What's the Matter with the CMB?

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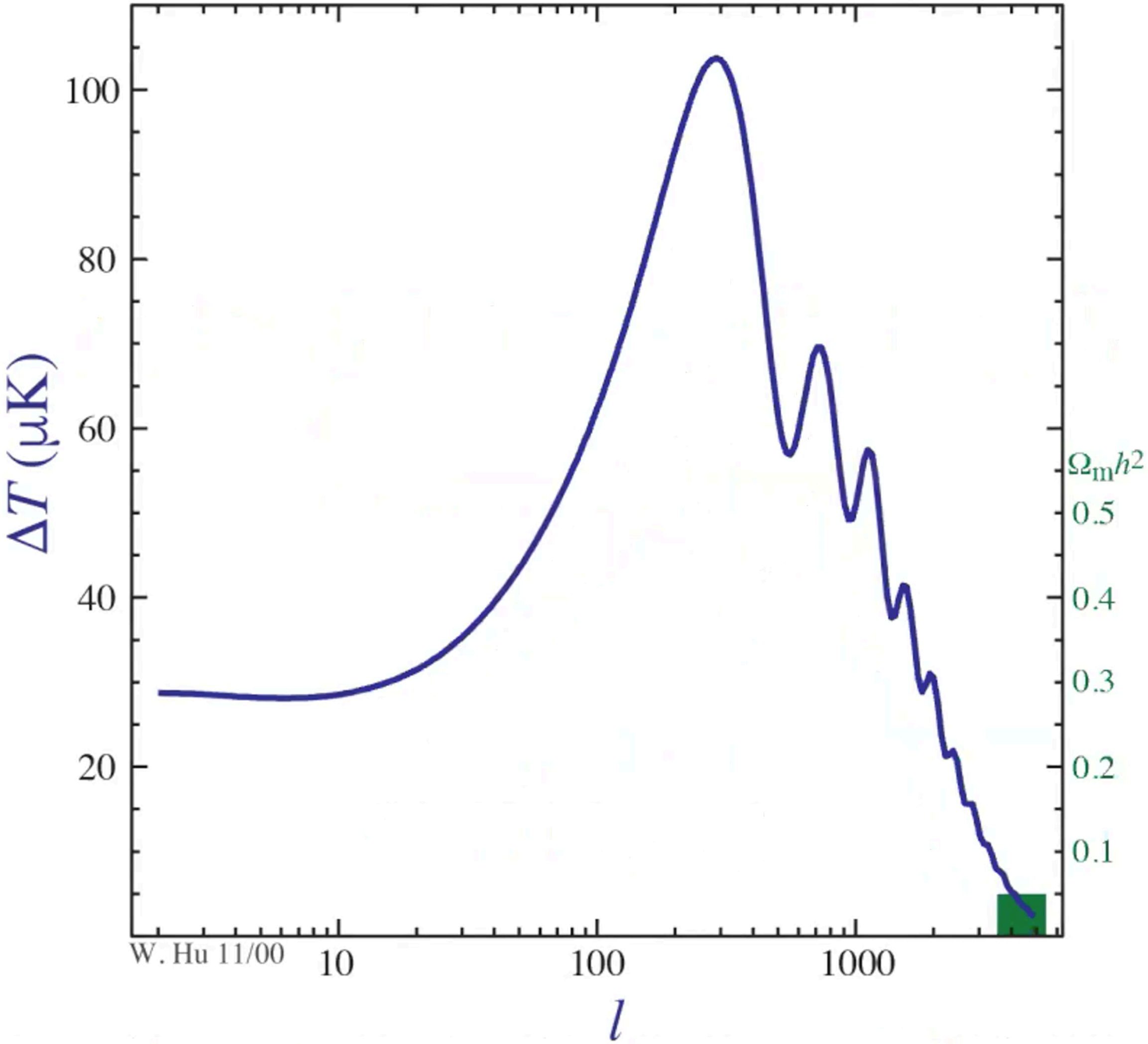
[Based in part on Hu & White (1997) astro-ph/9609079]

Cold Dark Matter in the CMB

- Hydrostatic equilibrium, gravitational forcing, diffusion damping



Cast of Characters

- $\Theta = \Delta T/T$: local CMB blackbody temperature fluctuation

  $p_\gamma = \rho_\gamma/3 \propto T^4$: radiation pressure, hydro equilibrium

  $n_\gamma \propto T^3$: photon number density, continuity equation

- Gravity:

  a: scale factor $T \propto a^{-1}$

  Ψ: Newtonian gravitational potential - hydro equilibrium

  $\Phi \sim -\Psi$: Local scale factor pert.- changes local number density

- Bulk (fluid) properties

  $\lambda_C \equiv 1/\dot{\tau}$: mean free path of γ to Thomson scattering

  $R = 3\rho_b/4\rho_\gamma$: baryon-photon (momentum density) ratio

  v_γ, v_b: photon and baryon bulk velocity

  π_γ: radiative viscosity or anisotropic stress

Exact (Linear) EOMs

- Linear fluctuations: solve in Fourier (or wavenumber k-) space
- Continuity: number density changes due to the local volume expansion and bulk velocity divergence

$$[a^3 \delta n_\gamma]^{\cdot} = -a^3 n_\gamma (k v_\gamma + 3\dot{\Phi}) \,,$$

- Momentum: momentum density $(\rho + p)v$ changes additionally with de Broglie wavelength, stress and potential gradients

$$[a^4(\tfrac{4}{3}\rho_\gamma)(v_\gamma + R v_b)]^{\cdot} = a^4 k[\delta p_\gamma - \tfrac{2}{3} p_\gamma \pi_\gamma + \tfrac{4}{3}\rho_\gamma (1+R)\Psi]$$

