Inclusive photon production in high energy pA collisions from the Color Glass Condensate

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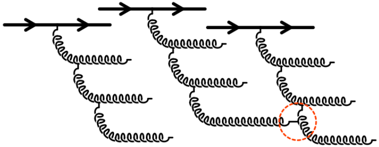
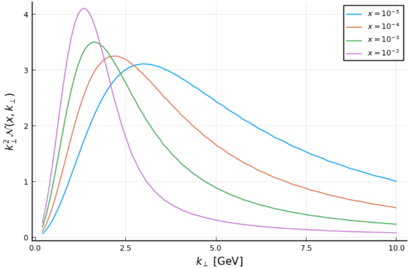
Motivation: The photon as a tool in pp and pA collisions

- $A + A \to$ thermal γ as a probe of QGP
- $e + p$, $p + p \to$ isolated γ as a pQCD benchmark
- $p + A \to \gamma$ as a probe of cold nuclear matter effects
- γ vs. hadrons as a probe:
  1. Better theoretical control
  2. Smaller cross sections by α_e
- Inclusive $\gamma^{(exp.)}$ = Direct $\gamma^{(exp.)}$ + Decay γ
- Inclusive $\gamma^{(th.)}$ = Direct $\gamma^{(exp.)}$ = Direct $\gamma^{(th.)}$ + Fragmentation $\gamma^{(th.)}$

Gluon saturation

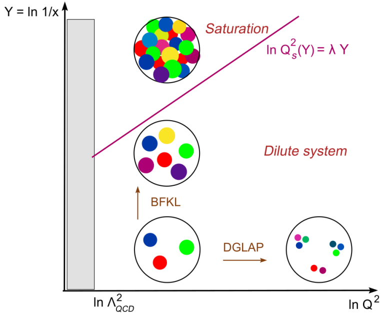


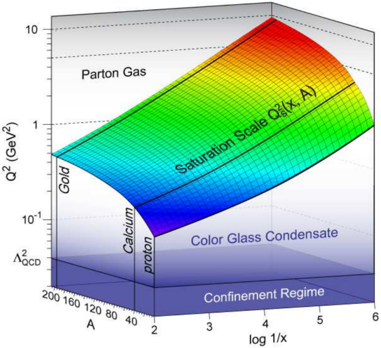


- At high energy, the parton density becomes large
- Gluon emission and recombination processes balance out
- Emergent saturation scale:

$$Q_s^2(x) \sim A^{1/3}/x^{0.3}$$

Phase space diagram of QCD

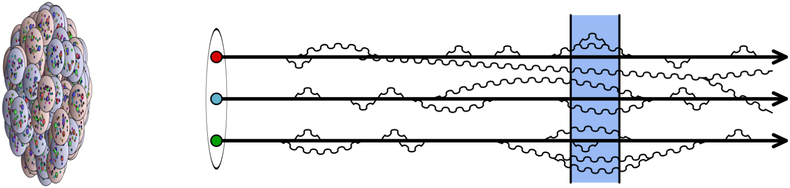




- Dependence of saturation on rapidity, transversal impulse and atomic number of the target

Venugopalan, J.Phys.G 35 (2008) 104003.

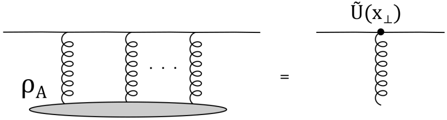
Color Glass condensate (CGC)



- EFT based on separating the gluonic degrees of freedom into frozen sources and dynamical fields
- Strong fields, but $\alpha_s \ll 1$ - saturation regime, classical chromodynamics

Gluon correlators

- High energy - eikonal scattering of the nucleus on partons
- Gluon distributions are correlators of Wilson lines:



$$\tilde{U}(x_\perp) = \mathcal{P}\exp\left[ig\int_{x^+} A_a^-(x)t_a\right]$$

$$\tilde{\mathcal{N}}_{A,Y_A}(k_\perp) = \frac{1}{N_c}\int_{\mathbf{y}_\perp} e^{ik_\perp y_\perp}\langle\tilde{U}(\mathbf{y}_\perp)\tilde{U}^\dagger(0)\rangle_{x_A}$$

