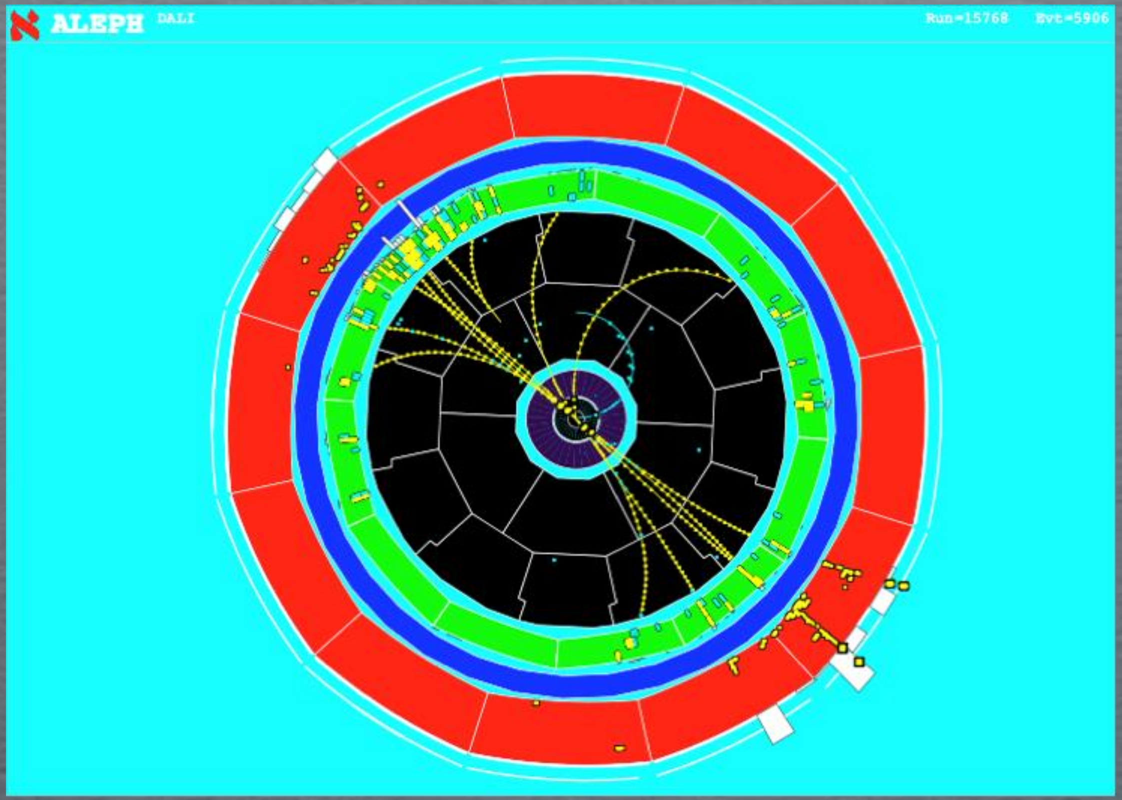
Resummation overview



Andrea Banfi

University of Sussex

Parton showers and resummation workshop — 25 May 2021

Outline

- General considerations: a personal perspective
- The precision frontier
- Spin correlations
- Resummation mode for public codes

Important progress that I will not cover here

- Resummation for jet substructure
- Next-to-leading power

A HUGE THANK YOU …

… to the founding fathers of the field…

Gribov, Lipatov,
Altarelli, Parisi,
Petronzio,
Collins, Soper,
Sterman,
Dokshitzer,
Webber,
Mueller,
Bassetto,
Ciafaloni,
Marchesini,
Catani,
Neubert,
Nason,
Seymour, …

A HUGE THANK YOU …

… to the second generation …

Salam,
Dasgupta,
Grazzini,
Magnea,
Laenen,
Kidonakis,
Oderda,
Stewart, Bauer,
Fleming, Pirjol,
Cacciari, AB,
Zanderighi,
Gardi, Berger,
Becher,
Schwartz,
Bozzi, …

A huge thank you …

… and to the new generations

Monni, Marzani, Larkoski, Thaler, Tackmann, Soyez, Moult, Neill, Ferrera, Re, Torrielli, Zhu, Rottoli, Vernazza, Dreyer, and many others ….

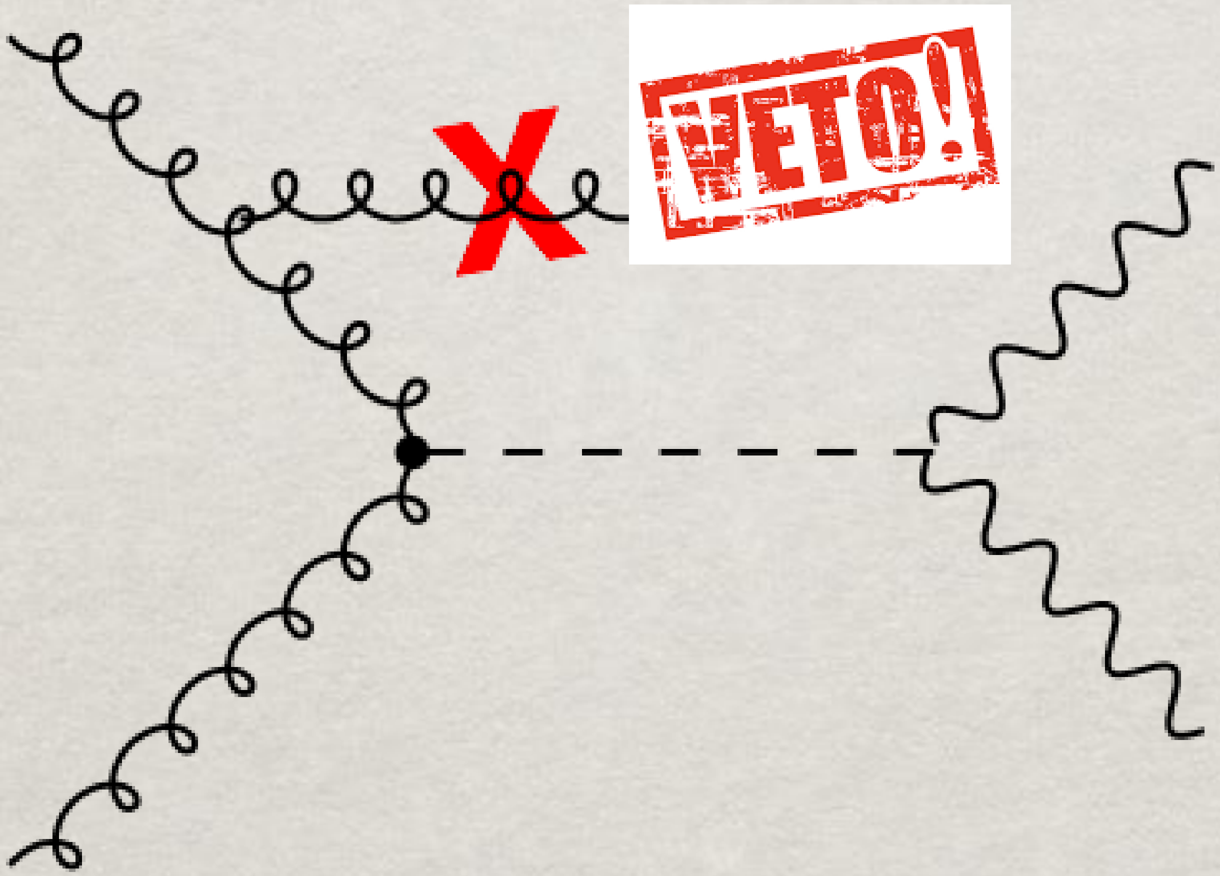
General considerations

Two-scale problems

- Resummation is all about taming two-scale problems, where an object characterised by a “hard scale” Q is accompanied by softer objects at a scale $Q_0 \ll Q$

Two-scale problems

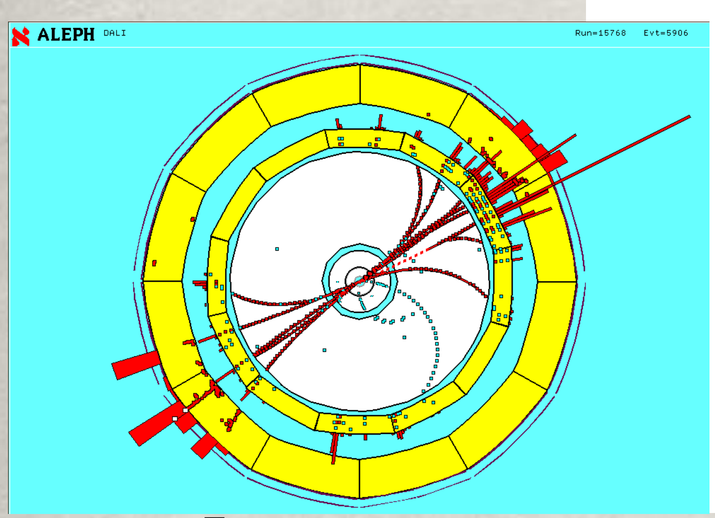
- Example of two-scale problem: an object with a large invariant mass Q accompanied by jet with low transverse momenta $Q_0 \ll Q$

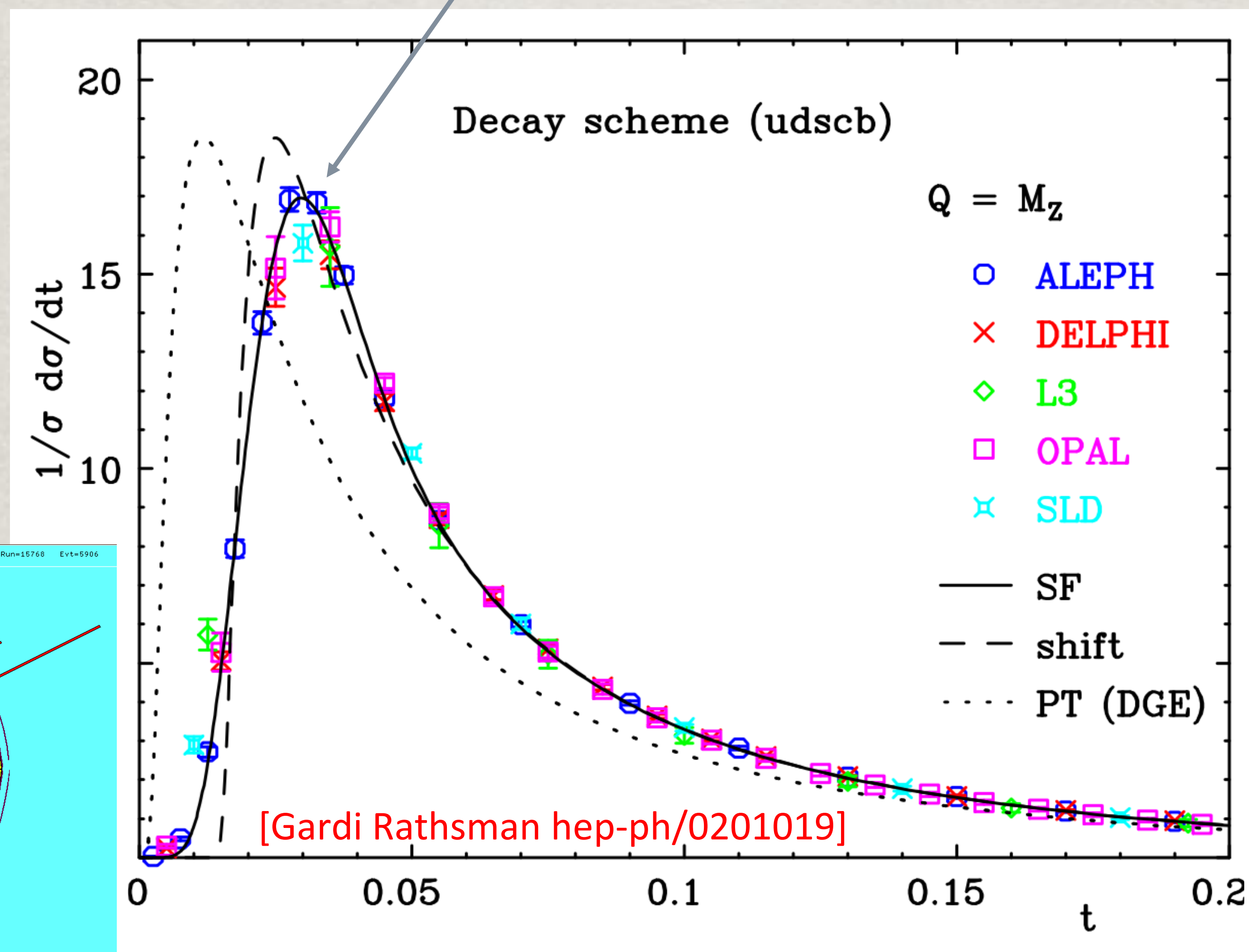


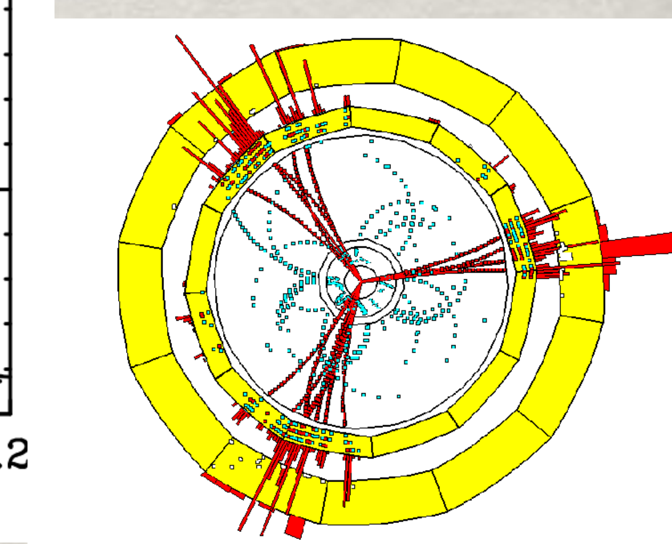
- In QCD, large logarithms such $\ln(Q/Q_0)$ appear whenever the phase space for the emission of soft and/or collinear gluons is restricted

Where do we resum?

