Two-loop master integrals for the single top production associated with W boson

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Introduction

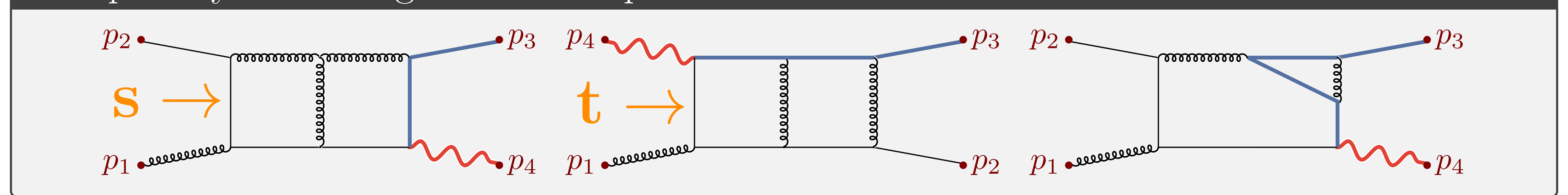
- Features of top quark: heavy as a gold atom, strong Yukawa coupling, decay before hadronization.
- LHC is a top factory: $\sigma_{t\bar{t}} > \sigma_{t-\text{channel}} > \sigma_{tW} > \sigma_{s-\text{channel}}$.
- Experimental highlights of single top production: probe of Wtb coupling, direct extraction of V_{tb}, top mass measurement, search for new physics.
- Top production up to NNLO QCD, all channels are available **except tW mode**. One of the missing ingredients [1]: **two-loop four-point master integrals** (MIs) for double-virtual corrections.

Setup

Consider the partonic process below at NNLO QCD,

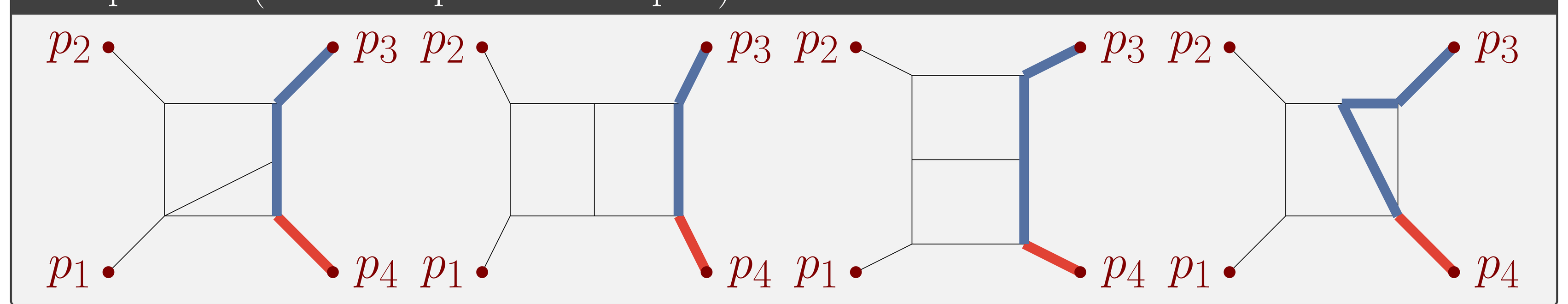
$$g(p_1) + b(p_2) \to t(p_3) + W(p_4), \text{ with } p_{1,2}^2 = 0, p_3^2 = m_t^2 \text{ and } p_4^2 = m_W^2.$$

Sample Feynman diagrams with up to 3 massive internal lines



We can identify two integral families. Family 1 (**60 MIs**) includes first two diagrams. Second family (**36 MIs**) contains the last diagram.

