Next-to-leading non-global logarithms in QCD

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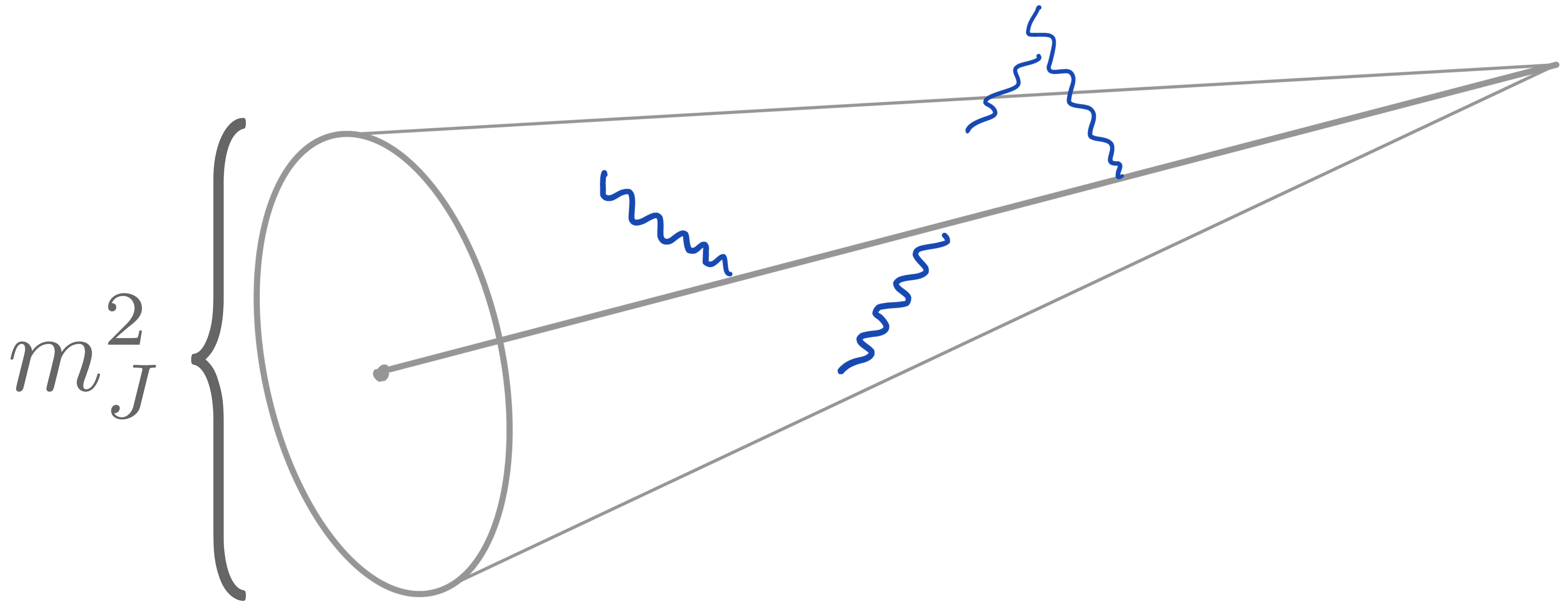
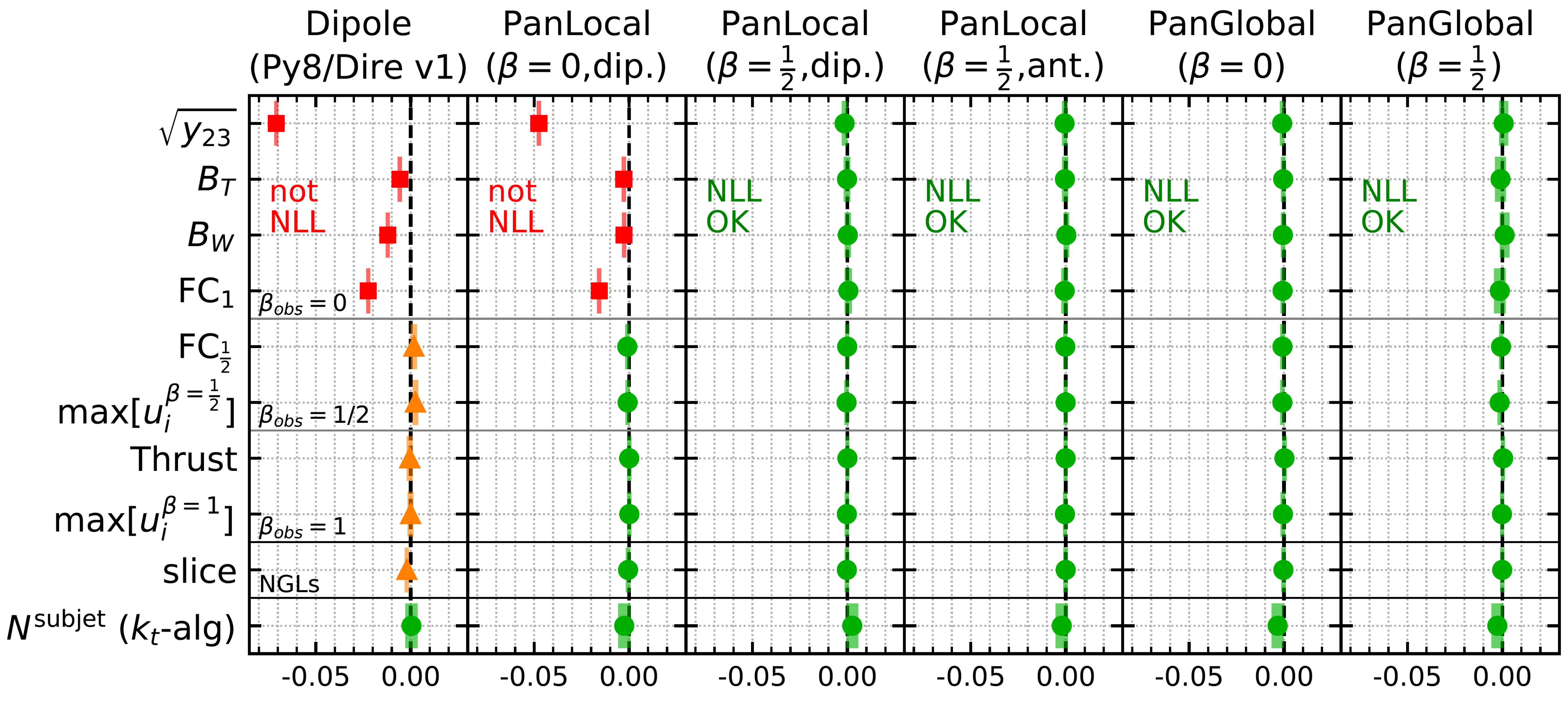
Parton Shower and Resummation - May 2021

