The role of small hard-core radius of Λ in $^{3}_{\Lambda}$H production puzzle

ACHT 2021

Oleksandr Vitiuk

April 22^{nd}, 2021

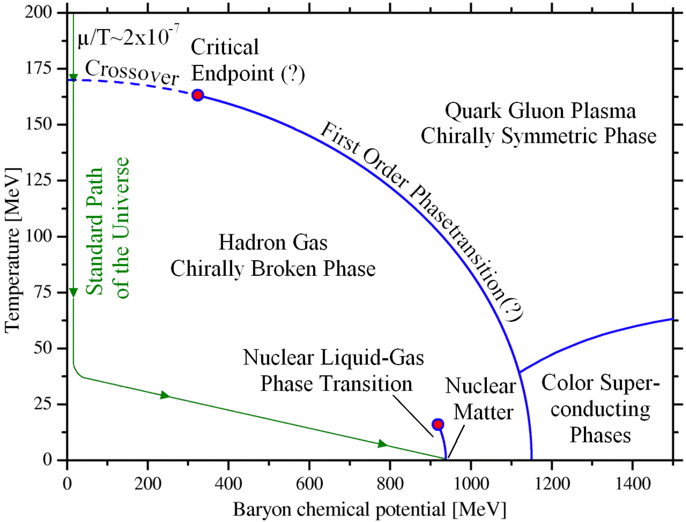
Taras Shevchenko National University of Kyiv

Outline

1. Introduction
2. Hadron Resonance Gas Model
3. Results
4. Summary

Introduction

Aims of Heavy Ion Collision Experiments

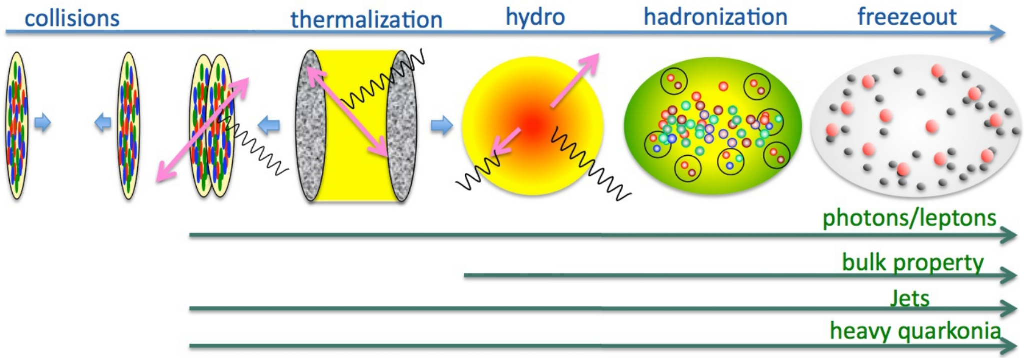


One of the major aims of HIC experiments is study of QCD phase diagram:

- Detect signals of deconfinement PT
- Detect signals of (partial) chiral symmetry restoration
- Locate (tri)critical endpoint(s) if such exists

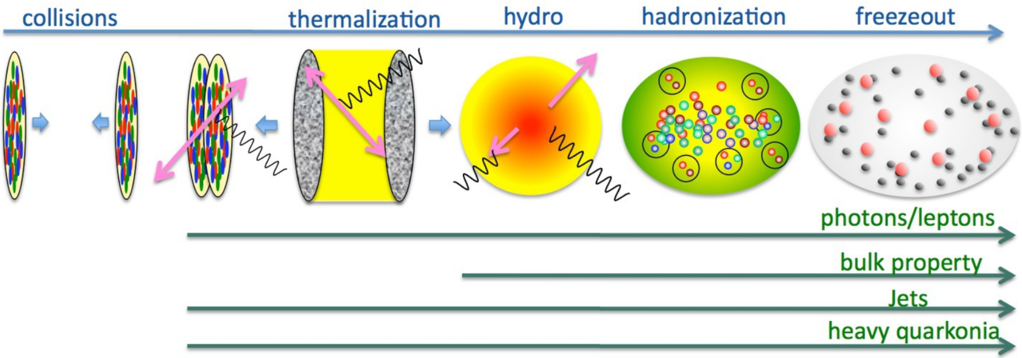
[Picture from: Universe 4 (2018) 52]

