BRST-restoring 1-loop Counterterms for the Standard Model in Dimensional Regularization & Renormalization

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Based on [arXiv:2004.14398 (JHEP 08 (2020) 08, 024)] and [Work In Progress],
with Amon Ilakovac, Marija Mađor-Božinović (PMF Zagreb),
and Dominik Stöckinger (IKT, TU Dresden)

Motivation: Dimensional Regularization and γ_5 (1/3)

- Observable nature is chiral $\Rightarrow$ Realistic 4D models contain **chiral** fermions (e.g. EW sector in Standard Model, extensions ...) $\rightsquigarrow \mathbb{P}_{\mathrm{R/L}} = (1 \pm \gamma_5)/2$.
- Dimensional Regularization (DReg, ['t Hooft,Veltman–1972]...): $\mu^{4-d} \int \mathrm{d}^d x$ widely used in calculations / literature / automated codes, etc.: doesn't break gauge and Lorentz symmetries *(as long as no γ_5 present, e.g. QCD)*.

$d = 4 - 2\epsilon$-"dimensions": $\mathbb{M}_d = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}$. Small $\epsilon > 0$ regularizes UV divergences ($\epsilon < 0$ for IR divs.).
Metrics: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{g}_{\mu\nu}$
More generally: $X_\mu = \overline{X}_\mu + \widehat{X}_\mu$ The $\widehat{\cdot}$ quantities are ***evanescent*** objects.

$$g_{\mu\nu} g^{\nu\mu} = d\,, \qquad \bar{g}_{\mu\nu} \bar{g}^{\nu\mu} = 4\,, \qquad \hat{g}_{\mu\nu} \hat{g}^{\nu\mu} = -2\epsilon\,.$$

Motivation: Dimensional Regularization and γ_5 (2/3)

- DimReg and Dirac γ^μ matrices? (see [Collins–1986])
- What about intrinsically 4-dimensional objects: γ_5 and $\epsilon_{\mu\nu\rho\sigma}$?

In 4D: $\{\gamma_5, \gamma^\mu\} = 0$ OK, but inconsistent in d-D:
$\mathrm{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) \propto (d-4)\,\mathrm{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) = 0$ when $d \to 4$ (using trace cyclicity), instead of $4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}$ ($\rightsquigarrow$ chiral anomaly, pion decay...).

't Hooft-Veltman-Breitenlohner-Maison ("BMHV") scheme:
$\gamma_5 = (i/4!)\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$, $\{\gamma_5, \bar{\gamma}^\mu\} = 0$, but $[\gamma_5, \hat{\gamma}^\mu] = 0$,
and: $\{\gamma_5, \gamma^\mu\} = \{\gamma_5, \hat{\gamma}^\mu\}$, $[\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu]$.
Cyclic trace, and $\mathrm{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) = 4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}$.

Axiomatically consistent (*unitarity/causality*) at all orders (proven by Breitenlohner and Maison [Breitenlohner,Maison–1975, Breitenlohner,Maison–1977]).
$1/\epsilon$-pole (e.g. MS(bar)) subtraction $\Rightarrow$ **"Dimensional renormalization"** (DimRen).

Motivation: Dimensional Regularization and γ_5 (3/3)

- $\exists$ numerous other γ_5 schemes (see e.g. the reviews [Gnendiger...–2017,Bruque...–2018], and [Larin–1993,Trueman–1995,Jegerlehner–2000] – *non-exhaustive*).
- However it is not clear what happens at all loop orders. E.g. semi-"naive" γ_5 treatment, with γ_5 traces replacements, supplemented with manual restoration of symmetries (using e.g. Ward IDs, etc.). Other schemes [Kreimer–1990,'94] even abandon the cyclicity property of the Trace operation ("reading-point prescription", ...).

Consistency wrt. unitarity/causality may not be always clear.

Our aim

- Apply our generic results [arXiv:2004.14398], based on [Martin,Sanchez-Ruiz–1999] and on ***algebraic renormalization*** techniques, to the (chiral) **massless Standard Model** in DReg/DimRen and BMHV scheme, at 1-loop.
- *Systematic* treatment of γ_5 matrix, **consistent by construction** at any loop order. Freedom of finite local counterterms in BMHV $\rightarrow$ we can **restore** BRST invariance.
- We obtain the singular and the **BRST-restoring SM counterterms** at 1-loop, as a basis for future **consistent ≥ 2 - loop calculations** in the BMHV scheme.

Outline

1. The SM formulated as a R-Model
   - SM action S_0; extension to d dimensions
   - BRST framework implementation
   - Completed SM in d dimensions
   - BRST invariance of SM @ tree-level?
2. Calculations @ 1-loop
   - 1-loop singular counterterm action
   - Restore BRST invariance @ loop-level
   - Finite contributions
   - Results: Gauge anomalies cancel!; 1-loop finite counterterm action
3. Discussion & Summary

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Usual Standard Model (1/2)

Group	Vector fields V_μ^A ; Field strengths $V_{\mu\nu}^A$	Couplings g_V	Structure ctes. f^{ABC}
$SU(3)_C$	$G_\mu^a\,,\quad a = 1\cdots 8\,,$ $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_S f^{abc} G_\mu^b G_\nu^c$	$g_3 \equiv g_S$	f^{abc}
$SU(2)_L$	$W_\mu^i\,,\quad i = 1,2,3\,,$ $W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk} W_\mu^j W_\nu^k$	$g_2 \equiv g$	ϵ^{ijk}
$U(1)_Y$	$B_\mu\,,$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$	$g_1 \equiv g'$	0

$N_f = 3$ families ($f = 1\cdots N_f$) of:

- Left Quark doublets: $Q_L^f = (u_L^f, d_L^f)^T$ + Right Quark singlets: u_R^f, d_R^f;
- Left Lepton doublets: $L_L^f = (\nu_L^f, e_L^f)^T$ + Right lepton singlets: e_R^f (no ν_R).

Higgs $SU(2)$ complex doublet ($\phi^\pm$ and ϕ^0: complex scalars) – **massless SM** $\rightsquigarrow$ no VEV:

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad ; \quad \widetilde{H} \equiv i\sigma_2 H^* (= \epsilon_{ij} H_j^*) = \begin{pmatrix} \phi^{0\,*} \\ -\phi^- \end{pmatrix} .$$

		$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
L. quark doublet $Q_L = (u_L, d_L)^T$		$\mathbb{3}$	$\mathbb{2}$	1/3
R. quark singlets	u_R	$\mathbb{3}$	$\mathbb{1}$	4/3
	d_R	$\mathbb{3}$	$\mathbb{1}$	-2/3
L. lepton doublet $L_L = (\nu_L, e_L)^T$		$\mathbb{1}$	$\mathbb{2}$	-1
R. charged lepton singlet e_R		$\mathbb{1}$	$\mathbb{1}$	-2
Higgs doublet	H	$\mathbb{1}$	$\mathbb{2}$	1
	$\widetilde{H}$	$\mathbb{1}$	$\widetilde{\mathbb{2}}$	-1

		T_3	Y	Q
Quarks	u_L	1/2	1/3	2/3
	d_L	-1/2	1/3	-1/3
	u_R	0	4/3	2/3
	d_R	0	-2/3	-1/3
Leptons	ν_L	1/2	-1	0
	e_L	-1/2	-1	-1
	e_R	0	-2	-1
Higgs boson	h	-1/2	1	0

Usual Standard Model (2/2)

