Some special EM physics topics:

II. Some notes on charged particle stepping

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Geneva (Switzerland), 23 March 2021

- Some special EM physics modelling topic to be discussed:
  - I. using secondary production thresholds (last time)
  - II. the corresponding stepping and its parameters (today)
  - III. multiple Coulomb scattering (next weeks)

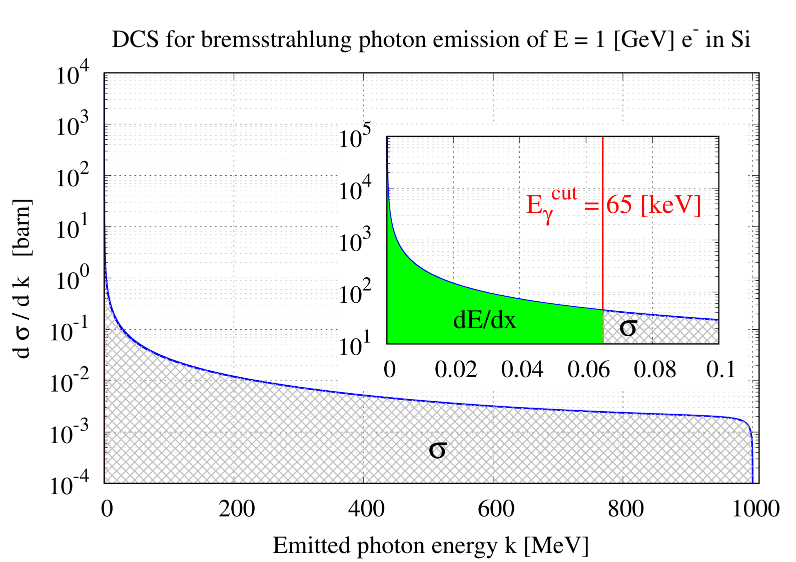
- There are some ongoing optimisations in which these might help
- We also have some pending issues (regarding the “cuts”, MSC)
- After some discussions with Marilena, it seemed to be a good idea to start working on these together by arranging these meetings
- These informal discussions might also help to tighten our collaboration

- II. Some notes on charged particle stepping (today)
  - **Continuous step limit** due to energy loss? Why?
  - How it is incorporated in `Geant4`. Why? Its parameters.
  - Relation to tracking cut or `G4UserLimits`.
  - How energy loss affects the **discrete** part of **the step limit**.
  - Some notes on the `G4GammaGeneralProcess`.

CONTINUOUS STEP LIMIT DUE TO ENERGY LOSS

Continuous step limit due to energy loss

- **Introduce secondary photon production threshold:**
  - ***secondary photons*, with** initial **energy below** a gamma production **threshold(k<E_γ^{cut}**), are **not generated**
  - ***E_γ^{cut}*** *is the* ***energy of the photon*** *that has* ***its absorption length equal to the cut (r****, i.e. the radius of the sphere)*
  - the corresponding **energy** (*that would have been taken away from the primary, but remained inside the sphere*) is **accounted as *CONTINUOUS* energy loss of the primary** particle **along its trajectory**
  - described by the **radiative** contribution of the (**restricted**) **stopping power (dE/dx)**: mean energy loss due to sub-threshold photon emissions in unit (path) length
  - i.e. when an electron makes a step with a given length ***L***, one can compute the **mean energy loss** (due to sub-threshold photon emissions) along the step as ***L*** x ***dE/dx*** (would be true only if *E = const* along the step)
  - ***secondary photons*, with** initial **energy above** a gamma production **threshold(k>E_γ^{cut})**, are **generated** (*since this energy is deposited outside of the sphere*) (***DISCRETE***)
  - the emission rate is determined by the corresponding (**restricted**) cross section(**σ**)



$$\frac{\mathrm{d}E}{\mathrm{d}x}(E, E_\gamma^{\mathrm{cut}}, Z) = \mathcal{N} \int_0^{E_\gamma^{\mathrm{cut}}} k \frac{\mathrm{d}\sigma}{\mathrm{d}k}(E, Z)\mathrm{d}k$$

$$\sigma(E, E_\gamma^{\mathrm{cut}}, Z) = \int_{E_\gamma^{\mathrm{cut}}}^{E} \frac{\mathrm{d}\sigma}{\mathrm{d}k}(E, Z)\mathrm{d}k$$

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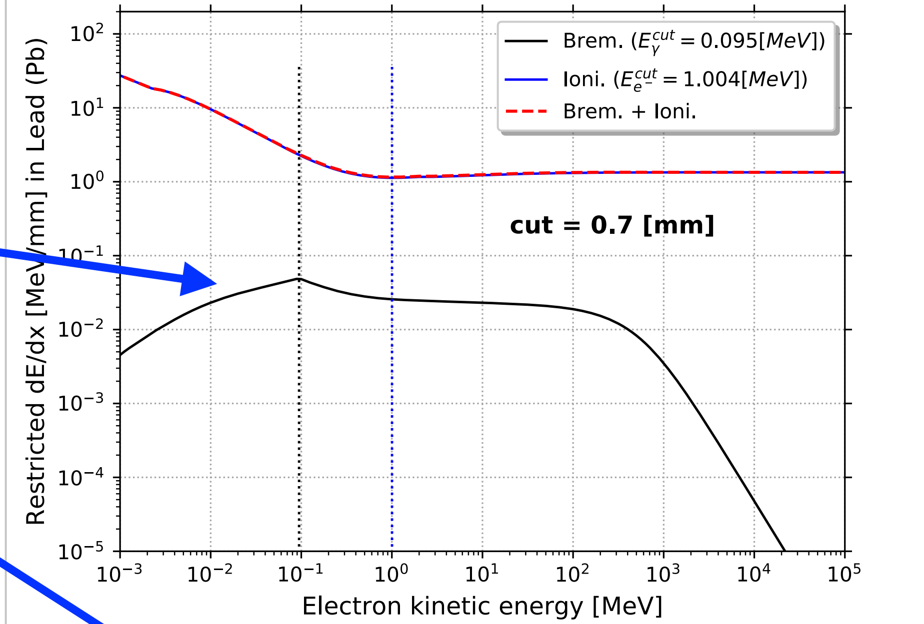

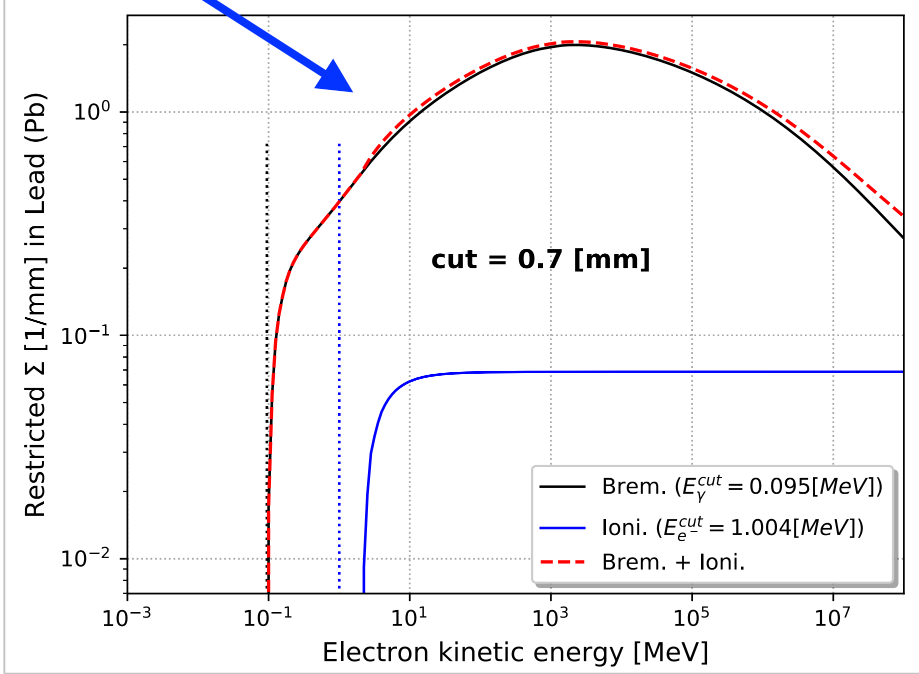

- **Note**
  - all quantities are "***restricted***" (covers only the sub/super secondary production threshold part of the interaction)
  - in case of ionisation, the minimum primary e^- energy to be able to produce secondary e^- with initial energy > $E_{e^-}^{cut}$ is 2x $E_{e^-}^{cut}$ (due to the indistinguishable two e^- at the final state; the one with the lower energy is considered to be the secondary e^-)