Heavy Ion Collision Time Evolution



Picture from PoS (KMI 2013) 025

Heavy Ion Collision Time Evolution



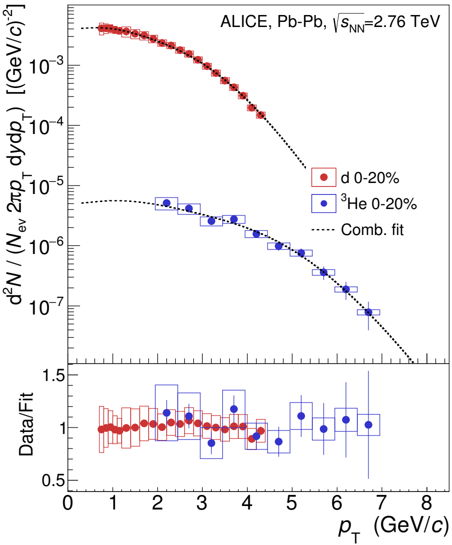
Picture from PoS (KMI 2013) 025

Where one should put light nuclei formation?

Kinetic or Chemical freeze-out?

Light Nuclei Production in Heavy-Ion Collisions: Experimental Overview

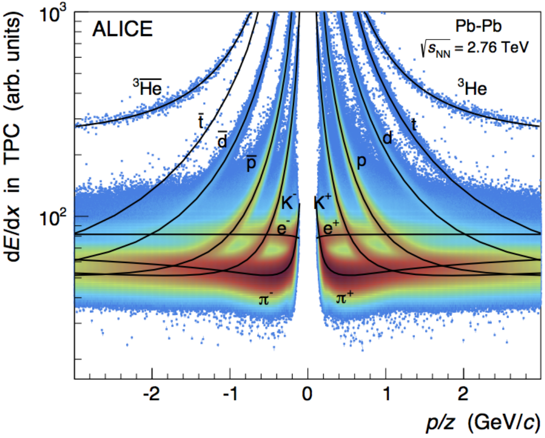
- Loosely-bound objects such as light nuclei are copiously produced in proton-proton and nuclear collisions
- Blast-Wave fit suggests that the kinetic freeze-out conditions for nuclei are identical to those of the other light flavour hadrons
- One of the key observations is the fact that the d/p and $^{3}He/p$ ratios are constant as a function of multiplicity. Such a behavior is expected from a thermal-statistical interpretation



[Picture from: Phys. Rev. C 93 (2015) 024917]

Light Nuclei Production in Heavy-Ion Collisions: Theoretician View

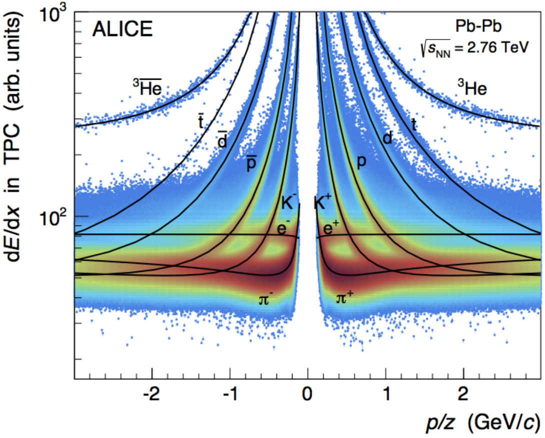
- In heavy ion collisions a fireball is observed at mid-rapidity



[Picture from: Phys. Rev. C 93 (2015) 024917]

Light Nuclei Production in Heavy-Ion Collisions: Theoretician View

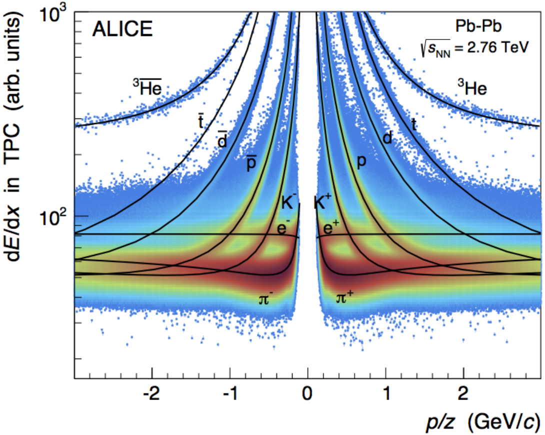
- In heavy ion collisions a fireball is observed at mid-rapidity
- Multiplicity and particle spectra suggest a temperature of $T \simeq 100 - 130$ MeV at KFO and $T \simeq 150 - 170$ MeV at CFO



[Picture from: Phys. Rev. C 93 (2015) 024917]