- Rewrite in terms of Θ: $\delta n_\gamma / n_\gamma = 3\Theta$, $\delta p_\gamma/\rho_\gamma = 4\Theta/3$

$$\text{Equilibrium} : \Theta + (1+R)\Psi = 0, \qquad \pi_\gamma = 0$$

- Observed temperature fluctuations are local Θ corrected for gravitational redshift: $\Theta + \Psi$

Fluid Approximation

- Near recombination $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$, the (comoving) mean free path of a photon

$$\lambda_C \equiv \frac{1}{\dot{\tau}} \sim 2.5\mathrm{Mpc}$$

  small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions

- Single bulk velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons $\pi_\gamma = 0$

- $\to$ No heat conduction or viscosity (anisotropic stress) in fluid

Acoustic Oscillations

- Combine continuity and momentum in fluid limit

$$[(1+R)\dot{\Theta}]^{\cdot} + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1+R)\Psi - [(1+R)\dot{\Phi}]^{\cdot}$$

- Acoustic oscillations of observed temperature $\Theta + \Psi$ around equilibrium point
- Toy example: if R, Ψ, Φ constant

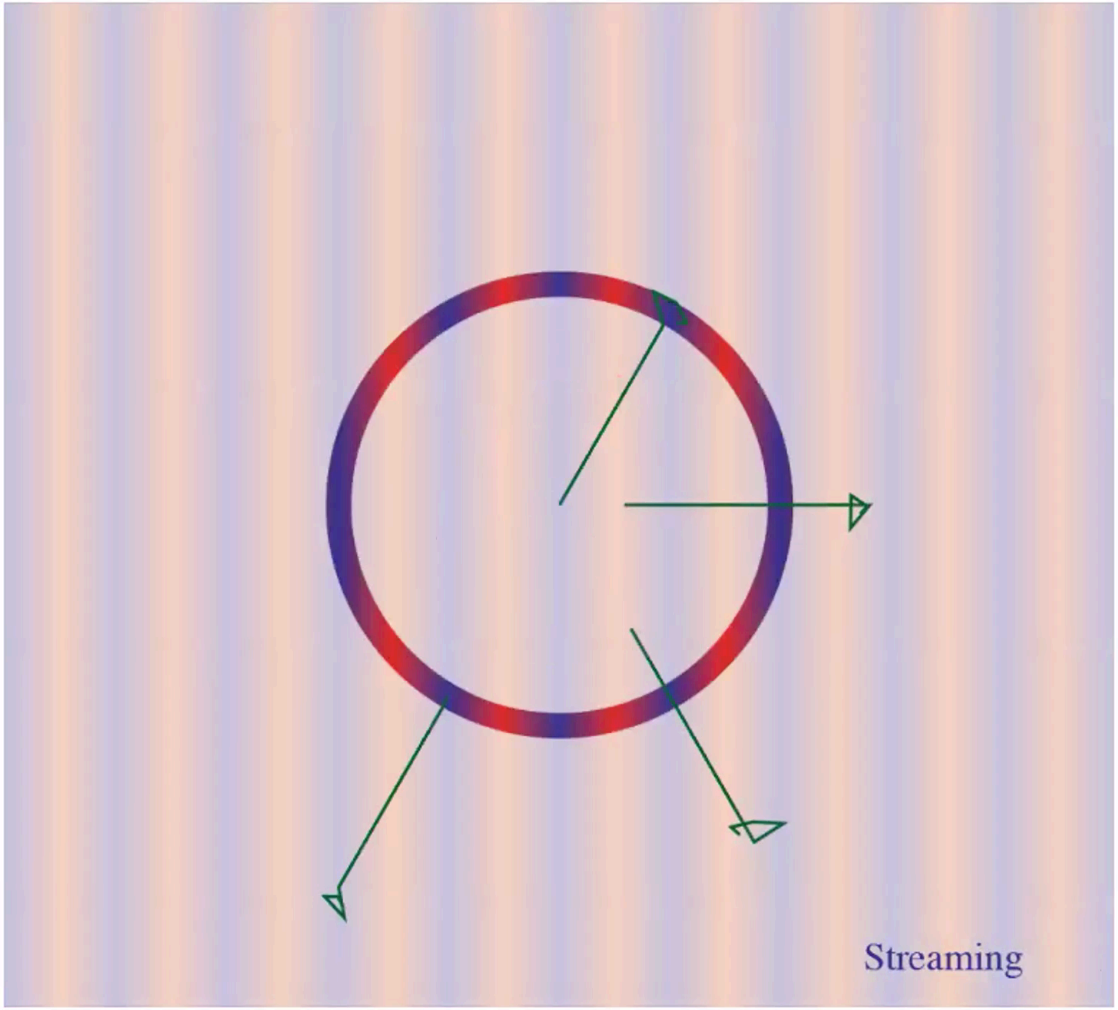
$$[\Theta + \Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks) - R\Psi$$

where s is the sound horizon $s = \int c_s d\eta = \int \frac{dt/a}{\sqrt{3(1+R)}}$

