Large N_c behavior of thermodynamical quantities from the (axial)vector meson extended PQM model

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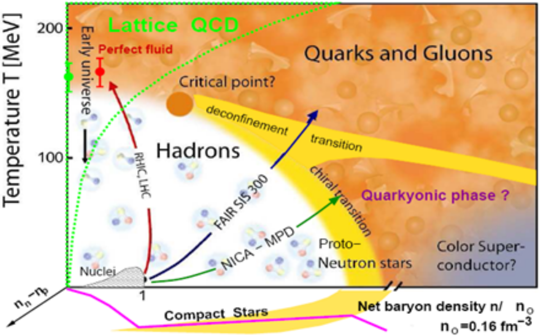
Overview

Envisaged phase diagram of QCD



Properties of the phase diagram can be investigated in large N_c $\longrightarrow$ scaling properties of various quantities $\longrightarrow$ can be different in diff. effective models

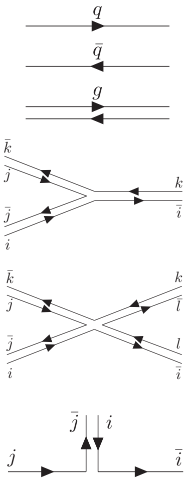
Some basics properties of Large N_c I.

G. 't Hooft. (1974), Nucl. Phys. B 72:461

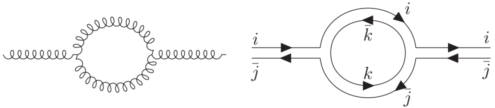
G. 't Hooft. (1974), Nucl. Phys. B 75:461–470

E. Witten. (1979), Nucl. Phys. B 160:57–115

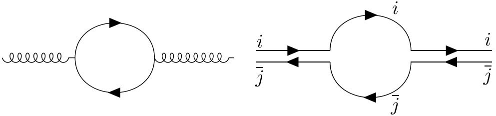
- No expansion parameter in QCD if $m_{u/d/s} \approx 0 \longrightarrow$ not so obvious expansion parameter: N_c
- $SU(3) \longrightarrow SU(N_c)$
- double line notation based on color structure of gluons: $A_j^{\mu;\,i} \sim q^i \bar{q}_j$
- 3-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} \partial^\mu A^{\nu;\,k}_i$
- 4-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} A^{\mu;\,k}_l A^{\nu;\,l}_i$
- quark - gluon vertex: $\bar{q}_i \gamma^\mu q^j A^i_{\mu;\,j}$



Some basics properties of Large N_c II.



N_c combinatorial factor due to closed color loop $\Longrightarrow$ $g \sim \frac{1}{\sqrt{N_c}}$



Quark loops are $1/N_c$ suppressed.

Leading diagrams are palanar diagrams with minimum number of quark loops

Investigation of N-point functions of quark bilinears ($J = \bar{q}q$, $\bar{q}\gamma^\mu q$) leads to the large N_c properties of mesons

Some properties of mesons and baryons for Large N_c

- mesons are free, stable, and non-interacting
- mesons are pure $q\bar{q}$ for Large N_c
- meson masses $\sim N_c^0$
- meson decay amplitudes $\sim 1/\sqrt{N_c}$
- for one meson creation: $< 0|J|m > \sim \sqrt{N_c}$
- k meson vertex $\sim N_c^{1-k/2}$. Specifically, the three- and four-meson vertices are $\sim 1/\sqrt{N_c}$ and $\sim 1/N_c$, respectively
- baryon masses $\sim N_c$. Consequently constituent quark masses $\sim N_c^0$

Lagrangian of the ePQM

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$
\begin{aligned}
\mathcal{L} = {} & \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} \mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i \frac{g_2}{2} (\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \mathrm{Tr}(\Phi^\dagger \Phi) \mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2 h_3 \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi,
\end{aligned}
$$

$$
\begin{aligned}
D^\mu \Phi &= \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - i e A_e^\mu [T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - i e A_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - i e A_e^\nu [T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - i e A_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - i e A_e^\nu [T_3, R^\mu]\}, \\
D^\mu \Psi &= \partial^\mu \Psi - i G^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}
$$

+ Polyakov loop potential (for $T > 0$)

Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar $(\sim \bar{q}_i q_j)$ and pseudoscalar $(\sim \bar{q}_i \gamma_5 q_j)$ meson nonets

$$\Phi_S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad \Phi_{PS} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

unknown assignment
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$

fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i} \right]^2 ,$$

$(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N) \longrightarrow$ from the model, $Q_i^{\mathrm{exp}} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Features of the planned approximation

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – u, d, s constituent quarks ($m_u = m_d$)
  – (Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\mathrm{YM}}$ or $\mathcal{U}_{\log}^{\mathrm{glue}}$)
- no mesonic fluctuations, only fermionic ones

$$\mathcal{Z} = e^{-\beta V \Omega(T,\mu_q)} = \int_{\mathrm{PBC}} \prod_a \mathcal{D}\xi_a \int_{\mathrm{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x \left(\mathcal{L} + \mu_q \sum_f q_f^\dagger q_f\right)\right]$$

approximated as $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$, $\tilde{\mu}_q = \mu_q - iG_4$

$$e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int_{\mathrm{APBC}} \prod_{f,g} \mathcal{D}q_g \mathcal{D}q_f^\dagger \exp\left\{\int_0^\beta d\tau \int_x q_f^\dagger \left[\left(i\gamma_0 \vec{\gamma}\cdot\vec{\nabla} - \frac{\partial}{\partial\tau} + \tilde{\mu}_q g - \gamma_0 \mathcal{M}_{fg}\big|_{\xi_a = 0}\right] q_g\right\}$$

- tree-level (axial)vector masses
- fermionic thermal fluctuations included in the (pseudo)scalar curvature masses
- 2 (or 4) coupled T/μ_B-dependent field equations for the condensates $\phi_N, \phi_S, (\Phi, \bar{\Phi})$
- Polyakov-loops and fermionic vacuum fluctuations will be included in the next step

N_c scaling of the Lagrange parameters

The parameters are: $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F,$ h_N, h_S

- m_0^2, m_1^2, $\delta_s \sim N_c^0$, because terms of tree level meson masses
- g_1, $g_2 \sim \frac{1}{\sqrt{N_c}}$, three couplings
- λ_2, h_2, $h_3 \sim \frac{1}{N_c}$, four couplings
- λ_1, $h_1 \sim \frac{1}{N_c^2}$, four couplings with different trace structure
- $c_1 \sim \frac{1}{N_c^{3/2}}$ $U_A(1)$ anomaly term has extra $1/N_c$ suppression
- $\Phi_{N/S} \sim \sqrt{N_c}$, $\Phi_N = Z_\pi f_\pi$, $f_\pi \sim \sqrt{N_c}$
- $h_{N/S} \sim \sqrt{N_c}$, Goldstone-theorem: $m_\pi^2 \Phi_N = Z_\pi^2 h_N$
- $g_F \sim \frac{1}{\sqrt{N_c}}$, $m_{u/d} = g_F \Phi_N$

practically: $g_1 \longrightarrow g_1\sqrt{\frac{3}{N_c}}$, $\Phi_{N/S} \longrightarrow \Phi_{N/S}\sqrt{\frac{N_c}{3}}$ ... *etc.*

Parameter sets

For lower $m_\sigma = 600$ MeV

Φ_N	0.092
Φ_S	0.095
m_0^2	-0.036
m_1^2	0.395
λ_1	-17.01
λ_2	82.47
h_1	-9.0
h_2	11.659
h_3	4.703
δ_S	0.153
c_1	0.0
g_1	-5.894
g_2	-2.996
g_F	6.494

For higher $m_\sigma = 1300$ MeV

Φ_N	0.162
Φ_S	0.124
m_0^2	-0.754
m_1^2	0.395
λ_1	0.0
λ_2	65.322
h_1	0.0
h_2	11.659
h_3	4.703
δ_S	0.153
c_1	1.121
g_1	-5.894
g_2	-2.996
g_F	4.943

Field equations, masses in Large N_c

Field equations:

