Transverse single spin asymmetry at two loops

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SB, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
SB, Hatta, Li (in preparation)

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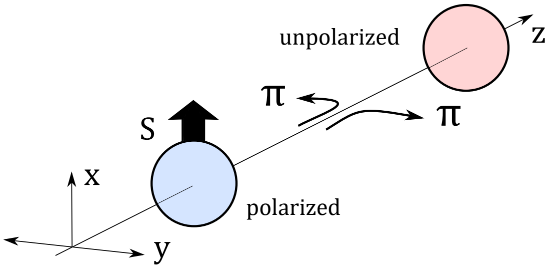
HRZZ
Croatian Science Foundation

Single spin asymmetry



$$A_N = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$
$$\sim (\boldsymbol{S} \times \boldsymbol{P}_h) \cdot \boldsymbol{P}$$

Initial pQCD expectation

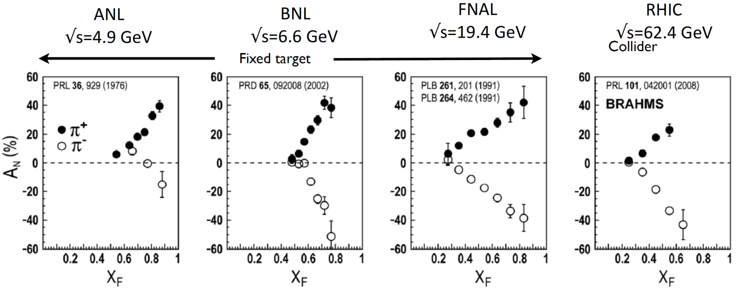
- helicity basis:

$$|\uparrow,\downarrow\rangle \sim |+\rangle \pm |-\rangle$$

- $A_N \sim \mathrm{Im}(\mathcal{M}_+^* \mathcal{M}_-)$
  $\rightarrow$ requires helicity flip

- pQCD expectation: $A_N \sim m_q/P_{hT}$
  SSA is universally suppressed at high energies!?

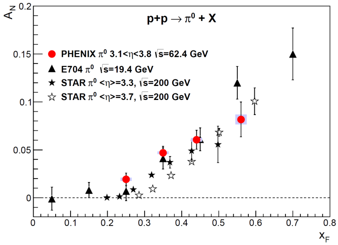
SSA in $pp^{\uparrow} \to hX$



- A_N from low to high energies! (puzzle!)
- A_N largest in the forward direction $x_F \to 1$

$$x_F = 2P_h^z/\sqrt{s}$$

SSA in $pp^{\uparrow} \to hX$



- A_N from low to high energies! (puzzle!)
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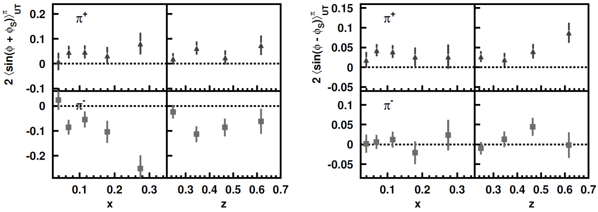
$$x_F = 2P_h^z/\sqrt{s}$$

PHENIX, Phys. Rev. D **90**, no. 1, 012006 (2014)

SSA in $ep^{\uparrow} \to ehX$ (SIDIS)

- measured at HERMES, COMPASS, JLAB
- future Electron Ion Collider (EIC)

- e. g. Sivers and Collins asymmetry at HERMES



$\to$ a few percent asymmetry (puzzle!)

HERMES, Phys. Rev. Lett. **94**, 012002 (2005)

Modern view on SSA

- large SSA generated by some non-perturbative matrix element
- helicity flip $\to$ chiral symmetry breaking

$$A_N \sim \frac{\Lambda_{\text{QCD}}}{P_{hT}} \qquad \left(\text{not } A_N \sim \frac{m_q}{P_{hT}}\right)$$

- what is the relevant non-perturbative matrix element?