Light Nuclei Production in Heavy-Ion Collisions: Theoretician View

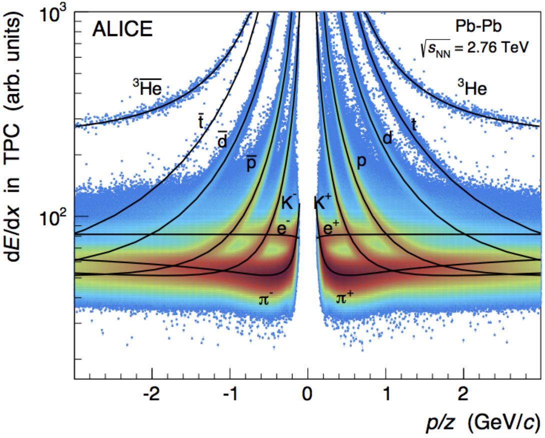
- In heavy ion collisions a fireball is observed at mid-rapidity
- Multiplicity and particle spectra suggest a temperature of $T \simeq 100-130$ MeV at KFO and $T \simeq 150-170$ MeV at CFO
- Binding energy per nucleon is $\lesssim 8$ MeV



[Picture from: Phys. Rev. C 93 (2015) 024917]

Light Nuclei Production in Heavy-Ion Collisions: Theoretician View

- In heavy ion collisions a fireball is observed at mid-rapidity
- Multiplicity and particle spectra suggest a temperature of $T \simeq 100 - 130$ MeV at KFO and $T \simeq 150 - 170$ MeV at CFO
- Binding energy per nucleon is $\lesssim 8$ MeV
- Nevertheless (anti-)(hyper-)nuclear clusters (d, t, ^{3}He, ^{4}He, $^3_\Lambda$H, $^4_\Lambda$He, ...) are observed

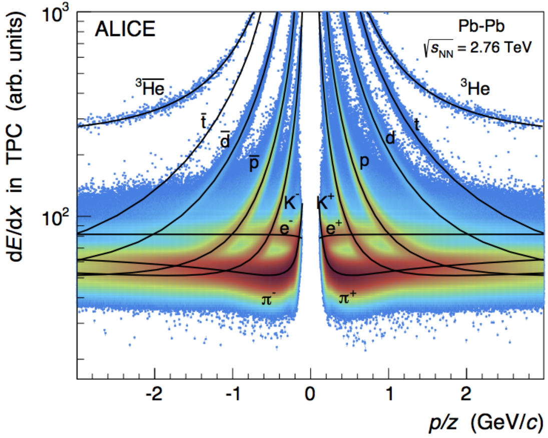


[Picture from: Phys. Rev. C 93 (2015) 024917]

Light Nuclei Production in Heavy-Ion Collisions: Theoretician View

- In heavy ion collisions a fireball is observed at mid-rapidity
- Multiplicity and particle spectra suggest a temperature of $T \simeq 100 - 130$ MeV at KFO and $T \simeq 150 - 170$ MeV at CFO
- Binding energy per nucleon is $\lesssim 8$ MeV
- Nevertheless (anti-)(hyper-)nuclear clusters (d, t, ^{3}He, ^{4}He, $^3_\Lambda$H, $^4_\Lambda$He, ...) are observed

Snowballs in hell! 🤯



[Picture from: Phys. Rev. C 93 (2015) 024917]

Theoretical Approaches

Analytical Coalescence

- Csernai, Kapusta, Phys. Rept. 131 (1986) 223-318
- Mrowczynski, Acta Phys. Polon. B48 (2017) 707
- Sun, Chen, Phys.Rev. C95 (2017) no.4, 044905

Dynamics + Coalescence

- Sombun et al, Phys.Rev. C99 (2019) no.1, 014901
- Liu et al., Physics Letters B 805, 135452 (2020)
- Kireyeu, V. et al. Bull. Russ. Acad. Sci. Phys. 84, 957–961 (2020)

Thermal HRG Models

- Vovchenko V. et al. Phys. Lett. B, 785, 171-174 (2018)
- P. Braun-Munzinger et al. Nucl. Phys. A 987, 144 (2019)
- Bugaev, K. et al. Eur. Phys. J. A 56, 293 (2020)

+ many others such us pure dynamic, Hagedorn mass spectrum ...

