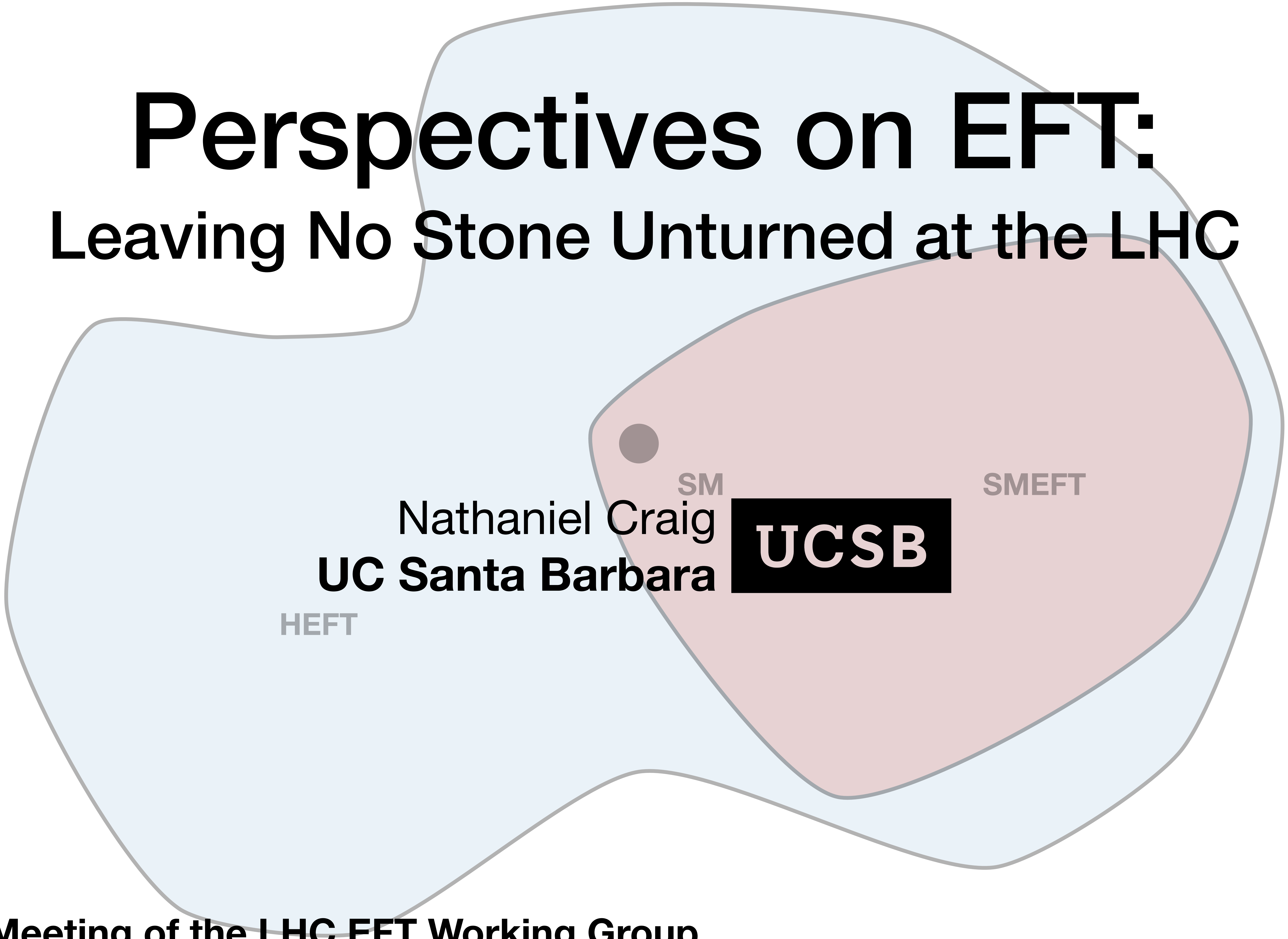
Perspectives on EFT:

Leaving No Stone Unturned at the LHC



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Disclaimers

(1) I would love to do justice to all of the amazing progress in LHC-relevant EFT in the last year(s), but in the interest of time will restrict my focus to two points. My apologies in advance, as there has been much progress!

(2) The purpose of this talk is not to suggest that the experimental collaborations stop doing any of their current (SM)EFT interpretations. It is only to point out ways in which we (especially theorists) might constructively broaden our horizons.

Part I: Which EFT?

(Apologies to Falkowski & Rattazzi for borrowing their title)

Higgs EFTs

SM ***SU(2)LxU(1)Y***

$$(D_\mu H)^\dagger (D^\mu H) - m^2 |H|^2 - \lambda |H|^4$$

HEFT* ***U(1)em***

[Feruglio '93, Bagger et al. '93, …]

$$\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left[vF(h/v)\right]^2 (\partial \vec{n})^2 - V(h) + \ldots$$

SMEFT ***SU(2)LxU(1)Y***

[Weinberg '79, Buchmuller, Wyler '86, …]

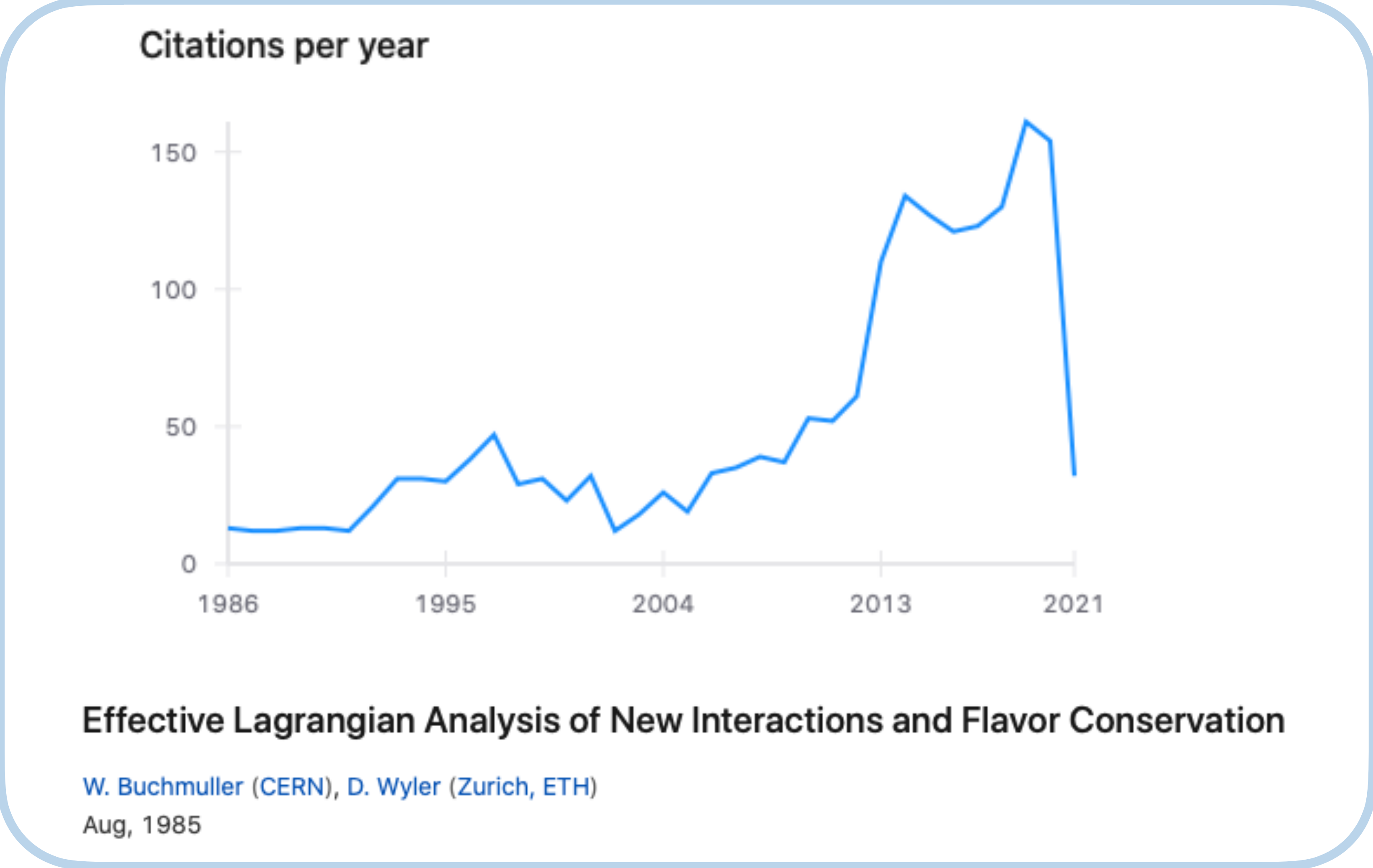
$$(D_\mu H)^\dagger (D^\mu H) - m^2 |H|^2 - \lambda |H|^4 + \frac{c_H}{2\Lambda^2}(\partial_\mu |H|^2)^2 + \frac{c_6}{\Lambda^2}|H|^6 + \ldots$$

