Role of inhomogeneities in the flattening of the quantum effective potential

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- Spontaneous symmetry breaking
- Constrained Monte Carlo simulations
- Summary

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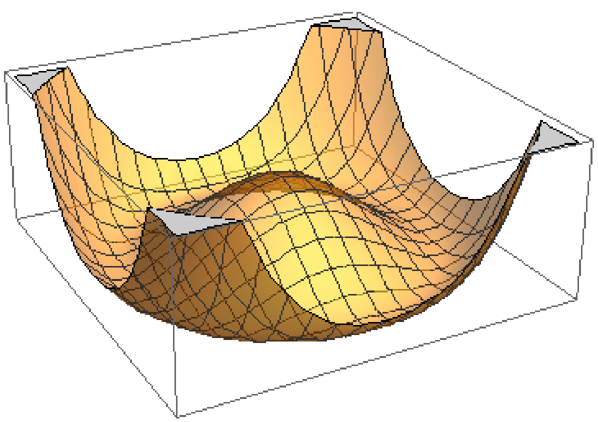
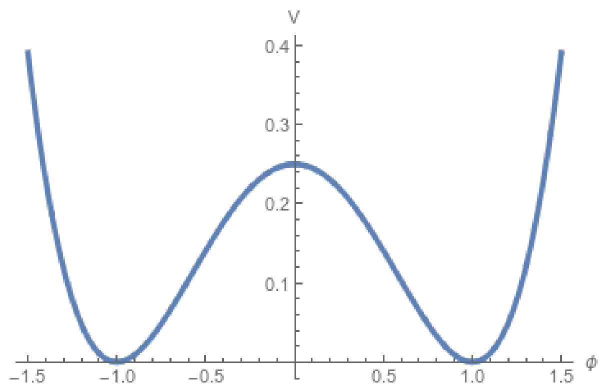
Spontaneous symmetry breaking

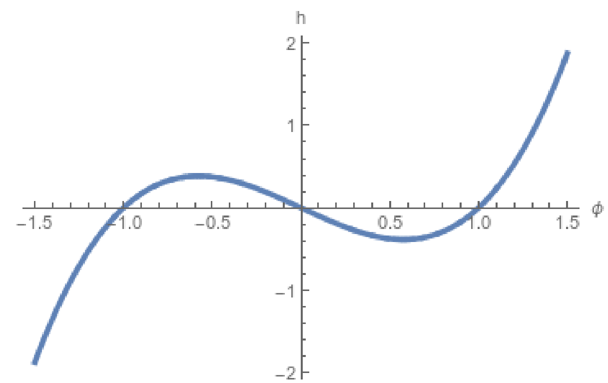
- Consider the 3D $O(2)$ symmetric scalar model

$$S = \int_{d^3x} \frac{(\partial_\mu \vec{\varphi})^2}{2} + \frac{m^2 \vec{\varphi}^2}{2} + \frac{g}{4!}(\vec{\varphi}^2)^2 \,.$$

- If $m^2 < 0$ the classical potential has **infinitely many degenerate minima** on a cricle of radius $\varphi_{\rm cl} = \sqrt{-6m^2/g}$.