Hadron Resonance Gas Model

HRG Models: General Remarks

Hadron Resonance Gas models are ultimate tools to connect experimental measurements with QCD phase diagram. Usually HRGM are based on vdW type excluded volume correction for all particles known from PDG.
Reasons to use vdW approximation:

- Hard-core repulsion EoS has the same energy per particle as ideal gas EoS
- Hard-core repulsion does not create problems with QGP existence, since such repulsion suppresses pressure compared to ideal gas EoS
- Almost in the whole hadronic phase the mixture of stable hadrons and resonances behaves as a mixture of ideal gases with small hard-core radii due to approximate cancellation of attraction and repulsion terms among the quantum second virial coefficients of hadrons [NPA 546 (1992) 718-760]

HRG Models: General Remarks

General algorithm of work of HRGM:

- For set of parameters $\{T, \mu_B, \mu_S, \mu_Q, \gamma_S, ...\}$ one can find all thermodynamic quantities
- Find particle thermal densities
- Perform particle decays according to PDG tables
- Compare obtained result with experimental data and calculate χ^2
- Do four items above in the loop until best set of thermodynamic parameters is found

With HRGM one can study system properties at chemical freeze-out (CFO) – moment at which hadronic composition is fixed but decays of resonances are allowed.

IST EoS based HRGM

Let us consider multicomponent, IST EoS based HRGM, which is a system of equations [K. Bugaev et al., NPA 970, 133 (2018)]:

$$\begin{cases} p = T \sum\limits_{k=1}^{N} \phi_k \exp\left[\frac{\mu_k - pV_k - \Sigma S_k}{T}\right], \\ \Sigma = T \sum\limits_{k=1}^{N} R_k \phi_k \exp\left[\frac{\mu_k - pV_k - \alpha \Sigma S_k}{T}\right]. \end{cases}$$

where ϕ_k, μ_k, R_k, S_k and V_k are thermal density, chemical potential, hard-core radius, eigen surface and volume of k-th sort particle.

From this system one can find particle thermal densities:

$$\rho_k^{th} = \frac{\partial p}{\partial \mu_k} = \frac{1}{T} \frac{p_k a_{22} - \Sigma_k a_{12}}{a_{11} a_{22} - a_{12} a_{21}},$$

$$a_{11} = 1 + \sum_{k=1}^{N} V_k \frac{p_k}{T}, \; a_{12} = \sum_{k=1}^{N} S_k \frac{p_k}{T}, a_{21} = \sum_{k=1}^{N} V_k \frac{\Sigma_k}{T}, \; a_{22} = 1 + \alpha \sum_{k=1}^{N} S_k \frac{\Sigma_k}{T}$$

HRGM and Light Nuclear Clusters

Severe problems with light nuclei:

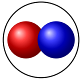
- The light nuclei yields maybe very sensitive to the properties of the phase in which they are formed [E. Shuryak and J. M. Torres-Rincon, PRC 100, 024903 (2019)]
- The quantum second virial coefficients of nuclei and hadrons are not known
- The classical second virial coefficients (excluded volume) of nuclei and hadrons were found recently in our work [EPJ A 56, 293 (2020)]
- After finding the excluded volumes one has to reformulate the HRGM completely, since the number of virial coeffieints is (Number of nuclei) × (number of hadronic hard-core radii)!
- There is no alternative to the classical approach!
- Hypertriton Puzzle: STAR data measured in 2011 related to (anti)hyper-triton were never described by HRGM or by coalescence

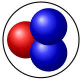
Second Virial Coefficient of Light Nuclear Clusters

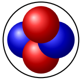
How does light nucleus look like?

Second Virial Coefficient of Light Nuclear Clusters

How does light nucleus look like?

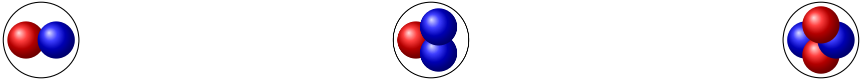




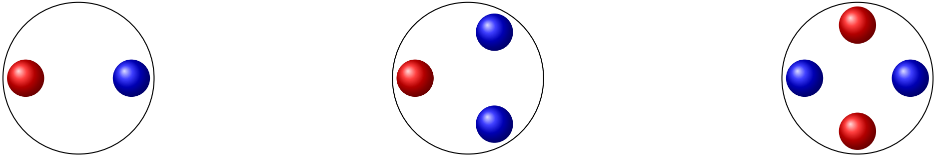


Second Virial Coefficient of Light Nuclear Clusters

How does light nucleus look like?



Or like this?



