Renormalization vs Causality in Finite-Time-Path Out of Equilibrium

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Abstract

We formulate the perturbative renormalization for the out-of-equilibrium $g\phi^3$, $g\phi^4$, QED ... quantum field theory in the formalism with the finite time path. We use the retarded/advanced (R/A) basis of out-of-equilibrium Green functions, in which time ordering plays a role.

We use the dimensional regularization method and find the correspondence of diverging contributions in the Feynman diagrams and their counterparts in R/A basis. We find that the Dimensional Renormalization works exactly the way it does within the S-matrix field theories with the same number of subtractions. Although we reveal the number of problems related to energy (non-)conservation and causality, they are kept under control thanks to the $D < 4$ sector.

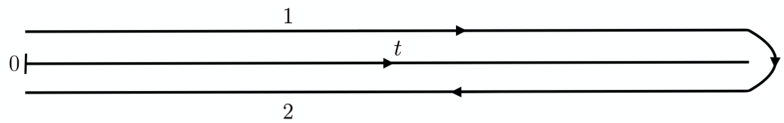

Figure 1: Finite Time Path

This talk is based mostly on the following work with D. Klabučar and D. Kuić:

I. D., D. Klabučar, Particles 2 (2019) 92-102, 2001.00124 hep-th

I.D., D. Klabučar, D. Kuić, Particles 3 (2020) 676-692, 2012.00863 hep-ph

We formulate the perturbative renormalization for the out-of-equilibrium quantum field theories such as $g\phi^3$, $g\phi^4$, QED , ...

in the formalism with the finite time path (see Figure 1),

which is necessary for time-evolution of two-point functions in out-of-equilibrium quantum field theories

The perturbation expansion is defined in matrix propagators $G_{ij}(p)$. The Wigner transforms of propagators, with transition to retarded/advanced basis leads to accumulation of all time dependence into vertex functions and time-independent (in time corresponding to $t=\infty$) lowest order propagators in the retarded/advanced basis:

$$G^{(0)}_{R(A)}(p) \;=\; \frac{-\mathrm{i}}{p^2 \;-\; m^2 \;\pm\; 2\,\mathrm{i}\,\epsilon\, p_0}, \tag{1}$$

and for the Keldysh component of the scalar propagator:

$$\begin{aligned}
&G^{(0)}_K(x,y) = \int d^4p 2\pi\delta(p^2-m^2)(1+2f(\omega_p))e^{-ip(x-y)},\\
&G^{(0)}_K(p) = G^{(0)}_{K,R}(p) - G^{(0)}_{K,A}(p)\\
&G^{(0)}_{K,R}(p) = -G^{(0)}_{K,A}(-p) = -[1+2f(\omega_p)]p_0/\omega_p G^{(0)}_{R,\infty}(p),\\
&\omega_p = \sqrt{\vec{p}^2+m^2}.
\end{aligned} \tag{2}$$

In the above expressions, time ordering plays a role.

The equal time limit of $G_K(p;t_1,t_2)$ plays the role of particle number:

$$N(\vec{p},t) \propto \lim\nolimits_{t_1\to t_2=t} G_K(p;t_1,t_2)\,.$$

Propagators are represented graphically in Figure 2.
Retarded propagators point downhill (opposite to the time direction)
and advanced propagators point uphill (in the time direction).
Retarded propagators become advanced if one interchanges points 1 and 2.