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{g-fix}} .$$

The covariant derivatives take the form:

$$D_\mu = \partial_\mu - i \sum_V g_V t_V^A V_\mu^A \equiv \partial_\mu - i g_S G_\mu^a \frac{\Lambda^a}{2} - i g W_\mu^i \frac{\sigma^i}{2} - i g' B_\mu \frac{Y}{2} .$$

and we have:

$$\mathcal{L}_{\text{gauge}} = \frac{-1}{4} (F_V^A)_{\mu\nu} (F_V^A)^{\mu\nu} \equiv \frac{-1}{4} \left(F_{\mu\nu} F^{\mu\nu} \big|_{U(1)} + F_{\mu\nu}^i F^{i\,\mu\nu} \big|_{SU(2)} + F_{\mu\nu}^a F^{a\,\mu\nu} \big|_{SU(3)} \right) ,$$

$$\mathcal{L}_{\text{fermions}} = \sum_{f=1}^{N_f} \overline{Q_L}^f i \not{D}_{Q_L} Q_L^f + \overline{L_L}^f i \not{D}_{L_L} L_L^f + \overline{u_R}^f i \not{D}_{u_R} u_R^f + \overline{d_R}^f i \not{D}_{d_R} d_R^f + \overline{e_R}^f i \not{D}_{e_R} e_R^f ,$$

$$\mathcal{L}_{\text{Higgs}} = |D_\mu H|^2 - V(H) \quad ; \quad \text{Potential:} \quad V(H) = \mu_H^2 H^\dagger H + \lambda (H^\dagger H)^2 ,$$

$$-\mathcal{L}_{\text{Yukawa}} = \sum_{f_1, f_2 = 1}^{N_f} y^u_{f_1 f_2} \overline{Q_L}_\alpha^{f_1} \widetilde{H}_\alpha u_R^{f_2} + y^d_{f_1 f_2} \overline{Q_L}_\alpha^{f_1} H_\alpha d_R^{f_2} + y^\ell_{f_1 f_2} \overline{L_L}_\alpha^{f_1} H_\alpha e_R^{f_2} + \text{h.c.} ,$$

$$\mathcal{L}_{\text{gh}} = \partial^\mu \bar{c}_A \cdot D_\mu^{AB} c_B ,$$

$$\mathcal{L}_{\text{g-fix}} = \frac{\xi_A}{2} B^A B_A + B^A \partial^\mu V_\mu^A \sim \frac{-(\partial^\mu V_\mu^A)^2}{2\xi_A} \leftsquigarrow \boldsymbol{R_\xi} \textbf{ gauge} .$$

SM reformulation as a R-Model

Based on the gauge transformations of the different SM fields, we define:

$$H_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{01} + i\phi_{02} \\ \phi_1 + i\phi_2 \end{pmatrix}_\alpha \Rightarrow \Phi_m = \begin{pmatrix} \phi_{01} & \phi_{02} & \phi_1 & \phi_2 \end{pmatrix}^T_m , \qquad \text{repres. "S", generators } \theta^A_{mn} .$$

$$D_\mu \Phi_m = (\partial_\mu \delta_{mn} - i g_V V^A_\mu \theta^A_{mn}) \Phi_n .$$

and the right-handed multiplets $\Psi = \boldsymbol{\mathcal{Q}}, \boldsymbol{\mathcal{L}}$ (generators $T^A_{\boldsymbol{\mathcal{Q}}\,IJ}, T^A_{\boldsymbol{\mathcal{L}}\,ij}$):

$$\mathbb{P}_\mathsf{R} \boldsymbol{\mathcal{Q}}_I = \begin{pmatrix} Q^C_L = \begin{pmatrix} u^C_L \\ d^C_L \end{pmatrix} \\ u_R \\ d_R \end{pmatrix}_{I=\{q,i,f\}} \qquad \mathbb{P}_\mathsf{R} \boldsymbol{\mathcal{L}}_i = \begin{pmatrix} L^C_L = \begin{pmatrix} \nu^C_L \\ e^C_L \end{pmatrix} \\ 0 \\ e_R \end{pmatrix}_{i=\{\ell,f\}}$$

(q: quark "flavour" $SU(2)$, i: $SU(3)$ color, f: family). (ℓ: lepton "flavour" $SU(2)$, f: family).

$$(D_{\boldsymbol{\mathcal{Q}}})_\mu = \partial_\mu \mathbb{1} - i g_V V^A_\mu T^A_{\boldsymbol{\mathcal{Q}}} , \qquad (D_{\boldsymbol{\mathcal{L}}})_\mu = \partial_\mu \mathbb{1} - i g_V V^A_\mu T^A_{\boldsymbol{\mathcal{L}}} .$$

$$\mathcal{L}_\mathsf{Higgs} = \frac{1}{2} (D_\mu \Phi_m)^2 - \cdots - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p , \quad \mathcal{L}_\mathsf{fermions} = \sum_{\Psi = \boldsymbol{\mathcal{Q}}, \boldsymbol{\mathcal{L}}} \overline{\Psi} \mathbb{P}_\mathsf{L} i \not{D}_\Psi \mathbb{P}_\mathsf{R} \Psi ,$$

$$-\mathcal{L}_\mathsf{Yukawa} = \sum_{\Psi = \boldsymbol{\mathcal{Q}}, \boldsymbol{\mathcal{L}}} \frac{(Y_\Psi)^m_{IJ}}{2} \overline{\Psi}^C_I \Phi_m \mathbb{P}_\mathsf{R} \Psi_J + \text{h.c.} \qquad \text{(with Yukawa matrices } Y_{\boldsymbol{\mathcal{Q}}}, Y_{\boldsymbol{\mathcal{L}}}) .$$

Extending the SM to d dimensions

Trivial for bosonic fields. Chiral fermions introduce **two problems**:

1. Kinetic term is **chiral** $\Rightarrow$ *non-regularized propagator* $\propto 1/\not{\overline{p}}$ in d-D [Bilal–2008]

$\Rightarrow$ We need an actual d-D kinetic term: $i\overline{\Psi}_I \not{\partial} \Psi_I$
$\Rightarrow$ Equivalent to introducing a "left-handed inert" component to the fermions.
(Inert because gets removed in interaction terms due to explicit presence of $\mathbb{P}_{\mathsf{R/L}}$.)

2. How to promote in d-D the $\overline{\Psi}\mathbb{P}_{\mathsf{L}}\not{G}\mathbb{P}_{\mathsf{R}}\Psi$ interaction term $\propto \overline{\Psi}_I \gamma^\mu \mathbb{P}_{\mathsf{R}} \Psi_J$?
While in 4D: $\gamma_\mu \mathbb{P}_{\mathsf{R}} = \mathbb{P}_{\mathsf{L}} \gamma_\mu = \mathbb{P}_{\mathsf{L}} \gamma_\mu \mathbb{P}_{\mathsf{R}}$, it is not so in d-D.

$\Rightarrow$ **NO unique way of extending the model to d-dimensions!**
$\Rightarrow$ Use the interaction term that makes calculations **the most simple**:

$$\overline{\Psi}_I \mathbb{P}_{\mathsf{L}} \gamma^\mu \mathbb{P}_{\mathsf{R}} \Psi_J \quad (\text{"symmetric chiral-projection"})\,.$$

(Explicitly conveys the fact that fermions were chiral.)
Equivalent to the Larin symmetrization prescription $\frac{1}{2}\left(\gamma^\mu - \gamma_5 \gamma^\mu \gamma_5\right) \mathbb{P}_{\mathsf{R}}$.