- Temporal oscillations around equilibrium point that measures gravitational potential: $-R\Psi$

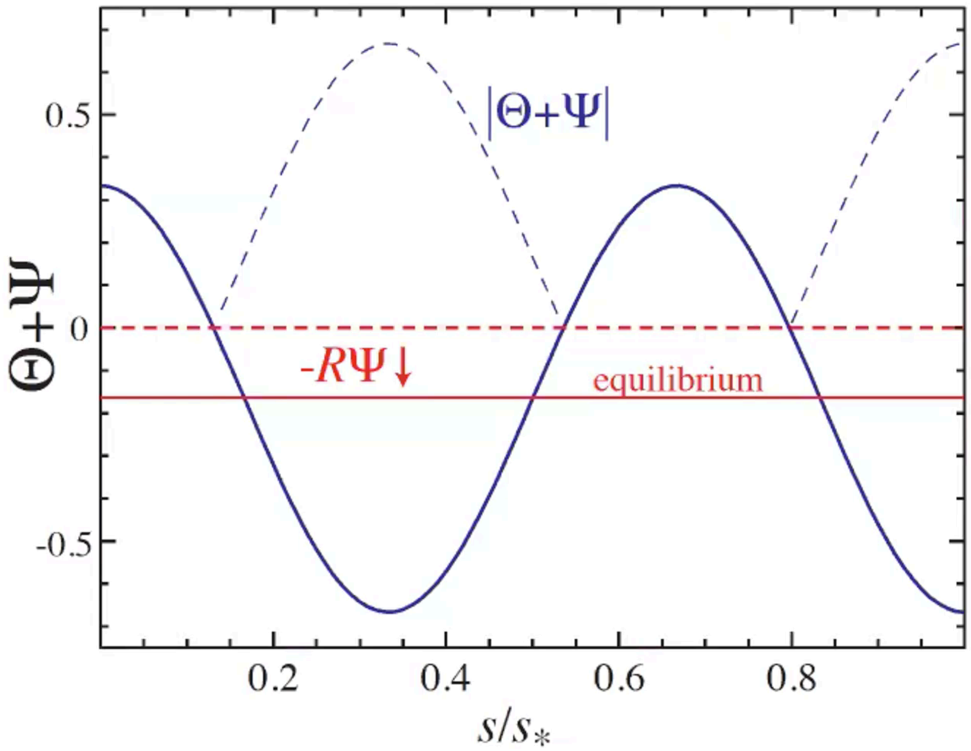
Temperature Anisotropy

- Spatial oscillations frozen at recombination; photons then stream
- Viewed at distance D_* as angular anisotropy $L \approx kD_*$