Second Virial Coefficient of Light Nuclear Clusters

Light nuclei are roomy clusters! Then one can evaluate second virial coefficient of nuclei with mass number A:

$$b_{Ah} = A\frac{2\pi}{3}(R_b + R_h)^3$$

From here one can find that IST EoS system is not modified and nuclear radii, eigen-surface and eigen-volume can be expressed through it constituents parameters. Here we use two approaches:

- Rigorously derived, named IST EoS:

$$R_A = AR_b, \; S_A = AS_b, \; V_A = AV_b$$

- Approximate and complementary to the first one, named Bag Model Radii (BMR):

$$R_A = A^{1/3}R_b, \; S_A = A^{2/3}S_b, \; V_A = AV_b$$

Second Virial Coefficient of Light Nuclear Clusters: Generalization

In general light nuclear cluster may be composed not of nucleons only. For example ${}^{3}_{\Lambda}H = pn\Lambda$. For this case one can perform simple generalization for nuclei with A constituents which can be divided into N_s different sorts ($A = \sum_{k=1}^{N_s} n_k$):

- For IST EoS:

$$R_A \to \sum_{k=1}^{N_s} n_k R_k, \; S_A \to \sum_{k=1}^{N_s} n_k S_k, \; V_A \to \sum_{k=1}^{N_s} n_k V_k$$

- Approximate and complementary to the first one, named Bag Model Radii (BMR):

$$R_A = \left[\sum_{k=1}^{N_s} n_k \left(R_k + \overline{R} \right)^3 \right]^{\frac{1}{3}} - \overline{R},$$

where $\overline{R}$ is the mean hard-core radius of hadrons

Model setup

Total particle number density and yield of k-sort of particles defined as:

$$\rho_k^{tot} = \rho_k^{th} + \sum_{l \neq k} \rho_l Br_{l \to k}, \; N_k = V \rho_k^{tot}$$

Then ratio of hadronic yields is $R_{kl}^{theo} = N_k^{tot}/N_l^{tot}$. Ratios are preferred for fit but for some hadrons and light nuclei only data on yields are available, hence the total $\chi^2(V)$ is:

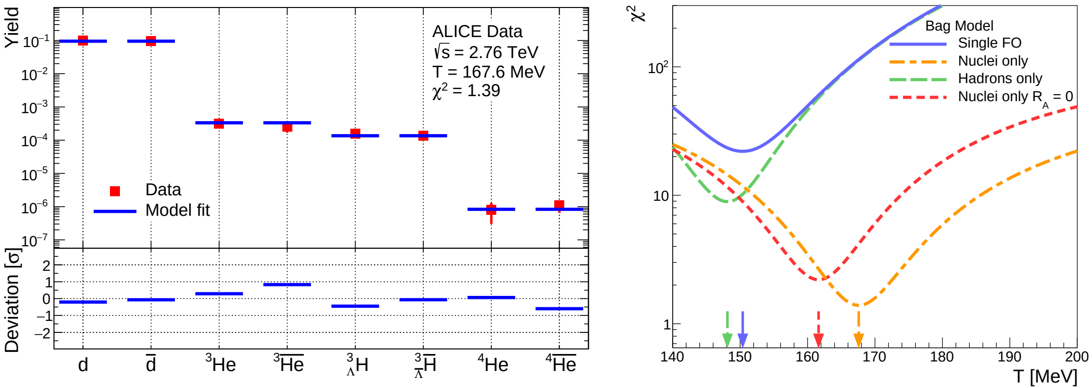
$$\chi^2_{tot}(V) = \chi^2_R + \chi^2_Y(V) = \sum_{k \neq l \in R} \left[\frac{R_{kl}^{theo} - R_{kl}^{exp}}{\delta R_{kl}^{exp}} \right]^2 + \sum_{k \in Y} \left[\frac{V \rho_k^{tot}(T) - N_k^{exp}}{\delta N_A^{exp}} \right]^2$$

Model parameters: $R_\pi = 0.15$ fm, $R_K = 0.395$ fm, $R_m = 0.42$ fm, $R_b = 0.365$ fm, $R_\Lambda = 0.085$ fm and $\alpha = 1.25$ [NPA 970, 133 (2018)]. $\gamma_S = 1$ and $\mu_{I3} = 0$. T, μ_B and V are set as fit parameters (for ALICE energy $\mu_B = 0$).

