Gauge-invariant description of the Higgs resonance

René Sondenheimer

University of Graz

Based on arXiv:2009.06671

In collaboration with A. Maas (& V. Afferrante, E. Dobson, B. Riederer, P. Törek)

ACHT2021
21st of April 2021

NAWI Graz
Natural Sciences

DFG Deutsche Forschungsgemeinschaft
German Research Foundation

UNIVERSITÄT GRAZ
UNIVERSITY OF GRAZ

BEH mechanism - weak subsector

- SU(2) non-Abelian gauge theory

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

$$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}, \quad D_\mu\phi = \partial_\mu\phi + \mathrm{i}gW_\mu\phi, \quad W^\mu = \begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3 \end{pmatrix}$$

BEH mechanism - weak subsector

- SU(2) non-Abelian gauge theory

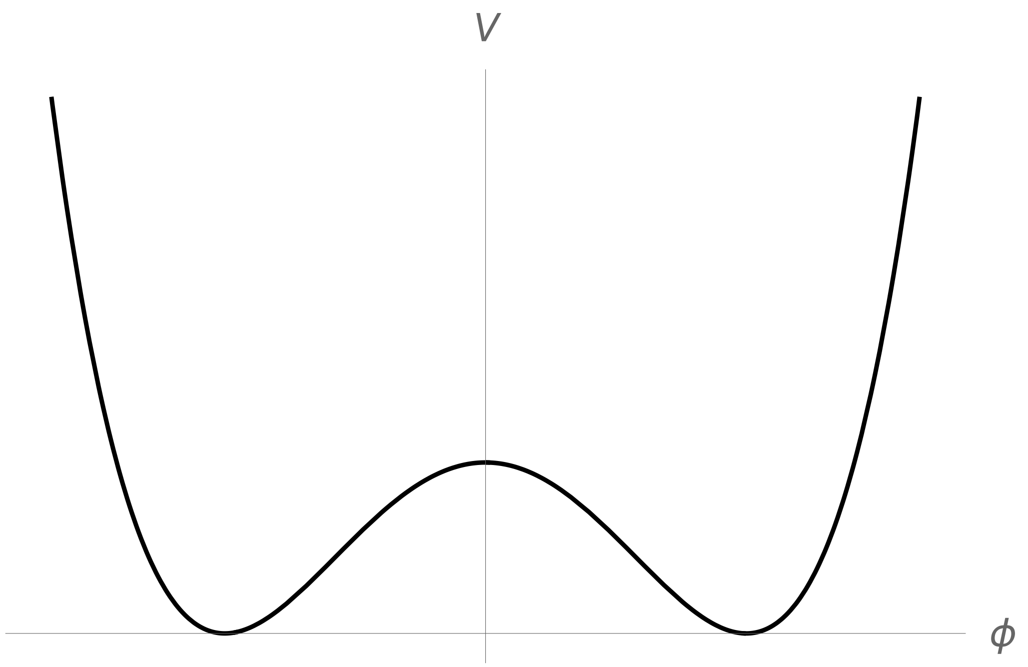
$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

$$\phi = \begin{pmatrix}\phi^1 \\ \phi^2\end{pmatrix}, \quad D_\mu\phi = \partial_\mu\phi + \mathrm{i}gW_\mu\phi, \quad W^\mu = \begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3\end{pmatrix}$$

- Higgs potential:

$$V = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}G_2(x) + \mathrm{i}G_1(x) \\ h(x) + \mathrm{i}G_3(x)\end{pmatrix}$$



BEH mechanism - weak subsector

- SU(2) non-Abelian gauge theory

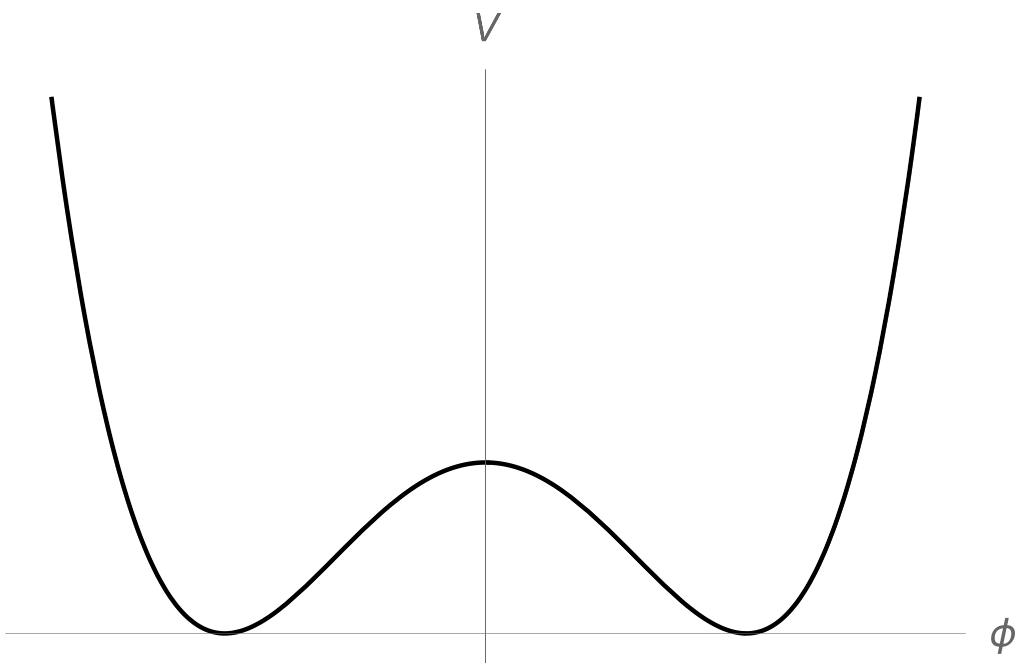
$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

$$\phi = \begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix}, \quad D_\mu\phi = \partial_\mu\phi + \mathrm{i}gW_\mu\phi, \quad W^\mu = \begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3\end{pmatrix}$$