Sample MIs (most complicate examples)



Differential equations

There are 80 independent MIs in total. After constructing a canonical basis [2] $\mathbf{F}$, one can build its differential equations w.r.t $\mathbf{r} = (s, t, m_t^2, m_W^2)$,

$$\partial_i \mathbf{F}(\mathbf{r}, \epsilon) = \epsilon \, \mathrm{A}_i(\mathbf{r}) \, \mathbf{F}(\mathbf{r}, \epsilon).$$

$d = 4 - 2\epsilon$ dimensions

Square roots in A_i

$$\lambda_1 = \sqrt{s(s - 4m_t^2)}$$
$$\lambda_2 = \sqrt{(s - m_t^2 - m_W^2)^2 - 4m_t^2 m_W^2}$$
$$\lambda_3 = \sqrt{s(t - m_t^2)(st - m_t^2(s + 4t - 4m_W^2))}$$

Optimization

- no roots: 29 MIs
- λ_2 : 32 MIs
- $\lambda_{1,2}$: 9 MIs, $\lambda_{2,3}$: 4 MIs
- $\lambda_{1,2,3}$: 6 MIs

Different rationalization schemes.

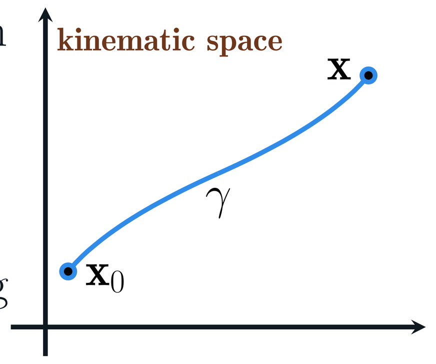
Solutions

By applying a suitable change of variables to every group, $\mathbf{g}$, according to their root dependences, its differential equation w.r.t introduced dimensionless variables, $\mathbf{x}$, fulfills the $d\log$ form,

$$d\,\mathbf{g}(\mathbf{x}, \epsilon) = \epsilon \, d\mathbb{A}(\mathbf{x}) \, \mathbf{g}(\mathbf{x}, \epsilon), \quad \mathbb{A}(\mathbf{x}) = \sum_i \mathbb{C}_i d\log[\beta_i(\mathbf{x})].$$

Formally, the solutions to such equation can be expressed in terms of a path-ordered integration,

$$\mathbf{g}(\mathbf{x}, \epsilon) = \mathcal{P}\exp\left(\epsilon \int_\gamma d\mathbb{A}(\mathbf{x})\right) \mathbf{g}(\mathbf{x}_0, \epsilon),$$



with $\mathbf{g}(\mathbf{x}_0, \epsilon)$ being the values at the boundary and γ being the path connecting $\mathbf{x}_0$ and $\mathbf{x}$ in kinematic variable space. Suppose that all MIs are normalized to be regular at $\epsilon = 0$, namely

$$\mathbf{g}(\mathbf{x}, \epsilon) = \sum_{i=0}^{\infty} \mathbf{g}^{(i)}(\mathbf{x}) \, \epsilon^i,$$

regularity in these limits

whose coefficient at each order is determined by an iterative manner,

$$\mathbf{g}^{(n)}(\mathbf{x}) = \begin{cases} \mathbf{g}^{(0)}(\mathbf{x}_0) & n = 0, \\ \mathbf{g}^{(n)}(\mathbf{x}_0) + \int_\gamma d\mathbb{A}(\mathbf{x})\mathbf{g}^{(n-1)}(\mathbf{x}) & n > 0, \end{cases}$$

evaluating to the Goncharov polylogarithms (GPLs) which are defined as

$$G(w_n, ..., w_1; a) = \int_0^a \frac{1}{t - w_n} G(w_{n-1}, ..., w_1; t) \, dt,$$

