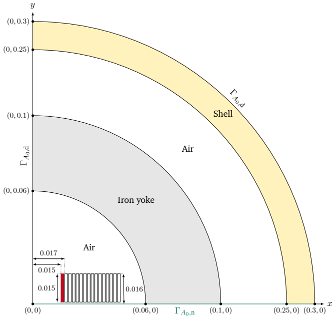
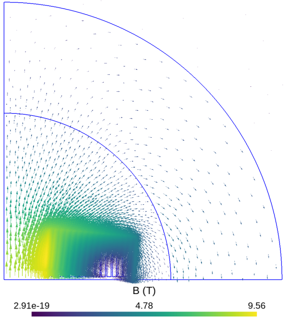
Magneto-Thermal Finite Element Models of Superconducting Magnets, Coils and Tapes in GetDP

Modeling, Relevant Equations and Implementation Hints/Code Snippets

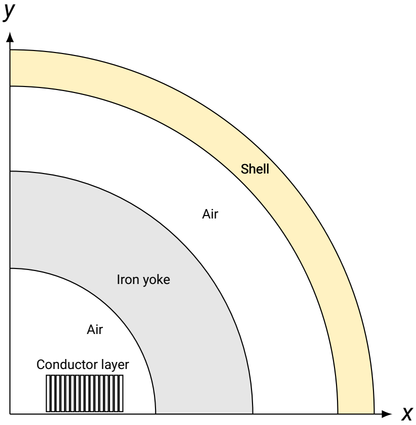
Erik Schnaubelt[1], **Lorenzo Bortot**[1,2], **Sebastian Schöps**[1]
[1]Computational Electromagnetics Group, TU Darmstadt [2]CERN





Introduction and Appetizer

First Model: Simple Nb_3Sn Dipole Accelerator Magnet With Circuit Coupling



- Rutherford cable

- Coupling to circuit

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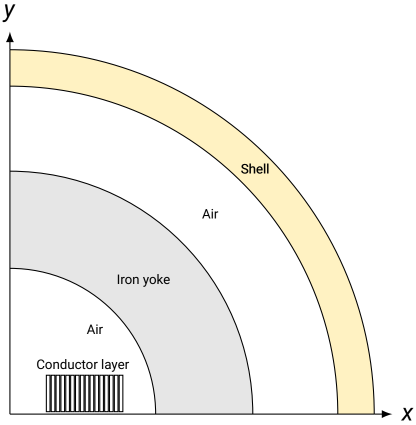

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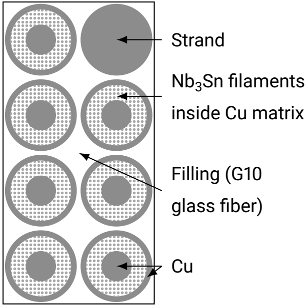

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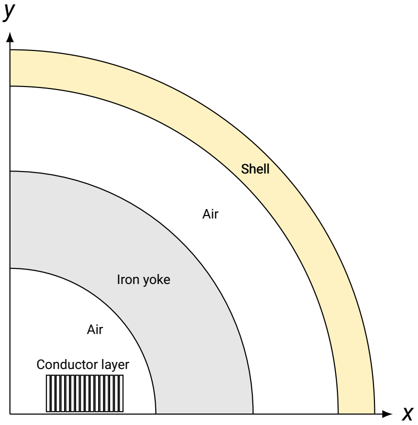

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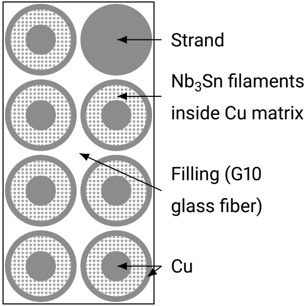

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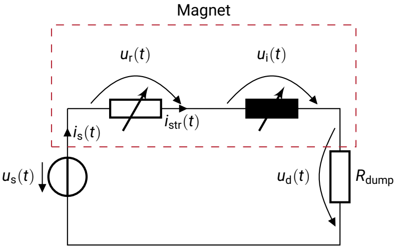

Introduction and Appetizer

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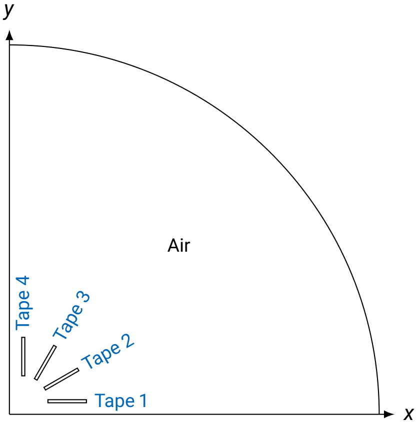

- High-temperature superconducting tape

- Different electrical connections
  - Tapes in parallel and series
  - End-coupled tapes

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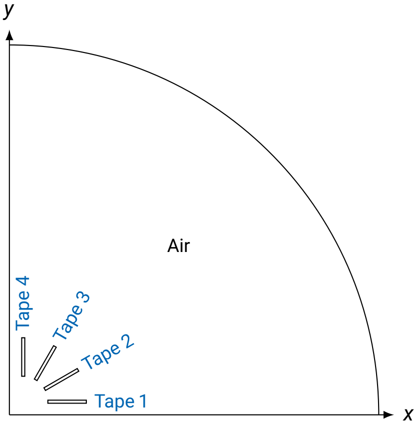

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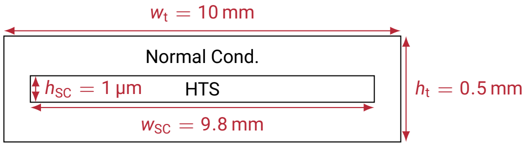

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Introduction and Appetizer

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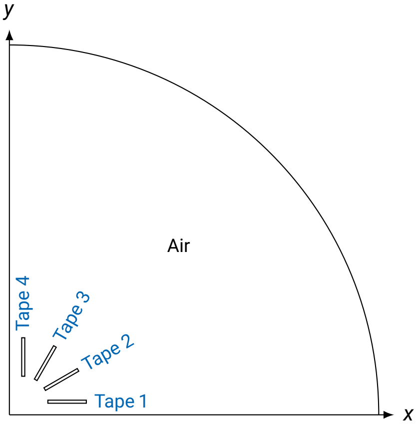

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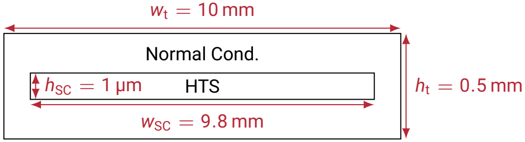