- Higgs potential:

$$V = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}G_2(x) + \mathrm{i}G_1(x)\\ h(x) + \mathrm{i}G_3(x)\end{pmatrix}$$



- Mass term for gauge bosons:

$$(D_\mu\phi)^\dagger D^\mu\phi = g^2\phi^\dagger W^2\phi + \cdots = \frac{1}{2}\frac{g^2v^2}{4}({W_1}^2 + {W_2}^2 + {W_3}^2) + \cdots$$

BEH mechanism - weak subsector

- SU(2) non-Abelian gauge theory

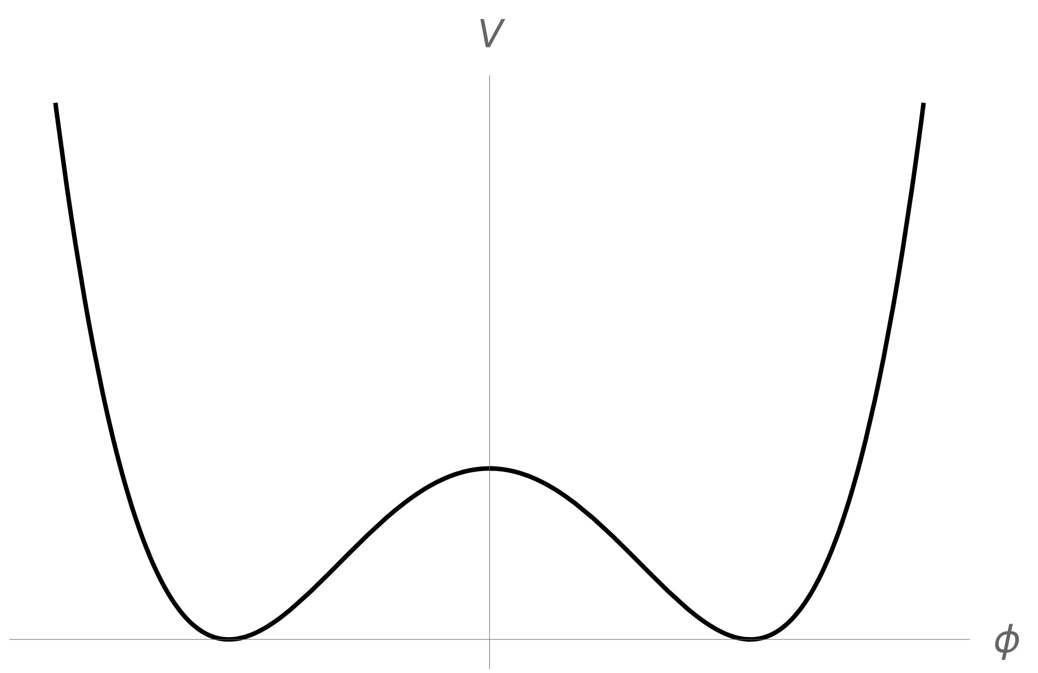
$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

$$\phi = \begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix}, \quad D_\mu\phi = \partial_\mu\phi + \mathrm{i}gW_\mu\phi, \quad W^\mu = \begin{pmatrix}W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2\\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3\end{pmatrix}$$

- Higgs potential:

$$V = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}G_2(x) + \mathrm{i}G_1(x)\\ h(x) + \mathrm{i}G_3(x)\end{pmatrix}$$



- Mass term for gauge bosons:

$$(D_\mu\phi)^\dagger D^\mu\phi = g^2\phi^\dagger W^2\phi + \cdots = \frac{1}{2}\frac{g^2v^2}{4}({W_1}^2 + {W_2}^2 + {W_3}^2) + \cdots$$