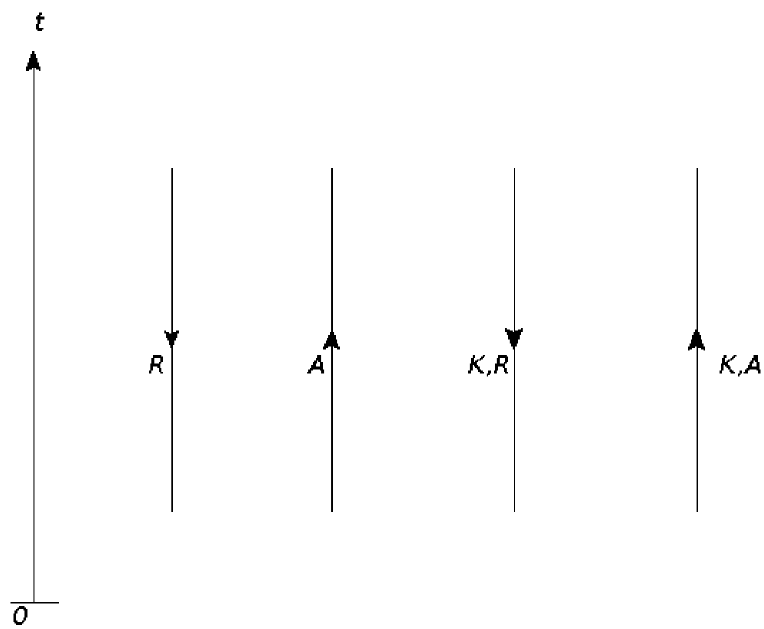

Figure 2: Graphical Representation of Propagators

VERTEX FUNCTION - at equal time limit

At each vertex (”j”) one collects the factors depending on the vertex and integrates over the space-time of the vertex ($\lambda_{i_j} = \pm 1$ for incoming/outgoing line):

$$\int_0^\infty \frac{dx_{0j} \int d^3x_j}{(2\pi)^4} e^{\mathrm{i} \sum_{i_j} \lambda_{i_j} p_{i_j} x_j} = \delta^3(\sum_{i_j} \lambda_{i_j} \vec{p}_{i_j}) \frac{\mathrm{i}}{2\pi(\sum_{i_j} \lambda_{i_j} p_{0 i_j} + \mathrm{i}\epsilon)} \ . \tag{3}$$

After integrating all internal times, all time dependence is contained in the external momenta $e^{\mathrm{i} \sum_i \lambda_k p_{0,k} x_{0,k}}$, where λ_k is equal to $+1$ (-1) if the propagator is incoming (outgoing) to vertex k.

As we are taking equal time limit $x_{0,k} \to t$, the factor becomes $e^{\mathrm{i}t(\sum_k \lambda_k p_{0,k})}$.

Thus, the total time dependence transforms into $e^{\mathrm{i}t(\sum_j \sum_{i_j} \lambda_{i_j} p_{0,i_j} + \sum_k \lambda_k p_{0,k})}$.

Vertices are represented graphically in Figure 3.

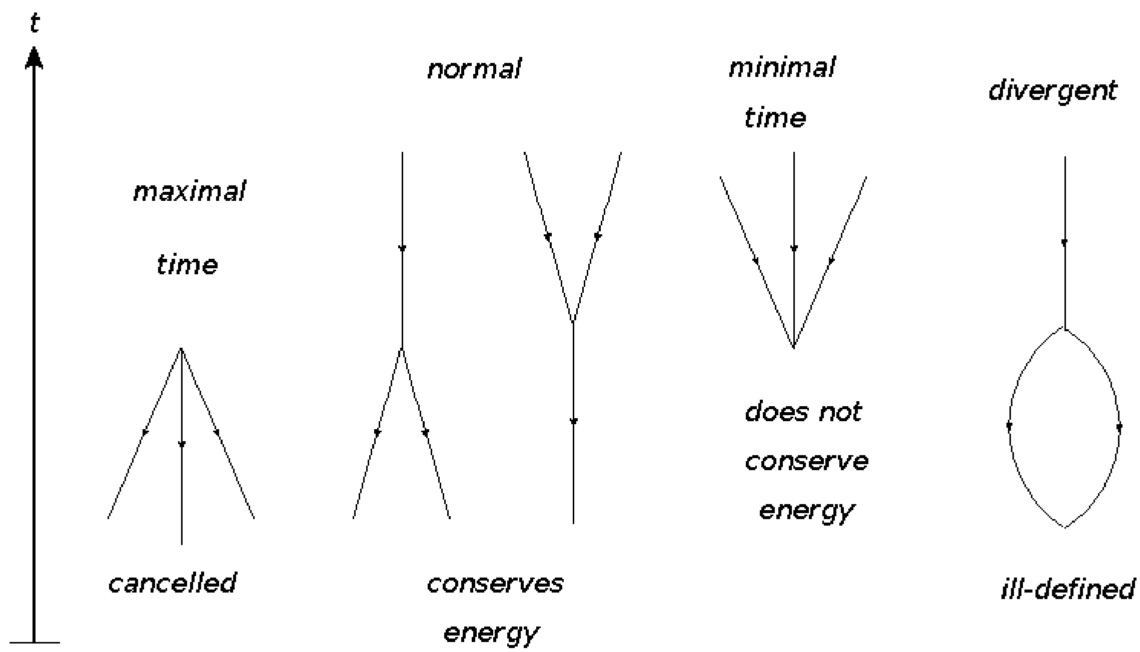

Figure 3: Vertices in Finite Time Path

There are four types of vertices:

A) At the far left in Figure 3 is the vertex with maximal time (with respect to the other ends of propagators). Those vertices are completely eliminated from perturbation expansion.

