# Precision Top Quark Physics at the Threshold at $e^+e^-$ Colliders

**André H. Hoang**

University of Vienna

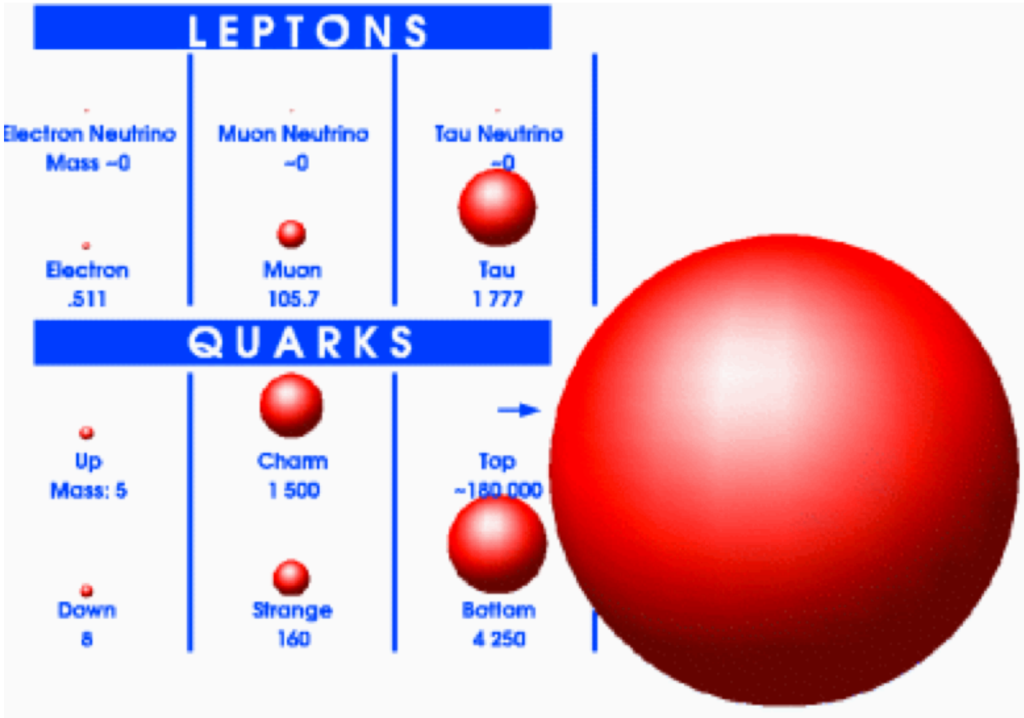
Doktoratskolleg Particles and Interactions

universität wien

FWF Der Wissenschaftsfonds.

14th International Workshop on Top Quark Physics, Sept 13-17 2021

# .. not just the heaviest SM particle




- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background it new physics searches
- One of crucial motivations for New Physics

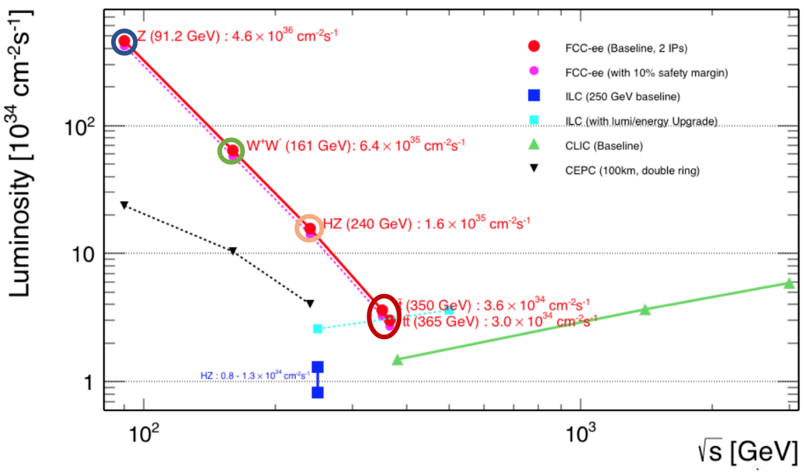
- Very special physics laboratory: $\Gamma_t \gg \Lambda_{QCD}$
  - Top treated a particle: $v_{part}$ , $p_{part}$ , spin, $\boldsymbol{\sigma}_{tot}$, $d\boldsymbol{\sigma}/d\, p_{part}$ ,.. $\rightarrow$ for scales $\gg \Gamma_t$
  - General: Top is quantum state sensitive low-E QCD and unstable particle effects: $m_t$, endpoint regions $\rightarrow$ scales ~ $\Gamma_t$
  - Multiscale problem: v, p, $m_t$, $\Gamma_t$, $\Lambda_{QCD}$, . . . (depends on resolution of observable)

# Outline

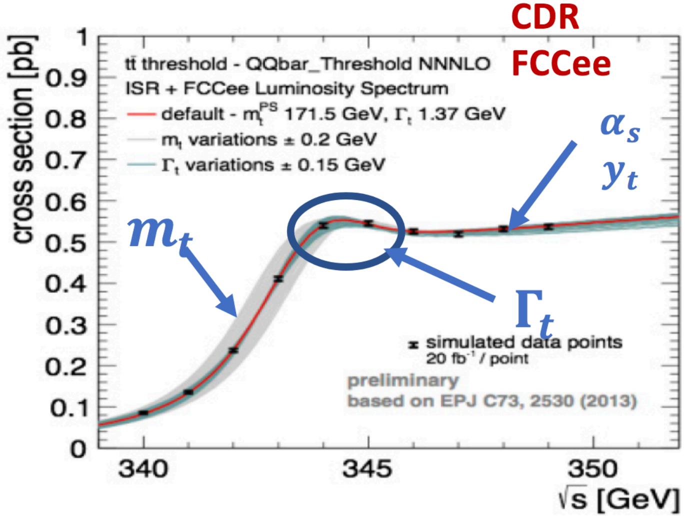
- Top Threshold in a Nutshell
- Threshold theory:  NRQCD + unstable particles
- Threshold continuum matching
- Fully differential treatment + MC simulations
- Associated threshold regions
- Top threshold at the LHC

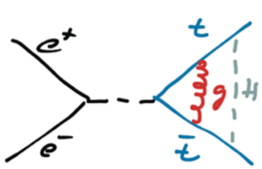
# $e^+e^-$ Top Threshold in a Nutshell

- Threshold runs: ILC, CLIC, FCC-ee
- Typical: 100 $fb^{-1}$ integrated luminosity
- Typical: $\sigma_{tot}$~ 1 pb → $10^5$ ttbar pairs
- Threshold scan: high precision method for top quark mass measurement in a well-defined scheme: ($\Delta m_t^{PS,1S,MSR}$ ~ 50 MeV)
- Sensitivity to $\Gamma_t$ , $\alpha_s$ , $y_t$
- Need: precise knowledge of beam effects