- Spontaneous symmetry breaking $\mathrm{SU}(2) \to 1$

BEH mechanism - weak subsector

- SU(2) non-Abelian gauge theory

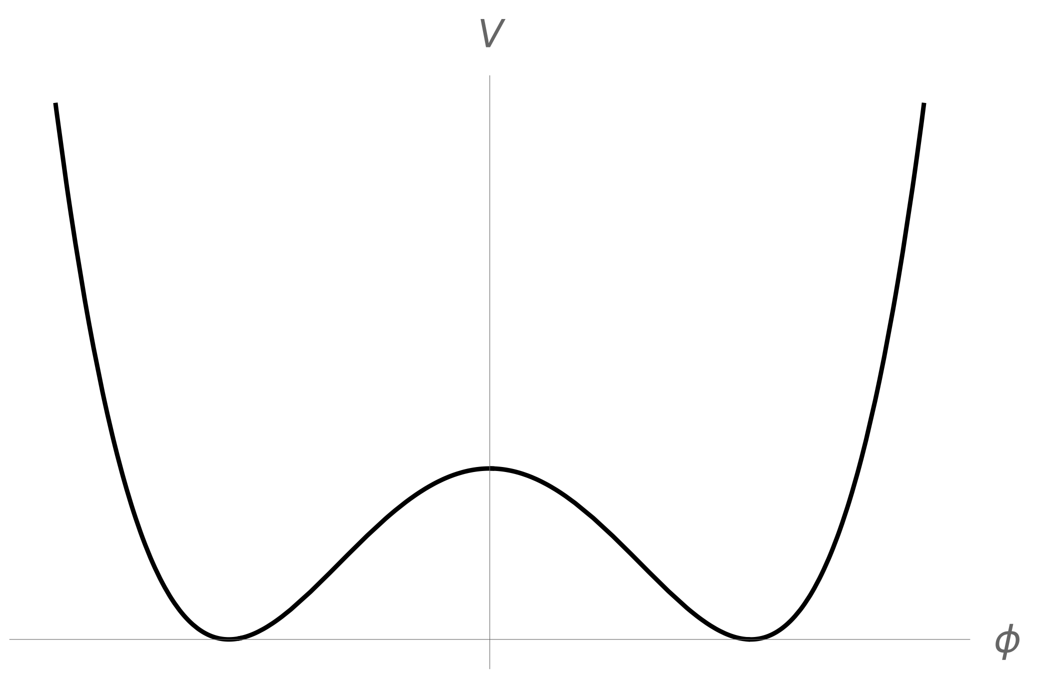
$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

$$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}, \quad D_\mu\phi = \partial_\mu\phi + \mathrm{i}gW_\mu\phi, \quad W^\mu = \begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3 \end{pmatrix}$$

- Higgs potential:

$$V = m^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} G_2(x) + \mathrm{i}G_1(x) \\ h(x) + \mathrm{i}G_3(x) \end{pmatrix}$$



- Mass term for gauge bosons:

$$(D_\mu\phi)^\dagger D^\mu\phi = g^2\phi^\dagger W^2\phi + \cdots = \frac{1}{2}\frac{g^2v^2}{4}({W_1}^2 + {W_2}^2 + {W_3}^2) + \cdots$$

- Spontaneous symmetry breaking $\mathrm{SU}(2) \to 1$?

BEH physics - ambiguities

- Spontaneous gauge symmetry breaking vs Elitzur's theorem
  $\rightarrow$ Higgs VEV is not a reliable order parameter

BEH physics - ambiguities

- Spontaneous gauge symmetry breaking vs Elitzur's theorem
  $\rightarrow$ Higgs VEV is not a reliable order parameter

- Spectrum qualitatively the same in QCD and BEH region
  [Osterwalder&Seiler'78, Fradkin&Shenker'79]

BEH physics - ambiguities

- Spontaneous gauge symmetry breaking vs Elitzur's theorem
  $\rightarrow$ Higgs VEV is not a reliable order parameter

- Spectrum qualitatively the same in QCD and BEH region
  [Osterwalder&Seiler'78, Fradkin&Shenker'79]

- Gribov-Singer ambiguity $\rightarrow$ no observable non-Abelian gauge charge
  [Gribov'77, Singer'78, Lenz et al'00, Lavelle&McMullan'97, Ilderton et al'07, Heinzl et al'08, Capri et al'13]

BEH physics - ambiguities

- Spontaneous gauge symmetry breaking vs Elitzur’s theorem
  $\rightarrow$ Higgs VEV is not a reliable order parameter

- Spectrum qualitatively the same in QCD and BEH region
  [Osterwalder&Seiler’78, Fradkin&Shenker’79]

- Gribov-Singer ambiguity $\rightarrow$ no observable non-Abelian gauge charge
  [Gribov’77, Singer’78, Lenz et al’00, Lavelle&McMullan’97, Ilderton et al’07, Heinzl et al’08, Capri et al’13]

- Mismatch conventional analysis vs lattice spectrum for
  SU(3) with fundamental and SU(2) with adjoint scalar
  [Maas&Törek’18, Afferrante et al’20]

BEH physics - ambiguities

- Spontaneous gauge symmetry breaking vs Elitzur's theorem
  $\rightarrow$ Higgs VEV is not a reliable order parameter

- Spectrum qualitatively the same in QCD and BEH region
  [Osterwalder&Seiler'78, Fradkin&Shenker'79]

- Gribov-Singer ambiguity $\rightarrow$ no observable non-Abelian gauge charge
  [Gribov'77, Singer'78, Lenz et al'00, Lavelle&McMullan'97, Ilderton et al'07, Heinzl et al'08, Capri et al'13]

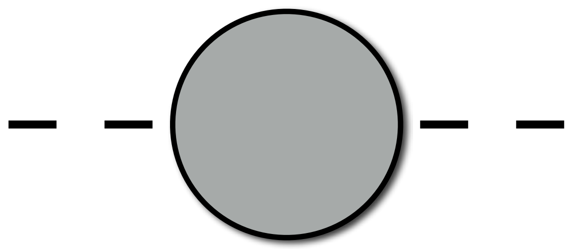
- Mismatch conventional analysis vs lattice spectrum for SU(3) with fundamental and SU(2) with adjoint scalar
  [Maas&Törek'18, Afferrante et al'20]

- Spectral function of the Higgs is gauge dependent
  [Maas&RS'20, Dudal et al'20]

BEH physics - ambiguities

- Spectral function of the Higgs is gauge dependent

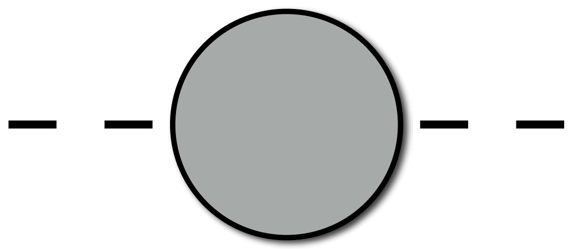
$$\langle h(p)h(-p)\rangle = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 - m^2}$$



