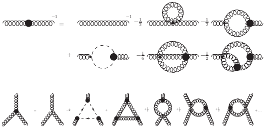
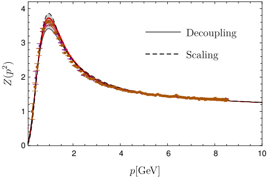
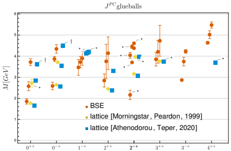
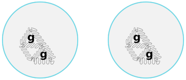
On the mass spectrum of glueballs with even charge parity









Markus Q. Huber

Institute of Theoretical Physics, Giessen University

MQH, Phys.Rev.D 101, arXiv:2003.13703

MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C 80, arXiv:2004.00415

ACHT2021

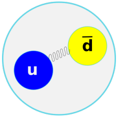
April 21, 2021

FWF Der Wissenschaftsfonds.

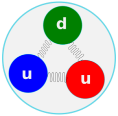
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Bound states in QCD

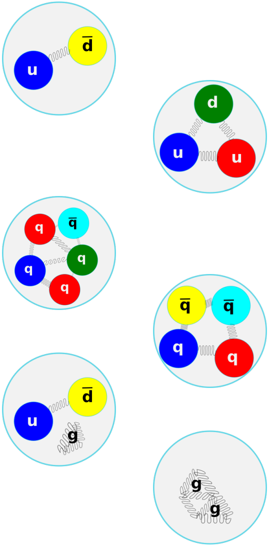


Mesons



Baryons

Bound states in QCD



Mesons	
Baryons	
Pentaquarks	First observations 2015 (LHCb)
Tetraquarks	Increasing number of confirmed states
Hybrids	
Glueballs	States of pure ’radiation’

Glueball observations

Experimental candidates, but situation not conclusive.

Scalar glueball: 0^{++}, mixing with scalar isoscalar mesons

Candidate reaction: $J/\psi \to \gamma + 2g$

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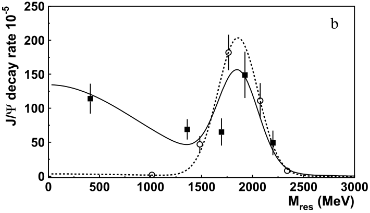
Candidate reaction: $J/\psi \to \gamma + 2g$

Recent analysis of BESIII data [Sarantsev, Denisenko, Thoma, Klempt '21]:

$$M = 1865 \pm 25^{+10}_{-30}\,\text{MeV},$$
$$\Gamma = 370 \pm 50^{+30}_{-20}\,\text{MeV}$$



Future experiments: e.g., PANDA, GlueX

Glueball calculations

Yang-Mills theory

- “Isolated” problem: only gluons
- Clean picture: well-established lattice results

Glueball calculations

Yang-Mills theory

- "Isolated" problem: only gluons
- Clean picture: well-established lattice results

Unquenching on the lattice [Gregory et al. '12]:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- Tiny (e.g., 0^{++}, 2^{++}) to moderate unquenching effects (e.g., 0^{-+}) found

Glueball calculations

Yang-Mills theory

- “Isolated” problem: only gluons
- Clean picture: well-established lattice results
- Functional methods: High quality input available for bound state equations

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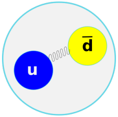
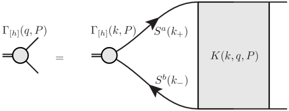
Hadrons from bound state equations

Example: Meson



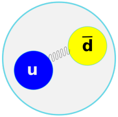


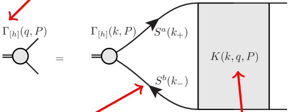
Integral equation: $\Gamma(q,P) = \int dk\, \Gamma(k,P)\, S(k_+)\, S(k_-)\, K(k,q,P)$

Hadrons from bound state equations

Bethe-Salpeter amplitude

Example: Meson

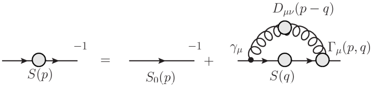




Integral equation: $\Gamma(q,P) = \int dk\, \Gamma(k,P)\, S(k_+)\, S(k_-)\, K(k,q,P)$

