ \sin \bar{\theta} \equiv \langle w|s\rangle = \frac{1}{\sqrt{d}} \]
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\[ \langle \text{queries} \rangle = \left[ \frac{\pi}{4} \sqrt{d-1} \right] \]

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Quantum Mechanics Helps in Searching for a Needle in a Haystack

Lov K. Grover*
3C-404A Bell Labs, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 4 December 1996)

Quantum mechanics can speed up a range of search applications over unsorted data. For example, imagine a phone directory containing $N$ names arranged in completely random order. To find someone’s phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database.

[SO031-9007/97/03564-3]

$|\psi\rangle$, $\hat{U}_{\text{oracle}}$

$$\langle \text{queries} \rangle = \left\lfloor \frac{\pi}{4} \sqrt{d - 1} \right\rfloor$$
Grover 1996

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\([S0031-9007(97)03564-3]\)

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- Zero total momentum
- Only light ball moving
- Equal velocities
- Only heavy ball moving
- \( \sqrt{d - 1} v_{i \neq w} \)
\[ |v\rangle = v_{i \neq w} \left( |1\rangle + |2\rangle + \ldots + |d\rangle \right) + v_w |w\rangle \]
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Diagram:
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\[ \hat{U}_s \]

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zero total momentum

only light ball moving

\[ |\bar{w}\rangle \quad |w\rangle \]

\[ |s\rangle \] equal velocities

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\[ \hat{U}_s \]

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Playing Pool with $|\psi\rangle$: from Bouncing Billiards to Quantum Search

Adam R. Brown

Galperin 1995

PLAYING POOL WITH $\pi$ (THE NUMBER $\pi$ FROM A BILLIARD POINT OF VIEW)

Counting collisions in a simple dynamical system with two billiard balls can be used to estimate $\pi$ to any accuracy.

Galperin 1995

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Quantum mechanics can speed up a range of search applications over unordered data. For example, imagine a phone directory containing $N$ names arranged in completely random order. To find someone’s phone number with a probability of 50%, any classical algorithm (whether deterministic or probabilistic) will need to access the database a minimum of $0.5N$ times. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ accesses to the database.

$\#\text{collisions} = \left\lfloor \pi \sqrt{M} \right\rfloor$

$\#\text{oracle calls} = \left\lfloor \frac{1}{4} \pi \sqrt{d - 1} \right\rfloor$

1912.02207
<table>
<thead>
<tr>
<th><strong>BOUNCING BILLIARDS</strong></th>
<th><strong>GROVER SEARCH</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>all kinetic energy in light ball</td>
<td>(</td>
</tr>
<tr>
<td>both balls equal velocity</td>
<td>(</td>
</tr>
<tr>
<td>all kinetic energy in heavy ball</td>
<td>(</td>
</tr>
<tr>
<td>total momentum zero</td>
<td>(</td>
</tr>
<tr>
<td>the ( M = d-1 ) billiards in big ball</td>
<td>the ( d-1 ) wrong answers</td>
</tr>
<tr>
<td>( \hat{O}_{\text{ball}} = )</td>
<td>( \hat{U}_s = 2</td>
</tr>
<tr>
<td>( \hat{O}_{\text{wall}} = )</td>
<td>( \hat{U}_w = \mathbb{1} - 2</td>
</tr>
<tr>
<td>small ball bounces back and forth</td>
<td>alternate ( \hat{U}_s ) and ( \hat{U}_w )</td>
</tr>
<tr>
<td>velocity ( v_i ) of ( i )th billiard</td>
<td>amplitude ( v_i ) of ( i )th eigenstate</td>
</tr>
<tr>
<td>2 \times kinetic energy of ( i )th billiard</td>
<td>probability (</td>
</tr>
<tr>
<td>conservation of kinetic energy</td>
<td>conservation of probability</td>
</tr>
<tr>
<td>conservation of phase space</td>
<td>unitarity</td>
</tr>
<tr>
<td>motion purely horizontal</td>
<td>wavefunction purely real</td>
</tr>
<tr>
<td>collision order matters</td>
<td>operators don’t commute</td>
</tr>
<tr>
<td>( \hat{O}_{\text{ball}} ) conserves total momentum</td>
<td>( [</td>
</tr>
<tr>
<td>( \hat{O}_{\text{wall}} ) conserves big-ball momentum</td>
<td>( [</td>
</tr>
</tbody>
</table>
Galperin’s π-calculating plan displays a wanton disregard for engineering practicalities. It requires that we overcome friction, overcome inelasticities, overcome the blurring effects of quantum mechanics, and then having overcome all these things it requires exceptional patience, because even pedestrian initial velocities provoke catastrophic corrections from special relativity. Nevertheless, whatever the shortcomings of billiard balls as tools for calculating π, the results of this paper suggest a tool that is even worse. We might start with a qu(100$$^N$$ + 1)it, and then step-by-step enact the quantum mirror of Galperin’s method, mirroring each velocity with an amplitude, mirroring each billiard collision with a unitary, before ending with a painstaking tomographic reconstruction of the final state and ushering in a new-if-pointless era of quantum arithmetic. It would not be easy, it would not be useful, but it would be a picturesquely quixotic way to seek π in the |ψ\rangle.
Quantum Gravity in the Lab
Quantum Gravity in the Lab
Quantum Gravity in the Lab

