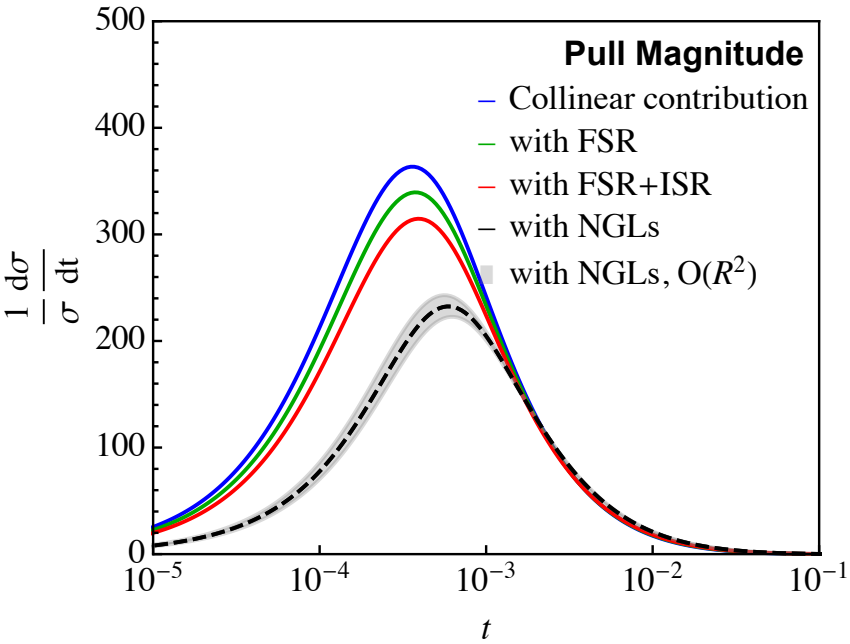
Non-global Logarithms Beyond Leading Colour

Chang Wu
Technion - Israel Institute of Technology

In collaboration with S. Marzani, S. Steffen and M. Spannowsky

PSR21 - Parton Showers and Resummation
May 25, 2021

Motivation



[A. Larkoski ,S. Marzani, C.W, 1911.05090]

NLL resummation formula:

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = t\int_0^\infty bdbJ_0(bt)\, e^{-C_F R(b)}\, S^{NG}(b)$$

Leading non-global logarithmic contribution:

$$\frac{d\sigma^{NG}}{dt} = \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{2\pi^2}{3}\frac{1}{t} log\frac{R^2}{t} + O\left(R^2\right)$$

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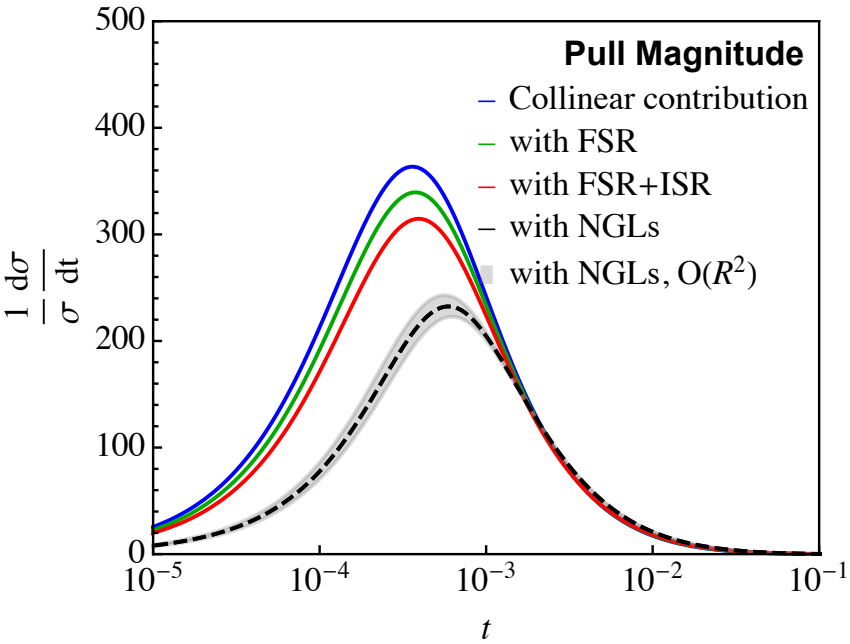

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Resummation of NGLs in large N_c limit

$$S^{NG} = exp\left[-C_F C_A \frac{\pi^2}{3}\frac{1+(a\tau)^2}{1+(b\tau)^c}\tau^2\right]$$

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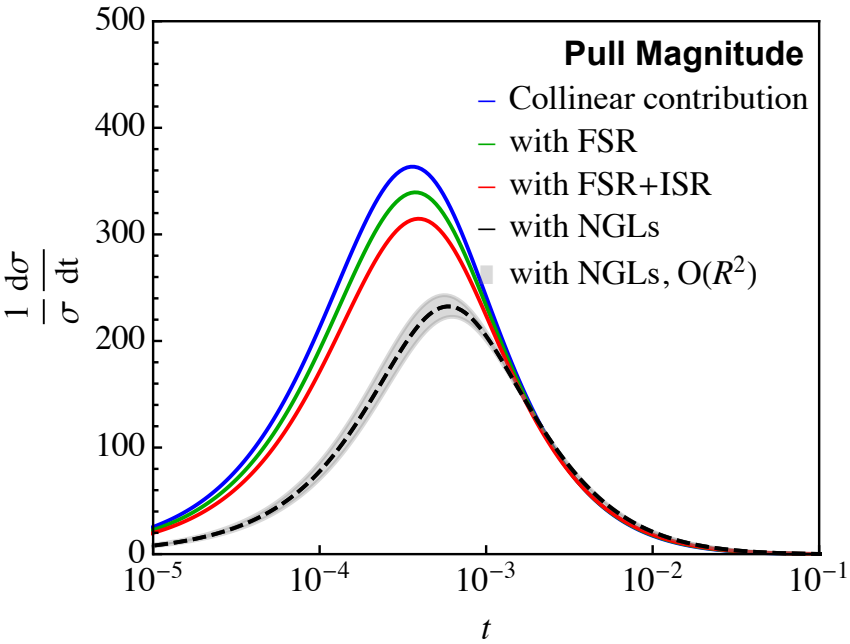

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R^2 corrections to the NGLs: $\frac{\pi^2}{3} \to \frac{\pi^2}{3}\left(1 \pm R^2\right)$

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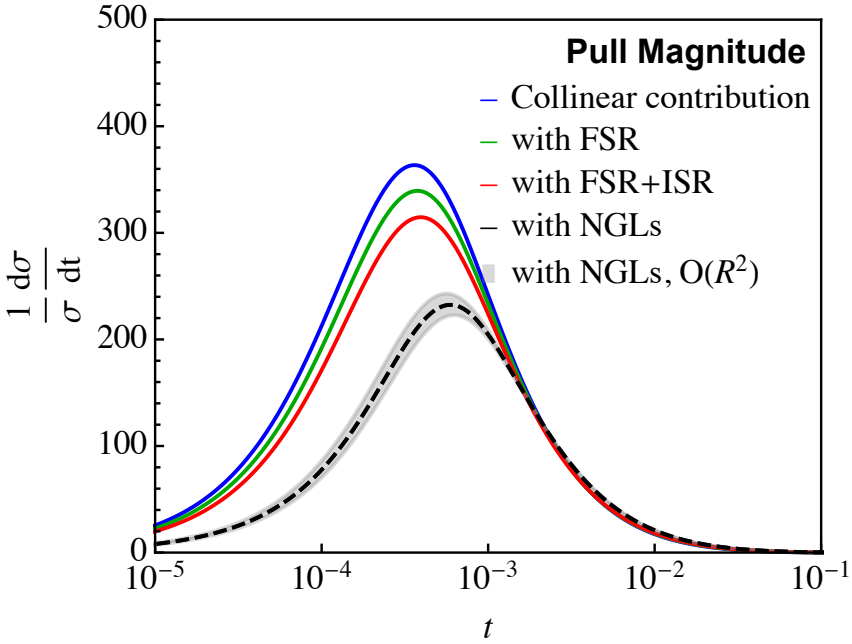

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Goals:

- Generalize the evolution equation for NGLs to finite N_c
- Resummation of non-global observables in the full color with the Sherpa resummation plugin.

Seeing in color with jet substructure

Processes with colour-singlet final states particle play an important role in high-energy physics

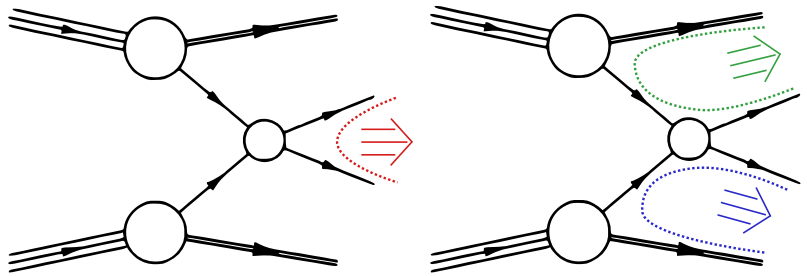
- Colour flow

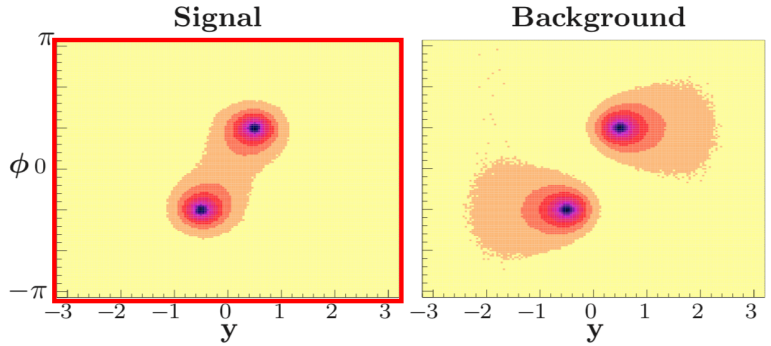
- Definition of pull vector:

$$\vec{t} = \sum_{i \in jet} \frac{p_T^i |r_i|}{p_T^{jet}} \vec{r}_i$$

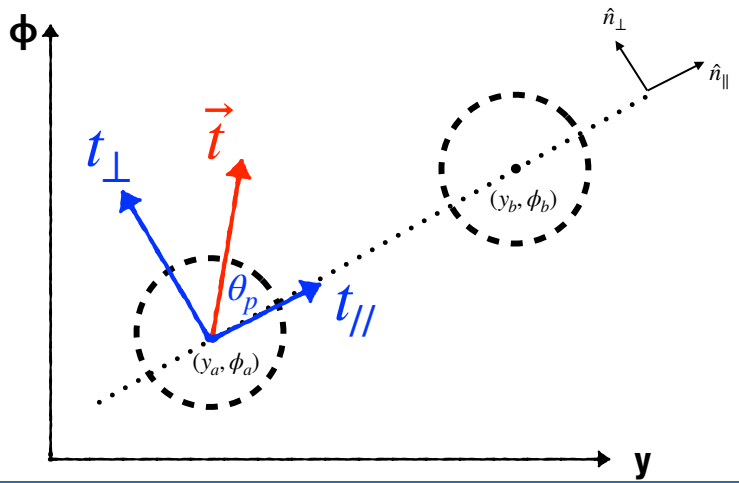
$$\vec{r}_i = (y_i - y_a, \phi_i - \phi_a)$$