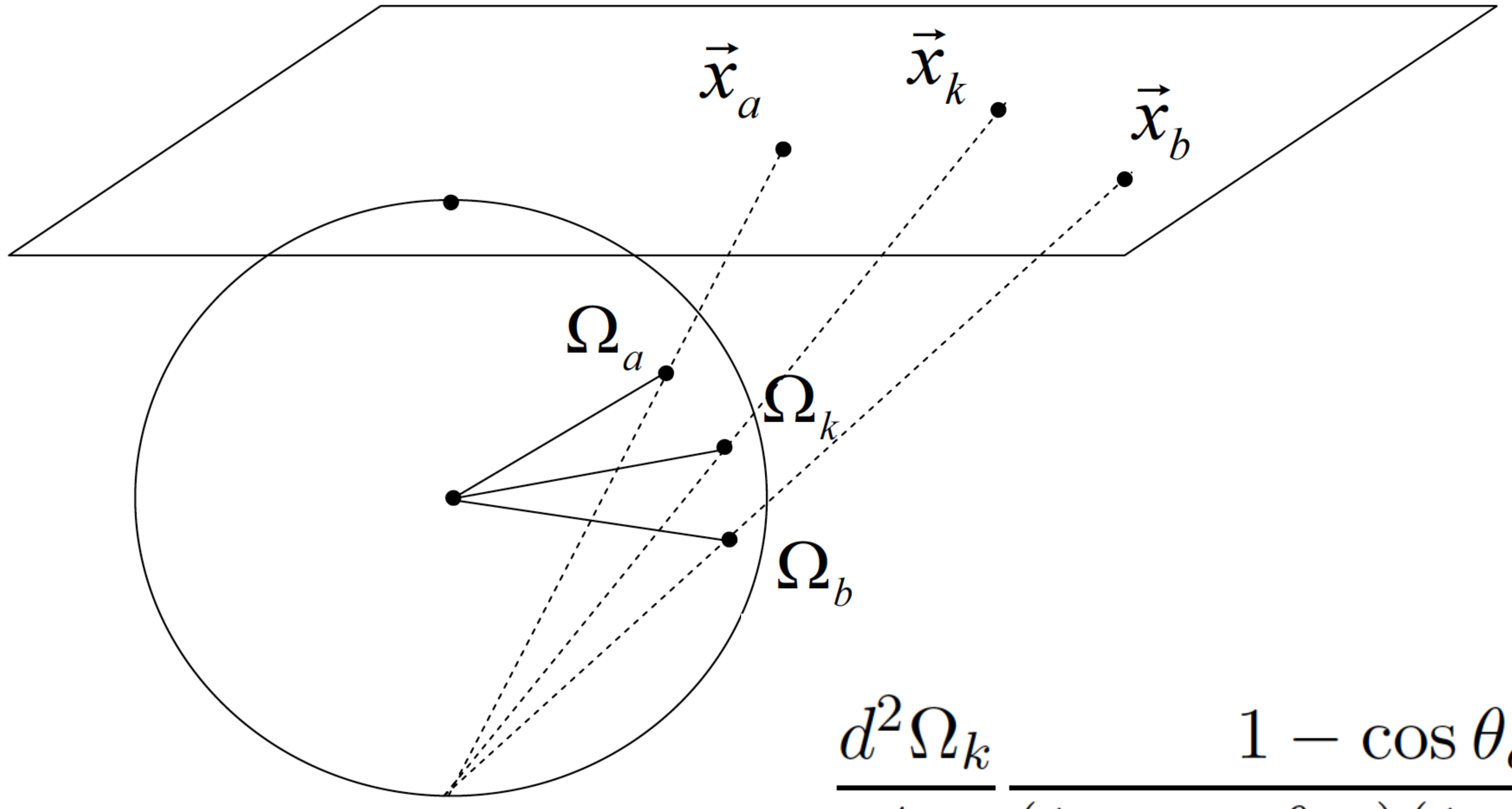
Non-global logarithms

- Ubiquitous in collider observables (e.g. use of jets and fiducial experimental cuts)
- Their understanding is an essential ingredient for parton showers (e.g. angular ordering vs. dipole showers)
- Stereographic projection relates them to high-energy (BK) evolution





[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]



[Weigert '03; Hatta '08]

$$\frac{d^2\Omega_k}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}=\frac{d^2\vec{x}_k}{2\pi}\frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$

Non-global logarithms

- Resummation of LL corrections known for a long time and studied in depth

[Dasgupta, Salam ’01-’02; Banfi, Marchesini, Smye ’02]
[Forshaw, Kyrieleis, Seymour ’06; Forshaw, Keates, Marzani ’09]
[Weigert ’03; Hatta, Ueda ’13-‘20 (+Hagiwara ’15)] …

- Revived interest more recently and new formulations with new QFT techniques

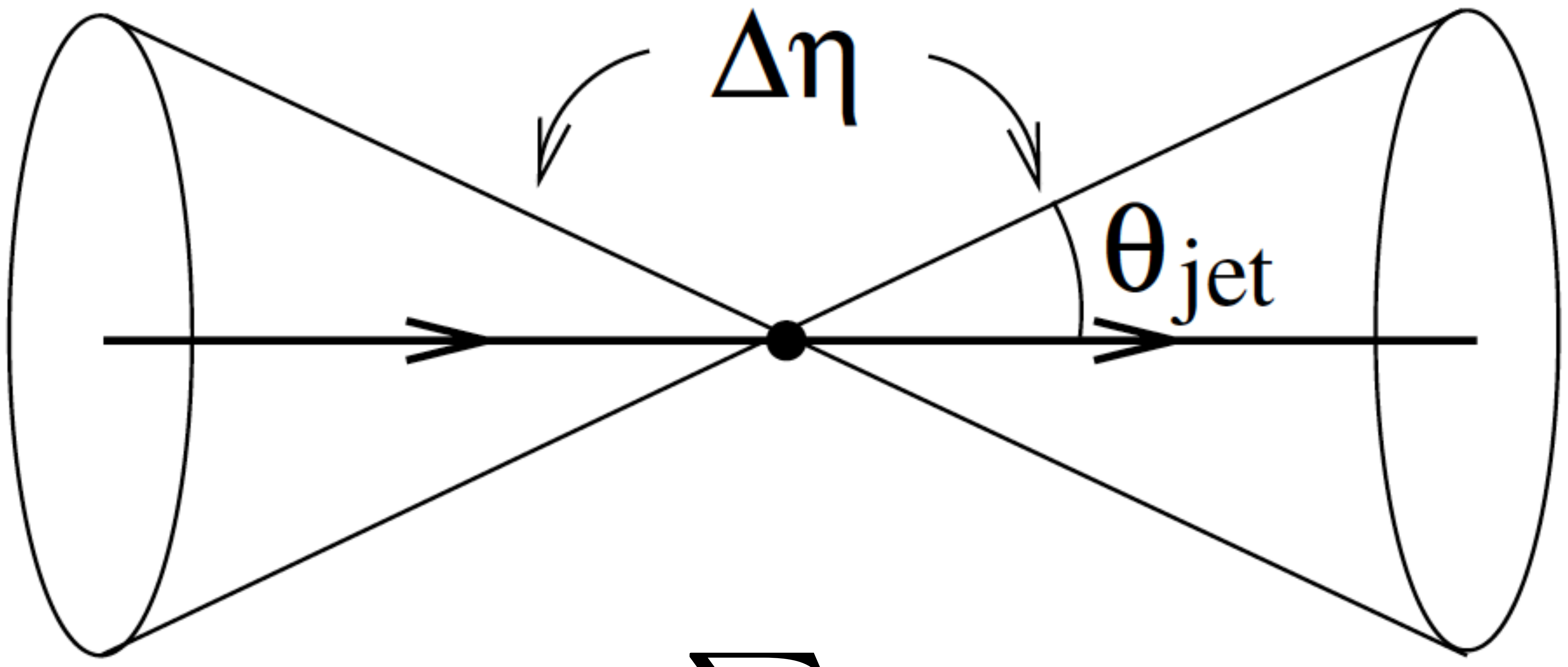
e.g. [Becher, Neubert, Rothen, Shao ’15-’16 (+ Pecjak ’16, Rahn ’17, Balsiger ’18-’19, Ferroglia ’20); Larkoski, Moult, Neill ’15-’16; Caron-Huot ’16; Angeles Martinez, De Angelis, Forshaw, Plaetzer, Seymour ’18] …

- Resummation of NLL corrections remains a great technical challenge due to the complexity of the geometry and colour structure of a typical NG problem