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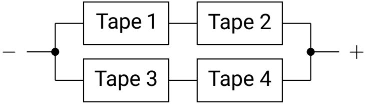

Outline of the Talk

1 Introduction

2 Nb_3Sn Dipole Accelerator Magnet
- Modeling Rutherford Cables: Inter-Filament Coupling Currents
- Reduced Vector Potential Formulation With Thermal Coupling
- Strong Field/Circuit Coupling
- Exemplary Results: Comparison Against Reference Solution

3 Coil of High-Temperature Superconducting Tapes
- Modeling HTS Tapes: Power Law
- Mixed $\boldsymbol{A}^{*}-\boldsymbol{H}$ Formulation With Thermal Coupling
- Exemplary Results: Current Redistribution Phenomenon

4 Conclusion

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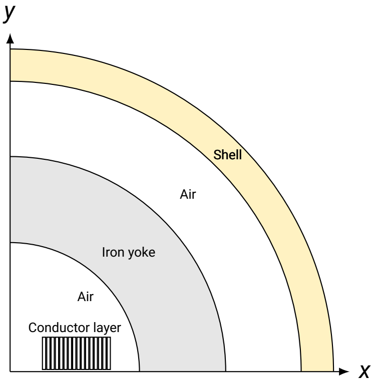
Nb_3Sn Dipole Acclerator Magnet Model

Numerical Analysis of Magneto-Thermal Phenomena



- Conducting layer with 16 turns
  - Rutherford cable: Nb_3Sn filaments in copper matrix
- Coupled to simple circuit
  - Consisting of dump resistor and voltage source
- Translational symmetry
  - → Reduction to computations on a 2D cross-section
- Symmetry boundary conditions
  - → Simulate only a quarter of the magnet
- Reference results available at CERN[1]

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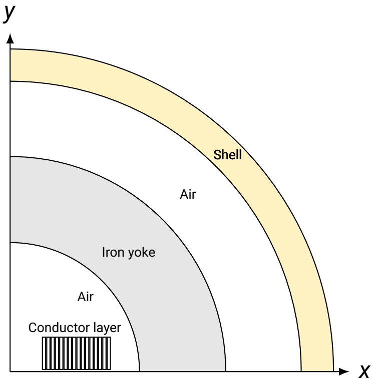

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[1] L. Brouwer et al. *Crosscheck of the Ansys-Comsol 2D FEM Implementations for Superconducting Accelerator Magnets*. Tech. rep. LBNL Eng. Note: SU-1010-4841. ATAP Division, Lawrence Berkeley National Laboratory and Technical Department, CERN, 2019.

Strong A^* Formulation for LTS Accelerator Magnets

- Magnetoquasistatic approximation

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}, \quad \nabla \times \boldsymbol{H} = \boldsymbol{J}, \quad \nabla \cdot \boldsymbol{B} = 0$$

- Material laws

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{src}} + \sigma \boldsymbol{E}, \quad \boldsymbol{H} = \nu \boldsymbol{B} - \boldsymbol{M}_{\mathrm{ifcc}}$$

- Reduced vector potential $\boldsymbol{A}^*$

$$\boldsymbol{A}^* = \boldsymbol{A} + \int_{t_0}^{t} \nabla \phi \, \mathrm{d}t' \quad \text{with} \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

- $\boldsymbol{A}^*$ formulation with $\boldsymbol{J}_{\mathrm{ifcc}} = \nabla \times \boldsymbol{M}_{\mathrm{ifcc}}$

$$\begin{aligned} \nabla \times \left(\nu \nabla \times \boldsymbol{A}^*\right) + \sigma \partial_t \boldsymbol{A}^* &= \boldsymbol{J}_{\mathrm{src}} + \boldsymbol{J}_{\mathrm{ifcc}} && \text{in } \Omega \\ \boldsymbol{n}_\Omega \times \boldsymbol{A}^* &= 0 && \text{on } \Gamma_{A_0,\mathrm{d}} \\ \boldsymbol{n}_\Omega \times \left(\nu \nabla \times \boldsymbol{A}^*\right) &= 0 && \text{on } \Gamma_{A_0,\mathrm{n}} \end{aligned}$$

1. Where to apply which boundary condition?
2. How to link $\boldsymbol{J}_{\mathrm{src}}$ and the circuit curr./volt.?
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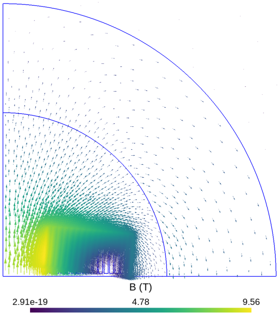
Boundary Conditions

Symmetry Conditions and Shell Transformation



- Boundary conditions by symmetry
  - Expected behavior at lower and left boundary

- Open bound. cond. by shell transformation[1]
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  - GetDP: special Jacobian `VolSphShell`

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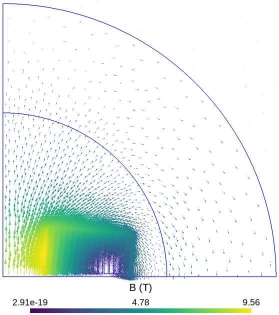

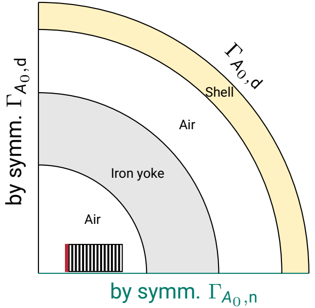

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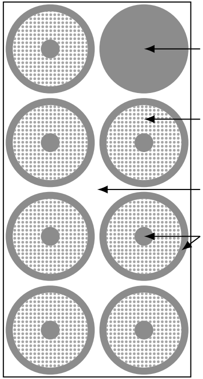
Modeling the Rutherford Cables

Inter-Filament Coupling Currents and Stranded Winding Functions



- Fine structure → homogenize material composition
  - Assume constant current density in cross-section

⇒ Stranded conductor with designated winding function χ_{str}

$$J_{src} = i_{str}\chi_{str}, \quad \text{for 2D case:} \quad \chi_{str} = \begin{cases} \frac{N_{turns}}{|\Omega_{src}|}\hat{\boldsymbol{z}} & \text{in } \Omega_{src} \\ 0 & \text{else} \end{cases}$$

- Wires modeled by inter-filament coupling currents (ifcc)

$$\boldsymbol{J}_{ifcc} = \nabla \times \boldsymbol{M}_{ifcc} \quad \text{with} \quad \boldsymbol{M}_{ifcc} = -\kappa\nu\tau\partial_t\boldsymbol{B} \quad \text{in } \Omega_{src}$$

