Chiral U(1) model and restoration of symmetry at the 2-loop level

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Based on work with Amon Ilakovac, Hermès Bélusca-Maïto, Paul Kühler & Dominik Stöckinger, upcoming paper *Two-Loop Renormalization of a Chiral QED in Dimensional Regularization and BRST Restoration*

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PRECIOUS

Dimensional regularization and chirality

- Precise predictions in Standard Model and beyond are possible due to renormalizability & knowledge of relevant input parameters.
- The formal proofs of renormalizability rely on assumption that gauge invariant regularization exists.
- Involved multiloop calculations feasible thanks to Dimensional Regularization (DR).
- DR preserves BRST symmetry of vector-like gauge theories (no chiral objects present).
- Multiplicative renormalization of the tree-level Lagrangian.

Chirality and dimensional regularization

- The existence of chiral fermions is a fundamental fact of nature.
- SM transformations of leptons under SU(2):

$$e_R \to e'_R = e_R\,,$$
$$\ell_L \to \ell'_L = e^{-i\omega^a T^a}\ell_L\,.$$

- γ_5 properties in $D = 4$:

$$\{\gamma^5, \gamma^\mu\} = 0\,,$$
$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}\,,$$
$$\mathrm{Tr}(ab) = \mathrm{Tr}(ba)\,.$$

Chirality and dimensional regularization

- γ_5 properties in $d \neq 4$ extension can not hold simultaneously $\rightarrow$ algebraic inconsistencies
- Naive scheme

$$\{\gamma^5, \gamma^\mu\} = 0, \quad \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) \neq 4i\epsilon^{\mu\nu\rho\sigma}, \quad \mathrm{Tr}(ab) = \mathrm{Tr}(ba).$$

- Non-cyclicity schemes

$$\{\gamma^5, \gamma^\mu\} = 0, \quad \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \quad \mathrm{Tr}(ab) \neq \mathrm{Tr}(ba).$$

- Breitenlohner-Maison-'t Hooft-Veltman scheme (BMHV)
  Breitenlohner&Maison'75,'76,'77.

$$\{\gamma^5, \gamma^\mu\} \neq 0, \quad \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \quad \mathrm{Tr}(ab) = \mathrm{Tr}(ba).$$

Breitenlohner-Maison-'t Hooft-Veltman scheme

- Mathematical rigor with perturbative all-order consistency with fundamental QFT properties - unavoidable in exact treatment of chiral theories.
- Extension to $d = 4 - 2\epsilon \to$ evanescent objects

$$d\text{-dim.}: g_{\mu\nu}\,, \quad 4\text{-dim.}: \bar{g}_{\mu\nu}\,, \quad (-2\epsilon)\text{-dim.}: \hat{g}_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}\,.$$

- Algebra properties:

$$\begin{aligned} &\{\gamma_5, \bar{\gamma}^\mu\} = 0\,, && [\gamma_5, \hat{\gamma}^\mu] = 0\,, \\ &\{\gamma_5, \gamma^\mu\} = \{\gamma_5, \hat{\gamma}^\mu\} = 2\gamma_5\hat{\gamma}^\mu\,, && [\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu] = 2\gamma_5\bar{\gamma}^\mu\,. \end{aligned}$$

- At what cost?

~~gauge invariance~~ ~~BRST~~ ~~multiplicative ren.~~

Chiral QED in $d = 4$

$$\mathcal{L} = \imath\overline{\psi_R}_i(\partial\!\!\!/^{\mu}\delta_{ij} + \imath e_A A\!\!\!/^{\mu}\mathcal{Y}_{Rij})\psi_{R_j} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \bar{c}\partial^2 c +$$
$$+\rho^\mu s A_\mu + \bar{R}^i s\psi_{R_i} + R^i s\overline{\psi_R}_i$$

- BRST transformations

$$sA_\mu = \partial_\mu c\,, \qquad s\psi_i = s\psi_{R_i} = \imath\, e_A\, c\,\mathcal{Y}_{Rij}\psi_{R_j}\,,$$
$$s\bar{c} = B \equiv -\frac{1}{\xi}\partial A, \qquad s\overline{\psi}_i = s\overline{\psi_R}_i = \imath\, e_A\,\overline{\psi_R}_j c\mathcal{Y}_{Rji}\,.$$