- One dimensional projections
  - (t, θ_p): similar to q_T and ρ
  - $\left(|\vec{t} \cdot \hat{n}_{\parallel}|, |\vec{t} \cdot \hat{n}_{\perp}|\right)$: similar to a_T and ϕ^*
  - $\left(\vec{t} \cdot \hat{n}_{\parallel}, \vec{t} \cdot \hat{n}_{\perp}\right)$: asymmetry





[Gallicchio, Schwartz, 1001.5027]



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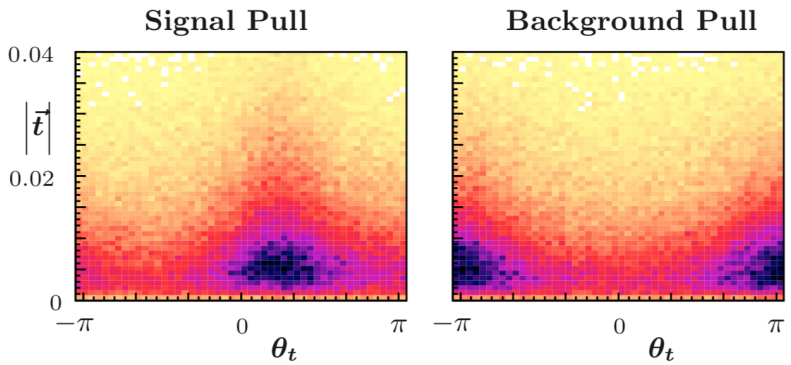
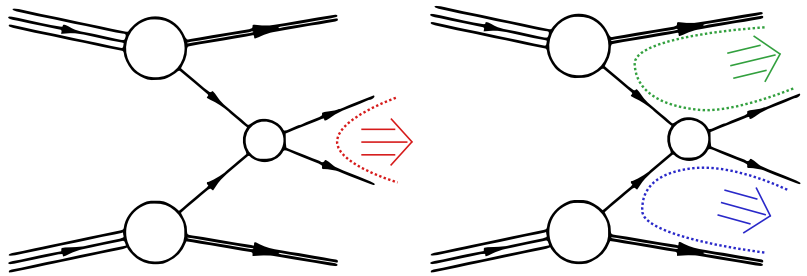
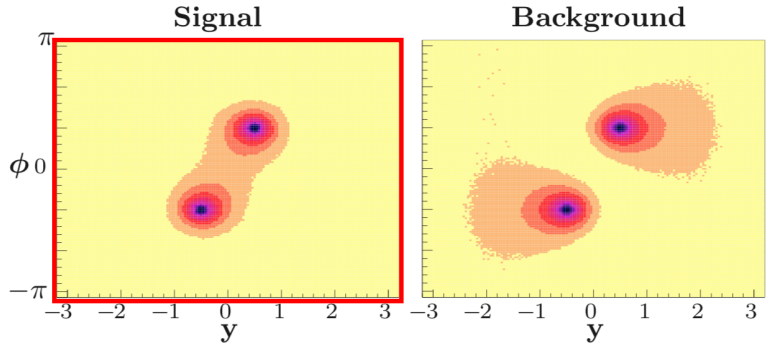
- One dimensional projections



[Gallicchio, Schwartz, 1001.5027]

- **Theoretical challenges:**
  IRC unsafe, Non-global logarithms





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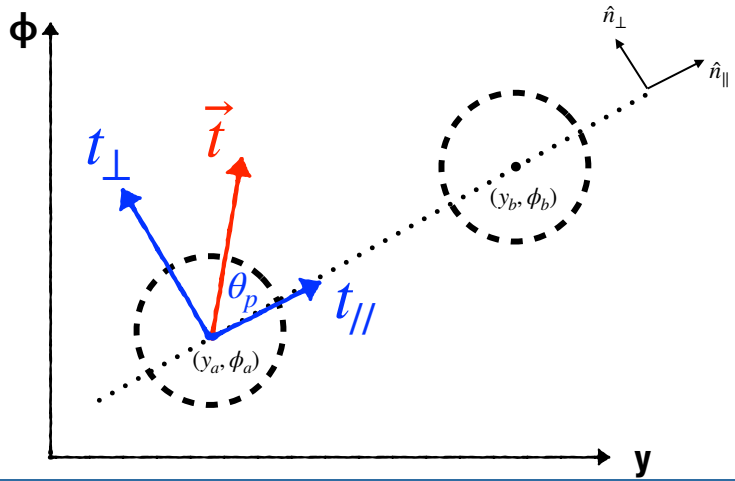

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1. Calculation for non-global logarithms
   - Introduction: history of NGLs
   - Symmetries of the BMS equation
   - Preliminary results
2. Soft radiation beyond leading colour
   - Soft radiation sensitive observable
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History of NGLs

- Dasgupta-Salam: Monte-Carlo simulation [hep-ph/0104277]
- Banfi-Marchesini-Smye (BMS) equation: [hep-ph/0206076]
  - Strong energy ordering
  - Large N_c limit

$$\partial_L G_{ab}(L) = \int \frac{d\Omega_j}{4\pi} W_{ab}^j \left[\theta_L(j)\, G_{aj}(L)\, G_{jb}(L) - G_{ab}(L)\right]$$

$$L = \frac{\alpha_s}{\pi} N_c \log \frac{1}{\rho}$$

- More recent progress:
  - Fixed-order up to 5-loop [Schwartz and Zhu, 1403.4949]
  - Dressed-gluon expansion [Larkoski, Moult, Neill 1501.04596]
  - Higher-logarithmic accuracy

[Caron-Huot, 1501.03754], [Banfi, Dreyer, Monni, 2104.06416]

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$$\partial_L g_{ab}(L) = \int \frac{d\Omega_j}{4\pi} W_{ab}^j \left[U_{abj}(L)\, g_{aj}(L)\, g_{jb}(L) - g_{ab}(L) \right]$$

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Applying neural network for perturbative expansion and all-order calculation

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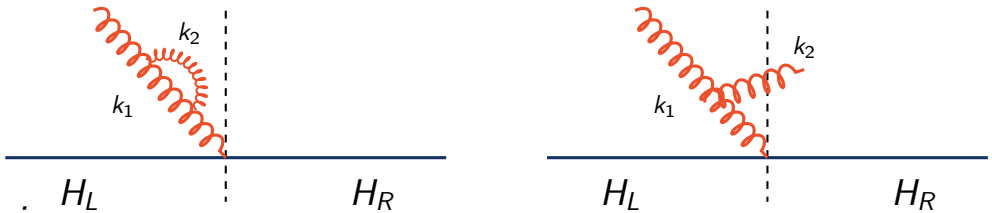

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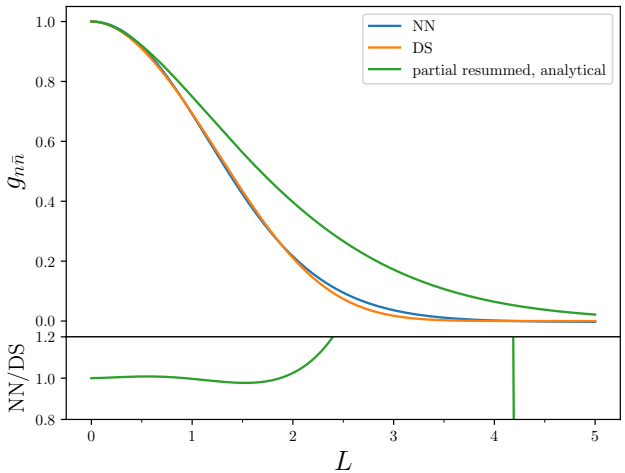


Applying neural n ... and all-order calculation

Symmetries of the BMS: stereographic projection

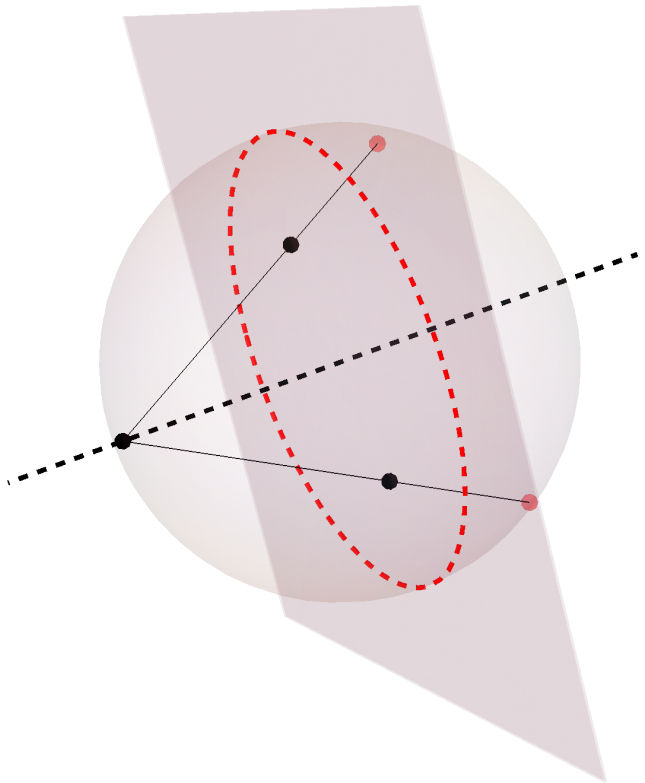
Recall the BMS equation under PSL(2, R): similar to BK equation
[Schwartz and Zhu, 1403.4949], [Hatta and Ueda, 0909.0056]

$$\partial_L g_{ab}(L) = \int d\Omega_j W^j_{ab} \times \left\{ \left[\frac{1+\langle ab\rangle}{(1+\langle aj\rangle)(1+\langle jb\rangle)} \right]^{L/2} g_{aj}(L)\, g_{jb}(L) - g_{ab}(L) \right\}$$

with

$$g_{ab}(L) = g(\langle ab\rangle, L) = g\left(\frac{1-cos\theta_{ab}}{2cos\theta_a cos\theta_b}, L \right)$$