- GOAL ⇒ formulate the problem in such a way that can be integrated with Monte Carlo technology for a variety of observables & processes at once

Cone-jet cross section with a veto

- A simple laboratory to study these radiative corrections is the production of two cone jets at lepton colliders, with a veto on radiation in the interjet region



$$\Delta\eta := \ln\frac{1+c}{1-c}\,, \qquad c = \cos\theta_{\rm jet}$$

$$v = \sum_{2|\eta_i|\leq\Delta\eta} v(k_i)$$

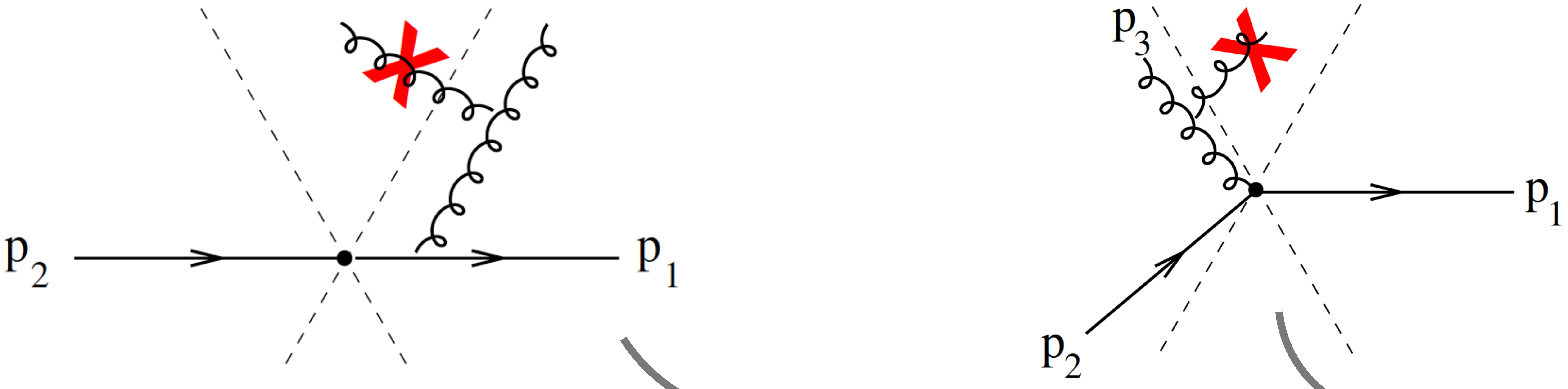
$$v(k_i) = E_i, k_{ti}$$

A veto can be set e.g. on the energy or transverse energy (scalar p_t sum) of the radiation in the gap

Factorisation of the cross section

$$\boxed{\mathcal{H}_n \otimes S_n(v) = \int \left(\prod_{i=i}^{n} d^2\Omega_i \right) \mathcal{H}_{1\ldots n} \times S_{1\ldots n}(v)}$$

- Cross section receives contributions from hard configurations with different multiplicity



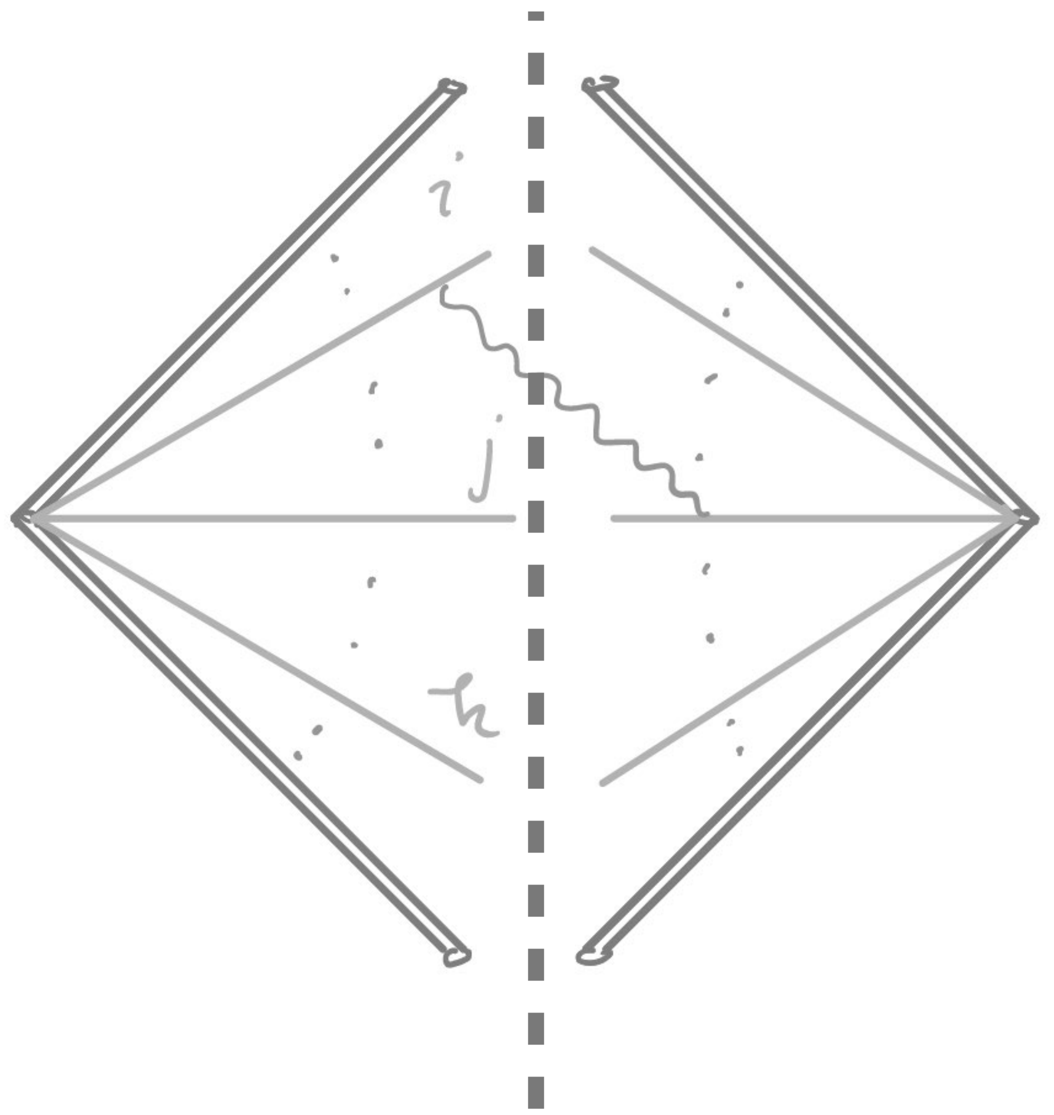
$$\Sigma(v) := \sum_{n=2}^{\infty} \mathcal{H}_n \otimes S_n(v) = \mathcal{H}_2 \otimes S_2(v) + \mathcal{H}_3 \otimes S_3(v) + \cdots$$

Use Laplace transform to factorise measurement, i.e.
(cumulative distribution)

$$\Theta\left(v - \sum_{2|\eta_i| \leq \Delta\eta} v(k_i)\right) = \frac{1}{2\pi i}\int_\gamma \frac{d\nu}{\nu} e^{\nu v} \prod_i \left(\underbrace{\Theta_{\rm out}(k_i) + \Theta_{\rm in}(k_i) e^{-\nu v(k_i)}}_{u(k_i)} \right) \Rightarrow \boxed{S_n(v) = \int_\gamma \frac{d\nu}{2\pi i \nu} e^{\nu v} G_{12\cdots n}[Q; u]}$$

Evolution of the soft factors (generating functionals)