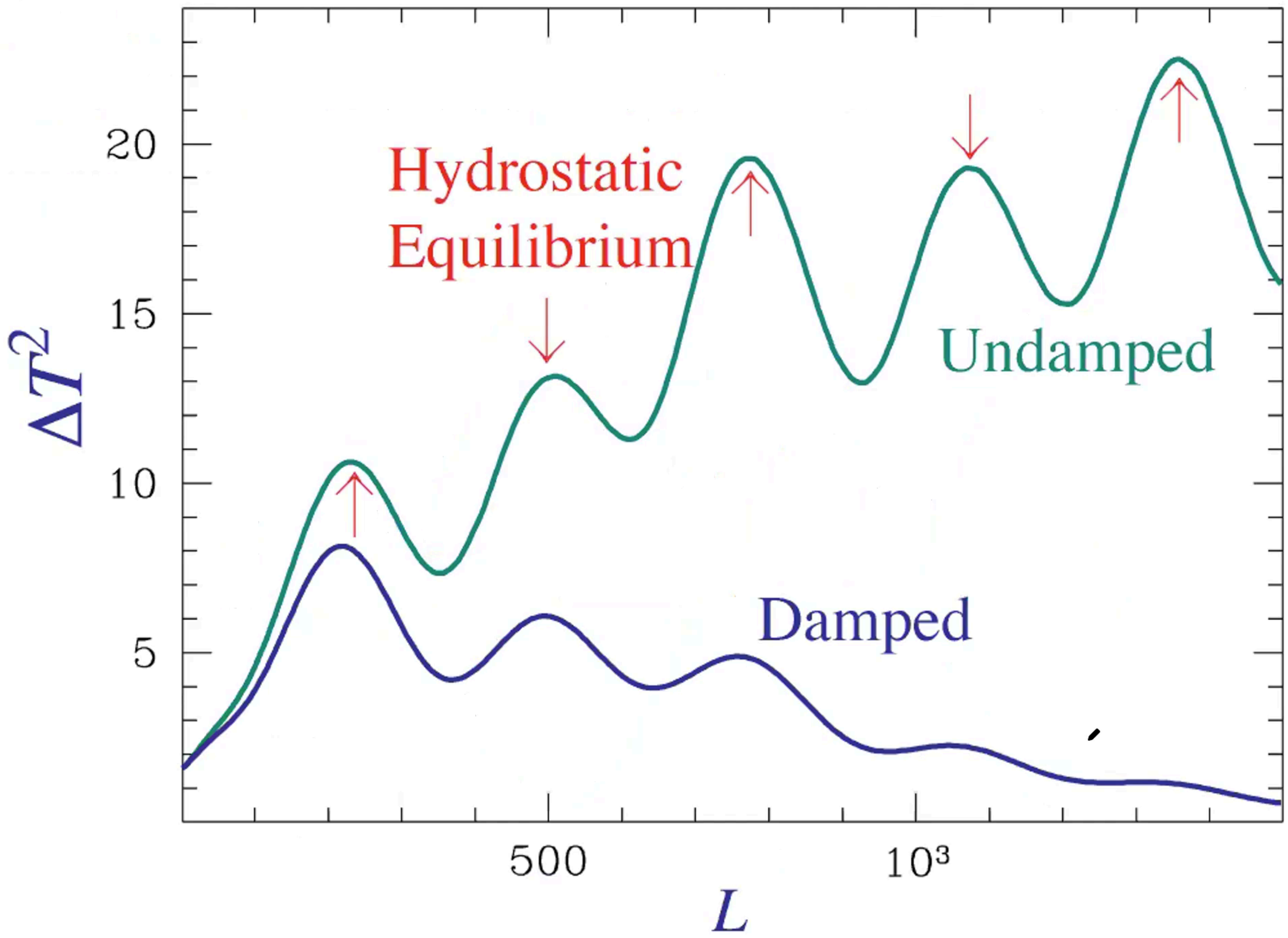
Peak Modulation

- Peaks in power when oscillations reach extrema at recombination; $ks_* = n\pi$
- Equilibrium offset adds to odd peaks, subtracts from even peaks
- Toy model:



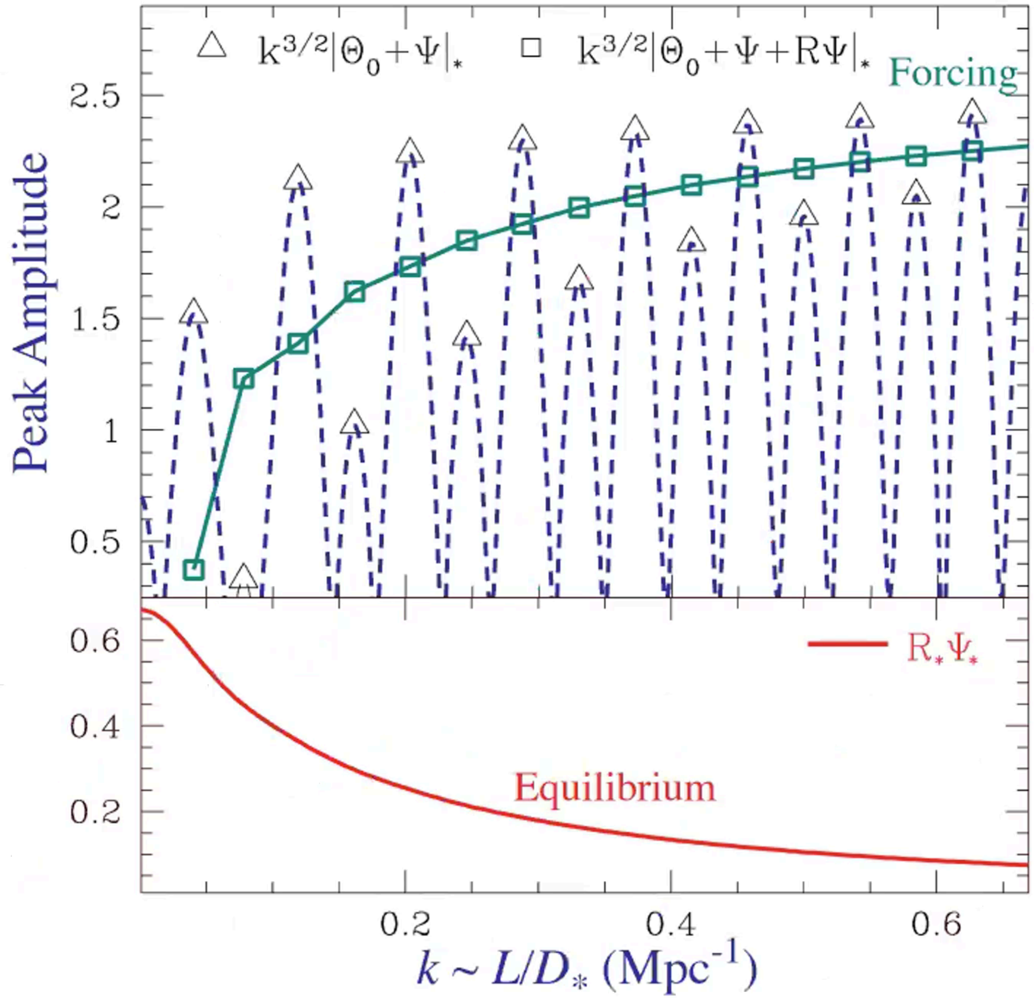
Peak Modulation

- Full calculation vs damping removed calculation $v_\gamma = v_b, \pi_\gamma = 0$