SSA comes from imaginary parts of Feynman diagrams

- consider the following products ($\chi = \uparrow, \downarrow$)

$$\mathcal{S}_{\chi\chi'} = \bar{u}_{\chi'}\gamma^{\mu_1} \dots \gamma^{\mu_n} u_\chi$$

- apply CPT

$$\mathcal{S}_{\chi\chi'} \to \chi\chi' \mathcal{S}^*_{-\chi,-\chi'}$$

- parametrize

$$\mathcal{S}_{\chi\chi'} = \delta_{\chi\chi'}(A + \chi B) + \delta_{\chi,-\chi'}(C + \chi D)$$

$$\to A^* = A \quad D^* = D \quad B^* = -B \quad C^* = -C$$

Sievert, 1407.4047

SSA comes from imaginary pieces of the amplitude

- make two copies

$$\mathcal{S}_{i,\chi\chi'} = \delta_{\chi\chi'}(A_i + \chi B_i) + \delta_{\chi,-\chi'}(C_i + \chi D_i) \quad i = 1, 2$$

- square it

$$\sum_{\chi'} \mathcal{S}_{1\chi\chi'} \mathcal{S}^*_{2\chi\chi'} = (A_1 A_2^* + B_1 B_2^* + C_1 C_2^* + D_1 D_2^*) + \chi(A_1 B_2^* + A_2^* B_1 + C_1 D_2^* + C_2^* D_1)$$

- spin independent part: real
- spin dependent part: imaginary

Sievert, 1407.4047

Imaginary parts comes from interference terms

- if $\mathcal{S}_1 = \mathcal{S}_2 \rightarrow$ spin dependent part is zero

  (amplitude squared is real)
- SSA is an interference between different Feynman diagrams
- we need two different contributions

- interference is purely imaginary, but cross section is real

  $\rightarrow$ we need another "i"

We need another "i"

- from soft factors

  $\rightarrow$ distribution/fragmentation functions become complex

- from hard factors

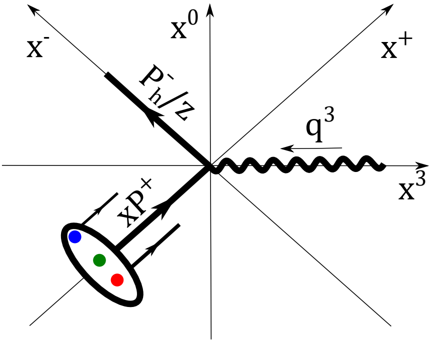
$$\frac{1}{k^2 + i\epsilon} = \mathcal{P}\frac{1}{k^2} - i\pi\delta(k^2)$$

  $\rightarrow$ on-shell intermediate particles

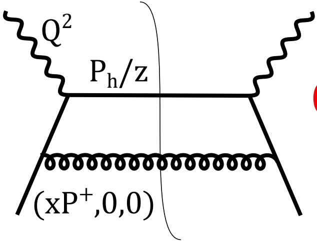
SIDIS

- Semi-Inclusive Deep Inelastic Scattering
- proton: $P^\mu = (P^+, 0, 0, 0)$
- photon: $q^\mu = (q^+, q^-, 0, 0)$ $(q^\pm = (q^0 \pm q^3)/\sqrt{2})$

$$d\sigma \sim L^{\mu\nu} W_{\mu\nu} \quad W_{\mu\nu} \sim \int_x e^{iq\cdot x} \langle P, \boldsymbol{S} | J^\mu(x) J^\nu(0) | P, \boldsymbol{S} \rangle$$



Collinear framework



$Q \sim P_{hT} \gg \Lambda_{QCD}$

- P_{hT} from 2-by-2 scatterings
- schematically

$$d\sigma \sim f(x, Q^2) \otimes D(z, Q^2) \otimes H$$

$f(x) \sim \int dy^- e^{ixP^+y^-} \langle P|\bar{\psi}(0)\gamma^+[0, y^-]\psi(y)|P\rangle$

gauge link: $[x^-, y^-] = \mathrm{P}\exp\left[ig\int_{y^-}^{x^-} dz^- A^+(z)\right]$