Ingredients:

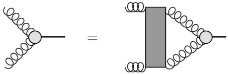
- Quark propagator S



Nonperturbative diagram: full momentum dependent dressings $\rightarrow$ numerical solution

- Interaction kernel K
- Constrained by symmetries

Glueball BSE



Need  and , solve for . $\rightarrow$ Mass

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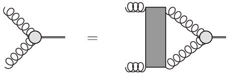
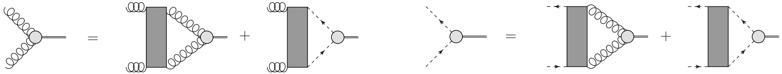


Need  and , solve for . $\rightarrow$ Mass

Not quite. . .

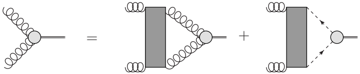
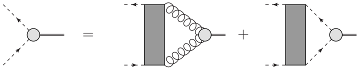
Glueball BSE



Gluons couple to ghosts → Include 'ghostball'-part.

(First step: no quarks
→ Yang-Mills theory)

Glueball BSE

Gluons couple to ghosts → Include 'ghostball'-part.

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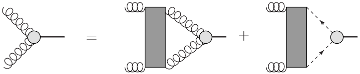
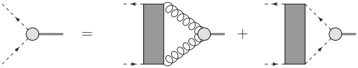
Need ,  and $4\times$, solve for  and . → Mass

Construction of kernel

Consistency with input: Apply same construction principle.

Glueball BSE





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Previous BSE calculations for glueballs:

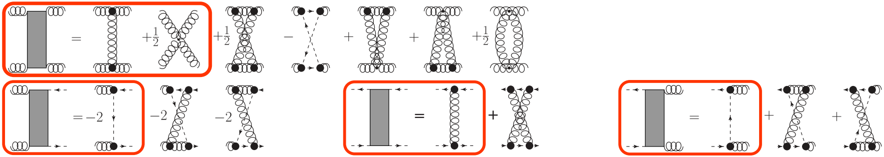
- [Meyers, Swanson '13]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal '15]
- [Souza et al. '20]
- [Kaptari, Kämpfer '20]

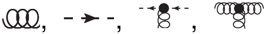
⇒ Input is important for quantitative predictive power!
[MQH, Fischer, Sanchis-Alepuz '20]

Kernel construction

From 3PI effective action truncated to three-loops:

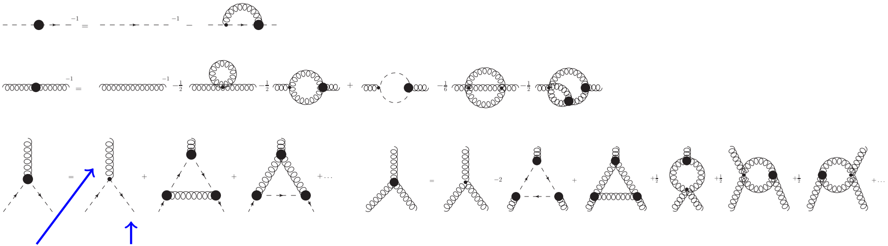
[Fukuda '87; McKay, Munczek '89; Sanchis-Alepuz, Williams '15; MQH, Fischer, Sanchis-Alepuz '20]



$\rightarrow$ Need .

- Some diagrams vanish for certain quantum numbers.
- Full QCD: Same for quarks $\rightarrow$ Mixing with mesons.

Equations of motion from 3-loop 3PI effective action

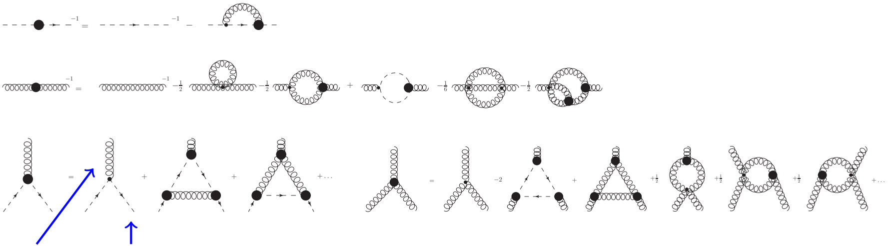


Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

Truncation?

