Parton showers beyond leading logarithmic accuracy

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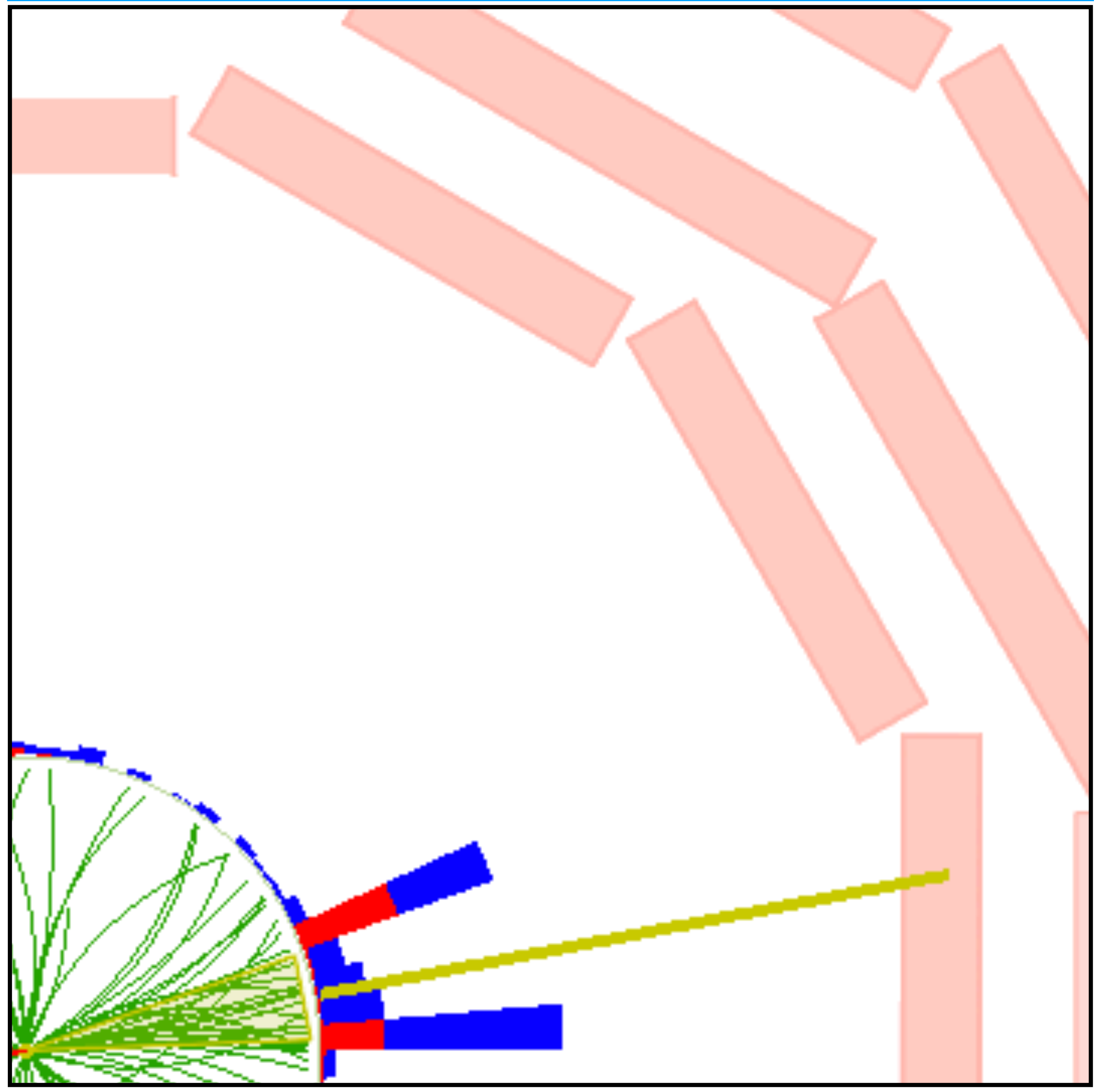
Parton Shower and Resummation - May 2021

Monte Carlo generators at colliders

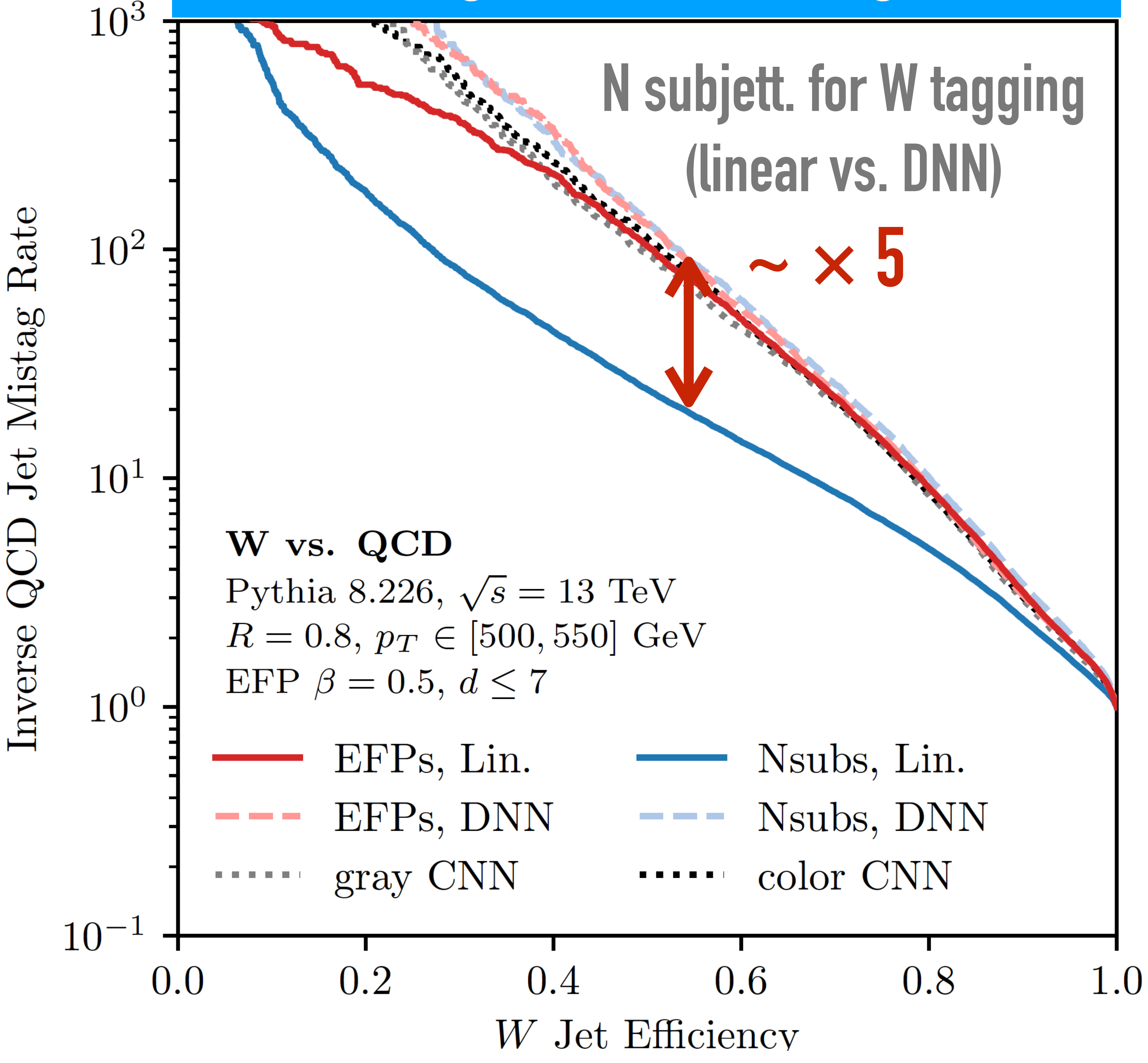
- MCs are vital tools to bridge between TH and EXP at colliders
- Advances in QFT perturbative calculations (fixed/all orders) hard to port to MC generators