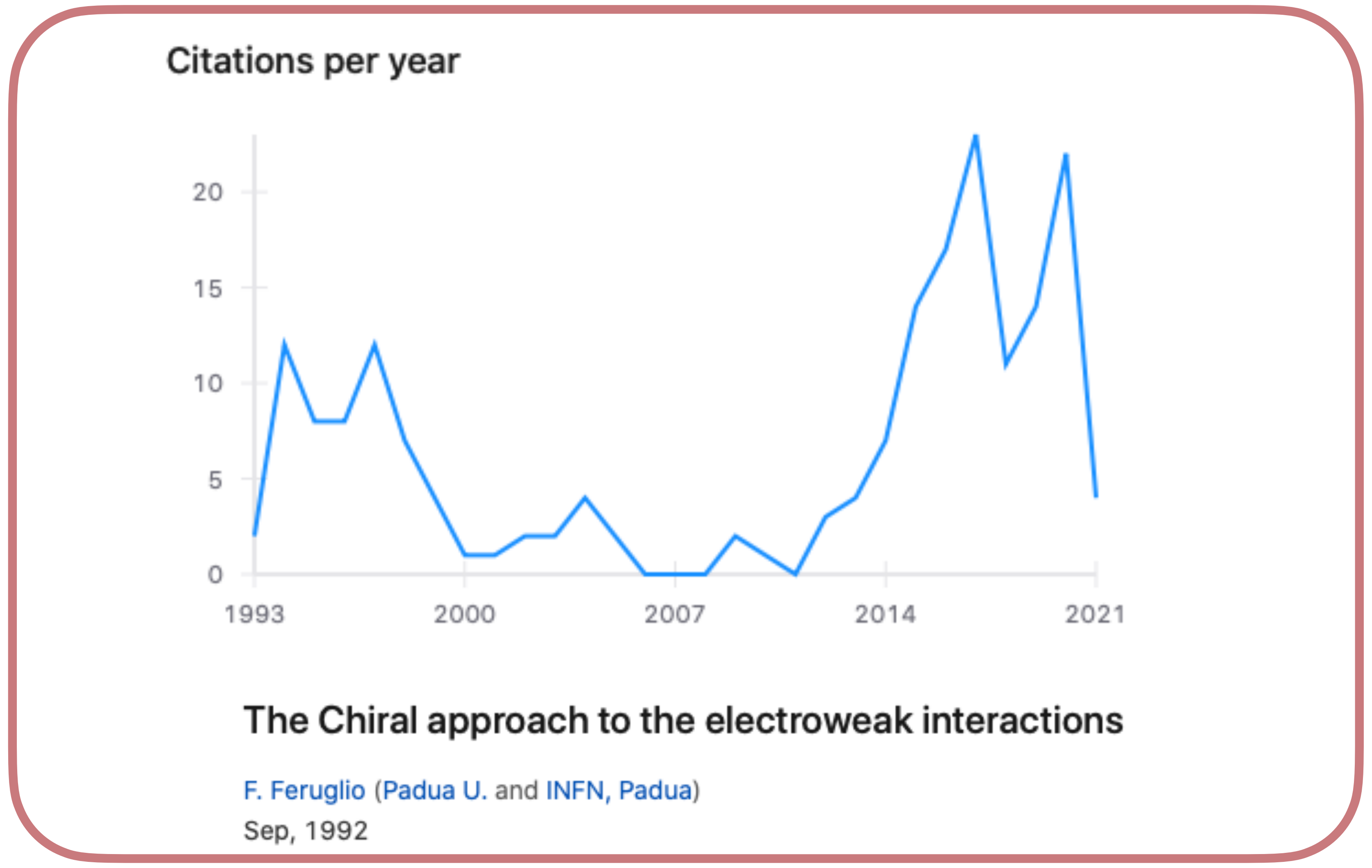
*Alternately, "Higgs-Electroweak Chiral Lagrangian", …

Comprehensive review: [Brivio, Trott 1703.10924]

Which EFT?

Vastly more progress in SMEFT since c. 2012 (precision, fits, projections, theorems,…)





Seems justified: SU(2)xU(1) an apparently good symmetry, no O(1) deviations or custodial symmetry violation

As far as I can tell, SMEFT is the preferred EFT framework for the LHC EFT WG.

(When) Is HEFT necessary?

See also: [Burgess, Matias, Pospelov '99; Grinstein & Trott '07; Alonso, Gavela, Merlo, Rigolin, Yepes '12; Espriu, Mescia, Yencho '13; Buchalla, Cata, Krause '13; Brivio et al. '13; Chang & Luty '19; Falkowski & Rattazzi ' 19; Abu-Ajamieh, Chang, Chen, Luty '20]

On-shell perspective: [Durieux, Kitahara, Shadmi, Weiss '19]

The Standard Model EFT

SMEFT: EFT where 4 scalar d.o.f. are arranged into an SU(2) doublet (equivalently, O(4) fundamental; assuming custodial symmetry):

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$

“Electroweak symmetry is linearly realized.”

$$\mathcal{L}_{\mathrm{SM}} = \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\mathcal{L}_{\mathrm{SMEFT}} = \frac{1}{2}A(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - V\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right)$$

Here and henceforth: assuming custodial symmetry & only worrying about scalars up to 2 derivatives…

The Higgs EFT

Alternately, HEFT:
construct EFT out of
singlet *h* and Goldstones π_i

No presumed relation between h, π

$$h \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}$$

$$h \to h\,, \qquad \vec{n} \to O\vec{n}\,, \qquad O \in O(4)$$

"Electroweak symmetry is nonlinearly realized."