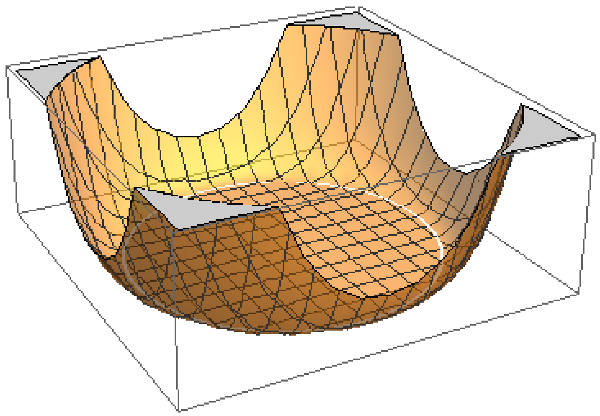


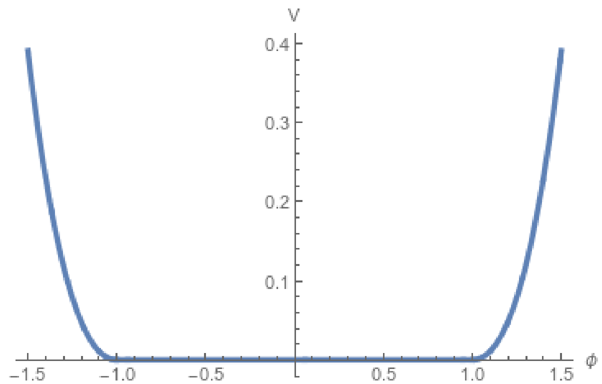
- Classically, there are zero-mass GS-bosons going around the circle, and massive excitations in the radial direction (see e.g. Rivers, *Path Integral Methods in QFT*).

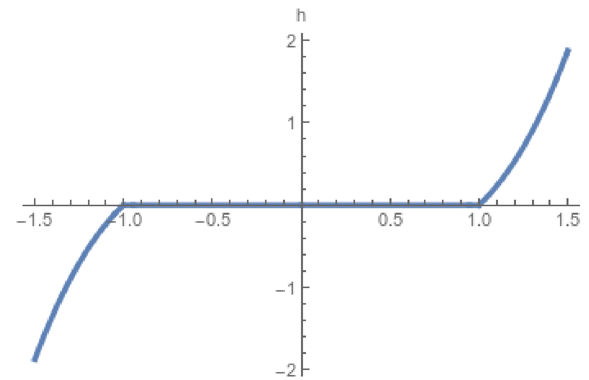
Spontaneous symmetry breaking

- In QFT the **effective potential** ($\gamma(\bar{\phi})$) is needed to find the **ground state**.

- Due to its **convexity** we end up with







- The **degeneracy** of the ground state is **enhanced**, $\dfrac{d\gamma(\bar{\phi})}{d\bar{\phi}} = 0$ in a region.

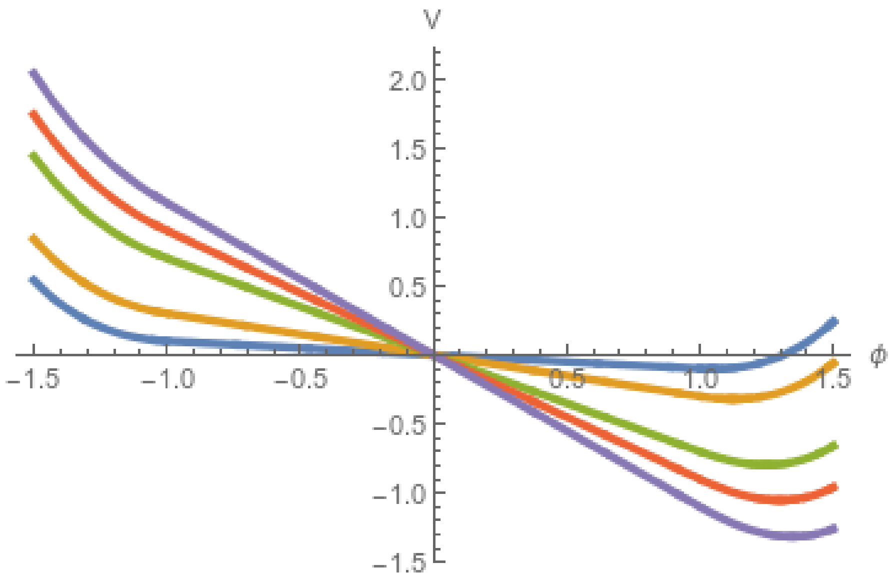
- The textbook way to a well defined groundstate is: **"SSB is a possible limit of linear symmetry breaking when the breaking parameter goes to zero."**
  Zinn-Justin, *QFT and Critical Phenomena*