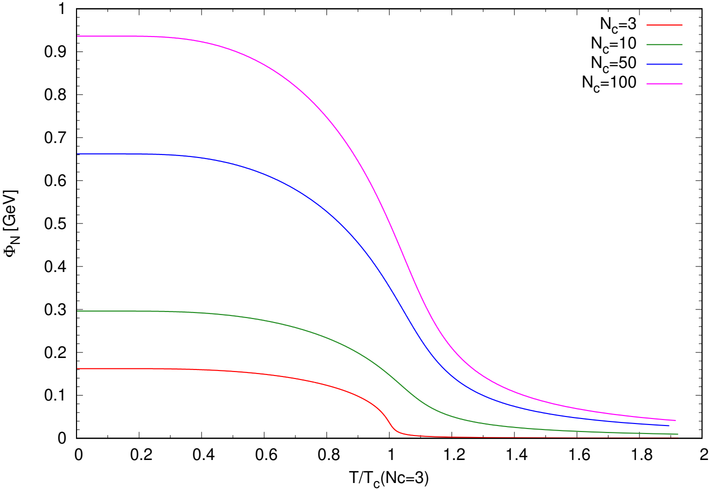
$$m_0^2\Phi_N\sqrt{\frac{N_c}{3}}+\left(\lambda_1\frac{3}{N_c}+\frac{\lambda_2}{2}\right)\Phi_N^3\sqrt{\frac{N_c}{3}}+\lambda_1\Phi_N\Phi_S^2\sqrt{\frac{3}{N_c}}-\frac{1}{\sqrt{2}}c_1\sqrt{\frac{3}{N_c}}\Phi_N\Phi_S -h_N\sqrt{\frac{N_c}{3}}+\frac{3}{2}g_F(\mathrm{Tad}_u+\mathrm{Tad}_d)=0$$

$$m_0^2\Phi_S\sqrt{\frac{N_c}{3}}+\left(\lambda_1\frac{3}{N_c}+\lambda_2\right)\Phi_S^3\sqrt{\frac{N_c}{3}}+\lambda_1\Phi_N^2\Phi_S\sqrt{\frac{3}{N_c}}-\frac{1}{2\sqrt{2}}c_1\sqrt{\frac{3}{N_c}}\Phi_N^2 -h_S\sqrt{\frac{N_c}{3}}+\frac{3}{\sqrt{2}}g_F\mathrm{Tad}_s=0$$

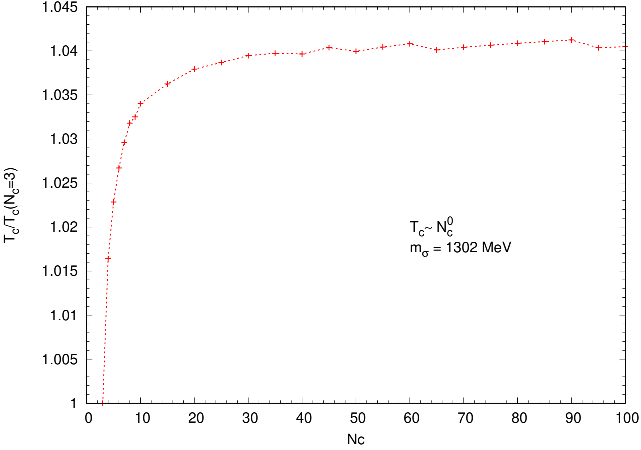
Pion three-level mass:

$$m_\pi^2=Z_\pi^2\left(m_0^2+\left(\lambda_1\frac{3}{N_c}+\frac{\lambda_2}{2}\right)\Phi_N^2+\lambda_1\frac{3}{N_c}\Phi_S^2-c_1\frac{3}{N_c}\frac{\Phi_S}{\sqrt{2}}\right)$$

$\phi_N(T)$ at diff. N_c values ($\mu_B = 0$, $m_\sigma = 1300$ MeV)