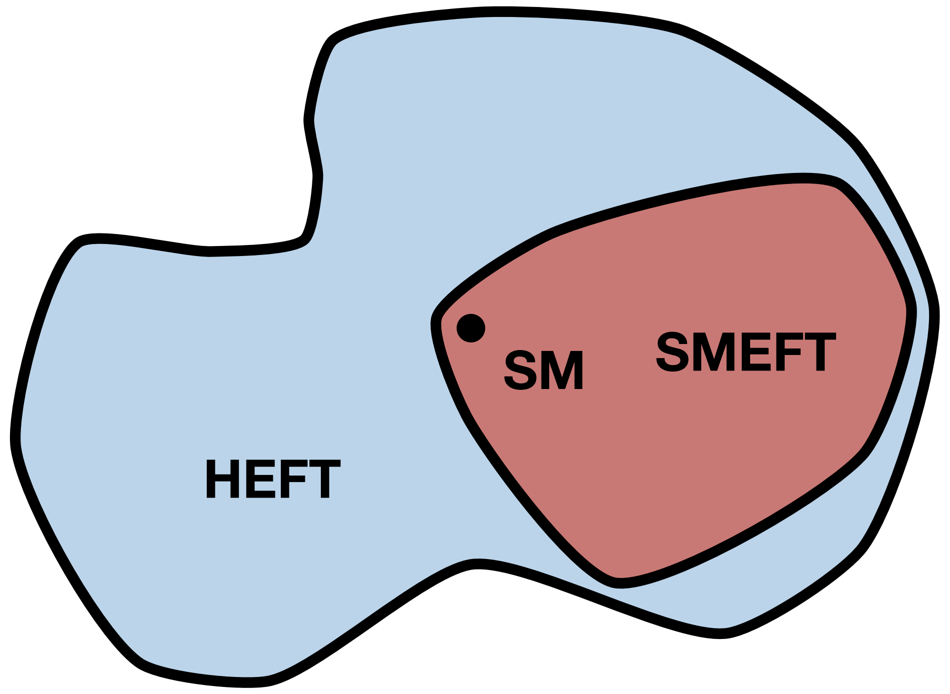
$$\mathcal{L}_{\rm SM} = \frac{1}{2}\left(\partial h\right)^2 + \frac{1}{2}(v+h)^2\left(\partial \vec{n}\right)^2 - \frac{1}{4}\lambda(h^2 + 2vh)^2$$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial \vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

(*K(h)* redundant, conventional to redefine h to set *K(h)* = 1)

SM ⊂ SMEFT ⊂ HEFT

[R. Alonso, E. Jenkins, A. Manohar 1511.00724 & 1605.03602]



Relate the two by field redefinition: $\vec{\phi} = (v+h)\,\vec{n}\,(\pi)\,; \quad \vec{\phi}\cdot\vec{\phi} = (v+h)^2$

SMEFT can always be written as HEFT:

$$\mathcal{L} = \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)$$
$$= \frac{1}{2}\left[A + (v+h)^2 B\right](\partial h)^2 + \frac{1}{2}(v+h)^2 A(\partial\vec{n})^2 - V$$

Correlations at every order between h, v

HEFT cannot always be written as SMEFT:

$$\mathcal{L} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$
$$= \frac{1}{2}\frac{v^2F}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi})^2 + \frac{1}{2}(\vec{\phi}\cdot\partial\vec{\phi})^2\frac{1}{\vec{\phi}\cdot\vec{\phi}}\left(K^2 - \frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}\right) - \tilde{V}(\vec{\phi}\cdot\vec{\phi})$$

Generically non-analytic at the origin

What defines the HEFTs that cannot be written as SMEFTs?
What is the UV physics that produces them?

A Geometric Perspective

Field redefinitions make it impossible to identify these "pure HEFTs" by inspection.
Instead: classify EFTs based on geometry, 2-derivative terms define a metric on the scalar manifold

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial \phi^i \partial \phi^j - V(\phi)$$

Field space corresponds to a (possibly curved) manifold with functions (e.g. V) defined on it; the field parameterization corresponds to charts on the manifold. Use geometric invariants to classify EFTs.

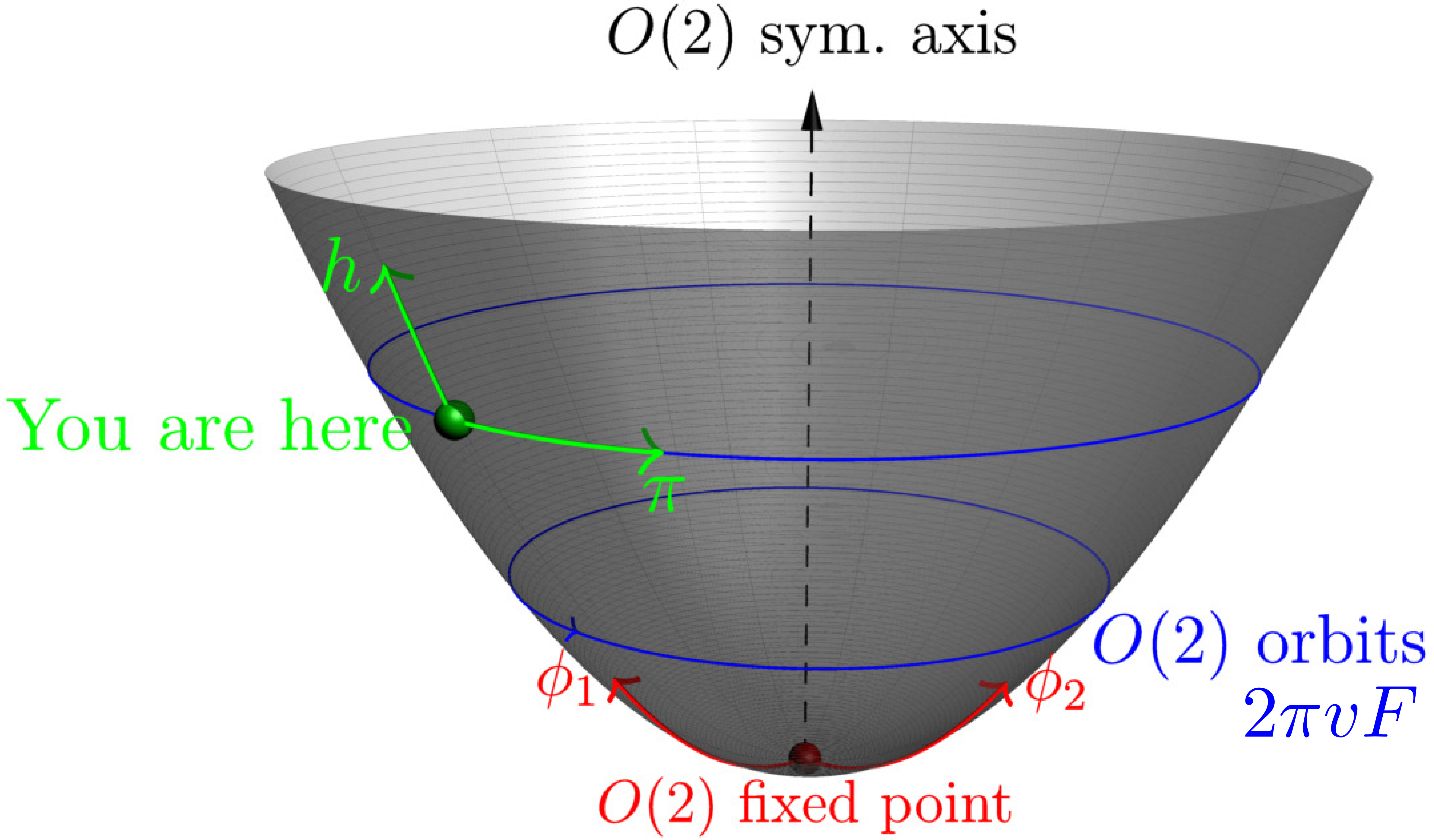
Long history (primarily) applied to nonlinear sigma models, e.g.
[Honerkamp '72; Tataru '75; Alvarez-Gaume, Freedman, Mukhi '81, …]