BRST symmetry [Becchi,Rouet,Stora–1975,Tyutin–1975]




BRST symmetry: **Residual** symmetry *after fixing the gauge* (≈ "generalized" version of gauge symmetry).

Infinitesimal gauge transfo. of fields: $\varphi_i \to \delta_\alpha \varphi_i$ linear in the (small) gauge parameter α $\xRightarrow{\alpha^a \to \theta c^a}$ θ: Grassmann parameter; c^a: (anticommuting) ghost. BRST transformation of φ:

$$\delta_{\mathsf{BRST}} \varphi = \theta s \varphi \equiv \delta_\alpha \varphi |_{\alpha^a \to \theta c^a} .$$

All-loop order BRST invariance?

Aim: Verifying/enforcing BRST invariance $\forall$ orders of perturbation.

- $\forall\phi$, introduce in S_0 external sources K_ϕ coupling linearly to $s_d\phi$.
  (Similar to **Batalin-Vilkovisky** "antifields" [Batalin,Vilkovisky–1977,'81,'84].)

- BRST invariance for **quantum effective action** Γ (up to $\mathcal{O}(\hbar^n)$) $\rightarrow$ **Slavnov-Taylor Identities** (STI) ($\sim$ Ward IDs with gauge transfos.):

$$\mathcal{S}(\Gamma) \equiv \int \mathrm{d}\,x \left(\sum_\Phi \mathrm{Tr}\, \frac{\delta\Gamma}{\delta K_\Phi(x)} \frac{\delta\Gamma}{\delta\Phi(x)} + B^a(x) \frac{\delta\Gamma}{\delta\bar{c}_a(x)} \right) = 0\,.$$

($\mathcal{S}(\Gamma_{\text{ren}})$: for 4 dims on renormalized Γ_{ren}; $\mathcal{S}_d(\Gamma_{\text{DReg}})$: for d dims on dim-reg Γ_{DReg}.)

From the **Quantum Action Principle** [Lowenstein–1971,Piguet,Sorella–1995], [Piguet,Rouet–1981], BRST/ST breaking as a local operator insertion in Γ:

$$\mathcal{S}(\Gamma) = \Delta_{\text{breaking}} \cdot \Gamma\,.$$

BRST restoration really matters only at the renormalized level (in 4D).

The completed SM in d dimensions

The complete defining SM action in d dimensions, including the BRST sources:

$$S_0 = \int \mathrm{d}^d x \, (\mathcal{L}_{\mathsf{SM}} \equiv \mathcal{L}_{\mathsf{gauge}} + \mathcal{L}_{\mathsf{fermions}} + \mathcal{L}_{\mathsf{Higgs}} + \mathcal{L}_{\mathsf{Yukawa}} + \mathcal{L}_{\mathsf{gh}} + \mathcal{L}_{\mathsf{g\text{-}fix}} + \mathcal{L}_{\mathsf{ext}}) \,,$$

with $\mathcal{L}_{\mathsf{gauge}}, \mathcal{L}_{\mathsf{Higgs}}, \mathcal{L}_{\mathsf{gh}}, \mathcal{L}_{\mathsf{g\text{-}fix}}$ as before, and:

$$\mathcal{L}_{\mathsf{fermions}} \Rightarrow \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} \overline{\Psi}_I i \not{D}_\Psi^{IJ} \Psi_J = \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} i\overline{\Psi}_I \not{\partial} \Psi_I + g_A {T_\Psi}^A_{IJ} \overline{\Psi}_I \mathbb{P}_\mathsf{L} \not{G}^a \mathbb{P}_\mathsf{R} \Psi_J \,,$$

$$\mathcal{L}_{\mathsf{Yukawa}} = \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} -\frac{(Y_\Psi)^m_{IJ}}{2} \Phi_m \overline{\Psi}^C_I \mathbb{P}_\mathsf{R} \Psi_J + \mathsf{h.c.} \,,$$

and the external BRST-source terms:

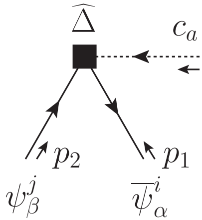
$$\mathcal{L}_{\mathsf{ext}} = \rho^\mu_A s_d V^A_\mu + \zeta_A s_d c^A + \mathcal{Y}^m s_d \Phi_m + \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} \bar{R}^I s_d \Psi_I + s_d \overline{\Psi}_I R^I \,.$$

BRST invariance of the SM @ tree-level?

- SM is BRST-invariant at tree-level in 4D due to gauge symmetry: $\mathcal{S}_4 S_0^{(4\mathrm{D})} = 0$.
- Is it still so in d-dimensions? $\Rightarrow$ **No!** $\exists$ BRST breaking $\widehat{\Delta}$ at tree-level:

$$\mathcal{S}_d S_0 = \int \mathrm{d}^d x \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} (g_A {T_\Psi}^A_{IJ}) c^A \left\{ \overline{\Psi}_I \left(\overleftarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_\mathsf{R} + \overrightarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_\mathsf{L} \right) \Psi_J \right\} \equiv \widehat{\Delta} \,.$$

Interpreted as an interaction vertex whose Feynman rule is:



$$= \sum_{\Psi=\boldsymbol{\mathcal{Q}},\boldsymbol{\mathcal{L}}} g_A {T_\Psi}^A_{IJ} \left(\widehat{p\!\!\!/}_1 \mathbb{P}_\mathsf{R} + \widehat{p\!\!\!/}_2 \mathbb{P}_\mathsf{L} \right)_{\alpha\beta} \,.$$

Outline

1-loop singular counterterm $S^{(1)}_{\text{sct}}$ action

1-loop SCT action evaluated from 1-loop diagrams (self-energies, vertices ...):

$$S^{(1)}_{\text{sct}} = -\Gamma^{(1)}\big|^{\text{BMHV}}_{\text{div}} = S^{(1)}_{\text{sct,inv}} + S^{(1)}_{\text{sct,evan}} \, .$$

First term arises from usual renormalization transformation (usual from literature):

$$S_{0,\text{inv}} \longrightarrow S_{0,\text{inv}} + S_{\text{ct,inv}} \, .$$

Second term *specific to BMHV scheme*, arises from fermion loops, & *evanescent*:

$$S^{(1)}_{\text{sct,evan}} = \frac{\hbar}{16\pi^2\epsilon} \sum_{\Psi=\mathcal{Q},\mathcal{L}} \int \mathrm{d}^d x \, \frac{1}{2} \Bigg\{ \sum_G \frac{g_G^2}{3} S_2^G(\Psi) \left(\overline{(F_G^A)_{\mu\nu}(F_G^A)^{\mu\nu}} - (F_G^A)_{\mu\nu}(F_G^A)^{\mu\nu} - \bar{G}^{A\,\mu}\widehat{\partial}^2 \bar{G}^A_\mu \right) - Y_2^\Psi(S) \left(\overline{(D_\mu \Phi^m)^2} - (D_\mu \Phi^m)^2 - \frac{2}{3} \Phi^m \widehat{\partial}^2 \Phi^m \right) \Bigg\} \, .$$

$$\sum_{\Psi=\mathcal{Q},\mathcal{L}} S_2^G(\Psi) = N_f \{ (3(2Y_{Q_L}^2 + Y_{u_R}^2 + Y_{d_R}^2) + 2Y_{L_L}^2 + Y_{e_R}^2)/4, \, (3+1)/2, \, 2+0 \}^G = N_f \{10/3, \, 2, \, 2\}^G \, ,$$

with $G = U(1), \, SU(2), \, SU(3)$,

$$\sum_{\Psi=\mathcal{Q},\mathcal{L}} Y_2^\Psi(S) = 2 \, \mathrm{Tr}(3 y^{u\,\dagger} y^u + 3 y^{d\,\dagger} y^d + y^{\ell\,\dagger} y^\ell) \qquad (3 = \text{number of "colors"}) \, .$$

