The effect of intermediate resonances in the quark interaction kernel on the time-like electromagnetic pion form factor[†]

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[†]based on arXiv:2102.12541

Motivation: Understanding hadrons from QCD

- Strong Interactions in Theory: QCD
  - The model quantum gauge field theory:
    Locality, Unitarity, Asymptotic Freedom
  - Non-perturbative phenomena:
    Dimensional Transmutation, Chiral Anomaly, DχSB, Confinement
- Strong Interactions in Experiment: Hadrons
  - Hadron spectroscopy:
    many "unexpected" resonances, many "missing" resonances
  - Hadron structure: surprising results
- Quark-hadron duality:
  orthogonality of quark-glue d.o.f. vs. hadronic states.

Motivation: Understanding hadrons from QCD

Hadron spectroscopy and hadron structure interrelated:
Microscopic understanding of
effect of resonances on form factors, structure functions, etc.?

Test case: **Pion form factor**[1]
Method: Functional method, in particular combination of
Dyson-Schwinger / Bethe-Salpeter eqs.

Important for the time-like pion form factor:

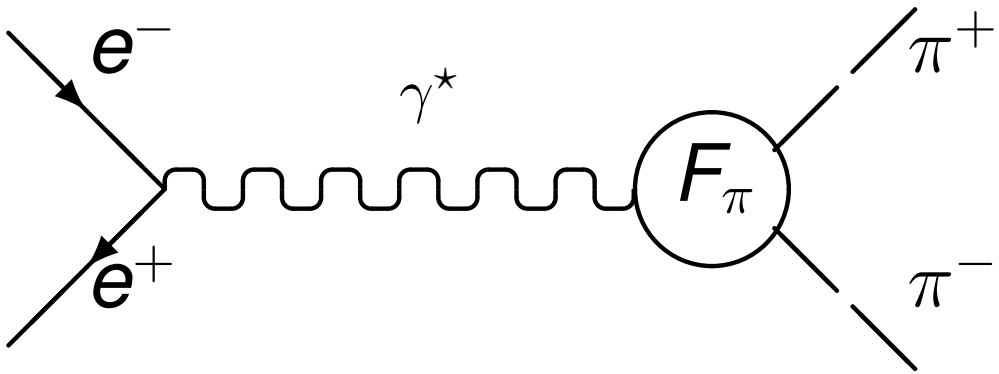
(i) Pion as $\bar{q}q$ bound state & as pseudo Goldstone Boson

(ii) Mixing of ρ-meson with virtual photon
(ρ as $\bar{q}q$ bound state in quark-photon vertex)