Application to SMEFT/HEFT: [Alonso, Jenkins, Manohar 1511.00724 & 1605.03602]
(Applied within SMEFT: [Helset, Martin, Trott 2001.01453])

SM: flat manifold **HEFT:** curved manifold **SMEFT:** curved manifold w/ O(4) invariant point

A Geometric Perspective

(Think O(4), but O(2) is easier to illustrate)

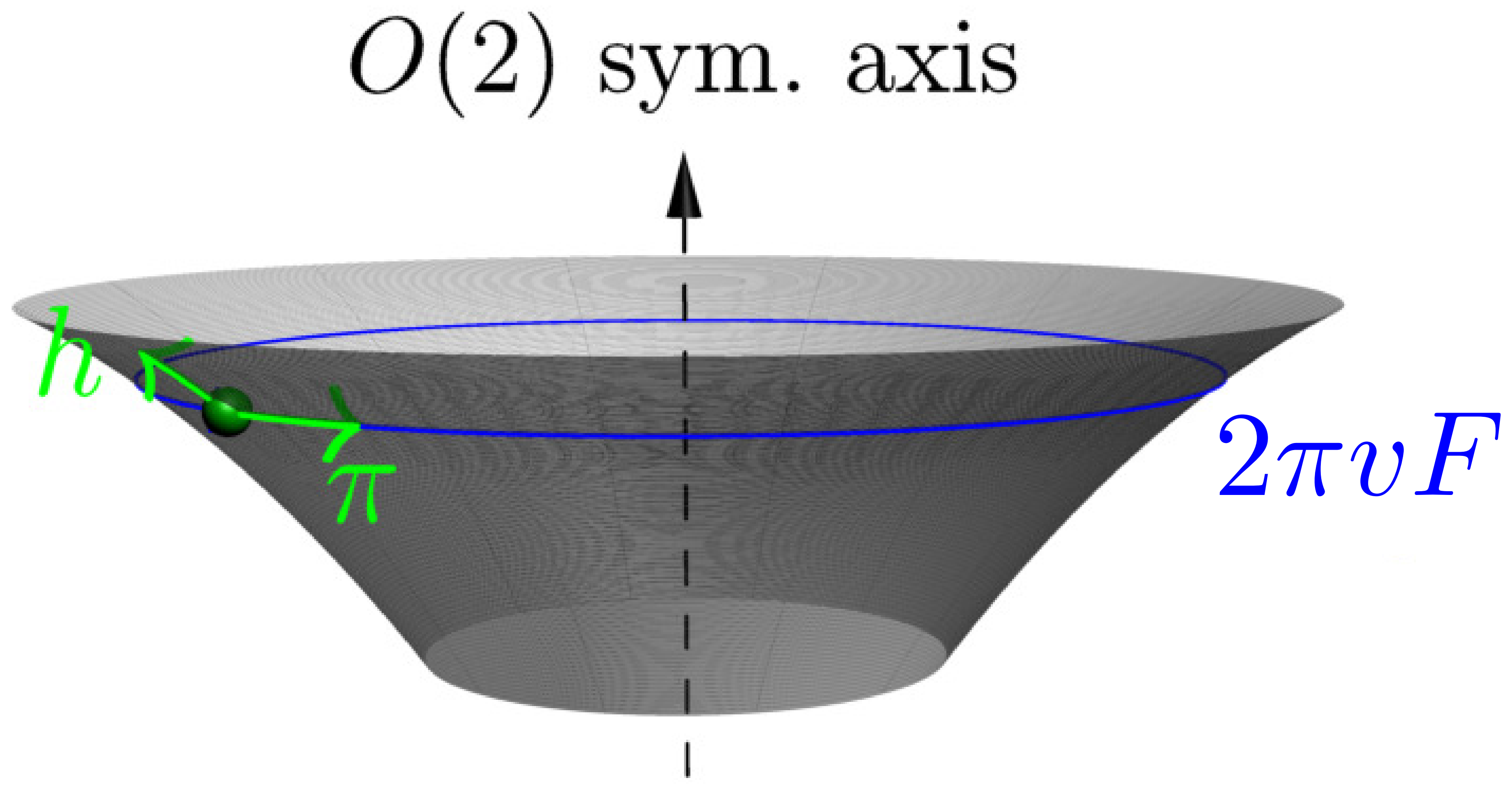


$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

SMEFT if O(4) fixed point on manifold → F(h) = 0 somewhere (say, *h* = -*v*, *i.e. H*=0)

HEFT not SMEFT

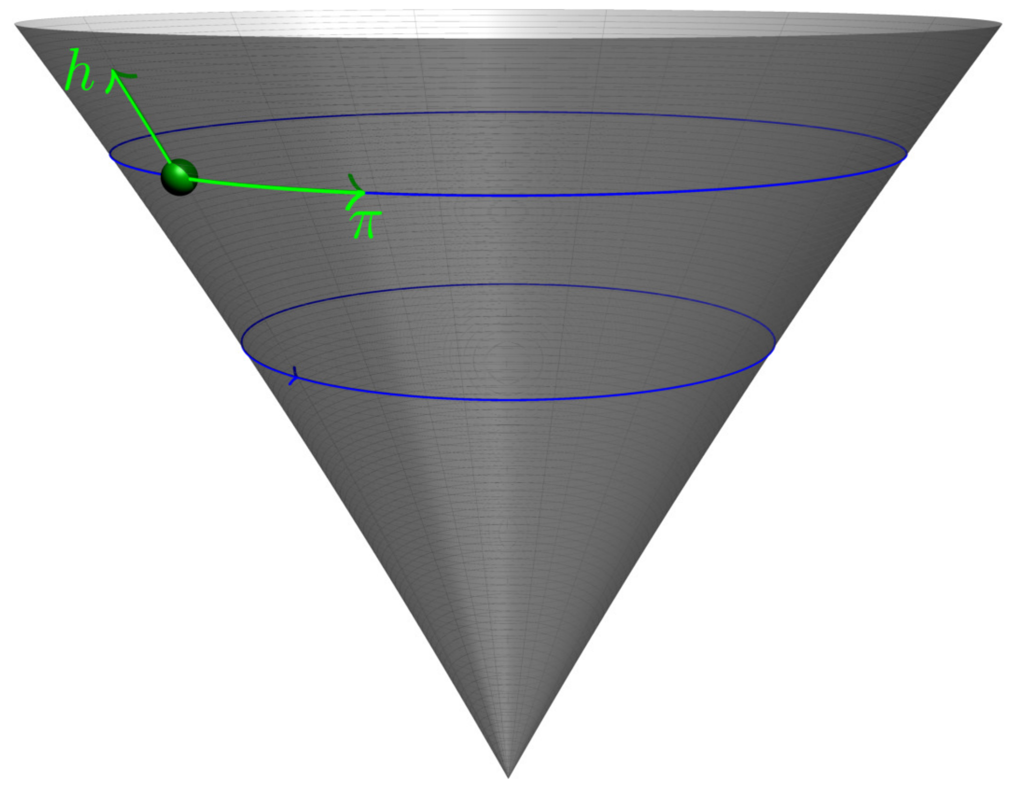
Case I: When there's a hole s.t. $h = -v$ is not on the manifold (no O(4) fixed point about which to expand in SMEFT coordinates). *Arises when UV physics also breaks the symmetry.* [Alonso, Jenkins, Manohar 1605.03602]



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

Corresponds to $F(h) \neq 0$ **everywhere**

Case II: When there's a cone or cusp at h=-v. *Arises when a field becomes massless.* [Cohen, NC, Lu, Sutherland 2008.08597]



Can diagnose singularities as in GR:

If $(\nabla^2)^n R$ and $(\nabla^2)^{n+1} V$

are finite at h=-v, then can write HEFT as SMEFT (gives the requisite infinite set of conditions!)