- The bulk of events lies in the region $\alpha_s \ln(Q/Q_0) \sim 1$







$$\sim \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots\right]$$

$$\sim \underbrace{\alpha_s}_{\mathrm{LO}} + \underbrace{\alpha_s^2}_{\mathrm{NLO}} + \underbrace{\alpha_s^3}_{\mathrm{NNLO}} \ldots$$

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with $L \equiv \ln(Q/Q_0)$

$$\Sigma(Q, Q_0) \sim e^{\underbrace{Lg_1(\alpha_s L)}_{\text{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}} + \dots \right)$$

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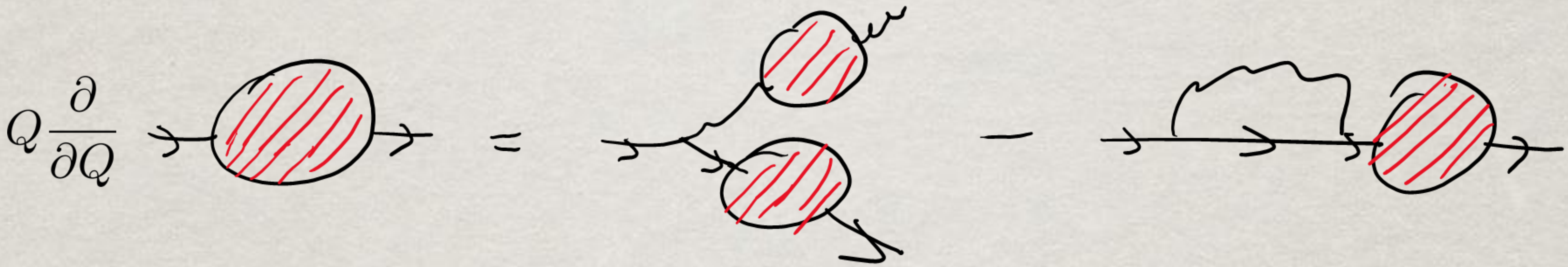
$$\Sigma(Q,Q_0) \sim e^{\overbrace{Lg_1(\alpha_s L)}^{\text{SLL}}} \times \left(\overset{1}{\underbrace{G_2(\alpha_s L)}_{\text{NLL}}} \overset{+}{+} \overset{\alpha_s}{\underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}}} \overset{+\dots}{+\dots} \right)$$

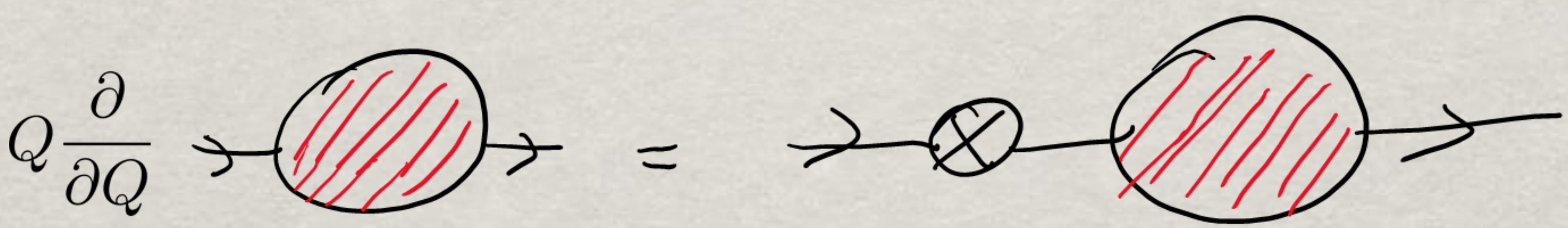
- This representation is independent of the formalism used for the resummation, so it can be used to compare different resummed predictions

QCD DYNAMICS IS NON-LINEAR

- Main problem of resummation: gluons and quarks radiate in the same fashion $\Longrightarrow$ QCD dynamics intrinsically non-linear



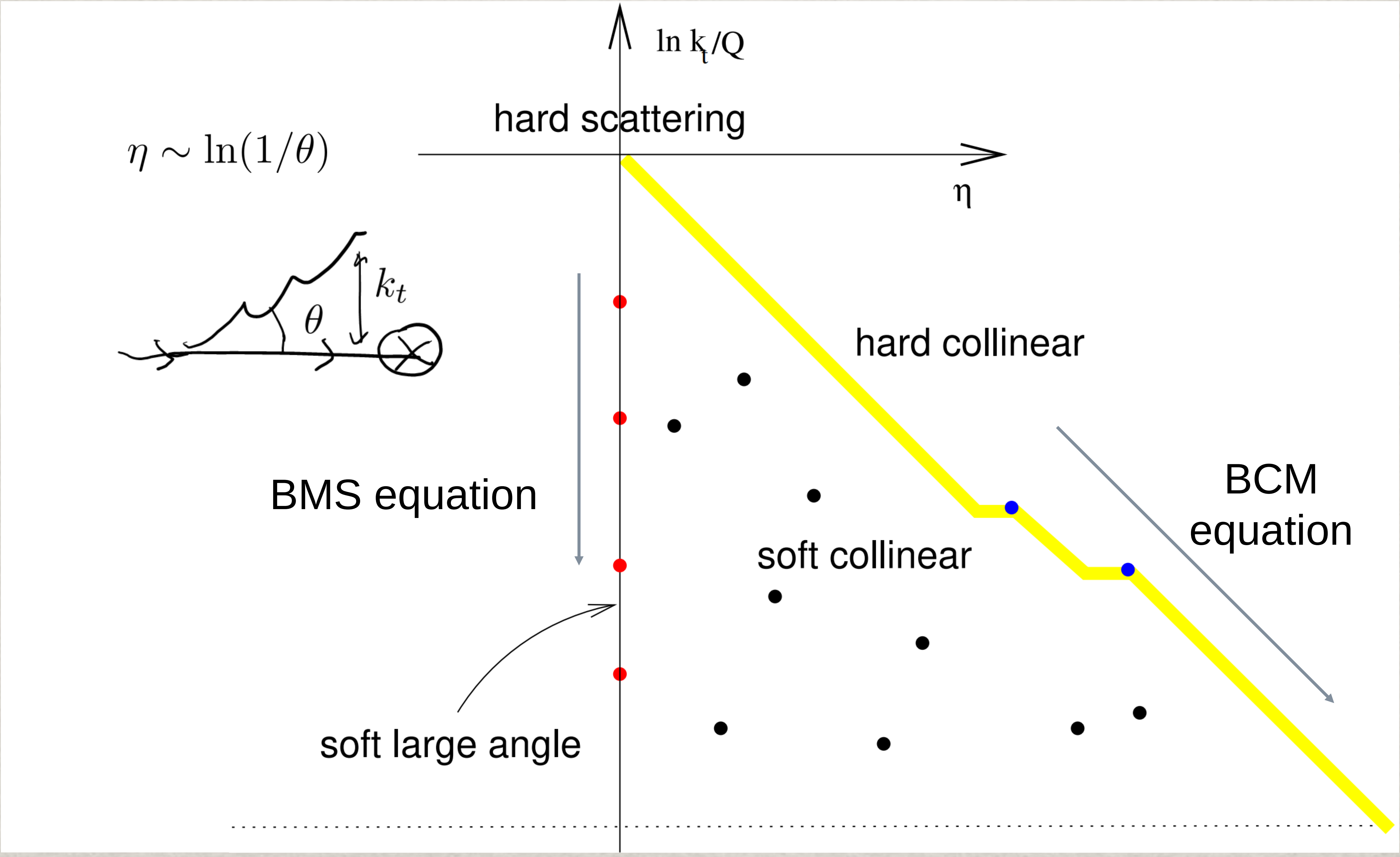
- The key feature of the most precise resummations is the possibility of following only one of the two branches of the splitting



- The main result of this operation is a set of linear RG equations

A Lund-plane overview

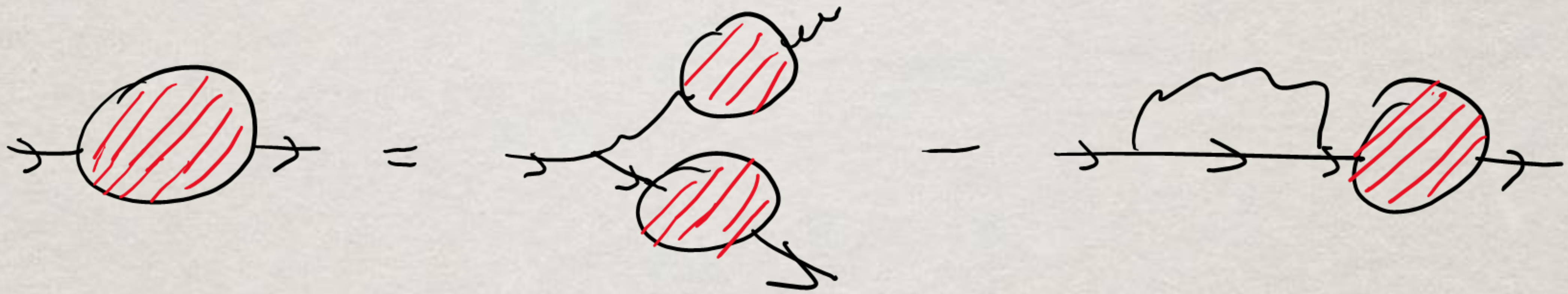
- Lund-plane representation of emissions from one incoming leg



BMS and BCM equations

- BMS and BCM non-linear equations give the evolution of single-logarithmic observables as functions of energy or angle

$$Q\frac{\partial}{\partial Q}$$

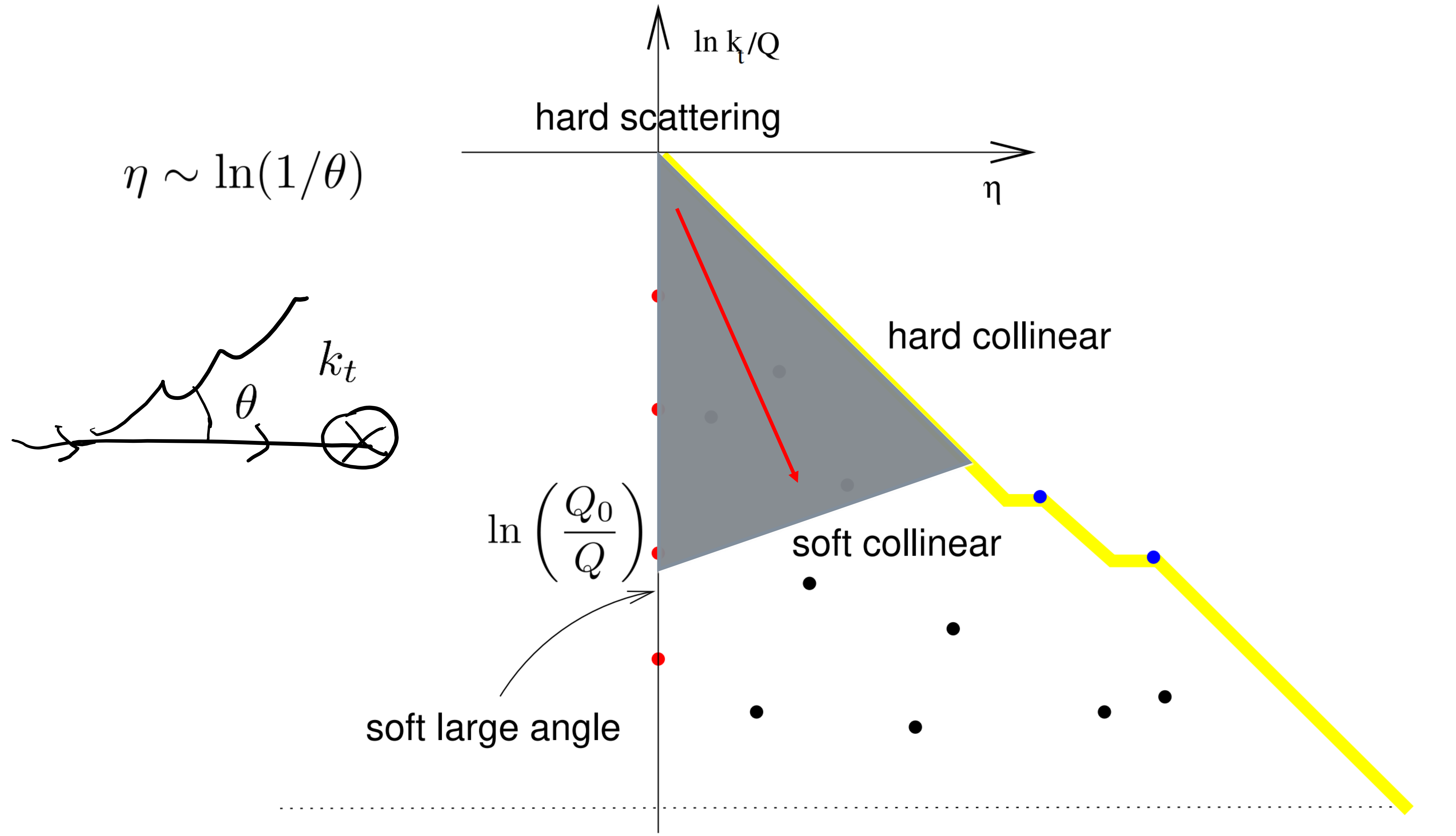


- Q = energy $\Longrightarrow$ BMS equation
  [AB Marchesini Smye hep-ph/0206076]
- Main application: resummation of non-global logarithms
- Linearisation $\Longrightarrow$ BFKL equation