(iii) ρ-meson decay $\rho \to \pi\pi$

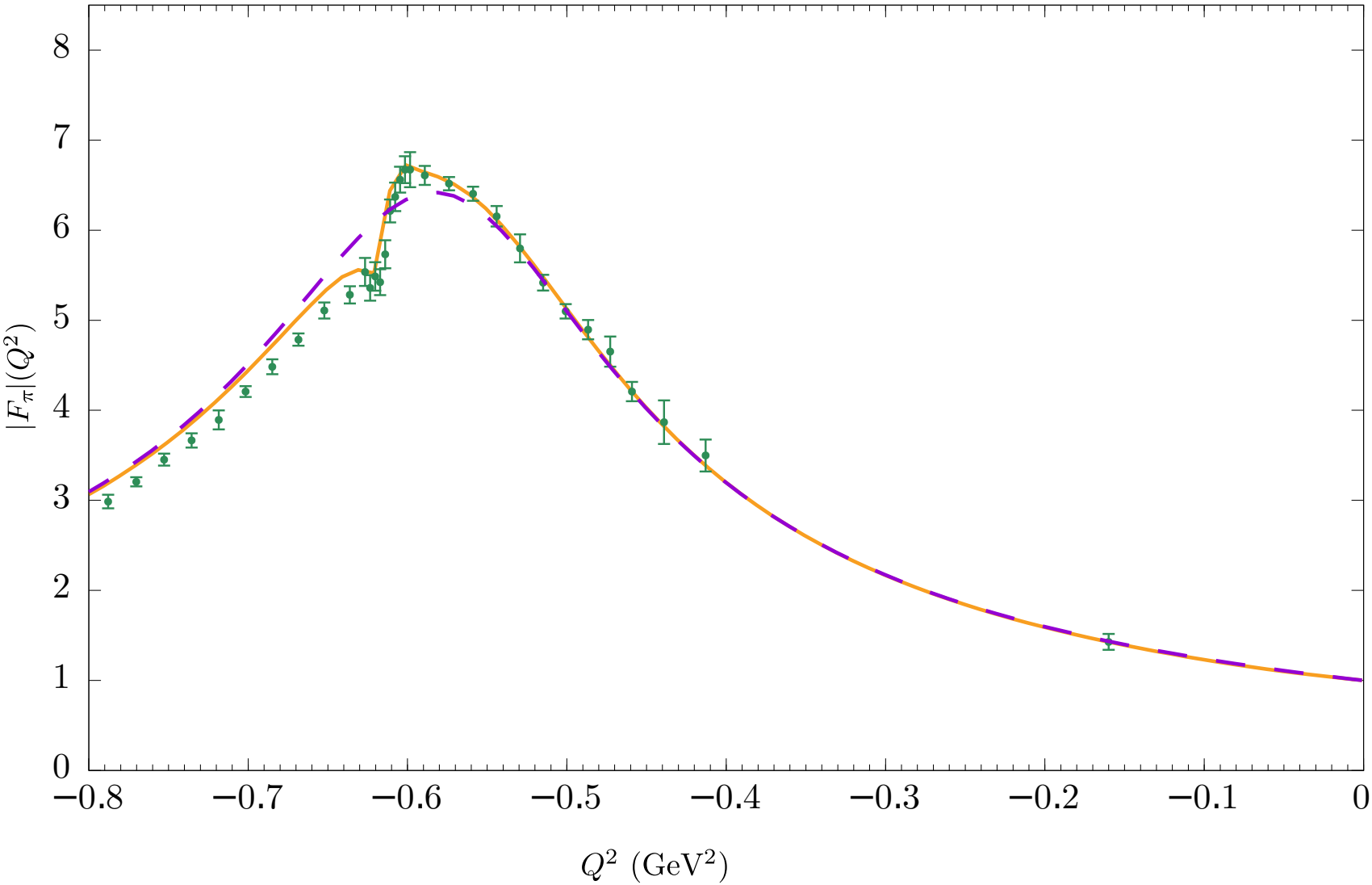
[1] A topic for me since the eighties [K. Langfeld *et al.*, Z. Phys. **C42 (1989)** 159]

Time-like pion form factor & Vector Meson Dominance



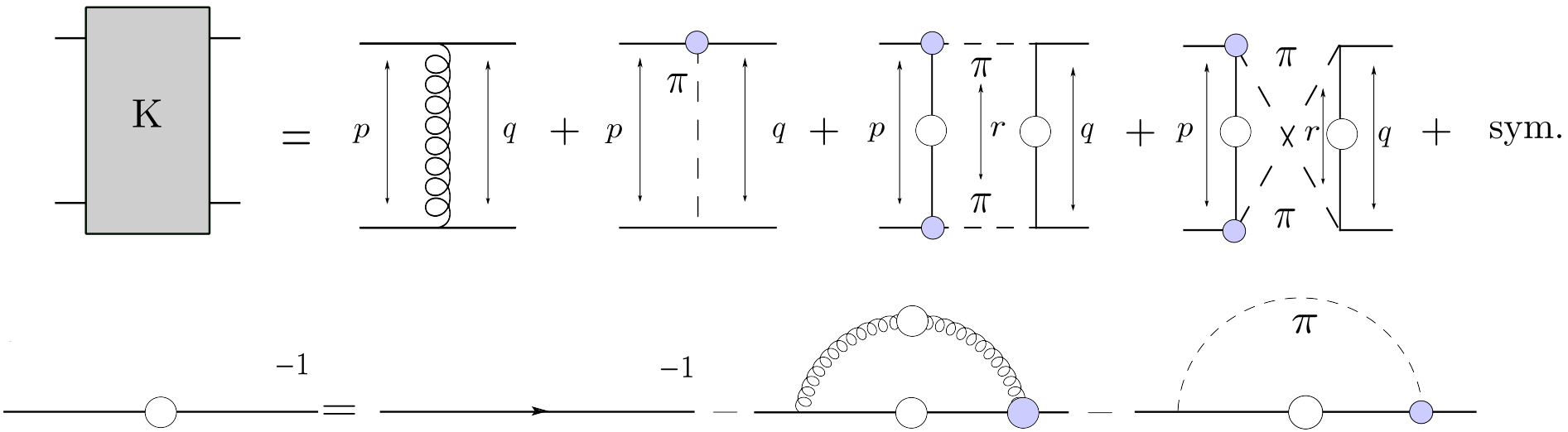
Experimentally, *e.g.*, from e^+e^- annihilation to $\pi\ \pi$