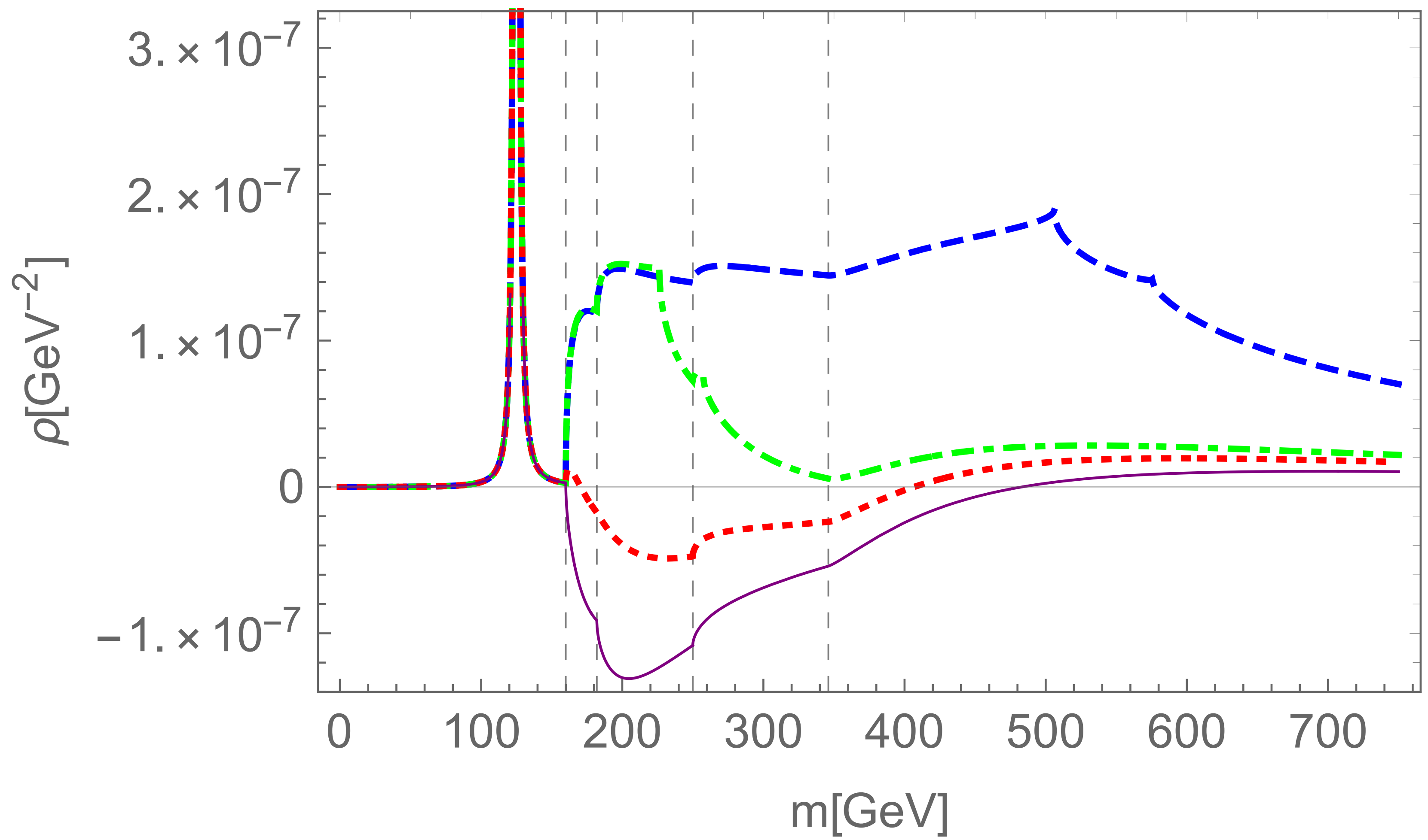
BEH physics - ambiguities

- Spectral function of the Higgs is gauge dependent

$$\langle h(p)h(-p)\rangle = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 - m^2}$$



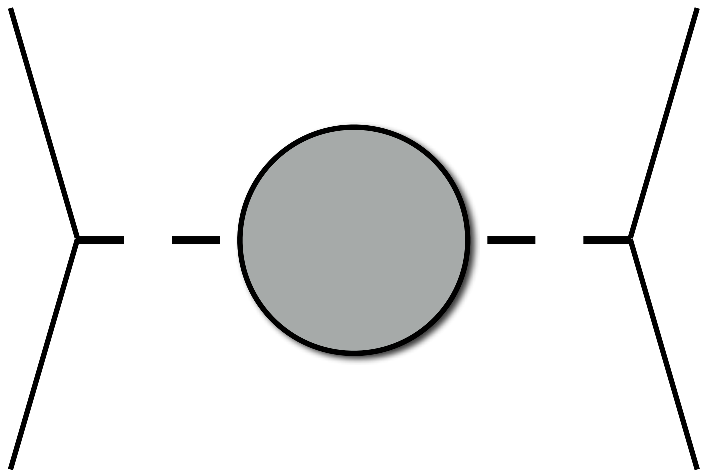
R_ξ gauge: $\xi = 1$, $\xi = 2$, $\xi = 10$