Computation time: $n_L n_j^3 n_\phi^2$



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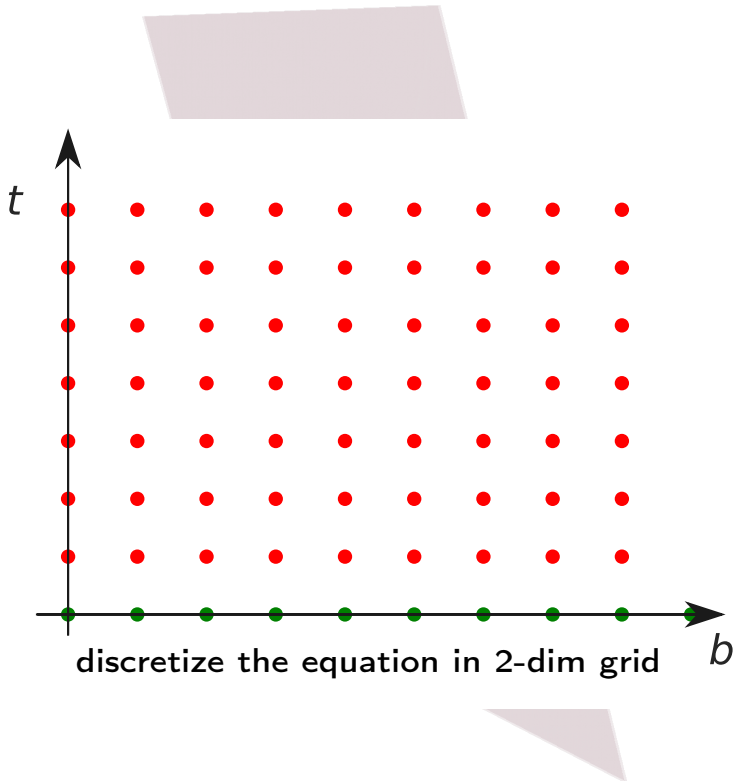


The BMS equation can be solved as coupled ODE system with the method of lines

Linearized BMS equation

Re-organize the BMS equation according to [Banfi, Marchesini and Smye, hep-ph/0206076]

$$\partial g_{ab}(L) = \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j g_{ab}(L)\left[U_{abj}(L) - 1\right] + \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}\left[g_{aj}(L)\, g_{jb}(L) - g_{ab}(L)\right]$$

Then expand it order by order up to 4-loop

$$\partial g_{ab}^{(2)}(L) = \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}^{(1)}(L)$$

$$\partial g_{ab}^{(3)}(L) = \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}^{(2)}(L) + \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j \left[g_{aj}^{(2)}(L) + g_{jb}^{(2)}(L) - g_{ab}^{(2)}(L)\right]$$

$$\begin{aligned}\partial g_{ab}^{(4)}(L) = &\int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}^{(3)}(L) + \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}^{(1)}(L)\, g_{ab}^{(2)}(L) \\ &+ \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}^{(1)}(L)\left[g_{aj}^{(2)}(L) + g_{jb}^{(2)}(L) - g_{ab}^{(2)}(L)\right] \\ &+ \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j \left[g_{aj}^{(3)}(L) + g_{jb}^{(3)}(L) - g_{ab}^{(3)}(L)\right]\end{aligned}$$

- Relation with BFKL evolution and dressed gluon approximation
- Fixed-order expansion up to 8-loop, with the analytical solution from [Schwartz and Zhu, 1403.4949]

Check with the fixed-order expansion

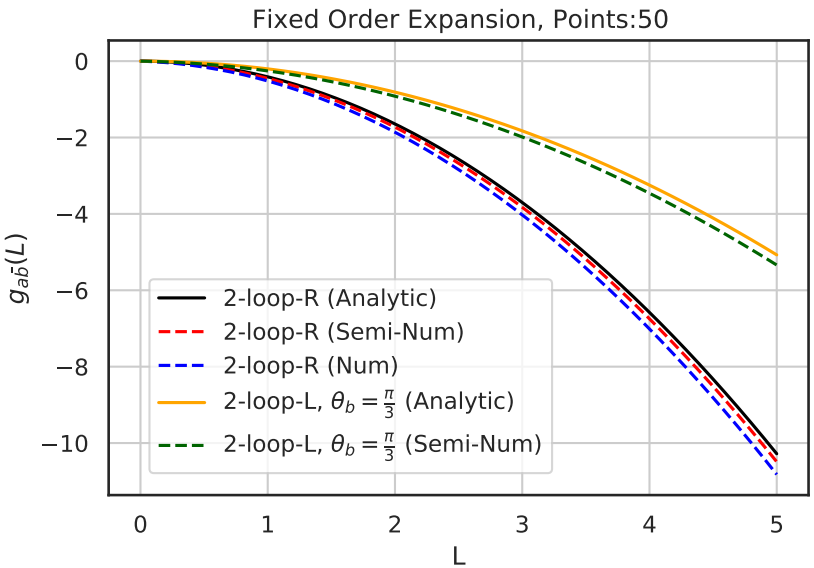
- Fixed-order result for $g_{n\bar{n}}$: [Schwartz and Zhu, 1403.4949]

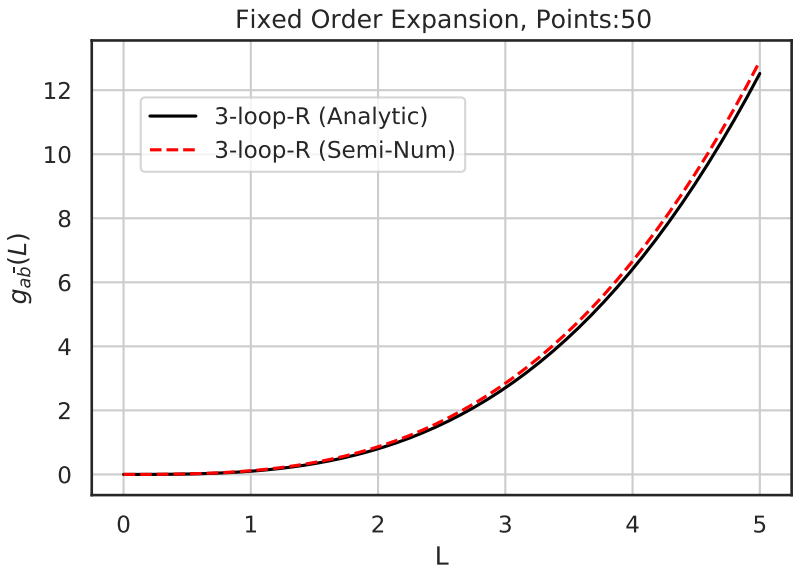
$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(\frac{17\zeta(5)}{480} - \frac{\pi^2\zeta(3)}{360}\right)L^5 + \cdots$$

$$L = \frac{\alpha_s}{\pi}N_c log\frac{m_L}{m_R}$$

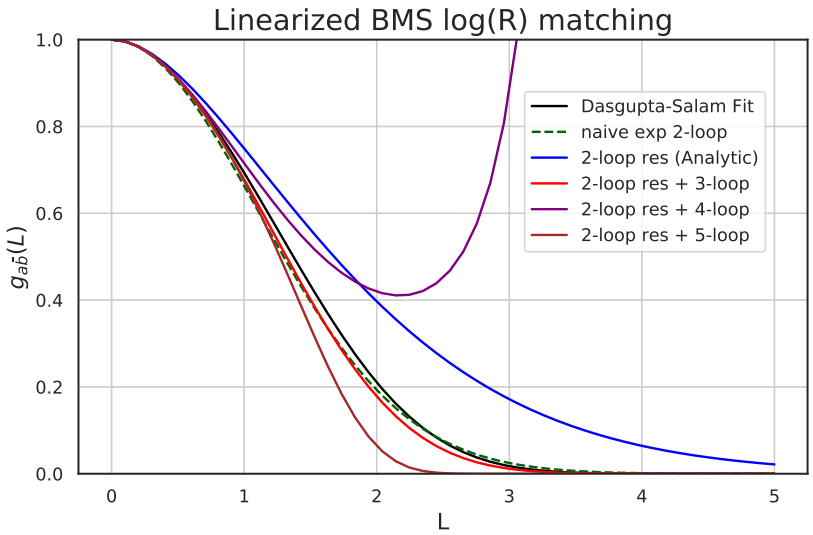
- a and b in left hemisphere:

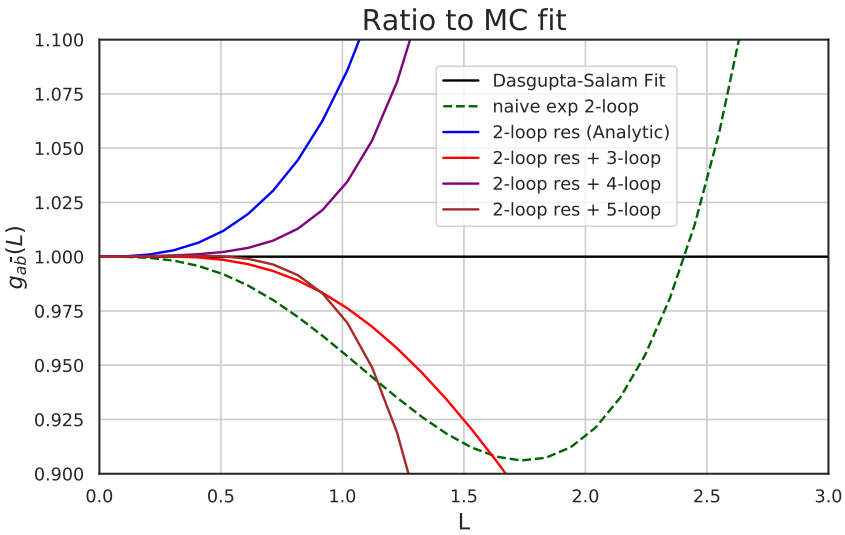
$$\frac{1}{L^2}g_{ab}^{(2)}(x) = \frac{1}{4}logxlog(1+x) - \frac{1}{8}log^2(1+x) + \frac{1}{4}Li_2(-x)$$





Partially resummed result of BMS





The analytical result is

$$g_{n\bar{n}}^{(2R)}(L) = exp^{-\frac{1}{2}(\gamma_E L + log\Gamma(1+L))}$$

$$= 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{6}L^3 - \frac{\pi^4}{1920}L^4 + O\left(L^5\right)$$

For 3-loop result, from the second part

$$\int_L \frac{d\Omega_j}{4\pi} W_{n\bar{n}}^j \left[g_{nj}^{(2)}(L) + g_{j\bar{n}}^{(2)}(L) - g_{n\bar{n}}^{(2)}(L)\right] = \int_L \frac{d\Omega_j}{4\pi} W_{\bar{n}n}^j g_{\bar{n}j}^{(2)}(L) = -\frac{\zeta(3)}{4}L^2$$

Comparison with dressed-gluon

Dressed gluon from BMS expansion

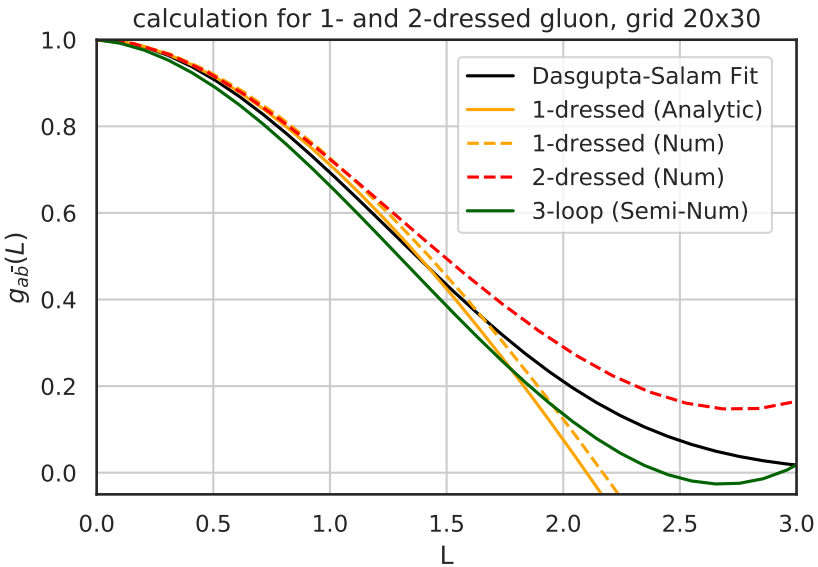
$$g_{ab} = 1 + g_{ab}^{(1)} + g_{ab}^{(2)} + \ldots$$

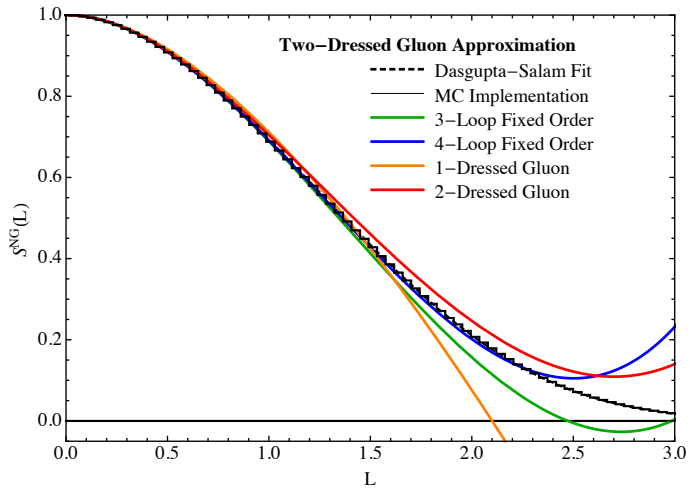
For one and two dressed gluon:

$$\partial_L g_{ab}^{(1)}(L) = \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j \left(U_{abj}(L) - 1\right)$$

$$\partial_L g_{ab}^{(2)}(L) = \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j U_{abj}(L) \left[g_{aj}^{(1)}(L) + g_{jb}^{(1)}(L)\right] - \int_L \frac{d\Omega_j}{4\pi} W_{ab}^j g_{ab}^{(1)}(L)$$

Compare with the original article: solve the 1-dressed gluon twice, and insert to the formula of 2-dressed gluon

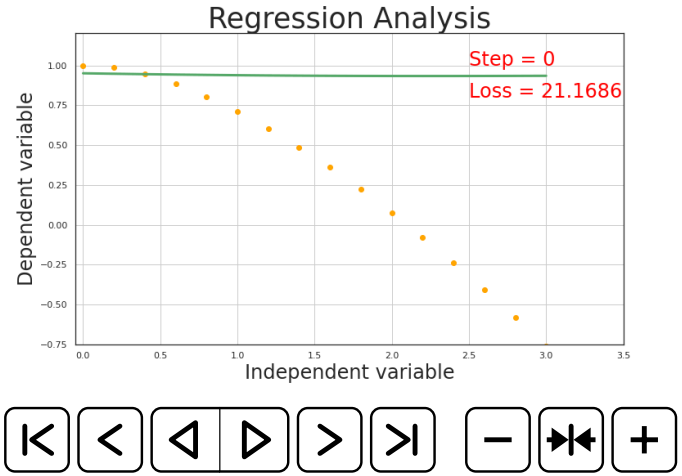




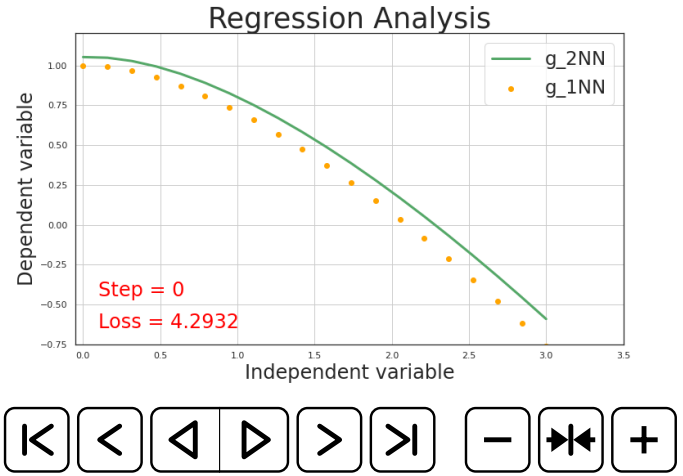
[Larkoski, Moult, and Neill, 1501.04596]

Preliminary NN results

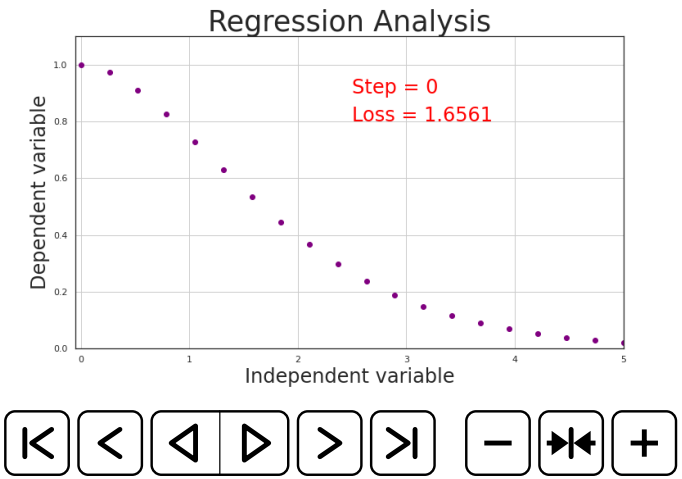
In collaboration with Marco Bellagente, Simone Marzani, and Michael Spannowsky.



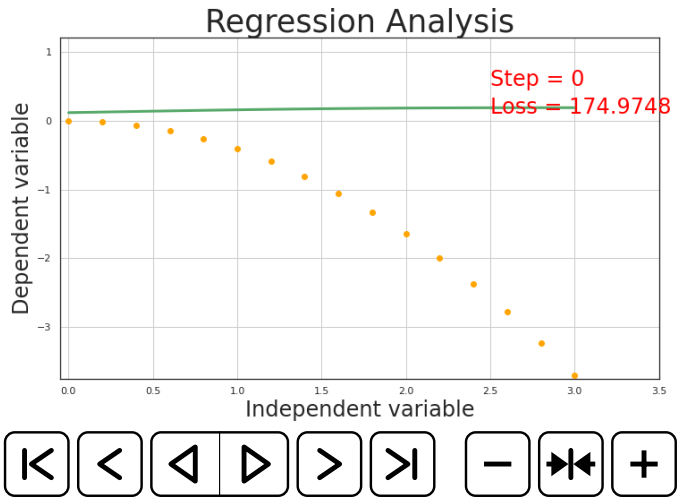
Calculation for 1-dressed gluon



Calculation for 2-dressed gluon



Calculation for partial resummation



Calculation for 2-loop R

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Safe uses of jet pull

The new variable $t_{\parallel}$ and $t_{\perp}$ are IRC safe: analog from a_T distribution

