Interplay between perturbative and non-perturbative effects with the ARES method

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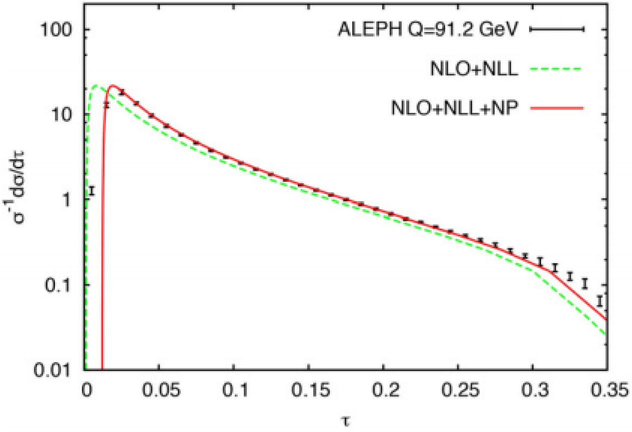
PSR21 - Parton Showers and Resummation

Motivation:

- Introduce a new method to compute leading hadronisation corrections to two-jet event shapes in e^+e^- annihilation.

- Hadronisation provides a shift in perturbative event-shape distributions:

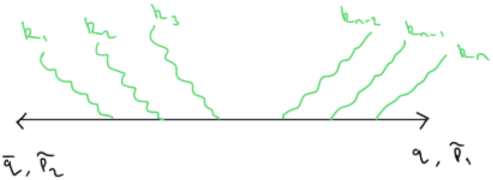


- Consider a generic recursive infrared and collinear safe (rIRC) observable in e^+e^- annihilation:

$$V\left(\{\tilde{p}\}, k_1, ..., k_n\right) \geqslant 0$$

- $\{\tilde{p}\} = \{\tilde{p}_1, \tilde{p}_2\}$ are the momenta of a hard quark-antiquark pair

- $k_1, ..., k_n$ are the subsequent emissions



- In the Born limit $V\left(\{\tilde{p}\}\right) = 0$
- We shall consider the region in which:

$$V\left(\{\tilde{p}\}, k_1, \ldots, k_n\right) = v \ll 1$$

- The observable cumulant is given by:

$$\Sigma(v) = \Sigma_{\mathrm{PT}}(v) + \delta\Sigma_{\mathrm{NP}}(v)$$

- We know that:

$$\Sigma_{\mathrm{PT}}(v) = e^{-R_{\mathrm{NLL}}(v)}\mathcal{F}_{\mathrm{NLL}}(v)$$

where:

$$\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta\left(1 - \frac{V_{\mathrm{sc}}\left(\{\tilde{p}\}, k_1, \ldots, k_n\right)}{v}\right)\right\rangle$$

- Leading hadronisation corrections have been successfully modelled so far as being due to the contribution of a very soft gluer (with momentum k)

- We define:

$$\delta V_{\mathrm{NP}} \equiv V\left(\{\tilde{p}\}, k, \{k_i\}\right) - V\left(\{\tilde{p}\}, \{k_i\}\right)$$

- We find that:

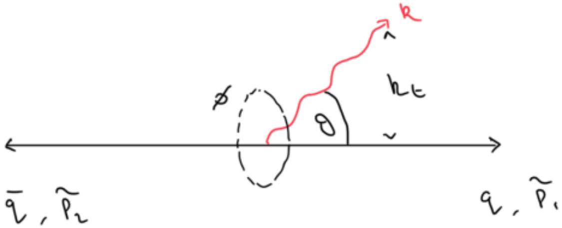
$$\delta \Sigma_{\mathrm{NP}} = -\left\langle \delta V_{\mathrm{NP}} \right\rangle \frac{d\Sigma_{\mathrm{PT}}}{dv}$$

- This gives us that:

$$\Sigma(v) = \Sigma_{\mathrm{PT}}\left(v - \left\langle \delta V_{\mathrm{NP}} \right\rangle\right)$$

- We shall restrict ourselves to event shapes for which:

$$\delta V_{\mathrm{NP}}\left(\{\tilde{p}\}, k, \{k_i\}\right) = \frac{k_t}{Q} f_V\left(\eta, \phi, \{k_i\}\right)$$