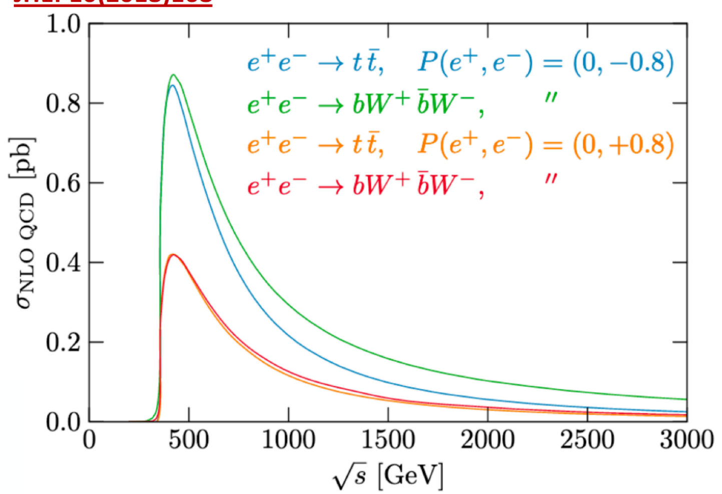



CDR FCCee



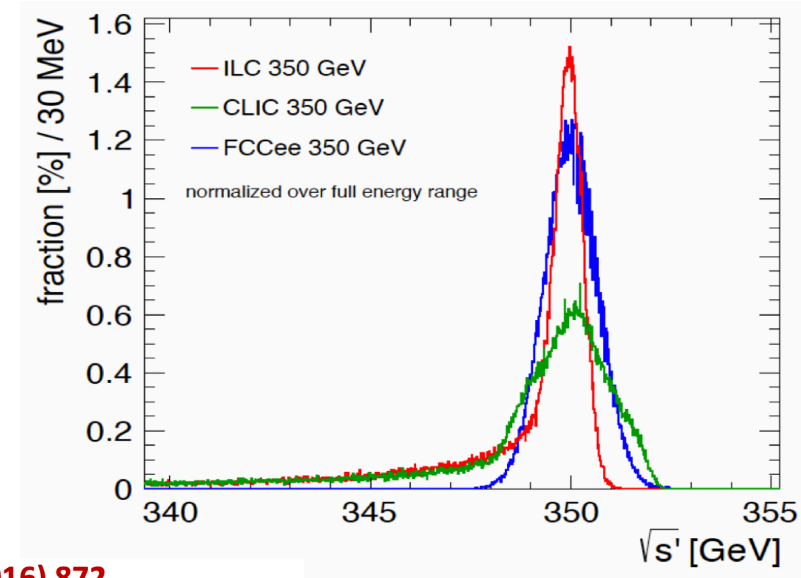




JHEP10(2018)168

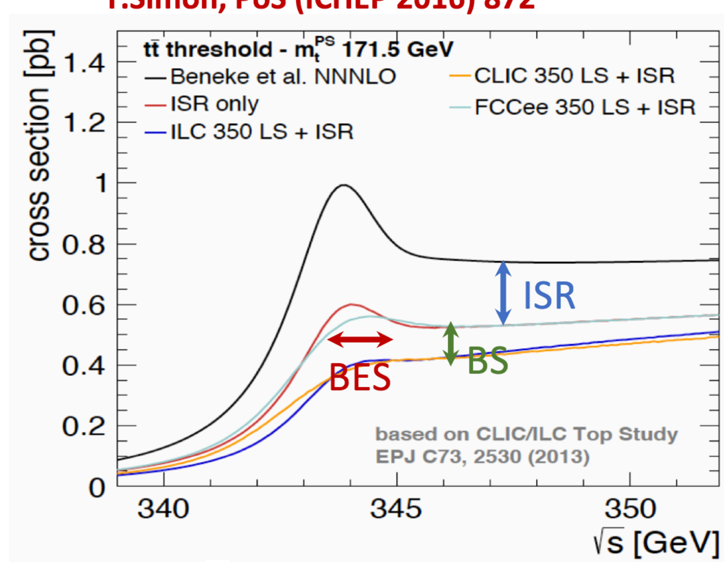


# $e^+e^-$ Top Threshold in a Nutshell

- Threshold runs: ILC, CLIC, FCC-ee
- Typical: 100 $fb^{-1}$ integrated luminosity
- Typical: $\sigma_{tot}$~ 1 pb $\rightarrow$ $10^5$ ttbar pairs
- Threshold scan: high precision method for top quark mass measurement in a well-defined scheme: ($\Delta m_t^{PS,1S,MSR}$ ~ 50 MeV)
- Sensitivity to $\Gamma_t$ , $\alpha_s$ , $y_t$
- Need: precise knowledge of beam effects

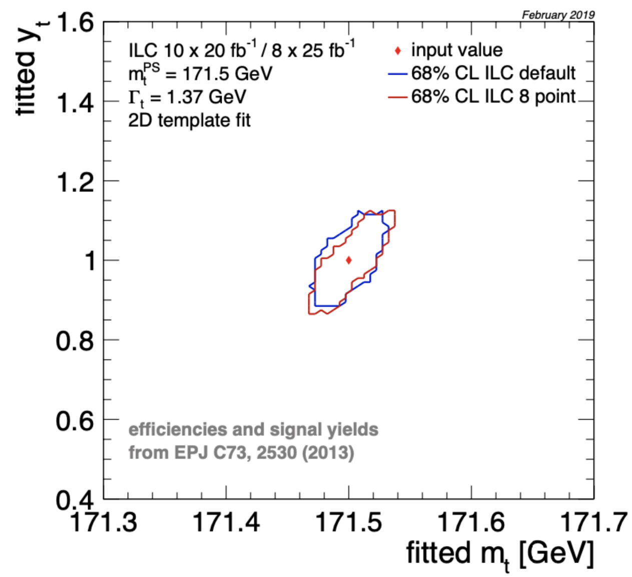



F.Simon, PoS (ICHEP 2016) 872



# $e^+e^-$ Top Threshold in a Nutshell

- Threshold runs: ILC, CLIC, FCC-ee
- Typical: 100 $fb^{-1}$ integrated luminosity
- Typical: $\sigma_{tot}$~ 1 pb $\rightarrow$ $10^5$ ttbar pairs
- Threshold scan: high precision method for top quark mass measurement in a well-defined scheme: ($\Delta m_t^{PS,1S,MSR}$ ~ 50 MeV)
- Sensitivity to $\Gamma_t$ , $\alpha_s$ , $y_t$
- Need: precise knowledge of beam effects




| error source | $\Delta m_t^{PS}$ [MeV] |
|---|---|
| stat. error (200 $fb^{-1}$) | 13 |
| theory (NNNLO scale variations, PS scheme) | 40 |
| parametric ($\alpha_s$, current WA) | 35 |
| non-resonant contributions (such as single top) | < 40 |
| residual background / selection efficiency | 10 – 20 |
| luminosity spectrum uncertainty | < 10 |
| beam energy uncertainty | < 17 |
| combined theory & parametric | 30 – 50 |
| combined experimental & backgrounds | 25 - 50 |
| total (stat. + syst.) | 40 – 75 |

- Many analyses with similar outcome
- Basic message: prospects confirmed and trustable (within exp+theo assumptions)
- No full simulations. Only $\boldsymbol{\sigma}_{tot}$ with high precision.

