The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at Third Resummed and Fixed Order in QCD

Johannes Michel
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The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at Third Resummed and Fixed Order in QCD

based on

[PRL 127 (2021) 7, 072001, 2102.08039]

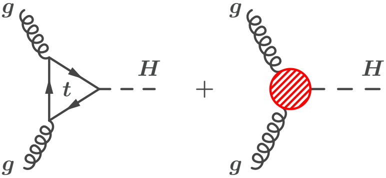
in collaboration with

G. Billis, B. Dehnadi, M. Ebert, F. Tackmann

Motivation

- Measure fiducial & differential Higgs cross sections at the LHC
  - Most model-independent way we have to search for BSM in the Higgs sector
- Can e.g. look for deviation from SM gluon fusion in total fiducial cross section



$$= \left(\frac{\alpha_s}{12\pi v} C_t + \frac{2v}{\Lambda^2} C_{HG}\right) H\, G^a_{\mu\nu} G^{a,\mu\nu}$$

Challenges for theory

- QCD corrections to $gg \to H$ are large: $\sigma/\sigma_{\mathrm{LO}} \approx 3$
  - Calculation of inclusive cross section has been pushed to N^{3}LO
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger ’15-’18]
- But LHC experiments apply kinematic selection cuts on Higgs decay products
  - Need complete interplay of QCD corrections and $\mathcal{O}(1)$ fiducial acceptance

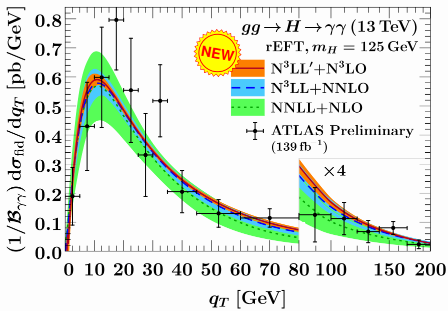
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

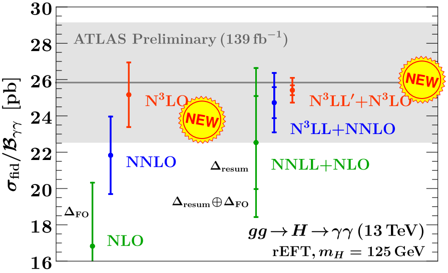
$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

Focus of this talk

[Billis, Dehnadi, Ebert, JM, Tackmann, PRL 127 (2021) 7, 072001, 2102.08039]

- Compute fiducial spectrum for $q_T \equiv p_T^H = p_T^{\gamma\gamma}$ at N^{3}LL′+N^{3}LO
- Compute total fiducial cross section at N^{3}LO, and improved by resummation





Fiducial predictions for $gg \to H \to \gamma\gamma$ at three loops

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- Previous state of the art was N^{3}LL(+NNLO$_1$) and NNLO, respectively
  [Chen et al. '18; Bizoń et al. '18; Gutierrez-Reyes et al. '19; Becher, Neumann '20]

Kicked off a recent push for fiducial color singlet at complete three-loop accuracy:

- Complementary N^{3}LO results for fiducial $Y_{\gamma\gamma}, \eta_{\gamma 1}, \Delta\eta_{\gamma\gamma}$ (with different method)
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]
- Fiducial N^{3}LL′ results for Higgs q_T spectrum
  [Re, Rottoli, Torrielli, 2104.07509]
  [For Drell-Yan, $\gamma\gamma$, see also 2103.04974, 2106.11260, 2107.12478, 2111.14509]

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- Com… t N… on
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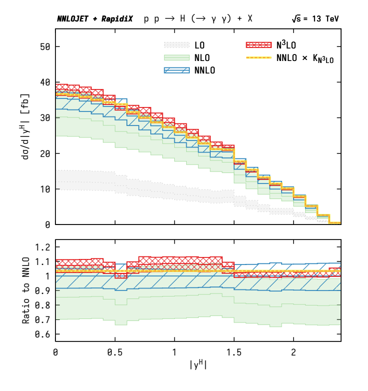

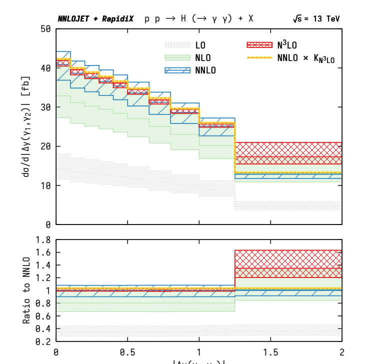

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Fiducial predictions for $gg \to H \to \gamma\gamma$ at three loops

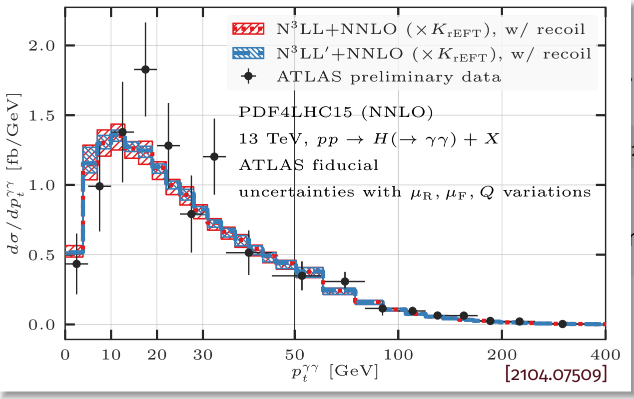
Consider gg

$p_T^{\gamma 1} \geq$ (.37, 1.52]

Focus of th 2001, 2102.08039]

- Compute
- Compute mmation

- Previous
  [Chen et al.