Equations of motion from 3-loop 3PI effective action



Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

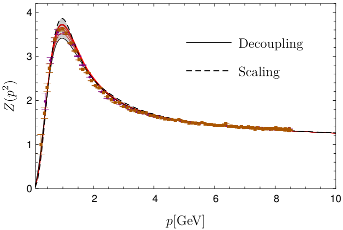
Self-contained system of equations with the scale as the only input.

Truncation→ 3-loop expansion of 3PI effective action [Berges '04]

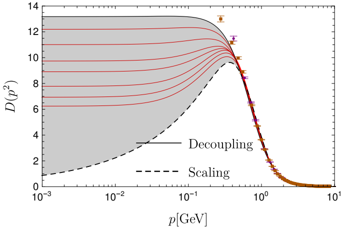
- 4 coupled integral equations with full kinematic dependence.
- Sufficient numerical accuracy required for renormalization.
- One- and two-loop diagrams [Meyers, Swanson '14; MQH '17].

Landau gauge propagators

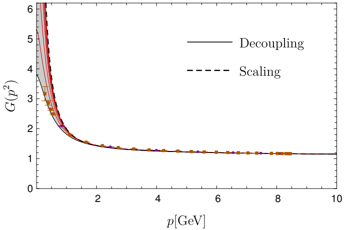
Gluon dressing function:



Gluon propagator:



Ghost dressing function:



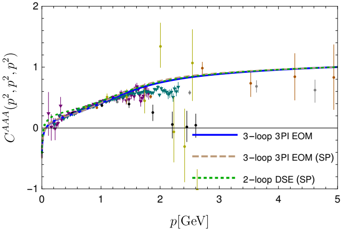
[Sternbeck '06; MQH '20]

- Family of solutions: Nonperturbative completions of Landau gauge [Maas '10]?
- Realized by condition on $G(0)$ [Fischer, Maas, Pawlowski '08; Alkofer, MQH, Schwenzer '08]

Concurrence of functional methods

Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; MQH ’20]

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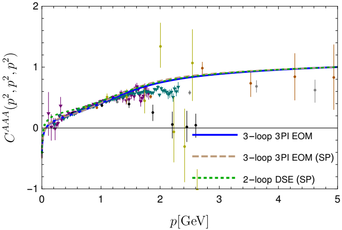

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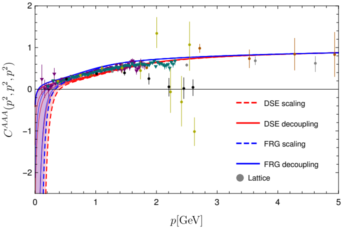
DSE vs. FRG:



[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; Cyrol et al. ’16; MQH ’20]

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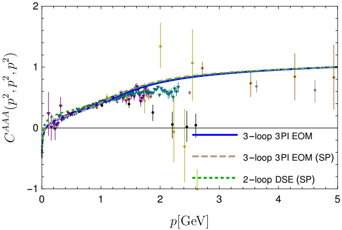

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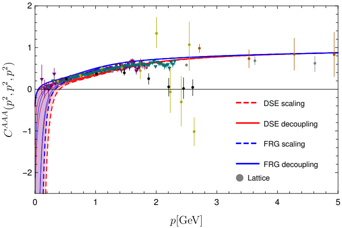
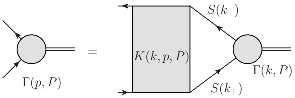


[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; Cyrol et al. '16; MQH '20]

Beyond this truncation

- Further dressings of three-gluon vertex [Eichmann, Williams, Alkofer, Vujinovic '14]
- Effects of four-point functions [MQH '16, MQH '17, Corell et al. '18, MQH '18]

