Stony Brook University

Topological origin of Mass and Spin

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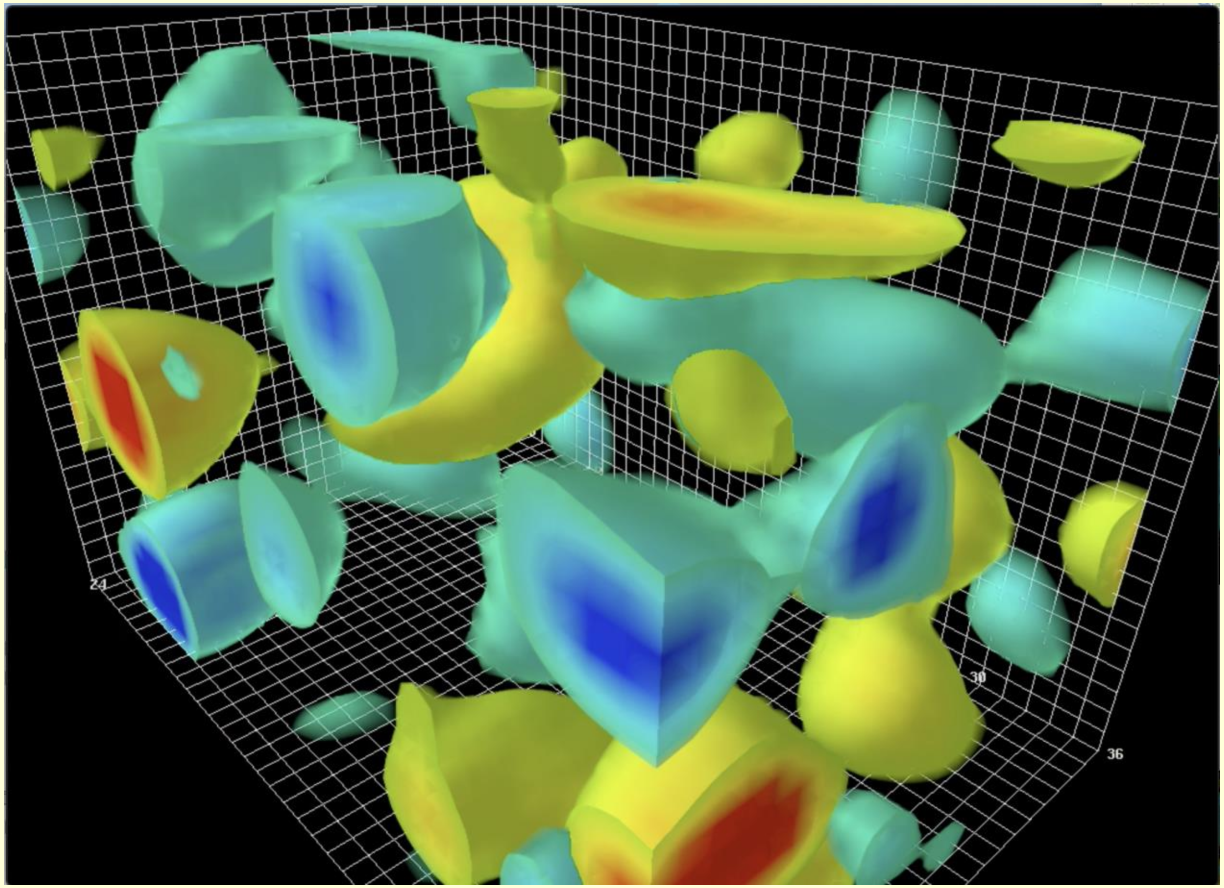
ACHT 21'

Outline

- Vacuum
- Mass
- Spin
- Summary

Vacuum

YM vacuum topologically active: primordial glue!