Continuous step limit due to energy loss

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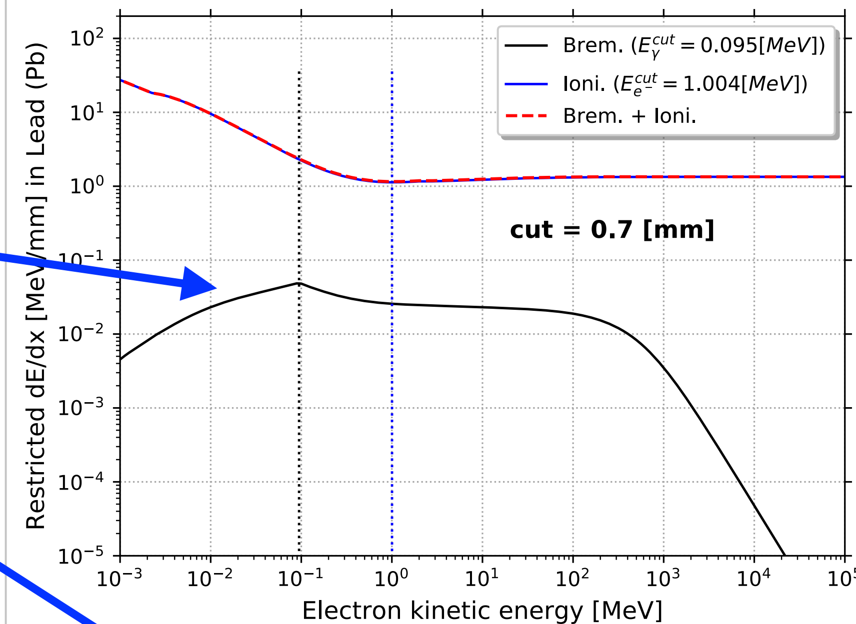
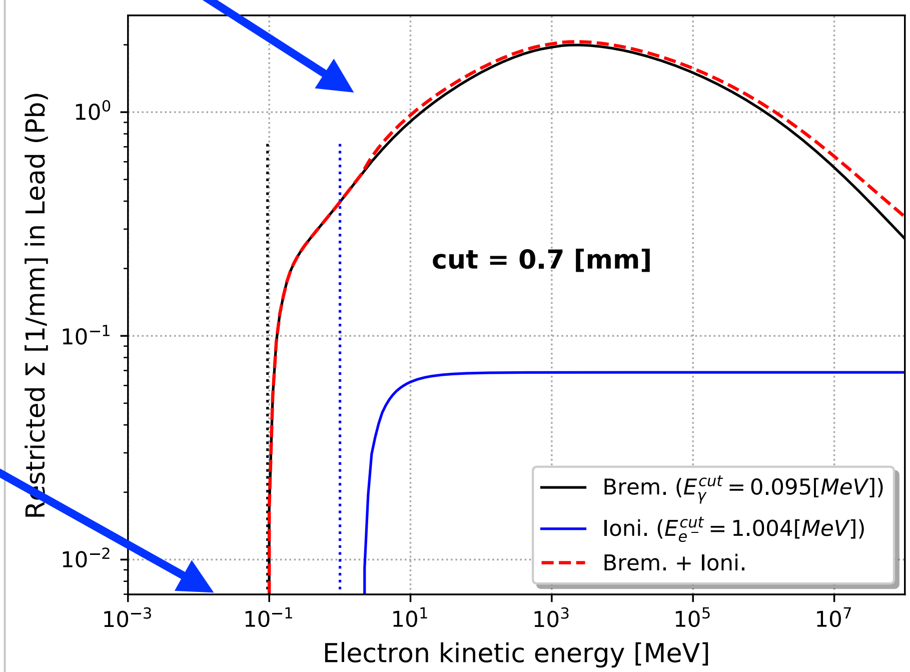
- **Consider the following:**
  - suppose, that an e⁻ is in a *large volume*
  - furthermore, the **step length** would be determined solely **by the discrete part**, i.e. by the macroscopic cross section
  - **how far** this particle, with an initial kinetic energy of 80 [keV] **would go**, till the next (super threshold) e⁻ or γ production?
  - **what would be its energy**, before the interaction, i.e. at the final position after removing the sub-threshold energy losses?
  - **?**





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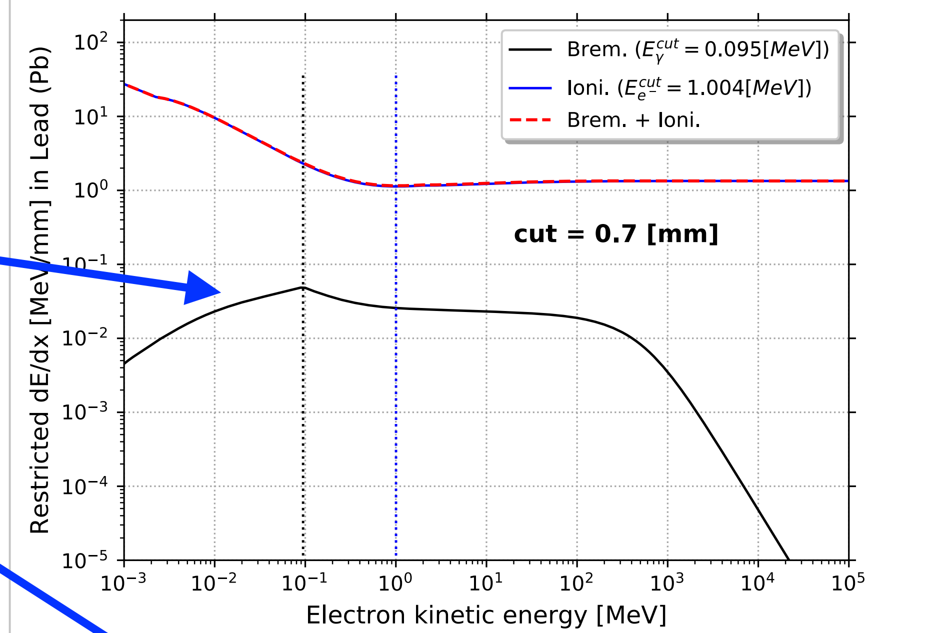
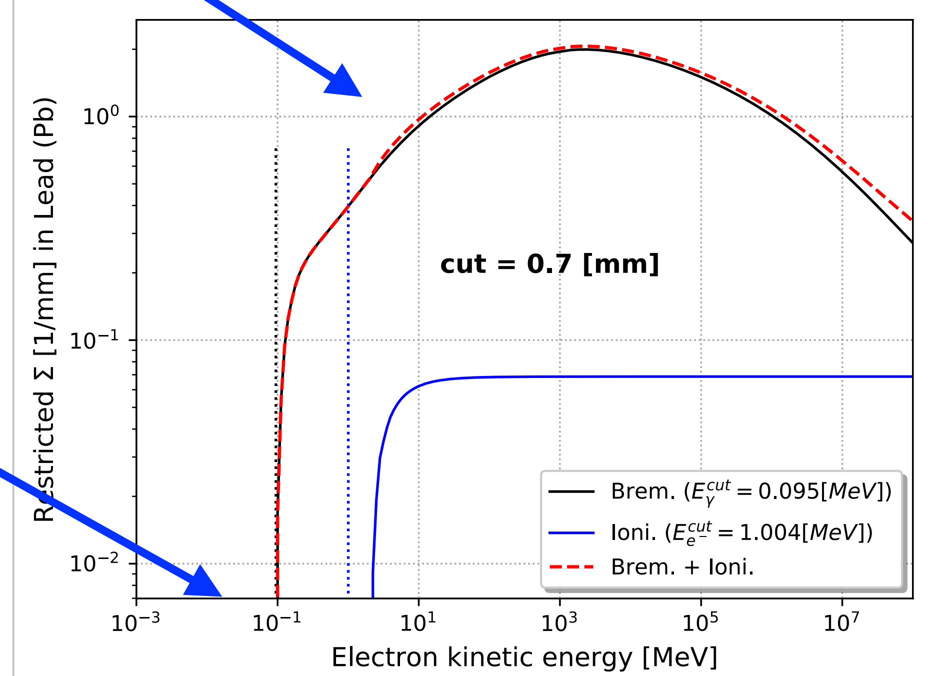
  - **till the boundary**; **could even be negative**