Time-like pion form factor & Vector Meson Dominance



Experimental data vs. VMD fit with / without ρ-ω mixing

Interactions in Dyson-Schwinger/Bethe-Salpeter eqs.



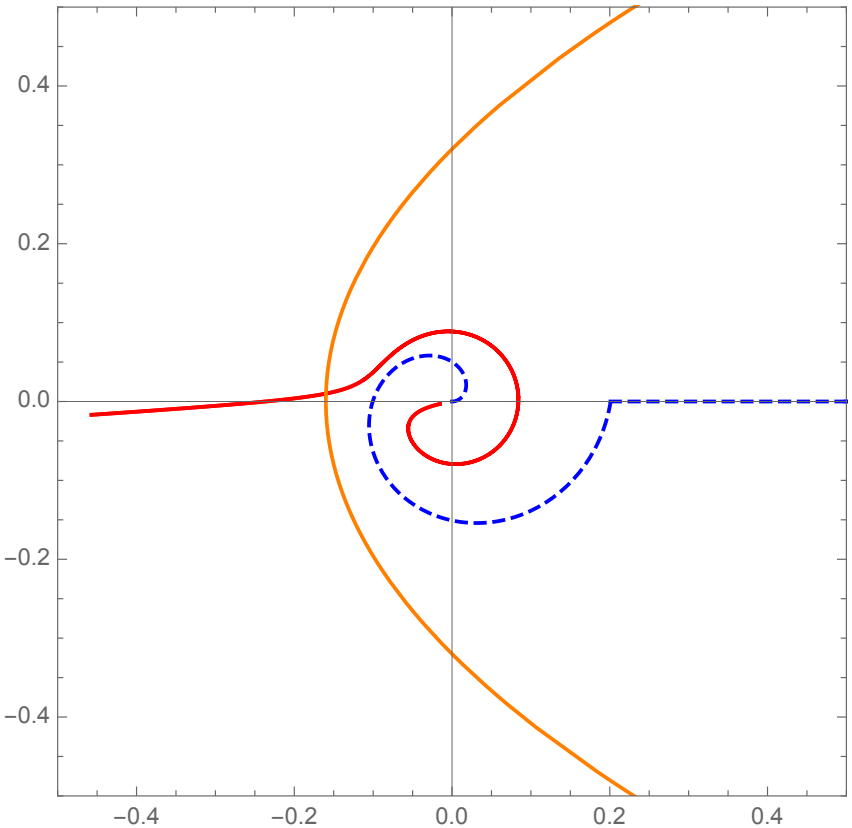
Interactions in this exploratory calculation:

- gluon exchange (Maris-Tandy model)
- pion exchange
- *s*- and *u*-channel pion decay contributions

Dyson-Schwinger/Bethe-Salpeter approach to time-like pion form factor

Disclaimer: To keep this calculation feasible a number of technically motivated approximations have been made,
see arXiv:2102.12541 for details.

Major technical challenge:
Find integration contour in presence of cuts generated by quark propagator poles, pion propagator pole as well as 2-pion cuts and ρ pole in quark-photon vertex!