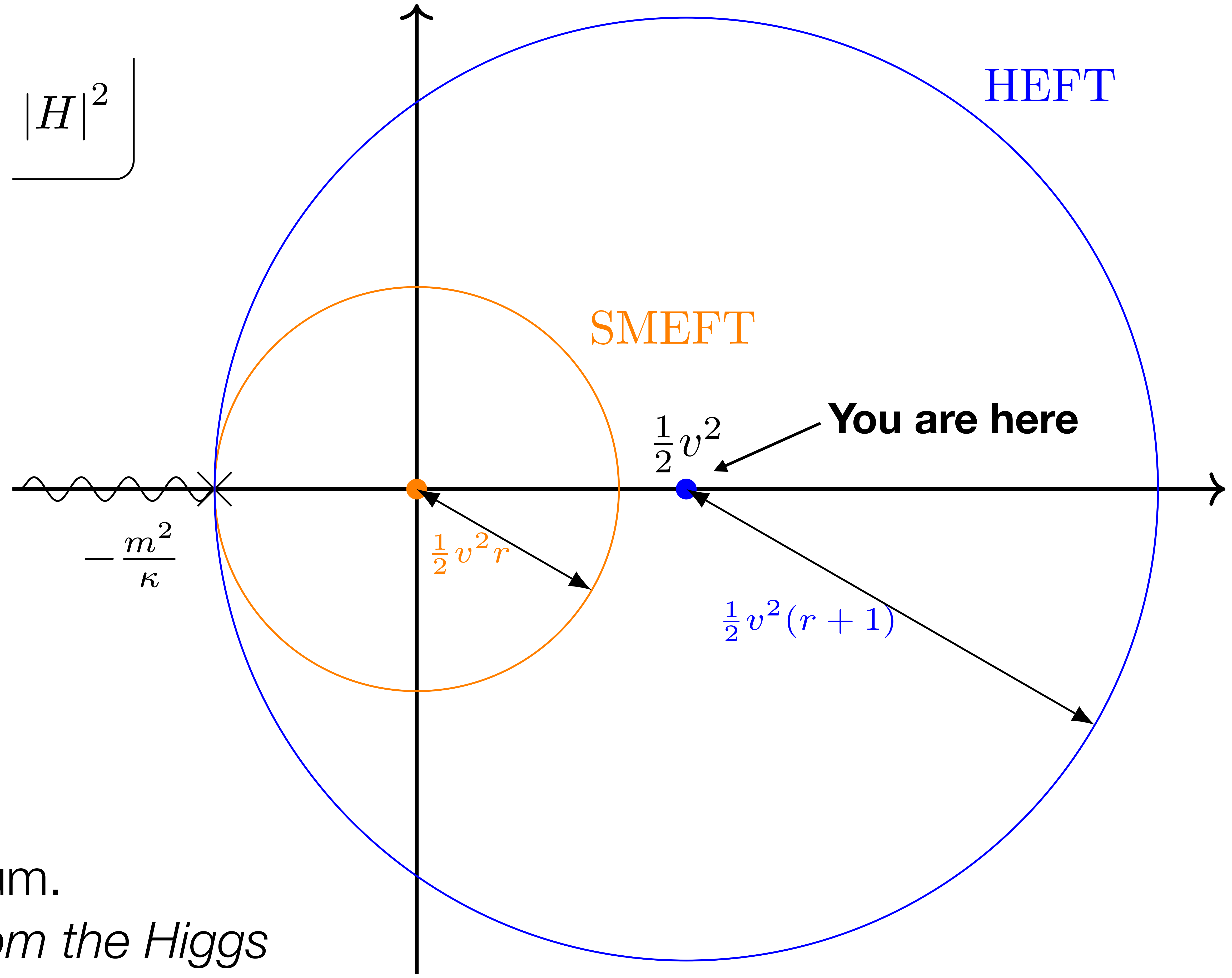
SMEFT Convergence

Even when SMEFT exists @ h=-v, the SMEFT expansion may not converge at our vacuum h=0.

For example: a singlet getting some mass from the Higgs via cross-quartic $\kappa S^2|H|^2/2$. Integrate it out, study analytic structure of the effective Lagrangian in the complex |H|² plane

$$r \equiv \frac{\text{bare mass}^2}{\text{mass}^2 \text{ from Higgs}} = \frac{m^2}{\frac{1}{2}\kappa v^2}$$

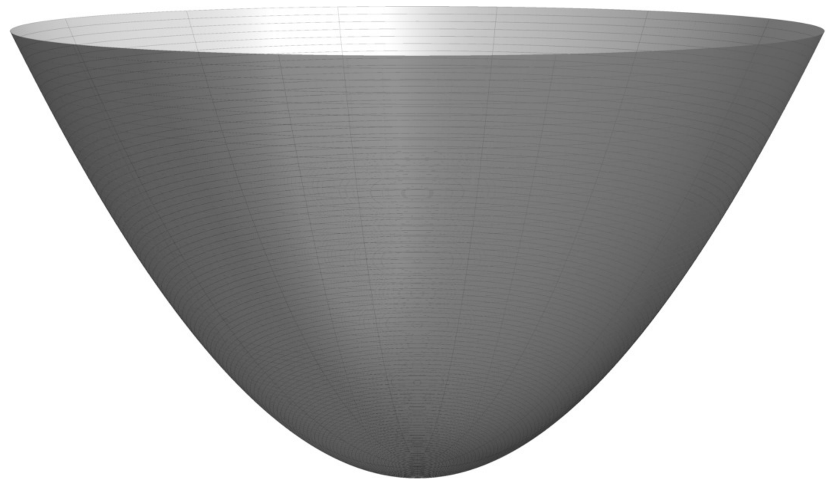
Branch cut at $|H|^2 = -m^2/\kappa \Rightarrow$
SMEFT radius of convergence is $v^2 r/2$
HEFT radius of convergence is $v^2(r+1)/2$



$r < 1$: SMEFT expansion does not converge at our vacuum.
HEFT required by states w/ more than half of their mass from the Higgs