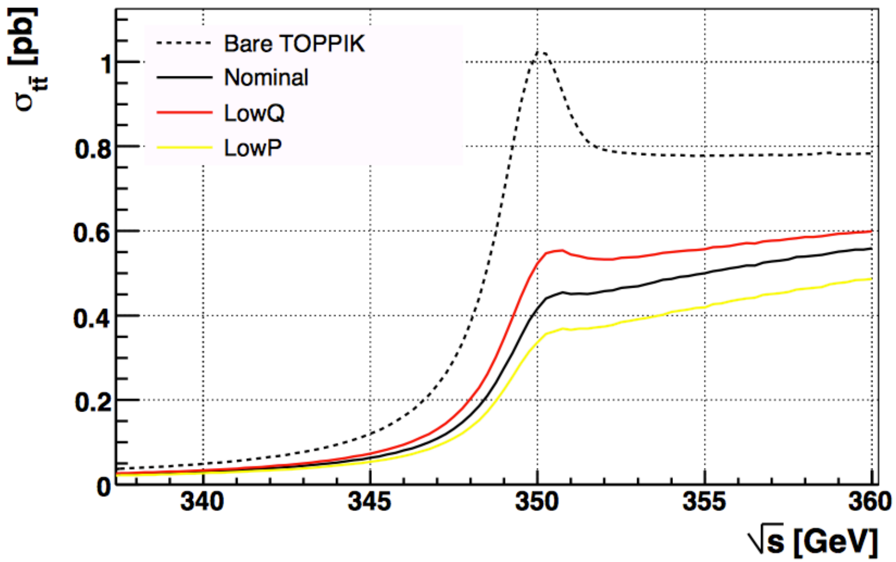
Martinez, Miquel '18 (arXiv:0207315)

| parameter | 8 point scan | | 10 point scan | |
|---|---|---|---|---|
| 2D fit $m_t$ and $y_t$ | | marg. | | marg. |
| $m_t$ | $(\pm 35(\text{stat}) \pm 45(\text{theo}))$ MeV | 17.0 MeV | $({}^{+34}_{-31}(\text{stat}) \pm 42(\text{theo}))$ MeV | 15.2 MeV |
| $y_t$ | ${}^{+0.120}_{-0.140}(\text{stat}) \pm 0.09(\text{theo})$ | 0.055 | ${}^{+0.128}_{-0.112}(\text{stat}) \pm 0.132(\text{theo})$ | 0.047 |

# We are very lucky

## Top pair total inclusive cross section:

$$\sigma(e+e- \rightarrow t\bar{t}+X) \text{ at } E_{cm} \approx 2m_t$$




### Advantages:

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

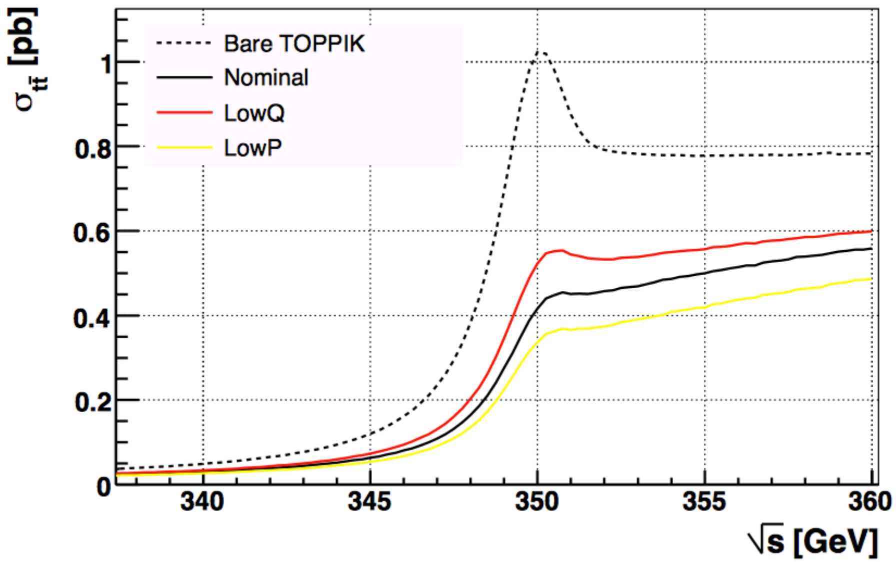
➢ Top decay protects from non-pert effects

- Remnant of a topionium resonance (“positronium of QCD”): $R_{bind} = m_t \alpha_S \sim 30$ GeV
- Crucial to control e+e- luminosity spectrum
- Binding energy about twice the top quark width: $$E_{\text{bind}} \approx \frac{\alpha_s^2 m_t}{2} \approx 2\Gamma_t$$
- $\sigma_{tot}$ can be calculated reliably in pQCD (nonrelativistic expansion)
- Non-resonant effects just tiny

# Top Threshold

## Top pair total inclusive cross section:

$$\sigma(e+e- \to t\bar{t} + X) \text{ at } E_{cm} \approx 2m_t$$




### Advantages:

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood

  (renormalons, summations)

Crucial difference to top pairs at LHC

- Top decay protects from non-pert effects

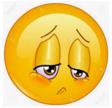
- $\sigma_{tot}$ only observable known where a well measurable threshold structure with resolution ≪ 1 GeV is generated by QCD dynamics at much larger scale:

  Quarkonium physics!   $R_{bind} = m_t \alpha_S \sim 30$ GeV ($\gg \Gamma_t, \Lambda_{QCD}$)

- Color single state protects from non-perturbative / ultra-soft effects ("non-jet like").

  We could not be more lucky!

Unfortunately no such observable at the LHC !