- Slavnov-Taylor identity for the tree-level action:

$$\mathcal{S}(S_0^{(4D)}) = \mathcal{S}\left(\int d^4x\mathcal{L}\right) = 0\,,$$

where the Slavnov-Taylor operator is given for a general functional $\mathcal{F}$ as

$$\mathcal{S}(\mathcal{F}) = \int d^4x\left(\frac{\delta\mathcal{F}}{\delta\rho^\mu}\frac{\delta\mathcal{F}}{\delta A_\mu} + \frac{\delta\mathcal{F}}{\delta\bar{R}^i}\frac{\delta\mathcal{F}}{\delta\psi_i} + \frac{\delta\mathcal{F}}{\delta R^i}\frac{\delta\mathcal{F}}{\delta\overline{\psi}_i} + B\frac{\delta\mathcal{F}}{\delta\bar{c}}\right)\,.$$

Chiral QED in $d \neq 4$

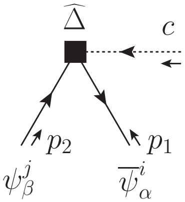
- Chiral 4-dimensional current $\overline{\psi_{R}}_i \gamma^\mu \psi_{R_j}$ can be extended in d in several ways:

$$\overline{\psi}_i \gamma^\mu P_R \psi_j \,, \qquad \overline{\psi}_i P_L \gamma^\mu \psi_j \,, \qquad \overline{\psi}_i P_L \gamma^\mu P_R \psi_j \,.$$

- Choice for fermionic part → left-chiral field is gauge singlet.

$$\begin{aligned}\mathcal{L}_{\text{fermions}} &= \imath \overline{\psi}_i \partial\!\!\!/ \psi_i + e_A \mathcal{Y}_{R ij} \overline{\psi_{R}}_i A\!\!\!/ \psi_{R_j} \\ &= \imath \overline{\psi}_i \overline{\partial\!\!\!/} \psi_i + e_A \mathcal{Y}_{R ij} \overline{\psi_{R}}_i A\!\!\!/ \psi_{R_j} + \imath \overline{\psi}_i \widehat{\partial\!\!\!/} \psi_i \,,\end{aligned}$$

- BRST breaking → $\mathcal{S}_d(S_0) = \mathcal{S}_d(S_{0,inv}) + \mathcal{S}_d(S_{0,evan}) = \widehat{\Delta}$



$$\begin{aligned}&= \frac{e_A}{2} \mathcal{Y}_{R ij} \left((\widehat{p\!\!\!/}_1 + \widehat{p\!\!\!/}_2) + (\widehat{p\!\!\!/}_1 - \widehat{p\!\!\!/}_2)\gamma_5 \right)_{\alpha\beta} \\ &= e_A \mathcal{Y}_{R ij} \left(\widehat{p\!\!\!/}_1 P_R + \widehat{p\!\!\!/}_2 P_L \right)_{\alpha\beta} \,.\end{aligned}$$

Symmetry restoration

- Total Lagrangian:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{0,\text{inv}} + \mathcal{L}_{0,\text{evan}} + \mathcal{L}_{\text{sct,inv}} + \mathcal{L}_{\text{sct,evan}} + \mathcal{L}_{\text{fct,inv}} + \mathcal{L}_{\text{fct,restore}} + \mathcal{L}_{\text{fct,evan}} \, .$$

- Gauge invariance and BRST symmetry are restored order by order by adding suitable counterterms $\rightarrow \mathcal{S}(\Gamma_{ren}^{(i)}) = 0 = s_d \Gamma_{ren}^{(i)}$.
- Upgrading to the multiloop case: where is breaking coming from?