- Q = angle $\Longrightarrow$ BCM equation
  [Bassetto Ciafaloni Marchesini PREP 100 (1983) 201-272]
- Main application: resummation of logarithms of jet radius
  [Dasgupta Dreyer Salam Soyez 1411.5182]
- Linearisation $\Longrightarrow$ DGLAP equation

Observable constraints

- Two-scale observables veto emissions in a portion of the Lund plane



$$\Sigma(Q, Q_0) \sim \underbrace{\mathcal{S}(Q_0)}_{\text{soft function}} \otimes \underbrace{\mathcal{H}(Q)}_{\text{hard function}} \otimes \underbrace{\mathcal{J}(Q_0)}_{\text{jet/beam function}}$$

Factorisation is the key

- Resummation would never be possible if we could not separate process-dependent hard matrix element from universal soft-collinear dynamics

[Collins Soper Sterman hep-ph/0409313]

- Factorisation is enough to write general resummation formulae for suitable observables in QCD [AB Salam Zanderighi hep-ph/0407286]

[AB El-Menoufi Monni 1807.11487]

- Using strategy of regions, factorisation formulae can include observable constraints $\Longrightarrow$ SCET factorisation theorems

[Bauer Fleming Pirjol Stewart hep-ph/0011336]

[Becher Schwartz 0803.0342]

[Becher Neubert 1007.4005]

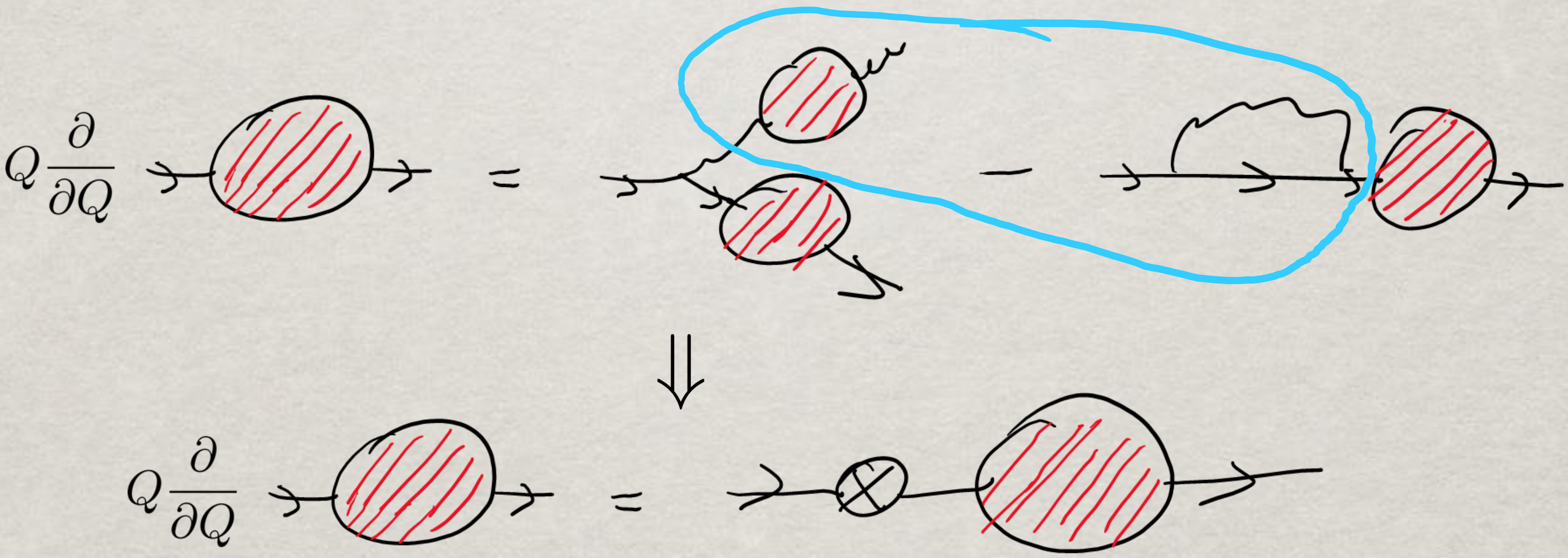
- Strategy of regions is the only way forward so far when no general principles such as gauge invariance can be invoked, e.g. quark-mass logarithms in loop amplitudes [Liu Mecaj Neubert Wang 2009.06779]

Inclusivity is even better

- Inclusive observables put indirect constraints on all emissions, e.g.

$$\vec{p}_{T,H} = \sum_i \vec{k}_{ti}$$

- In such cases one can integrate over all secondary splittings

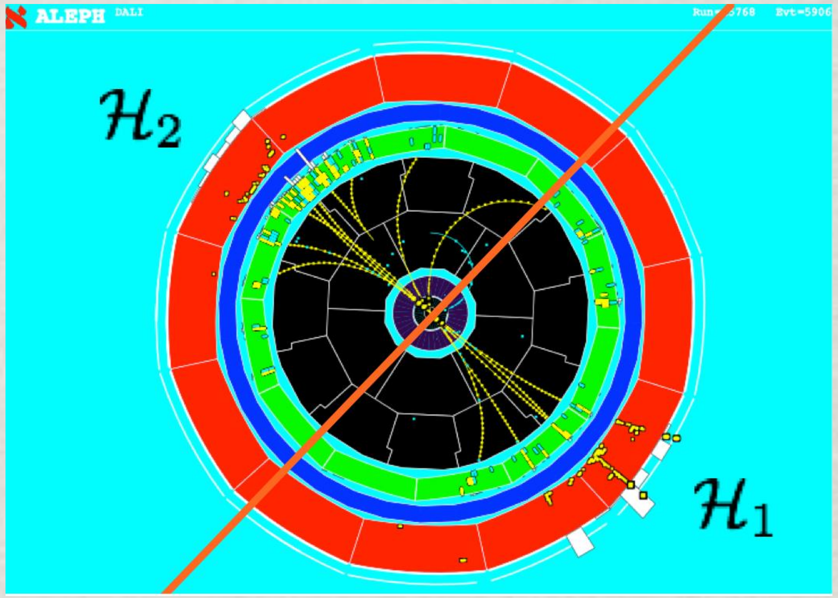


- Resummation of logarithms of an inclusive observable can be reduced to the problem of calculating (or borrowing) suitable anomalous dimensions

How inclusive can you be?

- Event-shape distributions and jet rates involve direct measurements of emissions, and are therefore quasi-inclusive, e.g.

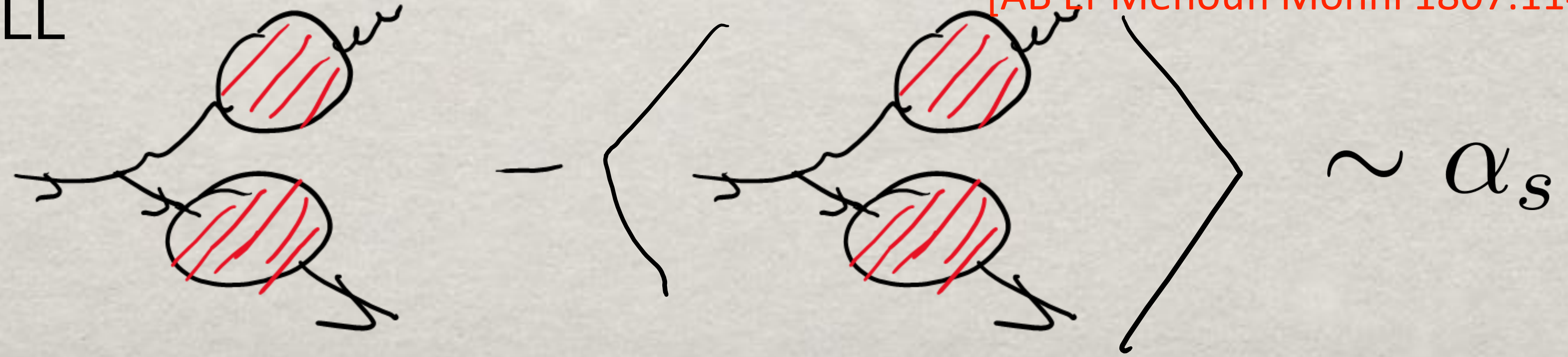
$$1 - T = \frac{M_1^2}{Q} + \frac{M_2^2}{Q}$$



- At NLL, there are observables for which one can integrate inclusively over secondary splittings. Such observables are called rIRC safe

- For rIRC safe observables, the difference from the inclusive approximation is at most NNLL

[AB El-Menoufi Monni 1807.11487]



Generating functionals

- Besides rIRC safety, the thrust is also additive

$$V(k_1, \dots, k_n) = V(k_1) + V(k_2) + \dots + V(k_n)$$

- Additive observables (e.g. angularities, moments of the EEC) enjoy remarkable factorisation properties [Becher Schwartz 0803.0342]

$$\Sigma(1 - T < \tau) = \int \frac{d\nu}{2\pi i \nu} e^{\nu\tau} \mathcal{J}[u_1]\, \mathcal{J}[u_2]\, S[u_s]$$

$$u_1(k) = e^{-\nu k^-} \qquad u_2(k) = e^{-\nu k^+} \qquad u_s(k) = e^{-\nu \omega}$$

- Since factorisation formulae in SCET are observable-dependent, resummation of non-factorizable observables is technically involved [Bauer Monni 1906.11258]

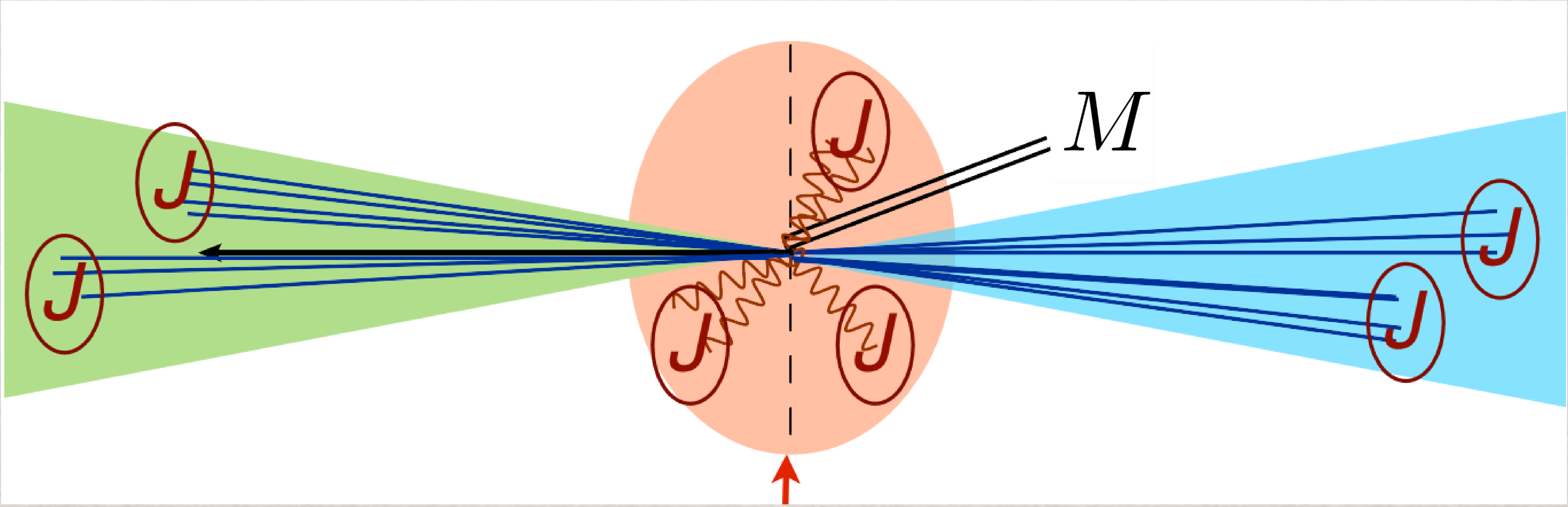
- The resummation of additive observables has the form of a generating functional. Can we obtain the resummation of non-additive observables by performing suitable functional differentiations?