Two CFO scenarios are considered: single and separate freeze-out of hadrons and light nuclei

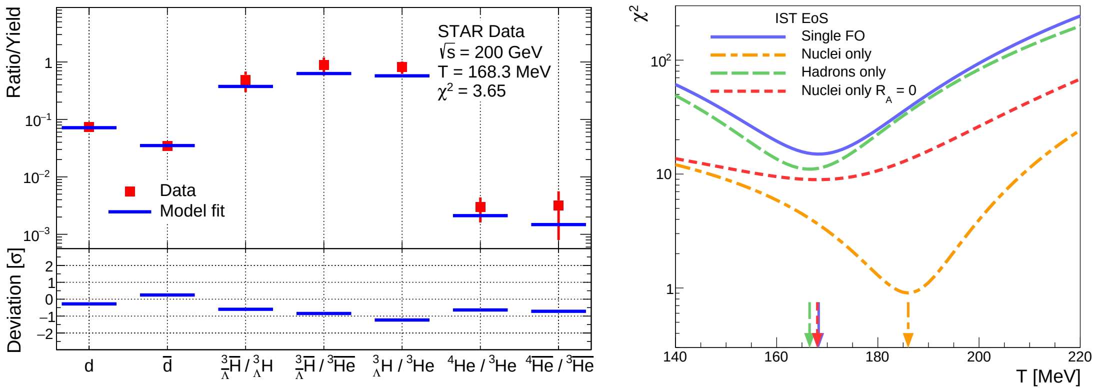
Results

Selected Results for ALICE



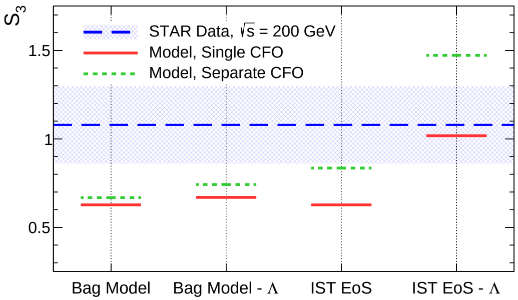
Left: The yields of nuclear clusters measured at $\sqrt{s_{NN}} = 2.76\ TeV$ by ALICE vs. theoretical description with BMR-Λ. Separate CFO scenario. **Right:** Temperature dependence of χ^2_{tot}, χ^2_h anf χ^2_A for fit of ALICE data measured at $\sqrt{s_{NN}} = 2.76\ TeV$

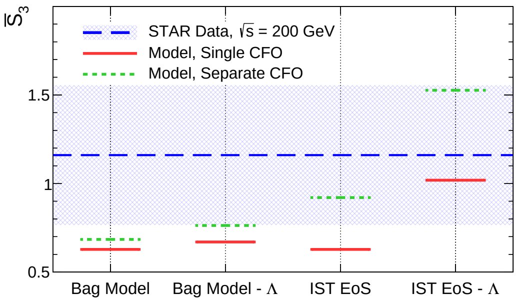
Selected Results for STAR



Left: The yields of nuclear clusters measured at $\sqrt{s_{NN}} = 200\ GeV$ by STAR vs. theoretical description with IST Λ EoS. Single CFO scenario. **Right:** Temperature dependence of χ^2_{tot}, χ^2_h anf χ^2_A for fit of STAR data measured at $\sqrt{s_{NN}} = 200\ GeV$

Selected Results for STAR: S_3 and $\bar{S}_3$





$$S_3 = \frac{{}^{3}_{\Lambda}H}{{}^{3}He \times \frac{\Lambda}{p}}$$

$$\bar{S}_3 = \frac{{}^{3}_{\bar{\Lambda}}\overline{H}}{{}^{3}\overline{He} \times \frac{\bar{\Lambda}}{\bar{p}}}$$

S_3 and $\bar{S}_3$ are reproduced automatically without fitting!

[O.V.V et al, Eur. Phys. J. A 57 (2021) 74]

Summary Tables

Description	T_h, MeV	T_A, MeV	V_A, fm^3	χ^2/dof
Single CFO, ISTΛ	150.29 ± 1.92	150.29 ± 1.92	13145 ± 2233	1.433
Single CFO, BMRΛ	150.39 ± 1.90	150.39 ± 1.90	11201 ± 2009	1.293
Separate CFO, ISTΛ	148.12 ± 2.03	169.25 ± 5.57	3898 ± 1272	0.753
Separate CFO, BMRΛ	148.12 ± 2.03	167.59 ± 5.39	3123 ± 1198	0.676

The results obtained by the advanced HRGM for the fit of ALICE data measured at $\sqrt{s_{NN}} = 2.76$ TeV