# Theoretical Background

- Strong scale hierarchy:

**Scales**

$v \ll 1$

$E \sim \Gamma_t$

$$m \gg \mathbf{p} \sim mv \gg E \sim mv^2 \,,\ \Lambda_{QCD}$$

| production decay | momentum inverse size | binding and kinetic energy | hadronization energy |
|---|---|---|---|

- Momentum regions:

| | | | |
|---|---|---|---|
| "hard": | $(k^0, \boldsymbol{k})$ | $\sim$ | $(m, m)$ |
| "soft": | $(k^0, \boldsymbol{k})$ | $\sim$ | $(m\,v, m\,v)$ |
| "potential": | $(k^0, \boldsymbol{k})$ | $\sim$ | $(m\,v^2, m\,v)$ |
| "ultrasoft": | $(k^0, \boldsymbol{k})$ | $\sim$ | $(m\,v^2, m\,v^2)$ |

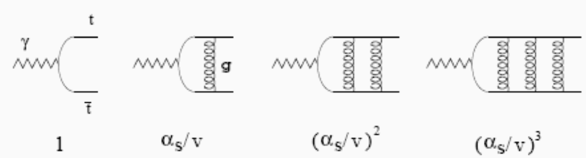
Ratardation effects "Lamb-shift" →

$$\Psi_{QCD} \sim \begin{pmatrix} \psi \\ \frac{\sigma \cdot \mathbf{p}}{E+m}\psi \end{pmatrix} \begin{matrix} \leftarrow \text{large} \\ \leftarrow \text{small} \end{matrix}$$

$v_t \sim \alpha_s$

- Usual fixed-order expansion does not apply

perturbation theory in $\alpha_s$ breaks down $(\alpha_s/v)^n$




"Coulombsingularities"

→ Schrödinger Equation

perturbation theory in $\alpha_s$ breaks down → large logs $(\alpha_s \ln v)^n$

$m_t = 175$ GeV, $\mathbf{p} \sim 25$ GeV, $E \sim 4$ GeV $\Rightarrow \ln\left(\frac{m_t^2}{E^2}\right) = 8$ → RGE's

FO approach:

$$\sigma_{t\bar{t}} \sim v \sum_n \left(\frac{\alpha_s}{v}\right)^n \Big[1,\ \{v, \alpha_s\},\ \{v^2, \alpha_s v, \alpha_s^2\},\ \ldots\Big]$$

LO NLO NNLO

RGI approach:

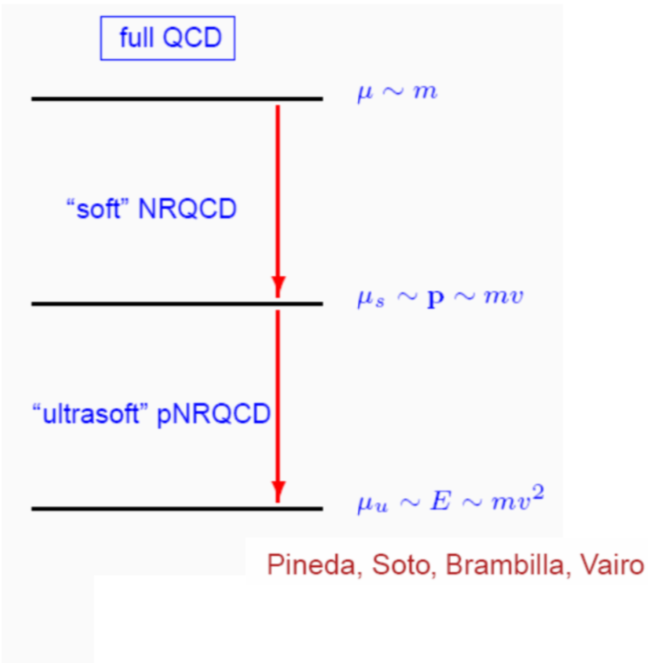
$$\sigma_{t\bar{t}} \sim v \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n \left(\alpha_s \ln v\right)^m \Big[1,\ \{v, \alpha_s\},\ \{v^2, \alpha_s v, \alpha_s^2\},\ \ldots\Big]$$

LL NLL NNLL

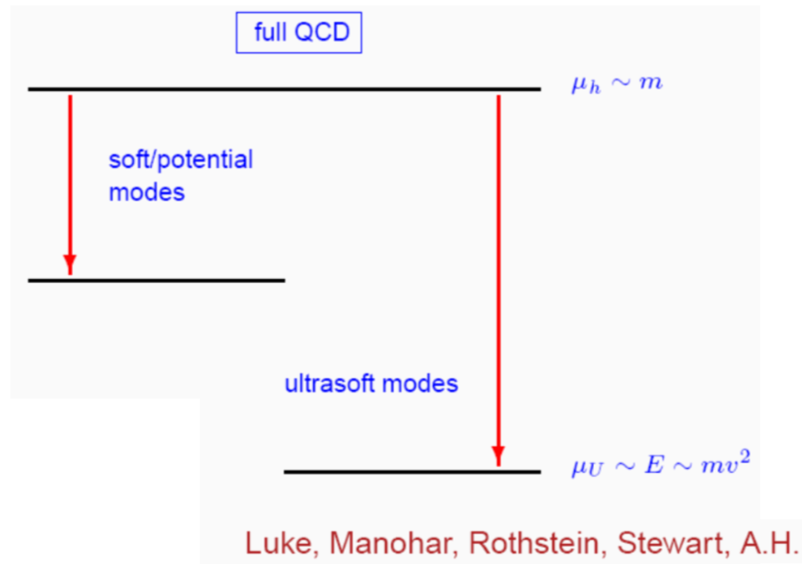
# Theoretical Background

- Two NRQCD formulations for RGI approach:

pNRQCD




vNRQCD




EFT construction: Soft (mv) and ultrasoft ($mv^2$) scales independent

EFT construction: Soft (mv) and ultrasoft ($mv^2$) scales correlated

Both theories are different, but obtain equivalent results at NNLL for top production.

# Total Cross Section $\sigma_{tot}$

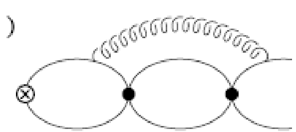
- Simple factorization formula up to NNLO/NNLL$_{QCD}$ + LO$_{EW}$ :

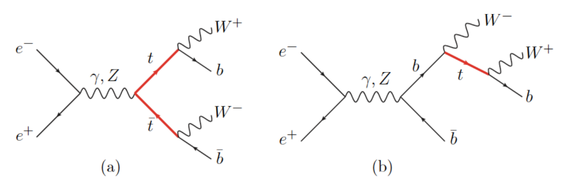
$$\sigma_{t\bar{t}} \propto c(\nu)^2 \, \text{Im}\, G(0,0,\sqrt{s}) + \ldots$$

$\longrightarrow$ More complicated beyond

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

- Total cross section at NNLO$_{QCD}$ (FO in $\alpha_S$~v) — AHH, Beneke, Melnikov, Nagano, Ota, Penin, Pivovarov, Signer, Smirnov, Sumion, Teubner, Yakovlev, Yekhovsky '01
- Total cross section NNLL$_{QCD}$ — AHH, Stahlhofen, '13
- Total cross section NNNLO$_{QCD}$ — Beneke, Kiyo, Marquard, Piclum, Steinhauser '13
- Non-resonant EW effects NNLL, NNNLO$_{partial}$ — AHH, Reisser, Ruiz-Femenia '04, '10
- Non-resonant EW effects NNNLO — Beneke, Maier, Rauh, Ruiz-Femenia '17,