- Representation of gluon shockwave through effective vertex

Fully differential $q \to q\gamma$ cross section

$$d\sigma_{q\gamma}^{(0)} = (\pi R_A^2) \frac{dk_\gamma^+ d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k_\gamma^+} \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} x_p f_{q,f}^{val}(x_p, Q^2) \times$$
$$\times \left[\frac{e^2}{q \cdot k_\gamma} P_{q\gamma}(y_q)\right] \left[\frac{4\pi}{\mathbf{k}_{\gamma\perp}^2} y_q^2 (\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp})^2\right] \tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp}),$$
$$y_q = \frac{k_\gamma^+}{q^+ + k_\gamma^+}, \quad Y_A = \ln\frac{1}{x_A}, \quad P_{q\gamma}(y_q) = \frac{1 + (1 - y_q)^2}{y_q}$$

Gelis, Jalilian-Marian, Phys.Rev.D 66 (2002) 014021

- Collinear limit: $q \cdot k_\gamma \to 0 \Longrightarrow \mathrm{q} + \mathrm{k}_\gamma \to k_\gamma / y_q$
$$\Longrightarrow \frac{4\pi}{\mathbf{k}_{\gamma\perp}^2} y_q^2 (\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp})^2 \to 4\pi$$

- Adding and subtracting the collinear singularity to the LO cross section:

$$d\sigma^{(0)}_{q\gamma} =(\pi R_A^2)\frac{dk_\gamma^+ d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k_\gamma^+}\frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} x_p f^{val}_{q,f}(x_p,Q^2)\times$$
$$\times\left[\frac{e^2}{q\cdot k_\gamma}P_{q\gamma}(y_q)\right]\left[\frac{4\pi}{\mathbf{k}^2_{\gamma\perp}}y_q^2(\mathbf{q}_\perp+\mathbf{k}_{\gamma\perp})^2-4\pi\right]\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp+\mathbf{k}_{\gamma\perp})$$
$$+(\pi R_A^2)\frac{dk_\gamma^+ d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k_\gamma^+}\frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} x_p f^{val}_{q,f}(x_p,Q^2)\times$$
$$\times\left[\frac{e^2}{q\cdot k_\gamma}P_{q\gamma}(y_q)\right]4\pi\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp+\mathbf{k}_{\gamma\perp})$$

- Next we integrate the second term over $\mathbf{q}_\perp$ in the collinear limit using the $\overline{\mathrm{MS}}$ scheme

- By defining $1/\hat{\epsilon} = 1/\epsilon - \gamma_E + \ln(4\pi)$ and $c_0 = 2e^{-\gamma_E}$, we may write

$$\int_{\mathbf{q}_\perp} \frac{\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp})}{q \cdot k_\gamma} = \frac{1-y_q}{2\pi y_q} \int_{\mathbf{x}_\perp} e^{i\mathbf{x}_\perp \mathbf{k}_\perp / y_q} \mathcal{N}_{A,Y_A}(\mathbf{x}_\perp) \times \times \left[-\frac{1}{\hat{\epsilon}} + \ln\left(\frac{c_0^2}{\mu^2 \mathbf{x}_\perp^2}\right)\right]$$

- Further calculation reveals that

$$\frac{1-y_q}{2\pi y_q} \int_{\mathbf{x}_\perp} e^{i\mathbf{x}_\perp \mathbf{k}_\perp / z} \mathcal{N}_{A,Y_A}(\mathbf{x}_\perp) \ln\left(\frac{c_0^2}{\mu^2 \mathbf{x}_\perp^2}\right) = \int_{\mathbf{q}_\perp} \frac{1}{q \cdot k_\gamma} \times \times \left[\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp}) - J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \frac{1-y_q}{y_q}\mathbf{k}_{\gamma\perp}\right|\right) \tilde{\mathcal{N}}_{A,Y_A}(\mathbf{k}_{\gamma\perp}/y_q)\right],$$

$$\frac{1-y_q}{2\pi y_q}\left(-\frac{1}{\hat{\epsilon}}\right) = \int_{\mathbf{q}_\perp} \frac{1}{q \cdot k_\gamma} J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \frac{1-y_q}{y_q}\mathbf{k}_{\gamma\perp}\right|\right)$$