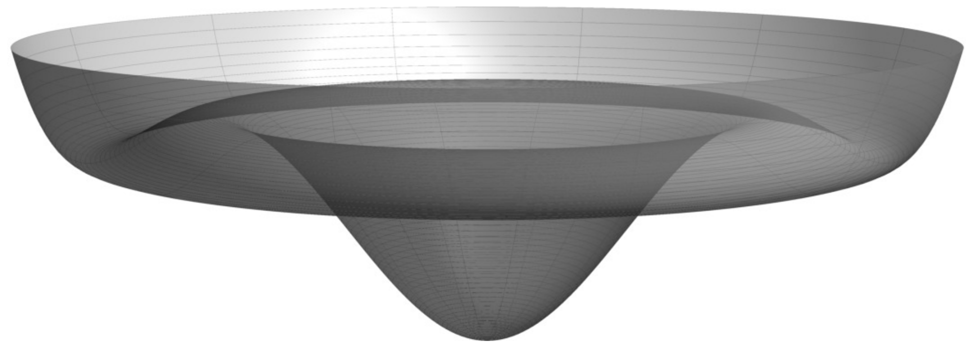
SMEFT Convergence

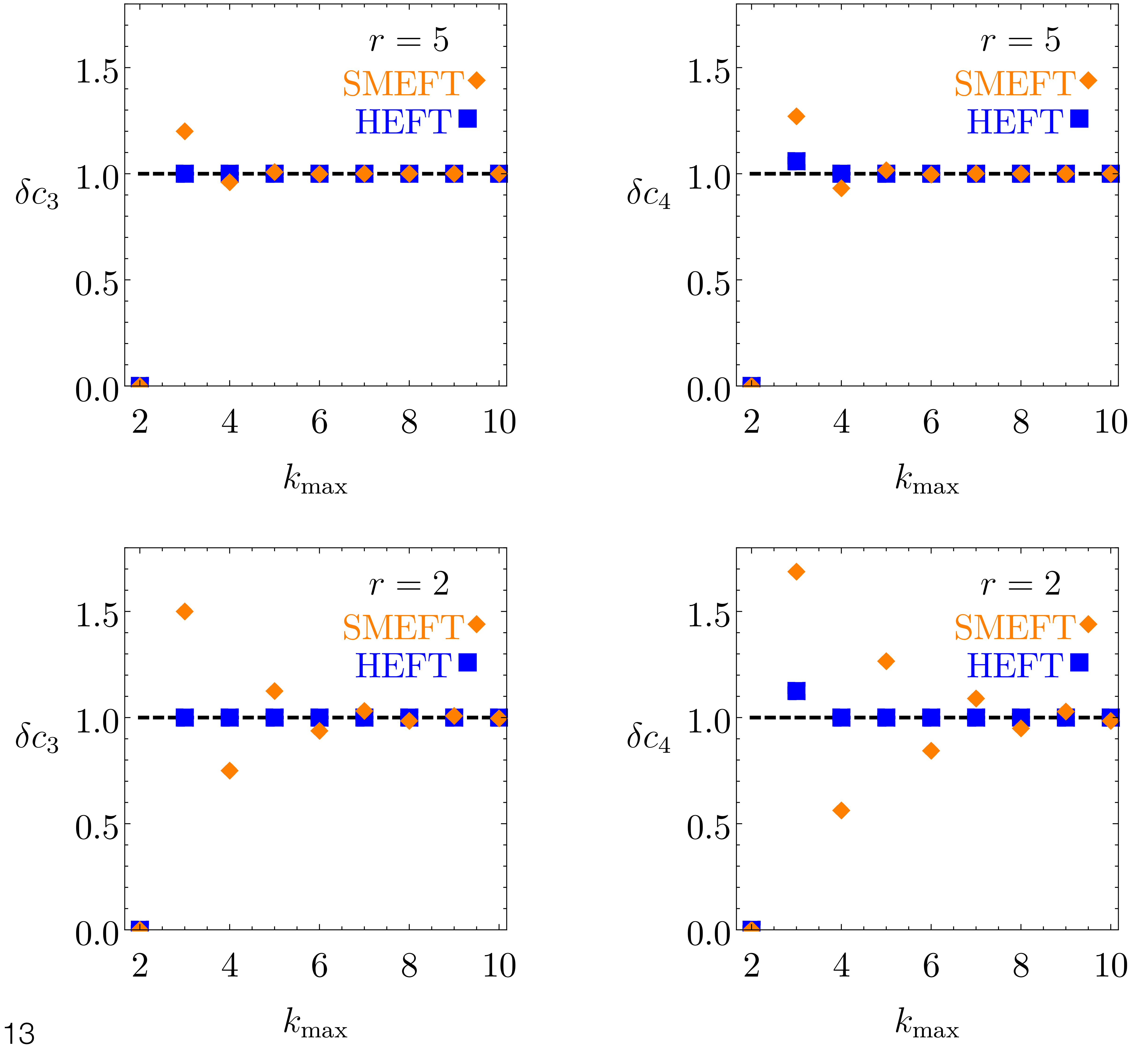
Even for $r \gtrsim 1$, HEFT can capture true corrections to SM using fewer terms in the relevant expansion than SMEFT.



A HEFT interpretation may allow faster identification of the underlying physics.

"v-improved matching" [Englert et al. 1403.7191; Brehmer et al. 1510.03443] in some sense matching in the broken phase (HEFT) and then converting to SMEFT coordinates.





Loryons*

*Following Gell-Mann, from *Finnegan's Wake*: "with Pa's new heft...see Loryon the comaleon."

HEFT required whenever a new particle ("Loryon") acquires more than half of its mass from the Higgs.

Many such Loryons viable, consistent with all existing data.
([Banta, Cohen, NC, Lu, Sutherland *to appear*], see also [Bonnefoy et al. 2011.10025])
Most likely to show up first in SM measurements / EFT fits.

The TL;DR

- There are many ways to start with an $SU(2)_L x U(1)_Y$ symmetric theory in the UV and end up with an infrared EFT that respects only $U(1)_{em}$ (HEFT), without doing great violence to precision electroweak physics, etc.

- Many of these ways are consistent with all known data.

- Many of these would first show up in indirect evidence (e.g. EFT fits), so it would be prudent to be prepared.

- Even new physics that admits SMEFT may be better fit by HEFT.

- Focusing exclusively on SMEFT is a **strong assumption** that is *not remotely justified by our current state of knowledge*.

But doesn't HEFT give up all the correlations?

SMEFT beloved for correlations. But HEFT deviations still spoil SM cancellations in many amplitudes. Measurements still connected, if not by $SU(2)_L \times U(1)_Y$

In pure HEFTs, unitarity always violated by ~$4\pi v$ in suitable channels (often not $2 \to 2$), even when coefficient is tiny. [Falkowski & Rattazzi 1902.05936; Chang & Luty 1902.05556; Abu-Ajamieh, Chang, Chen, Luty 2009.11293; Cohen, NC, Lu, Sutherland, *to appear*]

Still a strong link between Higgs measurements & high-energy behavior (a la "Higgs w/out Higgs" [Henning, Lombardo, Riembau, Riva 1812.09299])

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \delta_3 \frac{m_h^2}{2v} h^3 - \delta_4 \frac{m_h^2}{8v^2} h^4 - \sum_{n=5}^{\infty} \frac{c_n}{n!} \frac{m_h^2}{v^{n-2}} h^n + \cdots$$

$$+ \delta_{Z1} \frac{m_Z^2}{v} h Z^\mu Z_\mu + \delta_{W1} \frac{2m_W^2}{v} h W^{\mu+} W^-_\mu + \delta_{Z2} \frac{m_Z^2}{2v^2} h^2 Z^\mu Z_\mu + \delta_{W2} \frac{m_W^2}{v} h^2 W^{\mu+} W^-_\mu$$

$$+ \sum_{n=3}^{\infty} \left[\frac{c_{Zn}}{n!} \frac{m_Z^2}{v^n} h^n Z^\mu Z_\mu + \frac{c_{Wn}}{n!} \frac{2m_W^2}{v^n} h^n W^{\mu+} W^-_\mu \right] + \cdots$$

$$- \delta_{t1} \frac{m_t}{v} h \bar{t} t - \sum_{n=2}^{\infty} \frac{c_{tn}}{n!} \frac{m_t}{v^n} h^n \bar{t} t + \cdots$$