B) next two vertices from the left are normal vertices. Energy is conserved at these vertices.

C) the fourth from the left is the vertex with minimal time. Energy is not conserved at this vertex, in the way consistent with energy-time uncertainty relations. This type of vertex carries all the time dependence in the perturbation expansion.

D) the last vertex should be normal, but it is not, as its propagators close to form the divergent loop. We discuss this below.

We use the dimensional regularization method and find the correspondence of diverging contributions in the Feynman diagrams and their counterparts in R/A basis.

We find:

The Dimensional Renormalization works exactly the way it does within the S-matrix field theories with the same number of subtractions.

Nevertheless there is a number of problems:

1. The upper (in time) vertex of divergent loop does not conserve energy.
This failure is repaired while the loops converge at $D < 4$

2. Renormalized self-energies $\Sigma_R(p)$ and vacuum polarizations $\Pi_R(p)$ should be retarded (as indicated by the subscript $_R$), as they are obtained from two retarded propagators. But they are in fact not retarded since a divergence spoils this, thus creating the causality problem of the type $\Theta(t)\Theta(t) \neq \Theta(t)$.

$$\Sigma^1_F(p) = -\frac{g^2}{16\pi^2}\left\{\frac{1}{\kappa} - \frac{\gamma_E}{2} + 1 + \frac{1}{2}\ln(4\pi\frac{\mu^2}{m^2}) - \frac{1}{2}\sqrt{1-\frac{4m^2}{p^2+i\epsilon}}\ln\left[\frac{\sqrt{1-\frac{4m^2}{p^2+i\epsilon}}+1}{\sqrt{1-\frac{4m^2}{p^2+i\epsilon}}-1}\right]\right\}$$

with the limit of large p_0

$$\Sigma_F(p^2, m^2)_{p^2\to\infty} \approx -\frac{g^2}{16\pi^2}\left\{\frac{1}{\kappa} - \frac{\gamma_E}{2} + 1 + \frac{1}{2}\ln(4\pi\frac{\mu^2}{m^2}) - \frac{1}{2}\ln\left[-\frac{m^2}{p^2}\right]\right\}.$$

To verify causality of two point function one may try to project out the retarded part of finite (subtracted) part of $\Sigma^1_F(p)$, namely

$$-\mathrm{i}\int \frac{dp_0'}{2\pi}\,\frac{\Sigma^1_{F,finite}(p)}{p_0 - p_0' - \mathrm{i}\,\epsilon}\ ,$$

by integration $\int dp_0$ over large semicircle. But the contribution over very large semicircle does not vanish and the integral is ill-defined.

Indeed, we have started from the expressions for G_F (Σ_F) containing only retarded and advanced functions, and in the absence of divergence we expect this to be true at the end of calculation. Instead, the function Σ_F is not combination of R and A functions, otherwise it should vanish when $|p_0| \to \infty$ and κ chosen as arbitrarily small, such a behavior can be shifted to arbitrarily high scale. Specifically, it means that, the renormalized $\Sigma_{``R"}$