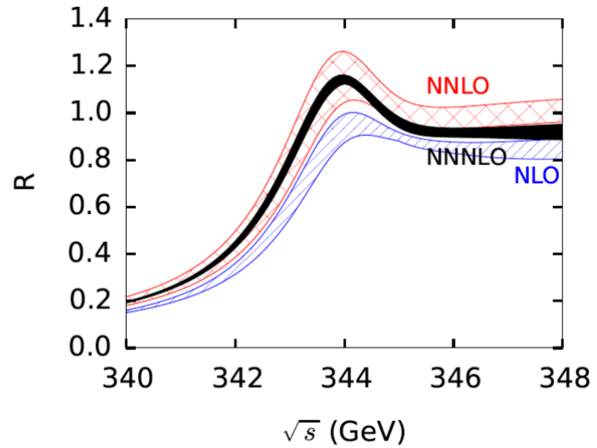
# Total Cross Section $\sigma_{tot}$

- Simple factorization formula up to NNLO/NNLL$_{QCD}$ + LO$_{EW}$ :

$$\sigma_{t\bar{t}} \propto c(\nu)^2 \, \text{Im}\, G(0,0,\sqrt{s}) + \ldots$$

→ More complicated beyond

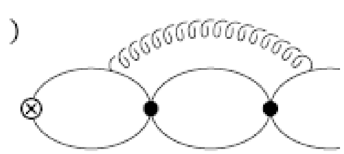
$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$




- Total cross section in good shape: $d\sigma_{tot}/\sigma_{tot}$ ~ 2-3% (for NNLL$_{QCD}$ / NNNLO$_{QCD}$)

- Combination of NNLL$_{QCD}$ and NNNLO$_{QCD}$ to be done.

- Why are ultrasoft gluon retardation effects not NLO ? They do contribute at NLO, but cancel in the total cross section for a ttbar pair in a color singlet state. Effects do not cancel if experimental cuts are applied, but effect small, if cuts are weak.



- Differential cross section in much worse state.

# Threshold Continuum Matching

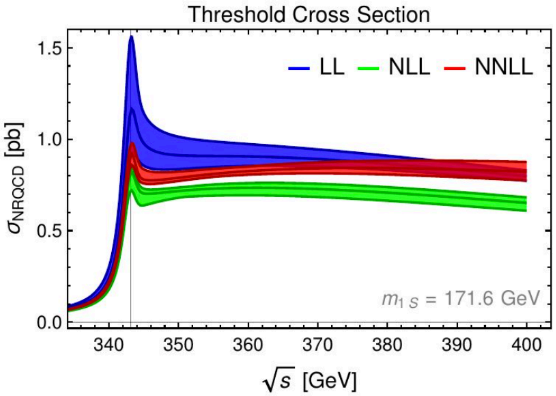
Widl, AHH to appear

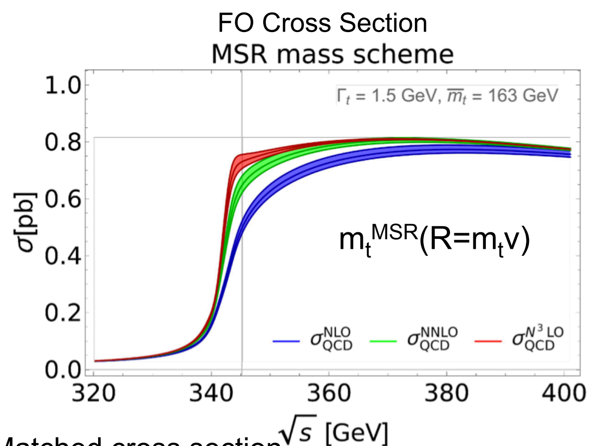
- Total cross section:

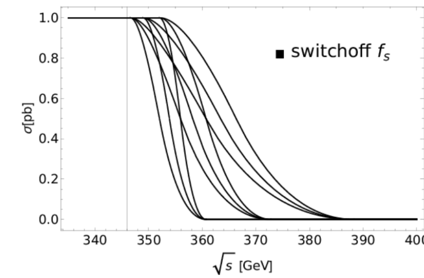
  Combination of the of NNNLO FO and NNLO+NNLL threshold cross section

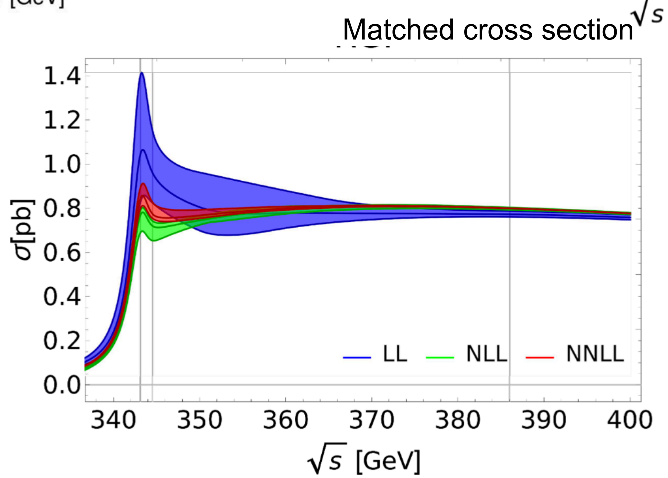
$$\sigma_{\mathrm{matched}} = \sigma_{\mathrm{QCD}} + (\sigma_{\mathrm{vNRQCD}} - \sigma_{\mathrm{double\text{-}counted}}) \cdot f_s$$

Switchoff- function




Scale-setting subtle!

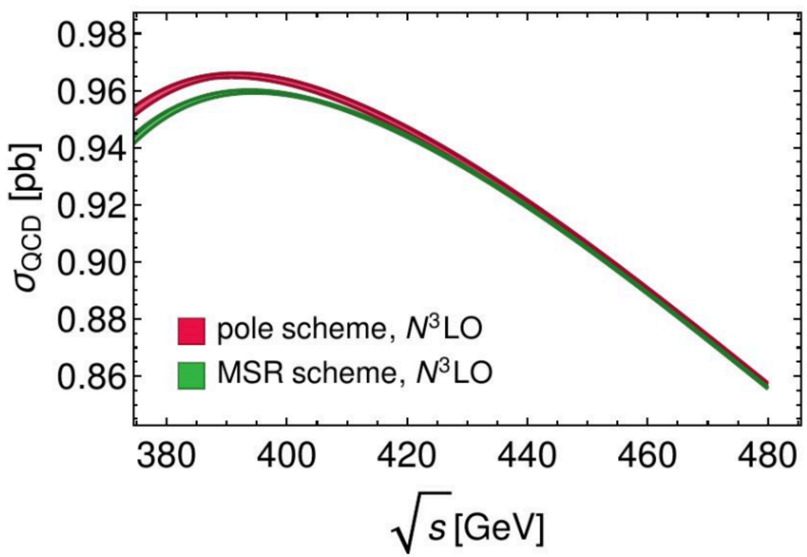
Dependence on matching function smaller than scale-dependence.