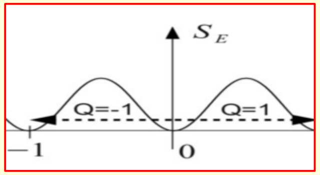
Leinweber 03'

Instantons and anti-instantons

QCD Instantons : few facts

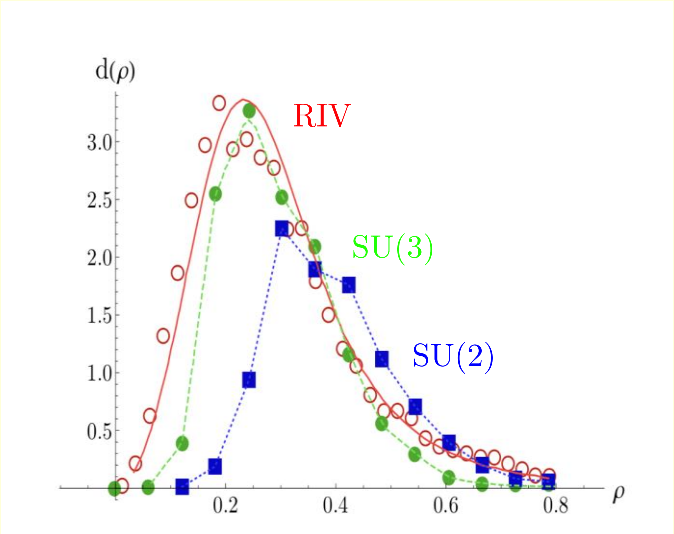
$$\mathrm{RIV} : d[\rho] \approx \frac{(\rho\Lambda)^{\beta_0}}{\rho^5} e^{-\#\rho^2/R^2}$$

Belavin et al. 75'



$$E = B = \frac{\sqrt{48}}{\rho^2} \approx 2.5\,\mathrm{GeV}^2$$

$$S_I = \frac{8\pi^2}{g_s^2} = \frac{2\pi}{\alpha_s} \approx 6\pi \gg 1$$



Many:

$$n_{I+\bar{I}} = 1/R^4 \approx 1\,\mathrm{fm}^{-4}, \quad \rho \sim 1/3\,\mathrm{fm} \sim 1/(0.6\,\mathrm{GeV})$$

Shuryak 82'

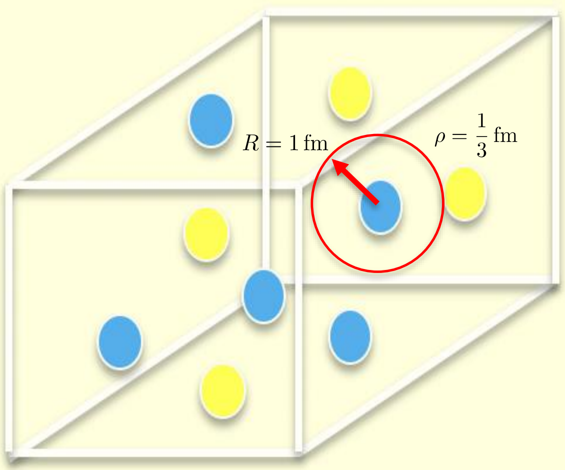
Small parameter:

$$\kappa = 4\pi\rho^4/R^4 \sim 10^{-1}$$

Michael-Spencer 95'

Schafer-Shuryak 98'

Instanton Vacuum: Primordial Glue



$$\langle N_\pm/V\rangle = \bar{N}/2V$$

@ 0.6 GeV

$$\langle F^2/(32\pi^2) = (N_+ + N_-)/V \equiv N/V\rangle = 1/\text{fm}^4$$

$$\langle F\tilde{F}/(32\pi^2) = (N_+ - N_-)/V \equiv \tilde{N}/V\rangle = 0$$

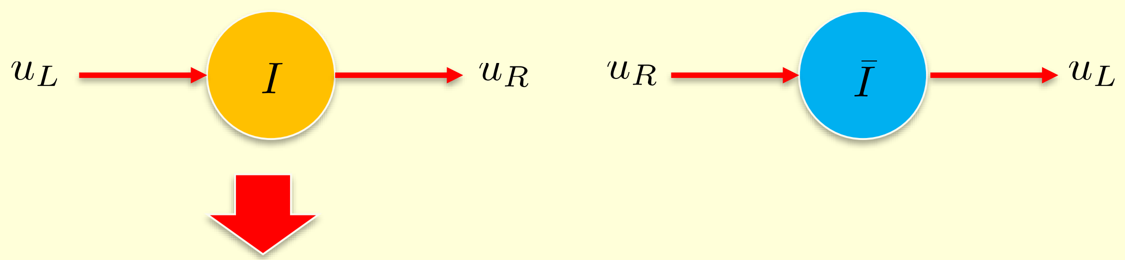
$$\mathbb{Q}(N_+, N_-) = \left[e^{\frac{bN}{4}}\left(\frac{\bar{N}}{N}\right)^{\frac{bN}{4}}\right]\left[\frac{1}{(2\pi\bar{N})^{\frac{1}{2}}}e^{-\frac{\tilde{N}^2}{2\bar{N}}}\right]$$