Results

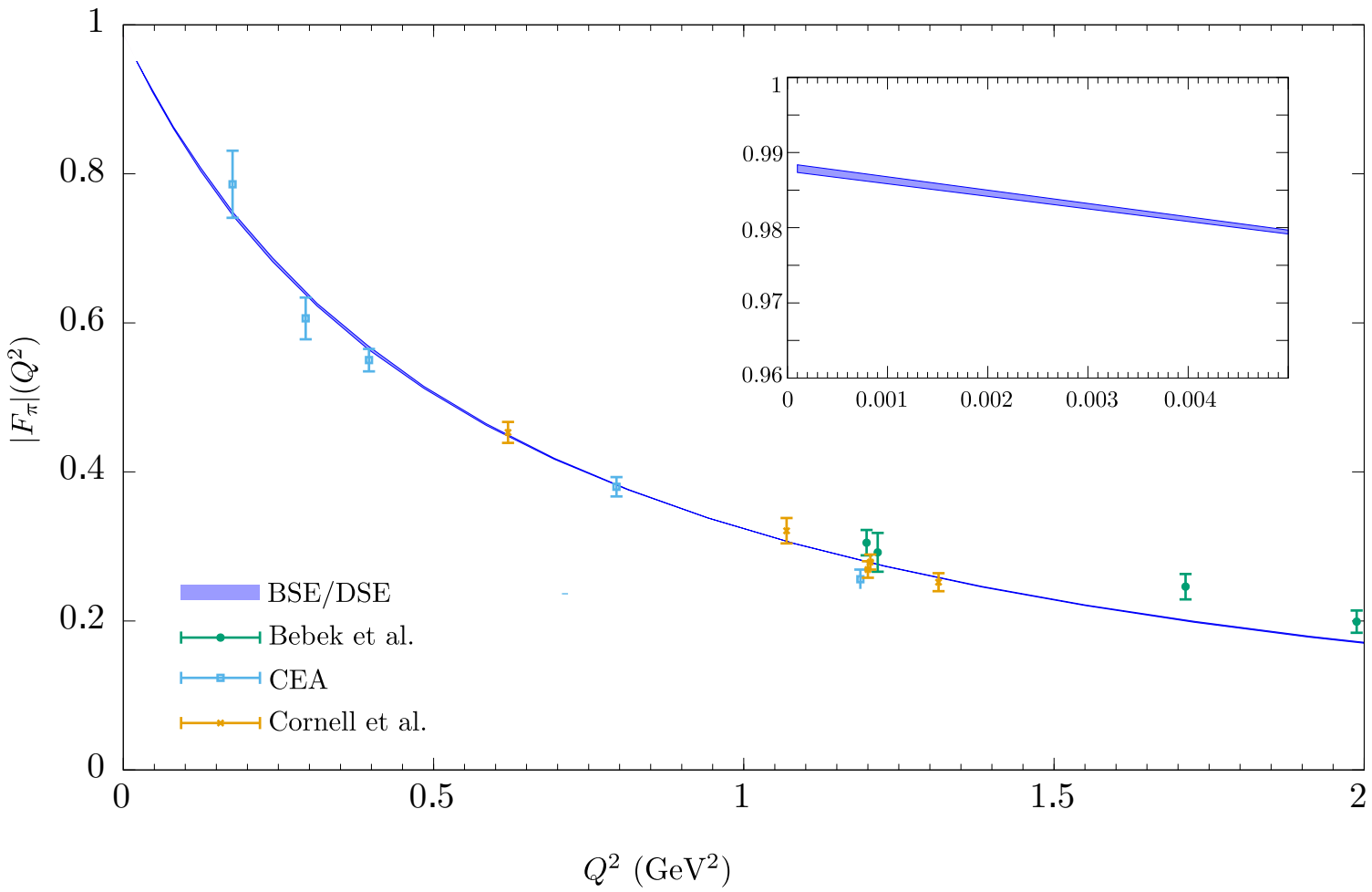
For two different parameters η of the Maris-Tandy model:

	m_π	f_π	m_ρ	m_ω	M_ρ	Γ_ρ
$\eta = 1.5$	0.139	0.138	0.768	0.778	0.750	0.100
$\eta = 1.6$	0.126	0.138	0.774	0.784	0.759	0.105

m_ρ and m_ω: Masses (in GeV) without two-pion decay kernel

M_ρ and Γ_ρ (in GeV) determined from ρ-meson pole position defined as $M^2_{pole} = M^2_\rho - iM_\rho\Gamma_\rho$ with two-pion decay kernel taken into account