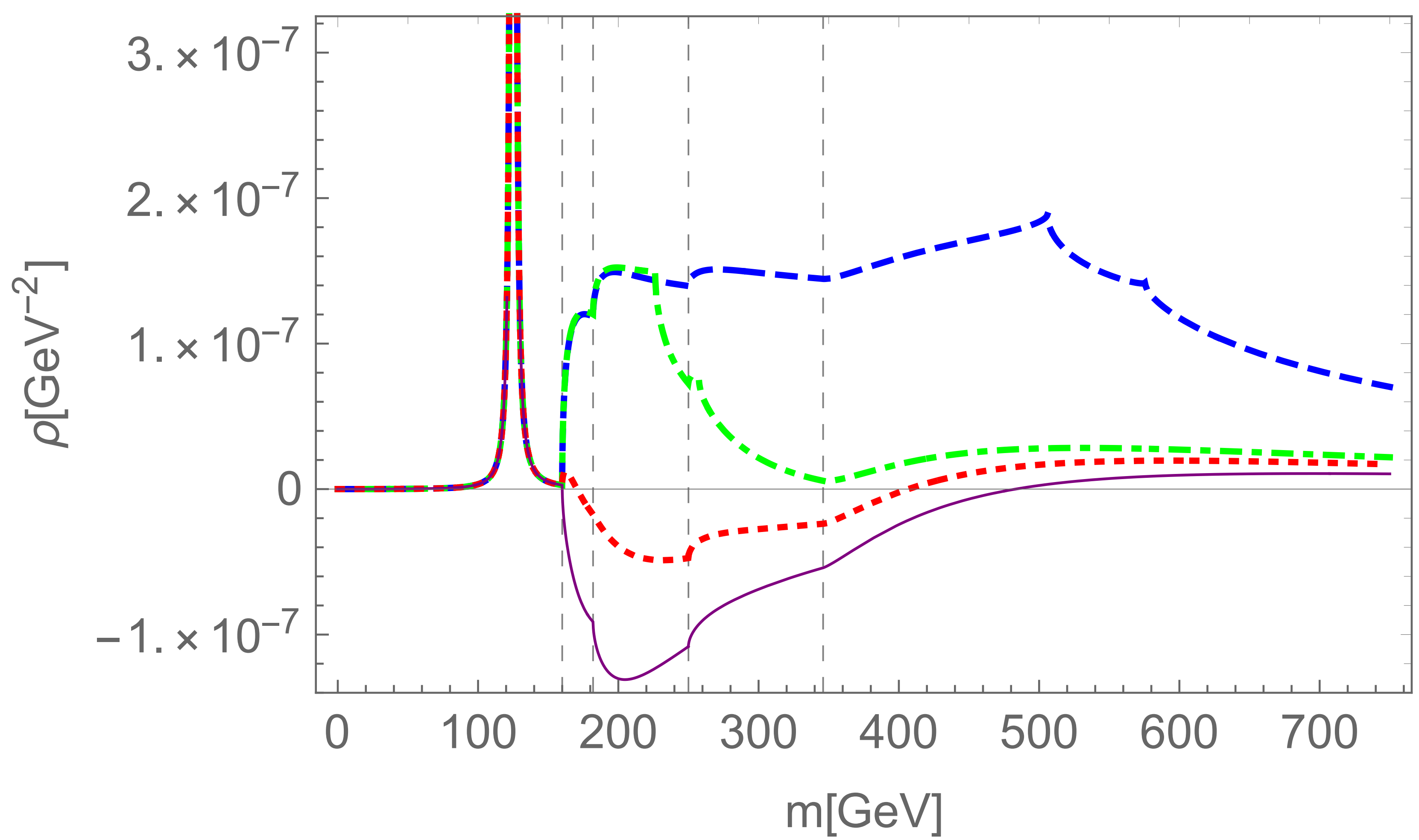
BEH physics - ambiguities

- Spectral function of the Higgs is gauge dependent

$$\langle h(p)h(-p)\rangle = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 - m^2}$$