$$\frac{\langle(N-\bar{N})^2\rangle_{\mathbb{Q}}}{\bar{N}} = \frac{4}{b} = \frac{12}{11N_c}$$

$$\frac{\langle(\tilde{N} = N_+ - N_-)^2\rangle}{\bar{N}} = \frac{\chi_{\mathbb{Q}}}{\bar{N}} \sim 1$$

Diakonov-Petrov 86',Shuryak 88',Nowak-Verbaarshot-Zahed 88', ...

Topological origin of mass: LR mixing and zero modes



$(i\partial + A_I)\psi_I = 0$ Atiyah-Patodi-Singer 63'

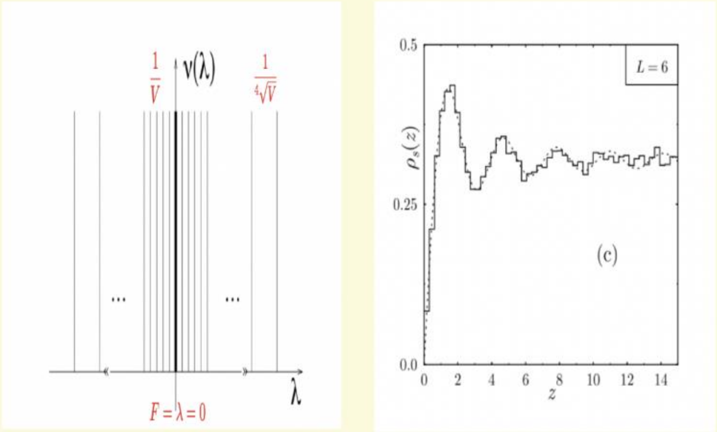
$$\psi^{\alpha}_{iI}(p) = \sqrt{2}\varphi'(p)(\hat{p}\epsilon U)^{\alpha}_{i} \equiv \sqrt{2}\left[\pi\rho^2\left(I_0K_0(z) - I_1K_1(z)\right)'_{z=\rho p/2}\right](\hat{p}\epsilon U)^{\alpha}_{i}$$ 't Hooft 76'

$$n_I\left\langle u^{\dagger}_{R}(p)p\left[\sqrt{2}\varphi'(p)\hat{p}\epsilon U\right]\frac{1}{m}\left[\sqrt{2}\varphi'(p)U^{\dagger}\epsilon\hat{p}\right]pu_L(p)\right\rangle_U + (I,L) \leftrightarrow (\bar{I},R)$$

$$M_u(p) = \tilde{\kappa}\frac{|p\varphi'(p)|^2}{m\rho^4} \rightarrow \frac{\sqrt{\tilde{\kappa}}}{\sqrt{2}\rho^2}\frac{|p\varphi'(p)|^2}{||q\varphi'^2||}$$

Diakonov-Petrov 86',Shuryak 88',Nowak-Verbaarshot-Zahed 88', ...

Lattice evidence for this mechanism



Wittig in book 20'

$$(iD[A] + im)q_n[A] = (\lambda_n[A] + im)q_n[A]$$

$$\nu(\lambda) = \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\left\langle \sum_n \delta(\lambda - \lambda_n[A])\right\rangle_A \equiv \frac{1}{V}\frac{1}{\Delta\lambda}$$

$$\langle\bar{\psi}\psi\rangle = -\pi\nu(0) \equiv -\sigma_C$$

Banks-Casher 80'

$$\sigma_C^n = \langle(\bar{\psi}\psi)^n\rangle_C$$

$$\nu_s(z = N\lambda) = \frac{z}{2}\Big(J^2_{N_f}(z) - J_{N_f+1}(z)J_{N_f-1}(z)\Big)$$

Verbaarschot-Zahed 93'