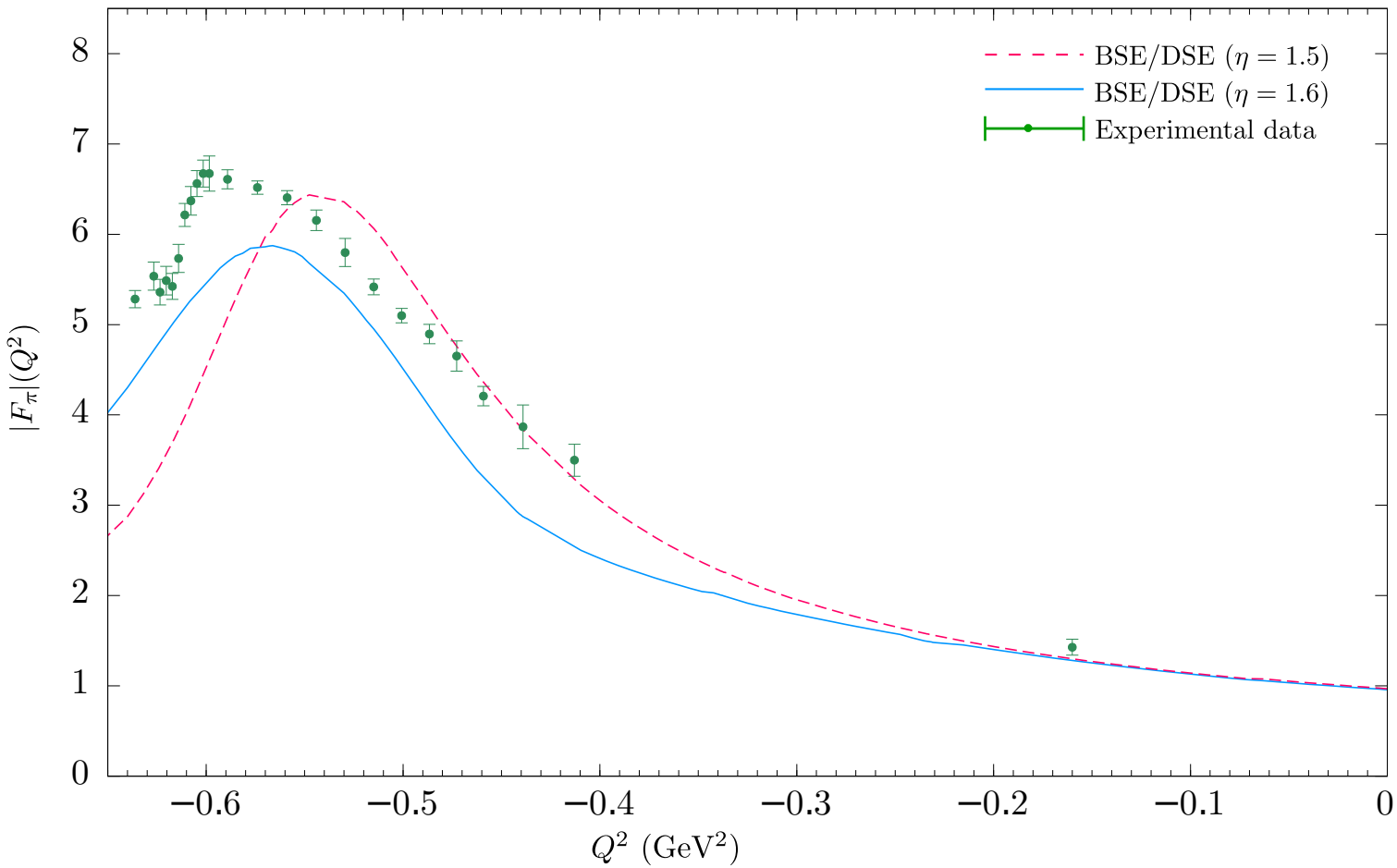
Results



Pion form factor in the space-like $Q^2 > 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$ compared to experimental data.

(The inset illustrates the impact of one of the technically motivated approximations.)

Results



Absolute value of the pion form factor in the time-like $Q^2 < 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$.