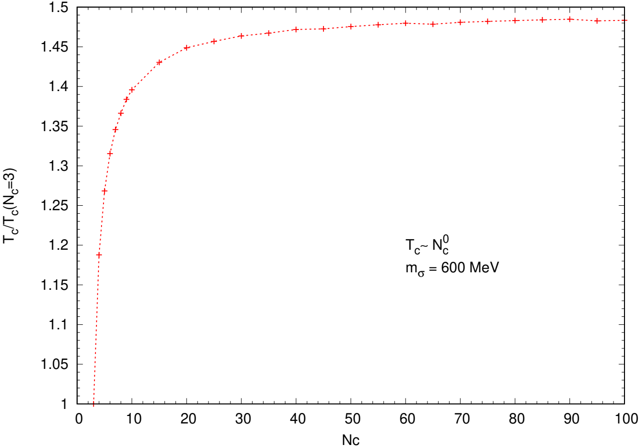


N_c scaling of the pseudocrit. T_c ($m_\sigma = 1300$ MeV)



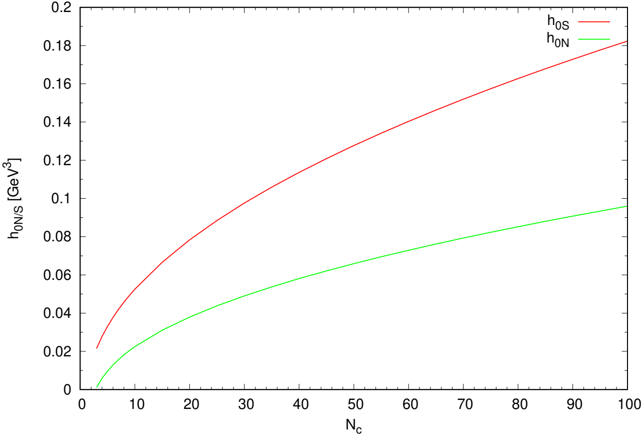
From $N_c = 3$ to $N_c = 100$: T_c changes $\approx 4\%$

N_c scaling of the pseudocrit. T_c ($m_\sigma = 600$ MeV)



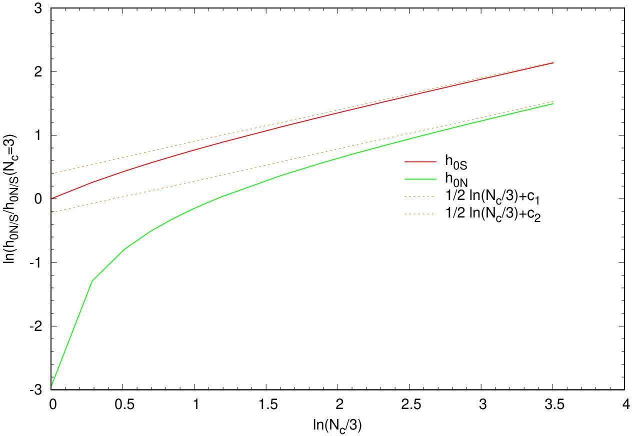
From $N_c = 3$ to $N_c = 100$: T_c changes $\approx 50\%$. In both cases $T_c \sim N_c^0$ as expected

N_c scaling of $h_{N/S}$ ($m_\sigma = 1300$ MeV)



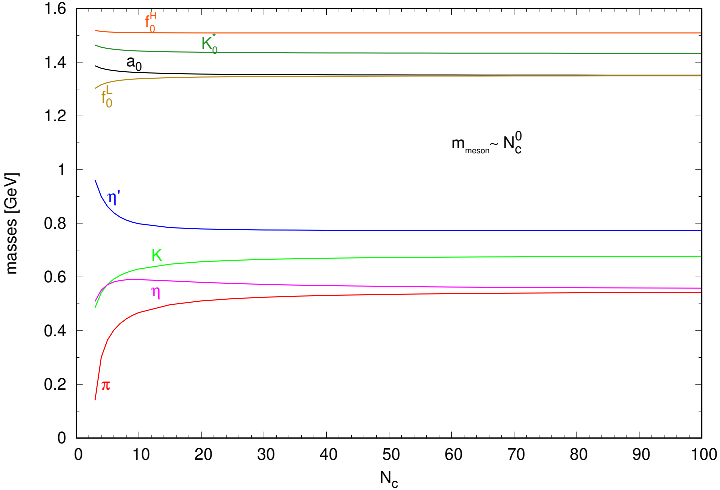
$h_{N/S}$ are calculated from the field equations at $T = \mu_B = 0$

N_c scaling of $h_{N/S}$ ($m_\sigma = 1300$ MeV)