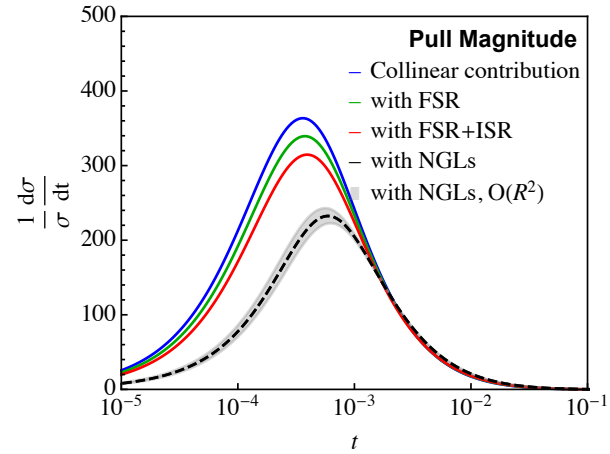
[Banfi, Dasgupta and Duran Delgado, 0909.5327]

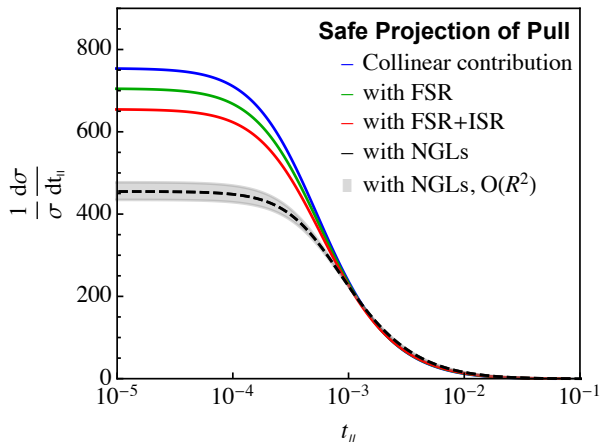
$$a_T = |\sum_i k_{ti} sin\phi_i|,\ t_{\parallel} = |\sum_i t_i cos\phi_i|$$

- The magnitude of safe projection leads to a loss of information
- Fully exploit the radiation pattern: difference between towards and away from the other jet

$$\mathcal{A}_{\parallel} = \frac{t_{\parallel}}{\sigma}\frac{d\sigma}{dt_{\parallel}}\Big|_{\vec{t}\cdot\hat{n}_{\parallel}>0} - \frac{t_{\parallel}}{\sigma}\frac{d\sigma}{dt_{\parallel}}\Big|_{\vec{t}\cdot\hat{n}_{\parallel}<0},$$

$$\mathcal{A}_{\perp} = \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}>0} - \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}<0}$$





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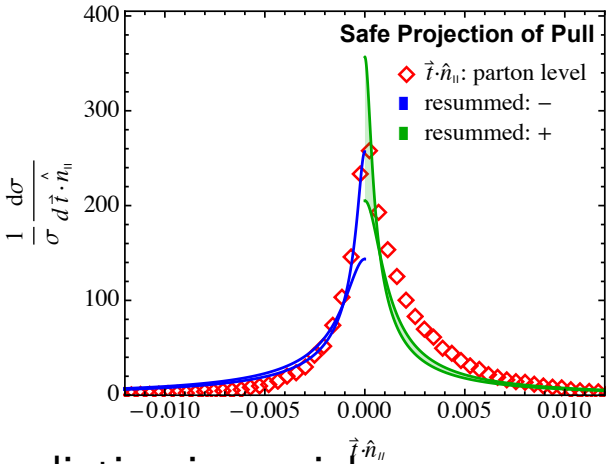

The soft radiation is crucial:

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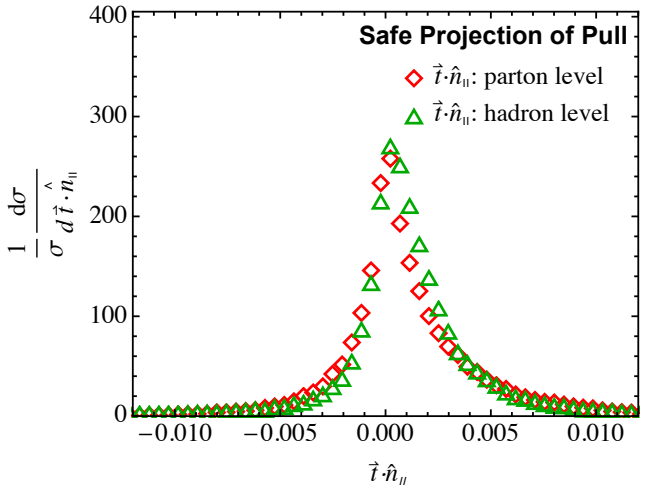

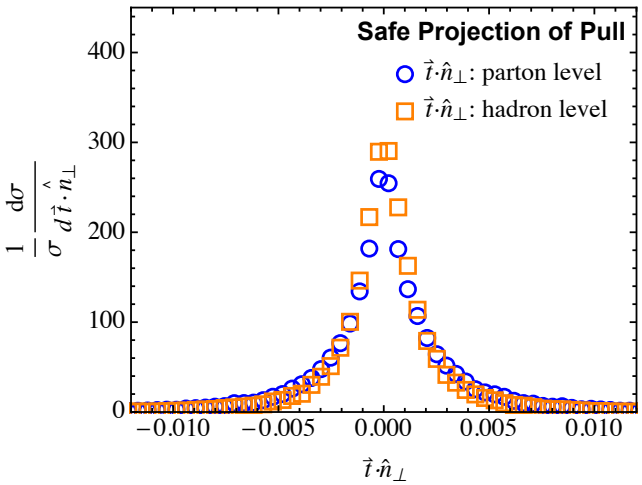

Safe uses of jet pull

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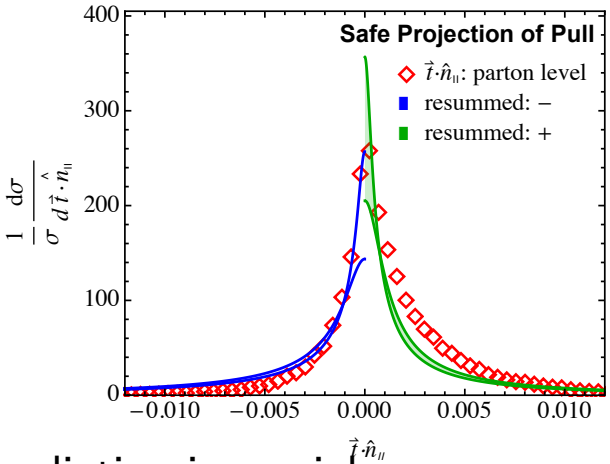

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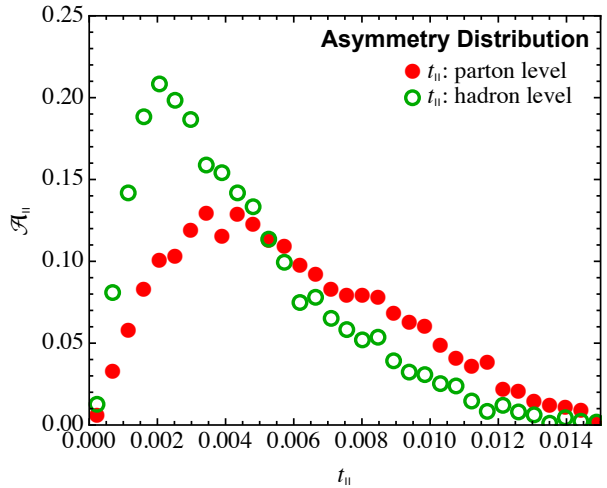

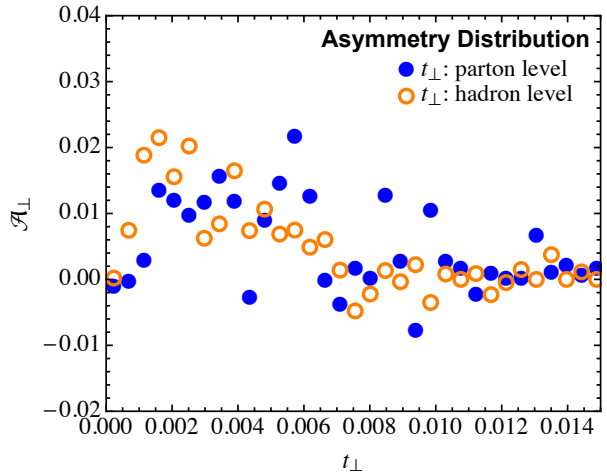

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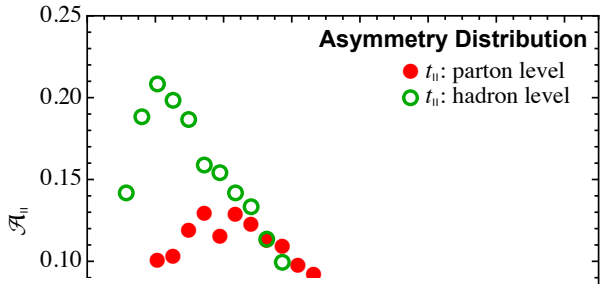

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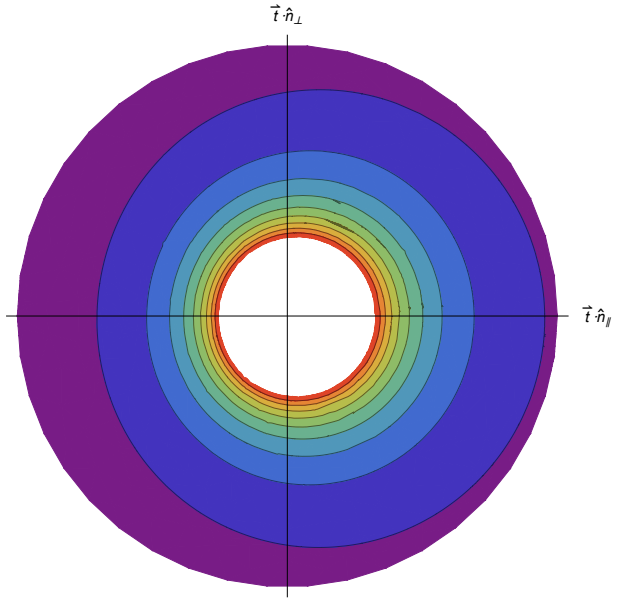