The precision frontier

Transverse momentum distributions

- The distribution in the transverse momentum of a colour singlet (e.g. Higgs, Z boson) is a fully inclusive observable and admits an all order factorisation

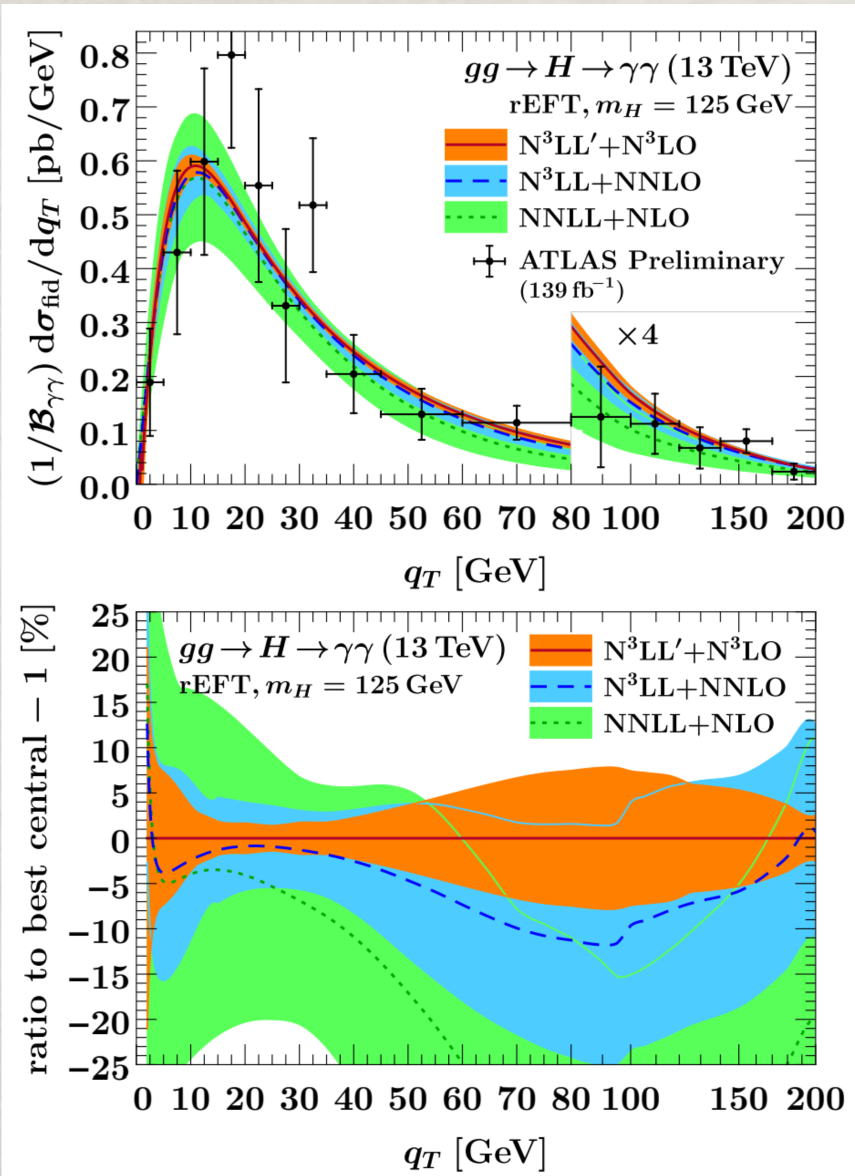
$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J) \otimes \mathcal{H}(M, \mu_H) \otimes \mathcal{B}(\mu_J)$$



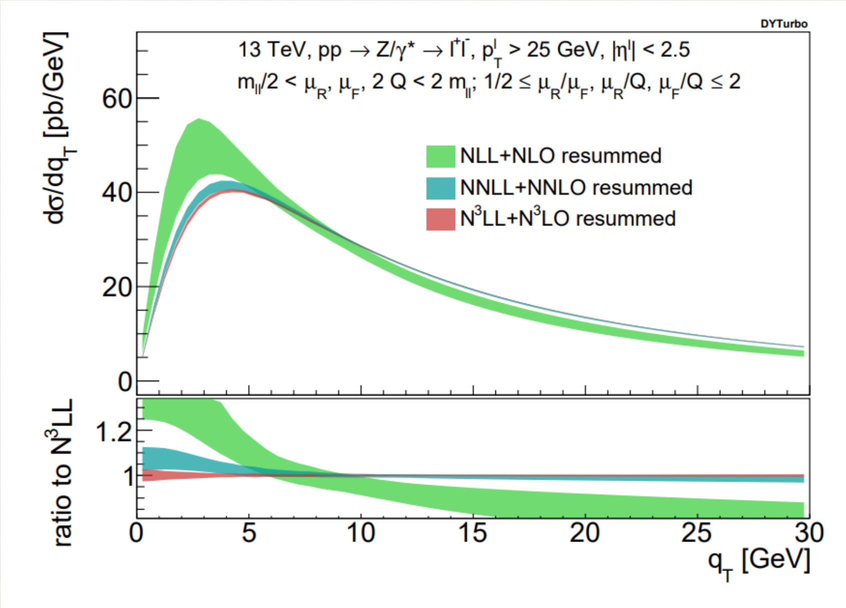
- Calculation of relevant anomalous dimensions makes it possible to achieve a remarkable N^{3}LL accuracy (times a constant at order α_s^3, a.k.a N^{3}LL')

B-space resummation

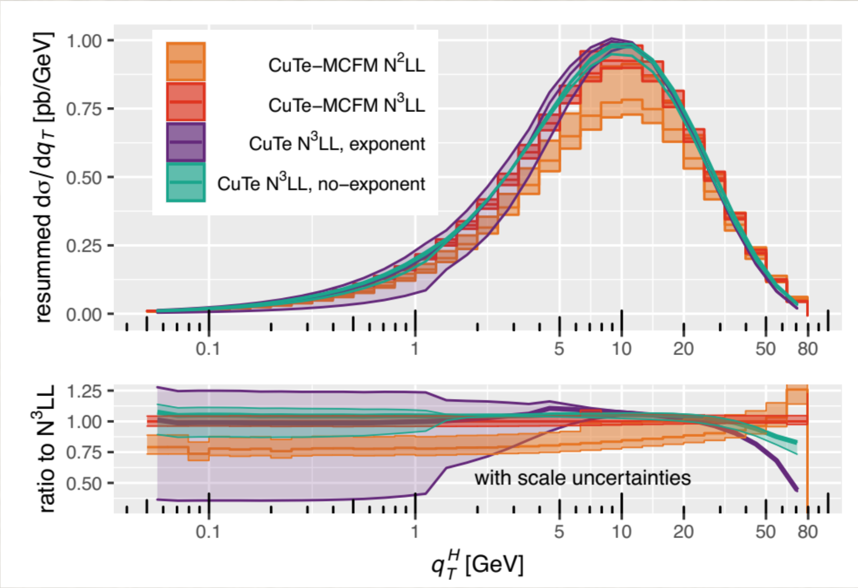
- Three calculations perform the resummation in impact-parameter or b-space, i.e. for the Fourier transform of $d\sigma/dp_T$



[Billis et al 2102.08039]



[Camarda et al 2103.04974]

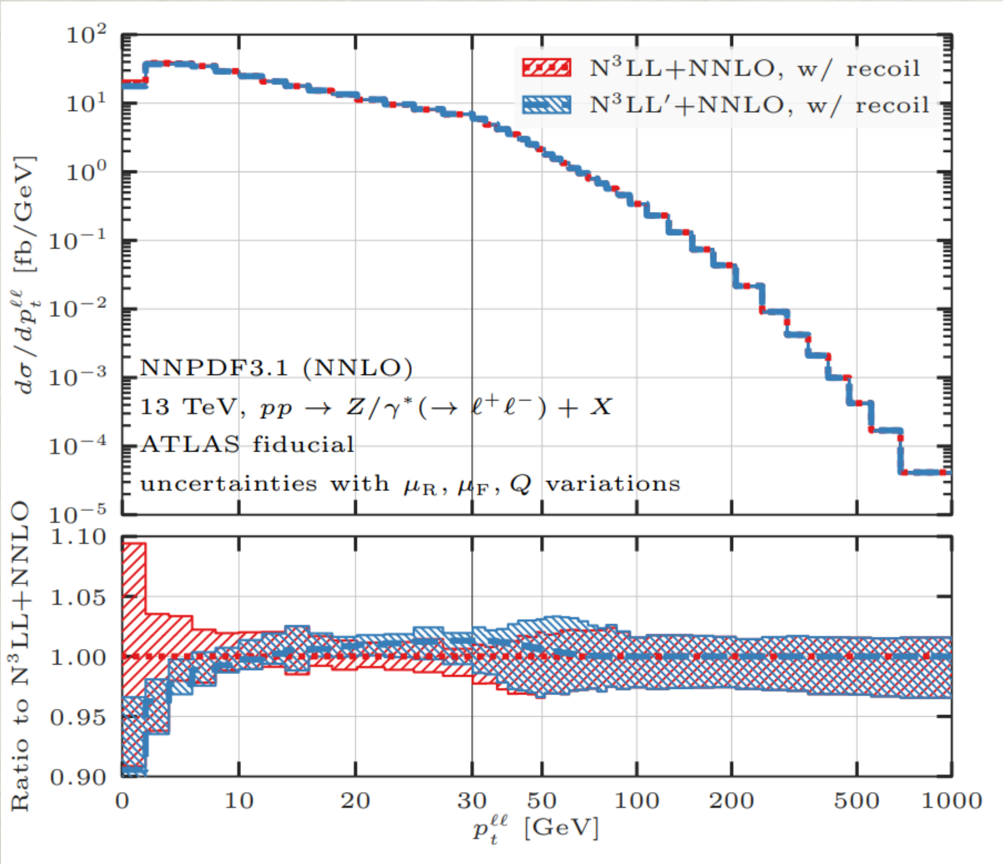


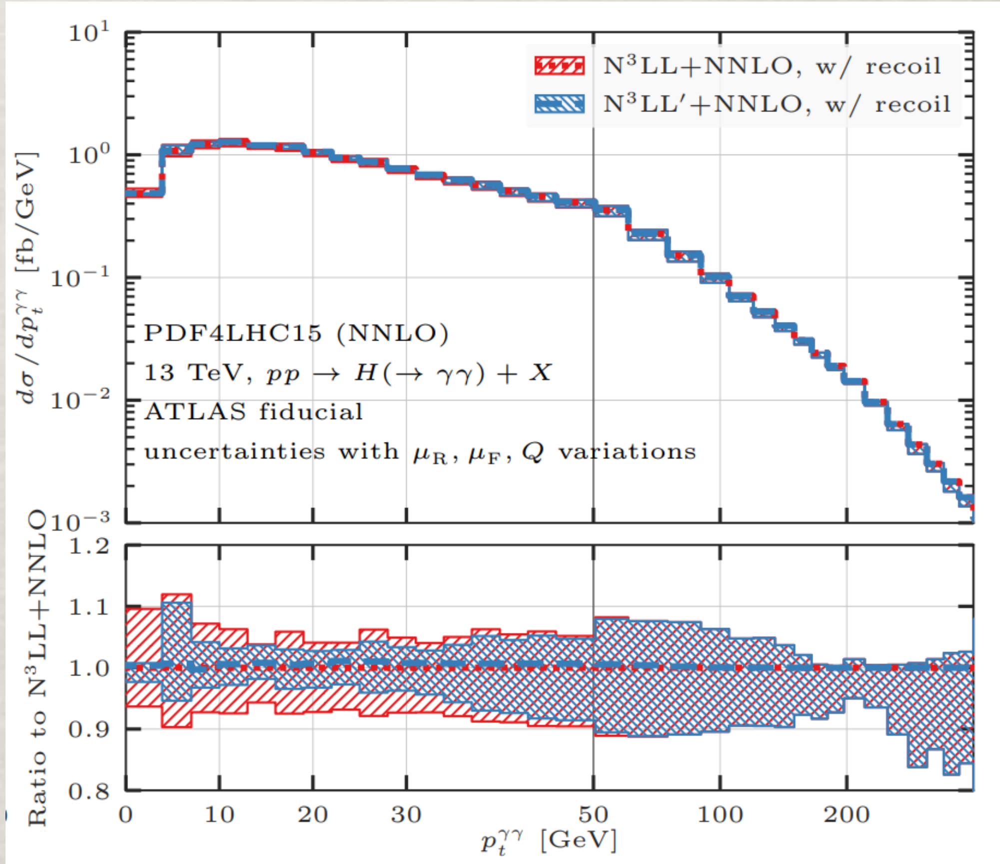
[Becher Neumann 2009.11437]