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Compute cross section from $\sigma = \int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$ and *power expand* around IR, $q_T \to 0$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}q_T} + \cdots$$
$$\sim \frac{1}{q_T}\left[\mathcal{O}(1) + \mathcal{O}\left(\frac{q_T}{m_H}\right) + \mathcal{O}\left(\frac{q_T^2}{m_H^2}\right) + \cdots \right]$$

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} = \sigma_{\mathrm{LO}} \, \delta(q_T) + \sum_n \alpha_s^n \left\{ \sigma_n^V \, \delta(q_T) + \sum_m \sigma_{n,m}^{(0)} \left[\frac{\ln^m(q_T/m_H)}{q_T} \right]_+ \right\}$$

- Contains LO contribution, virtual corrections, and log-enhanced singular terms

$$\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T} = \sum_n \alpha_s^n \sum_m \sigma_{n,m}^{(1)} \frac{1}{m_H} \ln^m(q_T/m_H)$$

- Still logarithmically divergent, only present if decay products are resolved

Compute cross section from $\sigma = \int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$ and *power expand* around IR, $q_T \to 0$:

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$$\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}q_T} = \sum_n \alpha_s^n \sum_m \sigma^{(2)}_{n,m} \frac{q_T}{m_H^2} \ln^m(q_T/m_H) + \cdots$$

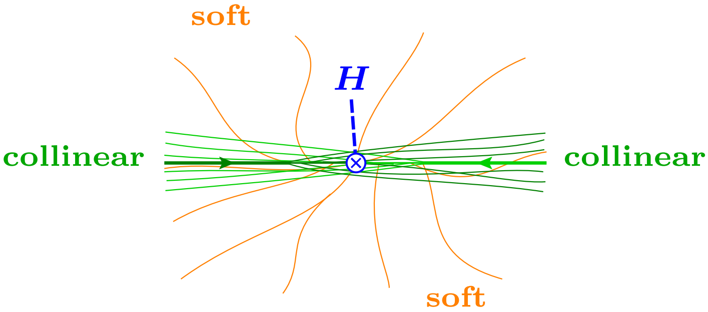
- Finite as $q_T \to 0$, extract by fitting known functional form to $H + 1j$ calculation
  - For the experts: Use a differential q_T subtraction accounting for fiducial power corrections
  - Avoids shortcomings of slicing [e.g. 1807.11501, 2103.04974] or Projection to Born [e.g. 2102.07607]
  - First complete application of q_T subtractions at N^{3}LO; see backup for details

Set up some notation, use that production and decay (acceptance) factorize:

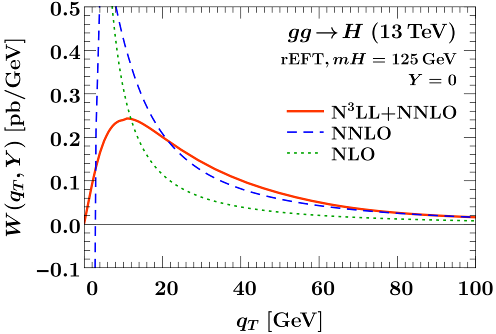
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \int \mathrm{d}Y \, A(q_T, Y; \Theta)\, W(q_T, Y)\,, \quad A_{\mathrm{incl}} = 1\,, \quad W(q_T, Y) = \frac{\mathrm{d}\sigma_{\mathrm{incl}}}{\mathrm{d}q_T\, \mathrm{d}Y}$$

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big)$$
$$\times\, B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$



- Renormalization group evolution between (e.g.) $\mu_S \sim q_T$ and $\mu_H \sim m_H$ resums large $\frac{\alpha_s^n}{q_T} \ln^{2n} \frac{q_T}{m_H}$ to all orders $\Rightarrow$ Sudakov peak $\sim \frac{1}{q_T} e^{-\alpha_s \ln^2 q_T/m_H}$



- N^{3}LL′ $\Leftrightarrow$ complete three-loop anomalous dimensions and boundary conditions
  [See backup for a complete list of ingredients and references]

▶ Apart from high order, this is completely standard for inclusive q_T spectrum

Fiducial power corrections...

... are the power corrections coming from the q_T-dependent acceptance:

$$\frac{d\sigma^{\rm fpc}}{dq_T} \equiv \int dY \Big[A(q_T, Y; \Theta) - A^{(0)}(Y; \Theta) \Big] W^{(0)}(q_T, Y)$$

- These uniquely predict all linear power corrections $d\sigma^{(1)}$ because

$$A(q_T, Y; \Theta) = A^{(0)}(Y; \Theta) \; \left[1 + \mathcal{O}\left(\frac{q_T}{m_H}\right)\right]$$

$$W(q_T, Y) = W^{(0)}(q_T, Y) \left[1 + \mathcal{O}\left(\frac{q_T^2}{m_H^2}\right)\right]$$

[Presence of linear terms pointed out in Ebert, Tackmann, 1911.08486]

[Factorization/resummation & use in subtractions: Ebert, JM, Stewart, Tackmann, 2006.11382]

[Analytic results in double-logarithmic approximation: Salam, Slade, 2106.08329; see talk by G. Salam]

[See also Alekhin et al., 2104.02400; Buonocore et al., 2111.13661; Camarda et al., 2111.14509]

Challenge [see talk by G. Salam]

Fiducial power corrections upset fixed-order perturbative convergence of *total* $\sigma_{\rm fid}$