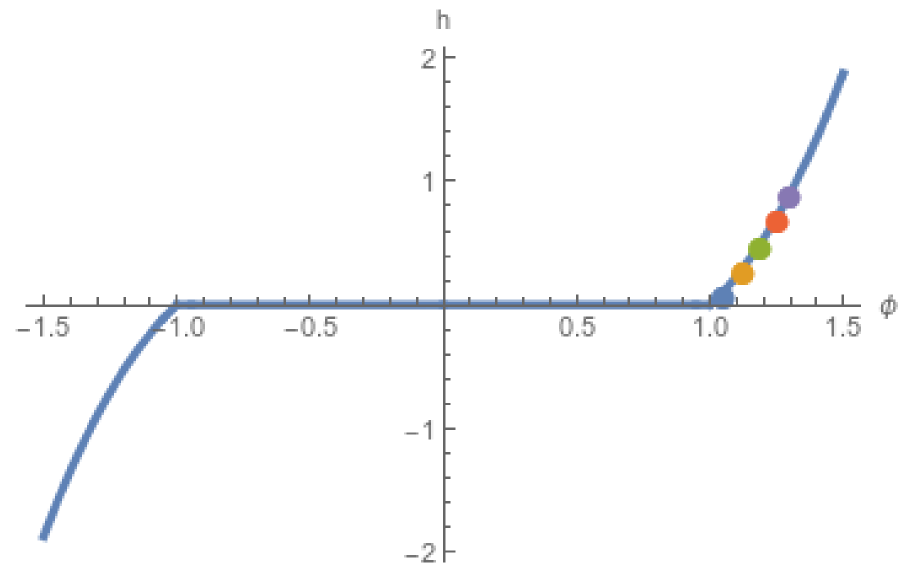
The textbook limit

- General linear **symmetry breaking** source term:

$$S_h = S - \int_{d^3x} \vec{h}(x)\vec{\varphi}(x)\,.$$

- Consider the effective potential $\gamma \Rightarrow$ restrict to $\vec{h}(x) = (h, 0)$ **constant**.
- Theory with SSB is then defined as the $\boldsymbol{h \to 0}$ limit of the **infinite volume** theory.





- $\bar{\phi}_{\min} := \lim\langle\varphi\rangle \neq 0$ will be on the **edge** of the flat region.
- Everything behaves well, the problem of a flat potential is cured, end of story.

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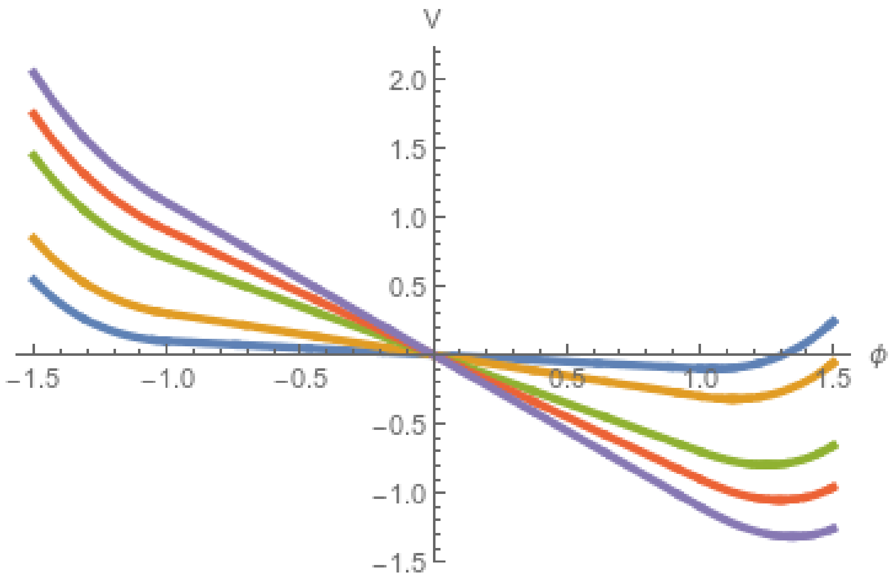