Running mass versus lattice

$$\frac{M(p)}{M(0)} = |p\varphi'(p)|^2 = |z(I_0(z)K_0(z) - I_1(z)K_1(z))'|^2_{z=\rho p/2}$$

$$M(0) = (2\pi\rho)^2 \frac{|\langle q^\dagger q\rangle|}{N_c}$$

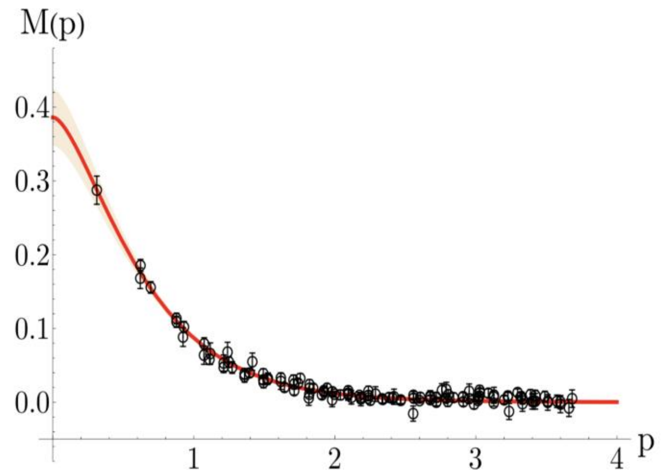
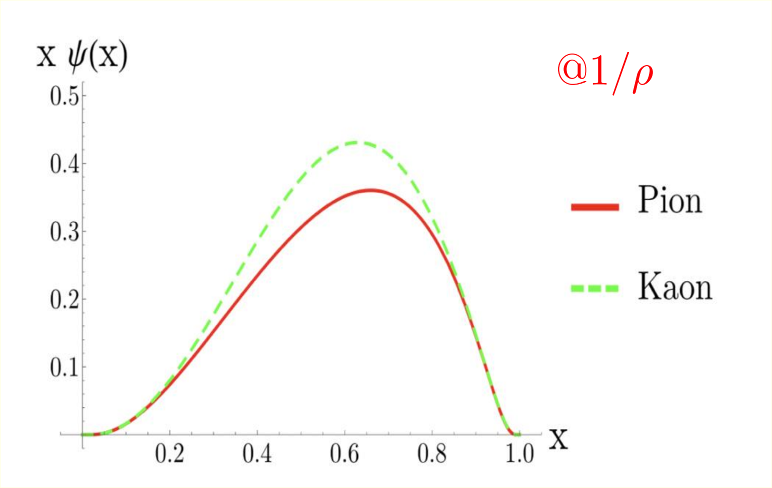


FIG. 2: Momentum dependence of the instanton induced effective quark mass in singular gauge (13) at LO (solid-curves), compared to the effective quark mass measured on the lattice in Coulomb gauge [21] (open-circles). The unit scale is GeV. We obtain a fitted parameter intervals $M(0) = 383 \pm 39$ MeV and $\rho = 0.313 \pm 0.016$ fm.

Lattice: Bowman et al. 04'

Diakonov-Petrov 86'
Shuryak 88'
Nowak-Verbaarschot-Zahed 88'
Pobylitsa 89'
....
Kock-Liu-Zahed 20'

Measurable: Pion and Kaon PDF



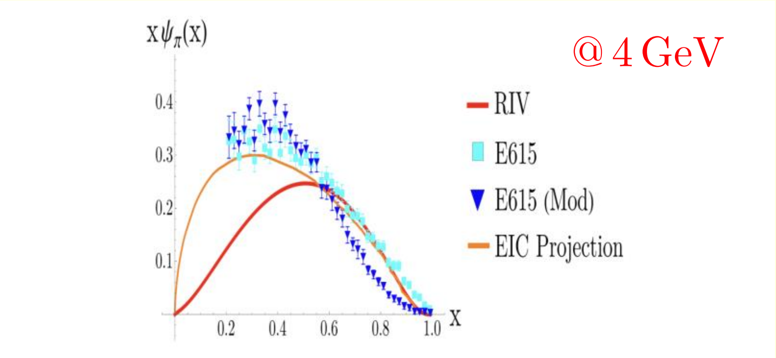


FIG. 12: Pion longitudinal momentum distribution in the QCD instanton vacuum RIV solid-red-curve (77), the E615 data blue-square [31] devolved to $Q = 4$ GeV with NNLO DGLAP in [11] (Lan et al.), improved E615 data inverse-blue-triangle [32], and the EIC projection in solid-orange-curve [33]. All are presented at factorization scale $Q^2 = 16\,\mathrm{GeV}^2$.