Solving a BSE



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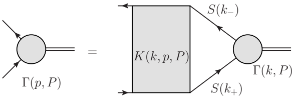
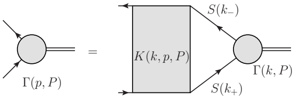


Consider the eigenvalue problem (Γ is the BSE amplitude)

$$\mathcal{K} \cdot \Gamma(P) = \boldsymbol{\lambda(P)}\, \Gamma(P).$$

$\lambda(P^2) = 1$ is a solution to the BSE $\Rightarrow$ Glueball mass $P^2 = -M^2$

Solving a BSE



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Calculation requires quantities for

$$k_\pm^2 = P^2 + k^2 \pm 2\sqrt{P^2\, k^2} \cos\theta = -M^2 + k^2 \pm 2\, \boldsymbol{i}\, M \sqrt{k^2}\, \cos\theta.$$

$\Rightarrow$ Complex momentum arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

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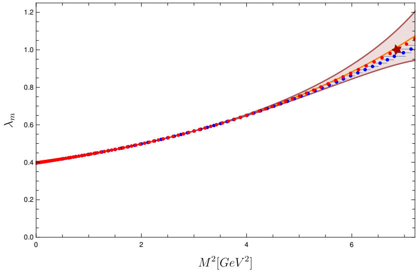
Test extrapolation for solvable system: Heavy meson



[MQH, Sanchis-Alepuz, Fischer '20]

Extrapolation of $\lambda(P^2)$ for glueballs

Higher eigenvalues: Excited states.

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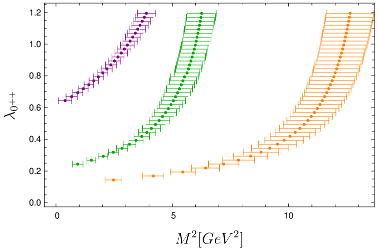
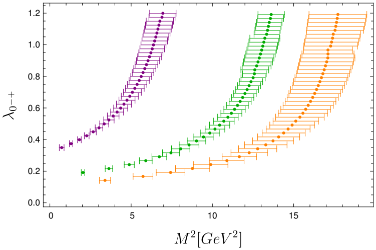
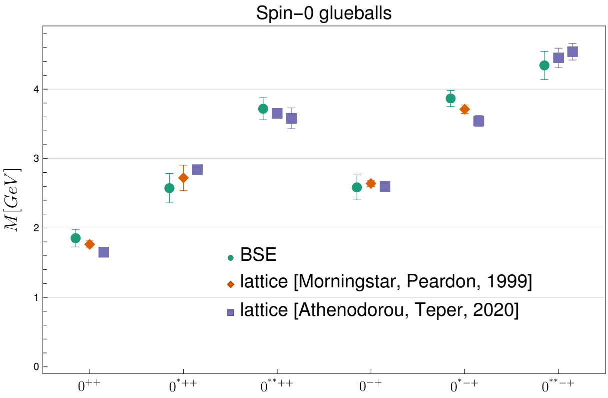
Higher eigenvalues: Excited states.





Physical solutions for $\lambda(P^2) = 1$.

Glueballs masses for $0^{\pm +}$



Lattice 0^{**++}: Conjectured based on irred. rep. of octahedral group

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz '20]

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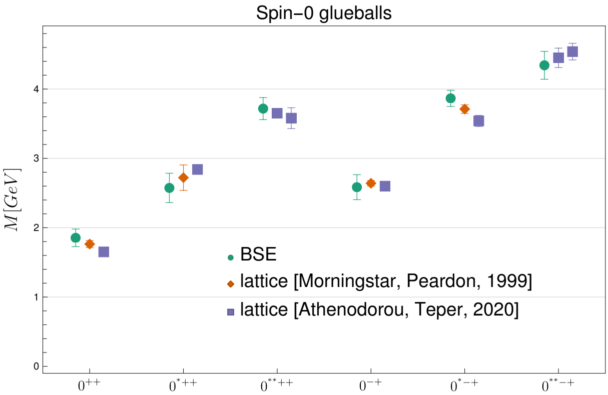