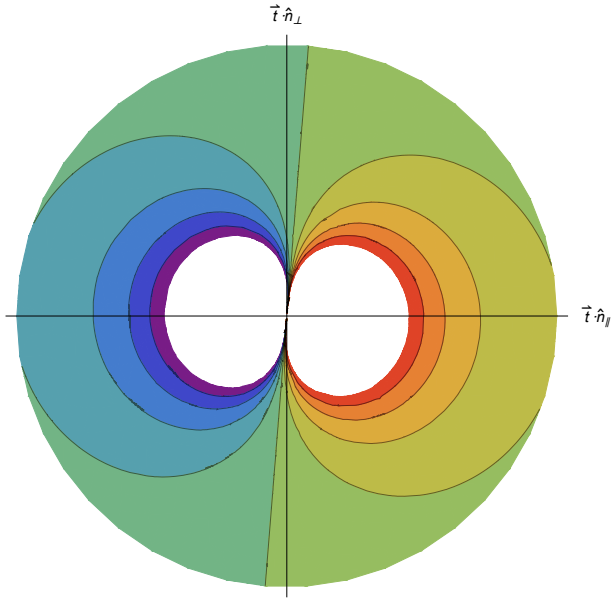

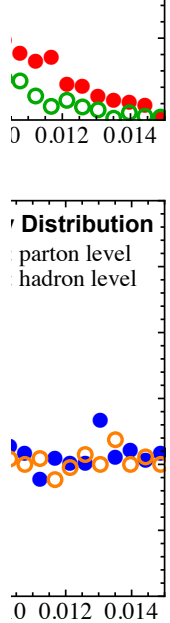

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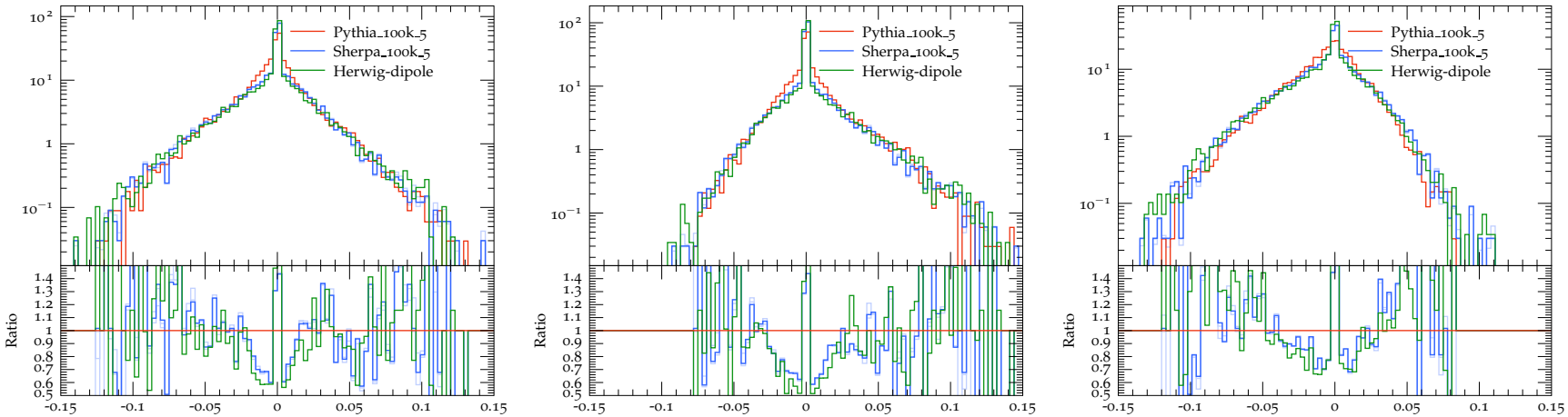
Asymmetries for $e^+e^- \to$ three jets

In collaboration with Daniel Reichelt and Steffen Schumann.

- Sub-leading colour correction: $1/N_c^2 \sim 10\%$
- Fully exploit the radiation pattern for b-, $\bar{b}$- and gluon jet, from left to right respectively



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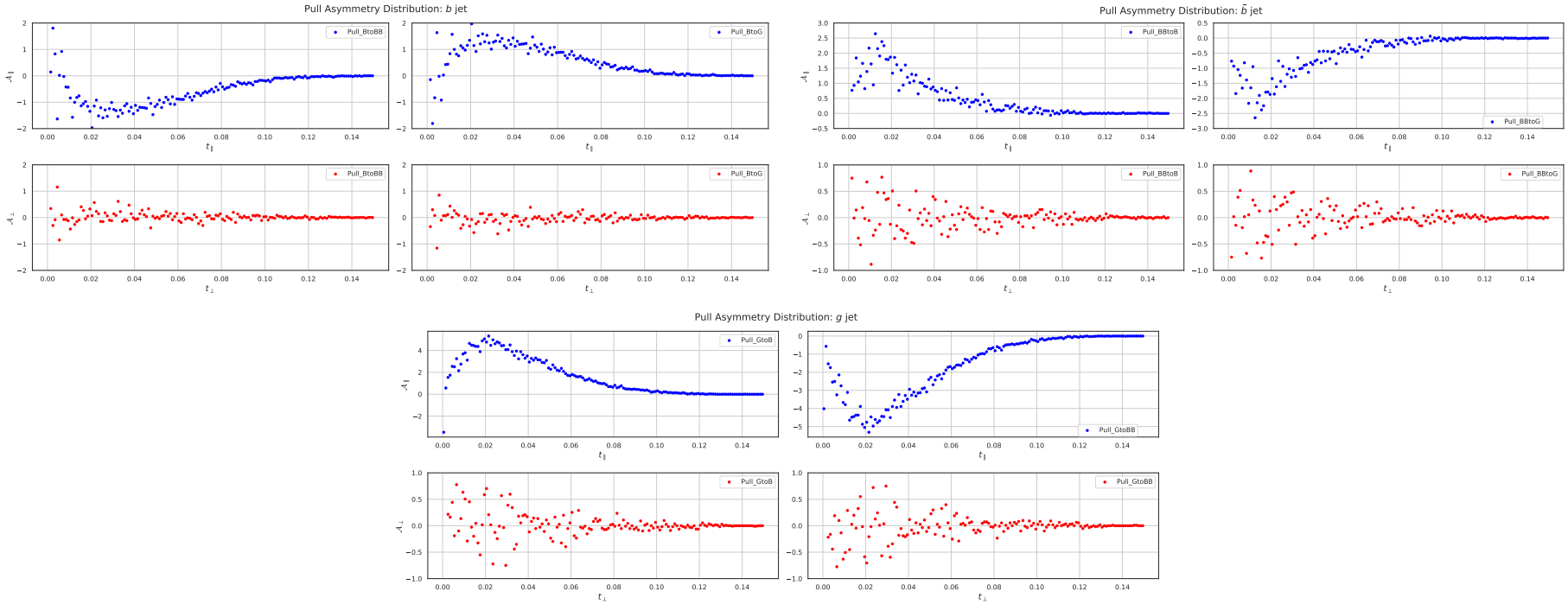

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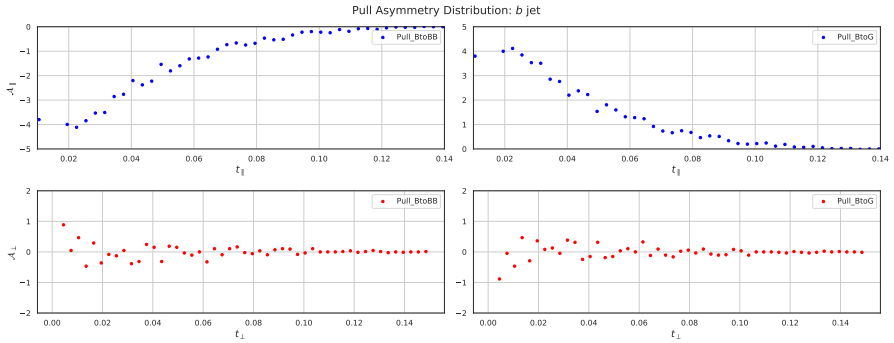
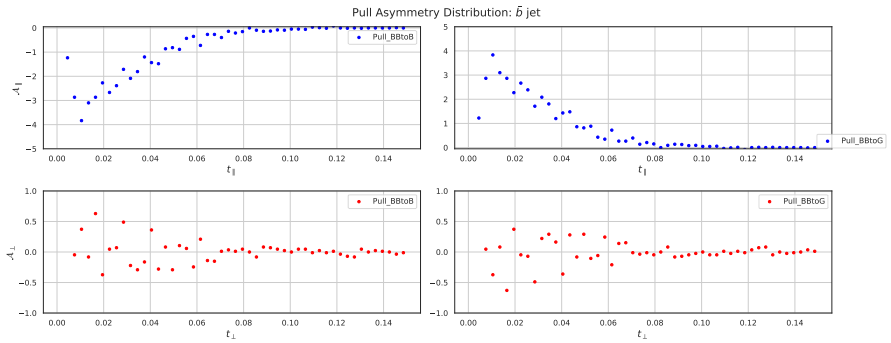
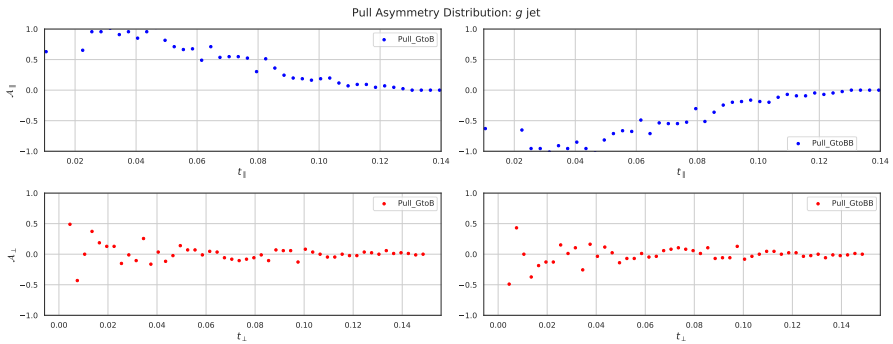
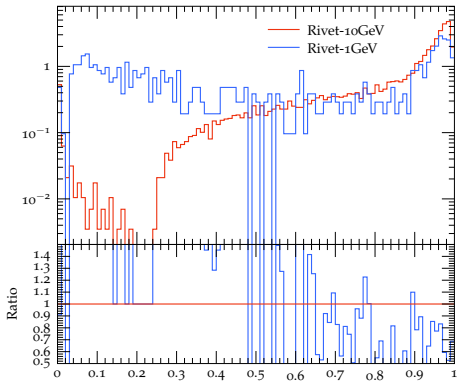