[ATLAS '19] see also
[Jaeger, Karlberg, Plaetzer, Scheller, Zaro '20]

e.g. extrapolation & unfolding

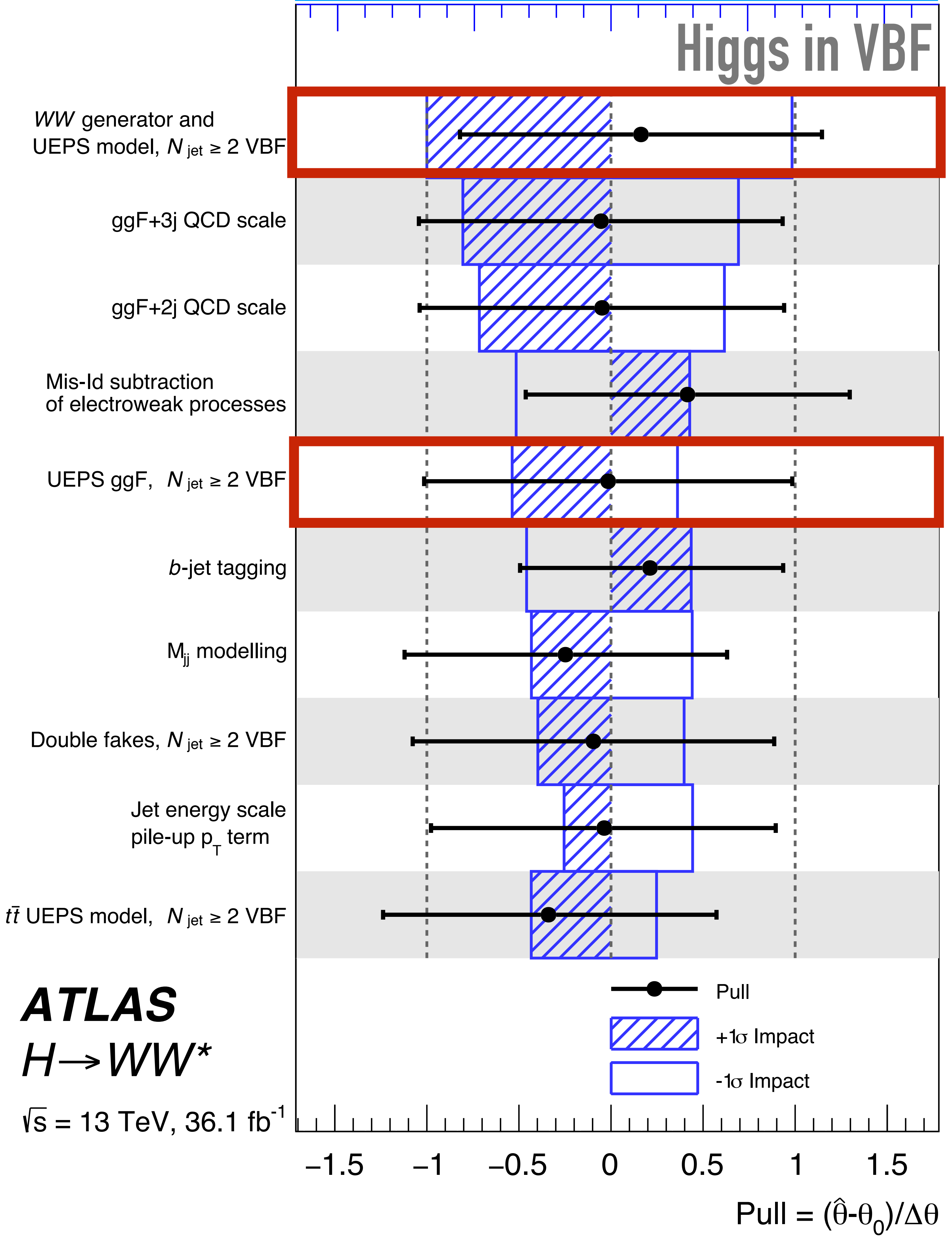


e.g. ML training

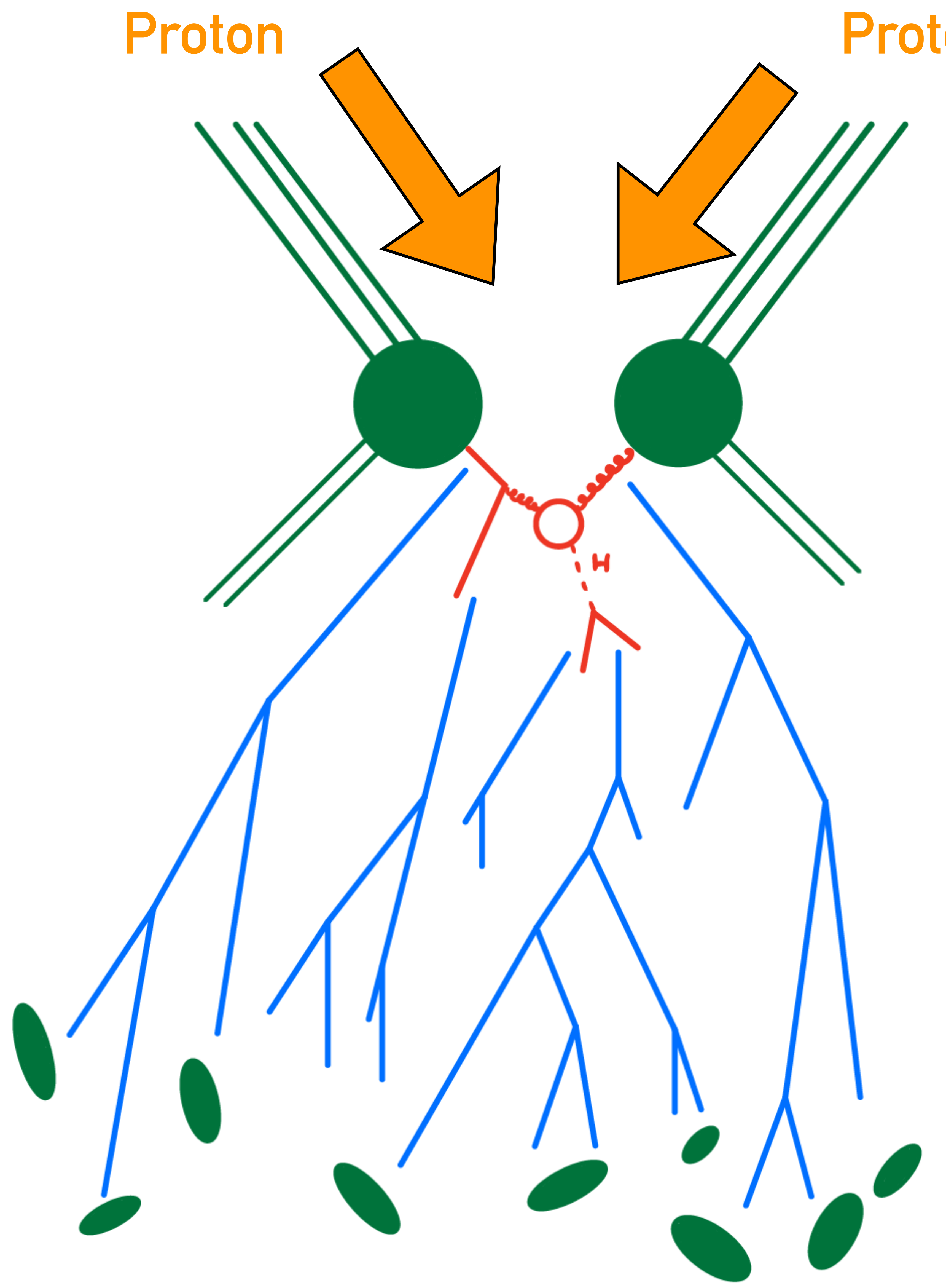


[Komiske, Metodiev, Thaler '17]

e.g. TH/EXP analysis



Monte Carlo generators at colliders



Scale

Hard scattering
(~10^2 - 10^3 GeV)

Hadron
formation
(~ 1 GeV)

Observation

Perturbation theory

Hard scattering: a lot of recent progress [e.g. N(N)LO+PS matching & multi-jet merging]

Parton Shower: multi-scale evolution & large hierarchy of scales: **perturbative accuracy ?**

Modelling of non-perturbative dynamics

Perturbation theory in multi scale regimes

- hierarchy of scales ⇒ perturbative accuracy ≡ logarithmic accuracy