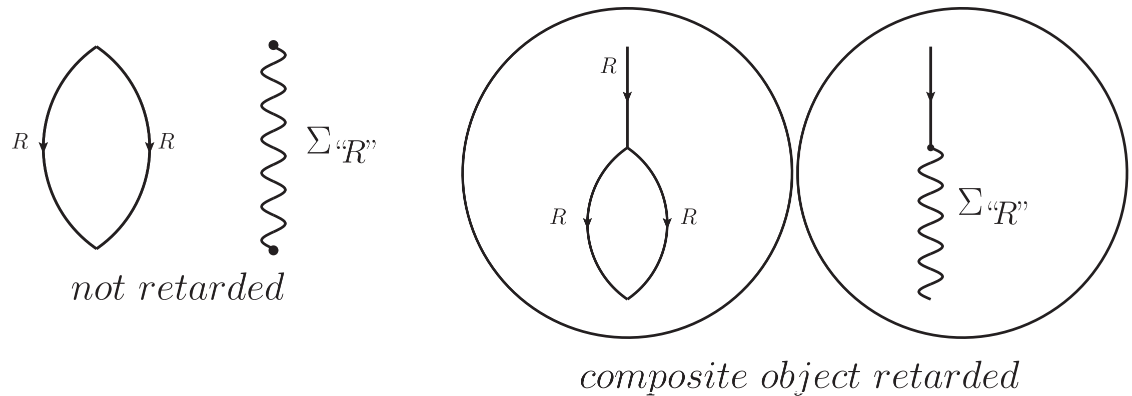

Figure 4: Composite Object with Correct Causality

is not really retarded (which we indicate by the quotation marks " "), and one cannot say which of its end points is earlier in time.

This failure is repaired by joining to the loop an upper leg so that the composite objects $G_R(p)\Sigma_R(p)$ and $D_R(p)\Pi_R(p)$ (see Figure 4), are true retarded functions.

This reparation is possible while $D < 4$.

3. In the $g\phi^4$ theory (and similarly for the gluon diagram in QCD) the $\Sigma_R(p)$ can be multiplied by retarded $G_R(p)$ or advanced $G_A(p)$ propagator (eventually of higher order to keep things finite), thus forming "sunset"-diagram (see Figure 5).

In this case, the $G_A(p)$-type contribution should be killed, but it is not, as $\Sigma_{\text{"}R\text{"}}(p)$ is not retarded. This creates causality problem of the type $\Theta(t)\Theta(-t) \neq 0$.

The contribution from sunset diagram with $G_A(p)$ is killed by performing the integral while $D < 4$

While we still get everything work, these points indicate the extended role of the $D < 4$ sector.

Notice the universality: the problems are the same in $\lambda\phi^3$, $\lambda\phi^4$, QED, and QCD.

The importance of $D < 4$ sector is growing, but things are under control, for now.

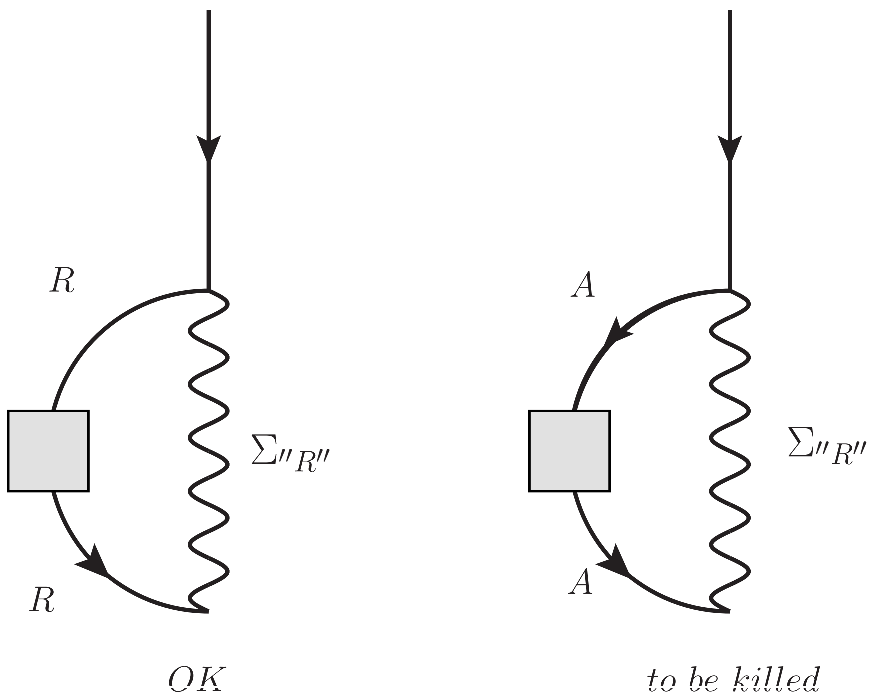

Figure 5: Sunset Diagram Incorporating the Divergent Loop

Prospects:

Complete renormalization (renormalization of vertex functions)

Renormalization Group in Finite-Time-Path out of equilibrium approach

Damping rates from Finite-Time-Path approach

Do more phenomenology

For out-of-equilibrium processes and decays calculated with Finite-Time-Path, reproduce/replace S-matrix results