Photon energy ratio: $z_\gamma = \dfrac{E_\gamma}{E_{Jet}}$

Outline

1. Calculation for non-global logarithms
   - Introduction: history of NGLs
   - Symmetries of the BMS equation
   - Preliminary results
2. Soft radiation beyond leading colour
   - Soft radiation sensitive observable
   - Asymmetries for $e^+e^- \to$ three jets
3. Conclusions and outlook

Conclusions and Outlook

Summary:

- Non-global logarithms provide an important contribution to precision calculations
- Fully exploit the radiation pattern can be achieved with the azimuthal asymmetry
- Asymmetries can play an important role in assessing subleading colour correlations

Work in Progress:

- The asymmetry introduces a new boundary in phase-space which renders the all-order structure of these observables richer
- ME+PS merging for the Monte-Carlo study
- Numerical resummation of non-global observables with the full color
- Extend BMS equation with finite N_c

Thank you for your attention

Extra Slides

Perturbative expansion of the BMS equation

Leading NGLs arise from the strongly ordered gluon emissions

$$E_1 \gg E_2 \gg E_3 \gg \cdots \gg E_n$$

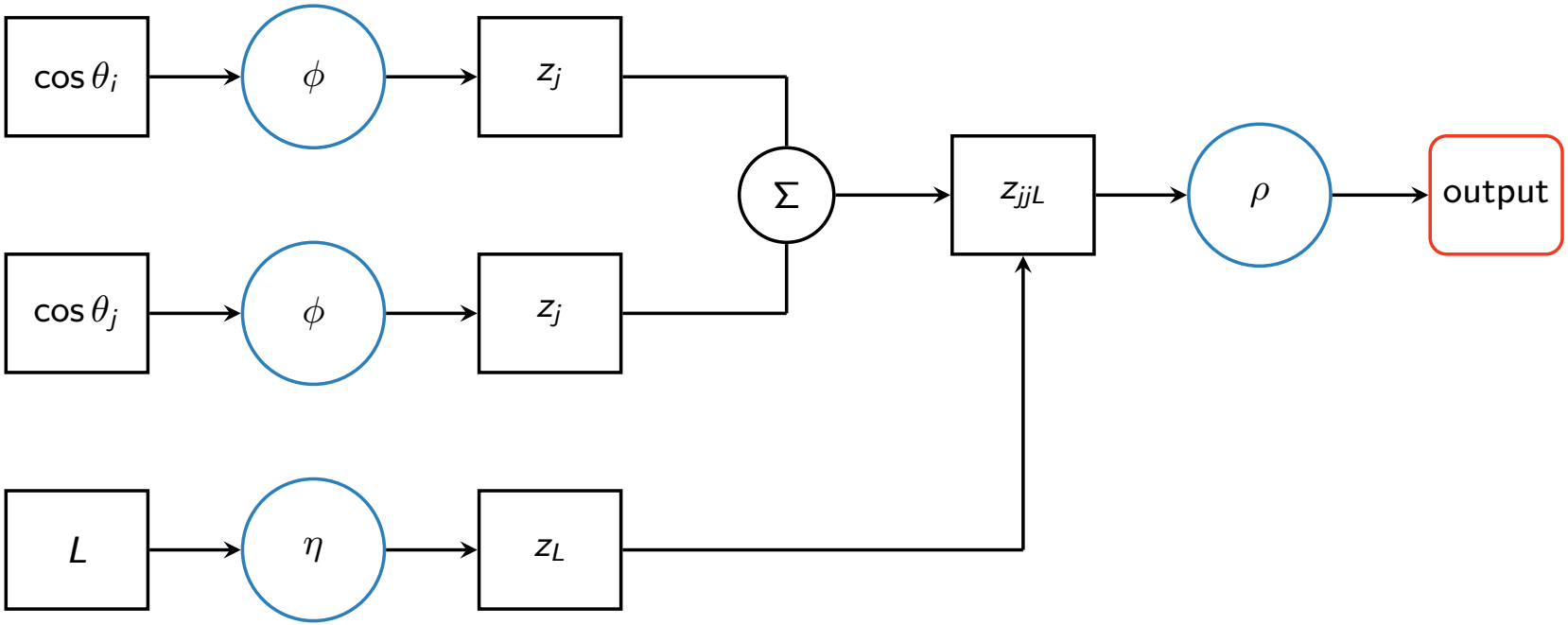
The multi-gluon emission amplitude is simplified, and at large N_c limit:

$$\begin{aligned}\left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 &= \left|\left\langle p_1 \cdots p_m \left| Y_a^\dagger Y_b \right| 0 \right\rangle\right|^2 \\ &= N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)(p_1 \cdot p_2)\cdots(p_m \cdot p_b)}\end{aligned}$$

And the expansion of BMS at 3-loop reads:

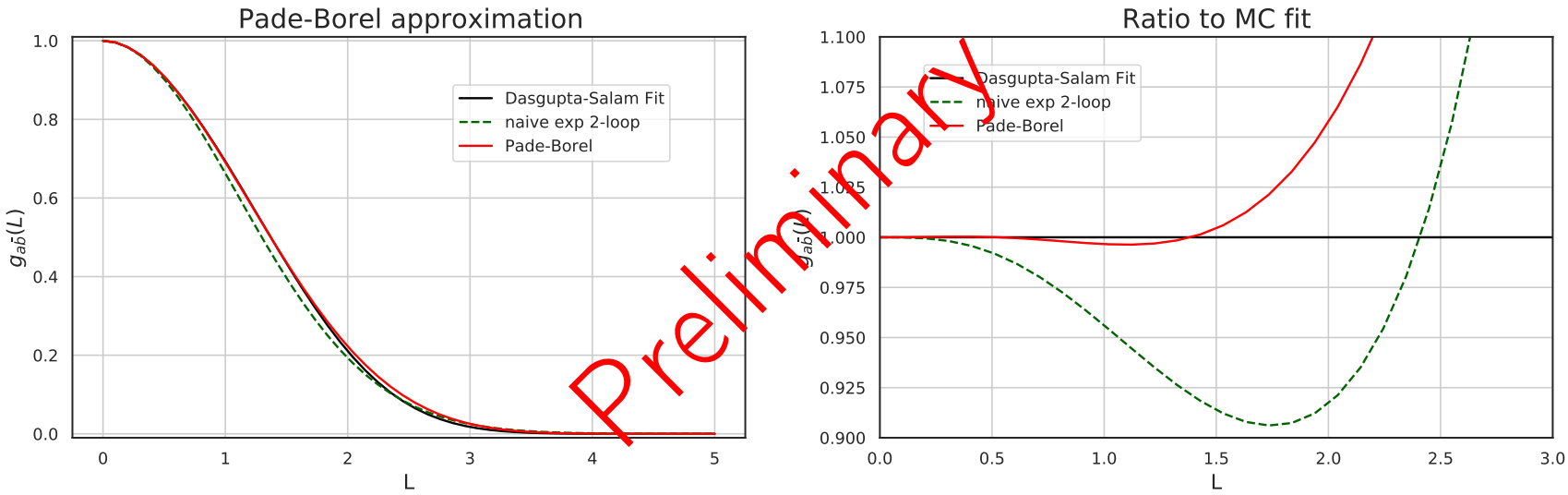
$$\begin{aligned}\partial g_{ab}^{(3)}(L) = \frac{L^2}{2}\int_\Omega 1_L 2_R 3_R W_{ab}^1 \left(W_{ab}^2 - W_{a1}^2 - W_{1b}^2\right)\left(W_{ab}^3 - W_{a1}^3 - W_{1b}^3\right) \\ + \int_\Omega 1_L W_{ab}^1 \left[g_{a1}^{(2)}(L) + g_{1b}^{(2)}(L) - g_{ab}^{(2)}(L)\right]\end{aligned}$$

Model used to solving BMS



Padé approximation and outlook

Finally, we want recover the large order behaviour:



This result agrees with [Larkoski, Moult, and Neill, 1609.04011]

Next Step:

- fixed-order result up to 7-loop (arbirary hemisphere) and 8-loop (back to back)
- full solution of BMS equation via NN

Future:

- apply to RGE and resummation formula
- exploit the BMS-BK duality
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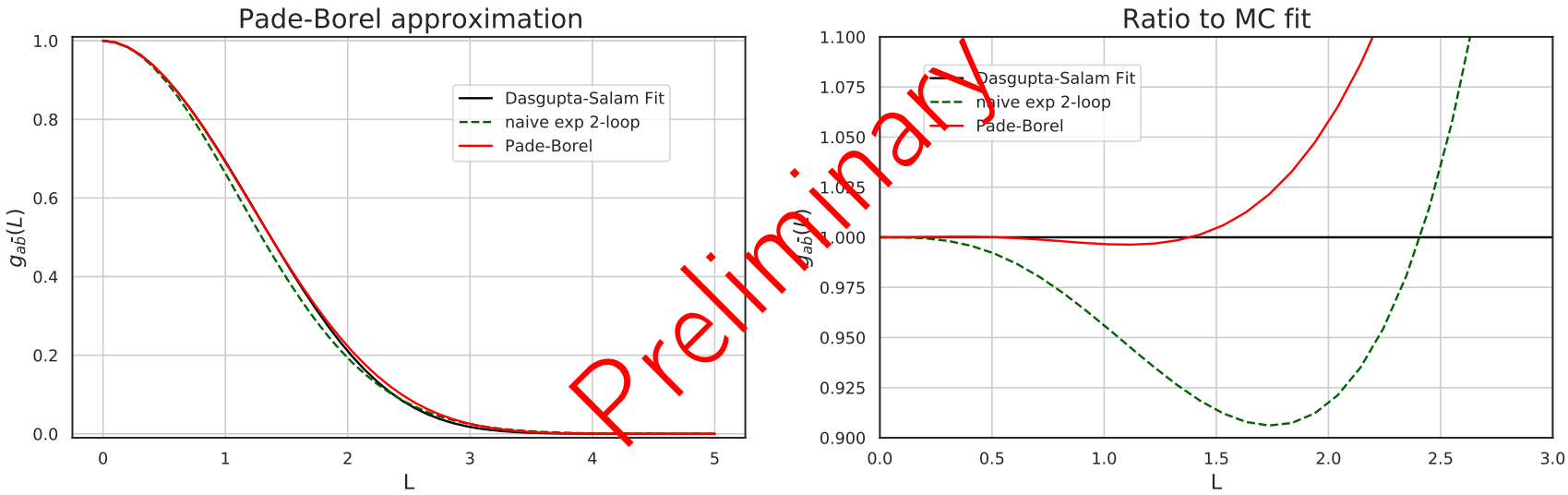