Some known sources of SSA

- Efremov-Teryaev-Qiu-Sterman function

- we need interference diagrams
  $\to$ a quark-gluon-quark correlation

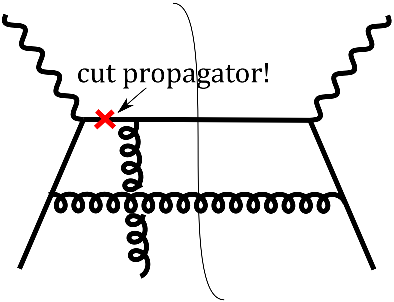
$$\int_{\lambda,\mu} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle PS_T | \bar{\psi}_j(0)[0,\mu n] g F^{\alpha n}(\mu n)[\mu n, \lambda n]\psi_i(\lambda n)|PS_T\rangle$$
$$= \frac{M_N}{4}(\not{P})_{ij}\epsilon^{\alpha P n S_T} G_F(x_1, x_2) + i\frac{M_N}{4}(\gamma_5 \not{P})_{ij} S_T^{\alpha}\tilde{G}_F(x_1, x_2)$$

- EQTS function does not have a simple number density interpretation

Efremov, Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982)
Qiu, Sterman, Phys. Rev. D **59**, 014004 (1999)

Some known sources of SSA

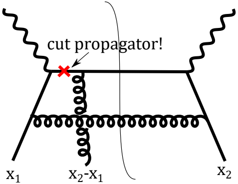
- where does the "i" come from?
- ETQS is a real function
  $\rightarrow$ we need an intermediate particle to go on shell



$$\frac{1}{k^2 + i\epsilon} \rightarrow -i\pi\delta(k^2)$$

Some known sources of SSA

$$((x_2 - x_1)P + P_h/z)^2 = 0$$



$\rightarrow x_1 = x_2 \rightarrow$ soft gluon pole

- twist-3 fragmentation functions

$$d\sigma \sim \boldsymbol{S} \cdot (\boldsymbol{P} \times \boldsymbol{P}_h) h_1(x)$$
$$\otimes \left[H(z) \otimes H_1 + H_1^{\perp(1)}(z) \otimes H_2 + \mathrm{Im}(\hat{H}_{FU}(z)) \otimes H_3 \right]$$

$\rightarrow$ can be complex functions (no need for cut propagators)

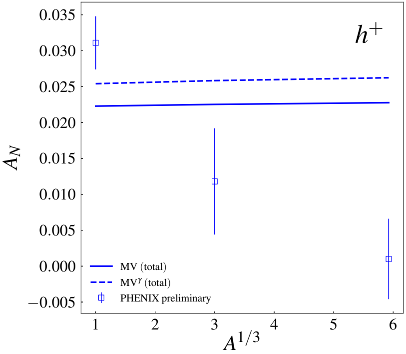
Kanazawa, Koike, Phys. Rev. D **88** (2013), 074022

ETQS vs twist-3 FF

- extract pdfs/ffs from SIDIS, apply to $p^{\uparrow}p$
- -'11 ETQS term dominates? ($dG_F(x,x)/dx$ term)
- '11 sign mismatch: ETQS function from SIDIS and from $p^{\uparrow}p$ have a different sign
  Kang, Qiu, Vogelsang, Yuan, Phys. Rev. D **83** (2011) 094001
- '10-'13 twist-3 fragmentation contribution
  Kang, Yuan, Zhou, Phys. Lett. B **691** (2010) 243
  Metz, Pitonyak, Phys. Lett. B **723** (2013) 365
  Kanazawa, Koike, Phys. Rev. D **88** (2013) 074022
- '14- fragmentation contribution dominates
  Kanazawa, Koike, Metz, Pitonyak, Phys. Rev. D **89** (2014) no.11, 111501
  Gamberg, Kang, Pitonyak, Prokudin, Phys. Lett. B **770** (2017) 242
  JLAB Collaboration, Phys. Rev. D **102** (2020), no.5, 054002 (2020)

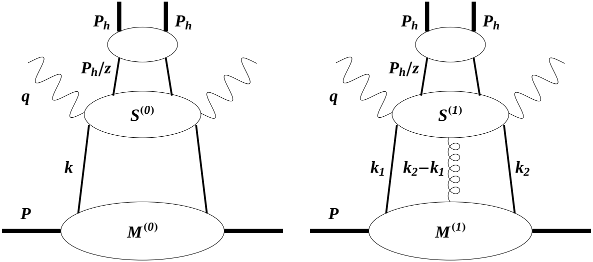
Twist-3 FF dominates?