- Two different (perturbatively related) definitions commonly used:

Perturbative order of anomalous dimensions & initial conditions

Counting used in e.g.[Collins, Soper, Sterman ’85; Catani, De Florian, Grazzini ’00, SCET factorisations, …]

Squared amplitudes in the relevant kinematic limits (ordering)

Counting used in e.g. [Catani, Trentadue, Turnock, Webber ’93; Banfi, Salam, Zanderighi ’04; Banfi, PM, Salam, Zanderighi ’12; Banfi, McAslan, PM, Zanderighi ’14; PM, Re, Torrielli ’16, …]

- e.g. cumulative distribution for an observable $v < e^{-L}$

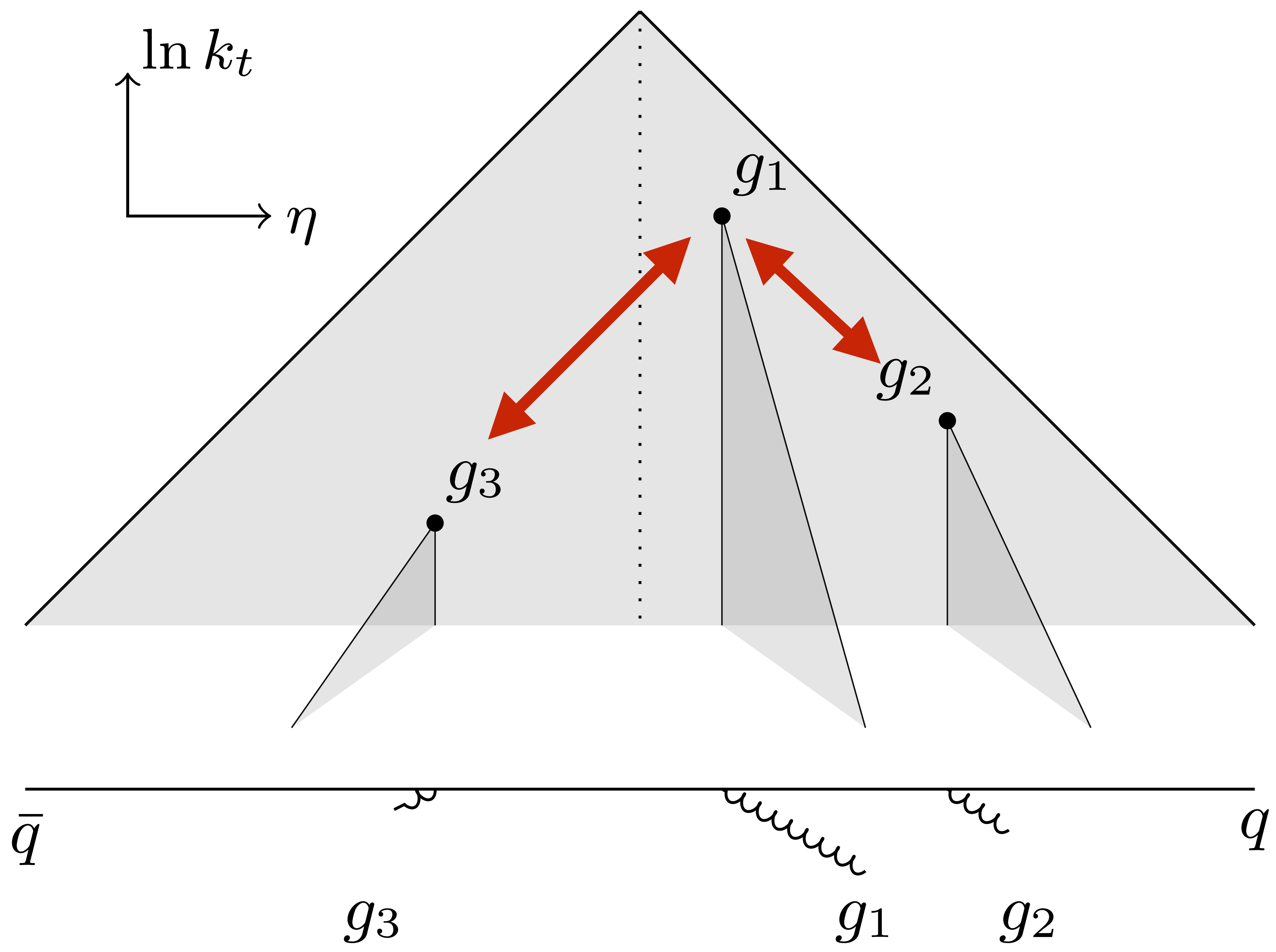
LL (=0 sometimes) NLL

$$\Sigma(\alpha_s, \alpha_s L) = \exp\left[\alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \mathcal{O}\left(\alpha_s^n L^{n-1}\right)\right]$$