- The fragmentation function is defined as

$$D_{q\gamma}(y_q) = q_f^2 \frac{\alpha_e}{2\pi} P_{q\gamma}(y_q) \left(-\frac{1}{\hat{\epsilon}}\right)$$

and has the property

$$\frac{1}{4\pi}\int_0^1 \frac{dy_q}{y_q^2} D_{q\gamma}(y_q) = \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} \frac{q_f^2 \alpha_e}{q \cdot k_\gamma} P_{q\gamma}(y_q) J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \frac{1-y_q}{y_q}\mathbf{k}_{\gamma\perp}\right|\right.$$

- This ultimately allows a clean physical interpretation, by using

$$d\sigma^{(0)}_{\gamma,\text{frag}} = \sum_f \frac{dy_q}{y_q^2} D_{q\gamma}(z) \left[2q^+(2\pi)^3 \frac{d\sigma_q^{(0)}}{dq^+ d^2\mathbf{q}_\perp}\right]_{q=k_\gamma/y_q}$$

- By integrating $d\sigma_{q\gamma}^{(0)}$ over the quark momentum space and summing over flavours we obtain

$$d\sigma_{\gamma,\mathrm{dir}}^{(0)} = (\pi R_A^2)\frac{dk_\gamma^+ d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k_\gamma^+}\sum_f \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} q_f^2 \alpha_e \frac{P_{q\gamma}(y_q)}{q\cdot k_\gamma} \times$$
$$\times x_p f_{q,f}^{val}(x_p, Q^2)\left[\frac{y_q^2(\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp})^2}{\mathbf{k}_{\gamma\perp}^2}\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{q}_\perp + \mathbf{k}_{\gamma\perp})\right.$$
$$\left. - J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \frac{1-y_q}{y_q}\mathbf{k}_{\gamma\perp}\right|\right)\tilde{\mathcal{N}}_{A,Y_A}(\mathbf{k}_{\gamma\perp}/y_q)\right]$$

- The result may be viewed directly as

$$d\sigma_{\gamma,\mathrm{dir}}^{(0)} = d\sigma_\gamma^{(0)} - d\sigma_{\gamma,\mathrm{frag}}^{(0)}$$

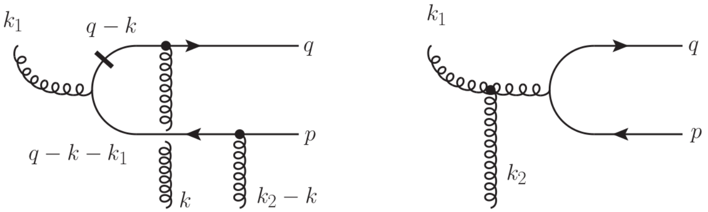
Definitions of quantities in NLO cross sections

- Gluon distributions:

$$\phi_A^{q\bar{q},g}(Y_A, \mathbf{k}_{2\perp}, \mathbf{k}_\perp) = \pi R_A^2 \frac{N_c \mathbf{k}_{2\perp}^2}{4\alpha_s} \tilde{\mathcal{N}}_{A,Y_A}(\mathbf{k}_{2\perp} - \mathbf{k}_\perp) \tilde{\mathcal{N}}_{A,Y_A}(\mathbf{k}_\perp),$$

$$\int_{\mathbf{k}_\perp} \phi_A^{q\bar{q},g}(Y_A, \mathbf{k}_{2\perp}, \mathbf{k}_\perp) = \varphi_A(Y_A, \mathbf{k}_{2\perp}), \quad \varphi_A(Y_A, \mathbf{k}_{2\perp}) \propto \mathbf{k}_{2\perp}^2 \mathcal{N}_{A,Y_A}(\mathbf{k}_{2\perp}),$$

$$\mathbf{k}_{2\perp} = \mathbf{q}_\perp + \mathbf{p}_\perp (+\mathbf{k}_{\gamma\perp}) - \mathbf{k}_{1\perp}$$



Initial state divergences?