BRST restoration, Renormalized action (1/2)

Restore BRST symmetry, i.e. **remove the irrelevant anomalies**, if possible.
The breaking equation $\mathcal{S}\Gamma_{\text{ren}} = \Delta_{\text{breaking}}$ is generalized for Γ_{DReg} by the *Regularized Quantum Action Principle* [Breitenlohner,Maison–1977]:

$$\mathcal{S}_d\Gamma_{\text{DReg}} = \widehat{\Delta}\cdot\Gamma_{\text{DReg}} + \widehat{\Delta}_{\text{ct}}\cdot\Gamma_{\text{DReg}} + \int \mathrm{d}^d x \sum_{\Phi} \mathrm{Tr}\,[\frac{\delta S_{\text{ct}}^{(n)}}{\delta K_\Phi(x)}\cdot\Gamma_{\text{DReg}}]\frac{\delta\Gamma_{\text{DReg}}}{\delta\Phi(x)}\,.$$

Corresponding 4D breaking: $\mathcal{S}\Gamma_{\text{ren}} = \underset{d\to4}{\text{LIM}}(\mathcal{S}_d\Gamma_{\text{DReg}}) = \Delta_{\text{breaking}}$.
($\underset{d\to4}{\text{LIM}}$ is by taking $d\to4$ and cancelling the remaining evanescent parts.)
At one-loop $\mathcal{O}(\hbar)$, using $\Gamma_{\text{ren}} = \underset{d\to4}{\text{LIM}}(\Gamma_{\text{DReg}})$: (with b_d, b_4: linearized ST operator)

$$\Delta^{(1)}_{\text{breaking}} = \underset{d\to4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}}\,.$$

Finite counterterm action $S^{(1)}_{\text{fct}}$: computed so that $b_4 S^{(1)}_{\text{fct}}$ cancels the irrelevant anomalies from $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$. $S^{(1)}_{\text{fct}}$ is contained in Γ_{ren}.

BRST restoration, Renormalized action (2/2)

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \, .$$

Procedure:

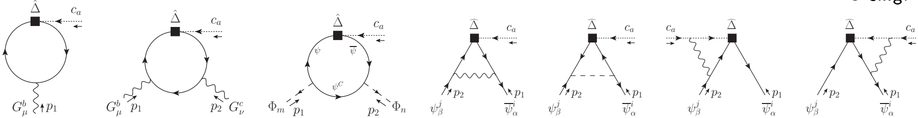
1. Evaluate $b_d S^{(1)}_{\text{sct}}$.
2. Evaluate $\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}}$ by computing 1-loop diagrams with insertion of $\widehat{\Delta}$. Check whether it cancels with $b_d S^{(1)}_{\text{sct}}$ (breaking is finite).
3. Evaluate the finite 4-dimensional part $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$.
4. Define $S^{(1)}_{\text{fct}}$ such that $b_4 S^{(1)}_{\text{fct}} \overset{\text{def.}}{=} -N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$ *("irrelevant anomalies")*, and verify the *absence of relevant anomalies* from $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$.

Evaluation of $b_d S^{(1)}_{\text{sct}}$ – Cancellation with $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

$b_d\, S^{(1)}_{\text{sct,inv}}\Big|_{\text{No ghost}} = 0$, except for $b_d\, S^{(1)}_{\text{sct,inv}}\Big|_{\text{Ghost}} = \widehat{\Delta}$, and $b_d S^{(1)}_{\text{sct,evan}} \neq 0$.

On the other hand we calculate the non-zero terms contributing to $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}}$



Terms from $b_d S^{(1)}_{\text{sct}} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$	Terms from $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$
$-g_A^2 \frac{S_2(\Psi)}{3} b_d \int \mathrm{d}^d x\, \frac{1}{2} \bar{G}^A_\mu \bar{\partial}^2 \bar{G}^A_\mu =$ $-g_A^2 \frac{S_2(\Psi)}{3} \int \mathrm{d}^d x\, (\bar{\partial}_\mu c_A)(\widehat{\partial}^2 \bar{G}^A_\mu)$ $-g_A^3 \frac{S_2(\Psi)}{3} \int \mathrm{d}^d x\, f^{ABC} c_A (\widehat{\partial}^2 \bar{G}^B_\mu) \bar{G}^C_\mu$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{BA,\mu}_{Gc}]^{(1)}_{\text{div}} + i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{CBA,\nu\mu}_{GGc}]^{(1)}_{\text{div}} \Rightarrow$ $S_{cG} = g_A^2 \frac{S_2(\Psi)}{3} \int \mathrm{d}^d x\, (\bar{\partial}_\mu c_A)(\widehat{\partial}^2 \bar{G}^A_\mu)$ $+S_{cGG} = g_A^3 \frac{S_2(\Psi)}{3} \int \mathrm{d}^d x\, f^{ABC} c_A (\widehat{\partial}^2 \bar{G}^B_\mu) \bar{G}^C_\mu$
$-\frac{2Y_2(S)}{3} b_d \widehat{S^0_{\Phi\Phi}}$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{nm,A}_{\Phi\Phi c}]^{(1)}_{\text{div}} \Rightarrow S_{c\Phi\Phi} = \frac{2Y_2(S)}{3} b_d \widehat{S^0_{\Phi\Phi}}$
$-g_A^2 \frac{\xi_G C_2(G)}{2} \widehat{\Delta}$	$i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}{}^{JI,A}_{\psi\bar{\psi}c}]^{(1)}_{\text{div}} \Rightarrow S_{c\bar{\psi}\psi} = g_A^2 \frac{\xi_G C_2(G)}{2} \widehat{\Delta}$

Evaluation of the finite part: $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

$$[N[\widehat{\Delta}]\cdot\Gamma_{\text{Ren}}]^{(1)} = \underset{d\to4}{\text{LIM}}[\widehat{\Delta}\cdot\Gamma^{(1)}]_{\text{fin}} \,,$$

finite part of $\widehat{\Delta}\cdot\Gamma_{\text{DReg}}$ after renormalization (removal of divs. and taking $\underset{d\to4}{\text{LIM}}$).

@ Fixed loop order: only **limited finite number** of UV-singular diagrams to evaluate → Main advantage of this method.

Shown by interpreting $N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$ using ***Bonneau Identities*** [Bonneau–1980]:

$$\text{At } \mathcal{O}(\hbar): \qquad [N[\widehat{\Delta}]\cdot\Gamma_{\text{Ren}}]^{(1)} = \underset{d\to4}{\text{LIM}}\left(-\text{r.s.p.}\left[\check{\Delta}\cdot\Gamma\right]^{(1)}_{\check{g}=0}\right) \,.$$

"r.s.p.": residue of simple pole in $\nu = 4 - d = 2\epsilon$.