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\rm fpc}}{\mathrm{d}q_T}$:

$$\begin{aligned} \sigma^{\rm FO}_{\rm incl} &= 13.80\,[1 + 1.291 + 0.783 + 0.299]\,\text{pb} \\ \sigma^{\rm FO}_{\rm fid} &= 6.928\,[1 + 1.429 + 0.723 + 0.481]\,\text{pb} \\ &= 6.928\,[1 + (1.300 + 0.129_{\rm fpc}) + (0.784 - 0.061_{\rm fpc}) + (0.331 + 0.150_{\rm fpc})]\,\text{pb} \end{aligned}$$

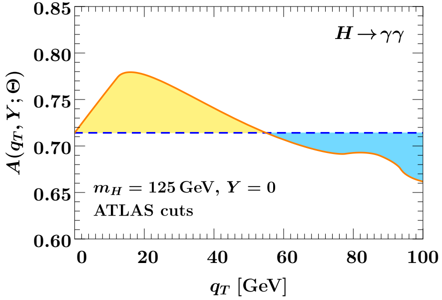
- Fiducial power corrections show no convergence, remainder is similar to inclusive

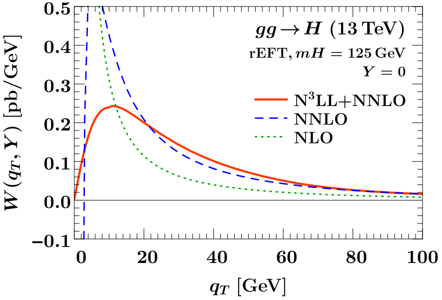
Challenge [see talk by G. Salam]

Fiducial power corrections upset fixed-order perturbative convergence of *total* σ_{fid}

Two ways to understand the effect of small q_T on total cross section:

1. Acceptance acts as a weight under the q_T integral





$$\sigma_{\mathrm{incl}} = \int \mathrm{d}q_T\, W(q_T) \qquad \sigma_{\mathrm{fid}} = \int \mathrm{d}q_T\, A(q_T)\, W(q_T)$$

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Two ways to understand the effect of small q_T on total cross section:

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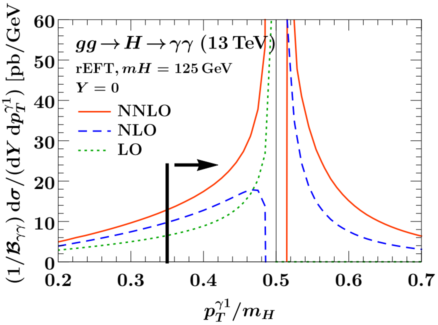
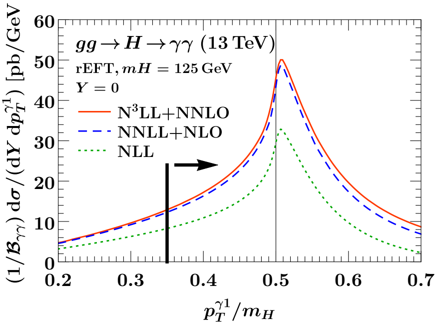
2. We're cutting on the resummation-sensitive photon p_T





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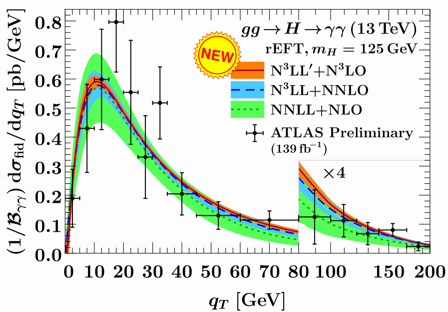
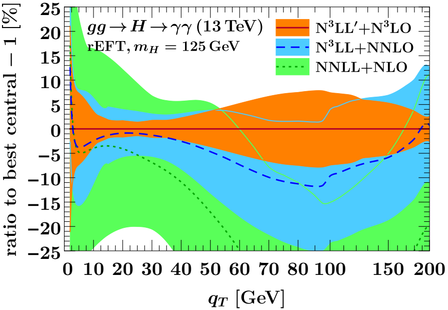
Solution

Resum fiducial power corrections to the same N^{3}LL$'$ accuracy as leading power by resumming $W^{(0)} = H\, B \otimes B \otimes S$ and keeping $A(q_T, Y; \Theta)$ exact:

$$\frac{\mathrm{d}\sigma_{\rm res}^{\rm fpc}}{\mathrm{d}q_T} = \int \mathrm{d}Y \Big[A(q_T, Y; \Theta) - A^{(0)}(Y; \Theta)\Big] W_{\rm res}^{(0)}(q_T, Y)$$

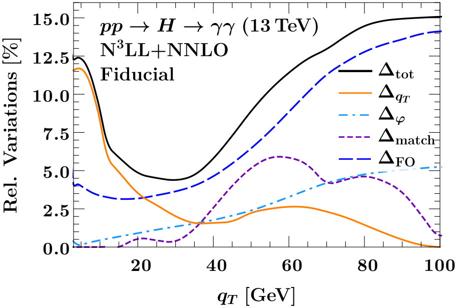
- Effect is fully predicted by resummed perturbation theory

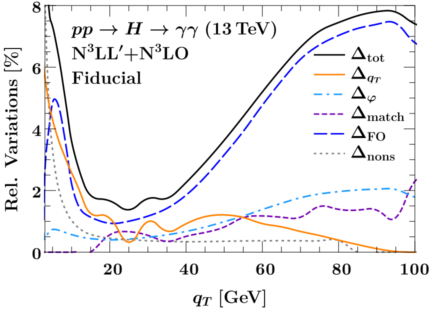




- Good agreement with ATLAS Run 2 data
  - Divide $H \to \gamma\gamma$ branching ratio $\mathcal{B}_{\gamma\gamma}$ out of data [LHC Higgs Cross Section WG, 1610.07922]
  - Data are corrected for other production channels, photon isolation efficiency [ATLAS, 1802.04146]
- Observe excellent perturbative convergence & uncertainty coverage