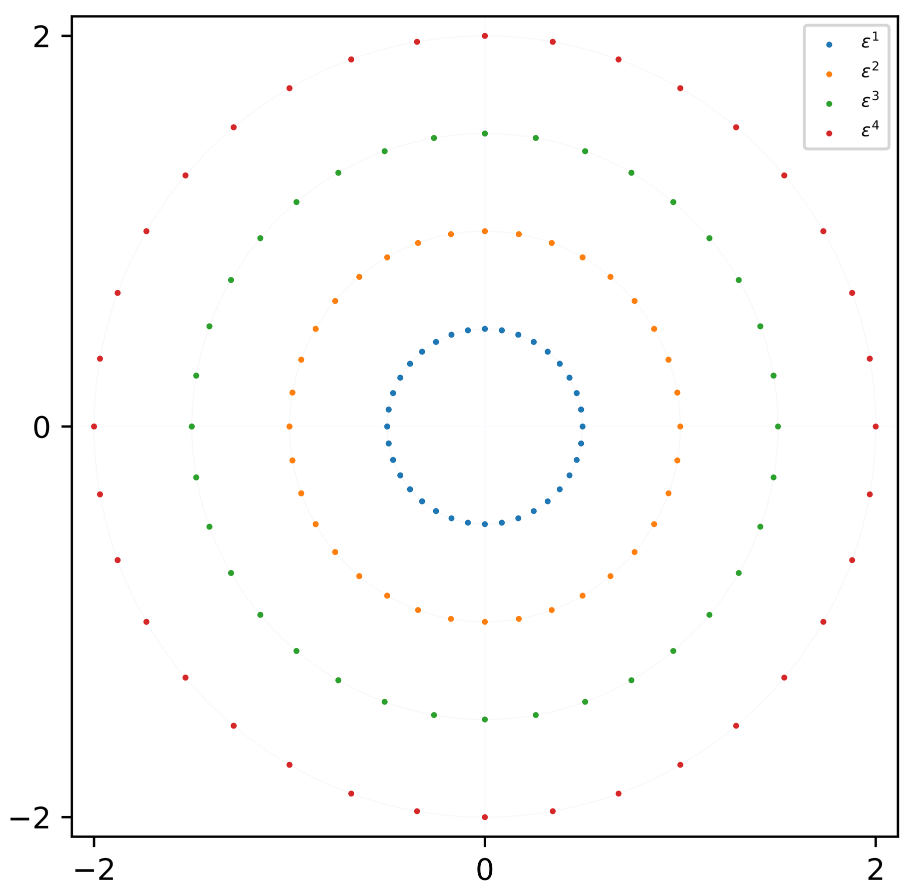
with

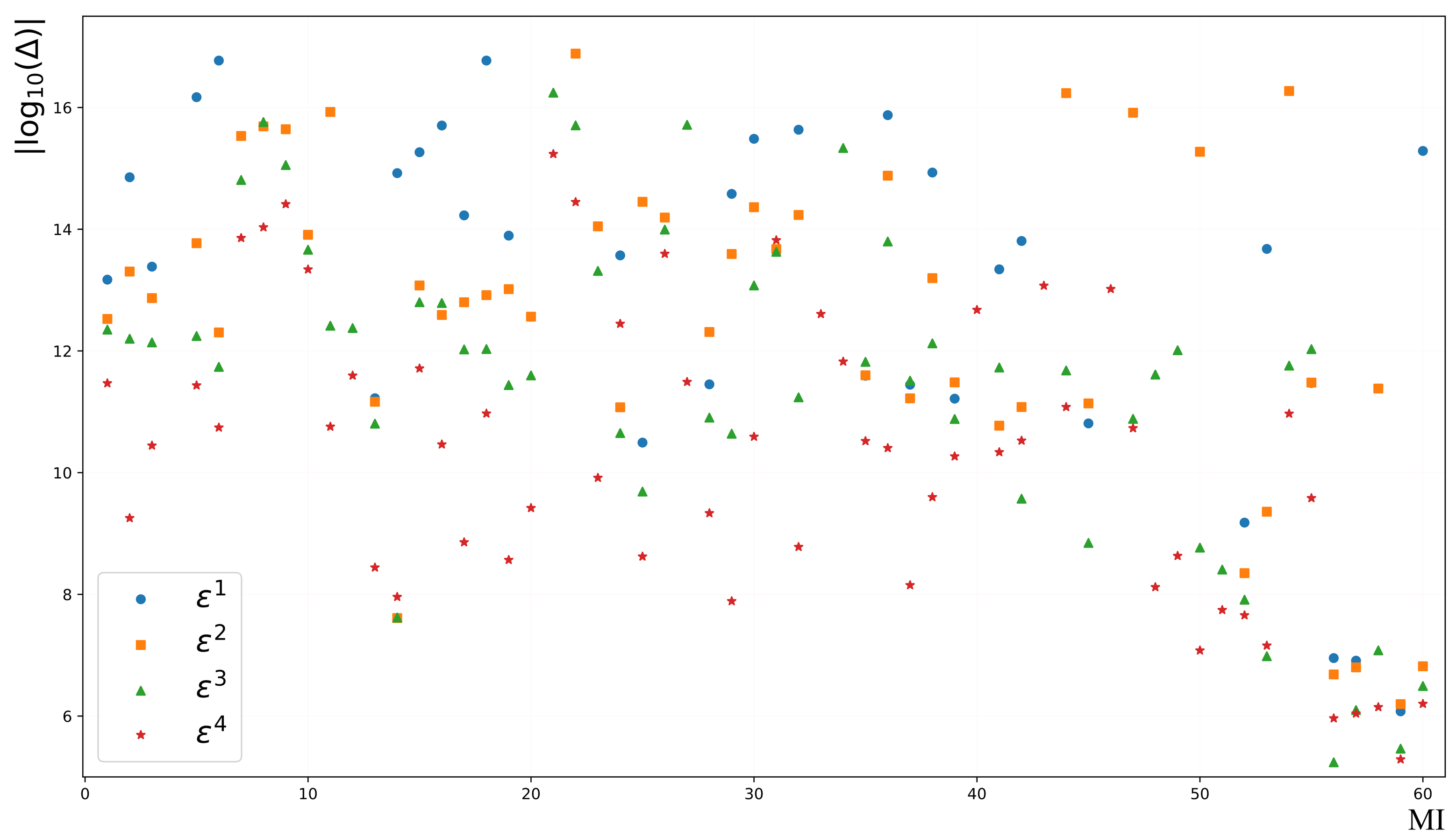
$$G(w_1; a) = \int_0^a \frac{1}{t - w_1} dt \quad w_1 \neq 0, \qquad G(\underbrace{0, ..., 0}_{n \text{ times}}; a) = \frac{\log^n(a)}{n!}.$$

Boundary conditions

	input	$m_W = m_t$	$t \to m_W^2 - s$	$t \to m_W^2$
Number of fixed boundaries	26	1	1	6
	$m_W \to 0$	$\lambda_2 \to 0$	$s \to (t - m_t^2)(m_W^2 - t)/t$	
	7	18	21	

Numerical validation

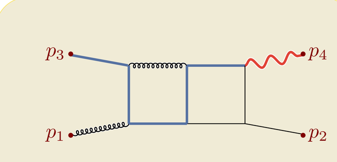




- Comparison with pySecDec [3] at $\mathbf{r} = (-6, -13, 5, 12)$ for MIs of family 1.
- In left plot, being closer to the circles means better agreement.
- In right plot, we define $\Delta = |1 - \textit{Numerical integration/Analytical evaluation}|$. Locating at higher means better agreement.
- Numerical routines struggle to give precise values for the most complicate MIs, even at lower ϵ order.

Conclusion and outlook

- Two-loop MIs (partial) for tW production are calculated analytically with results expressed in terms of GPLs. An optimization is realized by separating MIs into groups according to their root dependences.
- Other diagrams need further investigation in the future.



References

[1] Long-Bin Chen and Jian Wang. Analytic two-loop master integrals for the tW production at hadron collides: I. ArXiv:2106.12093.

[2] Johannes M. Henn. Multiloop integrals in dimensional regularization made simple. *Phys. Rev. Lett.*, 110:251601, 2013.

[3] S. Borowka, G. Heinrich, S. Jahn, S. P. Jones, M. Kerner, J. Schlenk, and T. Zirke. pySecDec: a toolbox for the numerical evaluation of multi-scale integrals. *Comput. Phys. Commun.*, 222:313–326, 2018.