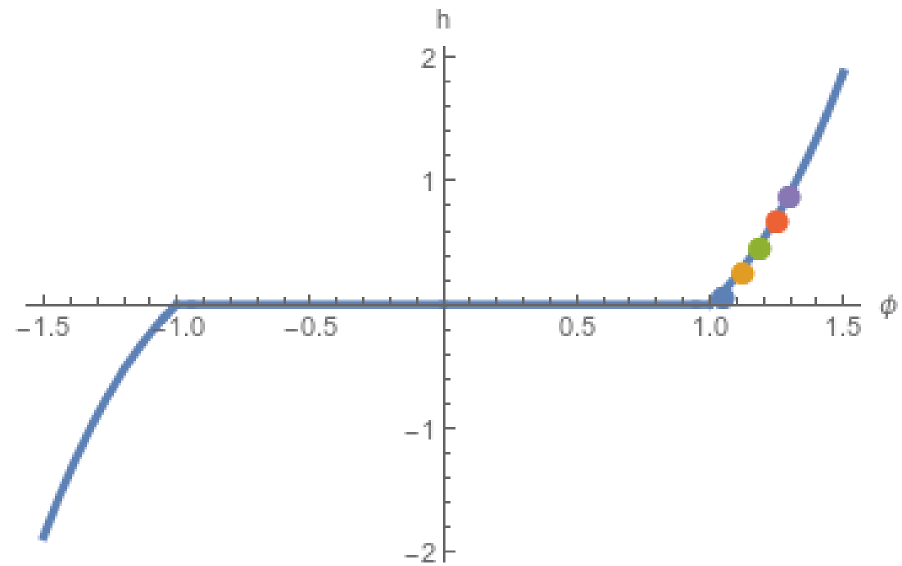

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Or is it?

The textbook limit

1. We implicitly took a **double limit** procedure: $\lim_{h\to 0}\lim_{V\to\infty}$. Changing the order leads to $\langle\varphi\rangle = 0$ for all V. How do we **access the flat region** of the potential?

2. What drives the flattening, what **configurations dominate the path integral**?

Accessing the flat region, constrained potential

Define the **constrained** effective potential

$$\exp\left(-V\Omega(\bar{\phi})\right) = \int \mathcal{D}\varphi \exp\left(-S[\varphi]\right)\, \delta\left(\int \varphi - V\bar{\phi}\right) .$$

- In the **infinite volume** limit (and only there) agrees with $\gamma(\bar{\phi})$.

  O'Raifeartaigh et al., NPB **271** (1986)

- Markov chain Monte Carlo techniques can be constructed which **respect** the constraint.

  Fodor et al., PoS LATTICE2007 056 (2007)

- Analogous to changing from **canonical** (fixed h) to **microcanonical** (fixed $\bar{\phi}$) ensemble.

- h can be recovered as

$$h = \frac{d\Omega(\bar{\phi})}{d\bar{\phi}} = m^2\bar{\phi} + \frac{g}{6V}\left\langle \int_x \varphi^3(x) \right\rangle_{\bar{\phi}} .$$

Constrained simulations

- We carry out constrained MC simulations on 3D lattices of size L^3 with **periodic** boundary conditions.
- In analogy to using $\vec{h} = (h, 0)$ we constrain both field directions:

$$V^{-1} \int \varphi_1 = \bar{\phi} \quad \text{and} \quad V^{-1} \int \varphi_2 = 0\,.$$

- We only study volume dependence towards the **infinite volume** limit, not the lattice spacing dependence.
- Based on the canonical (fixed h) simulations we expect homogeneous configurations for $\bar{\phi} \geq \bar{\phi}_{\rm min}$. But what about $\bar{\phi} < \bar{\phi}_{\rm min}$?
- Difficulty: find a simple observable which signals inhomogeneities and is not washed out by translational and rotational invariance.