A geometric criterion: the Lund plane First introduced in [Andersson, Gustafson, Lonnblad, Pettersson '89]

- Phase space: 3 variables per radiation, two of which are logarithmic, **e.g. $\{k_T, \eta\}$, $\{E, \theta\}$, …**

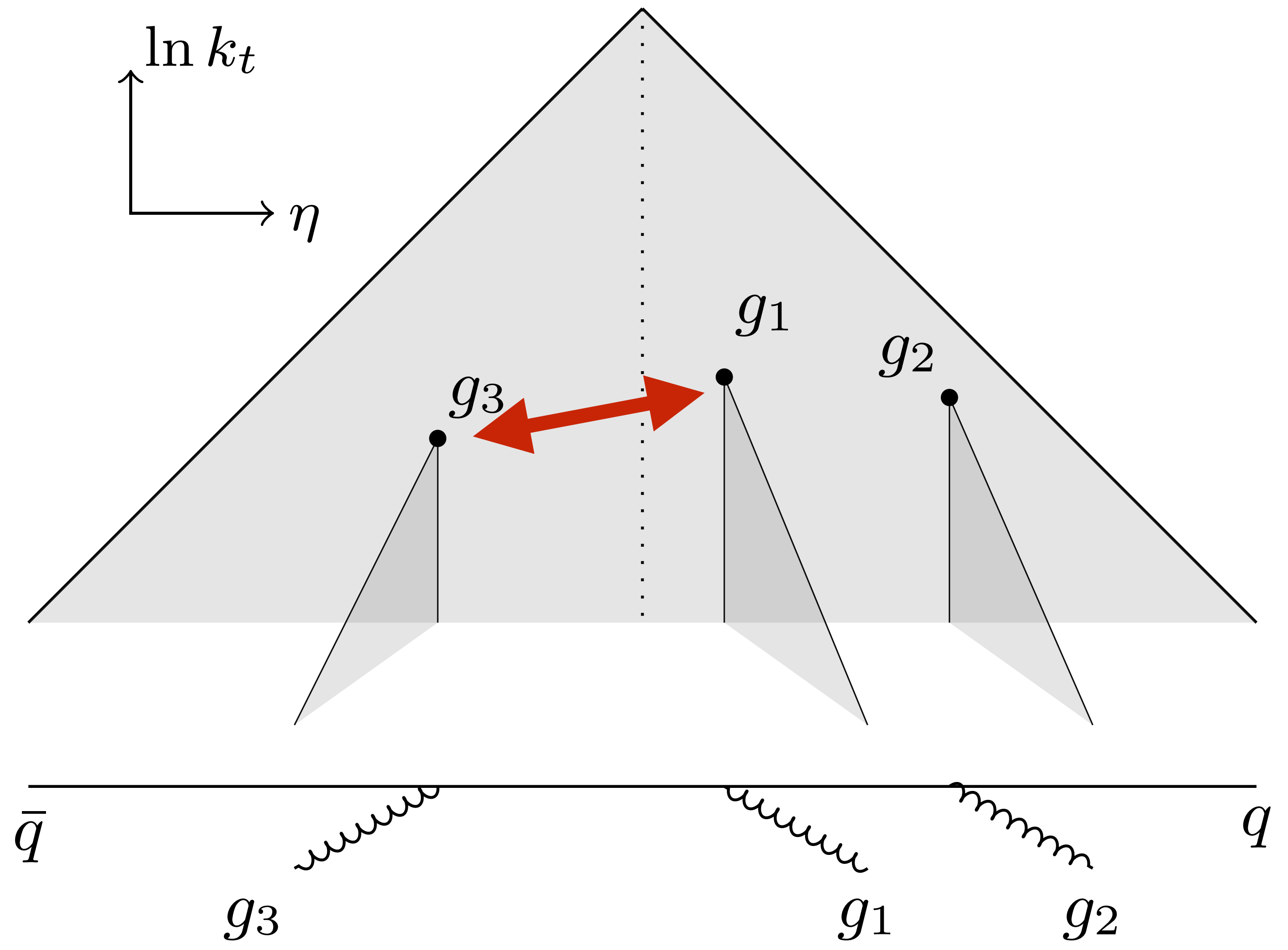
- LL ≡ squared amplitude in limits where **both logarithmic variables** are strongly ordered across emissions (**Area in the LP**)



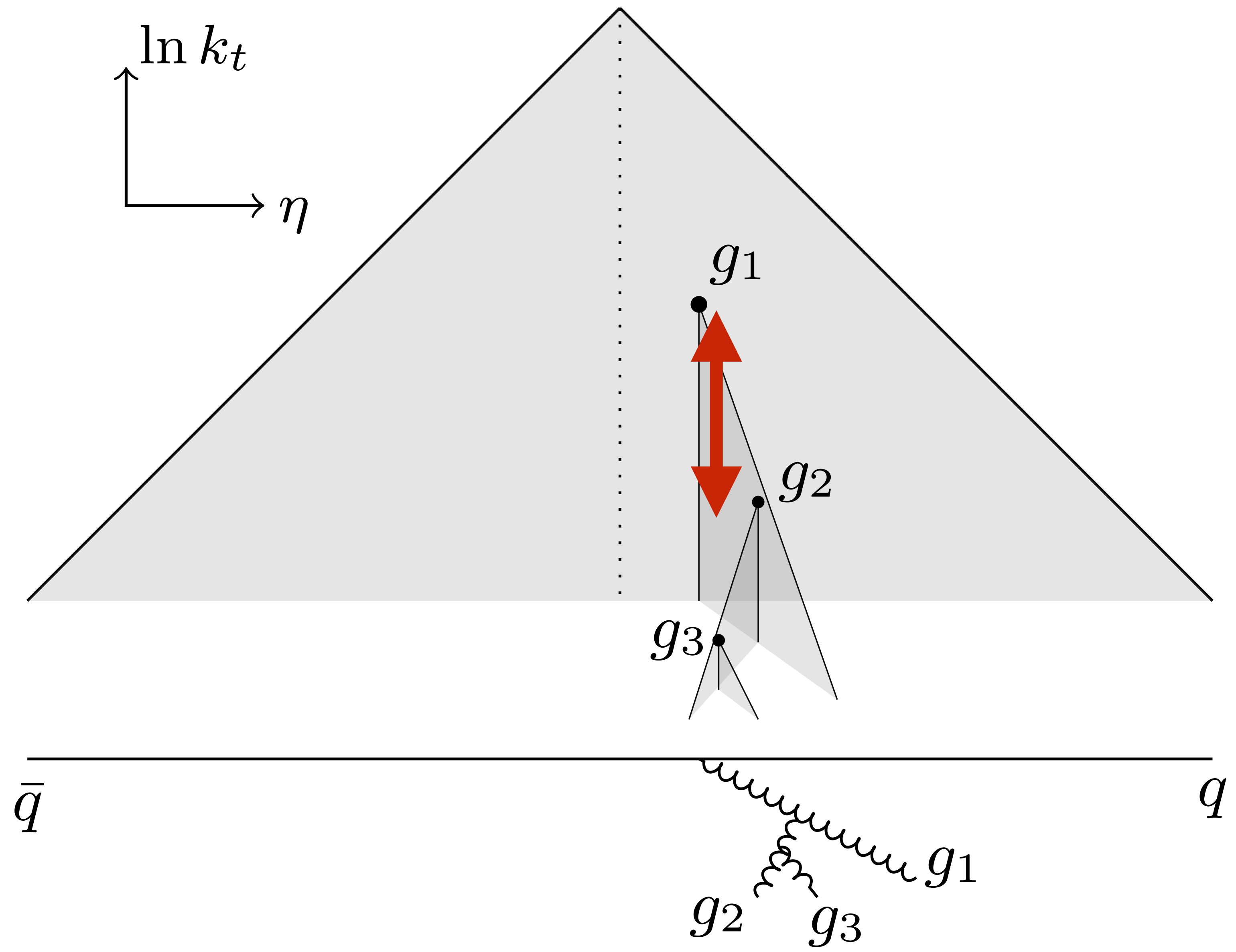
Strong ordering in one log variable ⇒ large Lund plane (LP) distance