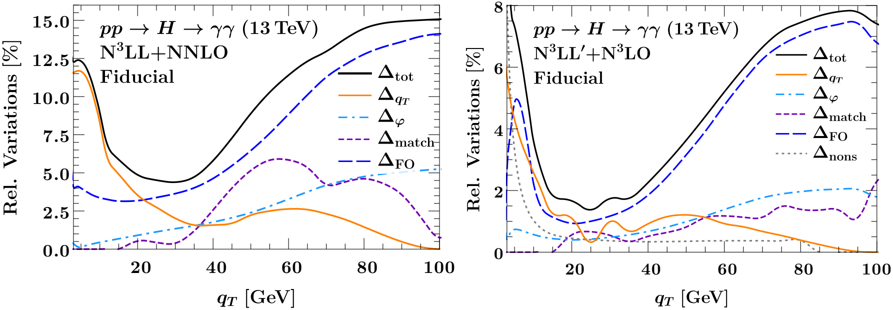
▶ Crucial to probe all parts of the prediction in uncertainty estimate!





$$\Delta_{\rm tot} = \Delta_{q_T} \oplus \Delta_\varphi \oplus \Delta_{\rm match} \oplus \Delta_{\rm FO} \oplus \Delta_{\rm nons}$$

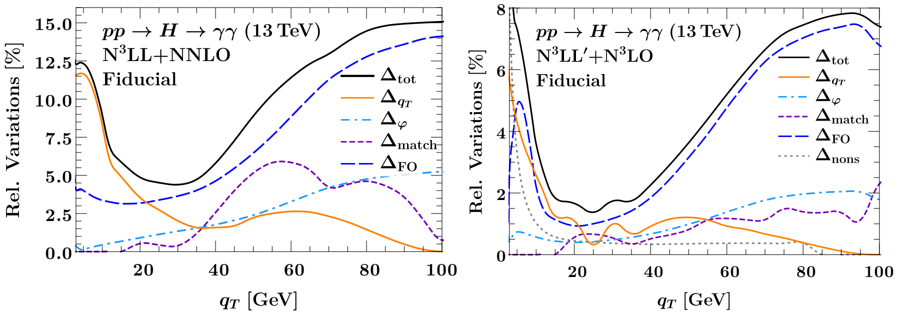
- Probes higher-order resummed terms $\sim \ln q_T/m_H$
- Estimated by envelope of 36 different combinations of independently varying $\{\mu_B, \mu_S, \ldots\}$ in $W^{(0)} = H\, B \otimes B \otimes S$



$$\Delta_{\text{tot}} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{\text{match}} \oplus \Delta_{\text{FO}} \oplus \Delta_{\text{nons}}$$

- Probes higher-order terms $\sim \ln \frac{-m_H^2 - i0}{\mu_H^2} = -i\pi$ in timelike gluon form factor
- Estimated by varying phase of complex hard scale over $\arg \mu_H \in \{\pi/4, 3\pi/4\}$

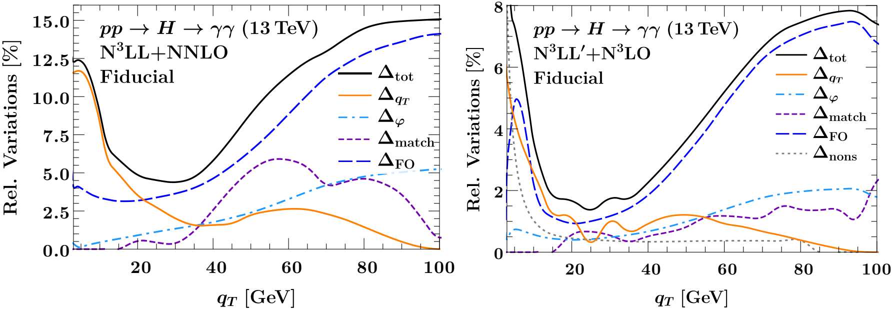
Uncertainty breakdown



$$\Delta_{\rm tot} = \Delta_{q_T} \oplus \Delta_\varphi \oplus \Delta_{\rm match} \oplus \Delta_{\rm FO} \oplus \Delta_{\rm nons}$$

- Uncertainty from matching scheme between resummed peak and fixed-order tail
- Estimated by varying the transition points governing resummation turn-off
- Turn-off implemented by *profile scales* $\mu_{B,S} \to \mu_{\rm FO}$
  - Ambiguity manifestly reduces at each order

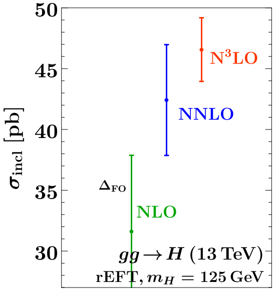
Uncertainty breakdown

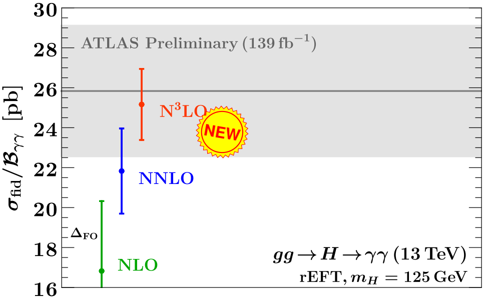


$$\Delta_{tot} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{match} \oplus \Delta_{FO} \oplus \Delta_{nons}$$

- Fixed-order uncertainty
- Estimated by standard variations of overall $\mu_{FO} = \mu_R$ (dominates over μ_F)
- Fit/MC uncertainty on extraction of nonsingular terms $\sim q_T^2/m_H^2$ ...see backup