Peak Modulation

- Once damping removed: equilibrium measures grav potential Ψ



- Reveals Ψ decays at high k and observed temperature rises

Matter-Radiation

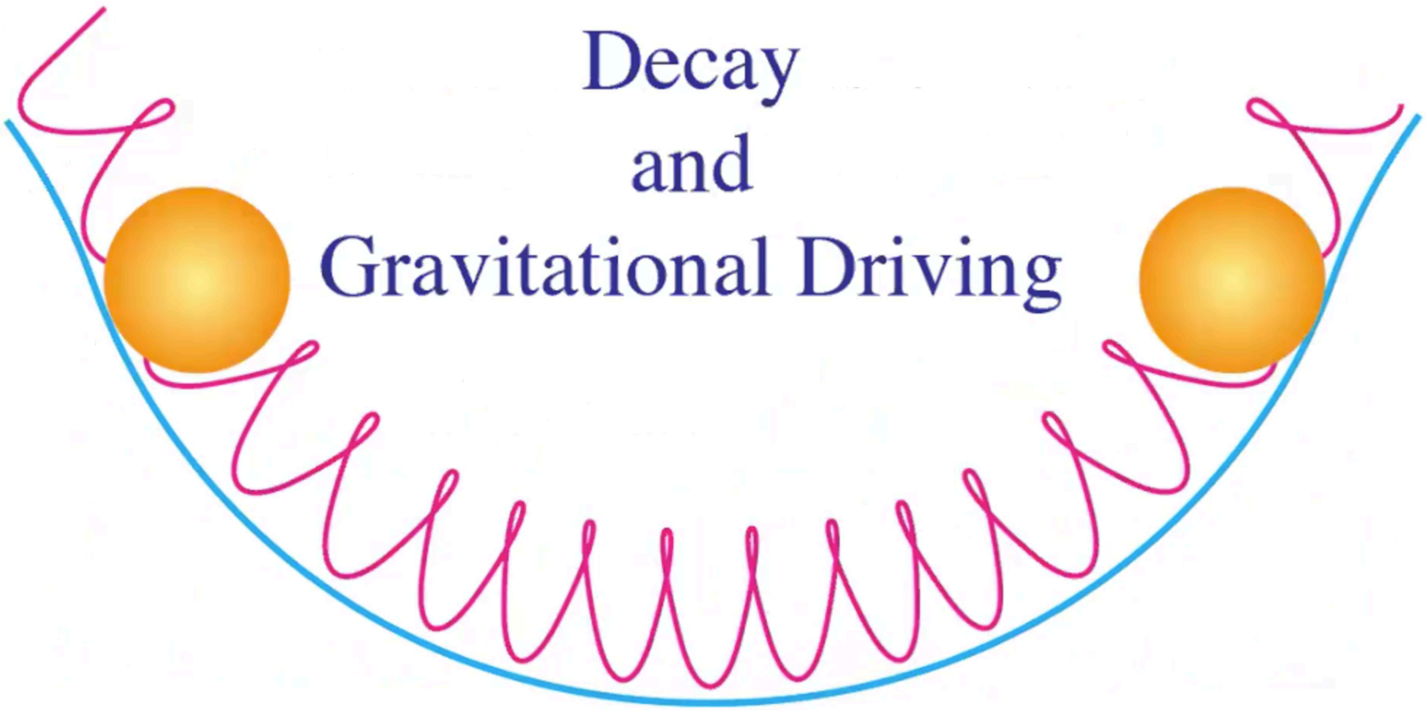
- At high k, acoustic oscillations begin in the radiation dominated regime
- Acoustic oscillations (Jeans) stabilizes (comoving) density fluctuations (Δ) leading to gravitational potential decay
- Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho\Delta$$

  in the radiation dominated era $\rho \propto a^{-4}$ and Δ oscillates at constant amplitude
- Decay is timed to beginning of acoustic oscillation acting as a coherent "push" – change in Φ doubles the effect to $2\Delta\Psi$.