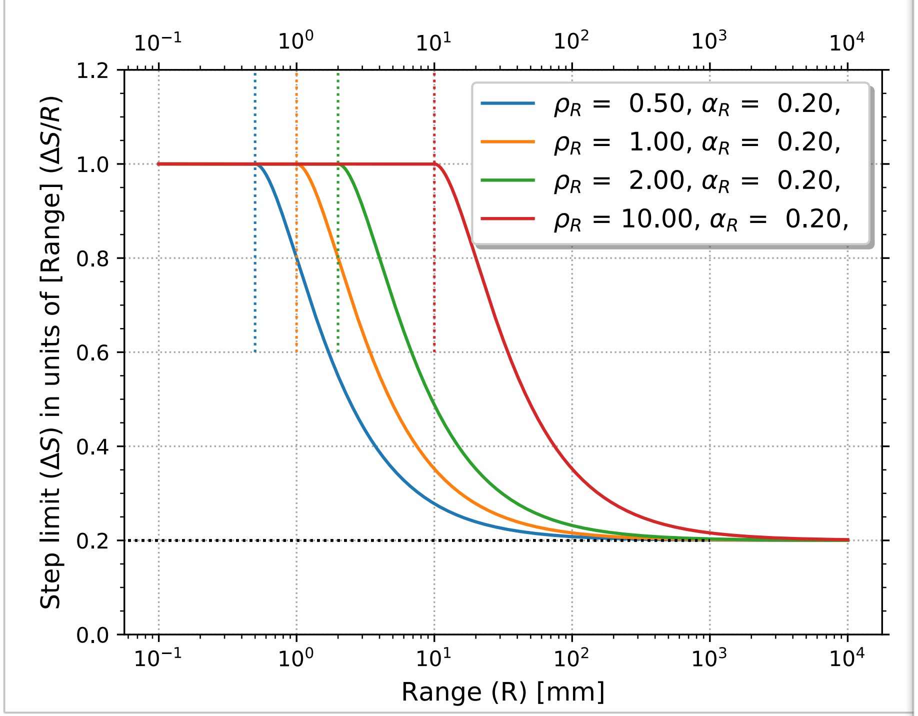


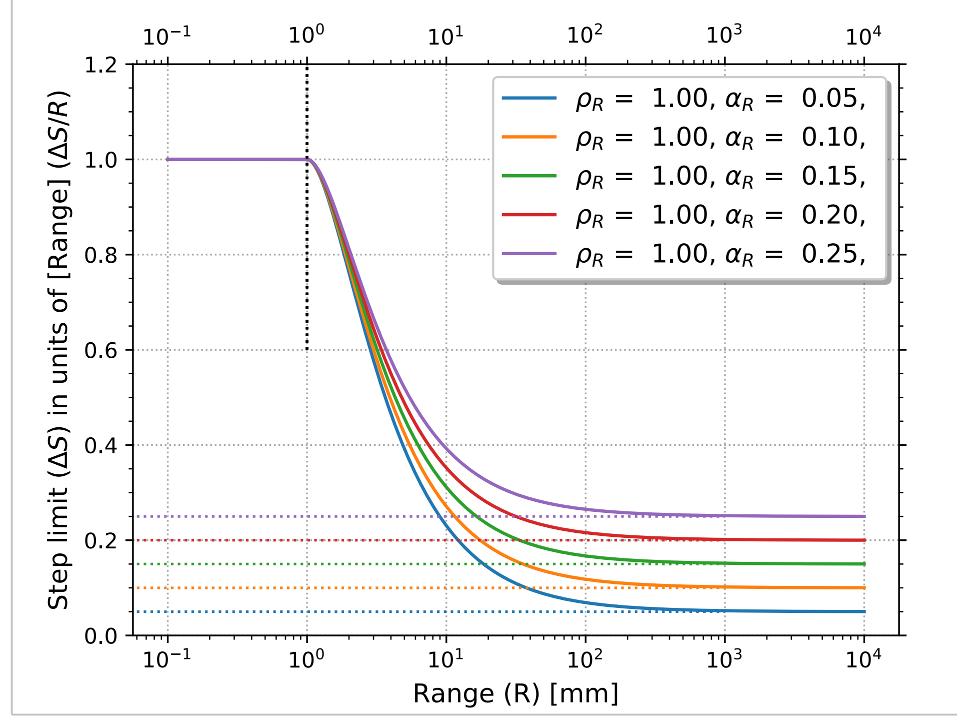


- **Continuous step limit due to energy loss:**
  - when using **condensed history** simulation, **continuous energy loss**es **impose a limit on the** charged particle **step** (beyond the discrete part)
  - an obvious choice of this limit **could be the** (pre-step, restricted) **range of the particle** (sum from collision and radiative, i.e. ionisation and bremsstrahlung)
  - this **would prevent** the particle **to go longer than the** (mean) **path length** at which **its energy** would **become zero due to** the (sub-threshold) **continuous energy loss**es
  - we **need to be even more strict** than this in order **to guarantee** the **stability** of the charged particle stepping: limit the allowed energy loss at most 20-25 % of the pre-step point energy of the particle (need to be relaxed at lower energies)
  - the ***loss function*** **below** ensures, that the **charged particle is stopped at the appropriate position** while its **energy is deposited in the correct volume**(**s**) **!**

$$\Delta S_{\text{eloss}}(E_0) = \begin{cases} R & \text{if } R < \rho_R \\ \alpha_R R + \rho_R(1-\alpha_R)(2-\rho_R/R) & \text{otherwise} \end{cases}$$

where $R := R(E_0)$ is the restricted range at energy E_0, $\boldsymbol{\rho_R}$ ***final range*** and $\boldsymbol{\alpha_R}$ ***rover range*** are two parameters.





