Transverse Beam Dynamics III

I) Linear Beam Optics
   Single Particle Trajectories
   Magnets and Focusing Fields
   Tune & Orbit

II) The State of the Art in High Energy Machines:
   The Beam as Particle Ensemble
   Emittance and Beta-Function
   Colliding Beams & Luminosity

III) Errors in Field and Gradient:
   Liouville during Acceleration
   The $\Delta p/p \neq 0$ problem
   Dispersion
   Chromaticity
Luminosity

Example: Luminosity at LHC

$$\beta_{x,y}^* = 0.55 \, m$$  \hspace{1cm}  $$f_0 = 11.245 \, kHz$$

$$\varepsilon_{x,y} = 5 \times 10^{-10} \, \text{rad} \, m$$  \hspace{1cm}  $$n_b = 2808$$

$$\sigma_{x,y} = 17 \, \mu m$$

$$I_p = 584 \, mA$$

$$L = 1.0 \times 10^{34} \, \frac{1}{cm^2 \, s}$$

Make $\beta^*$ as small as possible !!!
Mini-Beta-Insertions in phase space

A mini-β insertion is always a kind of special symmetric drift space.
→ greetings from Liouville

the smaller the beam size
the larger the beam divergence

Liouville: in reasonable storage rings area in phase space is constant.

\[ A = \pi \varepsilon = \text{const} \]
The LHC Insertions

**ATLAS R1**

**Inner Triplet**

- IP1
- TAS
- Q1 Q2 Q3
- D1 (1.38 T)

**Separation/Recombination**

- Tertiary collimator
- $s$
- 188 mm
- 1.9 K
- Warm

**Matching Quadrupoles**

- Q4
- D2 (3.8 T)
- Q5
- Q6
- Q7
- 4.5 K

**Mini β optics**

LHC Error Analysis MAD-X 3.00.03 03/12/08 10.35.00

Momentum offset = 0.00 %

$s$ (m) [10^10]
... finally ... let’s talk about acceleration

crab nebula,

burst of charged particles $E = 10^{20} \text{ eV}$
14.) Liouville during Acceleration

\[ \varepsilon = \gamma (s) x^2 (s) + 2\alpha (s)x(s)x'(s) + \beta (s)x'^2 (s) \]

**Beam Emittance** corresponds to the area covered in the \( x, x' \) Phase Space Ellipse

**Liouville:** Area in phase space is constant.

**But so sorry ... \( \varepsilon \neq \text{const} ! \)**

**Classical Mechanics:**

*phase space = diagram of the two canonical variables*

*position & momentum*

\[ x \quad p_x \]
According to Hamiltonian mechanics: phase space diagram relates the variables $q$ and $p$

**Liouville's Theorem:**

\[ \int p \, dq = \text{const} \]

\[ \int p_x \, dx = \text{const} \]

... referring to the hor. plane for convenience (i.e. because we are lazy bones) we use in accelerator theory:

\[ x' = \frac{d x}{d s} = \frac{d x}{d t} \frac{d t}{d s} = \beta_x = \frac{p_x}{p} \]

\[ \int x' \, dx = \int \frac{p_x}{p} \, dx = \text{const} \left( \frac{m_0 c}{\gamma \beta} \right) \]

\[ \Rightarrow \quad \varepsilon = \int x' \, dx \propto \frac{1}{\beta \gamma} \]

**the beam emittance shrinks during acceleration** \( \varepsilon \sim 1/\gamma \)

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \beta_x = \frac{v_x}{c} \]
Nota bene:

1.) A proton machine ... or an electron linac ... needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

$$\sigma = \sqrt{\epsilon \beta}$$

2.) At lowest energy the machine will have the major aperture problems,
   $\rightarrow$ here we have to minimise $\beta$

3.) we need different beam optics adopted to the energy:
   A Mini Beta concept will only be adequate at flat top.
Example: HERA proton ring

injection energy: 40 GeV \( \gamma = 43 \)
flat top energy: 920 GeV \( \gamma = 980 \)

emittance \( \varepsilon (40\text{GeV}) = 1.2 \times 10^{-7} \)
\( \varepsilon (920\text{GeV}) = 5.1 \times 10^{-9} \)

7 \( \sigma \) beam envelope at \( E = 40 \text{ GeV} \)

... and at \( E = 920 \text{ GeV} \)
The „not so ideal world“

15.) The „$\Delta p / p \neq 0$“ Problem

ideal accelerator: all particles will see the same accelerating voltage.
$\Rightarrow \Delta p / p = 0$

„nearly ideal“ accelerator: Cockroft Walton or van de Graaf
$\Delta p / p \approx 10^{-5}$
RF Acceleration

1928, Wideroe

Energy Gain per "Gap":

\[ W = n^* q U_0 \sin \omega_{RF} t \]

drift tube structure at a proton linac (GSI Unilac)

\( n \) number of gaps between the drift tubes
\( q \) charge of the particle
\( U_0 \) Peak voltage of the RF System
\( \Psi_s \) synchronous phase of the particle