R_ξ gauge: $\xi = 1$, $\xi = 2$, $\xi = 10$

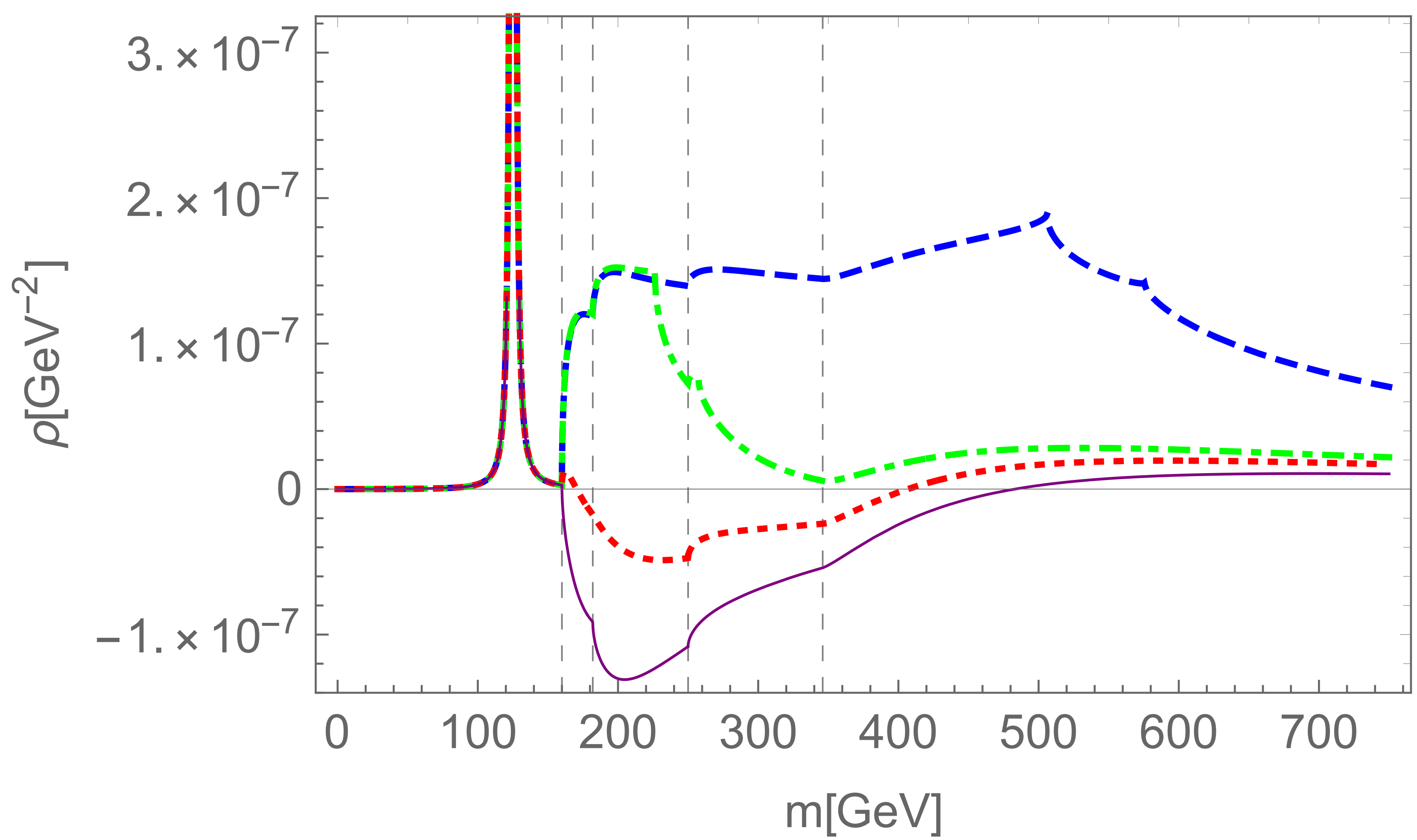


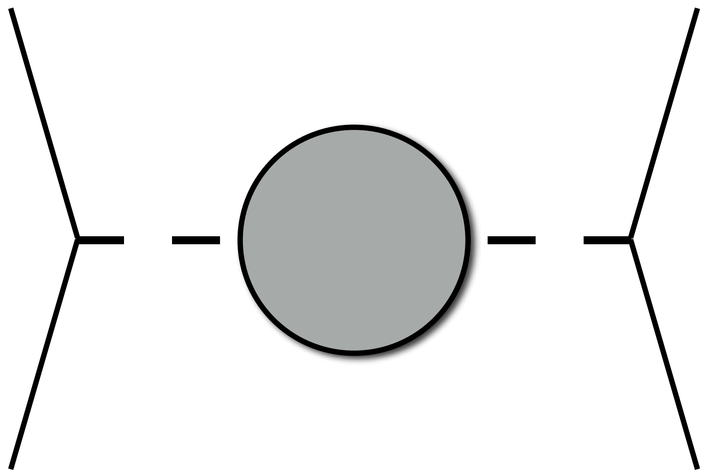
BEH physics - ambiguities

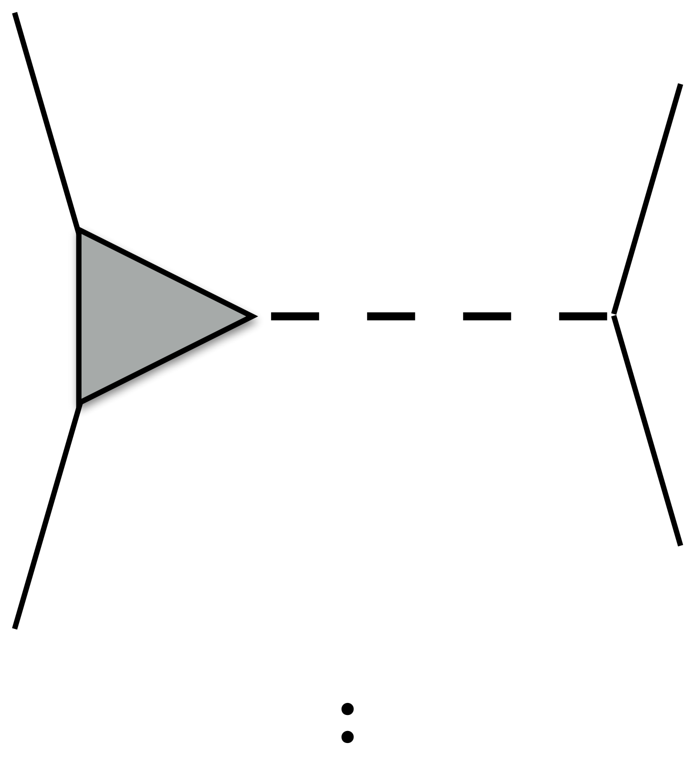
- Spectral function of the Higgs is gauge dependent

$$\langle h(p)h(-p)\rangle = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 - m^2}$$