Direct space resummation

- In the RadISH formalism, one resums $\ln(k_{t1}/M)$ where k_{t1} is the transverse momentum of the leading parton [Monni Re Torrielli 1604.02191]

- The resummation in $\ln(k_{t1}/M)$ is performed with the ARES philosophy and then the result is then integrated over k_{t1} and binned in q_T



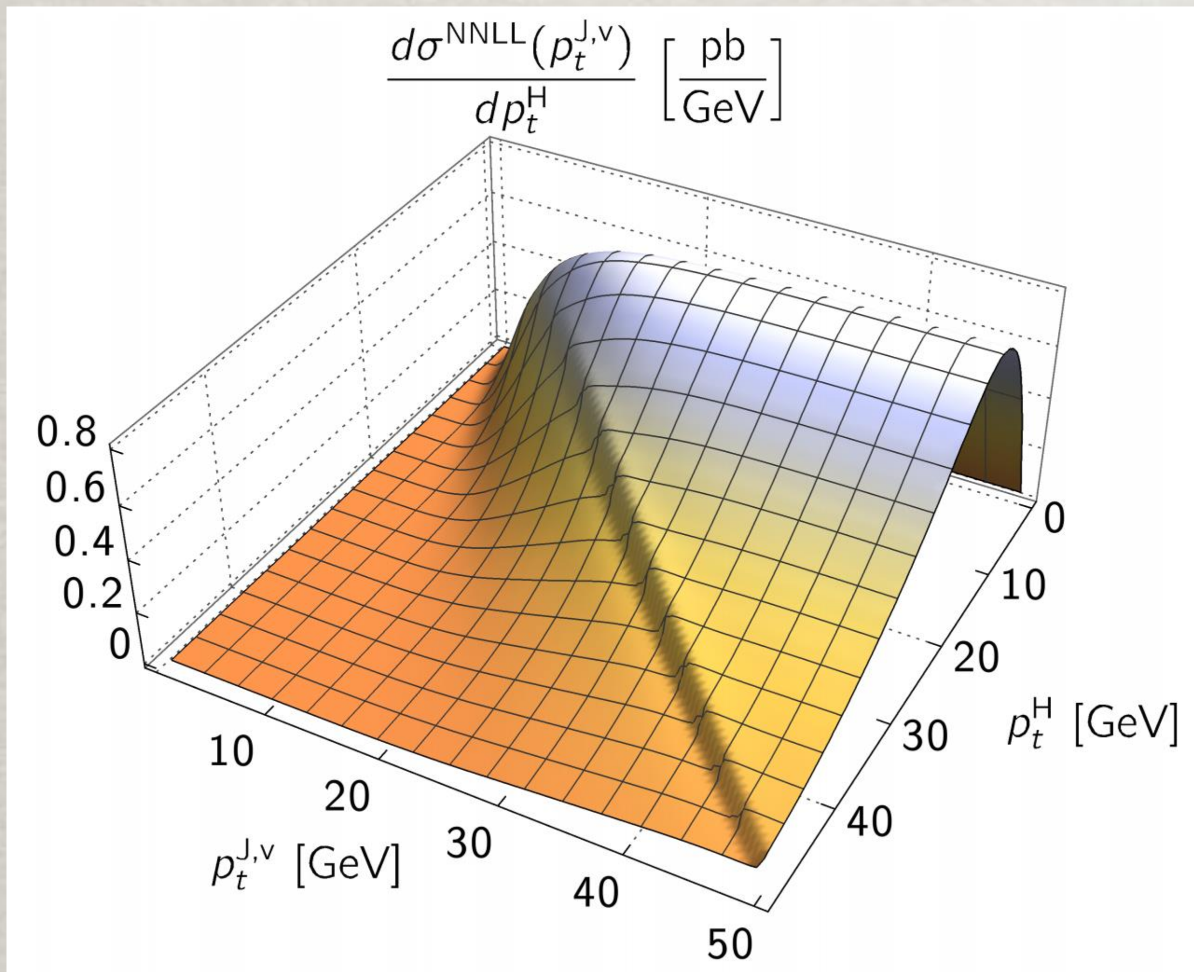


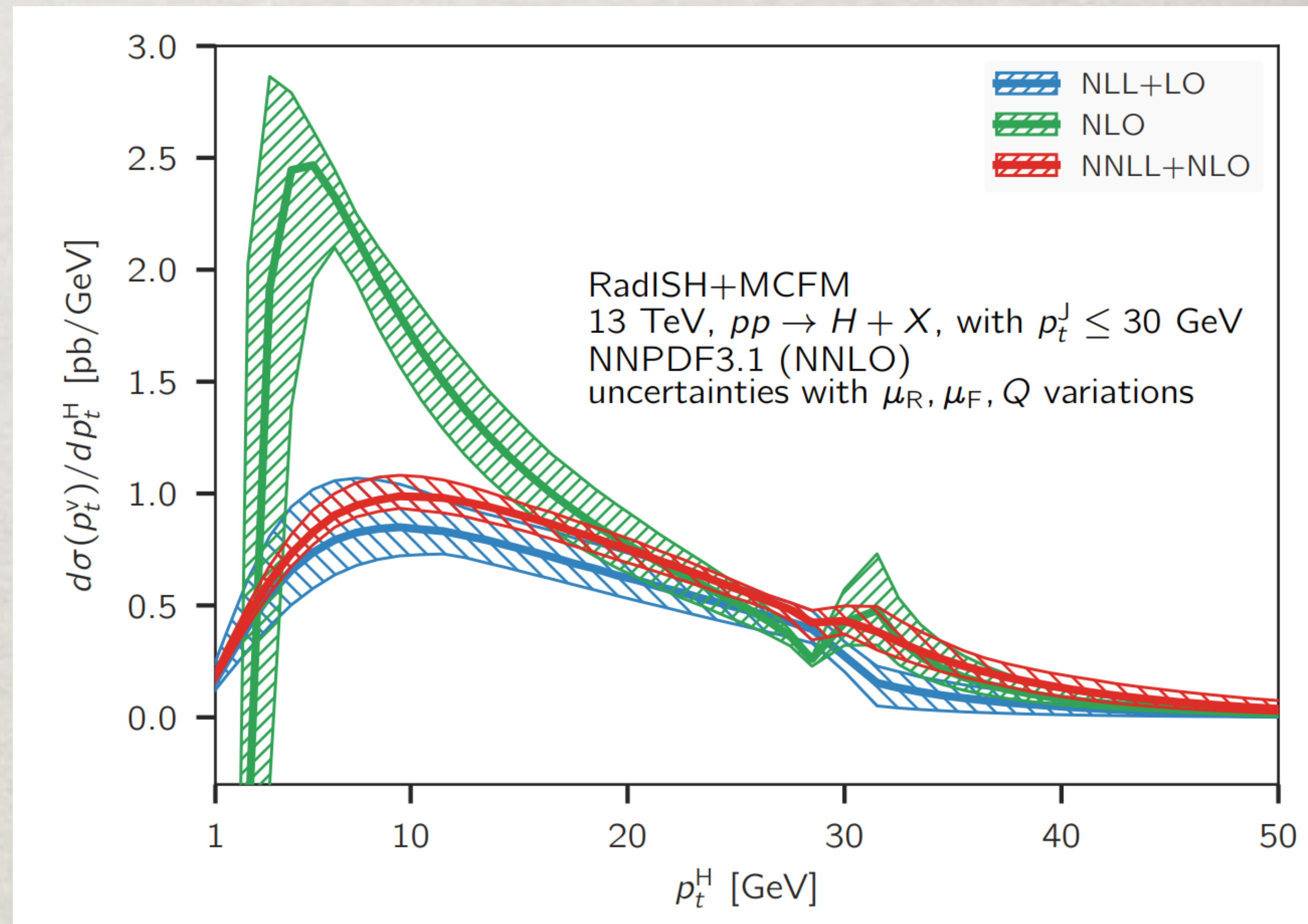
[Re Rottoli Torrielli 2104.07509]

- Uncertainties are reduced if one considers formally power-suppressed terms where the colour singlet acquires non-zero q_T via resummation

ADDING CONSTRAINTS

- Using the same set of soft-collinear emissions used for the q_T distribution, it is possible to add a constraint $p_{t,\mathrm{jet}} < p_{t,\mathrm{veto}}$ [Monni Rottoli Torrielli 1909.04704]





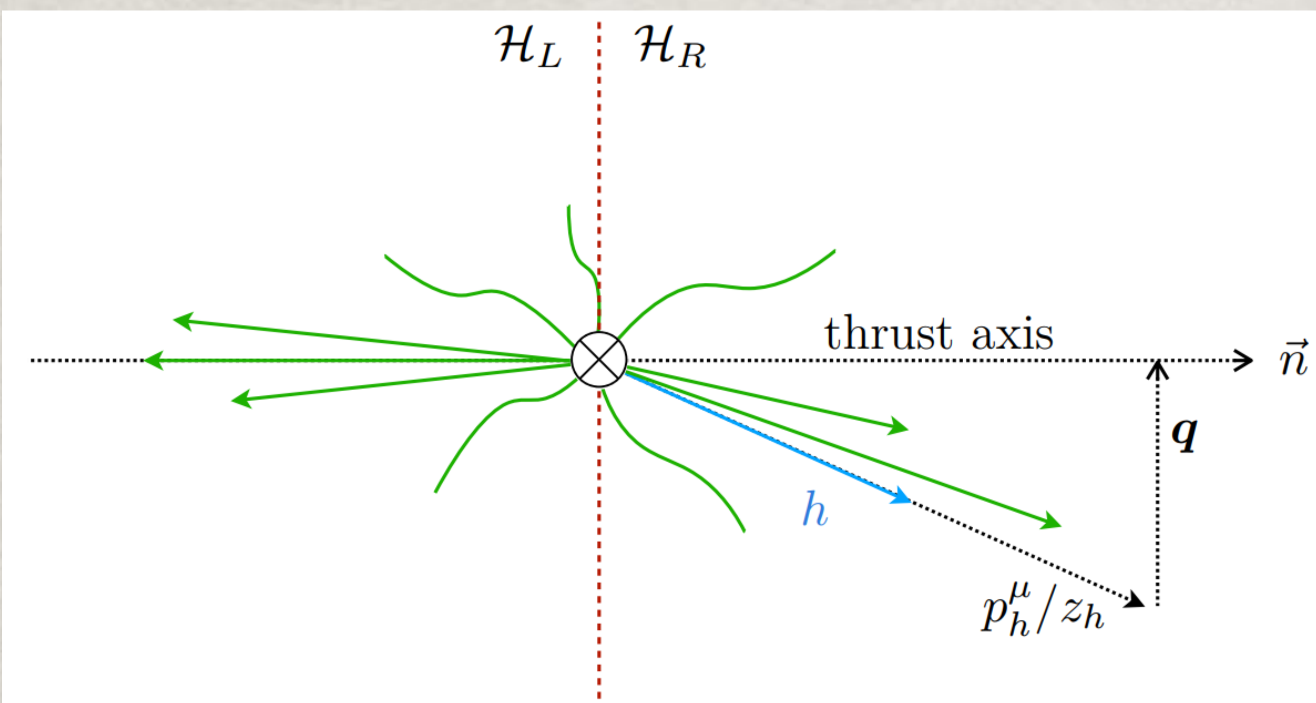
- The resummation accuracy here is limited to NNLL because of missing calculation of jet-veto effects