Process	$\times \frac{E^4}{1152\pi^3 v^4}$
$hZ^2 \to hZ^2$	$[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2}c_{V3}]$
$h^2 Z \to Z^3$	$-\frac{\sqrt{3}}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2}c_{V3}]$
$h^2 W^+ \to Z^2 W^+$	$-\frac{1}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2}c_{V3}]$
$h^2 Z \to Z W^+ W^-$	$-\frac{1}{\sqrt{2}}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2}c_{V3}]$
$h^2 W^+ \to W^+ W^- W^+$	$-[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2}c_{V3}]$
$hZW^+ \to hZW^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{Vc}]$
$hW^+W^+ \to hW^+W^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{V3}]$
$hW^+W^- \to hW^+W^-$	$-[28\delta_{V1} - 9\delta_{V2} + c_{V3}]$
$hZ^2 \to hW^+W^-$	$-\sqrt{2}[32\delta_{V1} - 11\delta_{V2} + \frac{3}{2}c_{V3}]$

Process	$\times \frac{\left(\frac{1}{2}c_{t2} - \delta_{t1}\right) m_t E^2}{32\pi^2 v^3}$
$\bar{t}_R t_R \to Zh^2$	$i\sqrt{N_c}$
$h^2 \to Z\bar{t}_L t_L$	$i\sqrt{\frac{N_c}{3}}$
$Zh \to h\bar{t}_L t_L$	$i\sqrt{\frac{2N_c}{3}}$
$t_R Z \to t_L h^2$	$\frac{i}{\sqrt{6}}$
$t_R h \to t_L Zh$	$\frac{i}{\sqrt{3}}$
$\bar{t}_R t_R \to Z^2 h$	$-\sqrt{N_c}$
$Z^2 \to \bar{t}_L t_L h$	$-\sqrt{\frac{N_c}{3}}$
$Zh \to \bar{t}_L t_L Z$	$-\sqrt{\frac{2N_c}{3}}$
$t_R h \to t_L Z^2$	$-\frac{1}{\sqrt{6}}$
$t_R Z \to t_L Zh$	$-\frac{1}{\sqrt{3}}$

[Abu-Ajamieh, Chang, Chen, Luty 2009.11293]

Our Mission

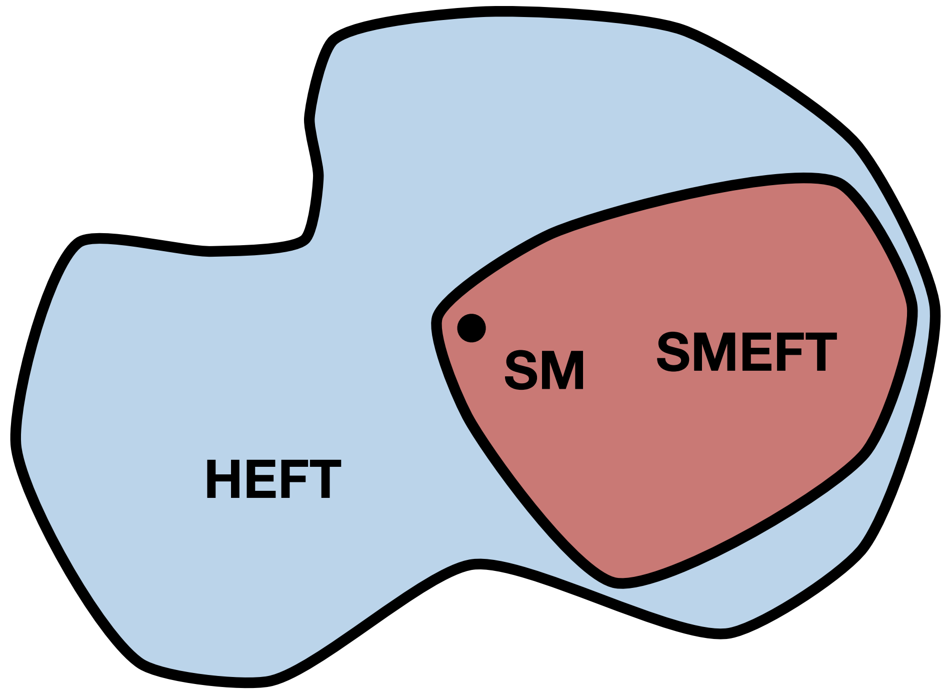
Should we choose to accept it…

To answer this question:

“Is electroweak symmetry linearly realized by the known fundamental particles?”

Equivalently: can we rule out pure HEFTs?

- *It is a sharply defined, bounded question.*
- *We don’t currently know the answer.*
- *We might be able to find out @ the LHC.*
- *Null results (agreement w/SM) only help.*



Top-down: rule out the perturbative scenarios forcing HEFT (less satisfying)

Bottom up: “check unitarity in a complete set of channels up to $4\pi v$” (specifics TBD)

This is a “big” question that we can potentially answer even if the LHC sees no departures from SM.

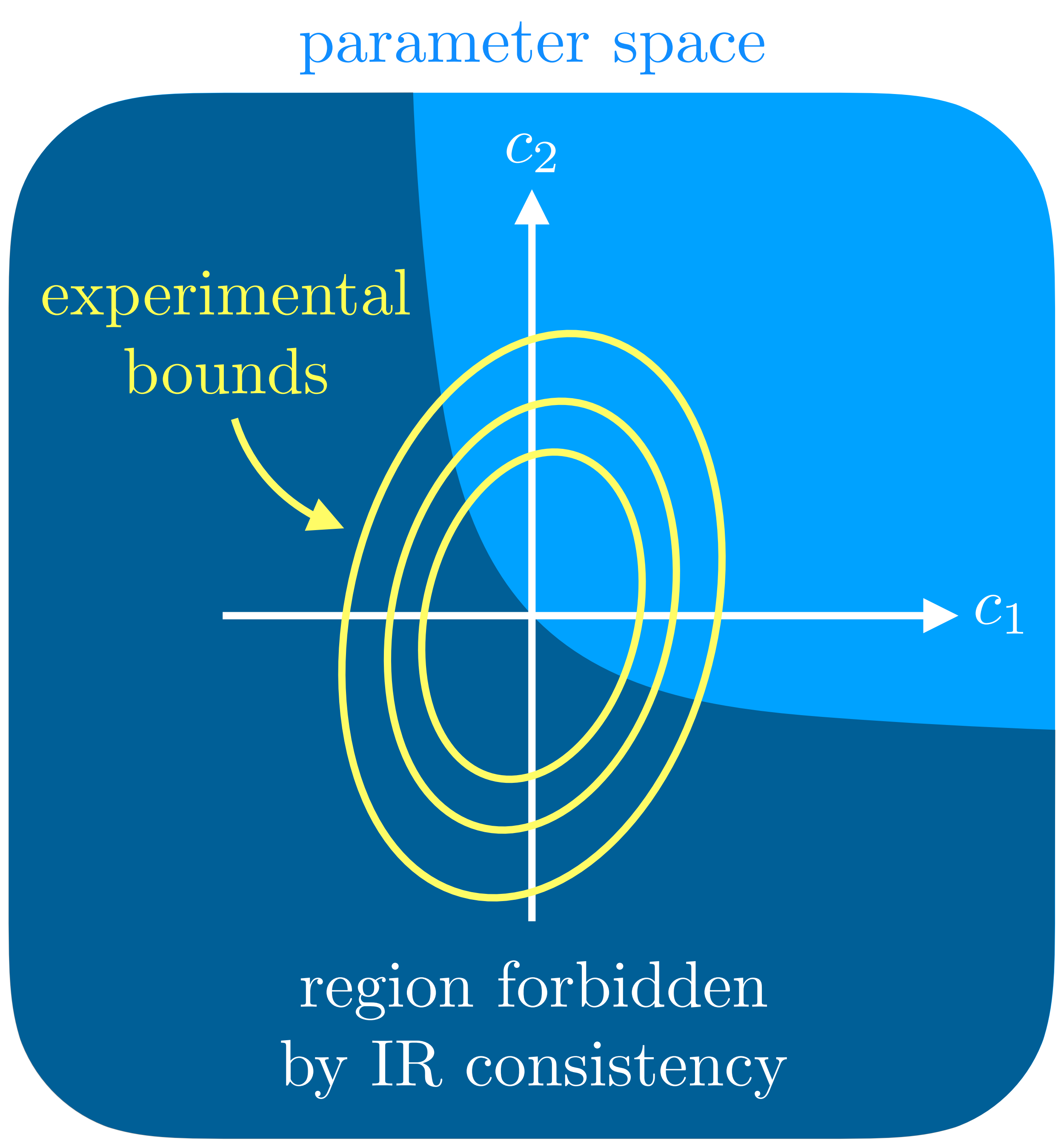
Part II: Thinking Positively

Thinking Positively

Causality, **unitarity**, and **analyticity** constrain EFT corrections to SM (“positivity bounds”)