R_ξ gauge: $\xi = 1$, $\xi = 2$, $\xi = 10$







Fröhlich-Morchio-Strocchi approach

[Fröhlich et al'80,'81]

1. Construct gauge-invariant operator, e.g., $\mathcal{O}(x) = \left(\phi^\dagger \phi\right)(x)$

Fröhlich-Morchio-Strocchi approach

[Fröhlich et al'80,'81]

1. Construct gauge-invariant operator, e.g., $\mathcal{O}(x) = \left(\phi^\dagger \phi\right)(x)$

2. Choose a gauge which allows for a nonvanishing VEV

$$\phi(x) = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \varphi(x) \qquad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} G_2 + \mathrm{i} G_1 \\ h + \mathrm{i} G_3 \end{pmatrix}$$

Fröhlich-Morchio-Strocchi approach

[Fröhlich et al'80,'81]

1. Construct gauge-invariant operator, e.g., $\mathcal{O}(x) = \left(\phi^\dagger \phi\right)(x)$

2. Choose a gauge which allows for a nonvanishing VEV

$$\phi(x) = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \varphi(x) \qquad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} G_2 + \mathrm{i}G_1 \\ h + \mathrm{i}G_3 \end{pmatrix}$$

3. Expand operator/correlator

$$\mathcal{O} = \frac{v^2}{2} + vh + \varphi^\dagger \varphi$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2 \langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle \left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

Fröhlich-Morchio-Strocchi approach

[Fröhlich et al'80,'81]

1. Construct gauge-invariant operator, e.g., $\mathcal{O}(x) = \left(\phi^\dagger\phi\right)(x)$

2. Choose a gauge which allows for a nonvanishing VEV

$$\phi(x) = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} + \varphi(x) \qquad \varphi = \frac{1}{\sqrt{2}}\begin{pmatrix}G_2 + \mathrm{i}G_1\\ h + \mathrm{i}G_3\end{pmatrix}$$

3. Expand operator/correlator

$$\mathcal{O} = \frac{v^2}{2} + vh + \varphi^\dagger\varphi$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

4. Compare properties on both sides

Fröhlich-Morchio-Strocchi approach

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

[Maas&RS'20]

Fröhlich-Morchio-Strocchi approach

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

- Similar constructions for other Standard Model particles [FMS'80,'81,Egger&RS&Maas'17]

[Maas&RS'20]

Fröhlich-Morchio-Strocchi approach

$$\langle \mathscr{O}(x)\mathscr{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

- Similar constructions for other Standard Model particles [FMS'80,'81,Egger&RS&Maas'17]
- Confirmed on the lattice [Maas&Mufti'13]

[Maas&RS'20]

Fröhlich-Morchio-Strocchi approach

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

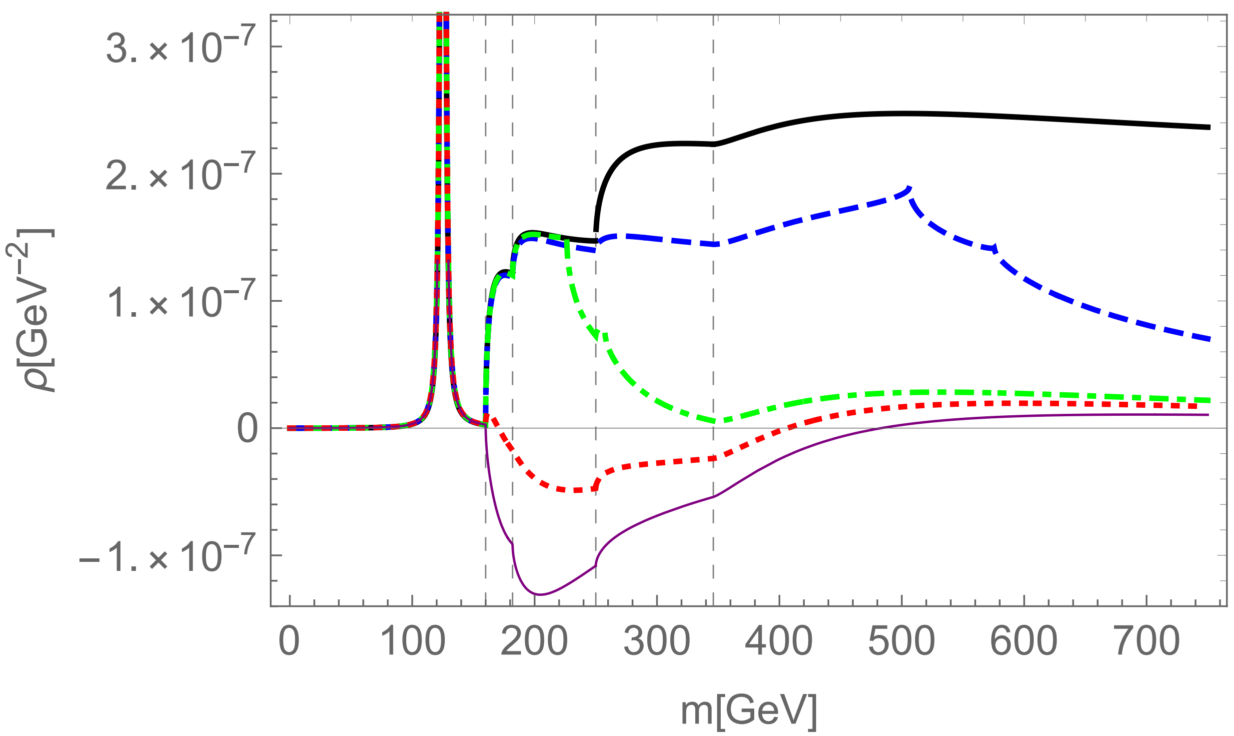
- Similar constructions for other Standard Model particles [FMS'80,'81,Egger&RS&Maas'17]
- Confirmed on the lattice [Maas&Mufti'13]
- Poles coincide to all orders in perturbation theory [Maas&RS'20]

[Maas&RS'20]

Fröhlich-Morchio-Strocchi approach

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

- Similar constructions for other Standard Model particles [FMS'80,'81,Egger&RS&Maas'17]
- Confirmed on the lattice [Maas&Mufti'13]
- Poles coincide to all orders in perturbation theory [Maas&RS'20]
- Spectral function of the bound state is positive!