Radiation Driving

- Cartoon version (doubled by local scale factor Φ effect):

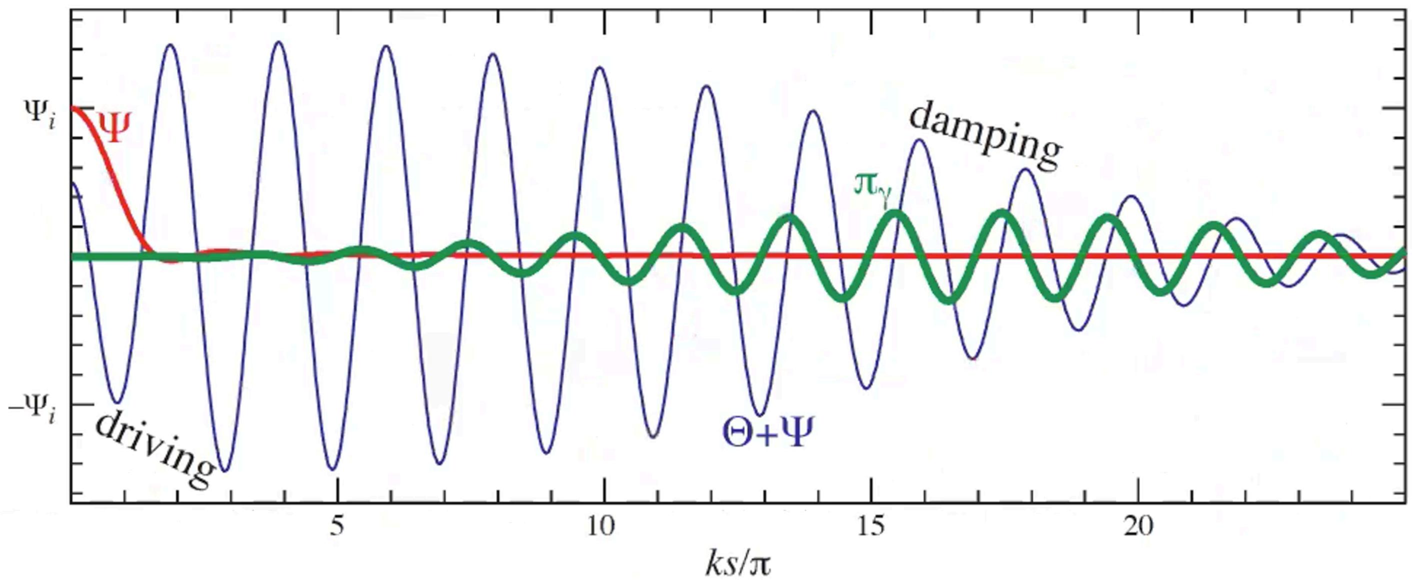


Radiation Driving

- Non-cartoon version: 25× power at low k limit (Sachs-Wolfe effect), lowered to $\sim$ 20 due to neutrino contribution

$$|[\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi| = |\tfrac{1}{3}\Psi(0) - 2\Psi(0)| = |\tfrac{5}{3}\Psi(0)|$$

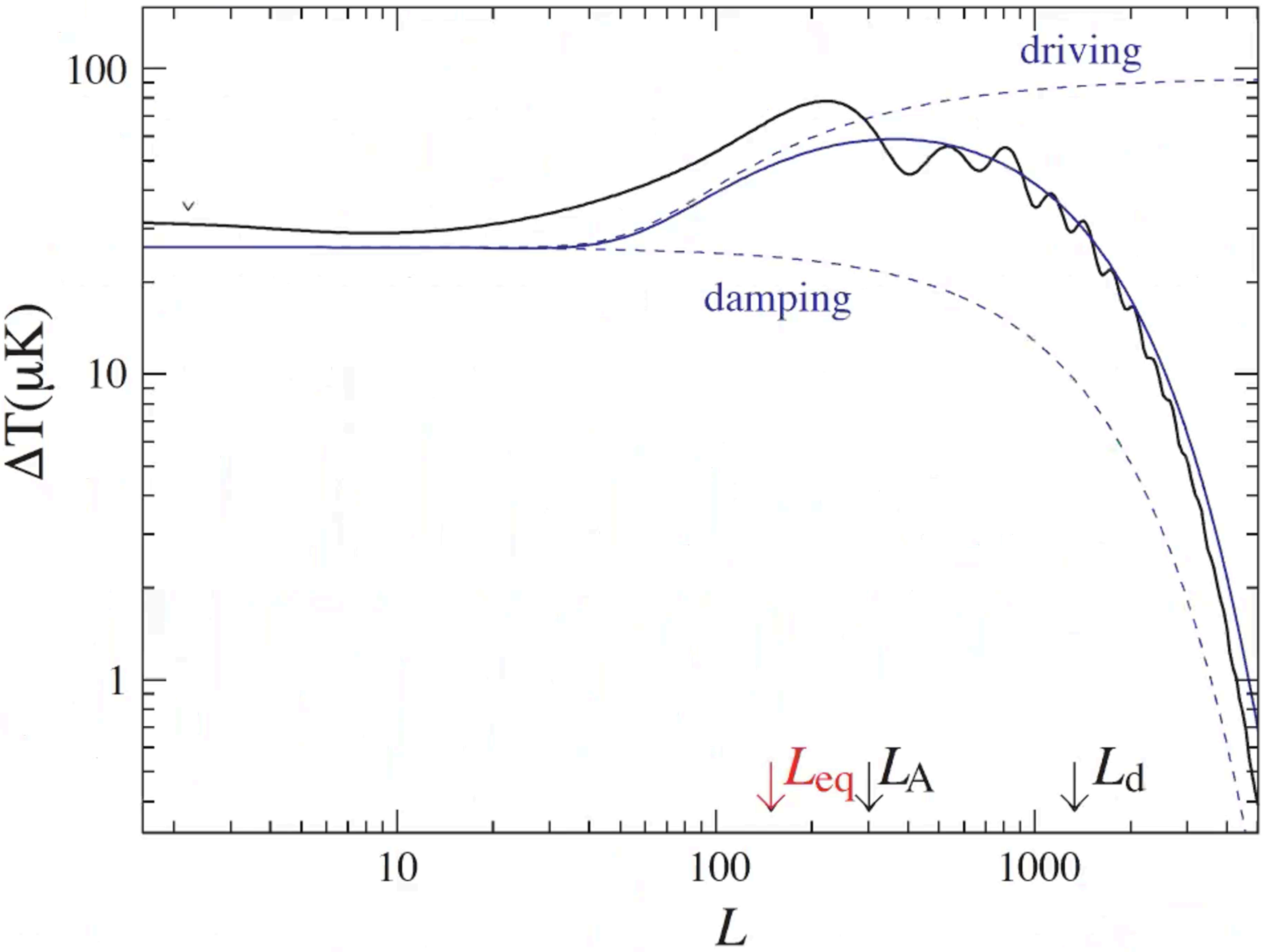
- Since baryons are also in acoustic oscillations driving goes away only when CDM dominates: $\Psi_*(k)$ measures CDM density