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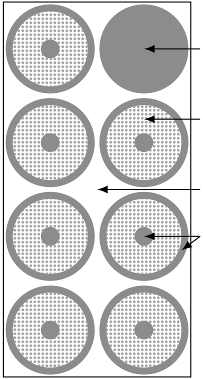

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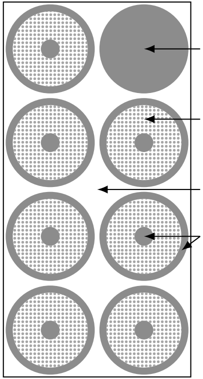

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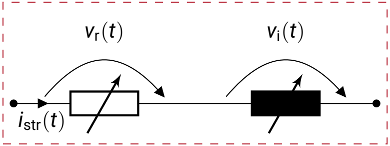
Coupling to a Surrounding Circuit
Current-Voltage Characteristic Using Winding Function



- Voltage drop over stranded conductor

$$u_{str} = u_r + u_i$$

$\Rightarrow$ Coupling to surrounding circuitry

- Resistive voltage (homogenized res. $\bar{\rho}$)

$$u_r = R_{str} i_{str} \quad \text{with} \quad R_{str} = \int_{\Omega_{src}} q_{flag}\, \bar{\rho} \left(\chi_{str} \cdot \chi_{str}\right) \,\mathrm{d}\Omega$$

- Quench status determined by

$$q_{flag}(\boldsymbol{J}, \boldsymbol{B}, \boldsymbol{T}) = \begin{cases} 0 & \text{if magnet is superconducting} \\ 1 & \text{else} \end{cases}$$

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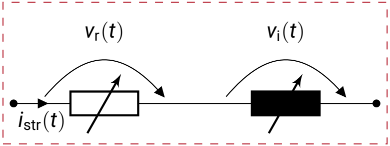
Magnet



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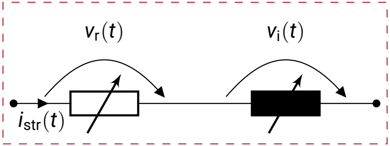

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Modeling Thermal Behavior

The Heat Equation

- Materials temperature-dependent
- Losses as heat sources
  - Joule losses

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  - Homogeneous Neumann conditions on Γ_{src}

$\Rightarrow$ Coupling between electromagnetic and thermal problem

- Via temperature- and field-dependent materials and heat sources

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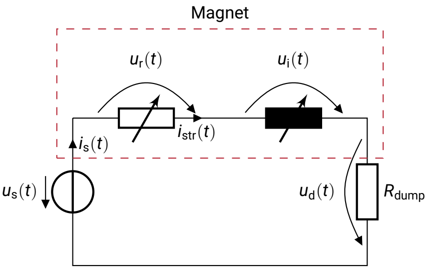
Strong Field/Circuit Coupling

Loop Analysis Using GetDP's Internal Circuit Coupling Capabilities



- Loop incidence matrix
  - Simplified for this case

$$\mathbf{B} = [\mathbf{B}_V, \mathbf{B}_R, \mathbf{B}_{str}]$$

- Kirchhoff's (branch) voltage law and loop currents $\mathbf{i}_l$

$$\mathbf{B}\mathbf{u} = \mathbf{0} \quad \text{and} \quad \mathbf{B}^\top \mathbf{i}_l = \mathbf{i}$$

- Constant diagonal resistance matrix $\mathbf{G}_R$

$$\mathbf{B}_R \mathbf{G}_R^{-1} \mathbf{B}_R^\top \mathbf{i}_l + \mathbf{B}_V \mathbf{u}_s + \mathbf{B}_{str}^\top \mathbf{u}_{str} = \mathbf{0} \quad \text{and} \quad \mathbf{B}_{str}^\top \mathbf{i}_l = \mathbf{i}_{str}$$

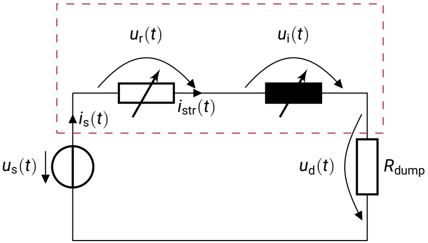
- For the simple model circuit

$$R_{dump} i_s + u_{str} = R_{dump} i_s + u_r + u_i = u_s \quad \text{and} \quad i_s = i_{str}$$

- Monolithic system
  - → Circ. and magneto-therm. eq. solved simultaneously
  - More sophist.: cosimulation (Lorenzo Bortot's talk)

Strong Field/Circuit Coupling

Loop Analysis Using GetDP's Internal Circuit Coupling Capabilities



- Loop incidence matrix
  - Simplified for this case

$$\mathbf{B} = [\mathbf{B}_\mathsf{V}, \mathbf{B}_\mathsf{R}, \mathbf{B}_\mathsf{str}]$$

- Kirchhoff's (branch) voltage law and loop currents $\mathbf{i}_\mathsf{l}$

$$\mathbf{B}\mathbf{u} = \mathbf{0} \quad \text{and} \quad \mathbf{B}^\top \mathbf{i}_\mathsf{l} = \mathbf{i}$$

- Constant diagonal resistance matrix $\mathbf{G}_\mathsf{R}$

$$\mathbf{B}_\mathsf{R}\mathbf{G}_\mathsf{R}^{-1}\mathbf{B}_\mathsf{R}^\top \mathbf{i}_\mathsf{l} + \mathbf{B}_\mathsf{V}\mathbf{u}_\mathsf{s} + \mathbf{B}_\mathsf{str}^\top \mathbf{u}_\mathsf{str} = \mathbf{0} \quad \text{and} \quad \mathbf{B}_\mathsf{str}^\top \mathbf{i}_\mathsf{l} = \mathbf{i}_\mathsf{str}$$

- For the simple model circuit

$$R_\mathsf{dump} i_\mathsf{s} + u_\mathsf{str} = R_\mathsf{dump} i_\mathsf{s} + u_\mathsf{r} + u_\mathsf{i} = u_\mathsf{s} \quad \text{and} \quad i_\mathsf{s} = i_\mathsf{str}$$

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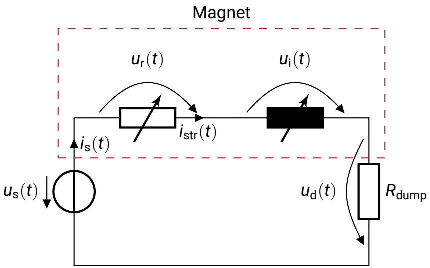