Results: The total fiducial cross section at N^{3}LO and N^{3}LL′+N^{3}LO





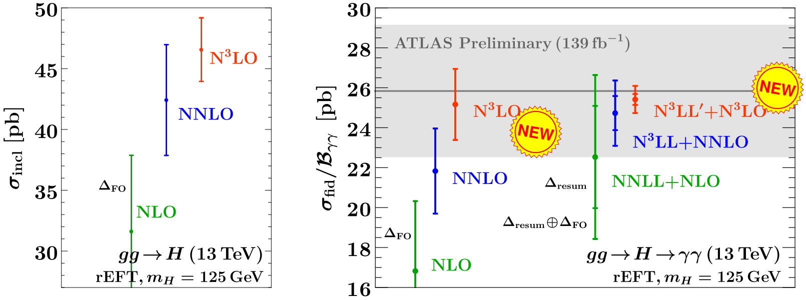
- Large N^{3}LO correction to fiducial cross section (larger than inclusive)
  - Caused by fiducial power corrections, *not* captured by rescaling inclusive N^{3}LO result
- Resummation restores convergence, gives detailed handle on uncertainty:

N^{3}LO: $$\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.16 \pm 1.78_{\rm FO} \pm 0.12_{\rm nons})\,\text{pb}$$

N^{3}LL′+N^{3}LO: $$\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.41 \pm 0.59_{\rm FO} \pm 0.21_{q_T} \pm 0.17_{\varphi} \pm 0.06_{\rm match} \pm 0.20_{\rm nons})\,\text{pb}$$

▶ Today's message: Theory can deal with it!

Results: The total fiducial cross section at N^{3}LO and N^{3}LL′+N^{3}LO



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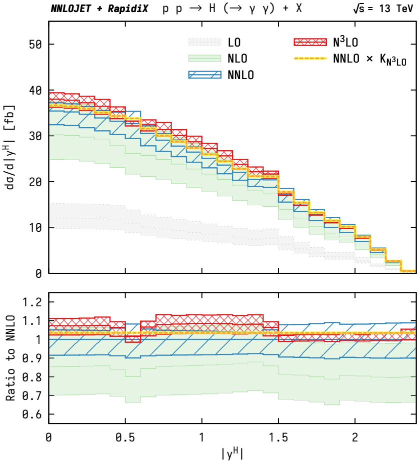
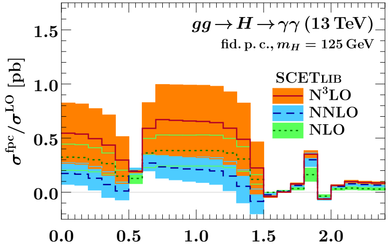
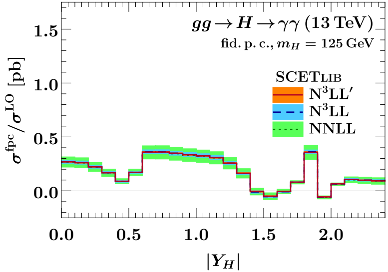
Outlook: Resummation effects in other $H \to \gamma\gamma$ observables

- "Infrared sensitivity" observed also in other Higgs observables at N^{3}LO
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

⇔ Precisely the fiducial power corrections we can analytically deal with and resum







Note: Plots on the right show *only* σ^{fpc}.

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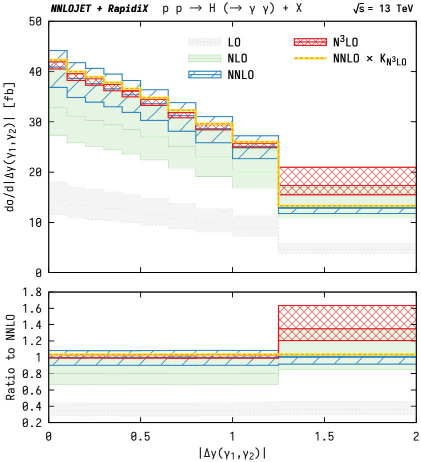

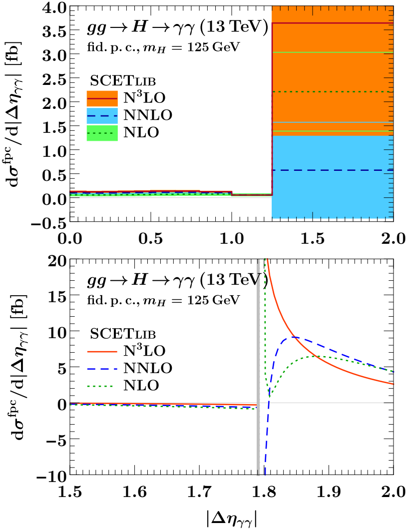

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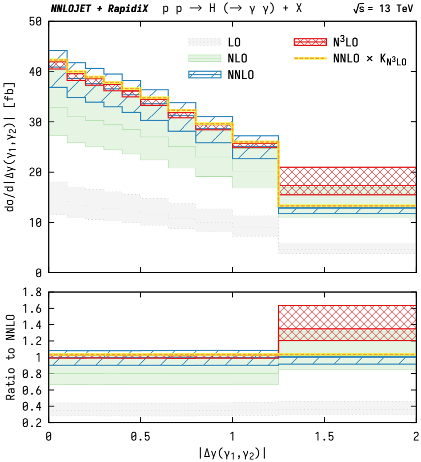
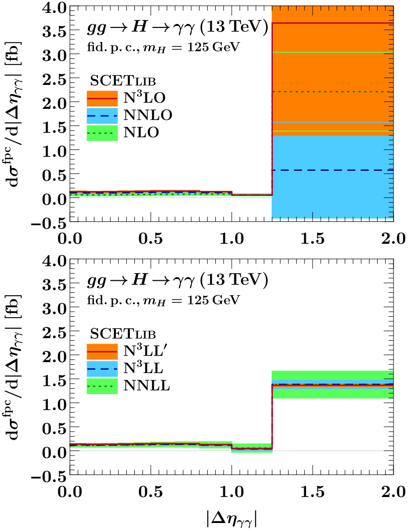
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