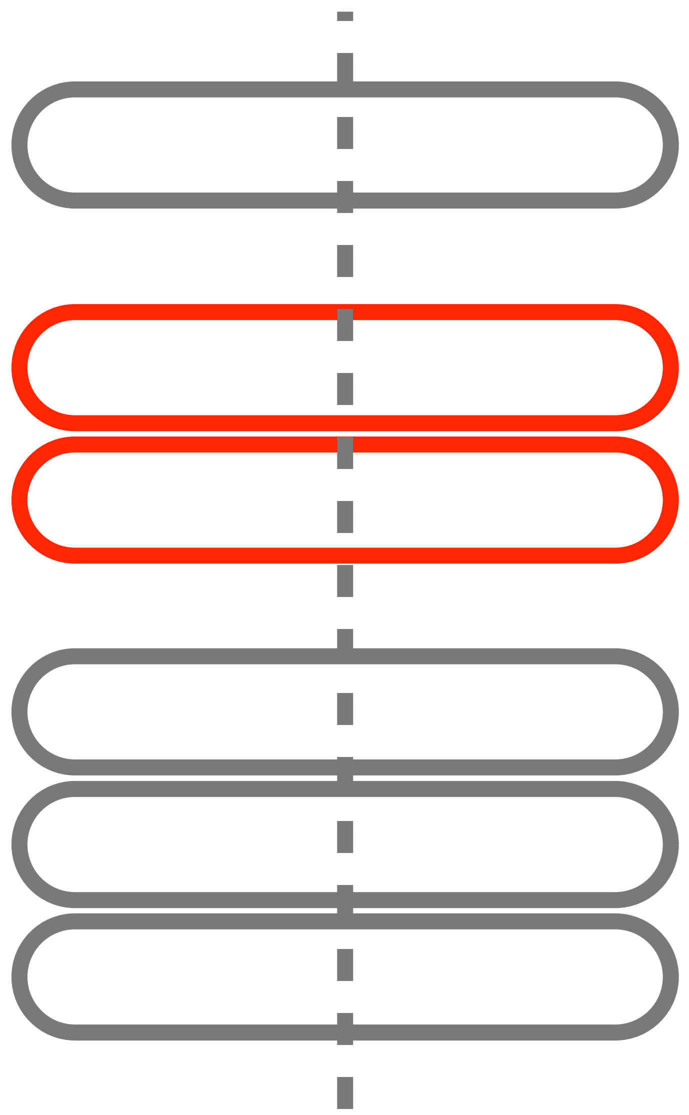
- Evolution equation describes the transition from a state with n to a state with n+1 partons
  - dynamics is non-linear already at LL (single gluon exchange)



$\Rightarrow$

['t Hooft '73]

$$\mathbf{T}_i \cdot \mathbf{T}_j \sim N_c\, \delta_{j,i\pm 1}$$



$$\mathcal{A}_{12}^2 = \bar{\alpha}^n(\mu)(2\pi)^{2n}(\mu^{2\epsilon})^n \sum_{\pi_n} \frac{(p_1 \cdot p_2)}{(p_1 \cdot k_{i_1})(k_{i_1} \cdot k_{i_2}) \ldots (k_{i_n} \cdot p_2)}$$

[Bassetto, Ciafaloni, Marchesini '83; Fiorani, Marchesini, Reina '88]

In the 't Hooft planar limit ($N_c \gg 1$, $\alpha_s N_c$ fixed) the evolution is expressed in terms of squared amplitude (i.e. colour dipoles) – treatable !

Evolution of the soft factors: LL (planar)

- Define a suitable (IRC safe) evolution variable (e.g. energy or dipole k_t)
- LL ≡ hard configurations with 2 legs (H_2) @ tree level ⊗ soft factor S_2 evolved at leading order (single gluon exchange between legs)
- Planar limit governed by a closed equation for colour dipoles (BMS equation)

[Banfi, Marchesini, Smye '02]

$$Q\partial_Q G_{12}[Q;u] = \mathbb{K}^{\mathrm{LL}}[G[Q,u],u]$$

$$\mathbb{K}^{\mathrm{LL}}[G[Q,u],u] := \int [dk_a]\bar{\alpha}(Q)\, w_{12}^{(0)}(k_a)\,(G_{1a}[Q;u]G_{a2}[Q;u]u(k_a) - G_{12}[Q;u])\, Q\delta(Q-k_{ta})$$

$$[dk] := \frac{d\eta}{2}\frac{d^{2-2\epsilon}k_t}{(2\pi)^{3-2\epsilon}}$$

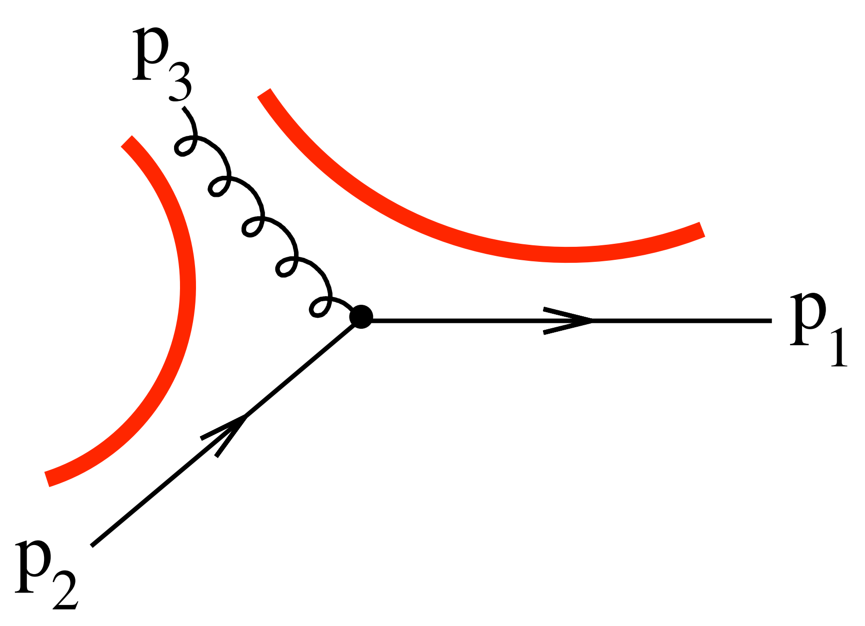
$$w_{12}^{(0)}(k) = 8\pi^2\frac{\mu^{2\epsilon}}{k_t^2}$$

Iteration reproduces squared amplitude for n emissions

Evolution of the soft factors: NLL (planar)

- NLL ≡ hard configurations with 3 legs (**H₃**) @ tree level ⊗ **S₃** with LL evolution …

$$G_{123}[Q;u] \xrightarrow{N_c \gg 1} G_{13}[Q;u]\, G_{32}[Q;u]$$



- … & hard configurations with 2 legs (**H₂**) @ 1-loop ⊗ **S₂** with NLL evolution

$$Q\partial_Q G_{12}[Q;u] = \mathbb{K}^{\mathrm{NLL}}[G[Q,u],u]$$
$$:= \mathbb{K}^{\mathrm{RV+VV}}[G[Q,u],u] + \mathbb{K}^{\mathrm{RR}}[G[Q,u],u] - \mathbb{K}^{\mathrm{DC}}[G[Q,u],u]$$