E615: Conway et al. 89'

EIC: Agilar et al. 19'

$$\psi^0_{f/P}(x) \to \frac{2N_c}{f_P^2}\int_{k_\perp \geq M(0,m_f)} \frac{d^2k_\perp}{(2\pi)^3}\frac{\theta(x\bar{x})\,k_\perp^2}{(k_\perp^2 - x\bar{x}m_P^2)^2} M^2(k_\perp/\lambda_P\sqrt{x\bar{x}})$$

RIV: Kock-Liu-Zahed 20'

Pion and Kaon TMD

$@1/\rho$

21

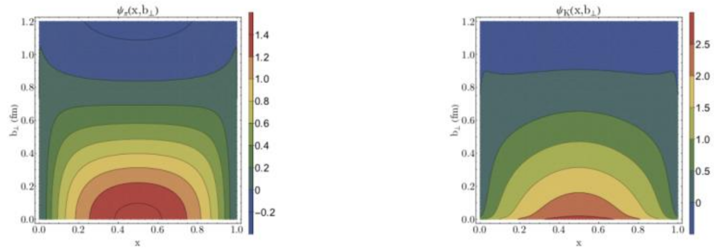


FIG. 14: Pion and Kaon transverse spatial distribution from the QCD instanton vacuum (87) with physical masses and at renormalization scale $Q_0 = 631\,\mathrm{MeV}$.

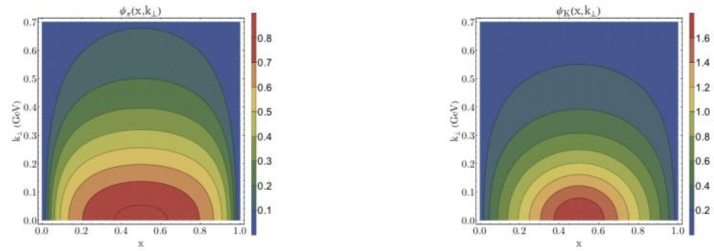


FIG. 15: Pion and Kaon transverse momentum distribution from the QCD instanton vacuum (87) with physical masses and at renormalization scale $Q_0 = 631\,\mathrm{MeV}$.

$$\psi^0_\pi(x,k_\perp) \to \frac{2N_c}{f_\pi^2}\frac{1}{(2\pi)^3}\frac{\theta(x\bar{x})\,(k_\perp^2+M^2(0))}{(k_\perp^2+M^2(0)-\bar{x}xm_\pi^2)^2}M^2\left(\frac{\sqrt{k_\perp^2+M^2(0)}}{\lambda\sqrt{x\bar{x}}}\right)$$

Kock-Liu-Zahed 20’

Mass

Ji Mass

$E^a = \pm i B^a$

$$H_G = \int d^3x\, \bar{T}^{00}_G = \int d^3x\, \frac{1}{2}(E^2 + B^2) \qquad \mathcal{O}(\kappa^2)$$

$$H_Q = \int d^3x\, \bar{T}^{00}_Q = \int d^3x \left(\frac{1}{2}\overline{\psi}\gamma \cdot i\overleftrightarrow{D}\psi\right)$$

$$H_A = \int d^3x\, \hat{T}^{00}_A = \int d^3x\, \frac{1}{4}\left(\frac{\beta(g^2)}{4g^4}F^2 \approx -\frac{b}{32\pi^2}F^2\right) \qquad \mathcal{O}(\kappa)$$

$\sigma_{\pi N} \sim 50\,\mathrm{MeV}$

$$H_m = \int d^3x\, \bar{T}^{00}_G = \int d^3x\, m\overline{\psi}\psi$$

$$M_N = \frac{\langle P|H_G + H_Q + H_A + H_m|P\rangle}{\langle P|P\rangle} \equiv M_G^N + M_Q^N + M_A^N + M_m^N$$

Ji 95'