$$\mathcal{S}_d(\Gamma_{DReg}) = (\widehat{\Delta} + \Delta_{ct}) \cdot \Gamma_{DReg} \, , \qquad RegQAP$$

where

$$\widehat{\Delta} \equiv s_d S_0 = \mathcal{S}_d S_0 \, ,$$

$$\Delta_{ct} \equiv s_d S_{ct} = \mathcal{S}_d(S_{ct}) = \sum_{i=1}^{\infty} \Delta_{ct}^i \, , \qquad \Delta_{ct}^i \equiv s_d S_{ct}^i \, .$$

Symmetry restoration

- Perturbative expansion of the effective action

$$\mathcal{S}_d(\Gamma_{DReg}) = \widehat{\Delta} + \sum_{i=1}^{\infty} \hbar^i \Big(\widehat{\Delta} \cdot \Gamma^i_{DReg} + \sum_{k=1}^{i-1} \Delta^k_{ct} \cdot \Gamma^{(i-k)}_{DReg} + \Delta^i_{ct}\Big)$$

in renormalization limit gives set of equations

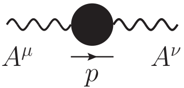
$$\mathrm{LIM}_{d\to 4}\, \widehat{\Delta} = 0 \; ,$$

$$\mathrm{LIM}_{d\to 4} \Big(\widehat{\Delta} \cdot \Gamma^i_{DReg} + \sum_{k=1}^{i-1} \Delta^k_{ct} \cdot \Gamma^{(i-k)}_{DReg} + \Delta^i_{ct}\Big) = 0 \; .$$

- Restoration example - back to U(1) Chiral QED (χQED).

Chiral QED in $d \neq 4$

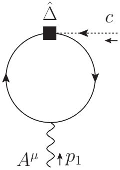
Gauge boson self-energy @1-loop



$$\imath\widetilde{\Gamma}^{\nu\mu}_{AA}(p)|^{(1)}_{\text{div, }\chi\text{QED}} = \frac{\imath e_A^2}{16\pi^2\epsilon}\frac{2\ \text{Tr}(\mathcal{Y}_R^2)}{3}(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) - \frac{\imath e_A^2}{16\pi^2\epsilon}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\widehat{p}^2\overline{g}^{\mu\nu},$$

$$\imath\widetilde{\Gamma}^{\nu\mu}_{AA}(p)|^{(1)}_{\text{div, QED}} = \frac{\imath e_A^2}{16\pi^2\epsilon}\frac{4\ \text{Tr}(\mathcal{Y}^2)}{3}(p^\mu p^\nu - p^2 g^{\mu\nu}).$$

Insertion of breaking

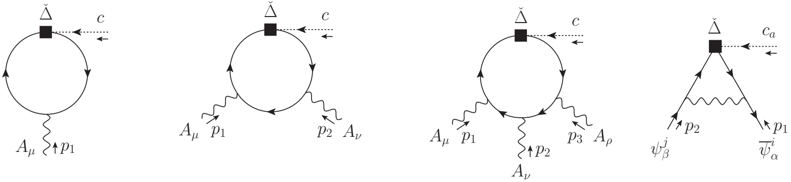


$$= \imath[\widehat{\Delta}\cdot\Gamma^\mu_{Ac}]^{(1)}_{\text{div}} = \frac{e_A^2}{16\pi^2\epsilon}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\widehat{p_1}^2\overline{p_1}^\mu,$$

$$\Delta^1_{sct} = \mathcal{S}_d\left(S^{(1)}_{\text{sct},\chi\text{QED}}\right) = -\frac{1}{16\pi^2\epsilon}\frac{e_A^2}{}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\int d^4x(\overline{\partial}_\mu c)\,\widehat{\partial}^2\bar{A}^\mu = -[\widehat{\Delta}\cdot\Gamma]^{(1)}_{\text{div}}.$$