- recently challenged by analysis of $p^{\uparrow}A$ data from RHIC



Hatta, Xiao, Yoshida, Yuan, Phys. Rev. D **95**, no. 1, 014008 (2017)
PHENIX, Phys. Rev. Lett. **123**, no. 12, 122001 (2019)
SB, Hatta, Phys. Rev. D **99**, no. 9, 094012 (2019)

Hadronic tensor



- hadronic tensor $W_{\mu\nu} = \int_z \frac{D(z)}{z^2} w_{\mu\nu}$

$$w_{\mu\nu} = \int_k M^{(0)}(k) S^{(0)}_{\mu\nu}(k) + \int_{k_1,k_2} M^{(1)}_\sigma(k_1,k_2) S^{(1)\sigma}_{\mu\nu}(k_1,k_2)$$

- two parton: $M^{(0)} \sim \langle P,S|\bar{\psi}\psi|P,S\rangle$
- three parton: $M^{(1)}_\sigma \sim \langle P,S|\bar{\psi}A_\sigma\psi|P,S\rangle$

Hadronic tensor

- general strategy:

1. collinear expansion of $S^{(0)}$ and $S^{(1)}$

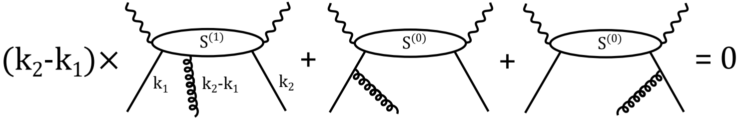
$$k^\mu = xP^\mu + k_T^\mu$$

2. make gauge invariant objects out of $M^{(0)}$ and $M_\sigma^{(1)}$
3. compute $S^{(0)}$ and $S^{(1)}$ up to two loops

Hadronic tensor

- key step: QCD Ward identity

$$(k_2 - k_1)_\sigma S^{(1)\sigma}(k_1, k_2) = S^{(0)}(k_2) - S^{(0)}(k_1)$$



- e. g. gauge link for $M^{(0)}$

$$P^+ S^{(1)-}(x_1, x_2) = -\frac{S^{(0)}(x_2)}{x_1 - x_2 + i\epsilon} + \frac{S^{(0)}(x_1)}{x_1 - x_2 + i\epsilon}$$

$$\int_{x_1,x_2} M^{(1)+} S^{(1)-} \sim \int_x \int_{\mu,\lambda} e^{i\mu x} \langle \bar{\psi}(0) igA^+(\lambda n) \psi(\mu n) \rangle S^{(0)}(x)$$

g_T - a new source for SSA?

- decomposition of $M^{(0)}$ up to twist-3

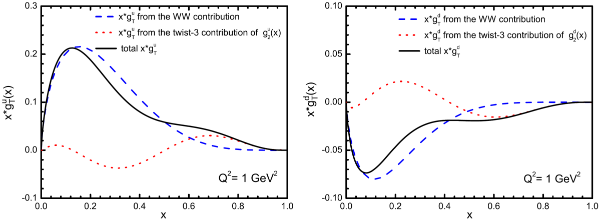
$$
\begin{aligned}
M^{(0)}(x) \sim \; & \not{P} f(x) + M_N e(x) + M_N (S \cdot n) \not{P} \gamma_5 g_1(x) \\
+ \; & M_N^2 (S \cdot n)[\not{P}, \not{n}]\gamma_5 h_L(x) + [\not{P}, \not{S}_T]\gamma_5 h_1(x) + \\
+ \; & M_N \not{S}_T \gamma_5 g_T(x)
\end{aligned}
$$

- $f(x)$ (unpolarized), $g_1(x)$ (longitudinal spin)..
- except transversity, $h_1(x)$, only $g_T(x)$ couples to transverse spin

g_T

- why is g_T interesting?
- we can calculate the twist-2 part of g_T

$$g_T(x) \sim \int_x^1 dx_1 \frac{g_1(x_1)}{x_1} + (G_F \text{ piece})$$



Wang, Mao, Lu, Phys. Rev. D **94**, no. 7, 074014 (2016)