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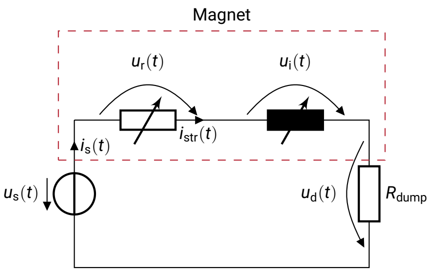

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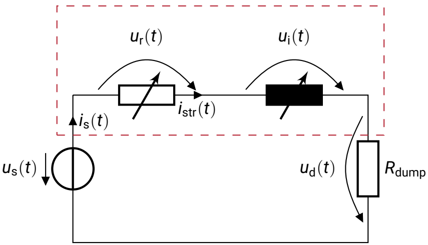

- Repr. based on netlists

```
Constraint{
    [...]

 { Name ElectricalCircuit; Type Network;
        Case Circuit1 { // Netlist of the circuit
            { Region voltageSource;  Branch {1,2} ; }
            { Region conductorLayer; Branch {2,3} ; }
            { Region dumpResistance; Branch {3,1} ; }
        }
    }

    { Name Voltage_Cir ; Type Assign;
        Case { // Apply source voltage
            { Region voltageSource; Value peakVoltage;
              TimeFunction source_timeDependency[];}
        }
    }

}
```

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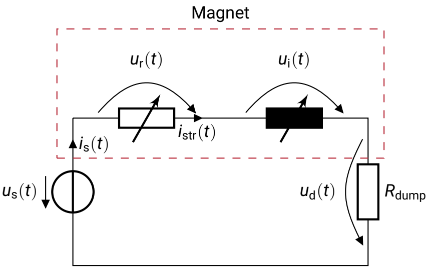
Loop Analysis Using GetDP's Internal Circuit Coupling Capabilities



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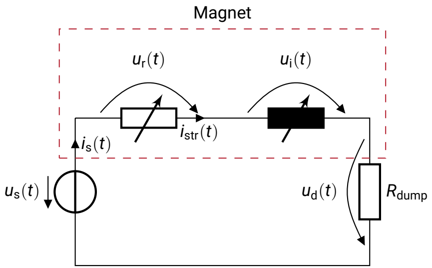

- Add resistor and netlist to formulation

```
Formulation{
    [...]

    // Current-voltage relationship of resistor U = RI
    GlobalTerm{[Dof{UCir}          , {ICir}]; In Cir_Resistor;}
    GlobalTerm{[Res[] * Dof{ICir}, {ICir}]; In Cir_Resistor;}

    // Add circuital constraints from the netlist
    GlobalEquation{
        Type Network; NameOfConstraint ElectricalCircuit;
        {Node {IStr}; Loop {UStr}; Equation {UStr};
            In Dom_source;}
        {Node {ICir}; Loop {UCir}; Equation {UCir};
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```

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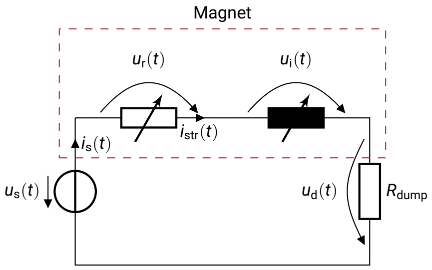


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- Add resistor and netlist to formulation

Interlude: Do I need to write GetDP code for every model?

Reminder: Templates and Interfaces

- No! As seen yesterday, GetDP can be parameterized!
  1. Parametrized templates developed by proficient GetDP users
  2. Used by application engineers, magnet designers etc.

- Without touching the templates, users can
  - change the geometry (e.g. in Gmsh)
  - change model parameters in Gmsh's GUI
  - embed the model in a larger framework (cosimulation, optimization, ...)
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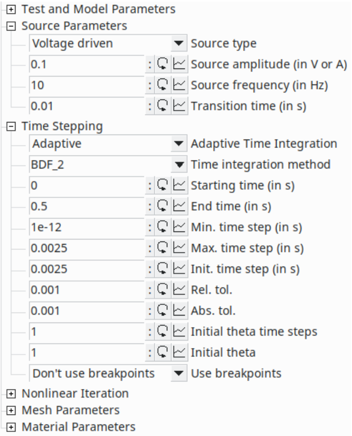

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```
getdp ltsMagnet.pro -post postop-id -solve resolution-id ...
```

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    - by using the ONELAB python module

      As seen in action in Nicolas Marsic's talk!

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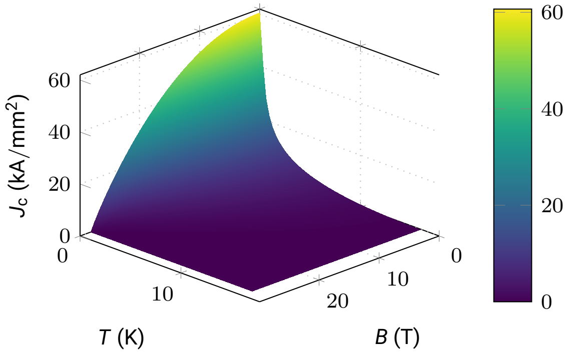
⇒ Inner workings of templates can be seen as a black box FE solver!
  - Similar to modules in COMSOL, e.g. the AC/DC module

Back to the Model: Quench Back Scenario Test Case

- Fit of critical surface of Nb_3Sn
  - $\rightarrow$ $q_{\text{flag}}(\boldsymbol{J}, \boldsymbol{B}, T)$ as GetDP function



- First 5 ms, steady state with $i_s = 18$ kA
- Then, linear ramp-down of voltage source in 0.1 ms
- Fast discharge over dump resistor
  - $\rightarrow$ High $\partial_t \boldsymbol{B}$ and $P_{\text{ifcc}} \propto \|\partial_t \boldsymbol{B}\|^2$
  - $\rightarrow$ Ifcc losses lead to quench
- Quench $\rightarrow$ high Joule losses
  - $\rightarrow$ More regions start to

$\Rightarrow$ More even distribution of stored magnetic energy

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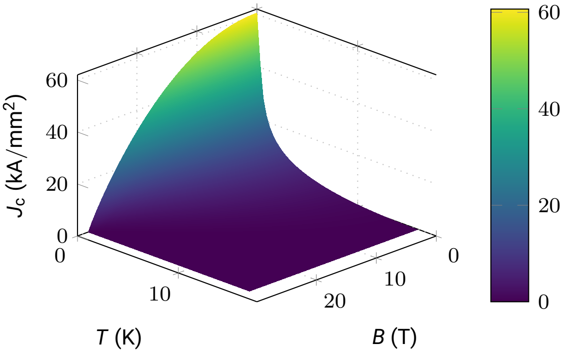