- Therefore:

$$\langle \delta V_{\mathrm{NP}} (\{\tilde{p}\}, k, \{k_i\}) \rangle = \frac{\langle k_t \rangle}{Q} c_V$$

with:

$$\boxed{c_{\mathrm{V}} = \frac{\left\langle f_{\mathrm{V}} (\eta, \phi, \{k_i\}) \, \delta \left(1 - \frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}\right) \right\rangle}{\left\langle \delta \left(1 - \frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}\right) \right\rangle}}$$

- The denominator may be written as $R'\mathcal{F}(R')$ with:

$$R' \equiv -v \frac{dR}{dv}$$

Method of computation:

- In a form suitable for evaluation numerically by a Monte Carlo procedure we can write:

$$
\begin{aligned}
&R'\mathcal{F}(R')c_{\mathrm{V}} \\
&= \int d\eta \frac{d\phi}{2\pi} R' \int_0^{2\pi} \frac{d\phi_1}{2\pi} \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} R' \int_{\epsilon}^{1} \frac{d\tilde{\zeta}_i}{\tilde{\zeta}_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \times \\
&\times \exp\left(-R' \ln \frac{V_{\mathrm{sc}}\left(\{\tilde{p}\}, \tilde{k}_1, \ldots, \tilde{k}_{n+1}\right)}{v} \right) f_{\mathrm{V}}\left(\eta, \phi, k_1, ..., k_n\right)
\end{aligned}
$$

- Where in $f_V(\eta, \phi, k_1, \ldots, k_n)$ the k_i's are functions of $\tilde{\zeta}_i$.

Additive Observables:

- For additive observables we have:

$$f_{\rm V}\left(\eta,\phi,k_1,\ldots,k_n\right)=f_{\rm V}\left(\eta,\phi\right)$$

$$c_{\rm V}=\int d\eta\frac{d\phi}{2\pi}f_{\rm V}\left(\eta,\phi\right)$$

- Example 1: Thrust

$$T\equiv\max_{\vec{n}}\frac{\sum_i\left|\vec{p}_i\cdot\vec{n}\right|}{Q}\quad,\quad\tau\equiv1-T$$

$$c_\tau=2$$

- Example 2: C-parameter

$$C\equiv3\left(1-\frac{1}{2}\sum_{i,j}\frac{\left(p_i\cdot p_j\right)^2}{\left(p_i\cdot Q\right)\left(p_j\cdot Q\right)}\right)$$

$$c_{\rm C}=3\pi$$

Heavy Jet Mass:

- Observable definition:

$$\rho_{\mathrm{H}} \equiv \max_{i=1,2} \frac{M_i^2}{Q^2} \quad , \quad M_i^2 \equiv \left(\sum_{j \in \mathcal{H}^{(i)}} p_j \right)^2$$

- A non-zero hadronisation correction to the heavy-jet mass arises only when the NP gluon is emitted in the heavier hemisphere. Therefore:

$$f_{\rho_H}(\eta, \phi, k_1, \ldots, k_n) = e^{-\eta}\Theta(\eta)\Theta(\rho_1 - \rho_2) + e^{\eta}\Theta(-\eta)\Theta(\rho_2 - \rho_1)$$

- We find that:

$$c_{\rho_H} = 1$$

Broadening-like observables:

- Broadening-type observables:

$$B_{\mathrm{L}} \equiv \sum_{i\in\mathcal{H}^{(1)}} \frac{|\vec{p_i} \times \vec{n_{\mathrm{T}}}|}{2Q} \quad , \quad B_{\mathrm{R}} \equiv \sum_{i\in\mathcal{H}^{(2)}} \frac{|\vec{p_i} \times \vec{n_{\mathrm{T}}}|}{2Q}$$

$$B_T \equiv B_L + B_R \quad , \quad B_W \equiv \max\{B_L, B_R\}$$

- We find that:

$$f_{B_W}(\eta, \phi, k_1, \ldots, k_n)$$
$$= \Theta(\eta^{(1)}) \left[\sqrt{1 + 2e^{\eta^{(1)}} \frac{p_{t,1}}{Q} \cos\phi + e^{2\eta^{(1)}} \left(\frac{p_{t,1}}{Q}\right)^2} - e^{\eta^{(1)}} \frac{p_{t,1}}{Q}\right] \Theta(B_1 - B_2)$$
$$+ 1 \leftrightarrow 2$$

$$f_{B_T}(\eta, \phi, k_1, \ldots, k_n)$$
$$= \sum_{\ell} \Theta(\eta^{(\ell)}) \left[\sqrt{1 + 2e^{\eta^{(\ell)}} \frac{p_{t,\ell}}{Q} \cos\phi + e^{2\eta^{(\ell)}} \left(\frac{p_{t,\ell}}{Q}\right)^2} - e^{\eta^{(\ell)}} \frac{p_{t,\ell}}{Q}\right]$$