- can also be extracted from double spin asymmetry A_{LT}

Hadronic tensor

- all-order gauge invariant result

$$w_{\mu\nu} = \frac{M_N}{2}\int_x g_T(x)\text{Tr}\left[\gamma_5 \not{S}_T S^{(0)}_{\mu\nu}(x)\right]$$
$$-\frac{M_N}{4}\int_x \tilde{g}(x)\text{Tr}\left[\gamma_5 \not{P} S_T^\alpha \left.\frac{\partial S^{(0)}_{\mu\nu}(k)}{\partial k_T^\alpha}\right|_{k=xP}\right]$$
$$+\frac{iM_N}{4}\int_{x_1,x_2}\text{Tr}\left[\left(\not{P}\epsilon^{\alpha P n S_T}\frac{G_F(x_1,x_2)}{x_1-x_2}+i\gamma_5\not{P}S_T^\alpha\frac{\tilde{G}_F(x_1,x_2)}{x_1-x_2}\right)S^{(1)}_{\mu\nu\alpha}(x_1,x_2)\right]$$

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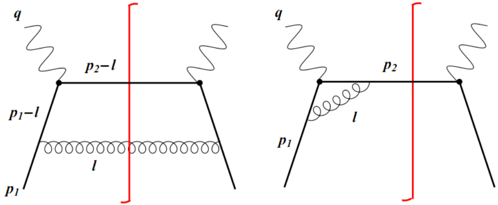
- intrinsic $g_T \sim \langle\bar{\psi}\psi\rangle$
- kinematical $\tilde{g} \sim \langle\bar{\psi}\partial^\mu\psi\rangle$
- dynamical $G_F \sim \langle\bar{\psi}F^{\mu\nu}\psi\rangle$

- remaining task: perturbative calculation of $S^{(0,1)}$

One loop analysis

- $g_T(x)$ associated with γ_5: i from the Dirac trace

  $\rightarrow$ we need another i from the cut propagator



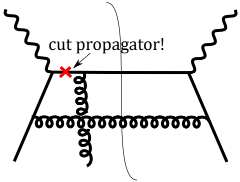
- $p_2 = q + p_1 = (p_2^+, p_2^-, 0, 0)$
- need one cut, but

$$(p_1 - l)^2 = -2p_1 \cdot l = -2p_1^+ l^- < 0$$

One loop analysis

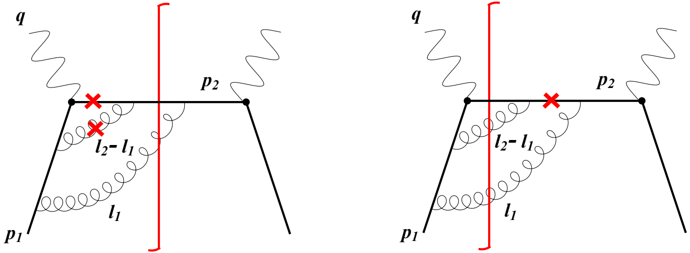
- $O(\alpha_S)$ (one-loop) for $S^{(0)}$
  $\rightarrow$ no inteference at Born level
  $\rightarrow$ no contribution from $g_T(x)$ or $\tilde{g}(x)$

- $O(\alpha_S)$ for $S^{(1)}$, there is intereference, recall:



- at one-loop only $G_F(x_1, x_2)$ contributes

Two loop analysis



- left: phase from $p_2 - l_2$ propagator
- right: phase from $p_2 - l_1$ propagator
- but, propagators are on the opposite side of the final state cut

$$\frac{1}{(p_2 - l_2)^2 + i\epsilon} \quad \text{vs.} \quad \frac{1}{(p_2 - l_1)^2 - i\epsilon}$$

$\rightarrow$ a real-virtual cancellation of the phase

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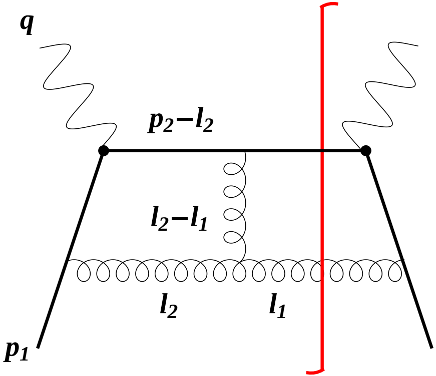
Two loop analysis