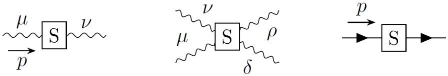
Chiral QED in $d \neq 4$

Finite BRST-restoration counterterms @1-loop



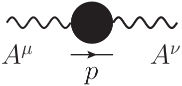
$$S^1_{\text{fct}} = \int d^4x \frac{1}{16\pi^2} \left(\frac{-e_A^2 \, \text{Tr}(\mathcal{Y}_R^2)}{6} \bar{A} \cdot (\bar{\partial}^2 \bar{A}) + \frac{e_A^4 \, \text{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 + \left(1 - \frac{1-\xi_A}{6}\right) e_A^3 \sum_j (\mathcal{Y}_R^3)_j \imath \overline{\psi}_j \overline{\gamma}^\mu \overline{\partial}_\mu \psi_j \right) + S_{\text{fct,BRSTinv}} \, .$$

- New operators in action $\rightarrow$ new counterterms $\rightarrow$ ~~multiplicative ren.~~



Chiral QED in $d \neq 4$

Gauge boson self-energy @2-loop



$$\imath\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\big|^{(2)}_{\text{div},\ \chi\text{QED}} = \frac{\imath e_A^4}{256\pi^4\epsilon}\frac{2\ \mathrm{Tr}(\mathcal{Y}_R^4)}{3}(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) + \frac{\imath e_A^4}{256\pi^4\epsilon}\frac{17\ \mathrm{Tr}(\mathcal{Y}_R^4)}{72}\widehat{p}^2\overline{g}^{\mu\nu} - \frac{\imath e_A^4}{256\pi^4\epsilon^2}\frac{\mathrm{Tr}(\mathcal{Y}_R^4)}{6}\widehat{p}^2\overline{g}^{\mu\nu},$$

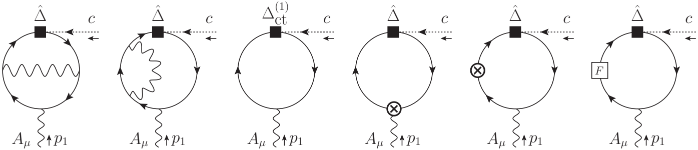
$$\imath\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\big|^{(2)}_{\text{div},\ \text{QED}} = \frac{\imath e_A^4}{256\pi^4\epsilon}2\ \mathrm{Tr}(\mathcal{Y}^4)(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}).$$

Insertion of breaking

$$\imath\left([\hat{\Delta} + \Delta^{(1)}_{\text{ct}}]\cdot\widetilde{\Gamma}\right)^{(2)}_{cA} = \frac{1}{256\pi^4}\frac{e_A^4\ \mathrm{Tr}(\mathcal{Y}_R^4)}{6}\left[\left(\frac{1}{\epsilon^2} - \frac{17}{12\epsilon}\right)\hat{p}_1^2\bar{p}_1^\mu - \frac{11}{4}\bar{p}_1^2\bar{p}_1^\mu + \mathcal{O}(\hat{\cdot})\right].$$

$$\left(\widehat{\Delta}\cdot\Gamma^2_{DReg} + \Delta^1_{ct}\cdot\Gamma^{(1)}_{DReg} + \Delta^2_{ct}\right)_{div} = 0$$

Chiral QED in $d \neq 4$



- BRST restoration in finite part

$$\left(\widehat{\Delta} \cdot \Gamma^2_{DReg} + \Delta^1_{ct} \cdot \Gamma^{(1)}_{DReg}\right)_{fin} + \Delta^2_{fct} = 0$$

$$S^{(2)}_{\text{fct}} = \frac{e_A^4}{256\pi^4} \left\{ - \text{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e_A^2 \frac{\text{Tr}(\mathcal{Y}_R^6)}{8} \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) \left(\bar{\psi}_j i \bar{\not{\partial}} P_R \psi_j \right) \right\}$$
$$+ \text{any BRST-symmetric term}\,.$$

Conclusion

- Chirality is the property of nature.
- Algebraicly consistent treatment of chiral theories achieved in BMHV scheme.
- BMHV breaks gauge invariance, BRST, and multiplicative renormalization is lost.
- Symmetries can be restored order by order in perturbation theory by adding suitable counterterms.
- Thanks to the Quantum action principle, breakings are local insertion in effective action.
- In the upcoming paper we used BMHV in the treatment of the chiral U(1) model and restored symmetries up to two-loop level.

Thank you!