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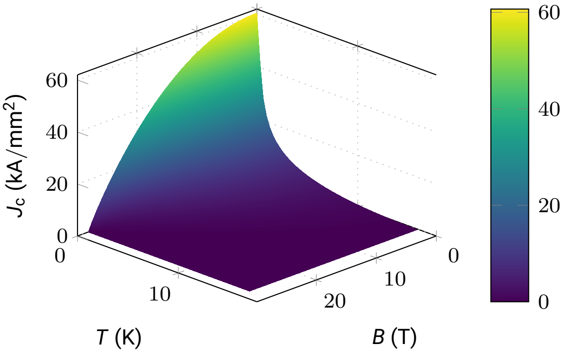

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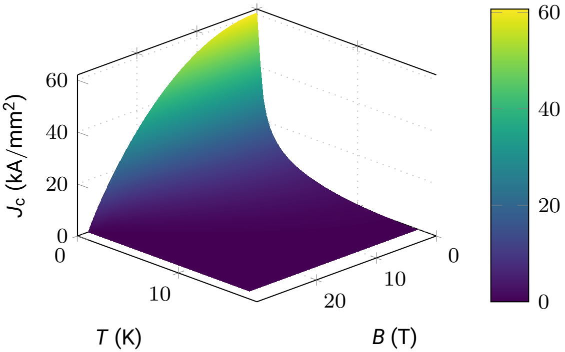

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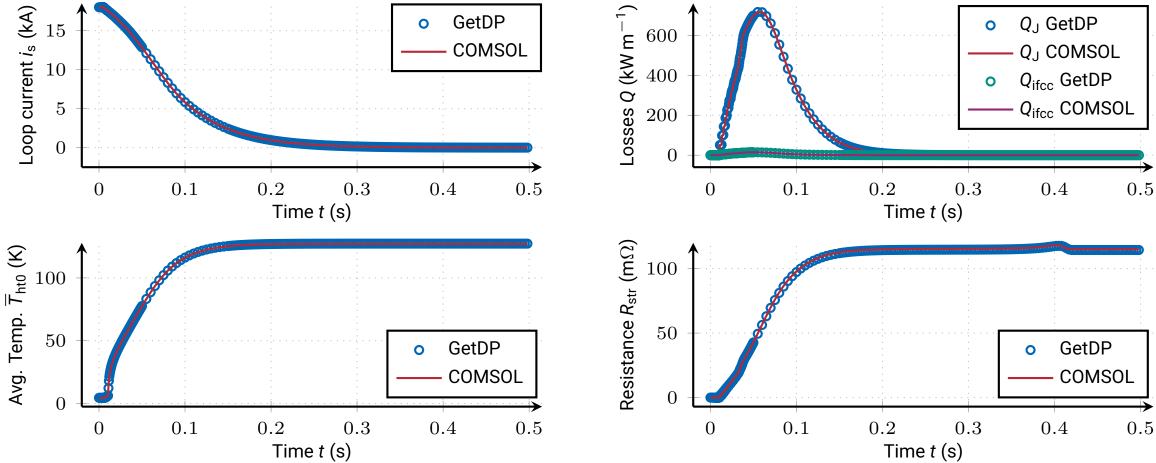
Numerical Results of GetDP Against Reference Solution

Subset of Investigated Post-Processing Quantities



Nb_3Sn Dipole Accelerator Magnet With Circuit Coupling

Summary

- Twisted multi-filament wires modeled by inter-filament coupling currents
- $\boldsymbol{A}^*$ formulation based on reduced vector potential
- Strong field/circuit coupling
- Good agreement with reference results from COMSOL MULTIPHYSICS® (and ANSYS®)
- Test case shown: $\sim$ 1 h simulation time on a laptop (no particular focus on performance)

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Outline of the Talk

1 Introduction

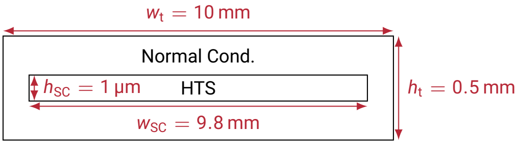
2 Nb_3Sn Dipole Accelerator Magnet

3 Coil of High-Temperature Superconducting Tapes

4 Conclusion

Modeling High-Temperature Superconducting Tapes

The Power Law



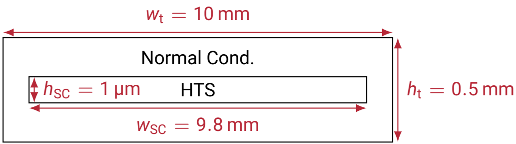
- Structure of HTS tapes significantly differs from Rutherford cables
  $\Rightarrow$ Modeling their behavior via ifcc magnetization not applicable
- Instead; phenomenological description via the power law

$$\mathbf{J}(\mathbf{E}) = \sigma(\|\mathbf{E}\|)\,\mathbf{E} \quad \text{with conductivity} \quad \sigma(\|\mathbf{E}\|) = \frac{J_{crit}}{E_{crit}}\left(\frac{\|\mathbf{E}\|}{E_{crit}}\right)^{(1-n)/n}$$

- J_{crit} and n material parameters
- Choose $E_{crit} = 10^{-4}\,\mathrm{V\,m^{-1}}$

Modeling High-Temperature Superconducting Tapes

The Power Law



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Coupled Formulation
Motivation

- As seen in Julien Dular's talk
  - In superconducting materials: $\boldsymbol{H}$ formulation more efficient than $\boldsymbol{A}$ formulation
  - In ferromagnetic materials: vice versa

$\Rightarrow$ Coupled (or mixed) formulation to combine their strengths

- Following the example of an accelerator magnet
  - Use $\boldsymbol{H}$ inside the source region Ω_H
    - Contains the excitation coil with superconducting and normal conducting material
  - Use $\boldsymbol{A}^*$ inside the passive region Ω_A
    - Contains the iron yoke, mechanical supports and the air regions

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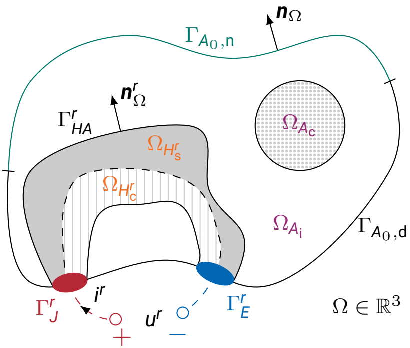
Coupled Formulation