- UI command to set the parameters:

```
/process/eLoss/StepFunction <roverRange-value> <finalRange-value> <unit>
/process/eLoss/StepFunctionMuHad" <roverRange-value> <finalRange-value> <unit>
```

- the lower the `finalRange` value the "later" (in range) the particle will be ranged-out (**default 1 [mm]**)
- the lower the `roverRange` value the smaller the allowed steps will be (**default 0.2**, i.e. 20 % of $R(E_0)$)

$$\Delta S_{\text{eloss}}(E_0) = \begin{cases} R & \text{if } R < \rho_R \\ \alpha_R R + \rho_R(1-\alpha_R)(2-\rho_R/R) & \text{otherwise} \end{cases}$$

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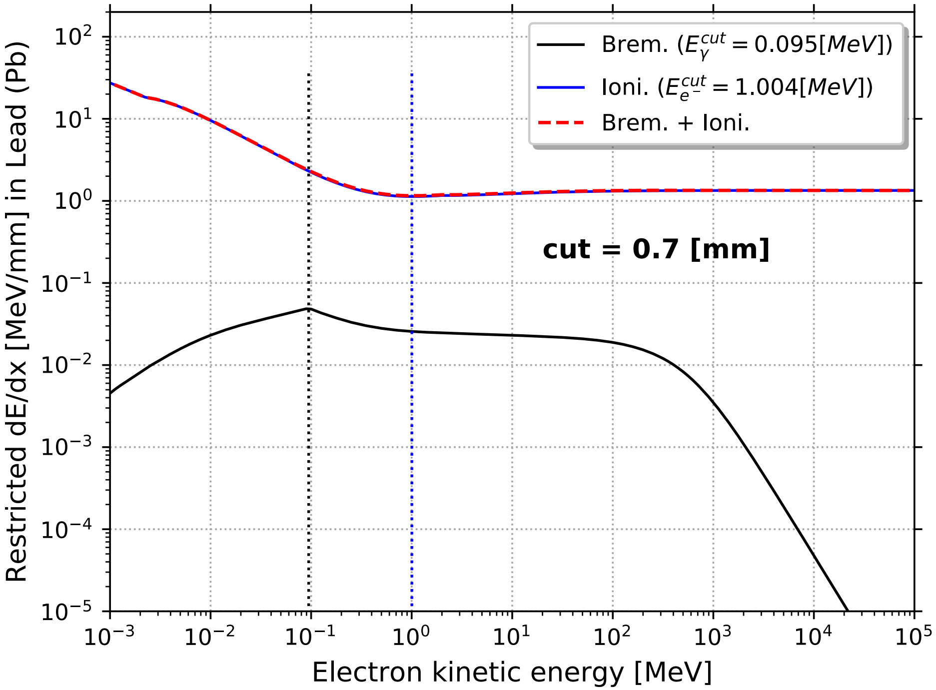

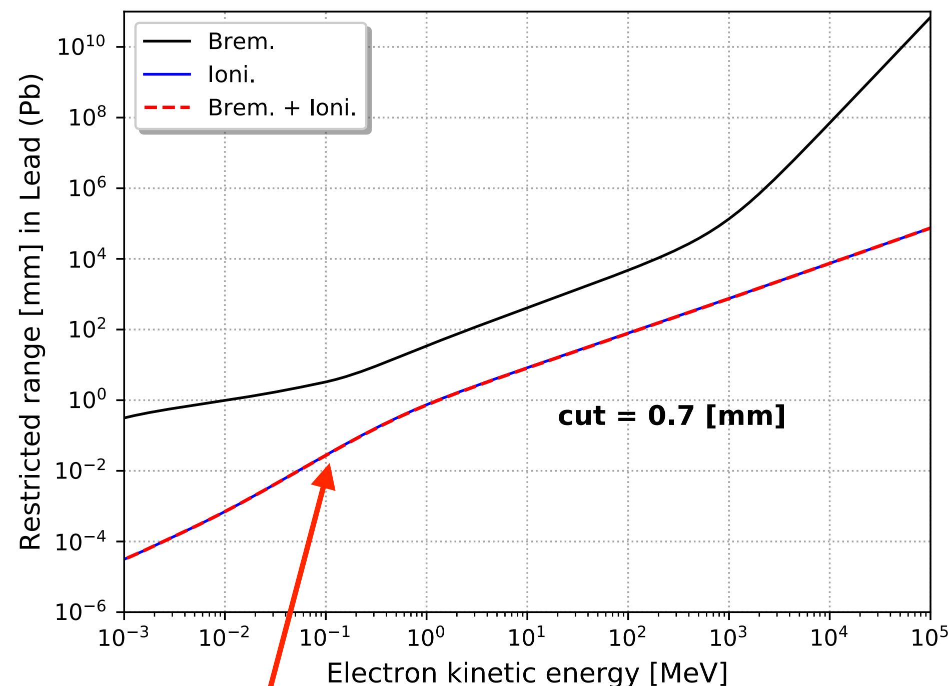

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R:= R(E₀): restricted range including both collision and radiative contributions

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TRACKING CUTS, USERLIMITS V.S. THE LOSS FUNCTION

- **Secondary production cuts vs tracking cut**:
  - **`Geant4` do not require** any **tracking cuts**: charged particles (like e^-/e^+) are “ranged-out” **thanks to the *loss function*** discussed before (photons are absorbed)
  - “ranged-out”: **appropriate final position** (resulting appropriate location of the energy deposit)
  - however, the user can easily introduce any limits on tracking by **`G4UserLimits`**
  - a **kinetic energy limit** (tracking cut in energy) has also been introduced recently:
    - ✦ only for computing performance reasons: to kill low energy “loopers” (low density material and field)
    - ✦ this kinetic energy limit can be set to any (even to zero) energy values
    - ✦ different values for e^-,e^+ and for hadrons, muons
    - ✦ UI commands:

```
/process/em/lowestElectronEnergy 100 eV
/process/em/lowestMuHadEnergy 10 keV
```

  - particles are **killed when their kinetic energy drops below the limit** and **their energy is deposited** (**at the given point!**)

- **Tracking cut (e.g. `G4UserLimits`):**
  - ***special user*** (limit) ***process***, can impose tracking cut in kinetic energy, range, etc.
  - **checked** at the step-limit phase of the step, i.e. **at the pre-step point**: if the given condition evaluates to be true, this special process is selected, the particle is **stopped immediately** and its kinetic **energy is deposited at the given point**
  - **+** many different type of limits even per detector region; **-** one extra step; inappropriate final position: the corresponding energy deposit (or a fraction of it) might be assigned to the wrong volume

- **Tracking cut (e.g. kinetic energy limit):**
  - checked within the step (after the continuous energy loss and the discrete interaction) and the particle is **stopped immediately** when its kinetic energy drops below the (global) **kinetic energy limit** and its kinetic **energy is deposited at the given point**
  - **+** no any extra steps; **-** global, inappropriate final position: the corresponding energy deposit (or a fraction of it) might be assigned to the wrong volume
  - (note: since the corresponding kinetic energy limit is usually small (default 1 [keV]) it do not cause any problems)

- **No tracking cut (e.g. relying on the loss function):**
  - the only way to **ensure appropriate final position of the charged particle** (and energy deposit location; see below)
  - whenever the **particle rage drops below** the ***final range*** parameter value, an extra step, with a continuous step limit equal to the particle range, is proposed
  - **+** only appropriate final position and energy deposit location; **-** requires an extra (last) step