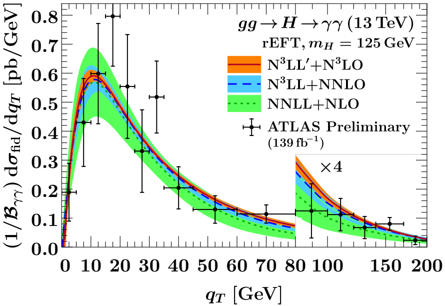
⇔ Precisely the fiducial power corrections we can analytically deal with and resum

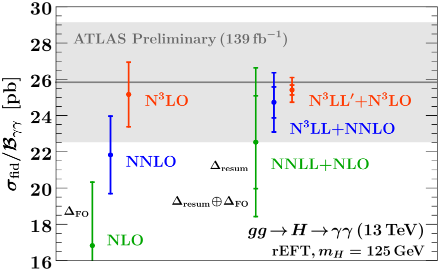




Note: Plots on the right show *only* σ^{fpc}.

- Presented third-order predictions for fiducial p_T^H spectrum and total fiducial cross section for $gg \to H \to \gamma\gamma$ at the LHC
  - First direct comparison to LHC data at genuine three-loop order





- Large fiducial power corrections induced by experimental acceptance
  - Fully accounted for by resummed perturbation theory
  - Enables best-possible combined predictions for other $H \to \gamma\gamma$ observables

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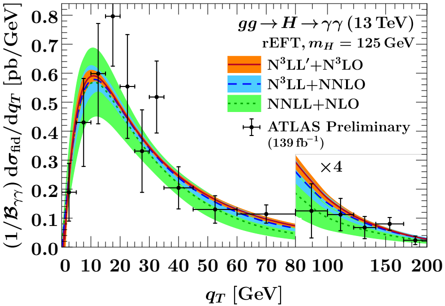

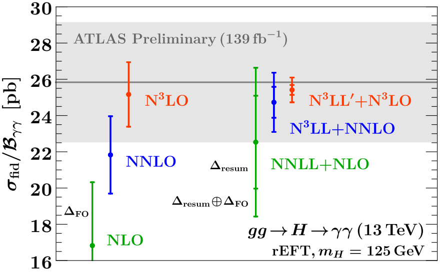

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  - Fully accounted for by resummed perturbation theory
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Thank you for your attention!

Backup

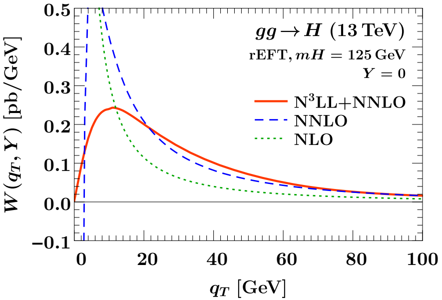
At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big) \times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

Ingredients satisfy 2D renormalization group equations, e.g. soft function:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_S^g(\mu, \nu) \qquad \nu \frac{\mathrm{d}}{\mathrm{d}\nu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_\nu^g(b_T, \mu)$$

- Solve recursively at fixed order
  - Complete log structure of $\mathrm{d}\sigma^{(0)}$
- Closed-form all-order solution
  - Resummed Sudakov peak
- Resummation order specified by perturbative order of anom. dims. and boundary conditions



Leading-power factorization & resummation to N^{3}LL$'$

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|)$$
$$\times\, B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

To reach N^{3}LL$'$ for $W^{(0)}$, implemented in `SCETlib`:

- Three-loop soft and hard function …includes in particular the three-loop virtual form factor [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20] [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- N^{3}LL solutions to virtuality/rapidity RGEs in b_T space
- Hybrid profile scales for fixed-order matching [Lustermans, JM, Tackmann, Waalewijn '19]

$$\sigma = \int_0^{q_T^{\rm off}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} + \int_0^{q_T^{\rm off}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T} + \int_{q_T^{\rm off}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T}$$

Include $\mathrm{d}\sigma^{\rm fpc}$ in differential subtraction:

$$\frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} = \int \mathrm{d}Y \, A(q_T, Y; \Theta) \, W^{(0)}(q_T, Y) = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\rm fpc}}{\mathrm{d}q_T}$$

Remaining (nonsingular) terms:

$$\frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T} = \int \mathrm{d}Y \, A(q_T, Y; \Theta) \left[W_{\rm FO}^{(2)}(q_T, Y) + \cdots \right] = \left[\frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma_{\rm FO}^{\rm sing}}{\mathrm{d}q_T} \right]_{q_T > 0}$$

Challenges:

- Obtaining stable $H + 1j$ results for $q_T \to 0$ is *hard* ...in particular at $\mathrm{NNLO_1}$
- Dropping the nonsingular below $q_T \leq q_T^{\rm cut}$ is not viable, either ...as we'll see shortly
  - Crucial to use differential subtraction, not slicing

Differential q_T subtractions

$$\sigma = \int_0^{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} + \int_0^{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T} + \int_{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T}$$