- Oscillations damp over time given viscosity π_γ – 3rd CDM effect

Potential Envelope

- Driving measures matter radiation scale L_{eq}



Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

- Dissipation related to diffusion length: random walk approx

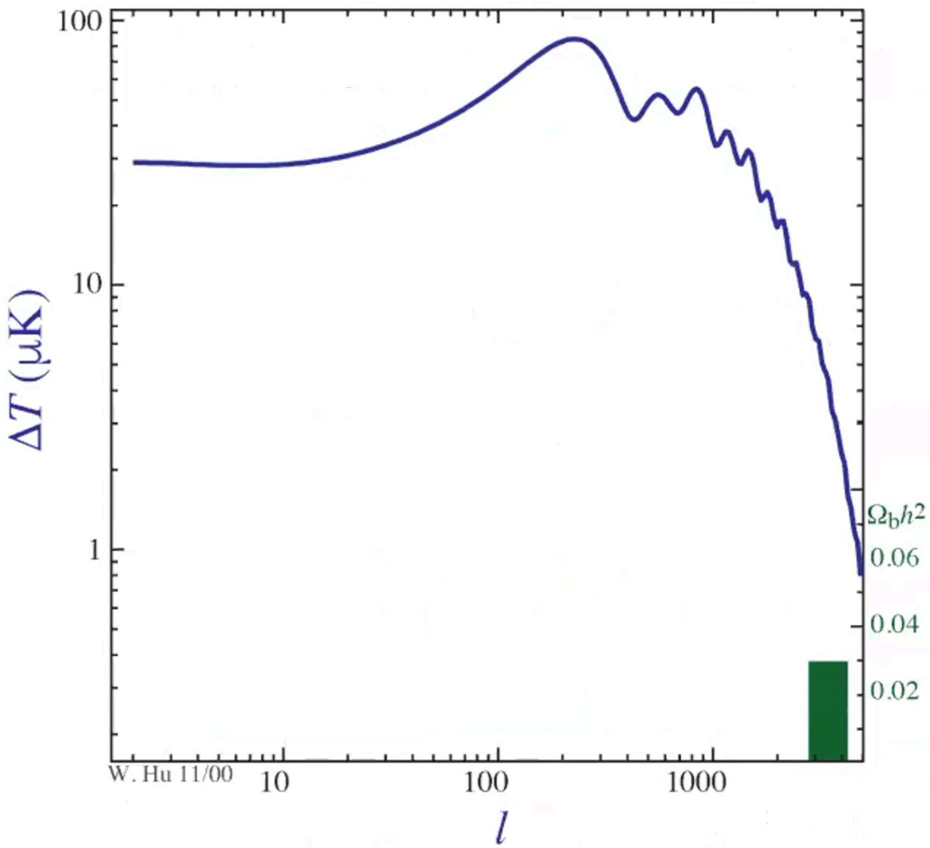
$$\lambda_D = \sqrt{N} \lambda_C = \sqrt{\eta / \lambda_C}\, \lambda_C = \sqrt{\eta \lambda_C}$$