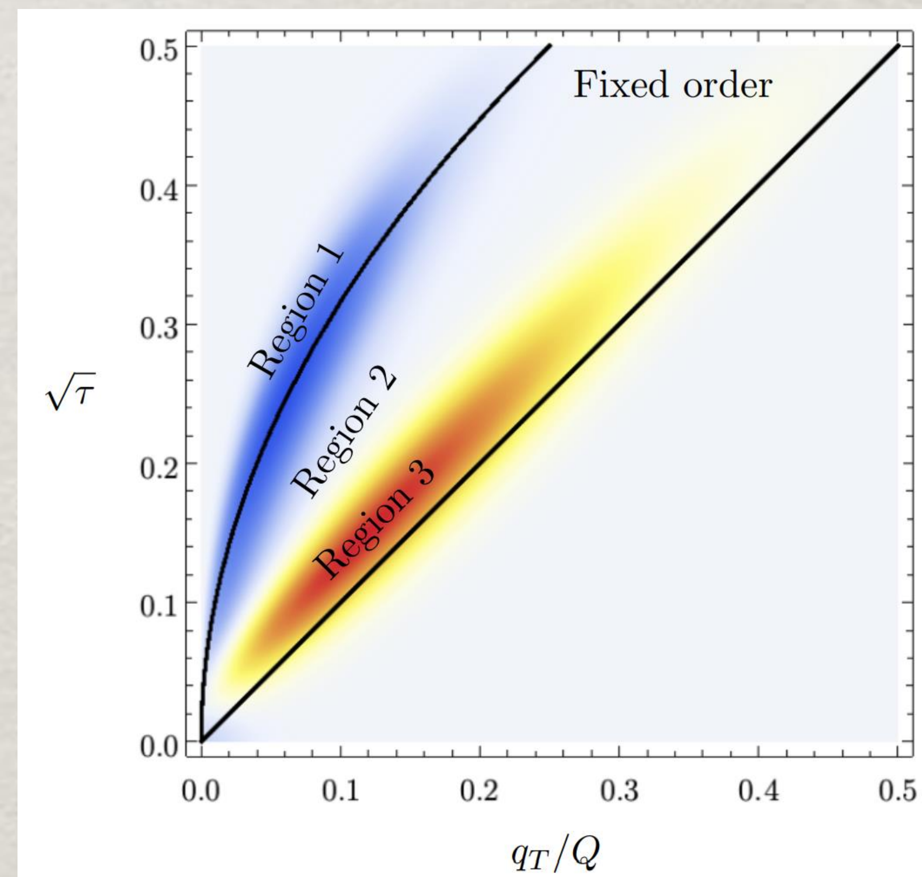
ADDING CONSTRAINTS

- An observable similar to q_T in e^+e^- is the transverse momentum distribution of hadrons with respect to the thrust axis

- Within SCET, a simultaneous resummation of TMD fragmentation function and thrust distribution has been achieved at NNLL accuracy

[Makris Ringer Waalewijn 2009.11871]



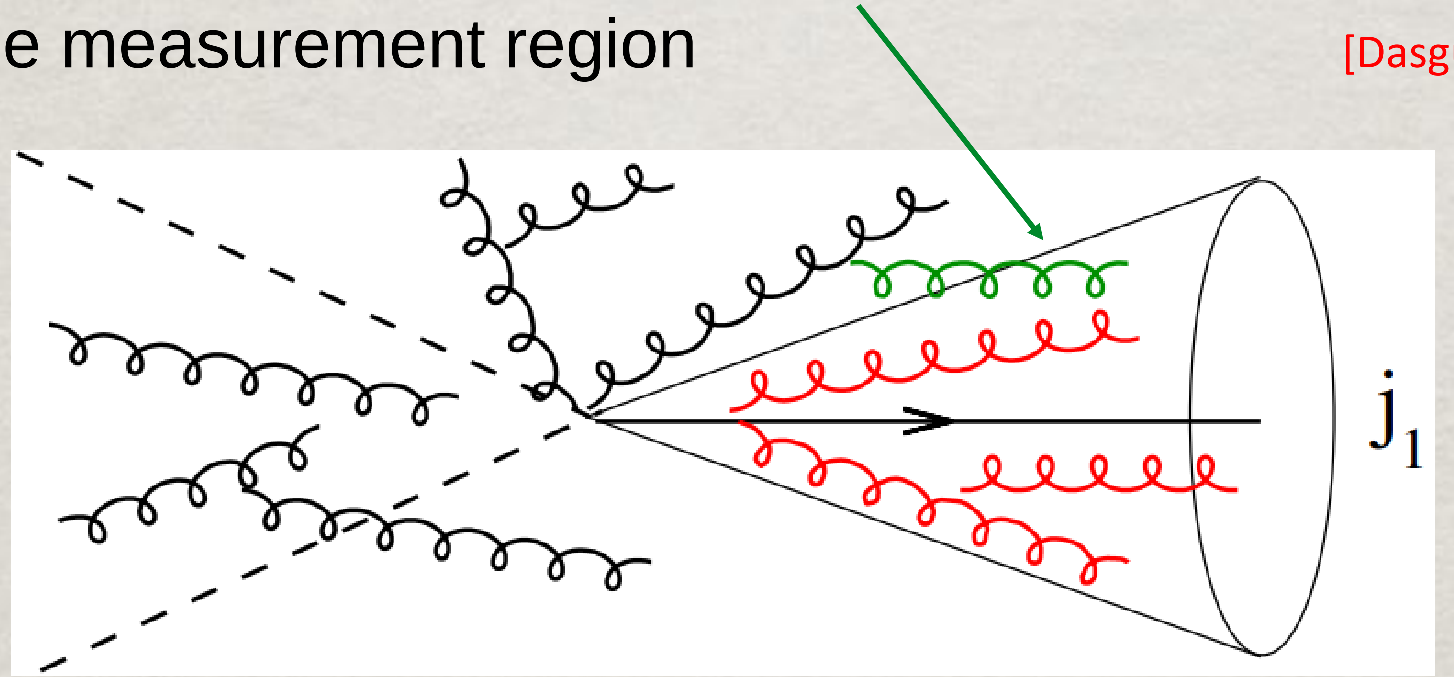


- For $q_T/Q \ll \tau$ one needs to consider non-global logarithms

Non-global logarithms

- Non-global logarithms arise whenever measurements are restricted to limited regions of phase space, e.g. single-jet mass distribution
- They originate when softest emission in a correlated cascade of soft gluons enters the measurement region

[Dasgupta Salam hep-ph/0104277]

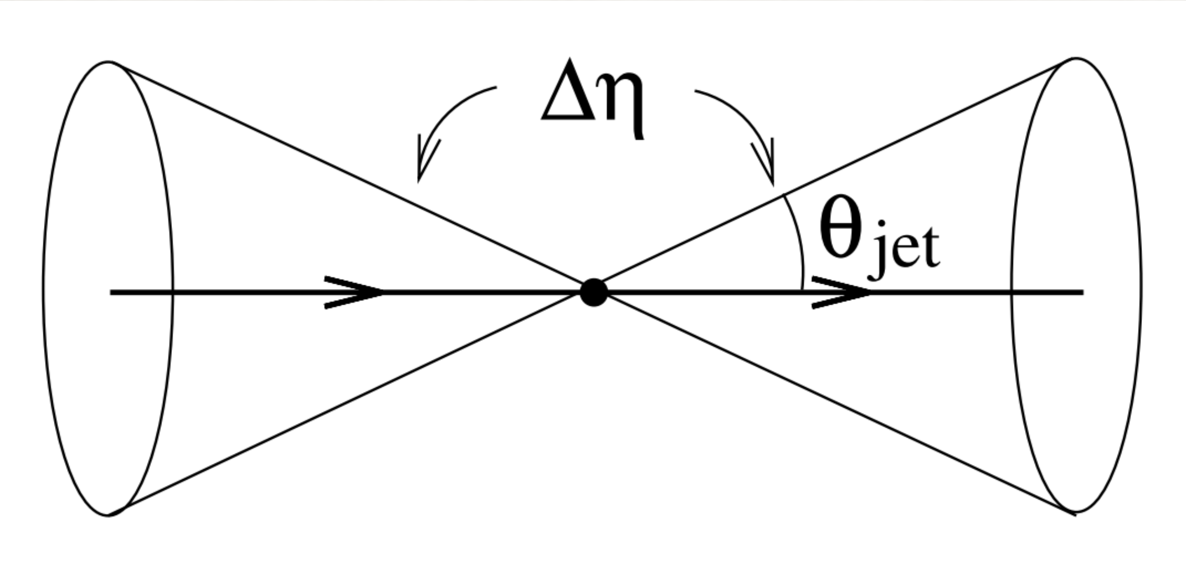


- Non-global logarithms arise from soft emissions at large angles, hence leading logarithms are single logarithms
- At LL accuracy, and in the large-N_c limit, they are resummed via the non-linear BMS equation

[AB Marchesini Smye hep-ph/0206076]

Leading non-global logarithms

- Case study: veto emissions inside a rapidity slice



$$\Sigma(Q, Q_0) = \text{Prob}\left[\sum_{|\eta_i|<\Delta\eta} E_i < Q_0\right]$$

- In SCET, one can separate soft and hard modes

[Becher Neubert 1605.02737]

$$\Sigma(Q, Q_0) = \sum_{n=2}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

$$\mu \frac{d}{d\mu} \mathcal{H}_m(Q, \mu) = -\sum_{l=2}^{m} \mathcal{H}_l(Q, \mu)\, \Gamma_{lm}$$

non-linear dynamics

$$\Gamma^{(1)} = \begin{pmatrix} V_2 & R_2 & 0 & 0 & \dots \\ 0 & V_3 & R_3 & 0 & \dots \\ 0 & 0 & V_4 & R_4 & \dots \\ 0 & 0 & 0 & V_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

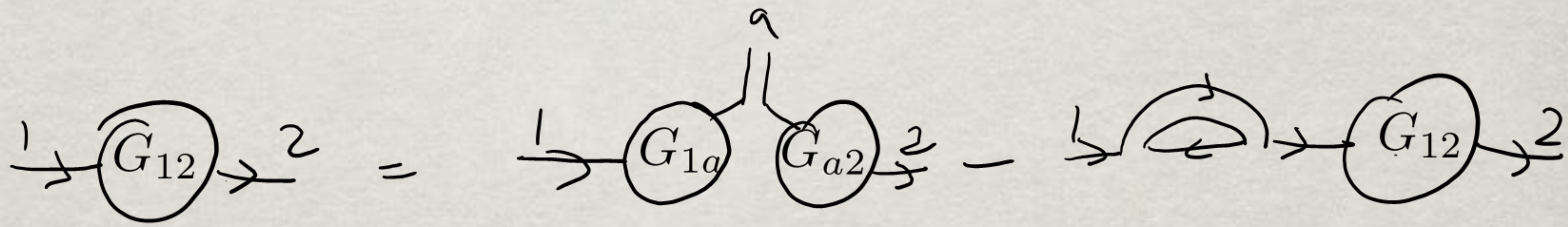
NL non-global logarithms

- In the large-Nc limit, one can construct a soft generating functional

$$\Sigma(Q, Q_0) = \int \frac{d\nu}{2\pi i\nu} e^{\nu Q_0} G_{12}[Q, u] \qquad u(k) = e^{-\nu E}\Theta_{\text{in}}(k) + \Theta_{\text{out}}(k)$$

- At LL accuracy, this functional satisfies BMS equation

$Q\frac{\partial}{\partial Q}$

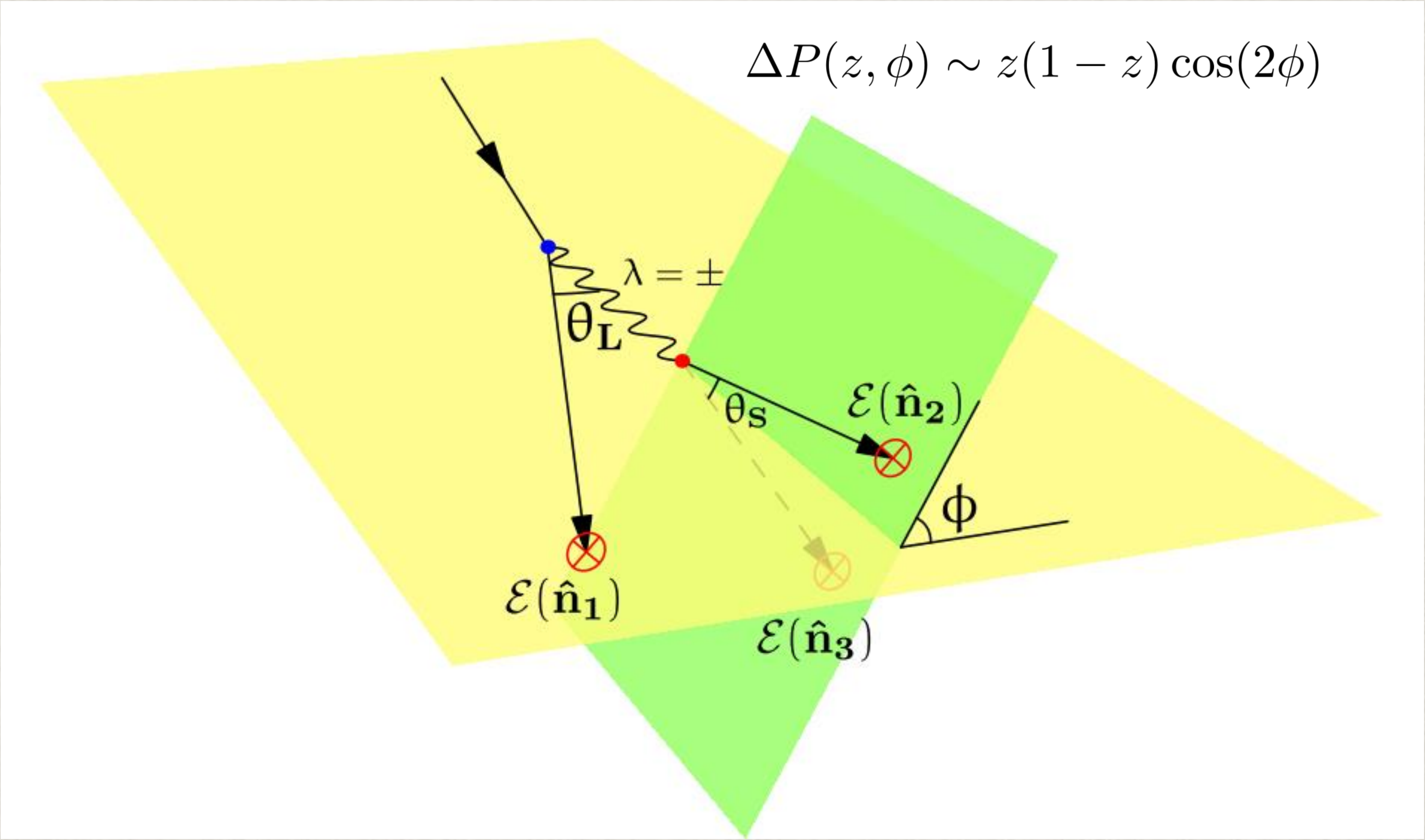


- The kernel of BMS equation has been recently improved at NLL
- Key strategy: start with real radiation and include virtual correction via unitarity [Banfi Dreyer Monni 2104.06416]
- Unlike at LL, at NLL different evolution variables lead to different equations, equivalent up to subleading terms