Description	T_h, MeV	T_A, MeV	μ_B^h, MeV	μ_B^A, MeV	V_A, fm^3	χ^2/dof
Single CFO, ISTΛ	168.30 ± 3.85	168.30 ± 3.85	30.12 ± 3.27	30.12 ± 3.27	2056 ± 375	1.069
Single CFO, BMRΛ	167.43 ± 3.84	167.43 ± 3.84	30.00 ± 3.26	30.00 ± 3.26	1667 ± 355	1.339
Separate CFO, ISTΛ	166.51 ± 4.07	185.99 ± 9.09	28.84 ± 5.37	34.30 ± 4.81	1093 ± 278	0.995
Separate CFO, BMRΛ	166.51 ± 4.07	182.69 ± 14.1	28.84 ± 5.37	33.30 ± 4.94	831 ± 455	1.459

The results obtained by the advanced HRGM for the fit of STAR data measured at $\sqrt{s_{NN}} = 200$ GeV

Summary

Summary

- (Anti-)(hyper-)nuclei are copiously produced in high energy proton-proton and nuclear collisions, which is a bit surprising
- Using advanced IST EoS based HRGM with small radii of Λ hyperon and correct second virial coefficient one can accurately describe experimental data on light nuclei
- From HRGM it is seen that light nuclei are better described with $T_{CFO} \approx 167\ MeV$ both at ALICE and STAR energies
- On the other hand, the chemical freeze-out of hadrons at these energies occurs under different conditions (see talk by E. Zherebtsova at 16:25)

THE
END

Questions?

Backup Slides

Relation Between IST EoS and Morphological Thermodynamics

IST EoS system:

$$\begin{cases} p = T \sum\limits_{k=1}^{N} \phi_k \exp\left[\frac{\mu_k - pV_k - \Sigma S_k}{T}\right], \\ \Sigma = T \sum\limits_{k=1}^{N} R_k \phi_k \exp\left[\frac{\mu_k - pV_k - \alpha \Sigma S_k}{T}\right]. \end{cases}$$

Main statement of Morphological thermodynamics:

For a convex rigid body r immersed into a fluid with pressure p, mean surface tension coefficient Σ, mean curvature tension coefficient K and mean Gaussian curvature tension Ψ one can express free energy of this body as follows:

$$\Omega = -pV_r - \Sigma S_r - KC_r - \Psi X_r,$$

where V_r is eigen volume of bode r, S_r its eigen surface, ...

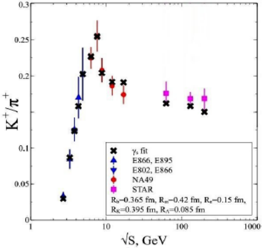
IST EoS is a truncated version of Morphological thermodynamics

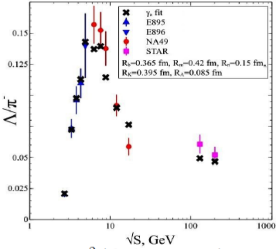
Relation Between IST EoS and Morphological Thermodynamics

In fact, in our recent works we generalized the morphological Thermodynamics to Grand Canonical Ensemble for Mixtures of:

- Hard spheres and hard discs
  – N. S. Yakovenko et al., Eur. Phys. J. ST 229 (2020)
- Quantum hard spheres
  – K. A. Bugaev, Eur. Phys. A 55 (2019)
- Hadrons and light nuclei
  – K. A. Bugaev et al, Eur. Phys. A 56 (2020)
  – O. Vitiuk et al, Eur. Phys. A 57 (2021)
- Small Systems (Induced Surface and Curvature Tensions)
  – K. A. Bugaev et al, arXiv:2104.05351 [hep-ph]

Most Problematic ratios at AGS, SPS and RHIC energies



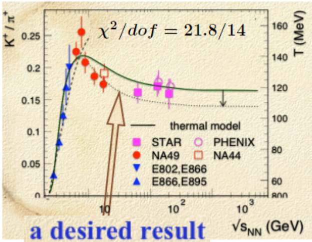


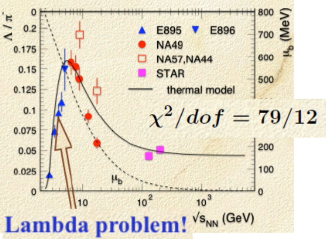
IST EOS: $\chi^2/dof \simeq 3.29/14$ $\chi^2/dof \simeq 11.62/12$

KAB et al., Nucl. Phys. A 970 (2018)

Note: RHIC BES I data have very large error bars and hence, are not analyzed!

Our IST EOS has 3 or 4 more fitting parameters compared to usual HRGM!