A geometric criterion: the Lund plane

- Phase space: 3 variables per radiation, two of which are logarithmic, **e.g. $\{k_T, \eta\}$, $\{E, \theta\}$, …**



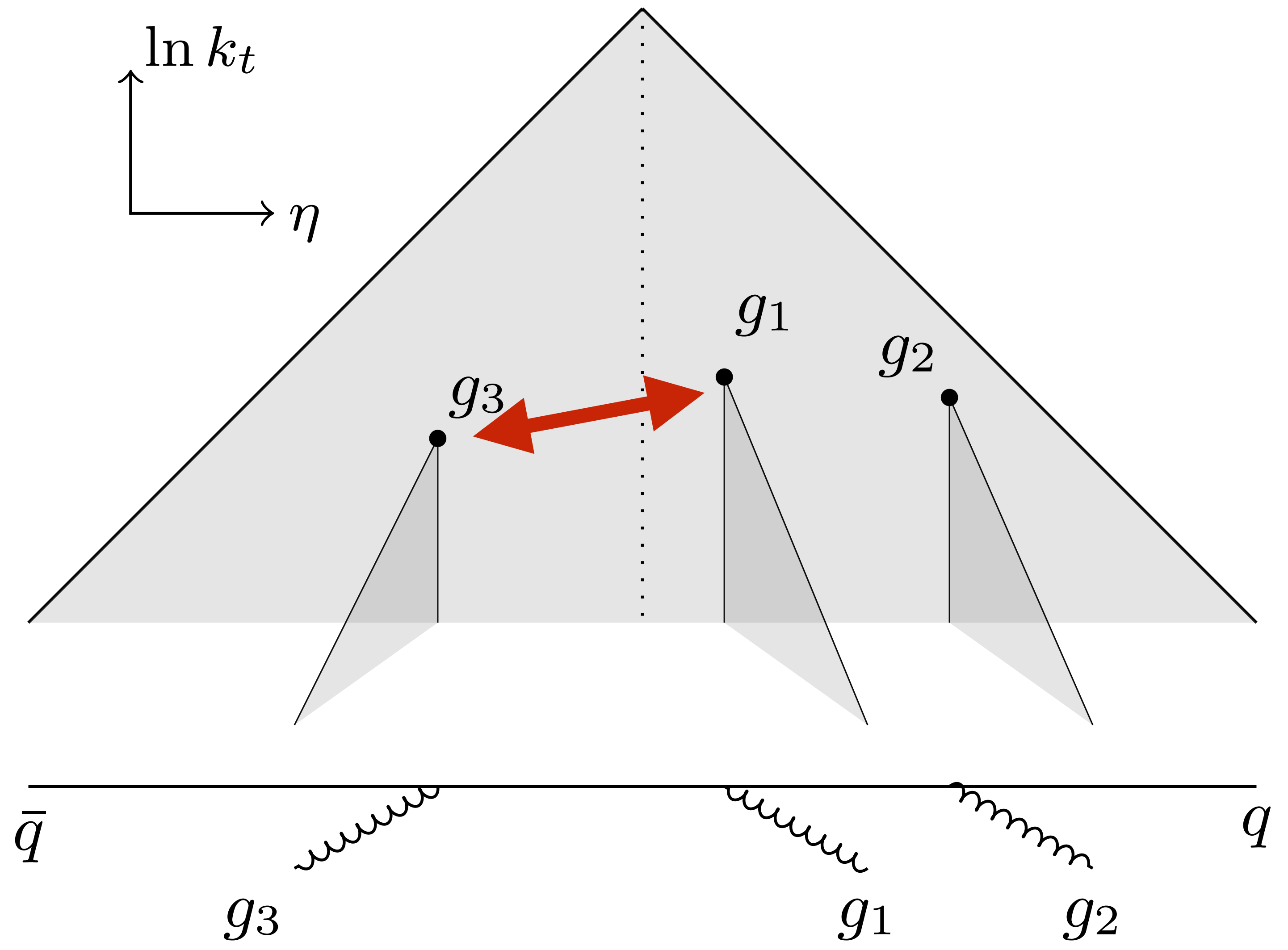
- NLL ≡ squared amplitude in limits where **at least one logarithmic variable** is strongly ordered across emissions (**Strip in the LP**). E.g.
  - **Similar k_t and ordered in angle**
  - **Nested collinear fragmentation**
    [specific observables also sensitive to spin correl.ns]
    ➥ **Talk by R. Verheyen**
  - **Similar angle and ordered in k_t/energy**
  - …



Two emissions close in both variables correspond to NNLL configurations, and so on …