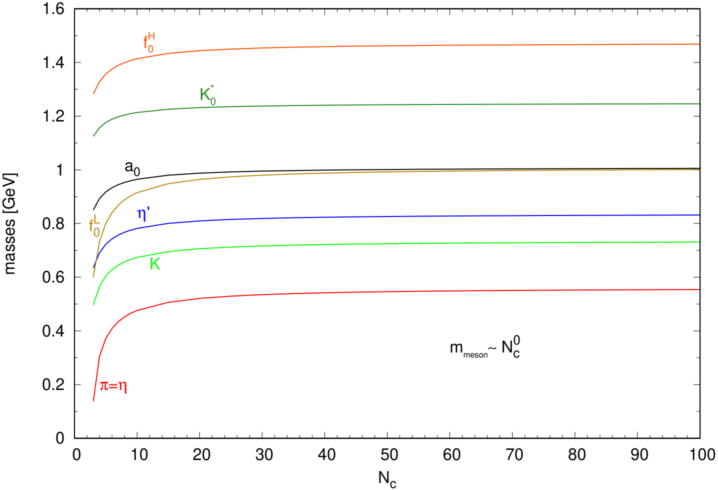


The expected N_c scaling ($h_{N/S} \sim \sqrt{N_c}$ are calculated from the field equations at $T = \mu_B = 0$

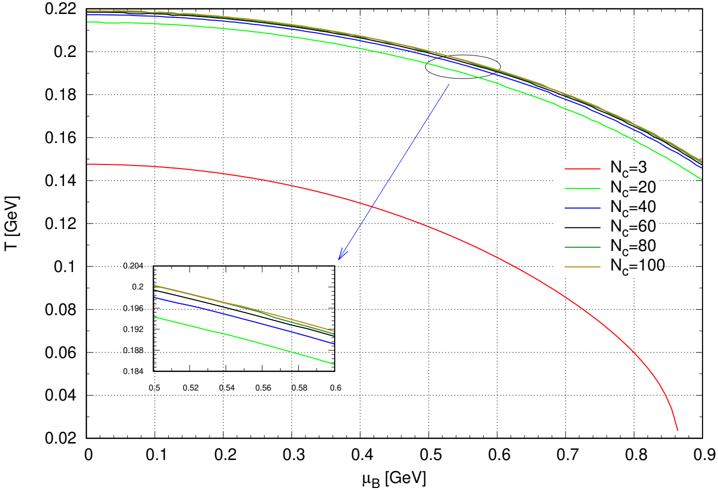
N_c scaling of meson masses ($m_\sigma = 1300$ MeV)



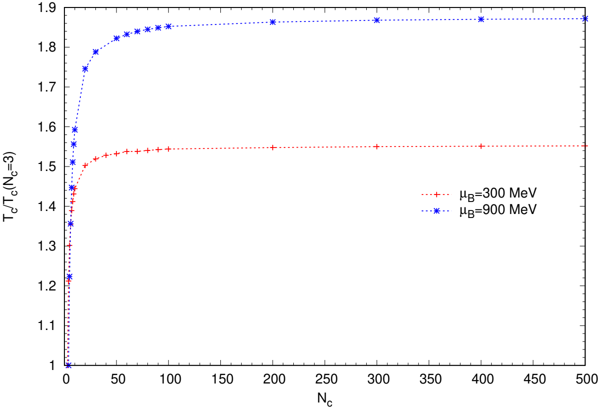
N_c scaling of meson masses ($m_\sigma = 600$ MeV)



Phase boundary at diff. N_c's ($m_\sigma = 600$ MeV)



T_c scaling at different μ_B's



For both low and high μ_B: T_c scales as $\sim N_c^0$. What happens if there is a CEP at large N_c?

Conclusion

Conclusion

- Large N_c scaling can be investigated in the ePQM model
- The pseudocritical temperature scales as expected ($\sim N_c^0$)
- Phase transition seems to get weaker with increasing N_c
- Changes of phase boundary is investigated as N_c changes

Plans

- Inclusion of vacuum fluctuation
- Inclusion of Polyakov-loop
- Scaling of the CEP
- Scaling of other thermodynamical qunatities
- Role of a more consistent approximation

Thank you for your attention!