Tracking cut, `G4UserLimits` v.s. the loss function

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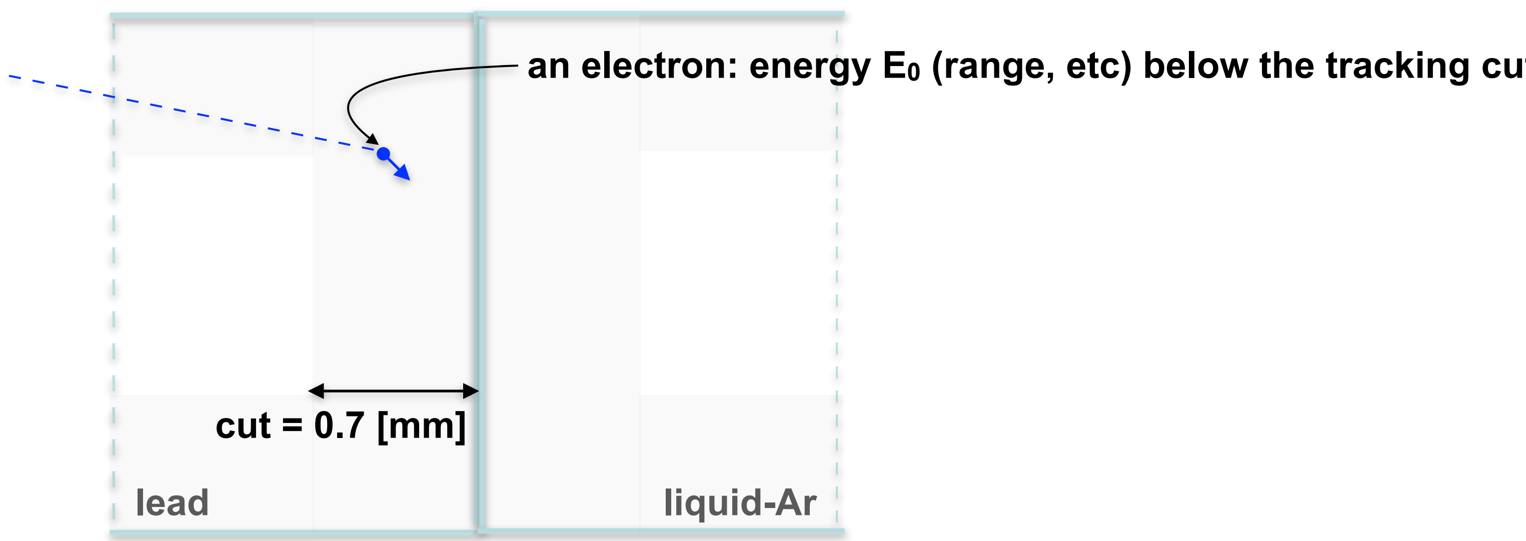
  - **+** `/process/eLoss/useCutAsFinalRange true` can be used to set ***final range*** = cut
  - **-** requires an extra (last) step



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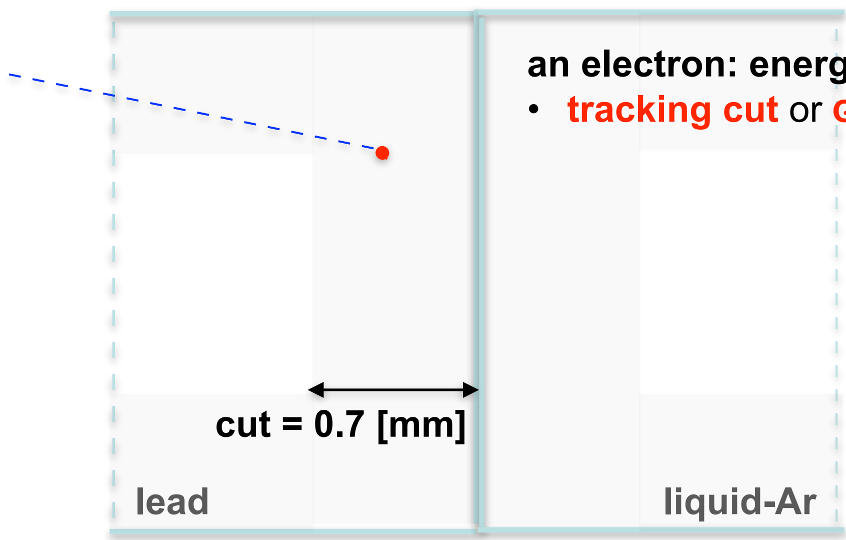

an electron: energy E_0 (range, etc) below the tracking cut

- **tracking cut** or **`G4UserLimits`**: stopped in **lead**, $\mathbf{E_{dep} = E_0}$

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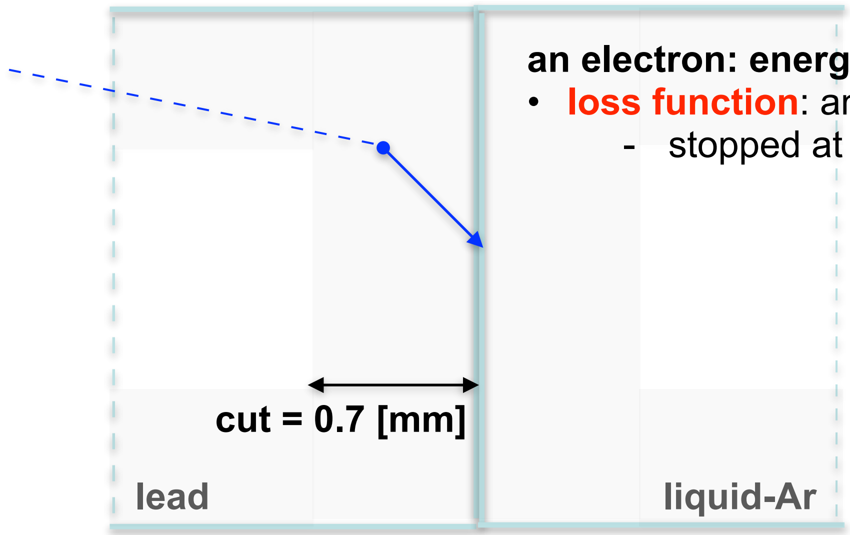

an electron: energy E_0 (range, etc) below the tracking cut

- **loss function**: an extra step with the range $R > d_{boundary}$
  - stopped at the boundary **lead**, $E_{dep} < E_0$

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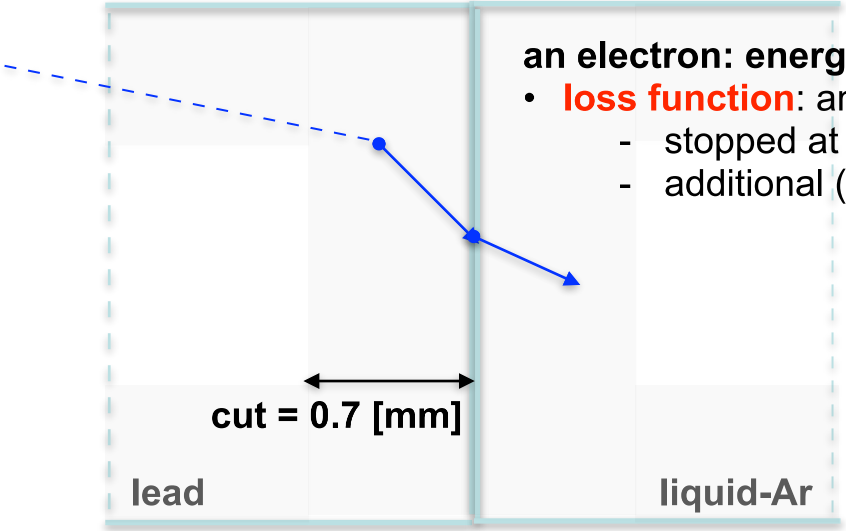

an electron: energy E_0 (range, etc) below the tracking cut

- **loss function**: an extra step with the range $R > d_{boundary}$
  - stopped at the boundary **lead: $E_{dep} < E_0$**
  - additional (rest of the) step in **liquid-Ar: $E_{dep} > 0$**