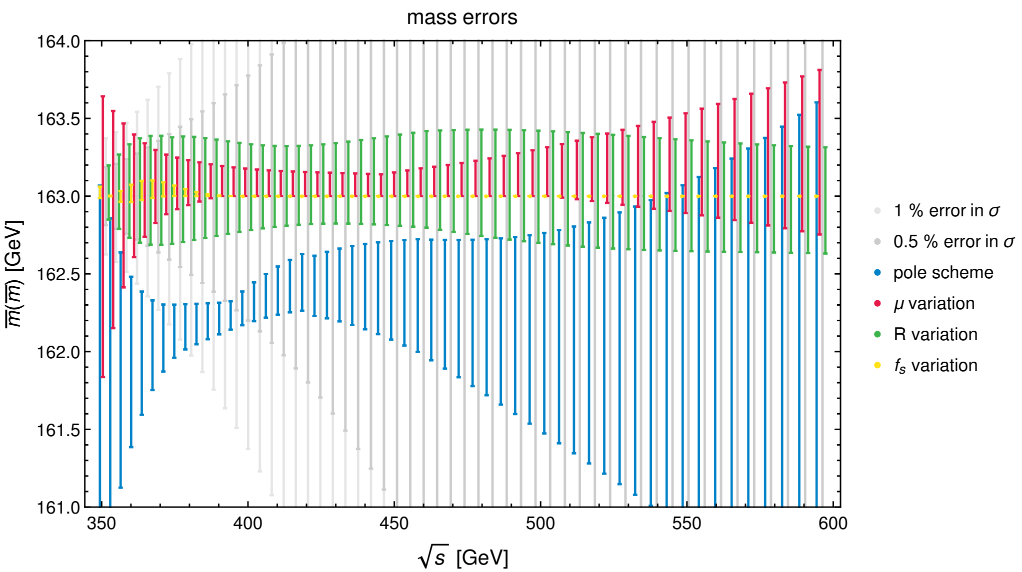
Uncertainties much larger in the pole mass scheme.

# More loops may be needed

Interesting observation: There is a mass sensitive region above the threshold region (at $E_{cm}$=360-380 GeV) where the renormalization scale uncertainty is much smaller than the mass scheme change uncertainty.

Widl, AHH to appear




Likely related to the pole mass renormalon, so that MSR certainly favoured.

$N^4$LO $O(\alpha_S^4)$ computation may resolve the issue.

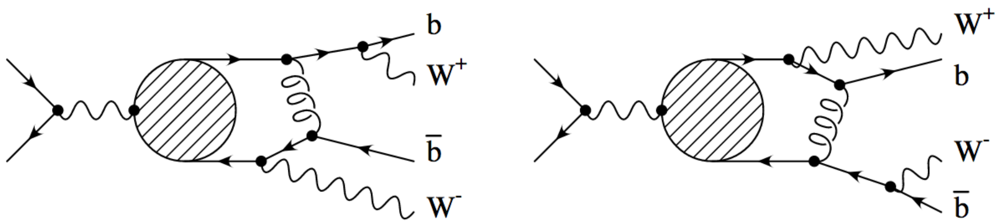
In principle mass measurement with uncertainty ~250 MeV possible at $E_{cm}$=360-380 GeV

# Differential Top Threshold

## Differential Cross Sections:

- Has not received much attention in the past, but important to correctly simulate experimental cuts and to interpret anything beyond the total cross section

- Very (!) hard problem due to ultrasoft ($E \lesssim \Gamma_t$) gluon exchange between the top quarks and their decay products.




- Large (non-factorizable) effects possible for selection cuts (size in principle unknown!!)

  Effects increase the more restrictive cuts are.

  Small for generous (wide) cuts

  Contribute at NLL/NLO order for differential cross sections.

AHH, Reisser, Ruiz-Femenia '10
Beneke, Maier, Rauh, Ruiz-Femenia '17,

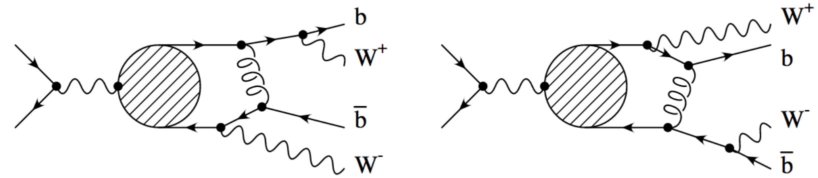
- Theoretically hard due to existence of Coulomb form factor that is defined in the non-relativisitic limit only (usual subtraction techniques known from NLO-revolution do not apply)

# Differential Top Threshold

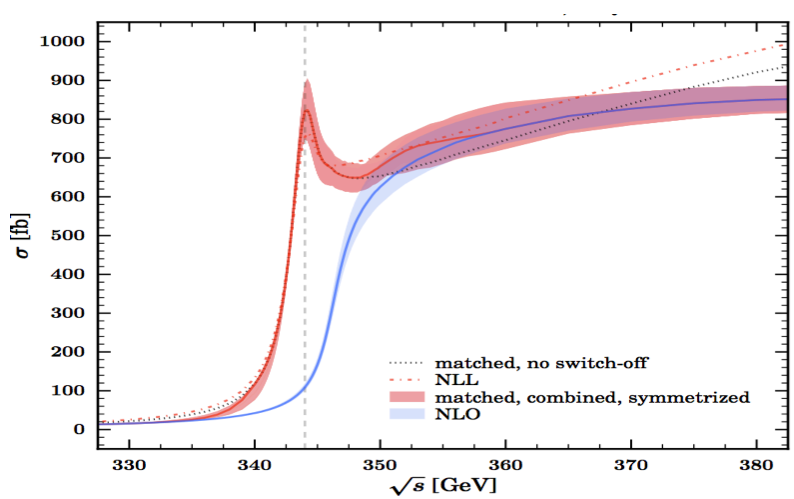
## Differential Cross Sections:

Differential cross sections available in Whizard

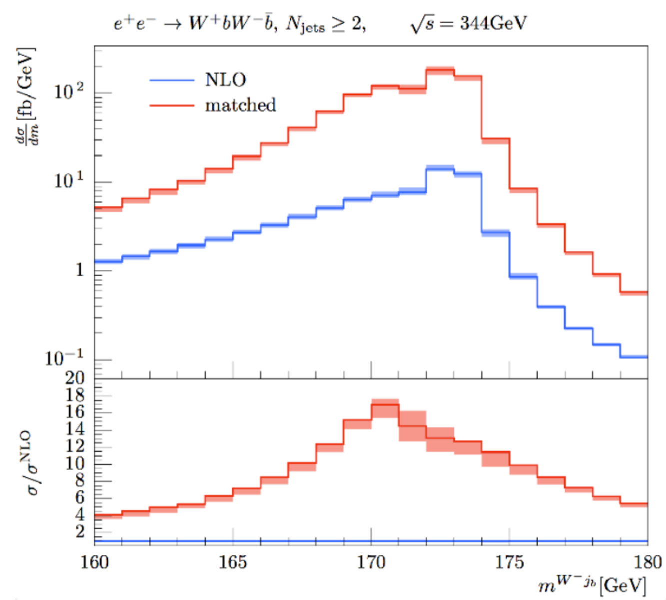
Bach, Neja, AHH, Kilian, Reuter ‘17




- Whizard threshold implementation does NOT contain these ultra-soft effects !
  Therefore $\mathrm{NLO_{FO}}$ + $\mathrm{NLL_{threshold}}$ only for total cross section, $\mathrm{NLO_{FO}}$ + $\mathrm{LL_{threshold}}$ otherwise.
  → Parton shower with coherent amplitude evolution needed und top decay needed.




Bach, Neja, AHH, Kilian, Reuter ‘17



# Differential Top Threshold

## Top quark anomalous coupling measurements:

Large QCD phases of the ttbar+γ/Z vertices due to Coulomb effects can interfere with CPV anomalous couplings of the top quark.

Reconstructed top quark spin affected.

Jezabek, Nagano, Sumino '00

(Happens also in the continuum.)

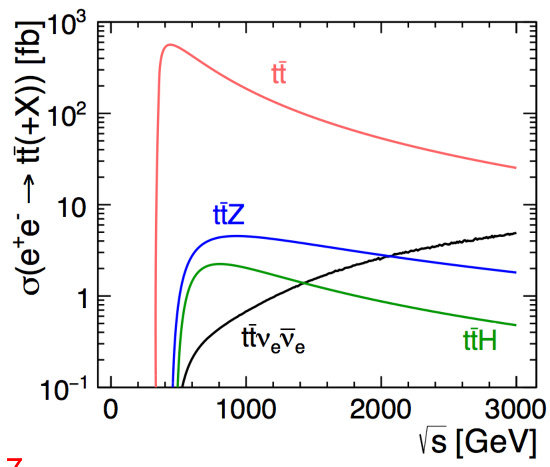
May supplement anomalous coupling analyses in the continuum, where threshold measurements are an important input.

See Durieux, Perello, Vos, Zhang '18

Ilaria Brivio's talk

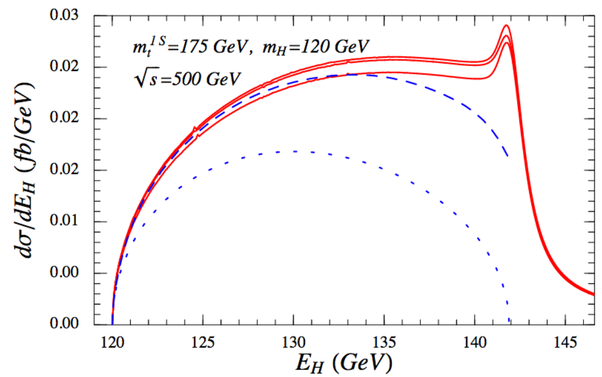
# Associated Top Threshold Physics (I)

- A future $e^+e^-$ collider with many associated ttbar thresholds
- Technology exists to extend ttbar threshold machinery to them, but much less event

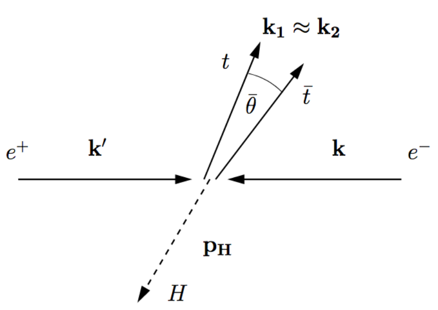



**ttbar H:** (similar story for ttbar Z)

- NLO QCD — Dawson, Reina '03,
- NLO EW corrections — Dener, etal,, Belanger, etal. You, etal '17,
- NLL ttbar threshold — Farrell, AHH '05
- CPV couplings — Hagiwara, Yokoya, Zheng '18




- Kinematic threshold enhancement reaching far into the continuum region for associated tt production, enhances cross section




| $\sqrt{s}$ [GeV] | $m_H$ [GeV] | $\sigma$(Born) [fb] | $\sigma(\alpha_s)$ [fb] | $\sigma$(NLL) [fb] | $\frac{\sigma(\text{NLL})}{\sigma(\text{Born})}$ | $\frac{\sigma(\text{NLL})}{\sigma(\alpha_s)}$ | $\frac{\sigma(\text{NLL})_{\lvert\beta\rvert<0.2}}{\sigma(\alpha_s)_{\beta<0.2}}$ |
|---|---|---|---|---|---|---|---|
| 500 | 120 | 0.151 | 0.263 | 0.357(20) | 2.362 | 1.359 | 1.78 |

Farrell, AHH '05

# Associated Top Threshold Physics (II)

**tt + γ:**

Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

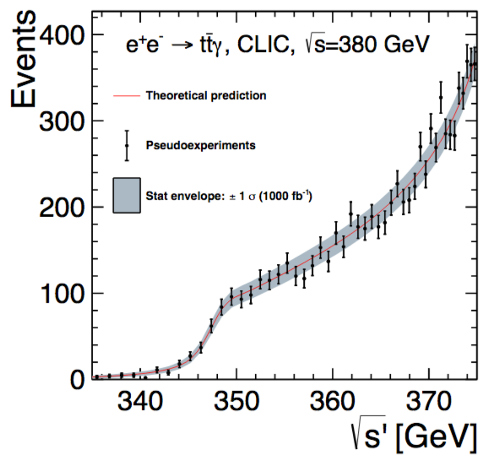
- Radiative return to the tt threshold allows for top threshold top mass measurements at higher energies.