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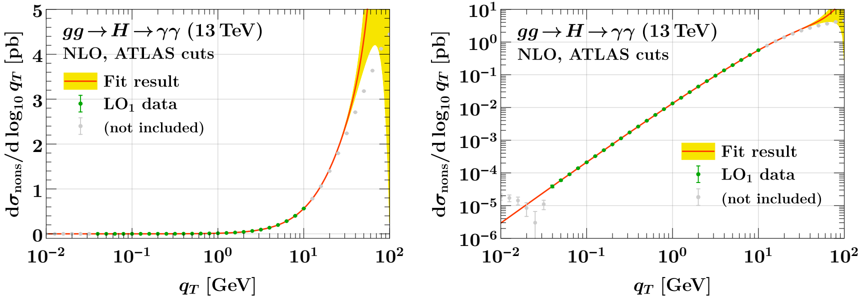
Key idea

Fit nonsingular data to known form at subleading power and integrate *analytically*:

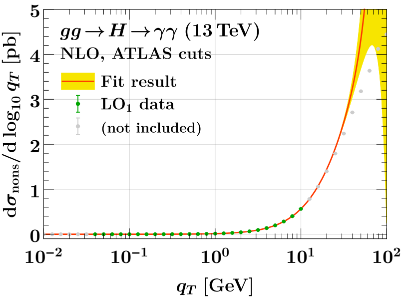
$$q_T \frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T}\bigg|_{\alpha_s^n} = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \left(a_k + b_k \frac{q_T}{m_H} + c_k \frac{q_T^2}{m_H^2} + \cdots \right) \ln^k \frac{q_T^2}{m_H^2}$$

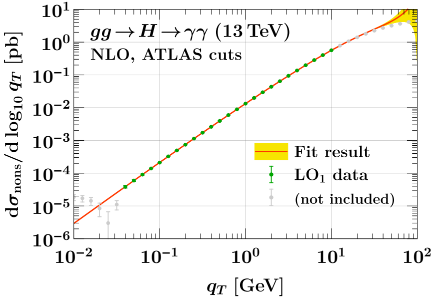
- Include higher-power b_k, c_k to get unbiased a_k
  - Allows us to use more precise data at higher q_T as lever arm in the fit



Fixed-order inputs:

- NLO contribution to $W(q_T, Y)$ at $q_T > 0$ (LO_1) is easy
- At NNLO (NLO_1), renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia ’17]

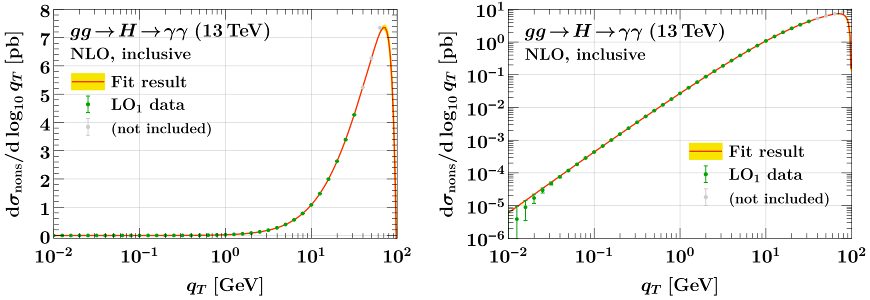




Fit procedure:

- Perform separate χ^2 fits of $\{a_k^{\text{incl,fid}}\}$ to inclusive and fiducial nonsingular data
  [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combinations to ensure fit is stable
  [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu ’15-’16]

Fit results at (N)NLO



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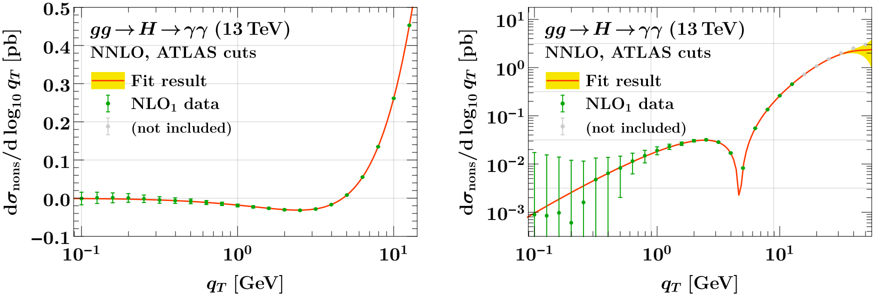

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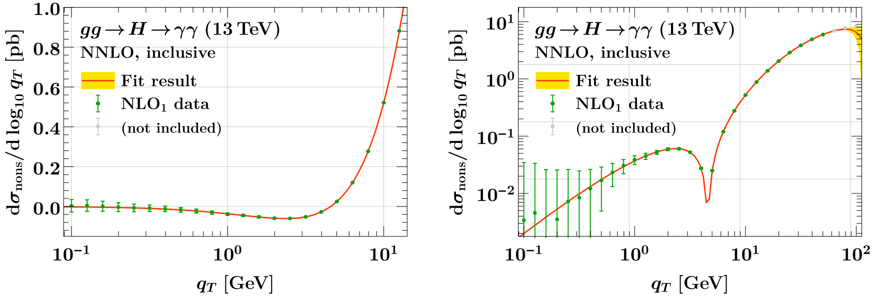