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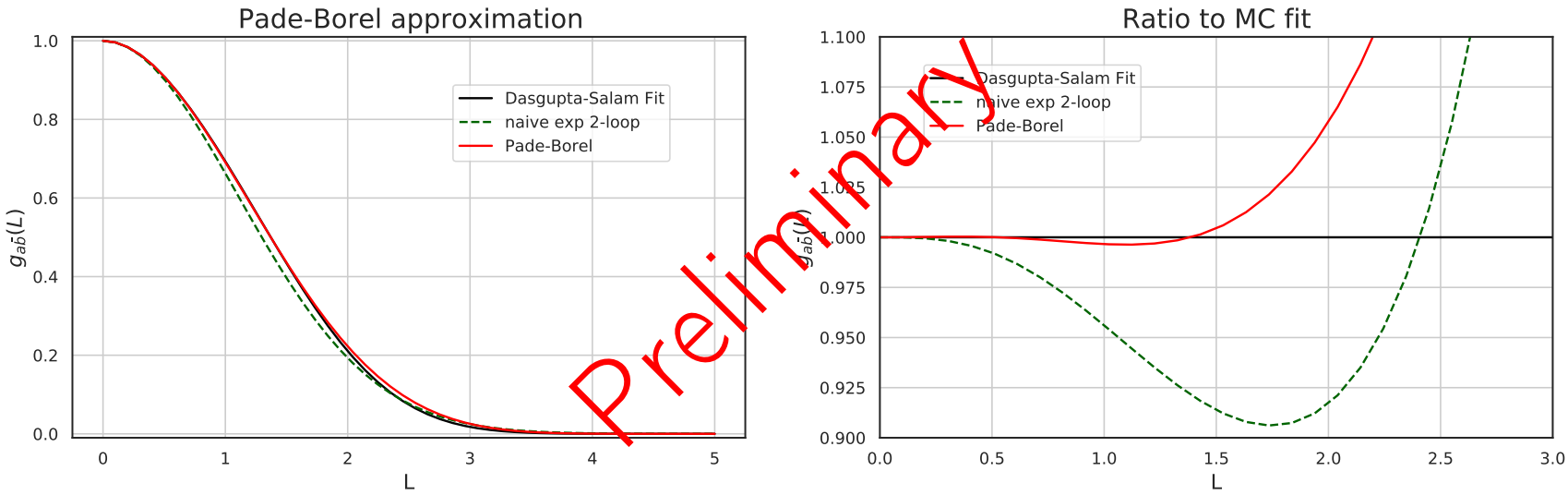

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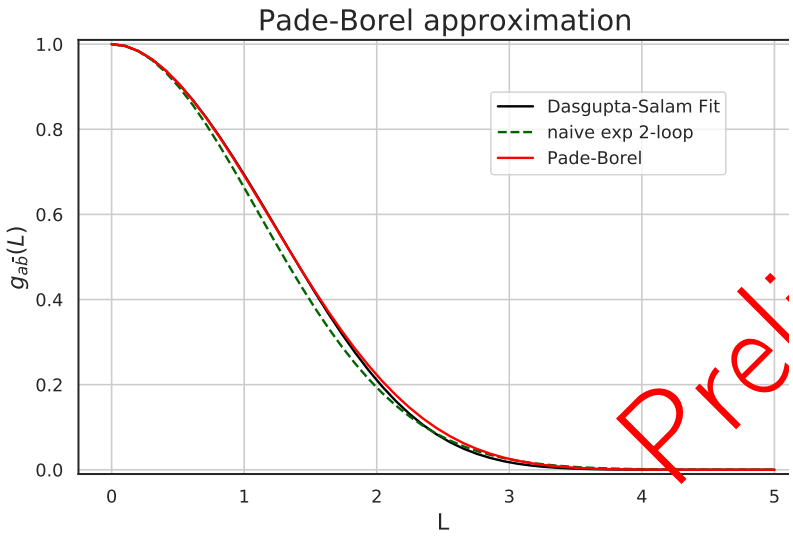

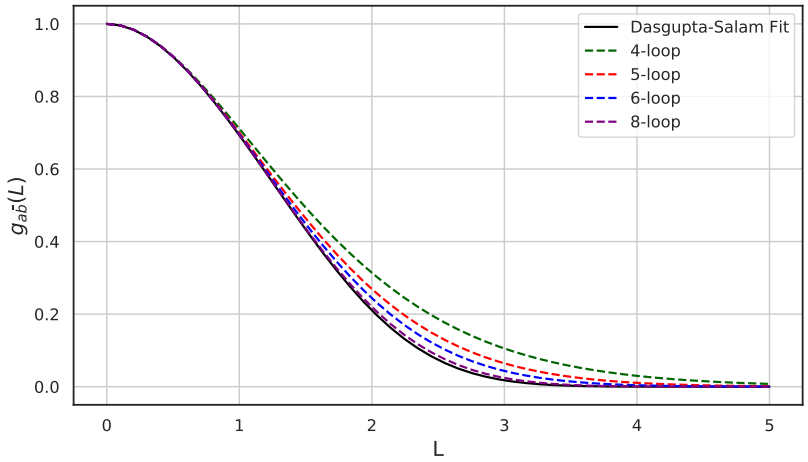

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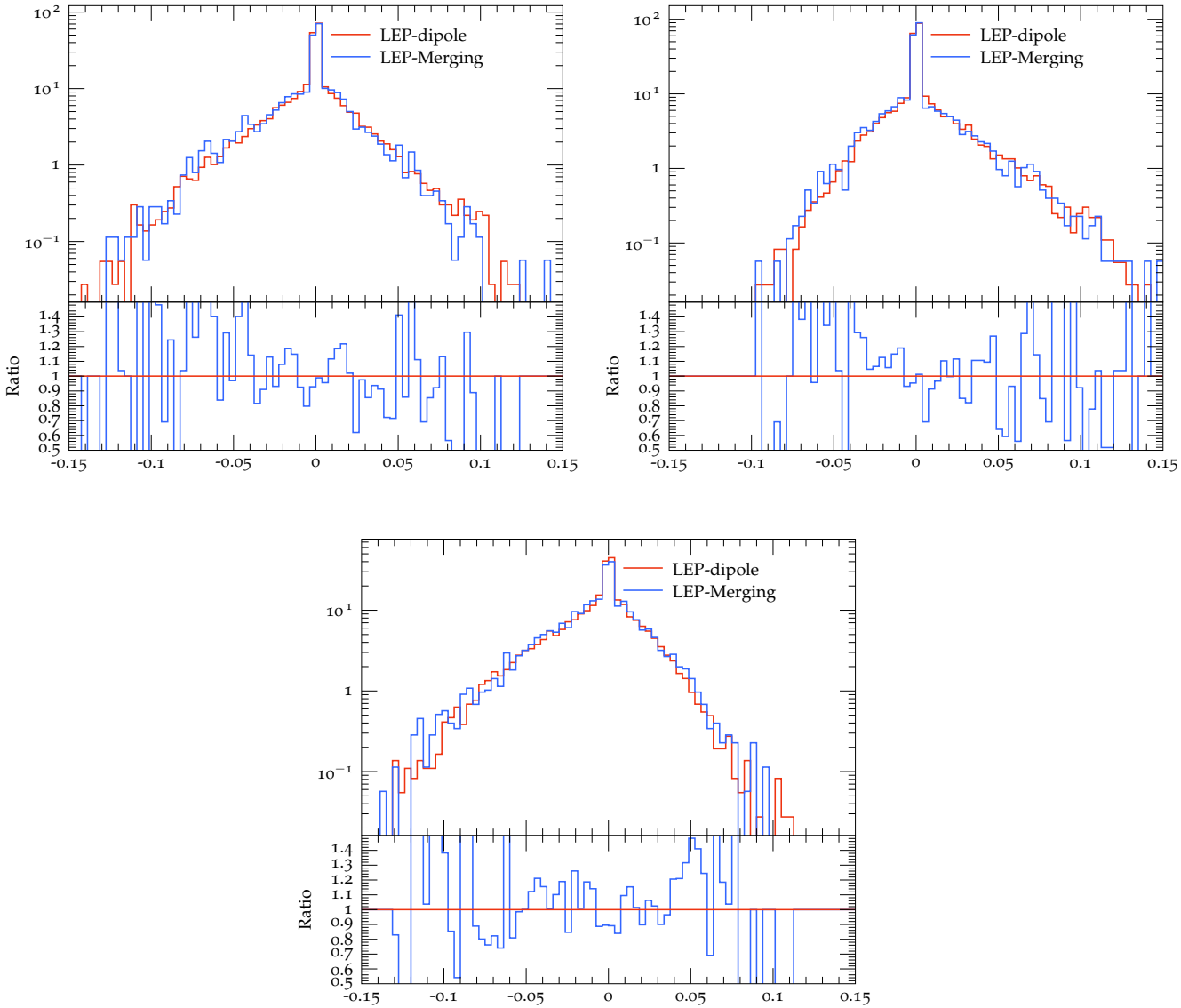
ME+PS merging



ME+PS merging of the asymmetry distributions measured on $e^+e^- \to b\bar{b}g$ events at parton level, generated with Herwig v7.1.0