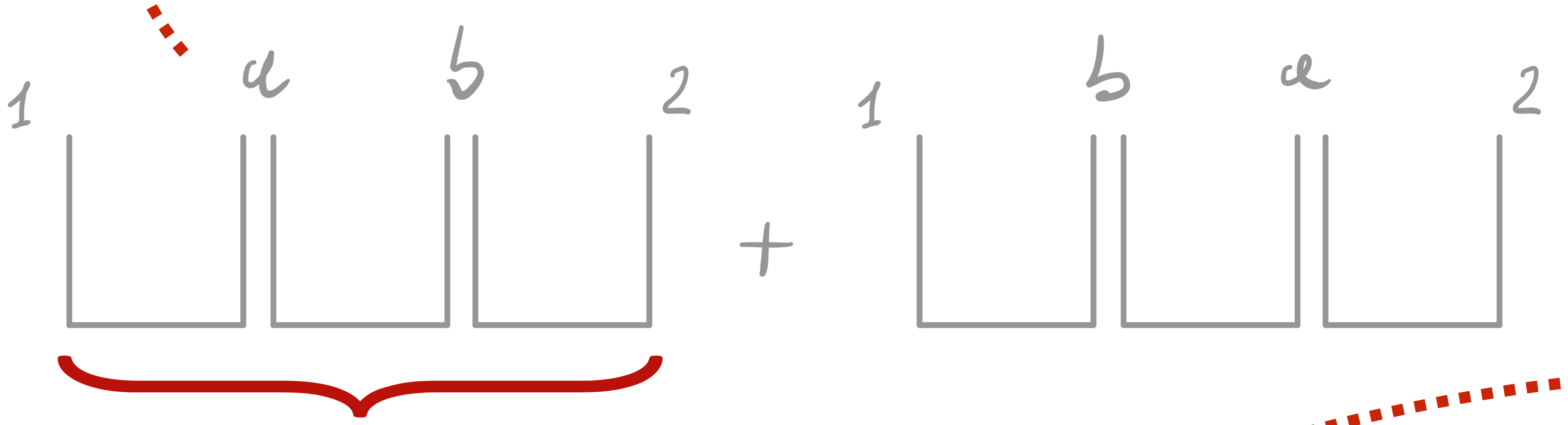
NB: We use dipole k_t as evolution variable; energy is possible too, with slightly different kernels at NLL.
Both variables are OK for problems with sensitivity to wide-angle soft radiation, i.e. no double logs!
Double logarithmic problems, require subtraction of primary radiation (to be handled separately)

NLL evolution equation

- e.g. real corr.$^{\text{ns}}$: contributions from two adjacent dipoles

$$\mathbb{K}^{\text{RR}}[G[Q,u],u] := \int [dk_a] \int [dk_b]\, \bar{\alpha}^2(Q) Q \delta(Q - k_{t(ab)}) \Theta(k_{ta} - k_{tb})$$
$$\times \Big[\bar{w}_{12}^{(gg)}(k_b,k_a) G_{1b}[Q;u] G_{ba}[Q;u] G_{a2}[Q;u] u(k_a) u(k_b)$$
$$\boxed{+\bar{w}_{12}^{(gg)}(k_a,k_b) G_{1a}[Q;u] G_{ab}[Q;u] G_{b2}[Q;u] u(k_a) u(k_b)}$$
$$- \left(\bar{w}_{12}^{(gg)}(k_b,k_a) + \bar{w}_{12}^{(gg)}(k_a,k_b) \right) G_{1(ab)}[Q;u] G_{(ab)2}[Q;u] u(k_{(ab)}) \Big]$$

Evolution variable must be adjusted (dipole k_t of the parent) to guarantee collinear safety for any u(k)



- Correct only for correlated contribution to squared amplitude (exponentiation of soft singularities)

$$\tilde{w}_{12}^{(0)}(k_a,k_b) = \frac{1}{2} w_{12}^{(0)}(k_a) w_{12}^{(0)}(k_b) + \boxed{\bar{w}_{12}^{(gg)}(k_a,k_b)}$$

- Independent contribution correctly treated in LL kernel

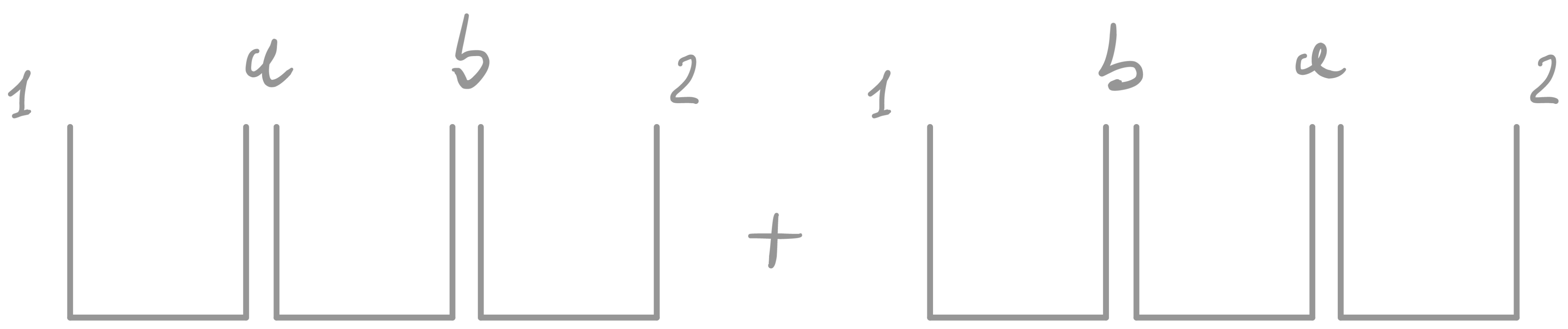
Ingredients from [Campbell, Glover '97]
also [Gehrmann-De Ridder, Gehrmann, Glover '05]

NLL evolution equation

- e.g. real corr.[ns]: contributions from two adjacent dipoles

$$
\begin{aligned}
\mathbb{K}^{\rm RR}[G[Q,u],u] \coloneqq \int [dk_a] \int [dk_b]\, \bar{\alpha}^2(Q) Q \delta(Q - k_{t(ab)}) \Theta(k_{ta} - k_{tb}) \\
\times \Big[\bar{w}_{12}^{(gg)}(k_b,k_a) G_{1b}[Q;u] G_{ba}[Q;u] G_{a2}[Q;u] u(k_a) u(k_b) \\
+ \bar{w}_{12}^{(gg)}(k_a,k_b) G_{1a}[Q;u] G_{ab}[Q;u] G_{b2}[Q;u] u(k_a) u(k_b) \\
- \left(\bar{w}_{12}^{(gg)}(k_b,k_a) + \bar{w}_{12}^{(gg)}(k_a,k_b) \right) G_{1(ab)}[Q;u] G_{(ab)2}[Q;u] u(k_{(ab)}) \Big]
\end{aligned}
$$

Evolution variable must be adjusted (dipole k_t of the parent) to guarantee collinear safety for any u(k)



- Counterterm defined by means of a collinear mapping that preserves the kinematics of the (pseudo-)parent