Results

Predicted by VMD (without ρ-ω mixing):

$$Re\, F_\pi(Q^2) - 1 = -\frac{a_1 Q^2 + a_2 (Q^2)^2}{b_0 + b_1 Q^2 + b_2 (Q^2)^2}$$

$$Im\, F_\pi(Q^2) = \frac{c_1 Q^2 + c_2 (Q^2)^2}{d_0 + d_1 Q^2 + d_2 (Q^2)^2}\,.$$

and verified by our "microscopic model" calculation

	η =1.5	η =1.6	VMD
a_1	0.5587	0.4149	0.72
a_2	0.8828	0.6827	1.2
b_0	0.3600	0.3600	0.36
b_1	1.2307	1.2517	1.2
b_2	1.0722	1.1000	1.0037
c_1	0.0591	0.0997	0
c_2	0.1295	0.2383	0.2308
d_0	0.3600	0.3600	0.36
d_1	1.1924	1.2464	1.2
d_2	0.9973	1.0916	1.0037

Results

- Other terms than VMD-predicted ones are tiny:
  Elaborated **calculation yields** within error margin
  the VMD predicted functional form.
- No significant impact from quark propagator poles!
  (Wanted in view of confinement! But why in this model-based calculation?)
- **The resulting time-like pion form factor**
  in the region $0 > Q^2 > 0.8\,\mathbf{GeV}^2$ **is determined by**
  the ρ**-meson pole and the two-pion cut!**

Conclusions & Outlook

- ☺ Exploratory DSE/BSE calculation of pion time-like form factor

  (. . . we can do time-like . . .)

- ☺ ρ-meson resonance & 2π cut determine time-like pion form factor: Detailed verification of VMD from microscopic model!
- ☺ Despite modelling and technical limitations: Remarkable agreement with experiment.

Outlook:

$\Longrightarrow$ Isospin breaking:

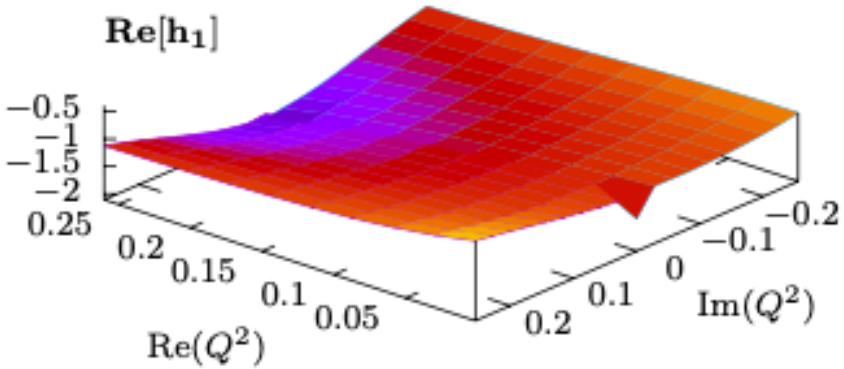
- Effect of different quark masses vs. electric charges
- ρ-ω mixing

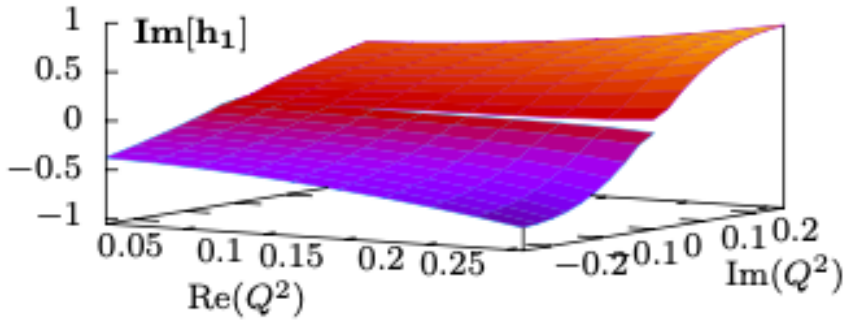
$\Longrightarrow$ $\gamma\ \pi\ \pi\ \pi$ form factor:

- Anomaly determining soft-point value vs.
- effect of hadron resonances (ρ & ω)

$\Longrightarrow$ Long-term wish list: Time-like form factors from first-principle "functional" calculations.