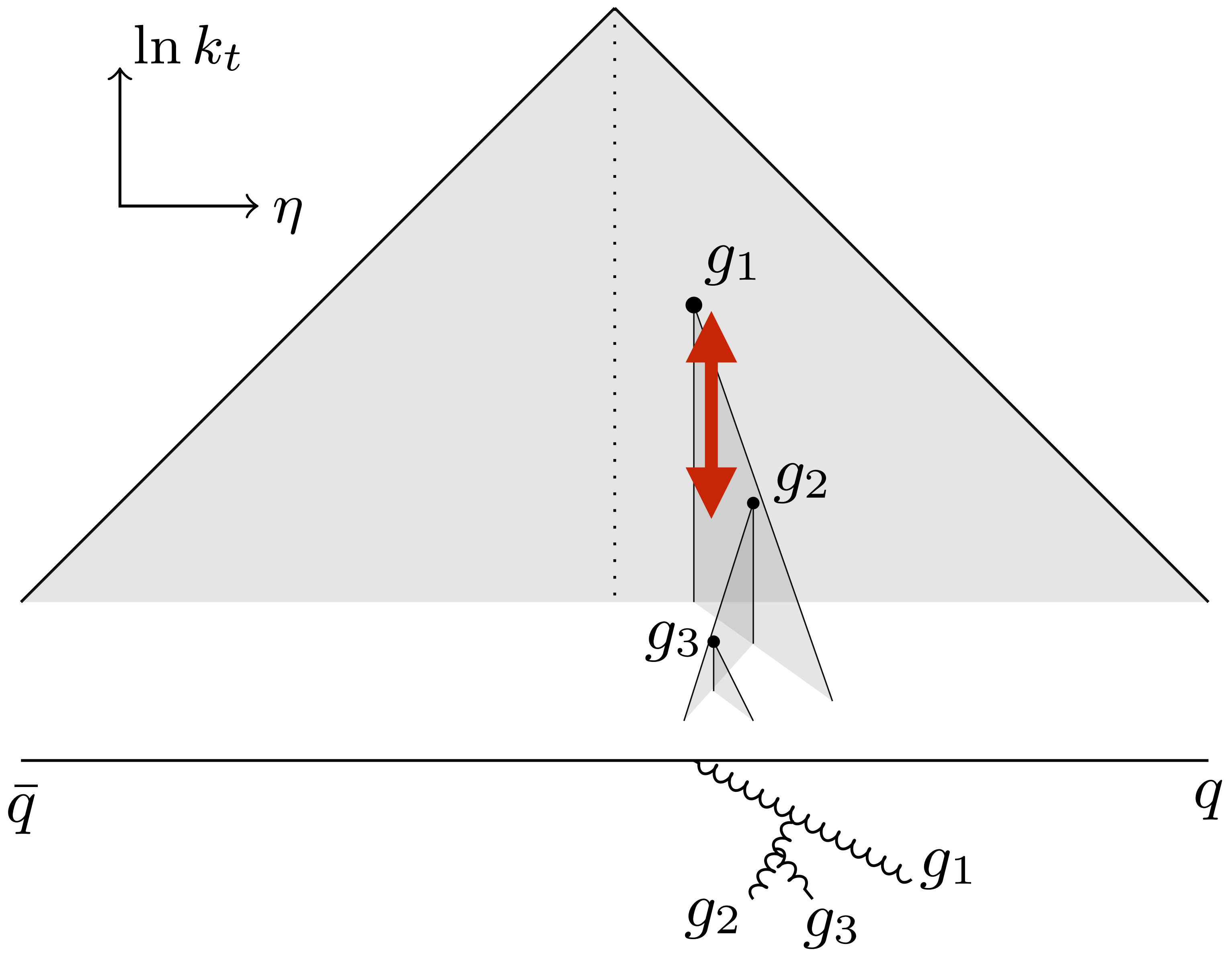
A geometric criterion: the Lund plane

- Phase space: 3 variables per radiation, two of which are logarithmic, e.g. {k_T, η}, {E, θ}, …



- NLL ≡ squared amplitude in limits where at least one logarithmic variable is strongly ordered across emissions (Strip in the LP).

Central criterion: <u>locality</u>, i.e. emissions horizontally distant in the Lund plane must be independent (i.e. kinematics and colour charge)



NB: also necessary to include running-coupling effects as well as radiative corr.ns for more ordered configurations (e.g. CMW scheme for DL observables)

[Catani, Marchesini, Webber '91]

(LO) Dipole showers in a nutshell

- Squared amplitudes built recursively via a Markovian chain of emissions (planar limit). Virtual corrections through unitarity

CMW scheme and running coupling effects

Evolution from a state with n particles S_n to S_{n+1}

$$\frac{d\mathcal{P}_{n\to n+1}}{d\ln v} = \sum_{\text{dipoles } \{\tilde{\imath},\tilde{\jmath}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t) + K\alpha_s^2(k_t)}{\pi} \times [g(\bar{\eta})a_k P_{\tilde{\imath}\to ik}(a_k) + g(-\bar{\eta})b_k P_{\tilde{\jmath}\to jk}(b_k)]$$

Evolution variable, e.g. k_T in the dipole c.o.m. frame

LO Splitting functions (azim. averaged here)

Some notion of rapidity of the emission within the dipole, deciding how the dipole is partitioned.

Recoil assigned according to a map $S_n \to S_{n+1}$

Dipole design necessary to correctly describe wide-angle soft radiation [Dasgupta, Salam '01; Banfi, Corcella, Dasgupta '06]

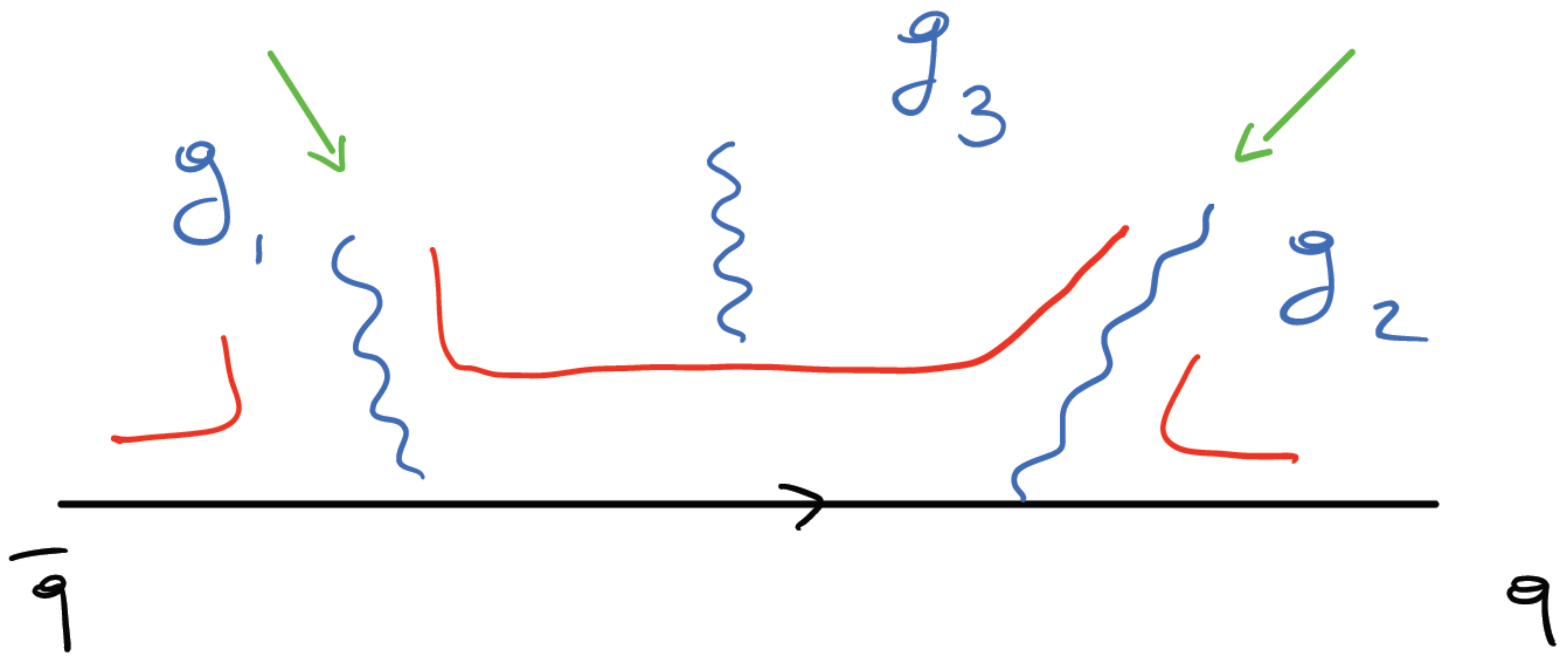
A case study: k_T ordering & local recoil