$\check{\Delta}$ Feynman rules: from $\widehat{\Delta}$ and formally replace evanescent structs. $\hat{g}_{\mu\nu}$ by $\check{g}_{\mu\nu}$ with properties:

$$\check{g}_{\mu\nu}g^{\nu\rho} = \check{g}_{\mu\nu}\hat{g}^{\nu\rho} = \check{g}_\mu^{\ \rho} \,, \qquad \check{g}_{\mu\nu}\bar{g}^{\nu\rho} = 0 \,, \qquad \check{g}_\mu^{\ \mu} = 1 \,.$$

Results: SM gauge anomalies $= 0$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \Delta^{(1)}_{\text{breaking}} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|^{(1)}_{\text{sing.}} + b_d S^{(1)}_{\text{sct}}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S^{(1)}_{\text{fct}} \,.$$

Relevant anomalies that cannot be absorbed into $S^{(1)}_{\text{fct}}$ (if they exist, BRST symmetry broken and model not renormalizable):

$$-\frac{\hbar}{16\pi^2}\sum_{\Psi=\mathcal{Q},\mathcal{L}}\int \mathrm{d}^4x\,\frac{g_A g_B g_C}{3}\epsilon^{\mu\nu\rho\sigma}c_A\left(d_\Psi^{ABC}(\partial_\rho G_\mu^B)(\partial_\sigma G_\nu^C) + g_D\frac{\mathcal{D}_\Psi^{ABCD}}{3!}\partial_\sigma\left(G_\mu^B G_\nu^C G_\rho^D\right)\right),$$

with: $\mathcal{D}_\Psi^{ABCD} = (-i)3!\,\mathrm{Tr}[{T_\Psi}^A {T_\Psi}^{[B} {T_\Psi}^C {T_\Psi}^{D]}] = \frac{1}{2}(d_\Psi^{ABE} f^{ECD} + d_\Psi^{ACE} f^{EDB} + d_\Psi^{ADE} f^{EBC})$
and: $d_\Psi^{ABC} = \mathrm{Tr}[{T_\Psi}^A\{{T_\Psi}^B, {T_\Psi}^C\}]$ the anomaly coefficient:

$$d_\Psi^{\varnothing\varnothing\varnothing} = d_{\mathcal{Q}}^{\varnothing\varnothing\varnothing} + d_{\mathcal{L}}^{\varnothing\varnothing\varnothing} = \frac{N_f}{4}\left[3\left(-2Y_{Q_L}^3 + Y_{u_R}^3 + Y_{d_R}^3\right) - 2Y_{L_L}^3 + Y_{e_R}^3\right],$$

$$d_\Psi^{\varnothing ii} = d_{\mathcal{Q}}^{\varnothing ii} + d_{\mathcal{L}}^{\varnothing ii} = \frac{-N_f}{2}\left(3Y_{Q_L} + Y_{L_L}\right),$$

$$d_\Psi^{\varnothing aa} = d_{\mathcal{Q}}^{\varnothing aa} + d_{\mathcal{L}}^{\varnothing aa} = \frac{N_f}{2}\left(-2Y_{Q_L} + Y_{u_R} + Y_{d_R}\right),$$

(+ indices permutations); All other components $= 0$.

With SM hypercharge values, for each $\mathcal{Q}+\mathcal{L}$ family:

$d_\Psi^{ABC} = 0\,;\ \mathcal{D}_\Psi^{ABCD} = 0$ ✓

$\Rightarrow$ **Anomaly cancellation!**

Results: Finite counter-terms $S_{\text{fct}}^{(1)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \Delta_{\text{breaking}}^{(1)} = \underset{d\to 4}{\text{LIM}}\{\widehat{\Delta}\cdot\Gamma_{\text{DReg}}|_{\text{sing.}}^{(1)} + b_d S_{\text{sct}}^{(1)}\} + N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)} + b_4 S_{\text{fct}}^{(1)} .$$

The **finite $\mathcal{O}(\hbar)$ counterterms** $S_{\text{fct}}^{(1)}$ such that $b_4 S_{\text{fct}}^{(1)} = -N[\widehat{\Delta}]\cdot\Gamma_{\text{ren}}|^{(1)}$:

$$\begin{aligned} S_{\text{fct}}^{(1)} = \frac{\hbar}{16\pi^2} \sum_{\Psi=\mathcal{Q},\mathcal{L}} \Bigg\{ & \sum_G \frac{g_G^2}{6} S_2^G(\Psi)\left(5S_{GG} + S_{GGG} - \int d^4x\, G^{A\,\mu}\partial^2 G^A_\mu\right) + \frac{Y_2^\Psi(S)}{3} S_{\Phi\Phi} \\ & + \int d^4x \sum_{A,B:G} \frac{g_A g_B}{6} G^A_\mu G^{B\,\mu}\left(\sum_{C,D:G} g_C g_D \frac{(T_\Psi)^{ABCD}}{2} G^C_\nu G^{D\,\nu} - (\mathcal{C}_\Psi)^{AB}_{mn}\Phi^m\Phi^n\right) \\ & + \sum_V g_V^2\left(1 + \frac{\xi_V - 1}{6}\right)\sum_{\psi\in\Psi} C_2^V(\psi) S_{\overline{\psi}\psi} - \frac{((Y_\Psi^m)^* T_{\overline{\Psi}}^A Y_\Psi^m)_{IJ}}{2}\int d^4x\, g_A \overline{\Psi}_I \not{G}^A \mathbb{P}_{\text{R}}\Psi_J \\ & - \sum_G g_G^2 \frac{\xi_G C_2(G)}{4}(S_{\bar{R}c_G\Psi_R} + S_{Rc_G\overline{\Psi_R}}) \Bigg\} \end{aligned}$$

+ any BRST-invariant terms.

- **Not** gauge invariant! ($\equiv -$breaking)
- **Non-zero**, even if all (relevant) gauge anomalies cancelled.

Outline

Discussion

- $S^{(1)}_{\text{sct,evan}}$ and $S^{(1)}_{\text{fct}}$ found and are $\neq 0$. No effect on the 1-loop-level RGEs of the model. However they matter for renormalization at higher orders (≥ 2 loops), from their insertion in loop diagrams. Compare with literature (e.g. RGEs: [Machacek,Vaughn–1983,'84,'85], ...)

- Any additional ***finite*** BRST-invariant terms in $S^{(1)}_{\text{fct}}$ $\Rightarrow$ freedom for different DimReg BMHV "schemes".

- $S^{(1)}_{\text{sct,evan}} \neq 0 \longrightarrow$ We cannot use straightforwardly the technique with bare φ's & g's, and the Z multiplicative renormalization factors for defining RGEs: β_g, γ_φ.
  - Either treat RGEs for the DimReg theory: define as well $\beta_{\widehat{\mathcal{O}}}$ for the (non-physical) evanescent operators $\Longrightarrow$ **All** β-functions need to be considered for consistency.
  - Or treat RGEs for the renormalized 4D theory: the effects of the evanescent operators dilute into the other non-evanescent ones [Schubert (Nucl.Phys.B323, 1989)]. This effect can be consistently described using Bonneau-like IDs for $\mu\partial_\mu \cdot \Gamma_{\text{ren}}$ [Bonneau–1980].

Summary

- Our 1-loop results [arXiv:2004.14398] on γ_5 treatment in DimReg with BMHV scheme have been applied to the **massless Standard Model** at 1-loop order.
- Systematic consistent treatment of γ_5 in d dimensions, backed by all-loop orders rigorously proven BMHV scheme in perturbative QFT and algebraic renormalization framework. BRST invariance restored.
- *Explicit documented* formulae for 1-loop **singular evanescent**, and **BRST-restoring finite counterterms** for the Standard Model in DimReg. Necessary for consistent higher 2+ - loop order calculations.