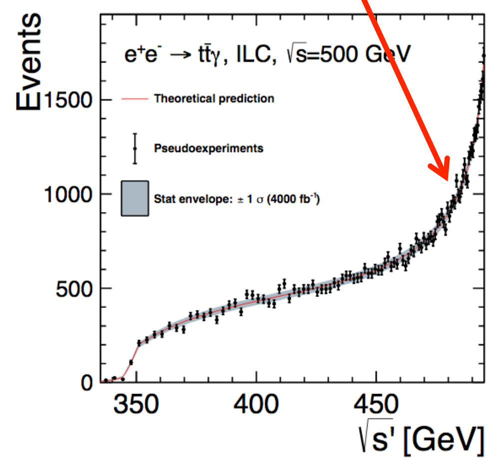
$$\frac{d\sigma_{t\bar{t}\gamma}}{d\cos\theta\, d\sqrt{s'}} = 2\, g(x,\theta)\sqrt{\frac{1-2x}{s}}\,\frac{\alpha_{\rm em}}{\pi}\,\sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}}\,,$$

$$s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

ISR enhancement




- Matched threshold (NNLL+NNLO)-continuum (NNNLO) cross section
- Realistic simulation experimental analysis
- Statistics dominated

| cms energy | CLIC, $\sqrt{s}=380$ GeV | | ILC, $\sqrt{s}=500$ GeV | |
|---|---|---|---|---|
| luminosity [fb$^{-1}$] | 500 | 1000 | 500 | 4000 |
| statistical | 140 MeV | 90 MeV | 350 MeV | 110 MeV |
| theory | 46 MeV | | 55 MeV | |
| lum. spectrum | 20 MeV | | 20 MeV | |
| photon response | 16 MeV | | 85 MeV | |
| total | 150 MeV | 110 MeV | 360 MeV | 150 MeV |

# Associated Top Threshold Physics

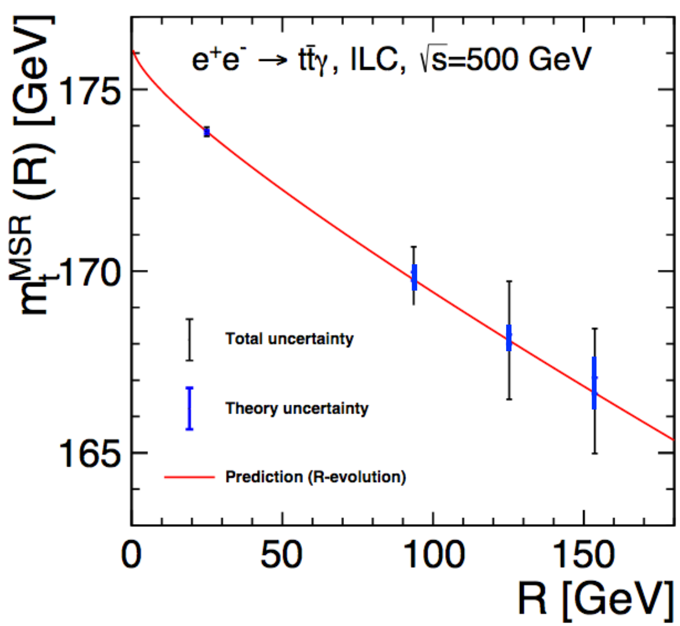
**tt + γ:**

Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

- Running MSR mass measurements

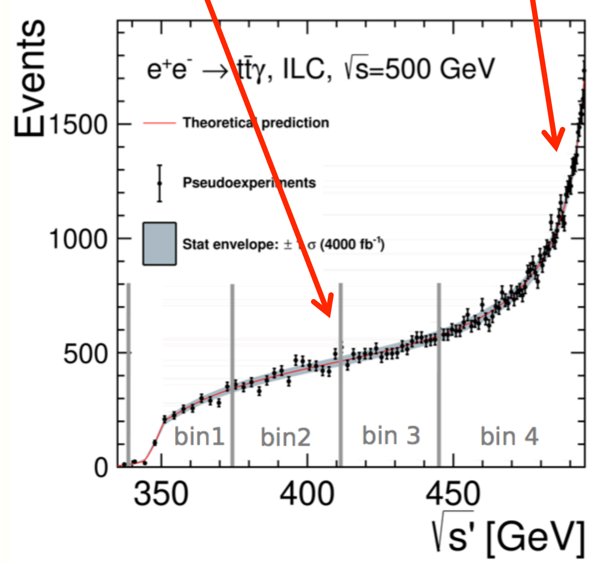
$$\frac{\mathrm{d}\sigma_{t\bar{t}\gamma}}{\mathrm{d}\cos\theta\,\mathrm{d}\sqrt{s'}} = 2\,g(x,\theta)\sqrt{\frac{1-2x}{s}}\,\frac{\alpha_{\rm em}}{\pi}\,\sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}}\,, \qquad s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$




Probes top mass sensitivity at scales $m_f\nu < m_t$

ISR enhancement



# Top Threshold at the LHC

⟶ Parton-parton collisions lead to significant complications compared to $e^+e^-$ collisions

## Parton level top pair threshold:

$$\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0 \quad \text{i.e.} \quad \hat{s} \to m_t^2$$

Parton energy just enough to produce the top pair + soft radiation.

⟶ Factorization formula for total cross section (straightforward extension compared to $e^+e^-$)

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{R=1,8} H^R_{pp'}(m_t,\mu) \int d\omega \; J_R(E - \frac{\omega}{2})\, W^R(\omega,\mu)$$

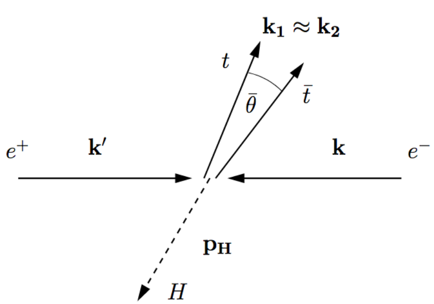
Beneke, Falgari, Klein, Schwinn ‘11

- $\sum_{R=1,8} H^R_{pp'}$: Color singlet + octet ttbar
- $J_R$: Color singlet + octet Coulomb Greens function
- $W^R$: Soft radiation function
  - ($O(\alpha_s)$ effects for pp collisions )
  - ($O(\alpha_s^3)$ effect for $e^+e^- \longrightarrow$ ttbar )

## $M_{ttbar}$ distribution for $M_{ttbar} \longrightarrow 2m_t$:

Yu, Wang, Wang, Xu, Xu, Yang ‘21

ttbar pair (with small relative velocity) can have any $p_T$ and can be color singlet or octet




It is problematic to identify $M_{ttbar}$ with $(p_t+p_{tbar})^2$ , when the ttbar is in a color octet state:

“ toponium-jet ”

Very challenging yet unresolved problem !

# Conclusions

- Gold-plated measurements at the top threshold based on the total cross section ($m_t$,$\Gamma_t$, $y_t$) are well-understood
- BUT: No full coherent simulations. + No consistent threshold MC
- Theoretical understanding adequate for total cross section
- Threshold continuum matching for total cross section
- Fully differential treatment + MC simulations still in infancy.
- Associated threshold regions important
- Top threshold in $d\sigma/dM_{ttbar}$ at the LHC not yet understood.