Constrained simulations

Looking at **typical configurations**:

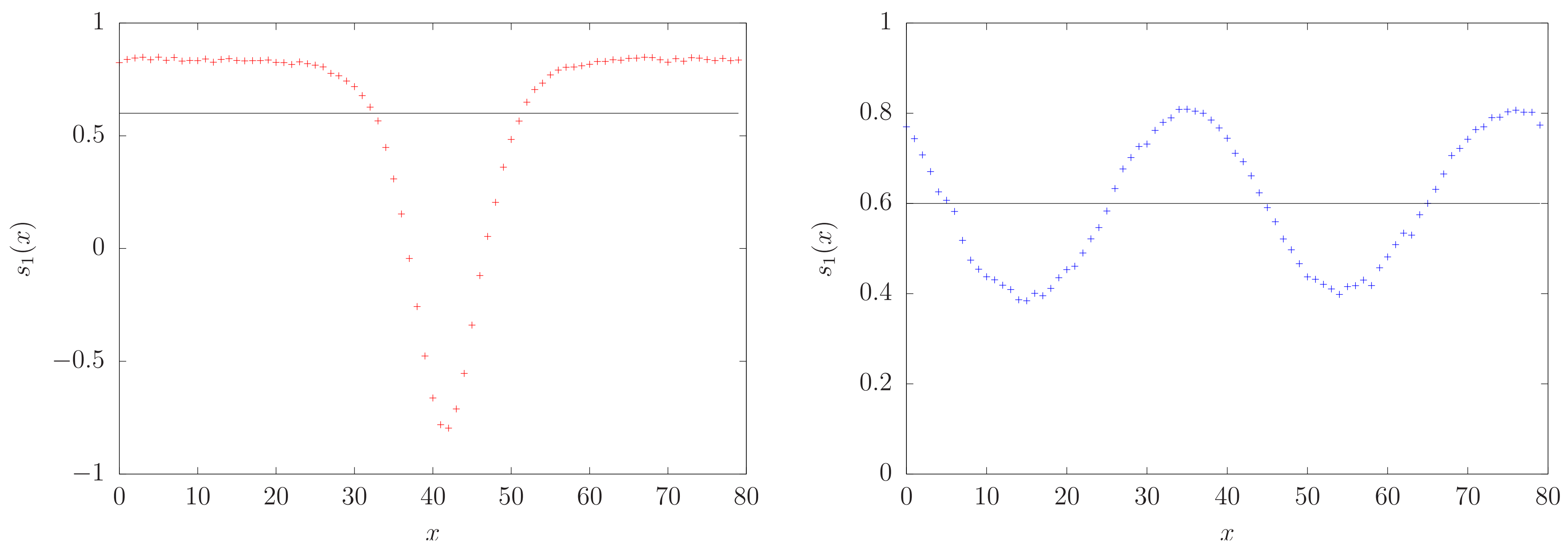
- Here slice averages

$$\vec{s}(x) = \frac{1}{L^2} \sum_{y,z \in L} \vec{\varphi}(x, y, z) \, .$$

- See GIF-s for more details.