Spin correlations

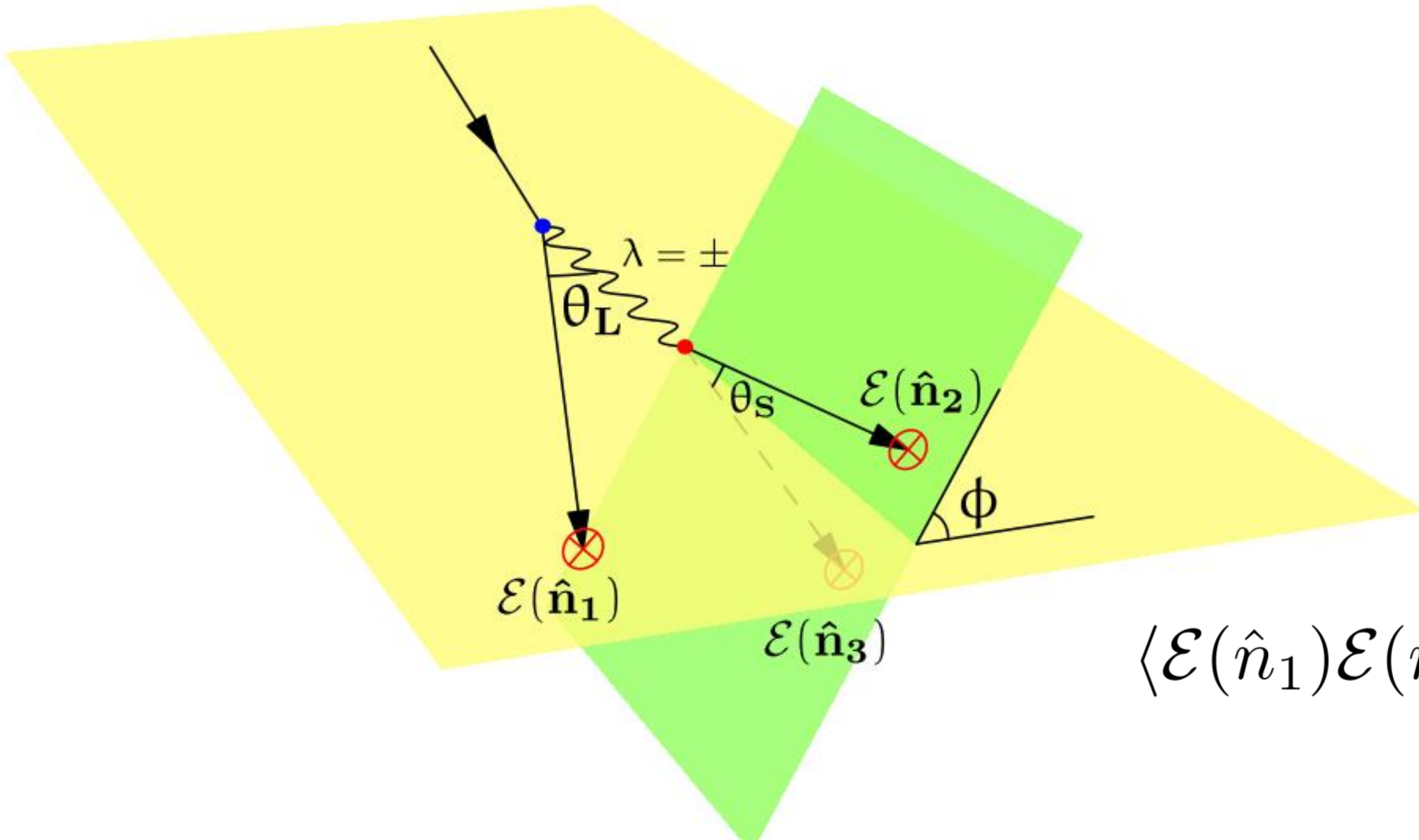
Gluon spin correlations

- Since gluons have spin-1, due to spin correlations their splittings depend on the azimuth with respect to a suitably defined plane



Triple-energy correlations

- Such spin correlations can be probed at single-logarithmic accuracy via triple-energy correlations



$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3)\rangle \sim \frac{d\Sigma}{d\theta_L^2 d\theta_S^2 d\phi}$$

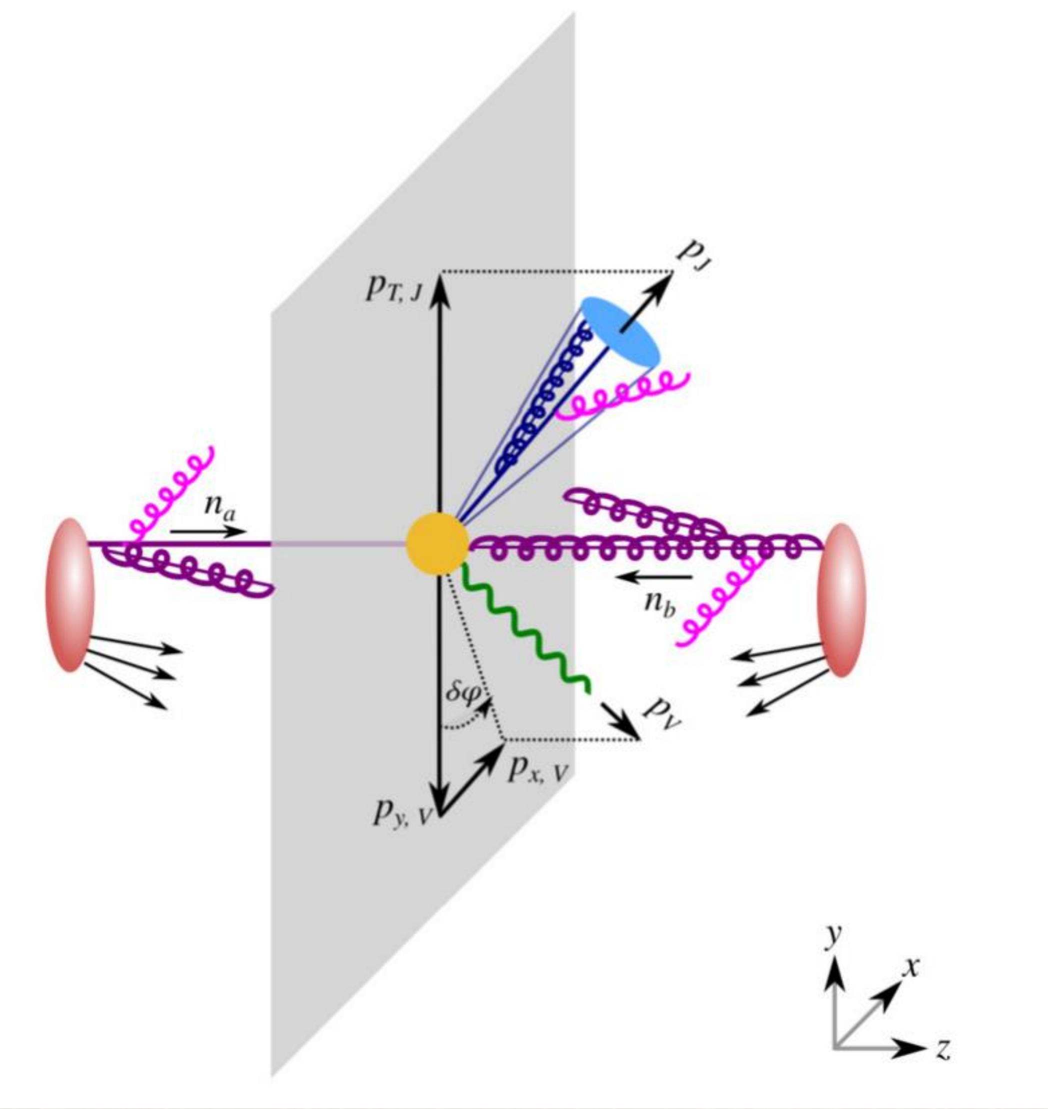
- Resummation of the triple-energy correlation is related to light-ray operators $\vec{\mathbb{O}}^{[J]}$ of twist J and their anomalous dimensions

[Chen Mout Zhu 2104.00009]

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = \frac{1}{(2\pi)^2}\frac{2}{\theta_L^2}\frac{2}{\theta_S^2}\vec{\mathcal{J}}\hat{C}(\phi)\left(\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\left(\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right)^{\frac{\hat{\gamma}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1)$$

Azimuthal correlations

- The plane of the splitting can be fixed by the beam and a hard particle, e.g. a vector boson recoiling against a jet



- WTA jet axis $\Longrightarrow$ no NGLs $\Longrightarrow$ SCET all-order factorisation formula

[Chien et al 2005.12279]

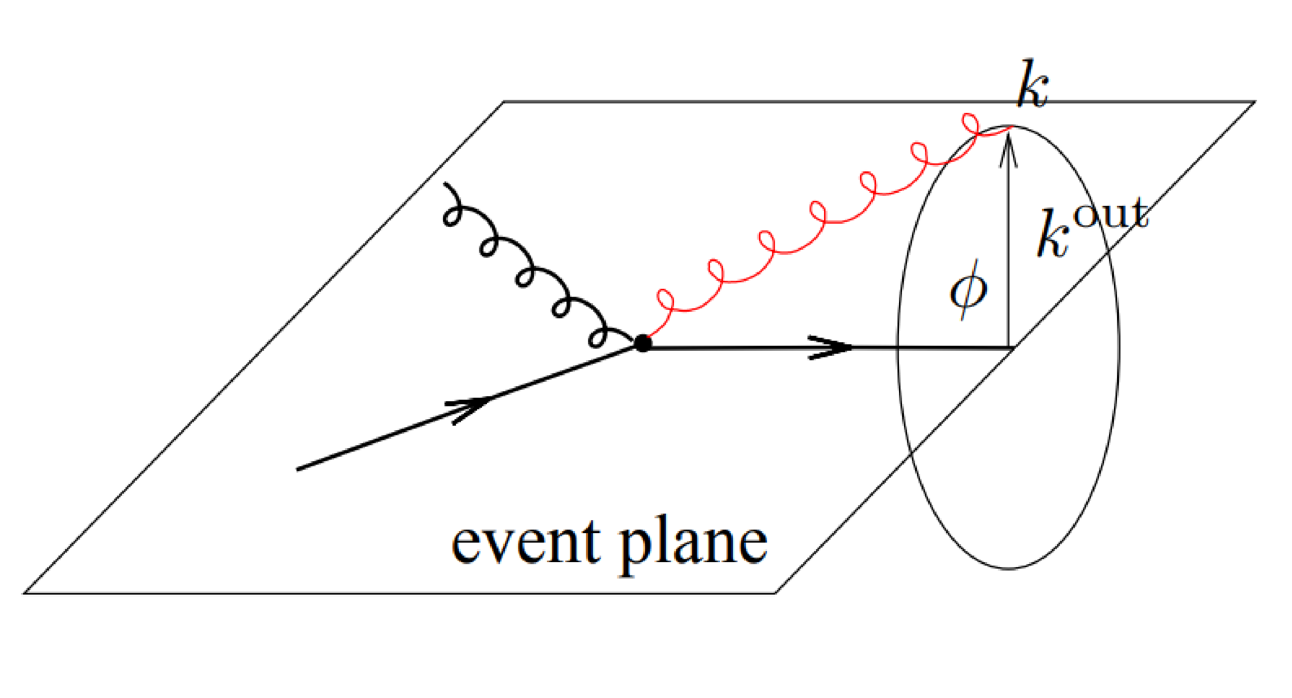
$$\frac{d\sigma}{dp_{x,V}\,dp_{T,J}\,dy_V\,d\eta_J}$$
$$= \int \frac{db_x}{2\pi} e^{ip_{x,V}b_x} \sum_{i,j,k} B_i(x_a,b_x)B_j(x_b,b_x)\times$$
$$\times S_{ijk}(b_x,\eta_J)\mathcal{H}_{ij\to Vk}(p_{T,V},\eta_V-\eta_J)\times$$
$$\times \mathcal{J}_k(b_x)$$