Epoxy or F^2 in Nucleon

$$\frac{\langle P|F^2|P\rangle}{\langle P|P\rangle} = \lim_{T\to\infty} \frac{\left\langle J_P^\dagger(T) F^2 J_P(-T)\right\rangle_C}{\left\langle J_P^\dagger(T) J_P(-T)\right\rangle}$$

Diakonov-Polyakov-Weiss 95', Kacir-Prakash-Zahed 96'

$$\frac{V}{32\pi^2}\frac{\langle P|F^2|P\rangle}{\langle P|P\rangle} \approx \left\langle (N-\bar{N})^2\right\rangle_{\mathbb{P}} \frac{\partial}{\partial \bar{N}} \mathrm{Log}\left(\lim_{T\to\infty}\left\langle J_P^\dagger(T) J_P(-T)\right\rangle\right)$$

$\frac{4\bar{N}}{b}$ Vaccuum compressibility

$$\frac{V}{2T}\frac{-b}{32\pi^2}\frac{\langle P|F^2|P\rangle}{\langle P|P\rangle} = 4\frac{\partial M_N}{\partial \mathrm{Log}\bar{N}} = M_{\mathrm{inv}}$$

$$M_N = M_{\mathrm{inv}} + \sigma_{\pi N} = C\left(\frac{\bar{N}}{V}\right)^{\frac{1}{4}} + \bar{C}m\Big(1+\mathcal{O}(mR)\Big)$$

$$\frac{\langle P|T_\mu^\mu|P\rangle}{2M_N} = M_{\mathrm{inv}} + \frac{\langle P|m\overline{\psi}\psi|P\rangle}{2M_N} = M_N$$

Ji Mass Sum Rule

$$\frac{M_Q^N}{M_N} \approx \frac{3}{4}\frac{1}{1+\kappa}\left(1-\frac{\sigma_{\pi N}}{M_N}\right) \approx 64\%$$

$$\frac{M_G^N}{M_N} \approx \frac{3}{4}\frac{\kappa}{1+\kappa}\left(1-\frac{\sigma_{\pi N}}{M_N}\right) \approx 7\%$$

$$\frac{M_A^N}{M_N} = \frac{1}{4}\left(1-\frac{\sigma_{\pi N}}{M_N}\right) \approx 24\%$$

$$\frac{M_m^N}{M_M} = \frac{\sigma_{\pi N}}{M_N} \approx 5\%$$

@ 0.6 GeV

LATTICE-Kentucky: 35%+35%+24% +5%=100% @ 2 GeV

$$\frac{M_Q^\pi}{m_\pi} \approx \frac{3}{8}\frac{1}{1+\kappa} \approx 34\%$$

$$\frac{M_G^\pi}{m_\pi} \approx \frac{3}{8}\frac{\kappa}{1+\kappa} \approx 3\%$$

$$\frac{M_A^\pi}{m_\pi} = \frac{1}{8} \approx 13\%$$

$$\frac{M_m^\pi}{m_\pi} = \frac{1}{2} \approx 50\%$$

@ 0.6 GeV

Zahed 21'

Spin

Ji Spin

$$\vec{J}_Q = \frac{1}{2}\vec{\Sigma} + \vec{L}_Q = \int d^3x \left(\frac{1}{2}\overline{\psi}\vec{\gamma}\gamma^5\psi + \psi^\dagger(\vec{x} \times i\vec{D}[A])\psi \right)$$

$$\vec{J}_G = \int d^3x \left(\vec{x} \times (\vec{E}^a \times \vec{B}^a) \right) \qquad E^a = \pm i B^a$$

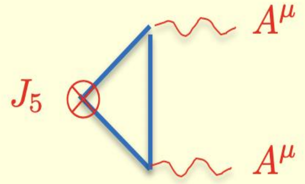
$\mathcal{O}(\kappa^2)$

Ji 97'

$$\vec{J}_N = \frac{\langle P|\vec{J}|P\rangle}{\langle P|P\rangle} \equiv \frac{1}{2}\vec{\Sigma}_Q^N + \vec{L}_Q^N + \vec{J}_G^N$$

$\mathcal{O}(\kappa)$ $\mathcal{O}(\kappa^2)$

Intrinsic spin and U(1) axial anomaly



Adler-Bardeen-Jackiw 69'

$$\partial_\mu \overline{\psi}\gamma^\mu\gamma^5\psi = \frac{N_f}{16\pi^2}F\tilde{F} + 2m\overline{\psi}i\gamma^5\psi$$

$$\Sigma_Q^N \approx \frac{N_f V_3}{32\pi^2}\frac{\langle P|F\tilde{F}(0)|P\rangle}{s_v^\uparrow m_N\langle P|P\rangle}$$

$$s_v^\uparrow = \chi_\uparrow^\dagger(\vec{\sigma}\cdot\vec{v})\chi_\uparrow$$

Altarelli 88', Carlitz-Collins-Mueller 88', ...