Visualization and Solid Conductor Winding Functions



Src. dom. $\Omega_H = \bigcup_r \Omega_H^r = \bigcup_r \left(\Omega_{H_\mathrm{s}}^r \cup \Omega_{H_\mathrm{c}}^r\right)$

Pass. dom. $\Omega_A = \Omega_{A,\mathrm{c}} \cup \Omega_{A,\mathrm{i}}$

- N_r windings with independent curr. and volt.

$$\boldsymbol{i}_\mathrm{s} = \left[i^1, \ldots, i^{N_r}\right]^\top \quad \text{and} \quad \boldsymbol{u}_\mathrm{s} = \left[u^1, \ldots, u^{N_r}\right]^\top$$

- Dedicated winding functions for solid conductors

$$\chi = \left[\chi^1, \ldots, \chi^{N_r}\right]$$

- Split $\boldsymbol{E}$ and $\boldsymbol{H}$ across the windings

$$\boldsymbol{E} = \sum_{r=1}^{N_r} \boldsymbol{E}^r = \sum_{r=1}^{N_r} \left(\rho \nabla \times \boldsymbol{H}^r - \underbrace{u^r \chi^r}_{\boldsymbol{E}_\mathrm{src}}\right)$$

- Add. integral constraint if $\mathbf{i}_\mathrm{s}$ known instead of $\mathbf{u}_\mathrm{s}$

Coupled Formulation

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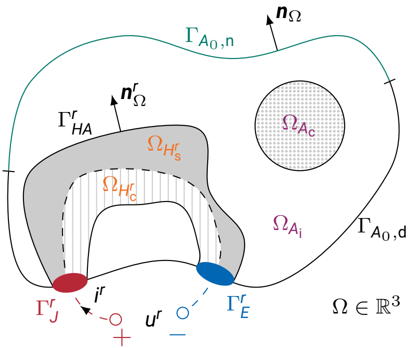

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Pass. dom. $\Omega_A = \Omega_{A,\mathrm{c}} \cup \Omega_{A,\mathrm{i}}$

- N_r windings with independent curr. and volt.

$$\boldsymbol{i}_\mathrm{s} = \left[i^1, \dots, i^{N_r}\right]^\top \quad \text{and} \quad \boldsymbol{u}_\mathrm{s} = \left[u^1, \dots, u^{N_r}\right]^\top$$

- Dedicated winding functions for solid conductors

$$\boldsymbol{\chi} = \left[\chi^1, \dots, \chi^{N_r}\right]$$

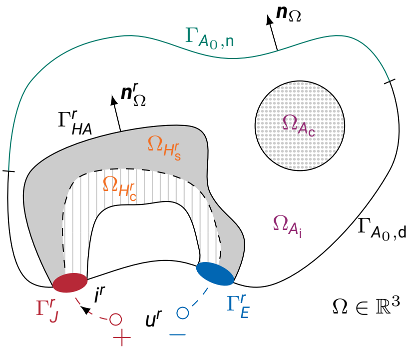
- Split $\boldsymbol{E}$ and $\boldsymbol{H}$ across the windings

$$\boldsymbol{E} = \sum_{r=1}^{N_r} \boldsymbol{E}^r = \sum_{r=1}^{N_r} \left(\rho \nabla \times \boldsymbol{H}^r - \underbrace{u^r \chi^r}_{\boldsymbol{E}_\mathrm{src}}\right)$$

- Add. integral constraint if $\mathbf{i}_\mathrm{s}$ known instead of $\mathbf{u}_\mathrm{s}$

Coupled Formulation

Visualization and Solid Conductor Winding Functions



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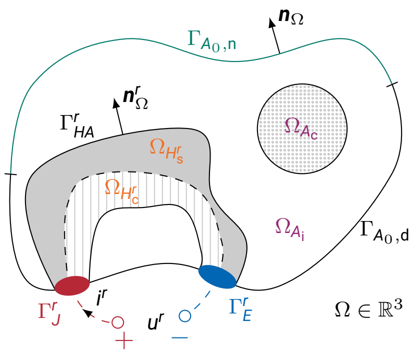
Coupled A^*–H Formulation

Strong Formulation



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$$\nabla \times (\nu \nabla \times \boldsymbol{A}^*) + \sigma \frac{\partial \boldsymbol{A}^*}{\partial t} = 0 \qquad \text{in } \Omega_A$$

$$\boldsymbol{n}_\Omega \times \boldsymbol{A}^* = 0 \qquad \text{on } \Gamma_{A_0,\mathrm{d}}$$

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for all windings r :

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$$\rho_\mathrm{m} C_\mathrm{p} \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = \rho \left\|\boldsymbol{J}\right\|^2 \qquad \text{in } \Omega_H$$

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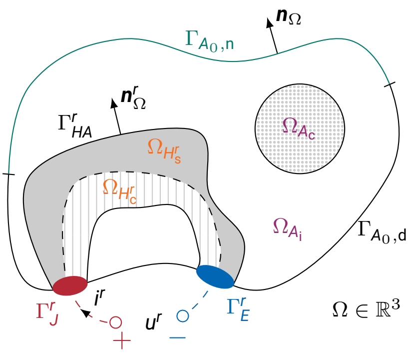
Coupled A^*–H Formulation

Strong Formulation



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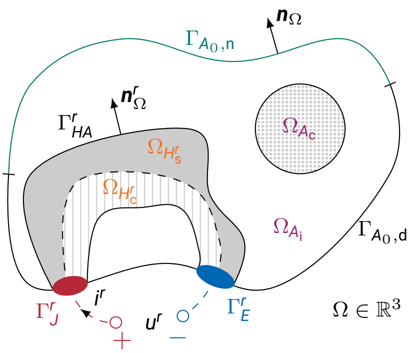

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Coupled A^*–H Formulation

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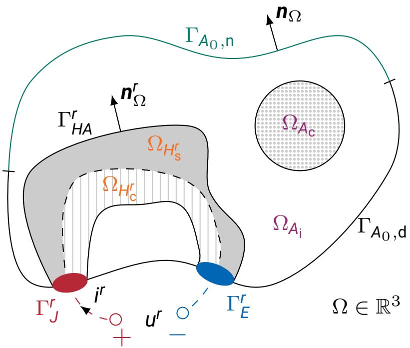

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- For 2D Cartesian case in *xy*-plane

$$\chi^r = \frac{\hat{\boldsymbol{z}}}{|l_\mathrm{z}|} \quad \text{in } \Omega^r_H, \qquad \chi^r = 0 \quad \text{else}$$