Long history, revived in [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi hep-th/0602178; Distler, Grinstein, Porto, Rothstein hep-ph/0604255; …]

More recently: extensive application directly to Wilson coefficients in SMEFT, e.g. [Bellazzini, Riva 1806.09640; Zhang, Zhou 1808.00010; Bi, Zhang, Zhou 1902.08977; Remmen, Rodd 1908.09845; Remmen, Rodd, 2004.02885; Zhang, Zhou 2005.03047; Fuks, Liu, Zhang, Zhou 2009.02212; Yamashita, Zhang, Zhou 2009.04490; Remmen, Rodd 2010.04723; Gu, Wang, Zhang 2011.03055; Trott 2011.10058; Bonnefoy, Gendy, Grojean 2011.12855; Li, Yang, Xu, Zhang, Zhou 2101.01191, …]



[Remmen & Rodd, 1908.09845]

Improve global fits by imposing positivity bounds

OR

Interpret as experimental tests of bedrock principles of QFT.

(Ideally do both)

Thinking Positively

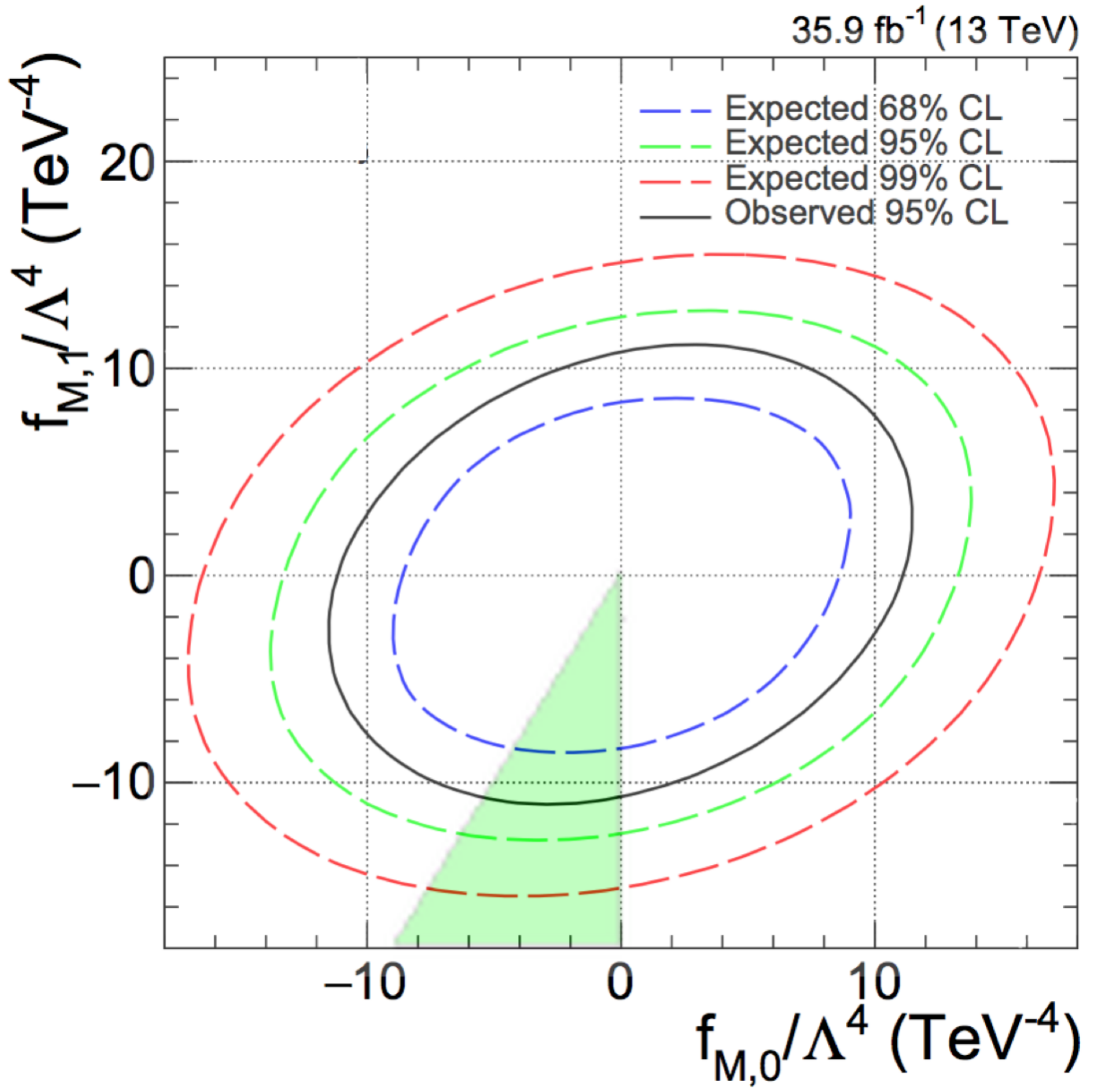
d=6: UV-sensitive positivity bounds, sum rules. **d=8**: UV-insensitive positivity bounds

Naive expectation: dim-8 operator effects always subleading

Reality: often leading effect due to non-interference thms and more pragmatic non-interference effects (color, phase space, …)

Thus far: primarily applied to aQGCs @ LHC
[e.g. C. Zhang, S-Y. Zhou, 1808.00010 et seq.]

Not yet fully understood: space of observables where dim-8 operators provide leading effects at LHC, prospects for constraints? Powerful opportunity for theory-experiment interplay.



[Bi, Zhang, Zhou 1902.08977]

Related: lack of positivity bounds @ dim-6 → null results don't preclude new particles below corresponding scale. Positivity bounds @ dim-8 robustly connect null results to confirmation of SM.

Conclusions

- *Many* ways to get $U(1)_{em}$ Higgs EFT starting from SU(2)xU(1) symmetry in the UV, consistent w/ data. HEFT can be the preferred EFT for data even when both HEFT & SMEFT expansions valid.

- Motivates giving HEFT more thorough attention, both as a theory and as an interpretation. Plethora of structural questions currently being explored in SMEFT can also be addressed in HEFT.

- There is a new "big" question we should ask, which to my knowledge is not being systematically explored: "Is electroweak symmetry linearly realized by the known fundamental particles?"

- Considerable progress in positivity bounds offers new "tests" of bedrock principles of QFT. Motivates looking for LHC measurements where dim-8 dominates, rather than just treating positivity bounds as subleading input to fits.

Thank you!