For the future:

- Application to 2-loop Standard Model?
- Massive case with non-zero VEV? 1-loop Abelian-Higgs by [Sanchez-Ruiz–2002].
- How does this translate in the Bkgd-field gauge formalism?

Thank you!

Backups

The R-Model defining action S_0

Model with generic gauge group $\mathcal{G}$ (usually $SU(N)$; can be something else...) with right-handed (RH) fermions in "right" (R) rep. of $\mathcal{G}$ and scalars in S rep. of $\mathcal{G}$, both coupling to gauge bosons.
Originally defined in 4 dimensions, using either Weyl, or Dirac fermions with projectors $\mathbb{P}_{\mathsf{R/L}} = (1 \pm \gamma_5)/2$.

$$S_0^{(4D)} = \int \mathrm{d}^4 x \, (\mathcal{L}_{\mathsf{YM}}^{(4D)} + \mathcal{L}_{\Psi}^{(4D)} + \mathcal{L}_{\Phi}^{(4D)} + \mathcal{L}_{\mathsf{Yuk}}^{(4D)} + \mathcal{L}_{\mathsf{gh}}^{(4D)} + \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)}) \,,$$

with:

$$\mathcal{L}_{\mathsf{YM}}^{(4D)} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \,,\ \mathcal{L}_{\Phi}^{(4D)} = \frac{1}{2} (D_\mu \Phi_m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p \,,$$

$$\mathcal{L}_{\Psi}^{(4D)} = i\overline{\Psi}_i \partial\!\!\!/ \mathbb{P}_{\mathsf{R}} \Psi_i + g_S T_{R\,ij}^{\,a} \overline{\Psi}_i G\!\!\!/^a \mathbb{P}_{\mathsf{R}} \Psi_j \equiv i\overline{\Psi}_i D\!\!\!\!/_R^{ij} \Psi_j \,,$$

$$\mathcal{L}_{\mathsf{Yuk}}^{(4D)} = -(Y_R)^m_{ij}/2\, \Phi_m \overline{\Psi}_i^C \mathbb{P}_{\mathsf{R}} \Psi_j + \mathsf{h.c.} \,,$$

$$\mathcal{L}_{\mathsf{gh}}^{(4D)} = \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b \,,\ \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu \,.$$

Note the Yukawa interaction with charge-conjugated fermion ($\neq$ Dirac model where left component couples to right component).

Nota about charge conjugation

While it is clear how to define the charge conjugation operation in 4D with e.g. an explicit construction: numerically $C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \sim i\gamma^0\gamma^2$ with the good properties,
In d-D we can define a similar operation only by its action on the fermions – such that it turns fermions to their charge-conjugate and back: $\Psi^C = C\overline{\Psi}^T$ –, and its action on Dirac 4-spinor bilinears:

$$(\Psi^C)^C = \Psi \, , \, C^T = -C \, ;$$

$$\overline{\Psi}_i^C \Gamma \Psi_j^C = -\Psi_i^T C^{-1} \Gamma C \overline{\Psi}_j^T = \overline{\Psi}_j C \Gamma^T C^{-1} \Psi_i = \eta_\Gamma \overline{\Psi}_j \Gamma \Psi_i \, ,$$

$$\text{with: } \eta_\Gamma = \begin{cases} +1 & \text{for } \Gamma = \mathbb{1} \, , \, \gamma_5 \, , \, \gamma^\mu\gamma_5 \, , \\ -1 & \text{for } \Gamma = \gamma^\mu \, , \, \sigma^{\mu\nu} \, , \, \sigma^{\mu\nu}\gamma_5 \, . \end{cases}$$

(See e.g. Appendix A of [Tsai–2011].)

BRST transformations of fields of R-Model

The d-dimensional BRST transformations on the fields are as follows:

$$s_d G^a_\mu = D^{ab}_\mu c^b = \partial_\mu c^a + g_S f^{abc} G^b_\mu c^c \,,$$

$$s_d \Psi_i = s_d \Psi_{Ri} = i c^a g_S T_R{}^a_{ij} \Psi_{Rj} \,, \; s_d \overline{\Psi}_i = s_d \overline{\Psi_{Ri}} = +i \overline{\Psi_{Rj}} c^a g_S T_R{}^a_{ji} \,,$$

$$s_d \Phi_m = i c^a g_S \theta^a_{mn} \Phi_n \,,$$

$$s_d c^a = -\frac{1}{2} g_S f^{abc} c^b c^c \equiv i g_S c^2 \,,$$

$$s_d \bar{c}^a = B^a \,, \; s_d B^a = 0 \;\leftrightsquigarrow\; (\bar{c}^a, B^a) \text{ is a BRST doublet} \,,$$

with a similar form (noted s in what follows) in 4D.
The BRST operator s_d is nilpotent: $s_d(s_d\phi) = 0$, similarly to its 4D counterpart.

The completed R-Model defining action S_0 in d-D

Our complete defining action in d dimensions, including the antifields, reads:

$$S_0 = \int \mathrm{d}^d x \, (\mathcal{L}_{\mathsf{YM}} + \mathcal{L}_\Psi + \mathcal{L}_\Phi + \mathcal{L}_{\mathsf{Yuk}} + \mathcal{L}_{\mathsf{gh}} + \mathcal{L}_{\mathsf{g\text{-}fix}} + \mathcal{L}_{\mathsf{ext}}) \, ,$$

with:

$$\mathcal{L}_{\mathsf{YM}} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \, , \; \mathcal{L}_\Phi = \frac{1}{2}(D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p \, ,$$

$$\mathcal{L}_\Psi \Rightarrow i\overline{\Psi}_i \not{D}_R^{ij} \Psi_j = i\overline{\Psi}_i \not{\partial} \Psi_i + g_S T_{R\,ij}^{\;a} \overline{\Psi_{R}}_i \mathbb{P}_{\mathsf{L}} \not{G}^a \mathbb{P}_{\mathsf{R}} \Psi_{Rj} \, ,$$

$$\mathcal{L}_{\mathsf{Yuk}} = -(Y_R)^m_{ij}/2 \, \Phi_m \overline{\Psi_{R}}_i^C \mathbb{P}_{\mathsf{R}} \Psi_{Rj} + \mathsf{h.c.} \, ,$$

$$\mathcal{L}_{\mathsf{gh}} = \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b \, , \; \mathcal{L}_{\mathsf{g\text{-}fix}} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu \, ,$$

$$\mathcal{L}_{\mathsf{ext}} = \rho^\mu_a s_d G^a_\mu + \zeta_a s_d c^a + \bar{R}^i s_d \Psi_{Ri} + s_d \overline{\Psi_{R}}_i R^i + \mathcal{Y}^m s_d \Phi_m \, .$$

Quantum numbers (mass dimension, ghost number and (anti)commutativity):

	G^a_μ	$\overline{\Psi}_i$, Ψ_i	Φ_m	c^a	$\bar{c}^a$	B^a	ρ^μ_a	ζ_a	R^i, $\bar{R}^i$	$\mathcal{Y}^m$	∂_μ	s
mass dim.	1	3/2	1	0	2	2	3	4	5/2	3	1	0
ghost #	0	0	0	1	-1	0	-1	-2	-1	-1	0	1
comm.	+	−	+	−	−	+	−	+	+	−	+	−

What about a L-Model?