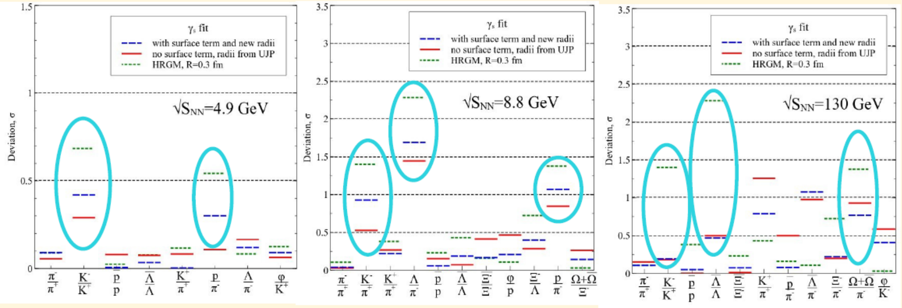




Conventional one component HRGM by PBM and Co:
A. Andronic, PBM, J. Stachel NPA (2006), PLB (2009)

Examples of Hadron Multiplicity Ratios for IST, Multicomponent and One component Van der Waals EoS (2018)

V.V. Sagun et al., Eur. Phys. J. A (2018) 54: 100



Blue bars **IST EoS**

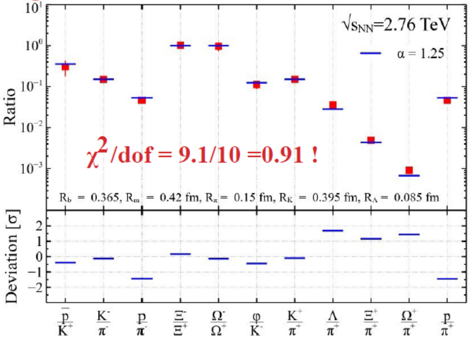
Red bars **Multicomponent Van der Waals EoS**

Green bars **One-component Van der Waals EoS (a la P. Braun-Munzinger et al),**

One-component Van der Waals EoS always gives the worst results!

IST EOS Results for LHC energy

Light (anti)nuclei are NOT included into fit V.V. Sagun et al., Eur. Phys. J. A (2018) 54: 100



Radii are taken from the fit of AGS, SPS and RHIC data => single parameter Tcfo=150+-4MeV

In all our fits (anti)protons and (anti)Ξ-s do not show any anomaly compared to J. Stachel et.al. fit, since we have right physics!

=> There is no proton yield puzzle in a realistic HRGM!

In contrast to J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich, J. Phys. Conf. Ser. 509, 012019 (2014) (anti)nuclei are NOT included into the fit!

Combined fit of AGS, SPS, RHIC and LHC data $\chi^2_{tot}/dof \simeq 64.8/60 \simeq 1.08$

Compare with J. Stachel et al. fit quality for Tcfo = 156 MeV χ^2/dof = 2.4 with our one!

BUT this does not resolve the puzzle of light (anti)nuclei!

Why Are Light Nuclei Thermalized?

Possible explanation: Hagedorn mass spectrum of QGP bags $dN/dM \propto e^{M/T_H}$

- System with such mass spectrum is a perfect thermostat and a perfect particle reservoir! Hadrons born from such bags will be in a full equilibrium [L. Moretto et al., EPL 76, 402 (2006)]
- Production of light nuclear clusters via Hagedorn resonances was recently considered in [K. Gallmeister and C. Greiner, EPJ A 57 (2021) 2, 62]
- In order to survive the (anti-)(hyper-)nuclei should be evaporated from the surface of QGP bags from the very beginning of their appearance. High T at CFO of light nuclei is a reflection of this scenario!

Why Are Light Nuclei Thermalized?

Picture by K. A. Bugaev

Why Are Light Nuclei so Hot?

- Possible explanation I: The analysis of microcanonical partition function of a system containing of one Hagedorn bag and N Boltzmann particles shows that at the end of mass spectrum (where it terminates) the temperature depends on the mass of particle and the mass of QGP bag: few heavier particles will be hotter than many light ones! [Europhys. Lett. 76, 402 (2006)]
- Possible explanation II: Effect of Laplace pressure. Under constant pressure the small and large QGP bags will have different T:

$$P^{tot} = const = P^{bulk}(T, \mu) - \frac{\sigma}{R}$$

For negative surface tension $\sigma < 0$ larger bags will be hotter. Hence after emitting a nucleus the bag gets smaller and cooler!
[K. A. Bugaev and G. M. Zinovjev, NPA 848 (2010) 443-453]