- solve loop for l_2, cut $p_2 - l_2$

$\rightarrow$ can generate a phase

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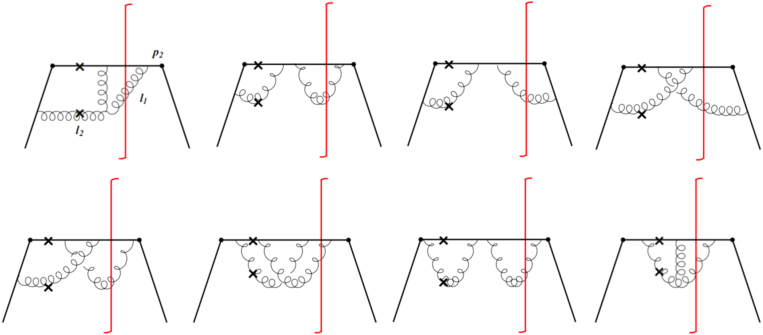
Two loop analysis



- diagrams in the same gauge invariant set
- total of 12 diagrams (mirrors of first, second and fifth omitted)

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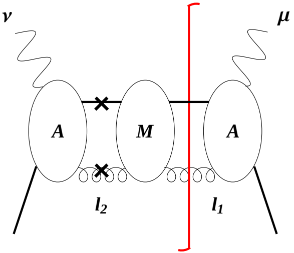
Two loop analysis



- final result for $S^{(0)}$ at two loops

$$S^{(0)\mu\nu}(x) = \frac{g^4}{N_c}(2\pi)\delta\left(\left(p_2 - \frac{P_h}{z}\right)^2\right)\int \frac{d^4 l_2}{(2\pi)^4}(2\pi)\delta(l_2^2)(2\pi)\delta((p_2 - l_2)^2)$$
$$\times \left\{iA^{\alpha\mu}(l_1)M_{\alpha\beta}(l_1, l_2)A^{\nu\beta}(l_2) - iA^{\alpha\mu}(l_2)M_{\alpha\beta}(l_2, l_1)A^{\nu\beta}(l_1)\right\}$$

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$$M_{\alpha\beta}(l_1, l_2) = (\not p_2 - \not l_1)t^a\left[-if^{abc}t^c\frac{V_{\alpha\beta\rho}\gamma^\rho}{(l_1 - l_2)^2} + t^a t^b \frac{\gamma_\alpha \not p_2 \gamma_\beta}{p_2^2} + t^b t^a \gamma_\beta \frac{\not p_2 - \not l_1 - \not l_2}{(p_2 - l_1 - l_2)^2}\gamma_\alpha\right]t^b(\not p_2 - \not l_2)$$

$$A^{\alpha\mu}(l_1) = \gamma^\alpha \frac{(\not p_1 - \not l_1)}{(p_1 - l_1)^2}\gamma^\mu + \gamma^\mu \frac{\not p_2}{p_2^2}\gamma^\alpha \qquad A^{\nu\beta}(l_2) = \gamma^\nu \frac{(\not p_1 - \not l_2)}{(p_1 - l_2)^2}\gamma^\beta + \gamma^\beta \frac{\not p_2}{p_2^2}\gamma^\nu$$

Factorization

$$S^{(0)}(p_1) \sim \int \frac{d^2 \boldsymbol{l}_{2T}}{(p_1 - l_2)^2} \sim \int \frac{d^2 \boldsymbol{l}_{2T}}{\boldsymbol{l}_{2T}^2}$$

$\rightarrow$ log divergence

- but, this is lowest order contribution for g_T so isn't it finite???
- g_T and $\tilde{g}$ are related

$$g_T(x) + \frac{\tilde{g}(x)}{2x} = \frac{1}{2x} \int_{x'} \frac{G_F(x, x') + \tilde{G}_F(x, x')}{x - x'}$$

Factorization

$$\int_x g_T(x)\mathrm{Tr}\left[\gamma_5 \not{S}_T S^{(0)}_{\mu\nu}(x)\right] - \frac{1}{2}\int_x \tilde{g}(x)\mathrm{Tr}\left[\gamma_5 \not{P} S_T^\alpha \left.\frac{\partial S^{(0)}_{\mu\nu}(k)}{\partial k_T^\alpha}\right|_{k=xP}\right]$$
$$= \int_x g_T(x)\left\{\mathrm{Tr}\left[\gamma_5 \not{S}_T S^{(0)}_{\mu\nu}(x)\right] + \mathrm{Tr}\left[\gamma_5 x\not{P} S_T^\alpha \left.\frac{\partial S^{(0)}_{\mu\nu}(k)}{\partial k_T^\alpha}\right|_{k=xP}\right]\right\} + (G_F \text{ piece})$$