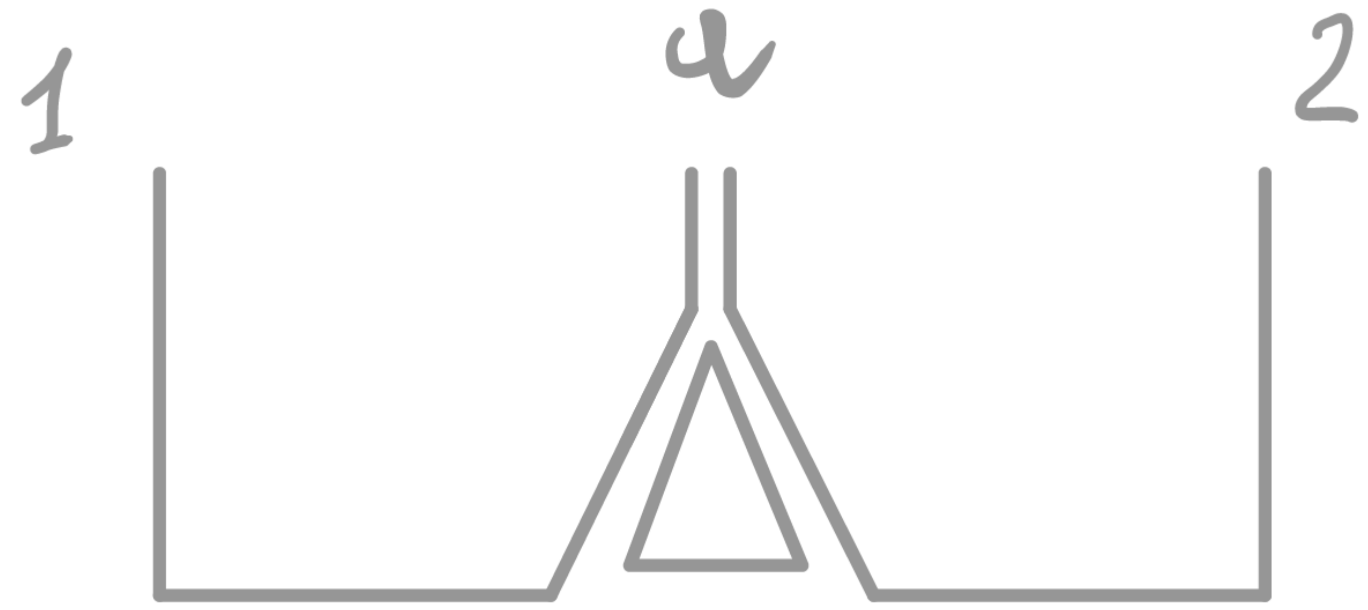
$$
\mathbb{P} : \{k_a, k_b\} \to k_{(ab)} = \left(|\vec{k}_a + \vec{k}_b|, \vec{k}_a + \vec{k}_b \right)
$$

NLL evolution equation

- e.g. virtual corr.ns: same dipole structure as in the LO kernel

[Catani, Marchesini, Webber '91]
see also [Banfi, El-Menoufi, PM '18;
Catani, De Florian, Grazzini '19]

$$\mathbb{K}^{\mathrm{RV+VV}}[G[Q,u],u] := \int [dk_a]\bar{\alpha}(Q) w_{12}^{(0)}(k_a) \left(1 + \bar{\alpha}(Q)\,\bar{K}^{(1)}\right)$$
$$\times \left(G_{1a}[Q;u]G_{a2}[Q;u]u(k_a) - G_{12}[Q;u]\right) Q\delta(Q - k_{ta})$$



- Subtract (iteration of) virtual corrns to LL kernel (cancel soft divergences)
  ➥ double-counting for reals in backup
- Double virtual contributions by unitarity
- Collinear divergences cancelled by means of a local counterterm (previous slide) $\Rightarrow$ limit to d=4

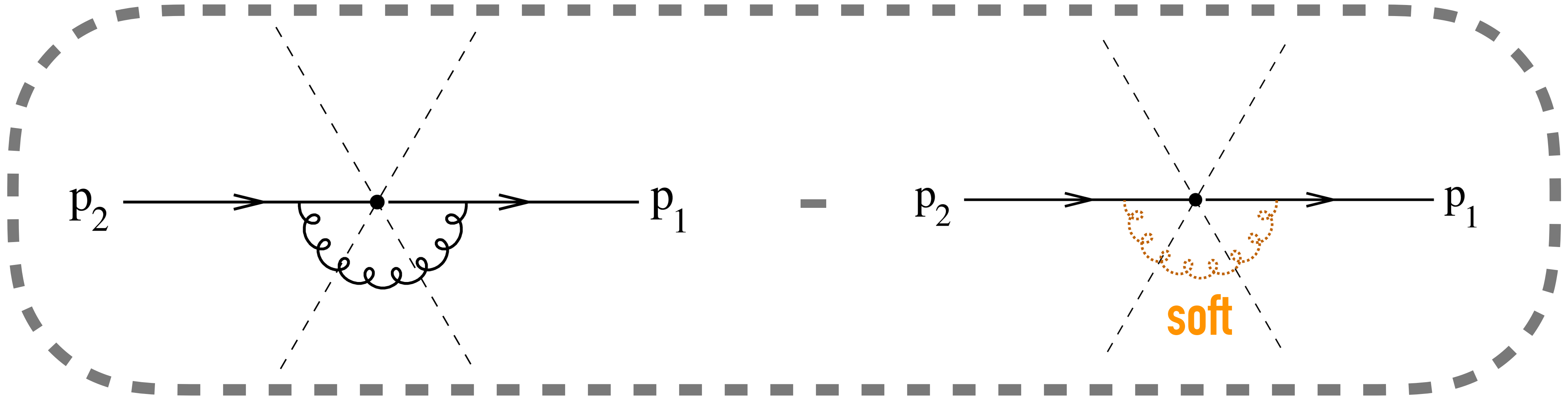
Ingredients from [Catani, Grazzini '00]
also [Angeles Martinez, Forshaw, Seymour '16]

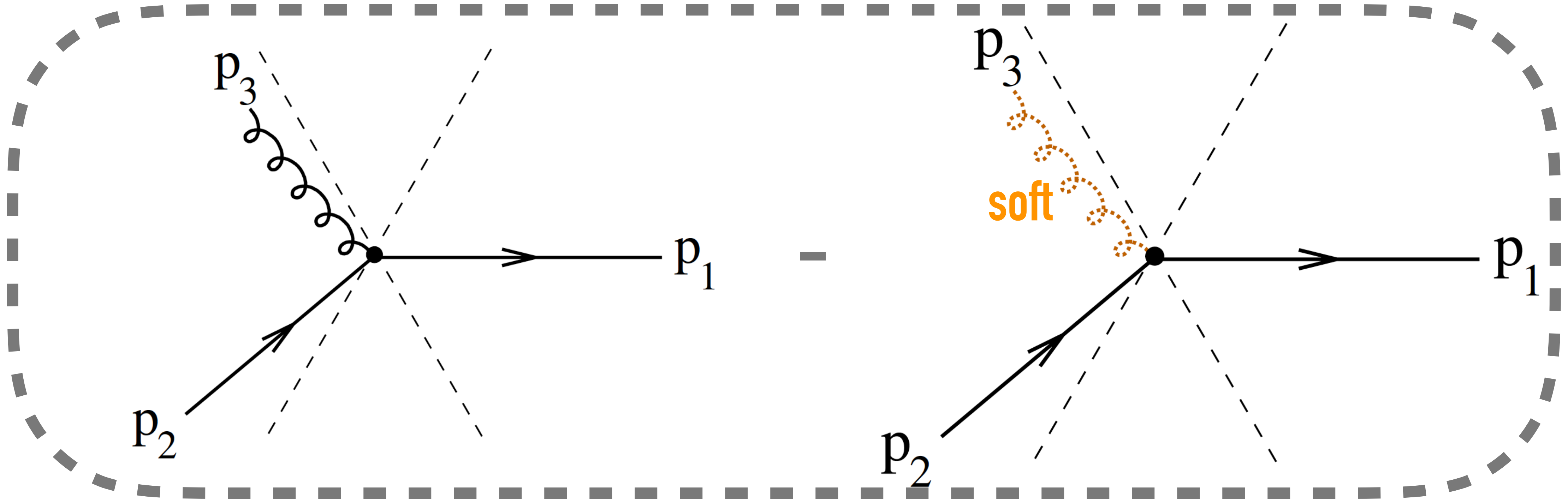
Hard factors at NLO

Recall:
$$\mathcal{H}_n \otimes S_n(v) = \int \left(\prod_{i=i}^{n} d^2\Omega_i \right) \mathcal{H}_{1\dots n} \times S_{1\dots n}(v)$$