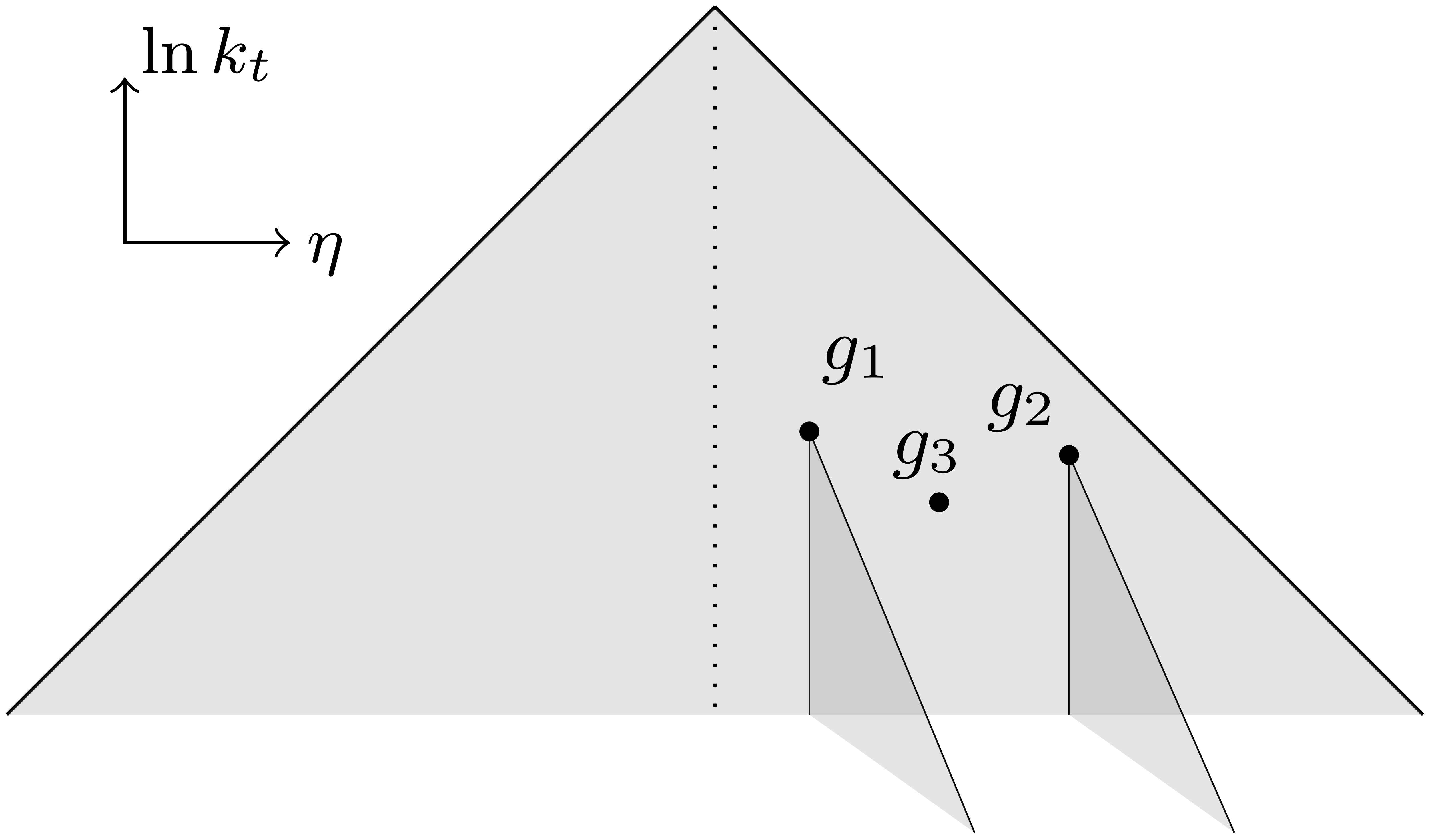
[Dasgupta, Dreyer, Hamilton, PM, Salam '18;
see also Bewick, Ferrario Ravasio, Richardson, Seymour '19]

- FSR algorithms based on k_T ordering and local recoil were found to violate the locality criterion: issues at <u>leading-colour at NLL (and subleading-colour at LL)</u>

➥ Talk by L. Scyboz

e.g. soft emission off {gg} dipole: recoil assigned to the soft ends regardless of distance





Unphysical super LL (starting at $O(\alpha_s^3)$ or $O(\alpha_s^4)$) related to long-range correlations also present in specific observables

Design of NLL dipole showers: some remarks

- GOAL: an algorithm that is NLL accurate simultaneously for rIRC-safe global, and non-global observables. Crucial to build a framework to demonstrate the formal accuracy in a solid manner

- Consider a clean theoretical laboratory: $e^+e^- \rightarrow$ 2 jets, large N_c limit, no spin correlations

  ➥ Talks by R. Verheyen, L. Scyboz

- NB: QCD resummation provides one with guidelines, therefore more than one architecture is possible. Difference between NLL solutions is a handle to assess the size of genuine subleading (NNLL) logarithmic corrections