- Gravitational process
- Quantum process
Quantum Gravity in the Lab

gravitational process ↔ holographic dual ↔ quantum process
Quantum Gravity in the Lab

Gravitational process

Holographic dual

Quantum process

Theorists like large "N"

Experimentalists like few qubits
Quantum Gravity in the Lab

Gravitational process

Holographic dual

Quantum process

Theorists like CHASM

Large “N”

Experimentalists like CHASM

Few qubits
quantum process

experimentalists like few qubits
quantum process

experimentalists like few qubits

now
quantum process

experimentalists like few qubits

“quantum supremacy”
experimentalists like few qubits

“quantum supremacy”

“do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
Experimentalists like few qubits

“quantum supremacy”

“do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
quantum process

experimentalists like few qubits

```
“quantum supremacy”

“do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
```
Quantum Gravity in the Lab?

“quantum supremacy”

“do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks
Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks

2. “Quantum gravitational supremacy”?
Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks

2. “Quantum gravitational supremacy”?
2. “Quantum gravitational supremacy”? 

IMAGE CREDIT: SEPEHR
2. “Quantum gravitational supremacy”?

Singularity

wormhole not traversable

“ER = EPR”
2. “Quantum gravitational supremacy”?

Singularity

wormhole not traversable

“ER = EPR”
2. “Quantum gravitational supremacy”?

wormhole traversable

Gao, Jafferis, Wall  1608.05687
Maldacena, Stanford, Yang 1704.05333
2. “Quantum gravitational supremacy”?

\[ \psi_{\text{out}} = e^{-iH_{R}t_{R}} \]

\[ e^{i \frac{\theta}{\pi} \sum_{i=1}^{n} z_{i}^{L} z_{i}^{R}} \]

\[ |TFD\rangle = \sum_{E_{i}} e^{-\frac{\theta}{\pi} E_{i}} |E_{i}\rangle_{L} |E_{i}\rangle_{R} \]

wormhole traversable

Gao, Jafferis, Wall 1608.05687
Maldacena, Stanford, Yang 1704.05333
AB, HG, SL, HL, SN, GS, LS, BS, MW 1911.06314
2. “Quantum gravitational supremacy”?

= “do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
2. “Quantum gravitational supremacy”?  

= “exhibit a phenomenon in quantum hardware that, while explainable just in terms of the boundary, is more easily explained by holographic gravity”

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theorists like
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= “do a computation that, while you could do it on a classical computer, is more easily done on a quantum computer”
Quantum Gravity in the Lab

1. Black-hole-flavored benchmarks

2. “Quantum gravitational supremacy”? 

3. Learn something new about quantum gravity?

theorists like
large “N”

experimentalists like
few qubits