- The corresponding hard factors are computed by matching the soft expansion to full QCD
- Cancellation of collinear divergences is scheme dependent $\left(x = \frac{2E_g}{\sqrt{s}}; \quad y = \cos\theta_{qg}\right)$

$$\mathcal{H}_2 = \left[1 + C_F \frac{\alpha_s}{2\pi}\left(\frac{5}{2} + \frac{\pi^2}{3}\right)\right] \delta^{(2)}(\Omega_1 - \Omega_q)\delta^{(2)}(\Omega_2 - \Omega_{\bar{q}})$$





$$\begin{aligned}\mathcal{H}_3 = C_F \frac{\alpha_s}{2\pi} \Bigg\{ &\int_0^1 \frac{dx}{x} \int_{-1}^1 dy \left[\frac{1}{(1-y)_+} + \frac{1}{(1+y)_+}\right] \\ &\times \left[\frac{(x-2)x\left((x-2)x(1-y)^2 - 4y + 8\right) + 8}{(2-(1-y)x)^2} - \boxed{2\mathbb{P}_{\text{soft}}}\right] \\ &- 4\left[\int_0^{\sqrt{s}} \frac{dk_t}{k_t} \int_{\ln\frac{k_t}{\sqrt{s}}}^{-\cosh^{-1}\frac{\sqrt{s}}{2k_t}} d\eta + \int_0^{\sqrt{s}} \frac{dk_t}{k_t} \int_{\cosh^{-1}\frac{\sqrt{s}}{2k_t}}^{\ln\frac{\sqrt{s}}{k_t}} d\eta + \int_{\frac{\sqrt{s}}{2}}^{\sqrt{s}} \frac{dk_t}{k_t} \int_{-\cosh^{-1}\frac{\sqrt{s}}{2k_t}}^{\cosh^{-1}\frac{\sqrt{s}}{2k_t}} d\eta\right] \boxed{\mathbb{P}_{\text{soft}}} \Bigg\} \\ &\times \boxed{\Theta_{\text{out}}(k)}\, \delta^{(2)}(\Omega_1 - \Omega_q)\delta^{(2)}(\Omega_2 - \Omega_{\bar{q}})\delta^{(2)}(\Omega_3 - \Omega_g)\end{aligned}$$

Projector onto soft kinematics

Acts on soft factor S_3

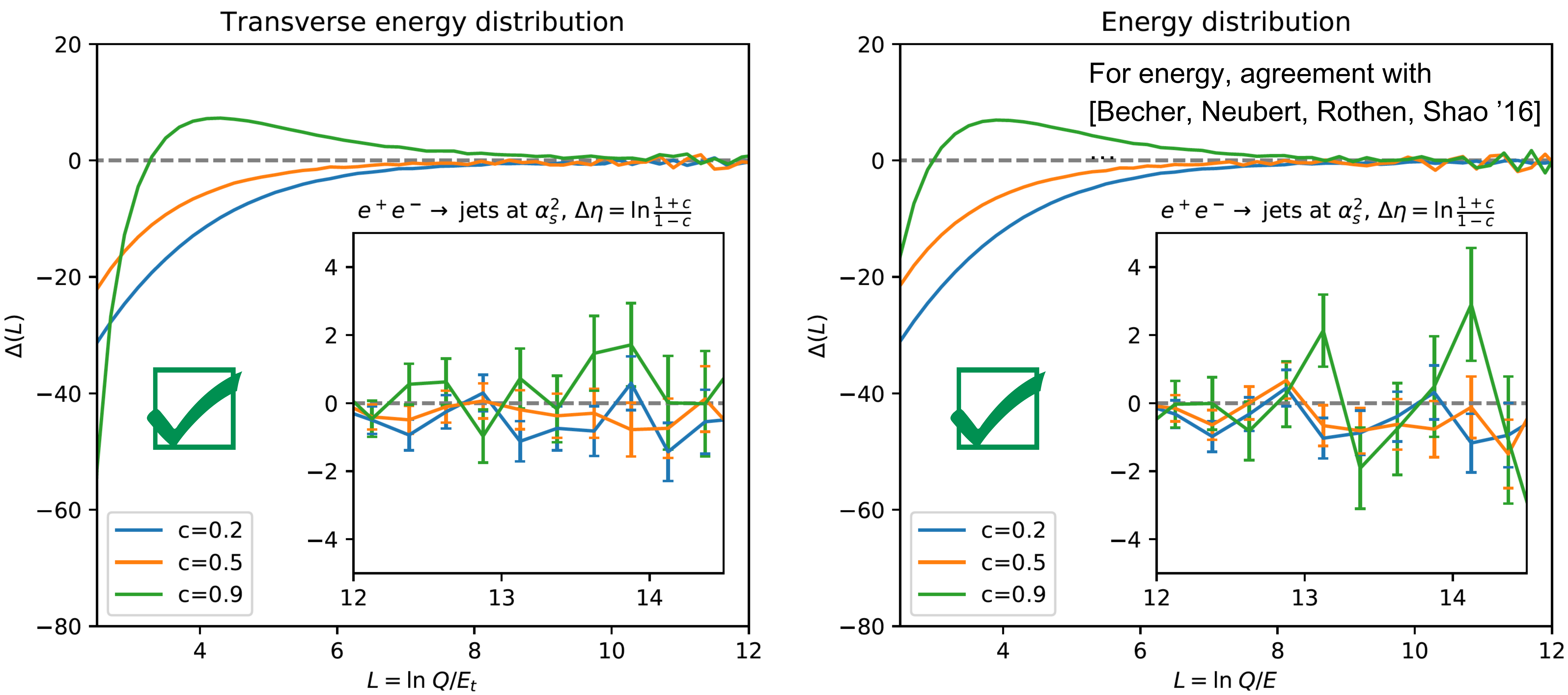
Fixed order expansion

- We expect to reproduce the correct logarithmic structure at $O(\alpha_S^2)$ by expanding

$$\Sigma(v) := \sum_{n=2}^{\infty} \mathcal{H}_n \otimes S_n(v) = \mathcal{H}_2 \otimes S_2(v) + \mathcal{H}_3 \otimes S_3(v) + \cdots$$

$$\Delta(L) := \frac{1}{\sigma_0}\left(\frac{d\Sigma^{\text{NLO}}}{dL} - \frac{d\Sigma^{\text{EXP.}}}{dL}\right)$$

➡ Expect: $\lim_{L\to\infty} \Delta(L) = 0$



Subleading colour corrections are straightforward to include at this order ⇒ compare to finite-N_c QCD (EVENT2) [Catani, Seymour '96]

Conclusions

- New formalism for resummation of non-global corrections beyond LL in the planar limit:
  - Soft evolution controlled by a set of closed non-linear evolution equations, solvable in terms of of combinations of colour dipoles
  - Integration can be carried out with Monte Carlo methods (ongoing)

- In the planar approximation this technology can be applied to proton-proton collision as well, avoiding complications that would arise at subleading N_c (e.g. SLL, Glauber modes)
  - Process dependence entirely encoded in the Hard factors and extraction is algorithmic