- Cross section for $q\bar{q}$ production (argument works even when we add γ):

$$d\sigma^{(1)}_{q\bar{q}} = \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} \frac{dp^+ d^2\mathbf{p}_\perp}{(2\pi)^3 2p^+} \times$$
$$\times \left[\frac{4\alpha_s}{C_F} \int_{\mathbf{k}_{1\perp}\mathbf{k}_\perp} \frac{\varphi_p(Y_p, \mathbf{k}_{1\perp})}{\mathbf{k}_{1\perp}} \frac{\phi_A^{q\bar{q},g}(Y_A, \mathbf{k}_{2\perp}, \mathbf{k}_\perp)}{\mathbf{k}_{2\perp}} \Xi(q,p) \right],$$

$$\Xi(q,p) = \Xi_{q\bar{q},q\bar{q}} + 2\Xi_{q\bar{q},g} + \Xi_{g,g}$$

Venugopalan et al, Nucl.Phys.A 780 (2006) 146-174

- $(q-k)^2 = 0$ follows from amplitude calculation
- $(q-k-k_1)^2$ is more complicated, but the potential divergences get screened by finite $\mathbf{k}_\perp$ from the proton.

Factorization of final state divergences for NLO quark production

- The collinear divergence which occurs here is due to $q \cdot p \to 0$
- On the surface level, $\Xi(p, q) \propto 1/(q \cdot p)^2$, but in the collinear limit it turns out that, for $z_q = q^+/(q^+ + p^+)$

$$\Xi(q, p) \to \frac{1}{q \cdot p} T_R P_{gq}(z_q) X(q/z_q),$$

$$P_{gq}(z_q) = z_q^2 + (1 - z_q)^2, \quad X(p) = \frac{8\mathbf{k}_{1\perp}^2 \mathbf{k}_{2\perp}^2}{\mathbf{p}_\perp^2}$$

- In a similar fashion as the LO cross section, denoting $P = q + p$, we obtain:

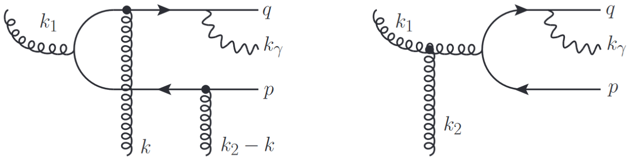
$$d\sigma_q^{(1)} = \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} \int \frac{dp^+ d^2\mathbf{p}_\perp}{(2\pi)^3 2p^+} \int_{\mathbf{k}_{1\perp}\mathbf{k}_\perp} \frac{\varphi_P(Y_p, \mathbf{k}_{1\perp})}{\mathbf{k}_{1\perp}^2} \frac{4\alpha_s}{C_F} \times$$
$$\times \left[\frac{\phi_A^{q\bar{q},g}(Y_A, \mathbf{k}_{2\perp}, \mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} \Xi(q,p) - \right.$$
$$-J_0\left(\frac{c_0}{\mu}\left|\mathbf{P}_\perp - \mathbf{q}_\perp/z_q\right|\right) \frac{T_R P_{gq}(z_q)}{q \cdot p} \times$$
$$\left. \times \left(\frac{\phi_A^{q\bar{q},g}(Y_A, \mathbf{k}_{2\perp}, \mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} X(P) \right)_{P = q/z_q} \right]$$

- Similar to before, we have the structure

$$d\sigma_{q,\mathrm{dir}}^{(1)} = d\sigma_q^{(1)} - d\sigma_{q,\mathrm{frag}}^{(1)}$$

Factorization of final state divergences for NLO inclusive photon production

- Inclusion of the photon emission results in the following diagrams:



- The collinear limit includes multiple divergences: $q \cdot k_\gamma \to 0$, $p \cdot k_\gamma \to 0$, $(q + k_\gamma) \cdot p \to 0$. We also need to integrate over both the quark and antiquark momenta to get $\sigma^{(1)}_{\gamma,\mathrm{dir}}$