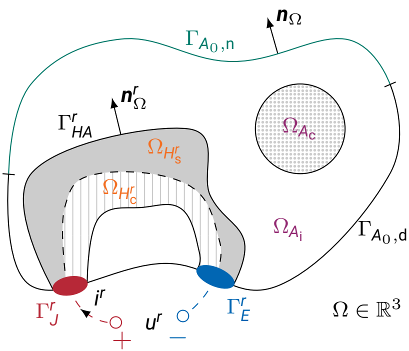
- In 3D, freedom of choice
  - E.g., $\chi^r = -\nabla\xi^r$ with scalar potential ξ^r

$$\begin{aligned} \nabla\cdot(\tilde{\sigma}\nabla\xi^r) &= 0 && \text{in } \Omega^r_H \\ \xi^r &= 1 && \text{on } \Gamma^r_J \\ \xi^r &= 0 && \text{on } \Gamma^r_E \\ \boldsymbol{n}^r_\Omega\cdot(\tilde{\sigma}\nabla\xi^r) &= 0 && \text{on } \Gamma^r_{HA} \end{aligned}$$

  - Find ξ^r before solving main coupled problem
    → GetDP: Own `Formulation` and `System` for ξ^r which is solved first in `Resolution`

Coupled A^*–H Formulation

Definition of Winding Functions



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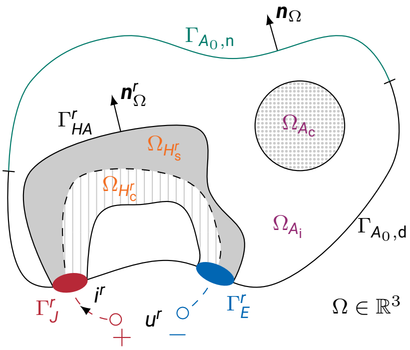
Coupled A^*–H Formulation

Interface Conditions



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- Interface conditions on Γ^r_{HA}
  - → Consistency of the overall solution
- Tangential continuity of $\boldsymbol{E}^r$ and $\boldsymbol{H}^r$
  - → Boundary integrals of the weak formulation as a natural coupling interface
- Normal continuity of $\boldsymbol{B}^r$ and $\boldsymbol{J}^r$
  - → By choosing appropriate function spaces
    - Curl-conforming Nédélec edge elements for A^*, H^r and test functions

Coupled A^*–H Formulation

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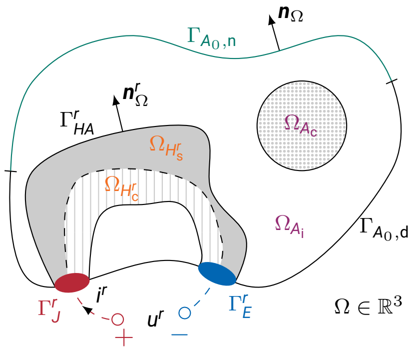

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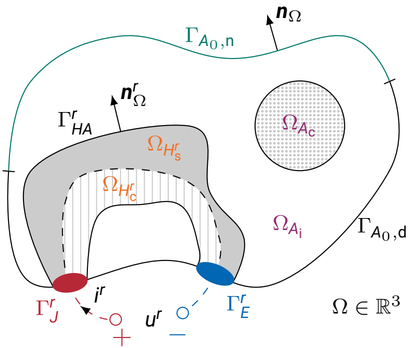

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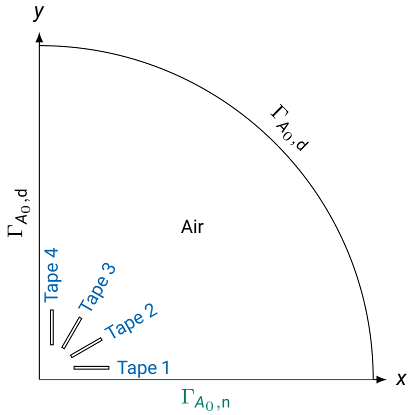
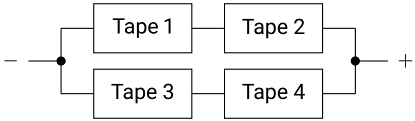
Model Problem: Four Connected HTS Tapes



- Different electrical connections
  - Tapes in parallel and series
  - End-coupled tapes



  - Using GetDP's circuit coupling capabilities
- Driven by applied source voltage or current
- Functionalities needed for real-world problems
  - Scalability? Attend Lorenzo Bortot's talk!

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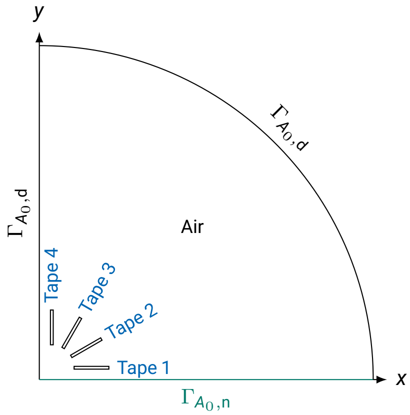

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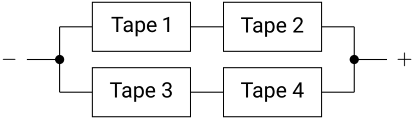

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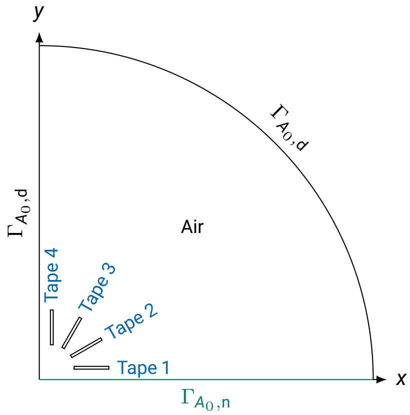

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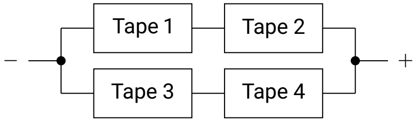

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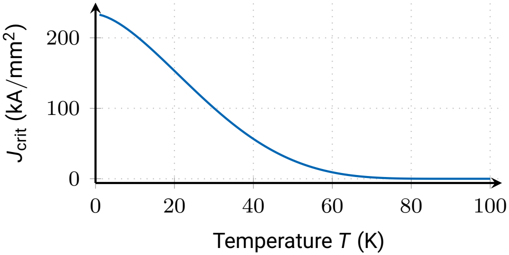
Model Problem: Four Connected HTS Tapes

- In general, $J_c(\|\boldsymbol{B}\|, T, \theta_{\text{field}})$[1]
- Assume $J_c(T) := I_{\text{crit,tape}}(T)/A_{\text{sc}}$
  - $I_{\text{crit,tape}}(10\,\text{K}) = 2\,\text{kA}$



- Parallel tapes $\rightarrow I_{\text{crit,cable}}(10\,\text{K}) = 8\,\text{kA}$
- Normal conductor: homogenized mix
  - Approx. 59 % Hastelloy, 39 % Cu, 2 % Ag

[1] J. Van Nugteren. "High Temperature Superconductor Accelerator Magnets". PhD thesis. University of Twente, 2016.