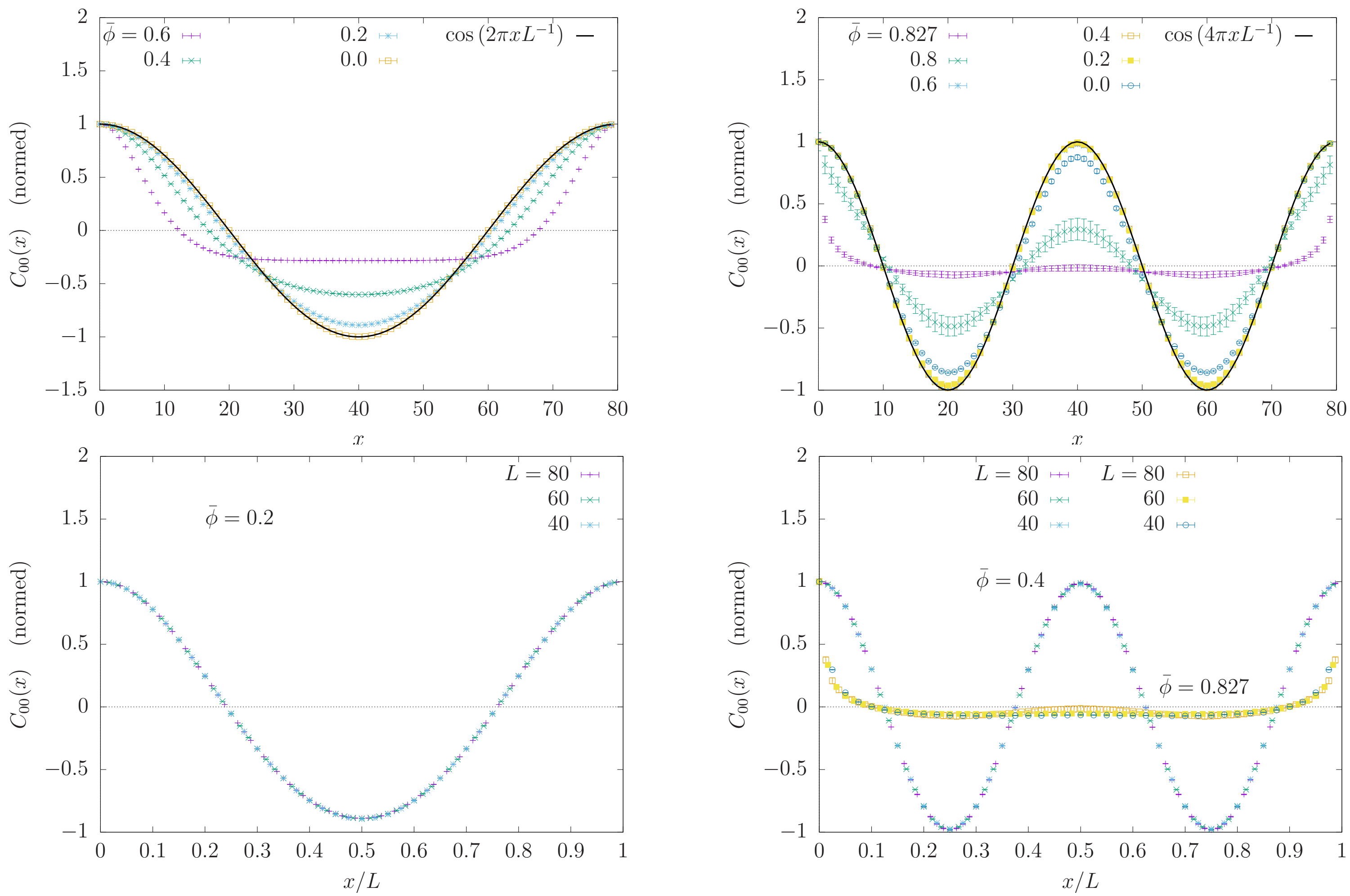
Two dominant types of configurations emerge for $\bar{\phi} < \bar{\phi}_{\rm min}$:

- a **topological**: the field winds around the full $O(2)$ space;
- a **non-topological**: the field oscillates in $O(2)$ space.