500 MHz cavities in an electron storage ring

* RF Acceleration: multiple application of the same acceleration voltage; brilliant idea to gain higher energies
RF Acceleration-Problem: panta rhei !!!
(Heraklit: 540-480 v. Chr.)

just a stupid (and a little bit wrong) example)

\[ \nu = 400 \text{ MHz} \]
\[ c = \lambda \nu \]
\[ \lambda = 75 \text{ am} \]

\[ \sin(90^\circ) = 1 \]
\[ \sin(84^\circ) = 0.994 \]
\[ \frac{\Delta U}{U} = 6.0 \times 10^{-3} \]

Bunch length of Electrons \( \approx 1 \text{ cm} \)

typical momentum spread of an electron bunch:

\[ \frac{\Delta p}{p} \approx 1.0 \times 10^{-3} \]
Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???
Sure there are !!!
16.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: prop. to magn. field & prop. zu $1/p$

**dipole magnet**

$$\alpha = \frac{\int B \, dl}{p/e}$$

**focusing lens**

$$k = \frac{g}{p/e}$$

$$x_D(s) = D(s) \frac{\Delta p}{p}$$

Particle having...

to high energy

to low energy

**ideal energy**
Dispersion

the typical Formula 1 effect:

Those who are faster (have higher momentum) ...
... are running on a larger circle.

BUT

they are focused nevertheless.
Dispersion

Example: homogeneous dipole field

Matrix formalism:

\[
x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}
\]

\[
x(s) = C(s) \cdot x_0 + S(s) \cdot x'_0 + D(s) \cdot \frac{\Delta p}{p}
\]

\[
\begin{pmatrix}
x \\
x'
\end{pmatrix}_{s} =
\begin{pmatrix}
C & S \\
C' & S'
\end{pmatrix}
\begin{pmatrix}
x \\
x'
\end{pmatrix}_{0} +
\frac{\Delta p}{p}
\begin{pmatrix}
D \\
D'
\end{pmatrix}_{0}
\]
or expressed as 3x3 matrix

\[
\begin{pmatrix}
  x \\
  x' \\
  \Delta p_p/p_s
\end{pmatrix} =
\begin{pmatrix}
  C & S & D \\
  C' & S' & D' \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  x' \\
  \Delta p_p/p_0
\end{pmatrix}
\]

Example

\[
x_\beta = 1 \ldots 2 \text{ mm}
\]
\[
D(s) \approx 1 \ldots 2 \text{ m}
\]
\[
\frac{\Delta p}{p} \approx 1 \cdot 10^{-3}
\]

Amplitude of Orbit oscillation contribution due to Dispersion \(\approx\) beam size

\[\rightarrow\] Dispersion must vanish at the collision point

Calculate \(D, D'\): ... takes a couple of sunny Sunday evenings!

\[
D(s) = S(s) \int_{s_0}^{s_1} \frac{1}{\rho} C(\tilde{s}) \, d\tilde{s} - C(s) \int_{s_0}^{s_1} \frac{1}{\rho} S(\tilde{s}) \, d\tilde{s}
\]
(proof see CAS proc.)
Dispersion is visible

**HERA Standard Orbit**

**HERA Dispersion Orbit**

A dedicated energy change of the stored beam → closed orbit is moved to a dispersions trajectory

\[ x_p = D(s) \ast \frac{\Delta p}{p} \]

Attention: at the Interaction Points we require \( D = D' = 0 \)
Periodic Dispersion:
„Sawtooth Effect“ at LEP (CERN)

In the arc the electron beam loses so much energy in each octant that the particles are running more and more on a dispersion trajectory.

In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.
17.) Chromaticity: 
*A Quadrupole Error for $\Delta p/p \neq 0$*

Influence of external fields on the beam: *prop. to magn. field & prop. zu $1/p$*

Remember the normalisation of the external fields:

- **focusing lens**
  
  \[
  k = \frac{g}{p/e}
  \]

A particle that has a higher momentum feels a weaker quadrupole gradient and has a lower tune.

**definition of chromaticity:**

\[
\Delta Q = Q' \frac{\Delta p}{p}
\]
Every individual particle has an individual momentum and thus an individual tune.

$Q'$ is a number indicating the size of the tune spot in the working diagram, $Q'$ is always created if the beam is focussed → it is determined by the focusing strength $k$ of all quadrupoles

$$\Delta Q = -\frac{1}{4\pi} \frac{\Delta p}{p_0} k_0 \beta(s) ds$$

$$Q' = -\frac{1}{4\pi} \oint k(s)\beta(s) ds$$

$k = \text{quadrupole strength}$
$\beta = \text{betafunction}$ indicates the beam size ... and even more the sensitivity of the beam to external fields

Example: LHC

$Q' = 250$
$\Delta p/p = +/- 0.2 \times 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

→ Some particles get very close to resonances and are lost

in other words: the tune is not a point it is a pancake
Tune signal for a nearly uncompensated cromaticity
\( Q' \approx 20 \)

Ideal situation: cromaticity well corrected,
\( Q' \approx 1 \)
Tune and Resonances

\[ mQ_x + nQ_y + lQ_s = \text{integer} \]

Tune diagram up to 3rd order

... and up to 7th order

Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive
Chromaticity Correction:

*We need a magnetic field that focuses stronger those individual particles that have larger momentum and focuses weaker those with lower momentum.*

... but that does not exist.