Results

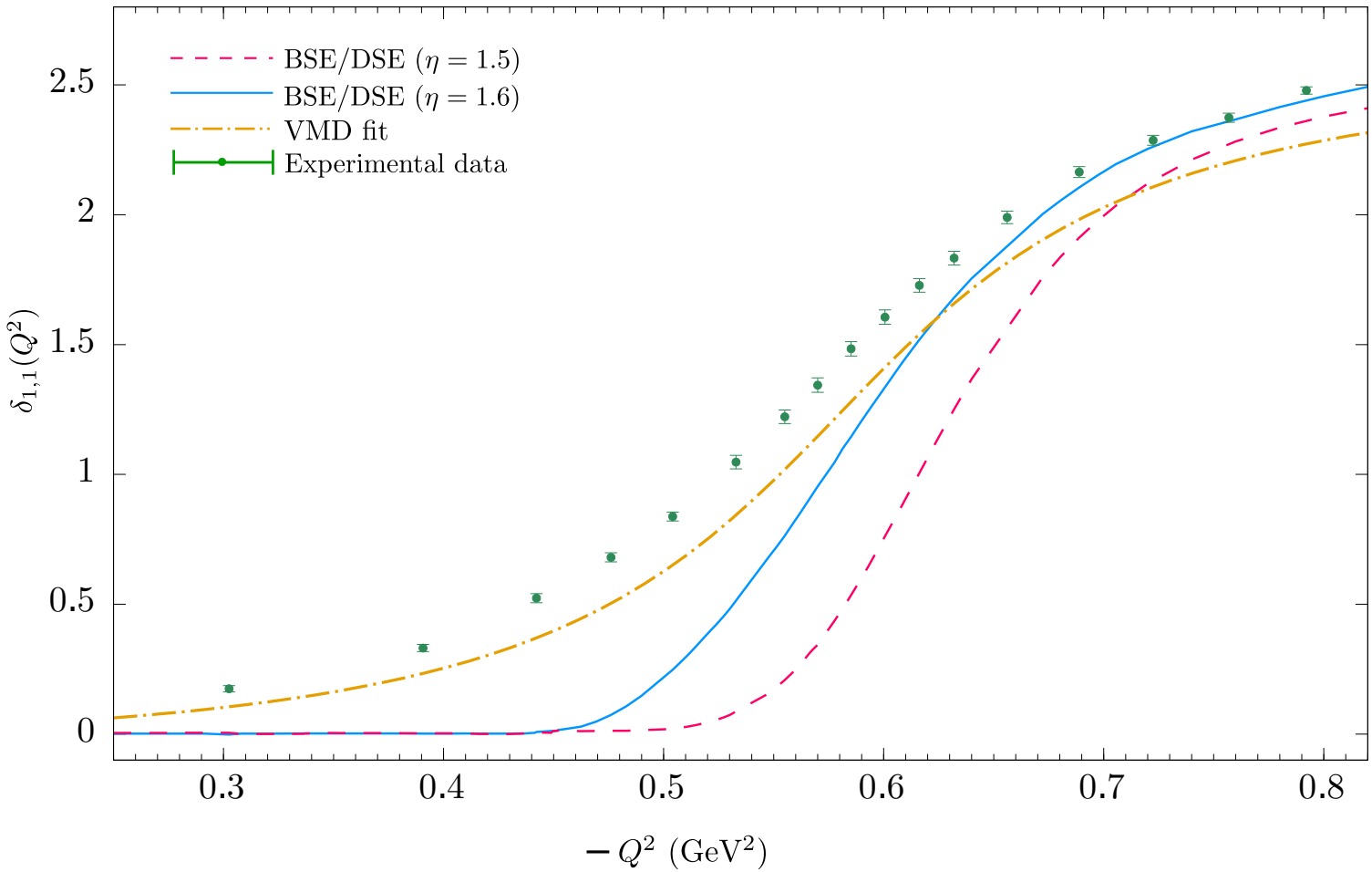




Real and imaginary part of the leading (transversely projected) amplitude of the quark-photon vertex for $p \cdot Q = 0$.
The two-pion branch cut starts at $Q^2 = -4m_\pi^2$.

[A. S. Miramontes,H. Sanchis-Alepuz, EPJA **55** (2019) 170 [arXiv:1906.06227].]

Results



Phase of the pion form factor in the time-like $Q^2 < 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$ compared to experimental data on pion-pion phase shift.