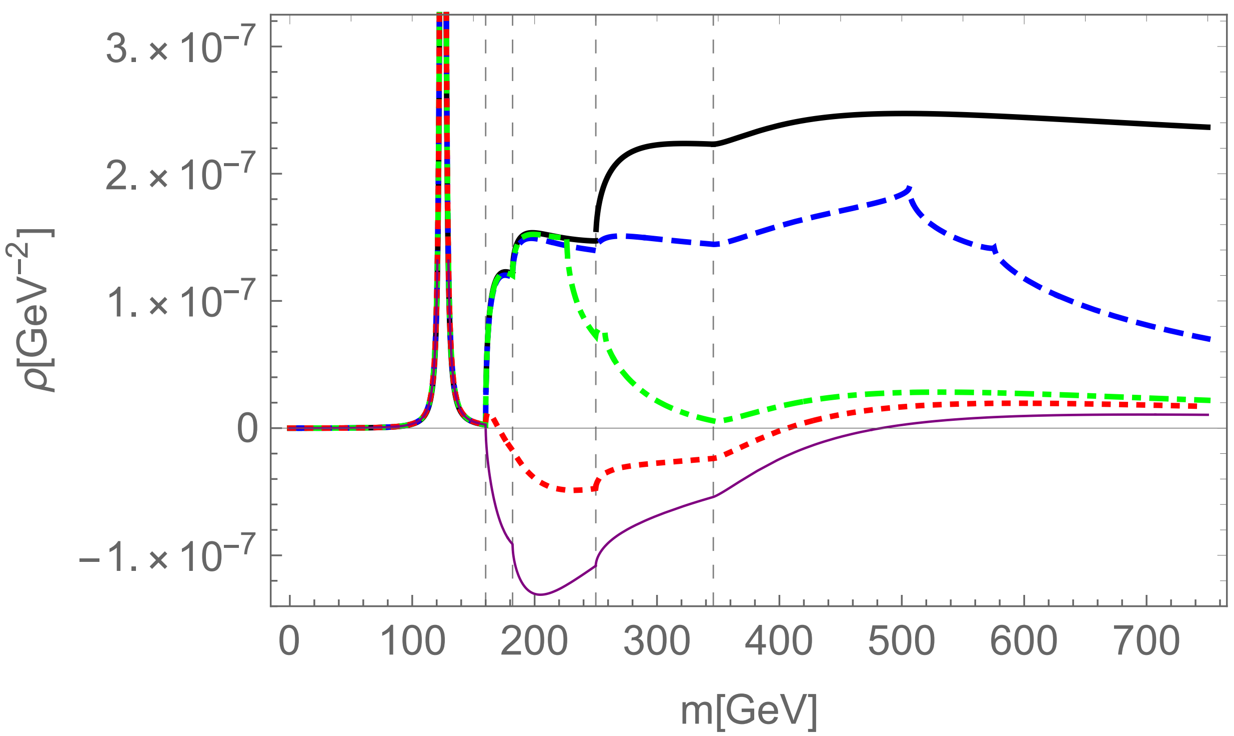


[Maas&RS'20]

Fröhlich-Morchio-Strocchi approach

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = v^2\langle h(x)h(y)\rangle + 2v\langle h(x)\left(\varphi^\dagger\varphi\right)(y)\rangle + \langle\left(\varphi^\dagger\varphi\right)(x)\left(\varphi^\dagger\varphi\right)(y)\rangle$$

- Similar constructions for other Standard Model particles [FMS'80,'81,Egger&RS&Maas'17]
- Confirmed on the lattice [Maas&Mufti'13]
- Poles coincide to all orders in perturbation theory [Maas&RS'20]
- Spectral function of the bound state is positive!



- Potential implications for hierarchy problem: $\delta M_h^2 \sim \Lambda^2$ vs $\delta M_{\phi^\dagger\phi}^2 \sim \log\Lambda$ [Maas&RS'20] (1-loop order)

Summary

- Observable spectrum must be gauge invariant
- Non-Abelian gauge theory: composite operator
- FMS approach explains duality induced by BEH mechanism
- Strict gauge-invariant description of electroweak phenomena
  BUT: Phenomenological implications
- Outlook: Spectrum of BSM models is altered!

[Maas&RS&Törek'17, RS'19, Maas&Törek'18, Afferrante et al.'20]

Summary

- Observable spectrum must be gauge invariant
- Non-Abelian gauge theory: composite operator
- FMS approach explains duality induced by BEH mechanism
- Strict gauge-invariant description of electroweak phenomena
  BUT: Phenomenological implications
- Outlook: Spectrum of BSM models is altered!
  [Maas&RS&Török'17, RS'19, Maas&Török'18, Afferrante et al.'20]

Thank you for your attention!