- Spin correlation effects occur at NNLL

- When the recoiling jet is a gluon jet, spin correlations give rise to a linearly polarised transverse momentum distribution $\mathcal{J}_g(b_\perp)$ in the final state

NEAR-TO–PLANAR SHAPES

- Out-of-plane event shapes, e.g. D-parameter, are also sensitive to spin correlations in gluon jets



$$D = \frac{27}{Q^3} \sum_{i<j<k} \frac{[\vec{p}_i \cdot (\vec{p}_j \times \vec{p}_k)]^2}{E_i E_j E_k}$$

- In the tree-jet region, spin correlations arise at NNLL

[Arpino AB El-Menoufi 1912.09341]

- In the two-jet region, when the hardest gluon is allowed to be soft and collinear, can spin correlation effects appear at NLL? [Larkoski Procita 1810.06563]

Implementation of resummation

Resummed vs fixed-order

- For resummations to have an impact on phenomenology, it is vital that predictions are available in the form of publicly available codes
- The great majority of resummed predictions are in the form of in-house programs, typically observable specific
- This is in contrast with fixed-order calculations, where a plethora of general tools is publicly available
  - NLO calculations are fully automated and implemented in general frameworks

    [MCFM https://mcfm.fnal.gov]

    [MADGRAPH_aMC@NLO http://madgraph.phys.ucl.ac.be/]

    [SHERPA https://sherpa-team.gitlab.io/]

    [HELAC https://helac-phegas.web.cern.ch/]
  - NNLO calculations are publicly available for a variety of 2->2 processes

    [MATRIX https://matrix.hepforge.org/]

    [NNLOJET https://nnlojet.hepforge.org/]
- Can we establish a common framework to implement resummed calculations?

NLL RESUMMATIONS

- NLL resummations for n hard emitting legs in any framework can be reduced to the CAESAR general formula [AB Salam Zanderighi hep-ph/0407286]

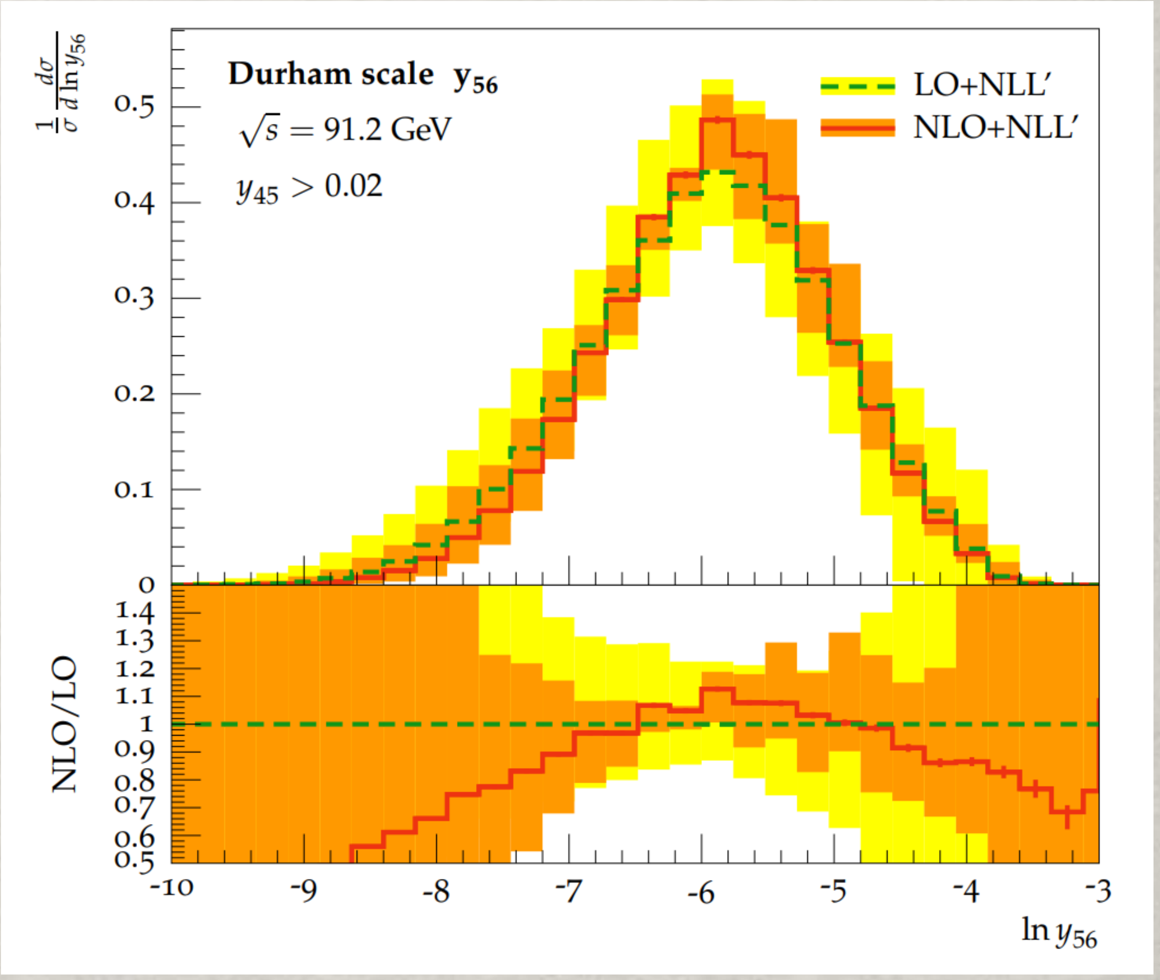
$$\Sigma(Q,Q_0) = e^{-R(Q,Q_0)} \prod_{i=1}^{n} J_i(Q/Q_0) \times \\ \times \frac{\langle \mathcal{B} | e^{-\Gamma^\dagger t(Q/Q_0)} e^{-\Gamma t(Q/Q_0)} | \mathcal{B} \rangle}{\langle \mathcal{B} | \mathcal{B} \rangle}$$

$$\Gamma = \sum_{i<j} \vec{T}_i \cdot \vec{T}_j \, \Gamma_{ij}$$

- When the colour correlators $\vec{T}_i \cdot \vec{T}_j$ are numbers, NLL resummations can be implemented by just reweighting any tree-level event generator

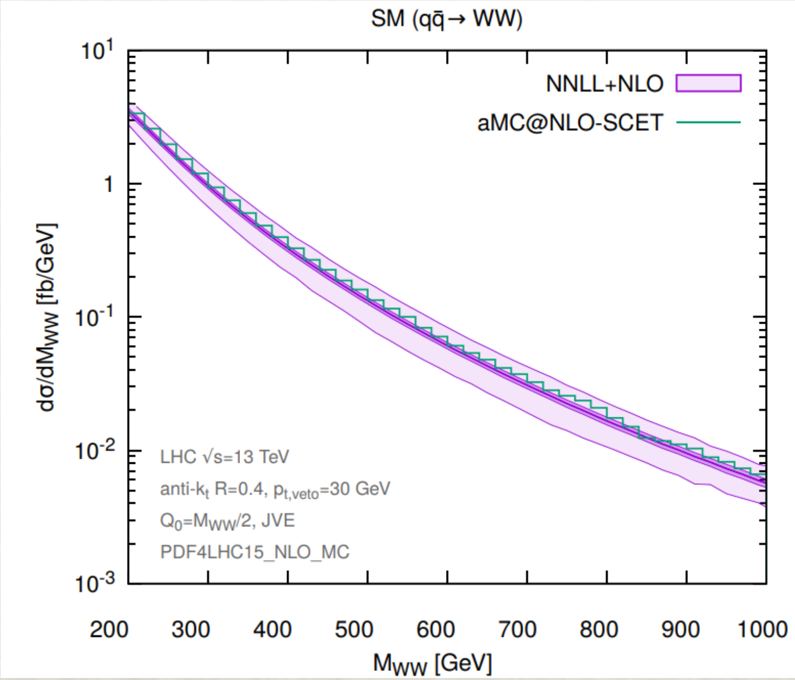
- For more than three-legs, $\vec{T}_i \cdot \vec{T}_j$ is a matrix and has to be decomposed in a colour basis [Baberuxki et al 1912.09396]

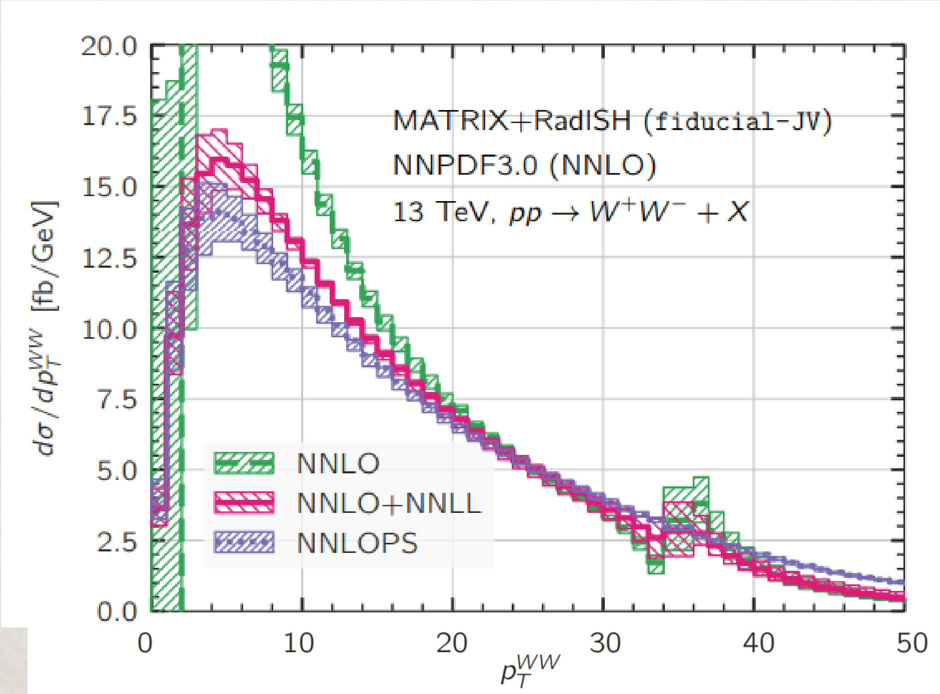
[SHERPA https://sherpa-team.gitlab.io/]



NNLL resummations

- NNLL resummations have been interfaced to fixed-order programs in the case of the production of a colour singlet





- All implementations are based on the reweighting of tree-level events, but with different strategies
  - aMC@NLO-SCET and CuTe-MCFM: beam functions as new LHAPDF tabs [CuTe-MCFM https://mcfm.fnal.gov/cute-mcfm.html]
  - MCFM-RE: NNLL corrections of collinear origin as new NLO integrated counterterms [MCFM-RE https://github.com/lcarpino/MCFM-RE]
  - MATRIX+RadISH: the tree-level matrix element is used as initial condition for the RadISH MC resummation [MATRIX+RadISH https://matrix.hepforge.org/matrix+radish.html]

RESUMMATION MODE

- As a basis for discussion, let's start from the CAESAR NLL formula and see how it can be upgraded to NNLL (and beyond)

$$\Sigma(Q,Q_0) = e^{-R(Q,Q_0)} \prod_{i=1}^{n} J_i(Q/Q_0) \frac{\langle \mathcal{B}| e^{-\Gamma^\dagger t(Q/Q_0)} e^{-\Gamma t(Q/Q_0)} |\mathcal{B}\rangle}{\langle \mathcal{B}|\mathcal{B}\rangle}$$

- If Sudakov form factor $R(Q,Q_0)$ is a function of Casimirs only it can be accounted for by a simple reweighting factor
- Jet functions $J_i(Q/Q_0)$ involve convolutions with tree-level matrix element squared $\Longrightarrow$ modified integrated counterterms
- For gluons, additional spin-dependent jet-function in near-to-planar kinematics $\Longrightarrow$ need spin decomposition of amplitudes

[HELAC https://helac-phegas.web.cern.ch/]

- For more than three emitters, soft functions and their anomalous dimensions involve colour matrices $\vec{T}_i \cdot \vec{T}_j$ $\Longrightarrow$ need colour decomposition of Born matrix elements

[SHERPA https://sherpa-team.gitlab.io/]

[DEDUCTOR https://pages.uoregon.edu/soper/deductor/]

Outlook

Resummation is a thriving field, with a plethora of new exciting results

- NNLL is the state-of-the-art, achievable both in QCD and with effective-theory methods
- Transverse momentum resummations have reached the impressive N3LL accuracy
- First NLL resummation of non-global logarithms
- Renewed interest in spin-correlation effects

Phenomenological impact of resummations: can be establish a common framework for public codes to run in “resummation mode”?

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Thank you for your attention!