How do the results modify for left-handed (LH) fermions? Two approaches:

1. Either note that $\mathbb{P}_\mathsf{R} \leftrightarrow \mathbb{P}_\mathsf{L}$, corresponding to the change $\gamma_5 \leftrightarrow -\gamma_5$, and related change $\epsilon^{\mu\nu\rho\sigma} \leftrightarrow -\epsilon^{\mu\nu\rho\sigma}$.

2. Or, view LH fermions in a "left" (L) representation of $\mathcal{G}$, as being the charge-conjugate of corresponding RH fermions that would belong to the conjugate representation of the "left" ones: $\mathbb{P}_\mathsf{L}\Psi_L \equiv (\mathbb{P}_\mathsf{R}\Psi_R)^C$, and $T_L \leftrightarrow T_R \equiv T_{\overline{L}}$.

WARNING! We have **not yet** taken into account possible mixings between these right-handed and left-handed fermions (in the Yukawa sector...)!

Effective action Γ: Interpretation & notation (1/2)

Effective action: Generating functional for 1-particle irreducible (1PI) Green's functions [Weinberg–1996]:

$$\Gamma[\Phi] = \sum_{n\geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \Gamma_{\phi_n \cdots \phi_1}(x_1, \ldots, x_n)$$

$$\left(\substack{\text{Fourier} \\ \text{transform}}\right) = \sum_{n\geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^{n} \frac{\mathrm{d}^4 p_i}{(2\pi)^4} \widetilde{\phi}_i(p_i) \right) \Gamma_{\phi_n \cdots \phi_1}(p_1, \ldots, p_n) \overbrace{(2\pi)^4 \delta^4(\textstyle\sum_{j=1}^{n} p_j)}^{\text{Momentum conservation}} ,$$

$\Gamma_{\phi_n \cdots \phi_1}$ are the 1PI Green's functions defined by:

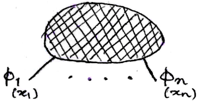
$$i\Gamma_{\phi_n \cdots \phi_1}(x_1, \ldots, x_n) = \left. \frac{i\delta^n \Gamma[\Phi]}{\delta\phi_n(x_n) \cdots \delta\phi_1(x_1)} \right|_{\phi_i = 0} = \langle \Omega | \mathbb{T}[\phi_n(x_n) \cdots \phi_1(x_1)] | \Omega \rangle^{\text{1PI}}$$

$$\equiv \langle \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}} ,$$

and $i\Gamma_{\phi_n \cdots \phi_1}(p_1, \ldots, p_n) \equiv \left\langle \widetilde{\phi_n}(p_n) \cdots \widetilde{\phi_1}(p_1) \right\rangle^{\text{1PI}}$ is defined similarly.

Effective action Γ: Interpretation & notation (2/2)

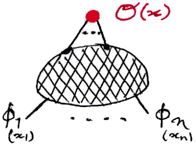
$$\Gamma[\Phi] = \sum_{n\geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \langle \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}}$$

$$= \sum_{n\geq 2} \frac{-i}{|n|!}$$ 

.

<u>Field-Operator insertion</u> in $\Gamma[\Phi]$ [Piguet,Rouet–1981]:
(e.g. counterterm insertions in loop diagrams...)

$$\textcolor{red}{\mathcal{O}(x)} \cdot \Gamma[\Phi] = \sum_{n\geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^{n} \mathrm{d}^4 x_i \; \phi_i(x_i) \right) \langle \textcolor{red}{\mathcal{O}(x)} \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{\text{1PI}}$$

$$= \sum_{n\geq 2} \frac{-i}{|n|!}$$ 

.

Notation:
$\textcolor{red}{\mathcal{O}} \cdot \Gamma[\Phi] = \int \mathrm{d}\, x \; \textcolor{red}{\mathcal{O}(x)} \cdot \Gamma[\Phi]\,.$

The BRST b_d invariants L used in $S_{\text{sct}}^{(1)}$ (1/2)

L quantities, invariant under the linear BRST transformation b_d, are defined:

$$
\begin{aligned}
L_G &= b_d \int \mathrm{d}^d x\, \widetilde{\rho}_a^\mu G_\mu^a = \left(N_G - N_{\bar{c}} - N_B - N_\rho + 2\xi \frac{\partial}{\partial \xi} \right) S_0 \\
&= \int \mathrm{d}^d x\, \left(2S_{GG} + 3S_{G^3} + 4S_{G^4} + \overline{S_{\overline{\Psi} G \Psi}} + S_{\Phi G \Phi} + 2S_{\Phi G G \Phi} - \widetilde{\rho}_a^\mu (\partial_\mu c_a) \right) , \\
L_c &= -\, b_d \int \mathrm{d}^d x\, \zeta_a c^a = (N_c - N_\zeta)\, S_0 \\
&= \int \mathrm{d}^d x\, \widetilde{\rho}_a^\mu s_d G_\mu^a + \zeta_a s_d c^a + \bar{R}^i s_d \Psi_i + s_d \overline{\Psi}_i R^i + \mathcal{Y}^m s_d \Phi_m , \\
L_\Phi &= b_d \int \mathrm{d}^d x\, \mathcal{Y}^m \Phi_m = (N_\Phi - N_{\mathcal{Y}})\, S_0 \\
&= \int \mathrm{d}^d x\, ((D_\mu \Phi^m)^2 \equiv 2S_{\Phi\Phi} + 2S_{\Phi G \Phi} + 2S_{\Phi G G \Phi}) + 4\lambda_{mnop} S_{\Phi^4_{mnop}} \\
&\quad + ((Y_R)^m_{ij} S_{\overline{\Psi_{R}}_i^C \Phi^m \Psi_{R j}} + \text{h.c.}) , \\
\overline{L_{\Psi_R}} &= -\, b_d \int \mathrm{d}^d x\, (\bar{R}^i \mathbb{P}_\mathsf{R} \Psi_i + \overline{\Psi}_i \mathbb{P}_\mathsf{L} R^i) - \int \mathrm{d}^d x\, i \overline{\Psi}_i \widehat{\partial\!\!\!/} \Psi_i = -(N_\Psi^R + N_{\overline{\Psi}}^L - N_{\bar{R}} - N_R) S_0 \\
&\quad - \int \mathrm{d}^d x\, i \overline{\Psi}_i \widehat{\partial\!\!\!/} \Psi_i = 2 \int \mathrm{d}^d x\, i \overline{\Psi}_i \overline{\partial\!\!\!/} \mathbb{P}_\mathsf{R} \Psi_i + \overline{S_{\overline{\Psi} G \Psi}} + ((Y_R)^m_{ij} S_{\overline{\Psi_{R}}_i^C \Phi^m \Psi_{R j}} + \text{h.c.}) ,
\end{aligned}
$$

The BRST b_d invariants L used in $S^{(1)}_{\text{sct}}$ (2/2)