$F\tilde{F}$ in Nucleon

$$\frac{\langle P|F\tilde{F}(0)|P\rangle}{\langle P|P\rangle} = \lim_{T\to\infty} \frac{\left\langle J_P^\dagger(T)F\tilde{F}(0)J_P(-T)\right\rangle_C}{\left\langle J_P^\dagger(T)J_P(-T)\right\rangle}$$

$$\frac{V}{32\pi^2}\frac{\langle P|F\tilde{F}|P\rangle}{\langle P|P\rangle} \approx \left\langle \tilde{N}^2\right\rangle_{\mathbb{Q}} \frac{\partial}{\partial\tilde{N}}\mathrm{Log}\left(\lim_{T\to\infty}\left\langle J_P^\dagger(T)J_P(-T)\right\rangle\right)$$

$\chi_{\mathbb{Q}}\bar{N} \sim \bar{N}$ Vaccuum susceptibility

$$\frac{V_3}{32\pi^2}\frac{\langle P|F\tilde{F}|P\rangle}{m_N\langle P|P\rangle} \approx -\langle\tilde{N}^2\rangle_{\mathbb{Q}}\left(\frac{\partial\mathrm{Log}\tilde{m}_N}{\partial\tilde{N}}\right)_{\tilde{N}=0}$$

Diakonov-Polyakov-Weiss 95', Kacir-Prakash-Zahed 96'

Nucleon intrinsic quark spin

$$n_I \left\langle u_R^\dagger(p) p \left[\sqrt{2}\varphi'(p)\hat{p}\epsilon U\right] \frac{1}{m} \left[\sqrt{2}\varphi'(p) U^\dagger \epsilon \hat{p}\right] p u_L(p) \right\rangle_U + (\bar{I}, L \leftrightarrow R)$$

$$M_u(p)\left(\frac{N}{\bar{N}}\, u^\dagger(p)u(p) - \frac{\tilde{N}}{\bar{N}}\, u^\dagger(p)\gamma^5 u(p)\right) \to u^\dagger(p)\left[M_u(p)\left(1 - \frac{\tilde{N}}{\bar{N}}\gamma^5\right)\right]u(p)$$

$N = u^\uparrow[ud]_0$ ➡ $\tilde{m}_N \approx m_N - M_u(0)s_v^\uparrow \frac{\tilde{N}}{\bar{N}}$

$$\frac{V_3}{32\pi^2}\frac{\langle P|F\tilde{F}|P\rangle}{m_N s_v^\uparrow \langle P|P\rangle} \approx \left(\frac{\chi_{\mathbb{Q}}}{\bar{N}}\right)\left(\frac{M_u(0)}{m_N}\right)$$

Zahed 21'

Ji Spin sum rule

$$\frac{\frac{1}{2}\Sigma_Q^N}{S_N} \approx \frac{N_f}{3}\frac{\chi_{\mathbb{Q}}}{\bar{N}} \approx 60\%$$

COMPASS: (30-40)%

@ 0.6 GeV

$$\frac{L_Q^N}{S_N} \approx \frac{1}{1+\kappa}\left(1-\frac{N_f}{3}\frac{\chi_{\mathbb{Q}}}{\bar{N}}\right) \approx 36\%$$

$$\frac{J_G^N}{S_N} \approx \frac{\kappa}{1+\kappa}\left(1-\frac{N_f}{3}\frac{\chi_{\mathbb{Q}}}{\bar{N}}\right) \approx 4\%$$

Zahed 21'

LATTICE-Cyprus: 40%+42%+17% =100% @ 2 GeV

LATTICE-Kentucky: 25%+47%+28% =100% @ 2 GeV

Summary

- Vacuum : topologically active
- Mass : topo-compressibility
- Spin : topo-susceptibility