- By denoting

$$y_p = \frac{k_\gamma^+}{k_\gamma^+ + p^+}, \quad y_q = \frac{k_\gamma^+}{k_\gamma^+ + q^+}, z = \frac{k_\gamma^+}{q^+ + p^+ + k_\gamma^+},$$

as well as defining the gluon-to-photon fragmentation function:

$$D_{g\gamma}(z) = \frac{\alpha_s}{2\pi}\left(-\frac{1}{\hat{\epsilon}}\right)\int \frac{dy_q}{y_q} T_R P_{gq}\left(\frac{z}{y_q}\right) D_{q\gamma}(y_q)$$

we can define the fragmentation photon cross section:

$$d\sigma^{(1)}_{\gamma,\text{frag}} = \frac{dk_\gamma^+ d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k_\gamma^+}\left[2\sum_f \int_0^1 \frac{dz}{z^2} D_{q\gamma}(z)\left(2q^+(2\pi^3)\frac{d\sigma_q^{(1)}}{dq^+ d^2\mathbf{q}_\perp}\right)_{q=k_\gamma/z}\right.$$
$$\left. + N_f \int \frac{dz}{z^2} D_{g\gamma}(z)\left(2q^+(2\pi)^3\frac{d\sigma_g^{(0)}}{dq^+ d^2\mathbf{q}_\perp}\right)_{q=k_\gamma/z}\right]$$

and the direct photon cross section:

$$
\begin{aligned}
d\sigma^{(1)}_{\gamma,\mathrm{dir}} &= \frac{dk^+_\gamma d^2\mathbf{k}_{\gamma\perp}}{(2\pi)^3 2k^+_\gamma} \sum_f \frac{16\pi\alpha_e\alpha_s^2 q_f^2}{C_F} \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3 2q^+} \frac{dp^+ d^2\mathbf{p}_\perp}{(2\pi)^3 2p^+} \times \\
&\times \int_{\mathbf{k}_{1\perp}\mathbf{k}_\perp} \frac{\varphi_P(Y_p,\mathbf{k}_{1\perp})}{\mathbf{k}_{1\perp}^2} \Bigg[\frac{\phi_A^{q\bar{q},g}(Y_A,\mathbf{k}_{2\perp},\mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} \Theta(p,q,k_\gamma) \\
&- J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \mathbf{k}_{\gamma\perp}\frac{1-y_q}{y_q}\right|\right) \frac{P_{q\gamma}(y_q)}{q\cdot k_\gamma} \left[\frac{\phi_A^{q\bar{q},g}(Y_A,\mathbf{k}_{2\perp},\mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} \Xi(q+k_\gamma,p)\right]_{q+k\gamma=k_\gamma/y_q} \\
&- J_0\left(\frac{c_0}{\mu}\left|\mathbf{p}_\perp - \mathbf{k}_{\gamma\perp}\frac{1-y_p}{y_p}\right|\right) \frac{P_{q\gamma}(y_p)}{p\cdot k_\gamma} \left[\frac{\phi_A^{q\bar{q},g}(Y_A,\mathbf{k}_{2\perp},\mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} \Xi(q,p+k_\gamma)\right]_{p+k\gamma=k_\gamma/y_p} \\
&+ J_0\left(\frac{c_0}{\mu}\left|\mathbf{q}_\perp - \mathbf{k}_{\gamma\perp}\frac{1-y_q}{y_q}\right|\right) J_0\left(\frac{c_0}{\mu}\left|\mathbf{p}_\perp - \mathbf{k}_{\gamma\perp}\frac{1-y_p}{y_p}\right|\right) \times \\
&\times \frac{P_{q,\gamma}(y_q)}{q\cdot k_\gamma} \frac{T_R P_{gq}(z/y_q)}{p\cdot k_\gamma/y_q} \left[\frac{\phi_A^{q\bar{q},g}(Y_A,\mathbf{k}_{2\perp},\mathbf{k}_\perp)}{\mathbf{k}_{2\perp}^2} X(P)\right]_{P=k_\gamma/z} \Bigg]
\end{aligned}
$$

Summary and future outlooks

- Computed formula for LO and NLO inclusive photon production cross section
- There are no initial state divergences
- Collinear final state divergences in the cross section have been isolated and factorized
- Further inquiries:
  1. Numerical implementation
  2. γ-jet angular correlation calculation
  3. Comparison with RHIC and LHC data