HOW ENERGY LOSS AFFECTS THE DISCRETE PART OF THE STEP LIMIT

- **assuming a** single interaction, e.g. bremsstrahlung with a **constant** restricted macroscopic **cross section Σ along the step**:

according to the *Beer-Lambert law*, the p.d.f. of the corresponding **interaction length** s i.e. the probability that the interaction will happen at $s, s+\mathrm{d}s$ is $p(s)\mathrm{d}s$

$$p(s) = \Sigma \exp[-s\Sigma] \rightarrow s \in \mathbb{E}\mathrm{xp}\{\Sigma\}$$

with a mean or expected value of

$$\mathbb{E}(s) = \frac{1}{\Sigma} \equiv \lambda \text{ mean free path}$$

- **however**, **the** restricted macroscopic **cross section is not constant along the step**: depends on the particle energy, that (in case of charged particles like e^{-}) is constantly changing along the ***s*** step!

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length s ... $s + \mathrm{d}s$ **is**
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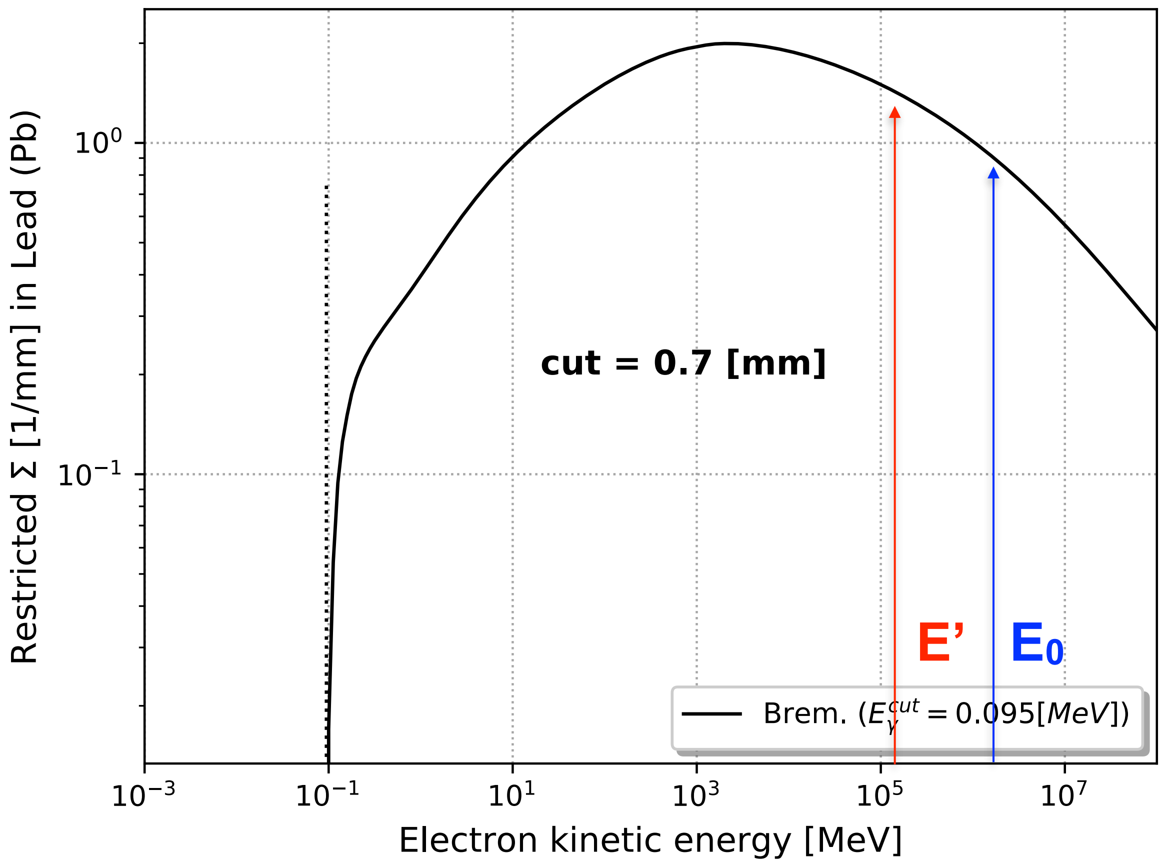

- **however, ... tant along the step:** ... particles like e^-, is

- having an energy E_0 at the pre-step point and **E' = E_0 - ΔE_{loss}** at the post-step point

Effects of the energy loss to the discrete part of the step limit

- energy E_0 at the pre-step point and $E' = E_0 - \Delta E_{loss}$ at the post-step point:

  instead of sampling the interaction till the next discrete *real* (e.g. brem.) event:

  - a fictive δ interaction (no effect) is introduced
  - its macroscopic cross section $\Sigma^{\delta}(E)$ is such that const. $= \Sigma = \Sigma^{\delta}(E) + \Sigma^{\text{real}}(E)$
  - Σ is already const. along the step and the interaction is either the *real* or the δ
  - at the post-step point, the probability of the *real* interaction is $p(\text{real}) = \Sigma^{\text{real}}(E')/\Sigma$
  - the only restriction is that $\Sigma \geq \Sigma^{\text{real}}(E)$ along the step (otherwise $p(\delta) < 0$)

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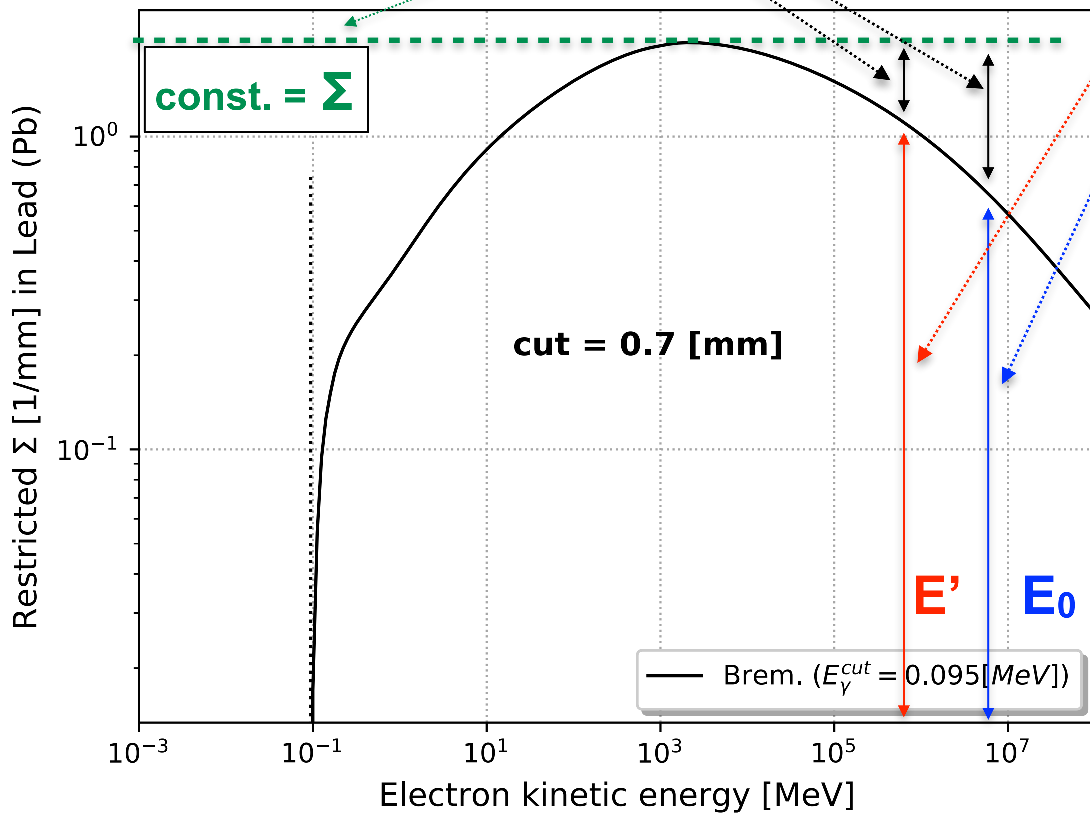

Good, but can results in a high fraction of “*delta*” steps (when far from the max.)!