The way the trick goes:

1.) sort the particle trajectories according to their energy
   we use the dispersion to do the job

2.) introduce magnetic fields that increase stronger than linear
    with the distance $\Delta x$ to the centre

3.) calculate these fields (sextupoles) in a way that the lack of focusing strength is exactly compensated.
Correction of $Q'$:

Need: additional quadrupole strength for each momentum deviation $\Delta p/p$

1.) sort the particles according to their momentum

\[ x_D(s) = D(s) \frac{\Delta p}{p} \]

... using the dispersion function

2.) apply a magnetic field that rises quadratically with $x$ (sextupole field)

\[
\begin{align*}
B_x &= \tilde{g}xy \\
B_y &= \frac{1}{2} g(x^2 - y^2)
\end{align*}
\]

\[
\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x
\]

--> amplitude dependent gradient
Correction of $Q'$:

Sextupole Magnets:

$k_1$ normalised quadrupole strength  
$k_2$ normalised sextupole strength

\[ k_1(\text{sext}) = \frac{\bar{g}x}{p/e} = k_2 \times x \]
\[ = k_2 \times D \frac{\Delta p}{p} \]

Combined effect of „natural chromaticity“ and Sextupole Magnets:

\[ Q' = -\frac{1}{4\pi} \left\{ \int k_1(s)\beta(s) \, ds + \int k_2(s)D(s)\beta(s) \, ds \right\} \]

You only should not forget to correct $Q'$ in both planes ...  
and take into account the contribution from quadrupoles of both polarities.
Chromaticity Correction:

schematical view
A word of caution: keep non-linear terms in your storage ring low.

\[ B_y + iB_x = B_{\text{ref}} \sum_{n=1}^{\infty} (b_n + ia_n) \left( \frac{x + iy}{r_0} \right)^{n-1} \]

"effective magnetic length"

\[ B \cdot l_{\text{eff}} = \int B ds \]
Clearly there is another problem …
… if it were easy everybody could do it

Again: the phase space ellipse

for each turn write down – at a given position „s“ in the ring – the single particle amplitude $x$ and the angle $x'$ ... and plot it.

\[
\begin{pmatrix}
  x \\
  x'
\end{pmatrix}_{s1} = M_{\text{turn}} \ast \begin{pmatrix}
  x \\
  x'
\end{pmatrix}_{s0}
\]

A beam of 4 particles – each having a slightly different emittance:
Installation of a weak (!!!) sextupole magnet

The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equations; instead: Computer simulation „particle tracking“
Effect of a strong (!!!) Sextupole ...

→ Catastrophy!

„dynamic aperture“
The Mini-Beta scheme ... 
... focusses strongly the beams to achieve smallest possible beam sizes at the IP. The obtained small beta function at the IP is called $\beta^*$. Don't forget the cat.

**Beam dimension during acceleration:** A proton beam shrinks during acceleration in both ytransverse dimensions. We call it unfortunately „adiabatic shrinking“. Nota bene: An electron beam in a ring is growing with energy!!

**Dispersion ...**
... is the particle orbit for a given momentum difference.

**Chromaticity ...**
... is a focusing problem. Different momenta lead to different tunes $\rightarrow$ attention ... resonances !!

**Sextupoles ...**
have non-linear fields and are used to compensate chromaticity. However we have to be careful: Strong non-linear fields can lead to particle losses (dynamic aperture)
2.) Where do we go?

* Physics beyond the Standard Model
* Dark Matter / Dark Energy
FCC-pp - Collider

The Next Generation Ring Collider
What’s next ???

Dark Matter & Dark Energy
Physics beyond the Standard Model
Reconstruction of Dark Matter distribution based on observations

Budget:  
Dark Matter: 26 %  
Dark Energy: 70 %  
Anything else (including us) 4 %
Bibliography

1.) Edmund Wilson: Introd. to Particle Accelerators
Oxford Press, 2001

2.) Klaus Wille: Physics of Particle Accelerators and Synchrotron
Radiation Facilities, Teubner, Stuttgart 1992

3.) Peter Schmüser: Basic Course on Accelerator Optics, CERN Acc.
School: 5th general acc. phys. course CERN 94-01

4.) Bernhard Holzer: Lattice Design, CERN Acc. School: Interm.Acc.phys course,
http://cas.web.cern.ch/cas/ZEUTHEN/lectures-zeuthen.htm

5.) Herni Bruck: Accelerateurs Circulaires des Particules,
presse Universitaires de France, Paris 1966 (english / francais)

6.) M.S. Livingston, J.P. Blewett: Particle Accelerators,

7.) Frank Hinterberger: Physik der Teilchenbeschleuniger, Springer Verlag 1997

8.) Mathew Sands: The Physics of e+ e- Storage Rings, SLAC report 121, 1970

9.) D. Edwards, M. Syphers: An Introduction to the Physics of Particle
Accelerators, SSC Lab 1990