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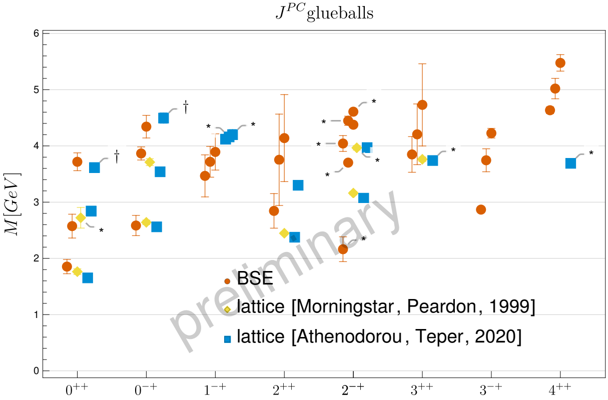
[MQH, Fischer, Sanchis-Alepuz '20]

Under conjecture that choice of solution is a gauge choice: Explicit test of gauge independence!

Tested that results are independent of family of solutions.

Glueball masses for $J^{\pm +}$



Lattice:
*: identification with some uncertainty
†: conjecture based on irred. rep of octahedral group

[MQH, Fischer, Sanchis-Alepuz, in preparation]

Summary

Parameter-free determination of glueball masses from functional methods.

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Thank your for your attention.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(x-y) = \langle O(x)O(y)\rangle$$

- $\rightarrow$ Lattice: Mass from this correlator by exponential Euclidean time decay.
- Complicated object in a diagrammatic language: 2-, 3- and 4-gluon contributions

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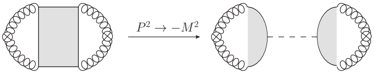
Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. $\rightarrow$ Each can have a pole at the glueball mass.

A^4-part of $D(x - y)$, total momentum on-shell:



Charge parity

Transformation of gluon field under charge conjugation:

$$A_\mu^a \to -\eta(a) A_\mu^a$$

where

$$\eta(a) = \begin{cases} +1 & a = 1,3,4,6,8 \\ -1 & a = 2,5,7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A_\mu^a A_\nu^a \to \eta(a)^2 A_\mu^a A_\nu^a = A_\mu^a A_\nu^a.$$

$\Rightarrow C = +1$

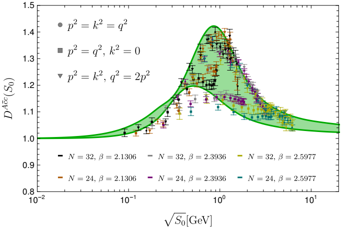
Negative charge parity, e.g.:

$$d^{abc} A_\mu^a A_\nu^b A_\rho^c \to - d^{abc} \eta(a)\eta(b)\eta(c) A_\mu^a A_\nu^b A_\rho^c = \\ - d^{abc} A_\mu^a A_\nu^b A_\rho^c.$$

Only nonvanishing elements of the symmetric structure constant d^{abc}: zero or two indices equal to 2, 5 or 7.

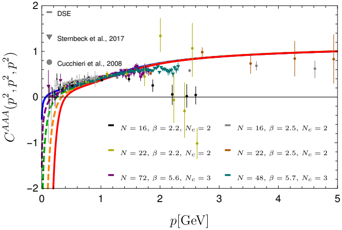
Landau gauge vertices

Ghost-gluon vertex:



[Maas '19; MQH '20]

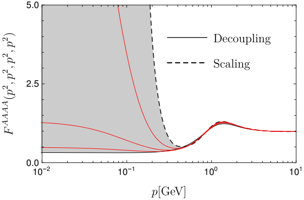
Three-gluon vertex:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

- Nontrivial kinematic dependence of ghost-gluon vertex
- Simple kinematic dependence of three-gluon vertex
- Four-gluon vertex from solution

Four-gluon vertex:

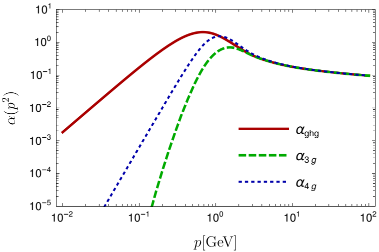


[MQH '20]

Some properties of the Landau gauge solution

[MQH '20]

- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime



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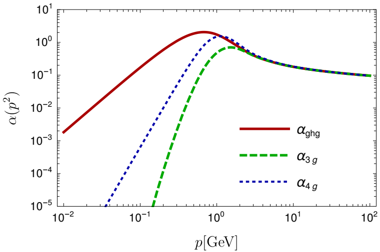
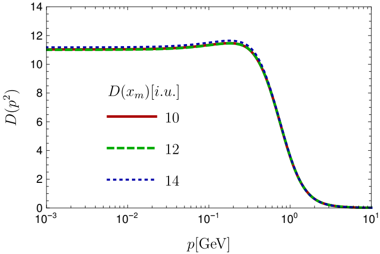


- Renormalization: First parameter-free subtraction of quadratic divergences
  $\Rightarrow$ One unique free parameter (family of solutions)



Landau gauge propagators in the complex plane

Propagators for complex momenta

- Reconstruction from Euclidean results: mathematically ill-defined, bias in solution
- Direct calculation from functional methods possible, e.g., contour deformation or spectral DSEs [Horak, Pawlowski, Wink '20]

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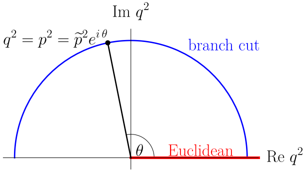
Contour deformation: Special technique to respect analyticity (avoid branch cuts in the integrand)

- QED3 [Maris '95 (QED)]
- Quark propagator [Alkofer, Fischer, Detmold, Maris '04]
- Self-consistent solution: Ray technique, YM propagators [Strauss, Fischer, Kellermann '12; Fischer, MQH '20]
- Glueball correlators [Windisch, Alkofer, Haase, Liebmann '13; Windisch, MQH, Alkofer '13]
- Meson decays [Weil, Eichmann, Fischer, Williams '17; Williams '18]
- Spectral functions at $T > 0$ [Pawlowski, Strodthoff, Wink '18]
- Quark-photon vertex [Miramontes, Sanchis-Alepuz '19]
- Scalar scattering amplitude [Eichmann, Duarte, Pena, Stadler '19]

Landau gauge propagators in the complex plane



Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

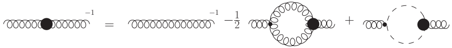
Deformation of integration contour necessary [Maris '95]. Recent resurgence: [Alkofer et al. '04; Windisch, MQH, Alkofer, '13; Williams '19; Miramontes, Sanchis-Alepuz '19; Eichmann et al. '19], . . .

Ray technique for self-consistent solution of a DSE: [Strauss, Fischer, Kellermann; Fischer, MQH '20].

Landau gauge propagators in the complex plane

Technique to resp. analyticity (avoid branch cuts in integrand): Contour deformation

Simpler truncation:



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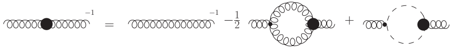
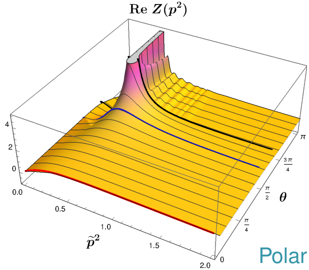
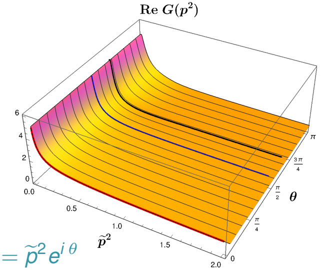




Polar coordinates: $p^2 = \tilde{p}^2 e^{i\,\theta}$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)
- No proof of existence of complex conjugate poles due to simple truncation.

[Fischer, MQH '20]