In the previous calculations the field-counting operators have been used; they are defined as:

$$N_\varphi = \int \mathrm{d}^d x \, \varphi(x)_i \frac{\delta}{\delta\varphi(x)_i} \,, \text{ for } \varphi_i = G^a_\mu, \Phi^m, c_a, \bar{c}_a, B^a, \rho^\mu_a, \zeta_a, R^i, \bar{R}^i, \mathcal{Y}^m \,,$$

$$N^{R/L}_\Psi = \int \mathrm{d}^d x \, (\mathbb{P}_{\mathsf{R/L}} \Psi_i(x))_s \frac{\delta}{\delta\Psi_i(x)_s} \,, \; N^{L/R}_{\overline{\Psi}} = \int \mathrm{d}^d x \, (\overline{\Psi}_i(x) \mathbb{P}_{\mathsf{L/R}})^s \frac{\delta}{\delta\overline{\Psi}_i(x)^s} \,.$$

Other b_d invariants are:

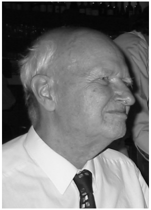
- the pure Yang-Mills term L_{gS} and its equivalent 4-dimensional version $\overline{L_{gS}}$:

$$L_{gS} = \frac{-1}{4} \int \mathrm{d}^d x \, F^a_{\mu\nu} F^{a\,\mu\nu} = S_{GG} + S_{G^3} + S_{G^4} \,;$$

- the Yukawa interaction: $L_{\bar\psi\phi\psi} = (Y_R)^m_{ij} S_{\overline{\Psi_{R_i}}^C \Phi^m \Psi_{R_j}} + \text{h.c.} \,,$
- the four-scalar interaction: $L_{\Phi^4} = \lambda_{mnop} S_{\Phi^4_{mnop}} \,.$

Since $L_\Phi = \int \mathrm{d}^d x \, 2S_{\Phi\Phi} + 2S_{\Phi G \Phi} + 2S_{\Phi G G \Phi} + 4L_{\Phi^4} + L_{\bar\psi\phi\psi}$ is a b_d invariant, it follows that the combination $L_{D\Phi} = \int \mathrm{d}^d x \, (D_\mu \Phi^m)^2 = L_\Phi - L_{\bar\psi\phi\psi} - 4L_{\Phi^4}$ is also a b_d invariant by itself.

Notation: "Normal Products" $N[\mathcal{O}(x)]$ [Zimmermann–1973]



Introduced by Zimmermann. (See also [Lowenstein–1971].)
For a field-product operator $\mathcal{O}(x)$, a normal product $N[\mathcal{O}(x)]$ is defined as the "finite part" of $\mathcal{O}(x)$, i.e. via the finite part of the time-ordered Green's functions of $\mathcal{O}(x)$:
$\langle N[\mathcal{O}] \prod_i \phi_i(x_i) \rangle^{\text{1PI}} = \text{Fin.} \left(\langle \mathcal{O} \prod_i \phi_i(x_i) \rangle^{\text{1PI}} \right)$.
[Piguet,Rouet–1981]

They depend on the chosen renormalization scheme:

- In BPHZ renormalization (original): done by subtracting the first terms of a Taylor expansion of loop integrands up to a given order ("degree" of subtraction). $\rightarrow \exists$ different normal products associated to the choice of the "degree" of subtraction. [Piguet,Rouet–1981]
- In dimensional renormalization (DimRen): the normal products are defined with respect to the ϵ-pole subtraction. [Collins–1974]

Bonneau Identities, graphical interpretation (1/2)

In DimRen, normal products $N[\widehat{\mathcal{O}}]$ of evanescent operators $\widehat{\mathcal{O}}$ of the form $\widehat{\mathcal{O}} \equiv (\hat{g}_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu})\mathcal{O}_{\mu\nu\rho\cdots}$ are interpreted [Bonneau–1980] as the difference between two ways of performing a "subtraction" in this renormalization scheme.
$\Rightarrow$ "Zimmermann-like" identities: **Bonneau Identities**.

$$N[\widehat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = - \sum_{n=2}^{n_{\text{max}}=4} \sum_{\substack{J=\{j_1,\cdots,j_n\}, \\ 0\le r\le \delta(J)}} \sum_{\substack{\{i_1,\cdots,i_r\}/ \\ 1\le i_j \le n}} \frac{(-i)^r}{r!} \frac{\partial^r}{\partial p_{i_1}^{\mu_1}\cdots\partial p_{i_r}^{\mu_r}} \cdot (-i\hbar)\text{r.s.p.} \left. \overline{\left\langle \prod_{i=1}^{n} \widetilde{\phi_{j_i}}(p_i) N[\check{\mathcal{O}}](q=-\textstyle\sum p_i) \right\rangle}^{\text{1PI}} \right|_{\substack{p_i=0 \\ \check{g}=0}}$$

$$\times N\left[\frac{1}{n!}\prod_{k=n}^{1}\left(\prod_{\{\alpha/i_\alpha=k\}} \partial_{\mu_\alpha}\right)\phi_{j_k}\right] \cdot \Gamma_{\text{ren}} + \text{similar with additional BV sources insertions}\,.$$

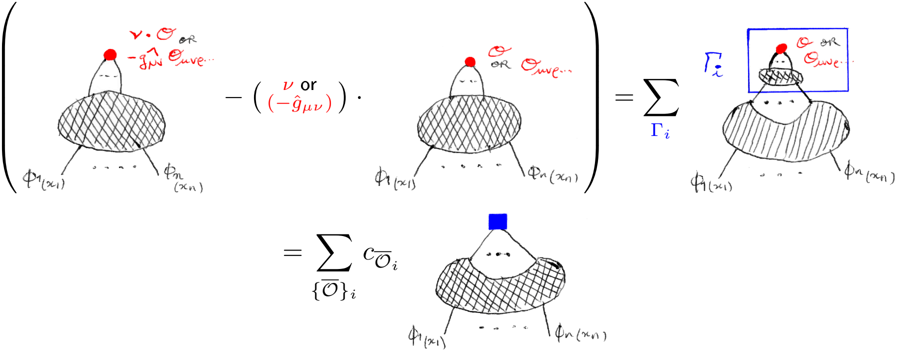
r.s.p.: residue of simple pole in $\nu = 2\epsilon = 4-d$. Overline: 1PI minimally subtracted.
$\check{g} \sim \hat{g}/\nu$, where this ν is not submitted to Laurent ν-expansion for the r.s.p..

$$N[\widehat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i] \cdot \Gamma_{\text{ren}}\,.$$

Expands evanescent operators $\widehat{\mathcal{O}_d}$ on a basis of quantum 4D operators of the renormalized 4D theory.

Bonneau Identities, graphical interpretation (2/2)

$$N[\widehat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i] \cdot \Gamma_{\text{ren}}$$



Calculation machinery

- Calculations performed with `Mathematica`.
- Model programmed using `FeynRules` [Christensen...–2009,Alloul...–2014] **(except with no BRST sources since unsupported)**. Manually patched for supporting arbitrary $SU(N)$ (N not limited to numerical values).
- Loop diagrams **(w/o BRST sources)** generated using `FeynArts` [Hahn–2000]. Amplitudes evaluated using `FeynCalc` [Mertig...–1990,Shtabovenko...–2016]; ϵ-"expansion" obtained using the `FeynCalc`'s interface `FeynHelpers` [Shtabovenko–2016] to `Package-X` [Patel–2017] **(for 1-loop only)**. WARNING! Using development version of `FeynCalc` that includes needed fixes (versions up to 17th June 2019 are OK).
- Diagrams with sources manually generated, then evaluated using `FeynCalc` as described above.
- Semi-automated (manually and computer) evaluation of group-structure invariants, using notations similarly defined as those in Machacek & Vaughn [Machacek,Vaughn–1983,'84,'85].

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