Broadening-like observables:

- Analytical expressions for c_{B_W} and c_{B_T} have been computed [Dokshitzer, Marchesini and Salam hep-ph/9812487v3]:

$$c_{B_W} = \frac{1}{2}\left[-2 - \psi(1) - \ln B + \eta_0 + \chi\left(\frac{R'}{2}\right) - \rho\left(\frac{R'}{2}\right) + \psi\left(1 + \frac{R'}{2}\right)\right]$$

$$c_{B_T} = 2c_{B_W} - \psi\left(1 + \frac{R'}{2}\right) + \psi\left(1 + R'\right) + \frac{1}{R'}$$

- c_{B_T} holds in the limit $R' \gg \sqrt{2R''}$

Next Steps:

- Having used the Broadening-like observables as a 'test of concept' we are now applying this method to the Thrust Major.

- Observable definition:

$$T_M \equiv \max_{\vec{n}\cdot\vec{n_T}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{Q}$$

- Thank you for listening

Additional Slides:

- First consider the denominator, $R'\mathcal{F}(R')$:
  - Zero emissions does not satisfy the observable constraint
  - Select the largest emission k_1 such that $V_{\rm sc}(\{\tilde{p}\}, k_1)$ is the largest of all $V_{\rm sc}(\{\tilde{p}\}, k_i)$
  - Neglect all emissions such that $V_{\rm sc}(\{\tilde{p}\}, k_i) < \epsilon V_{\rm sc}(\{\tilde{p}\}, k_1)$
- We find that:

$$\left\langle \delta\left(1 - \frac{V_{\rm sc}\left(\{\tilde{p}\}, \{k_i\}\right)}{v}\right)\right\rangle = \epsilon^{R'} R' \int_0^\infty \frac{d\zeta_1}{\zeta_1}\zeta_1^{R'} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \times$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} R' \int_{\epsilon\zeta_1}^{\zeta_1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi}\, \delta\left(1 - \frac{V_{\rm sc}(\{\tilde{p}\}, k_1, \dots, k_{n+1})}{v}\right)$$

- Introduce $\tilde{\zeta}_i = \frac{\zeta_i}{\zeta_1}$ with corresponding momenta $\tilde{k}_i$ such that $V_{\rm sc}(\{\tilde{p}\}, \tilde{k}_i) = \frac{V_{\rm sc}(\{\tilde{p}\}, k_i)}{\zeta_1}$

- Since our observable is rIRC safe:

$$V_{\rm sc}\left(\{\tilde{p}\}, k_1, ..., k_{n+1}\right) = \zeta_1 V_{\rm sc}\left(\{\tilde{p}\}, \tilde{k}_1, ..., \tilde{k}_{n+1}\right)$$

- We therefore find that:

$$\left\langle \delta\left(1 - \frac{V_{\rm sc}\left(\{\tilde{p}\}, \{k_i\}\right)}{v}\right)\right\rangle$$
$$= \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} R' \int_{\epsilon}^{1} \frac{d\tilde{\zeta}_i}{\tilde{\zeta}_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \times$$
$$\times R' \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_0^{\infty} \frac{d\zeta_1}{\zeta_1} \zeta_1^{R'} \delta\left(1 - \zeta_1 \frac{V_{\rm sc}(\{\tilde{p}\}, \tilde{k}_1, \ldots, \tilde{k}_{n+1})}{v}\right)$$

- Performing the ζ_1 integral gives:

$$\left\langle \delta\left(1 - \frac{V_{\rm sc}\left(\{\tilde{p}\},\{k_i\}\right)}{v}\right)\right\rangle$$
$$= R' \int_0^{2\pi} \frac{d\phi_1}{2\pi} \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} R' \int_\epsilon^1 \frac{d\tilde{\zeta}_i}{\tilde{\zeta}_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \times$$
$$\times \exp\left(-R' \ln \frac{V_{\rm sc}\left(\{\tilde{p}\},\tilde{k}_1,\dots,\tilde{k}_{n+1}\right)}{v}\right)$$

- This may be evaluated numerically using a Monte Carlo procedure