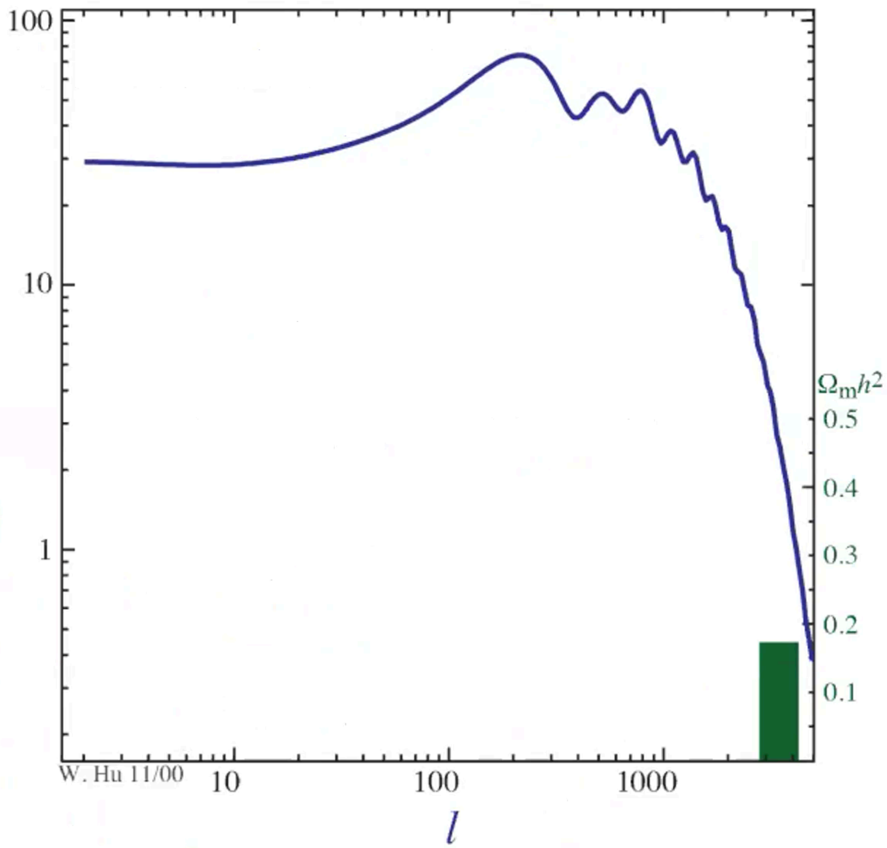
  the geometric mean between the horizon η and mean free path

- Comoving horizon scale $\eta = \int \frac{d \ln a}{H a}$: so CDM effect on the expansion $H^2 = \frac{8\pi G}{3}\rho$ changes damping scale relative to acoustic peak scale

CDM vs Baryons in the CMB

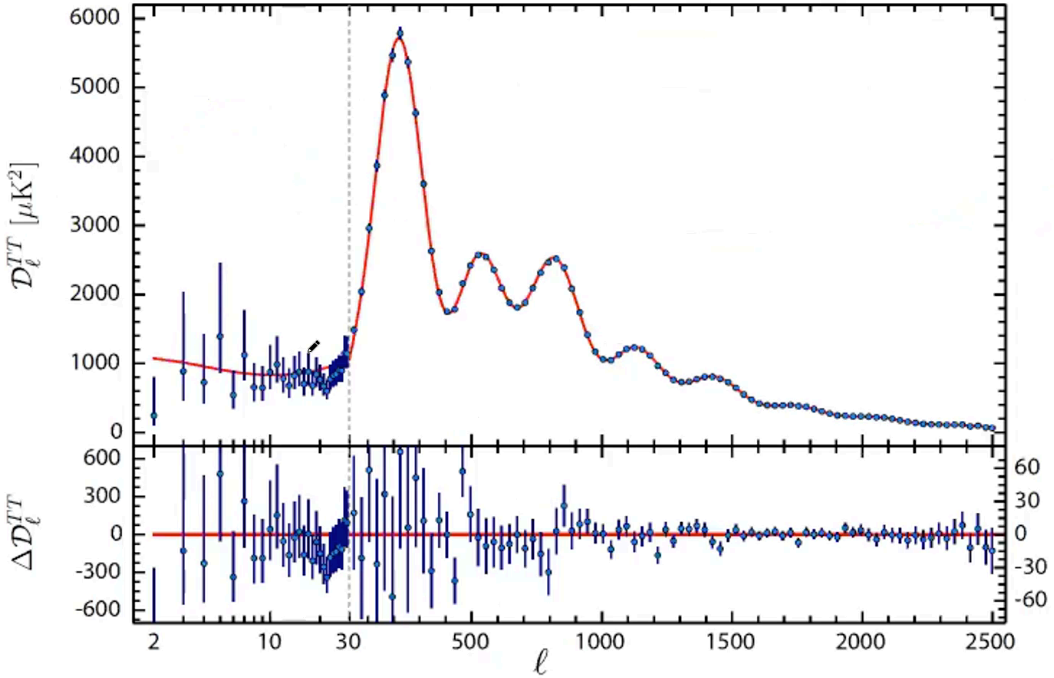
- Distinguishable effects of hydro equilibrium, forcing, damping





Planck Precision

- Planck 2018 parameter estimates:



Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54

Summary

- Three effects of CDM:

  Hydrostatic equilibrium: even-odd modulation measures $R\Psi$

  Radiation driving elimination: decrease in (undamped) peak amplitude measures matter radiation ratio: ρ_c/ρ_r

  Damping scale: change relative to sound horizon measures matter contribution to expansion rate H

- Cross checks and calibrates baryon density as well as fundamental assumption: only photons, neutrinos, baryons, CDM

  Passed with only $\sim 2\sigma$ "curiosities" (peaks slightly too smooth)

- What's the matter with the CMB?

  H_0 inference – requires robust calibration of the sound horizon

  Viable CMB H_0 explanations must be tuned to coincidentally mimic CDM behavior