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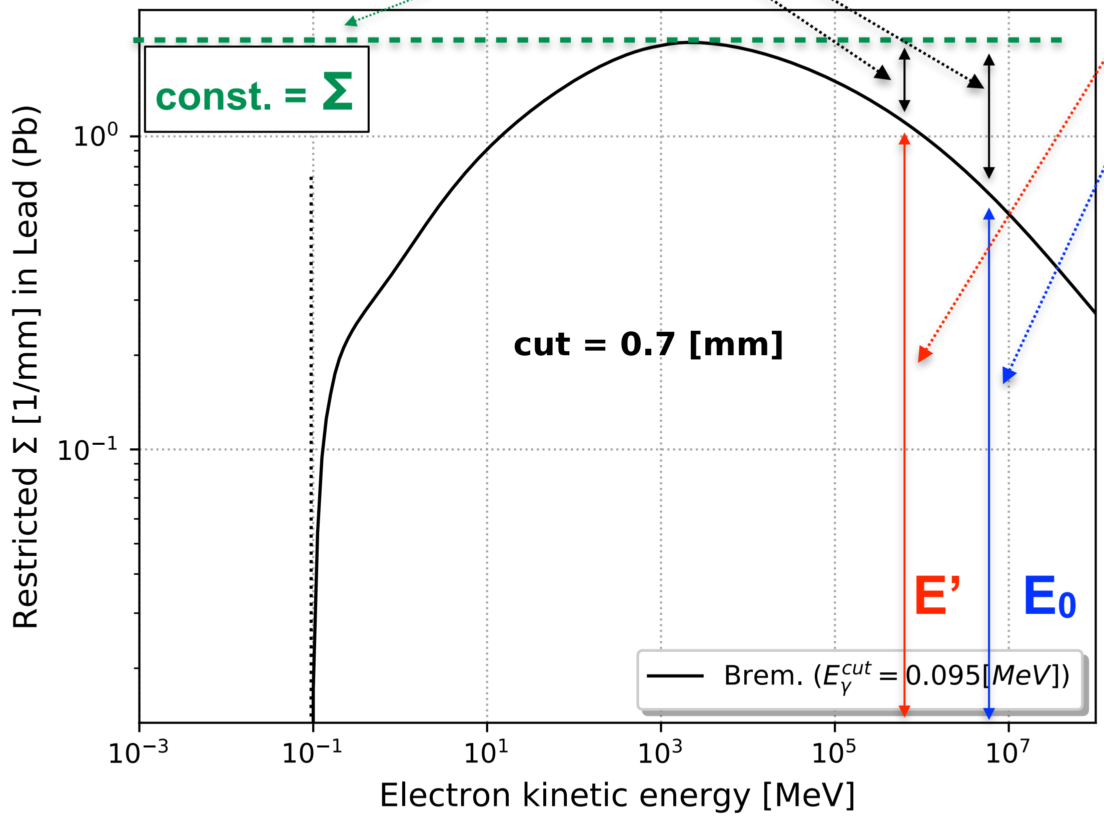

Using the "*maximum allowed energy loss*" along the step (from the step function) !

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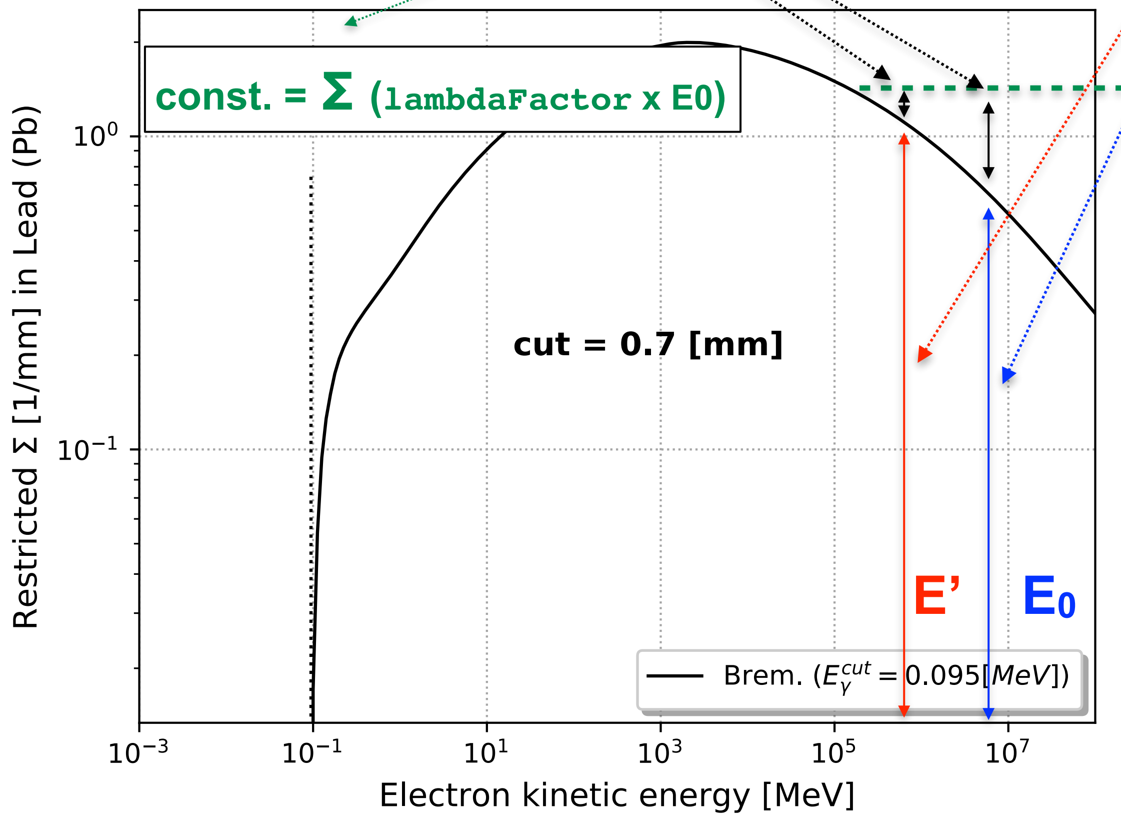