- $\partial S^{(0)}/\partial k_T^\alpha$ has the same collinear divergence but with a opposite sign!
- g_T contribution is completely finite
- the divergence resides in G_F pieces
  $\rightarrow$ evolution of G_F

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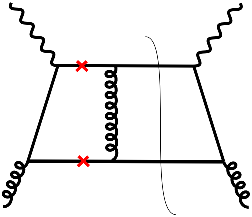
Gluon initiated channel

- there is also a gluon version of "g_T":
  $\mathcal{G}_{3T}(x) \sim \langle F^{\alpha+}(0) F^{\beta+}(z) \rangle$
- can be extracted from the "known" gluon helicity pdf

$$\mathcal{G}_{3T}(x) = \frac{1}{2} \int_x^1 dx_1 \frac{\Delta G(x_1)}{x_1} + \ldots$$

- generic diagram that contributes a phase



SB, Hatta, Li, in preparation

Gluon initiated channel

- hadronic tensor

$$w_{\mu\nu} = iM_N \int \frac{dx}{x} \mathcal{G}_{3T}(x) \epsilon^{n\alpha\beta S_\perp} S^{(0)\alpha'\beta'}_{\mu\nu}(p_1)\omega_{\alpha'\alpha}\omega_{\beta'\beta}$$

$$-iM_N \int \frac{dx}{x^2} \tilde{g}(x) \left(g_\perp^{\beta\lambda} \epsilon^{\alpha P n S_\perp} - g_\perp^{\alpha\lambda} \epsilon^{\beta P n S_\perp} \right) \left(\frac{\partial S^{(0)}_{\mu\nu\alpha\beta}(k)}{\partial k^\lambda} \right)_{k=p_1} + \dots$$

$$S^{(0)\mu\nu\alpha\beta}(p_1) = -\frac{ig^4}{2(N_c^2-1)}(2\pi)\delta\left(\left(p_2 - \frac{P_h}{z}\right)^2\right)$$

$$\times \int \frac{d^4 l_2}{(2\pi)^4}(2\pi)\delta((l_2^2) \times (2\pi)\delta((p_2 - l_2)^2)$$

$$\times \left[A^{\alpha\mu}_{lj}(l_1)\bar{M}_{jikl}(l_1, l_2)A^{\nu\beta}_{ik}(l_2) - A^{\beta\nu}_{lj}(l_1)\bar{M}_{jikl}(l_1, l_2)A^{\mu\alpha}_{ik}(l_2) \right]$$

SB, Hatta, Li, in preparation

Hard factors

- Dirac traces & loop integration

  $\rightarrow$ hard factors $\Delta\hat{\sigma}$
  $\rightarrow$ completely analytical!

$$\frac{d^6\Delta\sigma}{dx_B dQ^2 dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_S^2 M_N}{16\pi^2 x_B^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k\mathcal{S}_k \int_{x_{\rm min}}^1 \frac{dx}{x}\int_{z_{\rm min}}^1 \frac{dz}{z}$$
$$\times\,\delta\left(\frac{q_T^2}{Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$
$$\times \sum_{a,b=q,\bar{q},g} e_a^2\left[D_b(z)x^2\frac{\partial}{\partial x}\left(\frac{\tilde{g}_a(x)}{x}\right)\Delta\hat{\sigma}_{Dk}^{ab} + D_b(z)\tilde{g}_a(x)\Delta\hat{\sigma}_k^{ab}\right],$$

- for ex.

$$\Delta\hat{\sigma}_1^{qq} = \frac{(N_c^2-1)\,\hat{z}\,\left(N_c^2\left(3\hat{x}^2-3\hat{x}-1\right)+3\hat{x}^2-3\hat{x}+1\right)}{4N_c^2 q_T(\hat{z}-1)\hat{z}}$$

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Conclusions

- SSA requires helicity flip

  $\rightarrow$ insight into chiral symmetry breaking!

- what is the origin of SSA?

- g_T contribution suppressed by α_S but g_T has a twist-2 piece

  $\rightarrow$ can be numerically significant
- numerical evaluation underway