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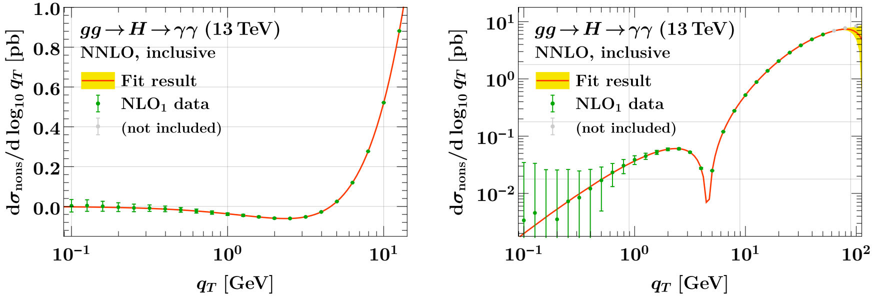
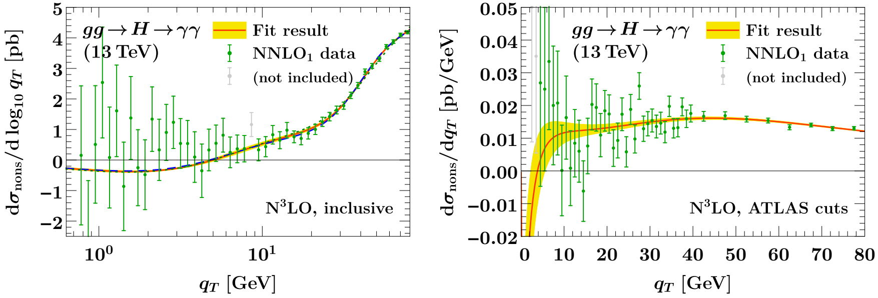


- Check the purely hadronic $a_k^{\rm fid}$ by directly fitting them to

$$q_T \int dY\, A^{(0)}(Y;\Theta)\,[W - W^{(0)}] = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \Big(a_k^{\rm fid} + c_k' \frac{q_T^2}{m_H^2} + \cdots \Big) \ln^k \frac{q_T^2}{m_H^2}$$ ✓

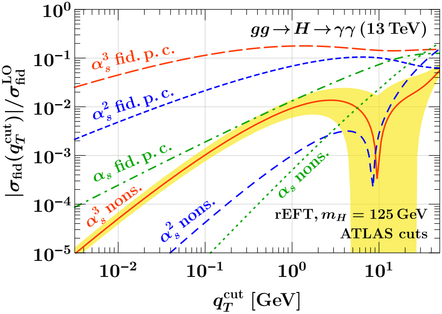
- Recover analytic (N)NLO coefficient of $\sigma_{\rm incl}$ at 10^{-5} (10^{-4}) ✓
- Analytic implementation gives us awesome precision on *all* NLP coefficients (all logs at NLO *and* NNLO, also differential in Y, broken down by color structure, ...)
  - Can serve as benchmark for q_T resummation at subleading power



Setup:

- Combined fit to existing binned inclusive and fiducial NNLO$_1$ data from `NNLOjet`
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier ’15-16; as used in Chen et al. ’18; Bizoń et al. ’18]
- Empirically find $0.4 \leq a_k^{\mathrm{fid}}/a_k^{\mathrm{incl}} \leq 0.55$ at (N)NLO $\Rightarrow$ use as weak 1σ constraint
- Add $\sigma_{\mathrm{incl}}(q_T \leq q_T^{\mathrm{cut}}) = \sigma_{\mathrm{incl}}^{\mathrm{N^3LO}} - \sigma_{\mathrm{incl}}(q_T > q_T^{\mathrm{cut}})$ as additional incl. data point
  [Mistlberger ’18]

Comparison to other methods: q_T slicing

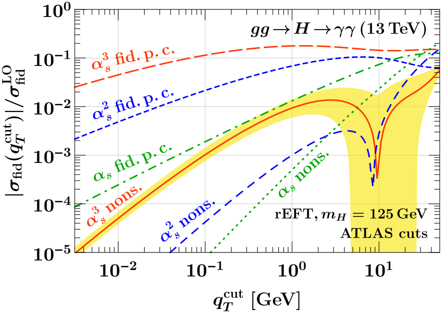


Slicing approach to q_T subtractions:

[used e.g. in Cieri, Chen, Gehrmann, Glover, Huss, 1807.11501; Camarda, Cieri, Ferrera, 2103.04974]

$$\sigma = \sigma^{(0)}(q_T^{\rm cut}) + \sigma^{\rm fpc}(q_T^{\rm cut}) + \sigma^{\rm nons}(q_T^{\rm cut}) + \int_{q_T^{\rm cut}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T}$$

- Slicing uses finite $q_T^{\rm cut} \sim 2\,\mathrm{GeV}$ and neglects *both* $\sigma^{\rm fpc}(q_T^{\rm cut}), \sigma^{\rm nons}(q_T^{\rm cut}) \approx 0$
- This is a catastrophic approximation even at α_s^2, and definitely at α_s^3
- Even without $\sigma^{\rm fpc}$ (e.g., without cuts), this is a bad approximation at α_s^3
  - $q_T^{\rm cut}$ variations only scan local maximum around $2\mathrm{GeV}$ …



Projection-to-Born method:

[used e.g. in Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

$$\frac{d\sigma}{dY} = A(0,Y)\,\frac{d\sigma_{\rm incl}}{dY} + \int_{\approx q_T^{\rm cut}} dq_T\,\big[A(q_T,Y) - A(0,Y)\big]\,W(q_T,Y)$$

- First term from analytic (threshold expansion of) inclusive rapidity spectrum
- Second term numerically from $H+1j$ MC, dominated by $\sigma^{\rm fpc}$ at small q_T

▸ Need to integrate down to $q_T^{\rm cut} \ll 0.1\,{\rm GeV}$ to get error below 10% of $\sigma_{\rm LO}^{\rm fid}$!
[See also Salam, Slade, 2106.08329 for an explicit/analytic estimate at double-logarithmic level]