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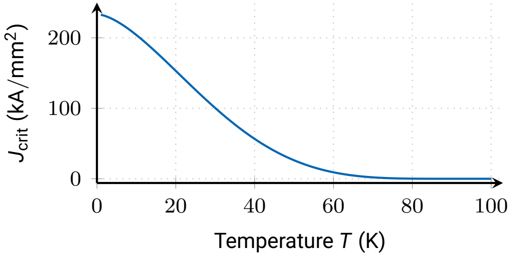

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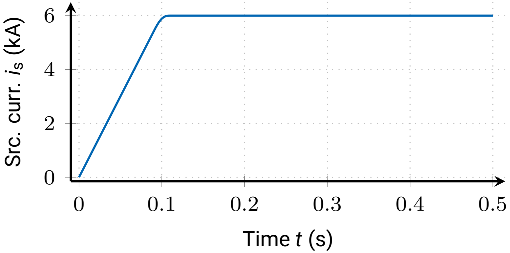

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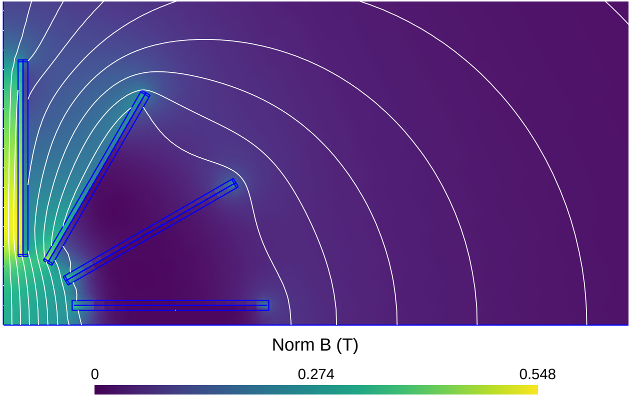
Numerical Results for Current-Driven Tapes in Parallel

Visualization of the Magnetic Field



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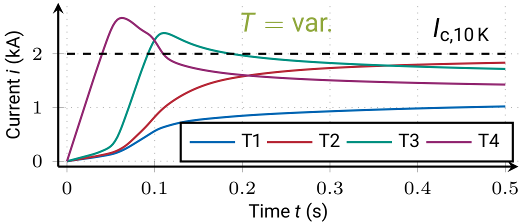
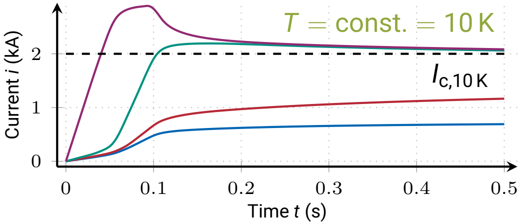
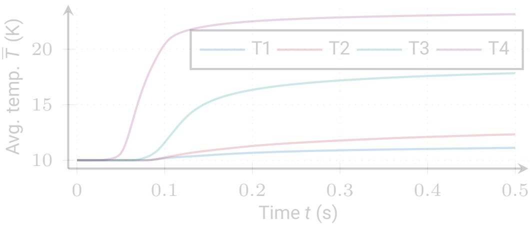
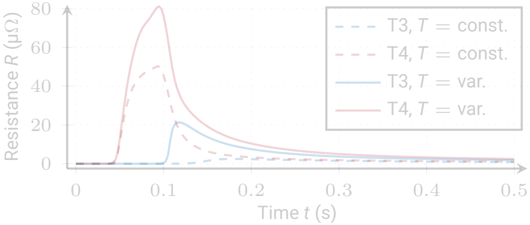
Current Redistribution Phenomena









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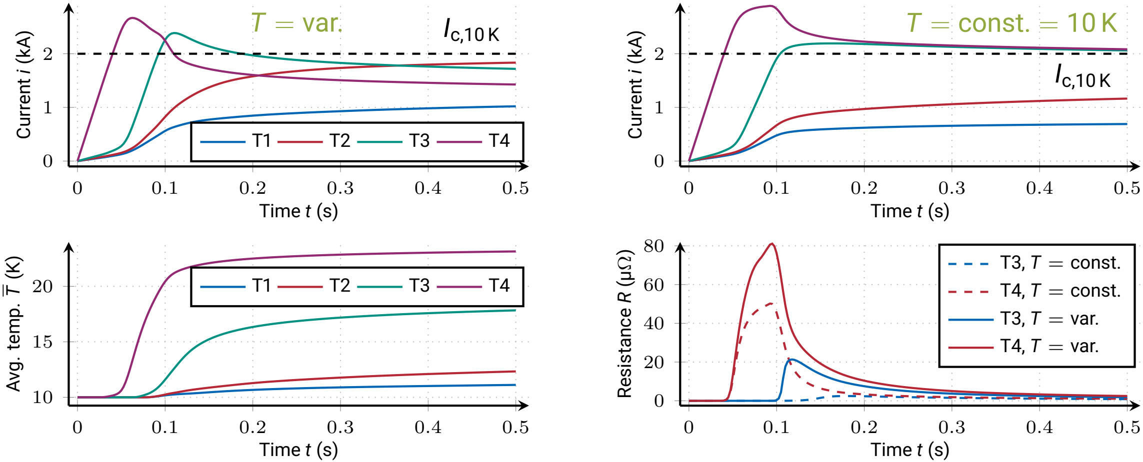

Coil of High-Temperature Superconducting Tapes

Summary

- HTS tapes modeled by power law
- Coupled $\boldsymbol{A}^{*}$–$\boldsymbol{H}$ formulation
- Different electrical connections by GetDP's circuit coupling features
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Conclusion

⇒ GetDP is a flexible tool

- Multiphysics → magnetoquasistatics coupled with heat equation
- Coupled formulations → $\boldsymbol{H}$ in source regions, $\boldsymbol{A}^*$ everywhere else
- Field/circuit coupling capabilities
- Embedding into bigger frameworks via command line, Python interface or Gmsh's API
- Thin shell approximation → Julien Dular's and Lorenzo Bortot's talk

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