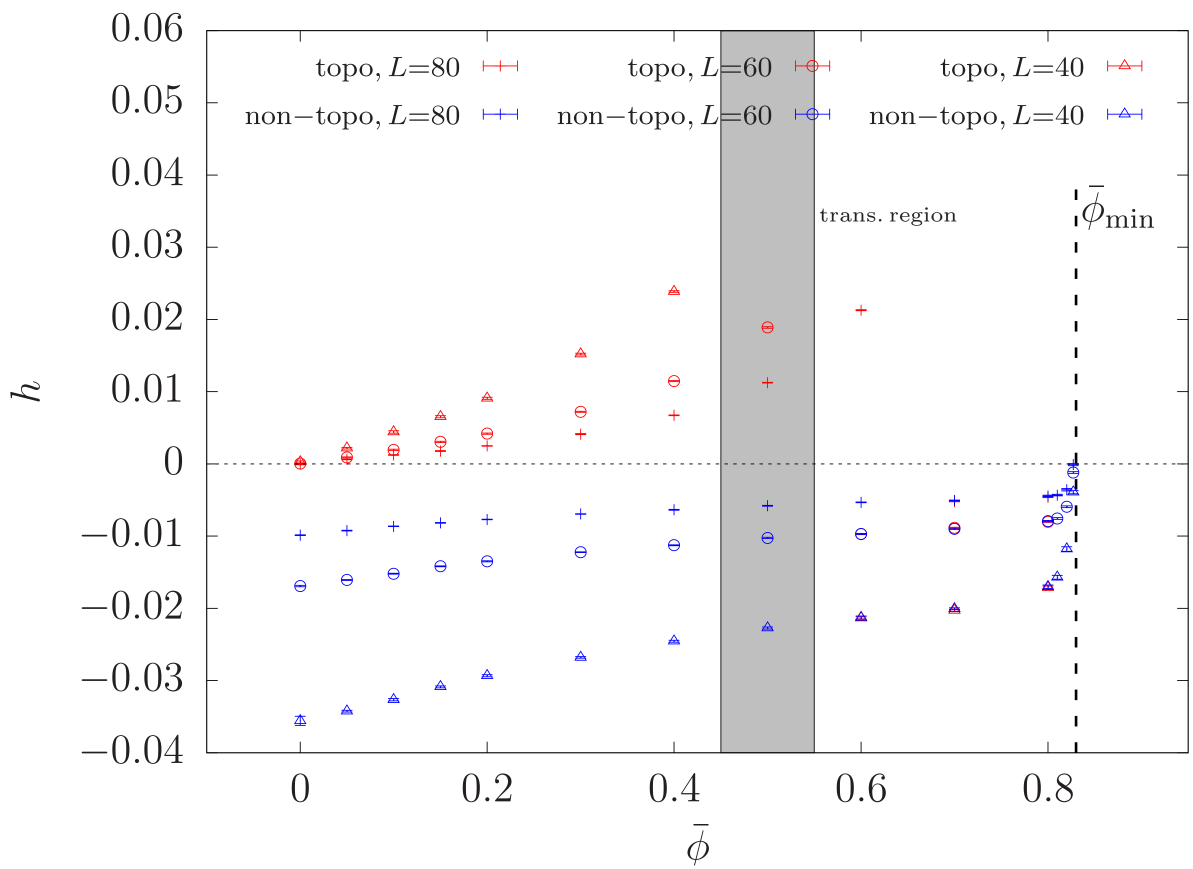
Constrained simulations

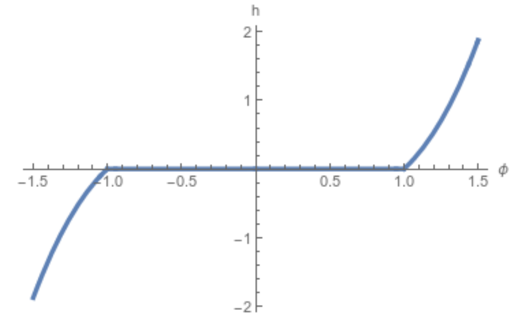
Ensemble averaged slice **correlators**: $C_{ij}(\Delta x) = \frac{1}{L}\left\langle \sum_{x\in L} s_i(x+\Delta x)s_j(x)\right\rangle_{\bar{\phi}}$.



- Topological cfgs **disappear** for $\bar{\phi} > \bar{\phi}_c$.
- Non-topological cfgs **continously connect** to homogeneous cfgs.
- Both can be loosely interpreted as **connecting different vacua**.
- Trivial volume dependece: length scale proportional to L.

Constrained simulations





- To obtain $\Omega(\bar{\phi})$, $h(\bar{\phi})$ should be integrated.
- Lowest energy cfg **changes** as a function of $\bar{\phi}$.
- At finite volume there is a transition from topological to non-topological.
- At infinite volume both set of curves tend to zero.

Summary

We wanted to understand 2 questions:

1. How to access the flat region of the effective potential?

2. What configurations cause the flatness?

We found the following answers:

1. One can construct and simulate the constrained potential which coincides with the effective potential in the $V \to \infty$ limit.

2. The flat region is dominated by inhomogeneous spin wave configurations (topological or not depending on $\bar{\phi}$).