- Integration by MC closely related to a parton-shower implementation

Extra material

NLL evolution equation

- Cancellation of IRC singularities can be made manifest by introducing a counterterm in the NLO kernel, equation can be directly solved in d=4

  e.g. double counting term: (minus) iteration of LO kernel

$$
\begin{aligned}
\mathbb{K}^{\mathrm{DC}}[G[Q,u],u] \coloneqq & \int [dk_a] \int [dk_b] \bar{\alpha}^2(Q) Q \delta(Q-k_{ta}) \Theta(k_{ta}-k_{tb}) \\
\times & \Bigg[w_{12}^{(0)}(k_a) \left(w_{1a}^{(0)}(k_b) - \frac{1}{2} w_{12}^{(0)}(k_b) \right) G_{1b}[Q;u] G_{ba}[Q;u] G_{a2}[Q;u] u(k_a) u(k_b) \\
& + w_{12}^{(0)}(k_a) \left(w_{a2}^{(0)}(k_b) - \frac{1}{2} w_{12}^{(0)}(k_b) \right) G_{1a}[Q;u] G_{ab}[Q;u] G_{b2}[Q;u] u(k_a) u(k_b) \\
& - w_{12}^{(0)}(k_a) \left(w_{1a}^{(0)}(k_b) + w_{a2}^{(0)}(k_b) - w_{12}^{(0)}(k_b) \right) G_{1a}[Q;u] G_{a2}[Q;u] u(k_a) \Bigg]
\end{aligned}
$$

Fixed order expansion

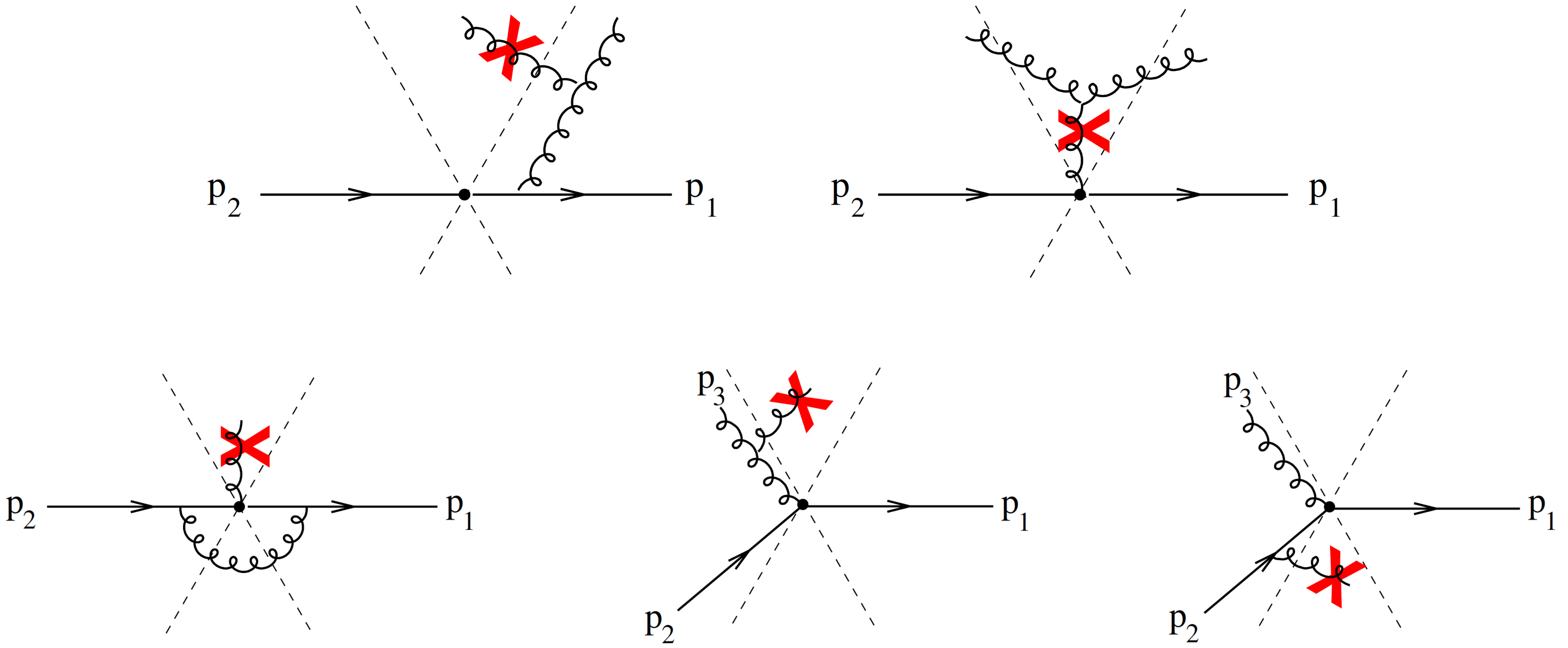
- We expect to reproduce the correct logarithmic structure at $O(\alpha_S^2)$
  - Expand our master formula

$$\Sigma(v) := \sum_{n=2}^{\infty} \mathcal{H}_n \otimes S_n(v) = \mathcal{H}_2 \otimes S_2(v) + \mathcal{H}_3 \otimes S_3(v) + \cdots$$

  - Keep only terms up to NLL (neglect pure $O(\alpha_S^2)$ constants)

$$\text{X} \longrightarrow -\Theta(v(k) - v)$$

Subleading colour corrections are straightforward to include at this order, so we can compare to finite-N_c QCD (Event2)



Fixed order expansion (full colour)

$$
\begin{aligned}
\Sigma(v) \simeq 1 &+ \left(\frac{\alpha_s}{2\pi}\right)\left(\mathcal{H}_2^{(1)} - 4C_F\int [dk] w_{12}^{(0)}(k)\Theta_{\rm in}(k)\Theta(v(k)-v)\Theta(Q-k_t) + \mathcal{H}_3^{(1)}\otimes \mathbb{1}\right) \\
&- 4C_F\left(\frac{\alpha_s}{2\pi}\right)^2 \int [dk] w_{12}^{(0)}(k)\Theta_{\rm in}(k)\Theta(v(k)-v)\Theta(k_t-Q)\left(K^{(1)} - 4\pi\beta_0 \ln\frac{k_t}{Q}\right) \\
&+ 8C_F^2\left(\frac{\alpha_s}{2\pi}\right)^2 \left(\int [dk] w_{12}^{(0)}(k)\Theta_{\rm in}(k)\Theta(v(k)-v)\Theta(Q-k_t)\right)^2 \\
&- 8C_F\left(\frac{\alpha_s}{2\pi}\right)^2 \int [dk_a]\int [dk_b]\left[C_A\left(\bar{w}_{12}^{(gg)}(k_a,k_b) + \bar{w}_{12}^{(gg)}(k_b,k_a)\right)\right. \\
&\qquad\qquad\qquad\qquad\qquad\qquad \left. + n_f\left(\bar{w}_{12}^{(q\bar{q})}(k_a,k_b) + \bar{w}_{12}^{(q\bar{q})}(k_b,k_a)\right)\right] \\
&\times \Theta(Q - k_{t(ab)})\Theta(k_{ta}-k_{tb})\left\{\Theta_{\rm out}(k_{(ab)})\left[\Theta_{\rm in}(k_a)\Theta_{\rm out}(k_b)\Theta(v(k_a)-v)\right.\right. \\
&\left.\left. +\Theta_{\rm out}(k_a)\Theta_{\rm in}(k_b)\Theta(v(k_b)-v)\right] - \Theta_{\rm in}(k_{(ab)})\Theta_{\rm out}(k_a)\Theta_{\rm out}(k_b)\Theta(v(k_{(ab)})-v)\right\} \\
&- 2\left(\frac{\alpha_s}{2\pi}\right)^2 \int [dk]\Theta_{\rm in}(k)\Theta(v(k)-v)\Theta(Q-k_t) \\
&\times\left[2C_F\mathcal{H}_2^{(1)} w_{12}^{(0)}(k) + \mathcal{H}_3^{(1)}\otimes\left(C_A(w_{13}^{(0)}(k) + w_{32}^{(0)}(k)) + (2C_F - C_A)w_{12}(k)\right)\right]
\end{aligned}
$$

Neglecting NNLL terms in the expansion