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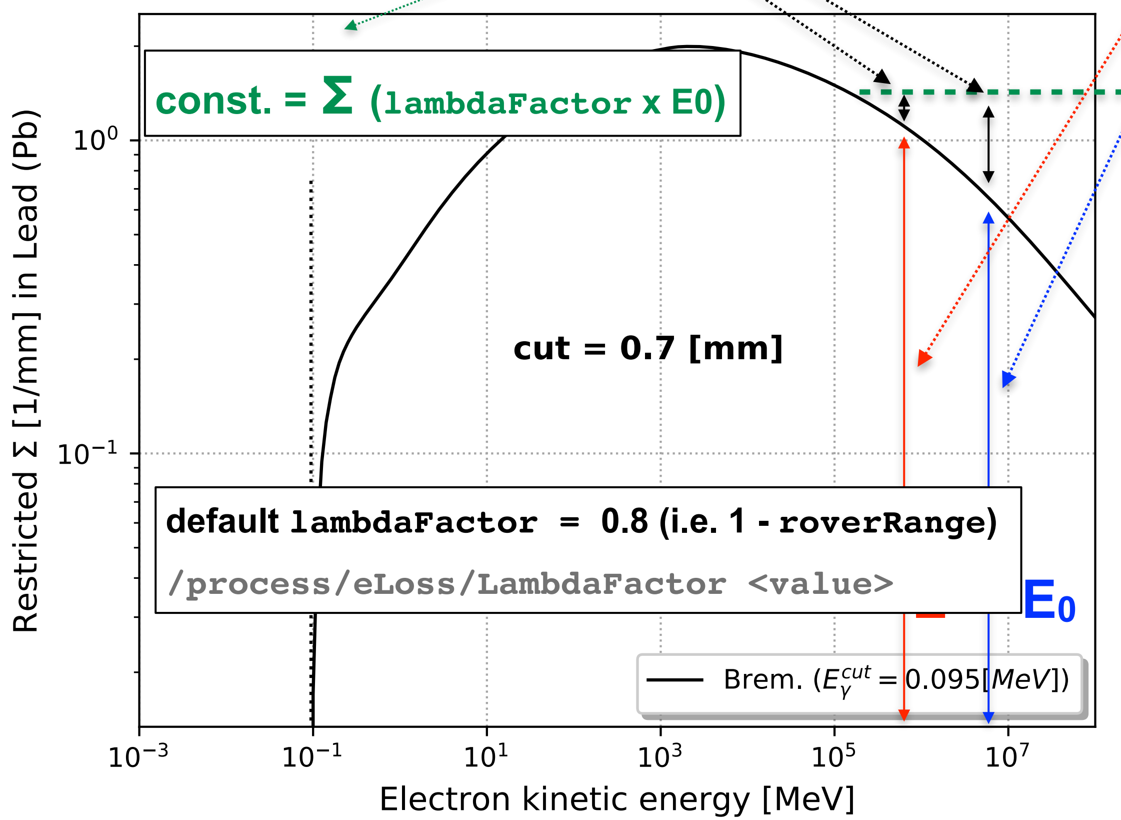


SOME NOTES ON THE
G4GAMMAGENERALPROCESS

- Why we could use the "sum" of the two ("*real*" and "*δ*") processes above?

if $s_i = \{s_1, \ldots, s_M\}$ *independent* stochastic variables, such that $s_i \in \mathbb{E}\text{xp}\{1/\lambda_i\}, \forall i = 1, \ldots, M$, then $\eta \equiv \min\{s_1, \ldots, s_M\} \in \mathbb{E}\text{xp}\left\{\sum_{i=1}^{M} 1/\lambda_i\right\}$

Proof: by definition, the cumulative distribution function of η at any x is equal to the probability that

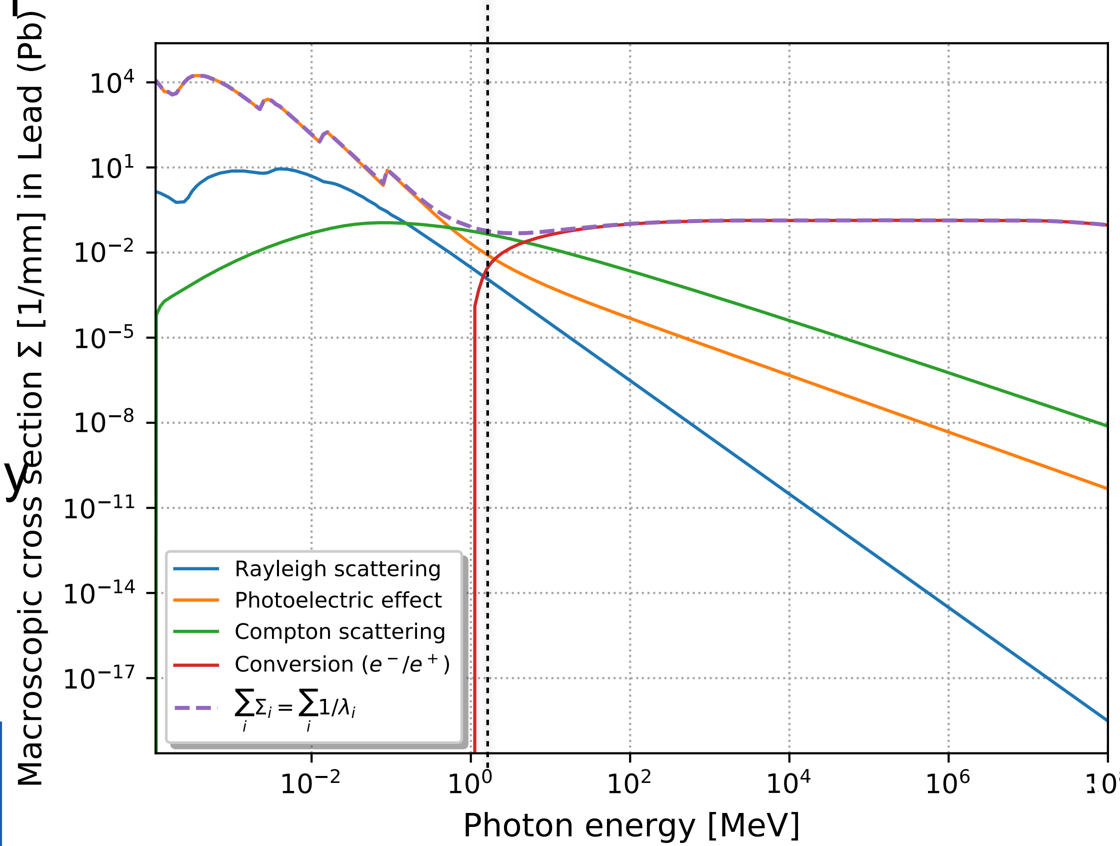
$$P(\eta < x) = 1 - P(\eta \geq x) = 1 - P(s_1 \geq x, \ldots, s_M \geq x) = 1 - \prod_{i=1}^{M} P(s_i \geq x)$$

$$= 1 - \prod_{i=1}^{M}[1 - P(s_i < x)] = 1 - \prod_{i=1}^{M}[1 - (1 - \exp(-x/\lambda_i))]$$

$$= 1 - \exp\left[-x\sum_{i=1}^{M} 1/\lambda_i\right] \rightarrow \eta \in \mathbb{E}\text{xp}\left\{\sum_{i=1}^{M} 1/\lambda_i\right\}$$

- This can be applied in other cases!

- This can be applied in other cases: above a given gamma energy, the sum of the 4 (macroscopic) cross sections is smooth:
  - instead of evaluating the 4 macroscopic cross sections, sampling the corresponding 4 interaction lengths (**s_1, s_2, s_3, s_4**) and taking their minimum **s** as the current physics step length at each step
  - one might use the single sum of macroscopic cross section value to sample directly the minimum **s** length
  - then evaluate the 4 interactions only if needed, i.e when eventually physics limits the step
  - this can save up several run time interpolations (especially in highly granular detectors) without altering the results



THAT'S IT FOR TODAY