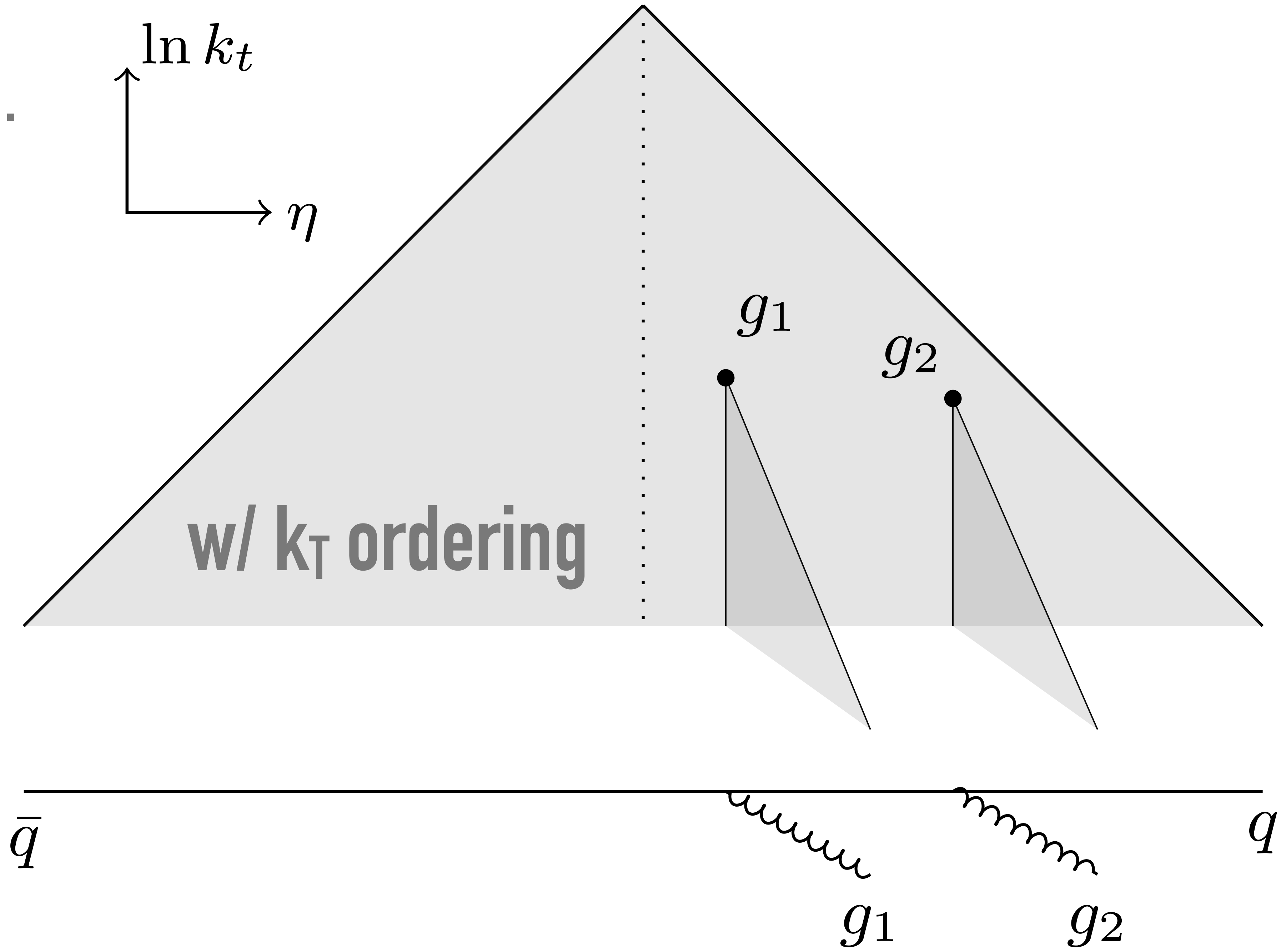
The PanLocal shower (local recoil)

- Keep the recoil dipole-local, i.e. for each new emission

$$\text{dipole } \{\widetilde{p}_i, \widetilde{p}_j\} \longrightarrow \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp\,, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp\,, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1-f) k_\perp \end{aligned}$$

- **Key element #1**: partitioning of the dipole (at $\bar{\eta} = 0$) occurs at equal angles between the emission and the dipole ends in the **event c.o.m. frame***

e.g.



- In the limit of strong angular ordering and commensurate k_T's, g_2 takes the recoil from the hard quark

* Partitioning in the dipole frame would lead to violations of locality

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- Keep the recoil dipole-local, i.e. for each new emission

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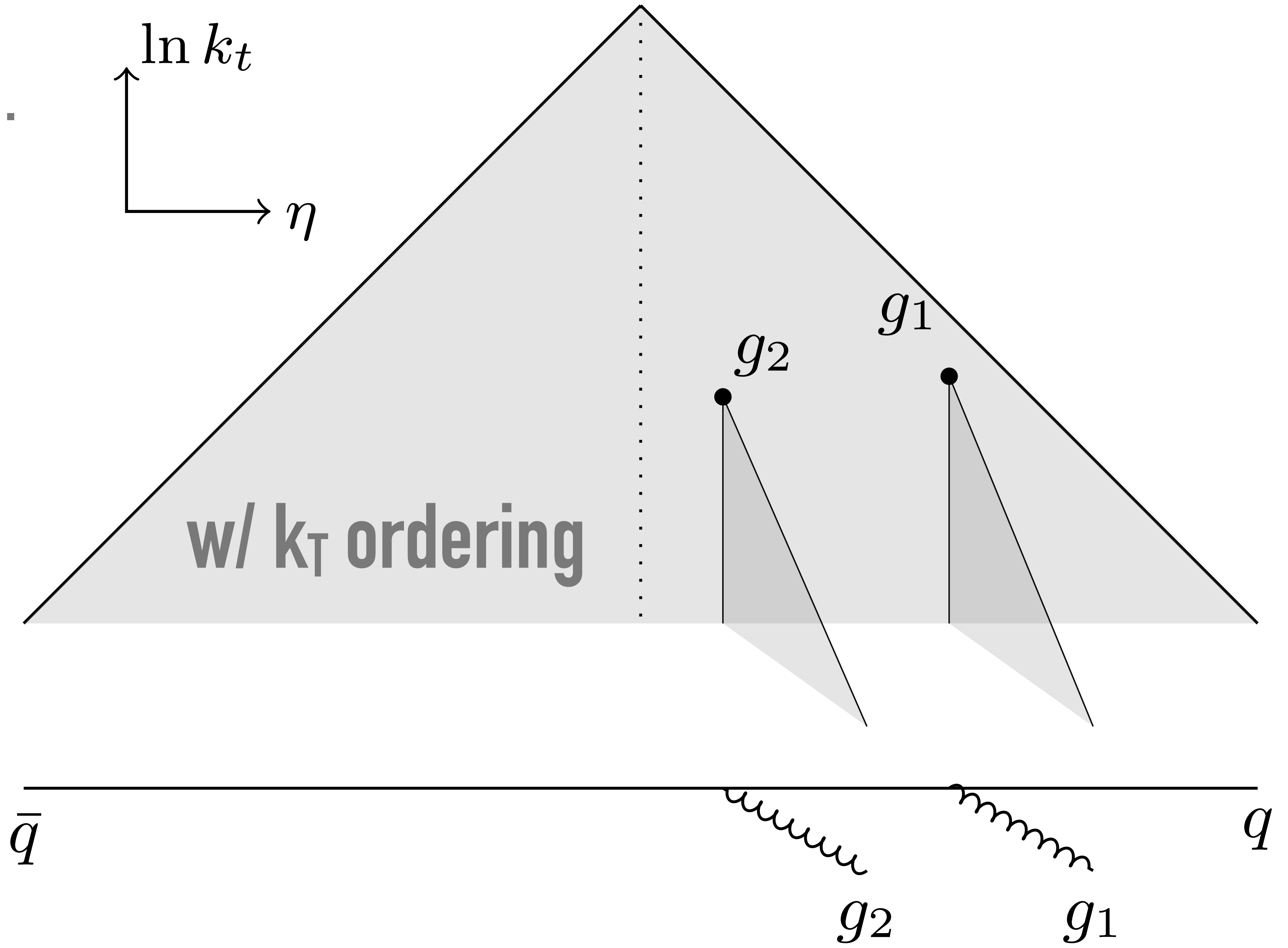

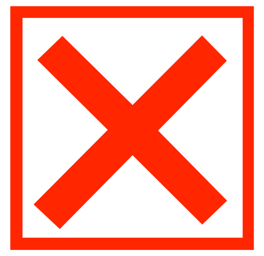

- Instead, if g_2 is produced at larger angles than g_1, and they are both collinear to the quark, the recoil is still taken from g_1 in a logarithmic (NLL) region of phase space

The PanLocal shower (local recoil)

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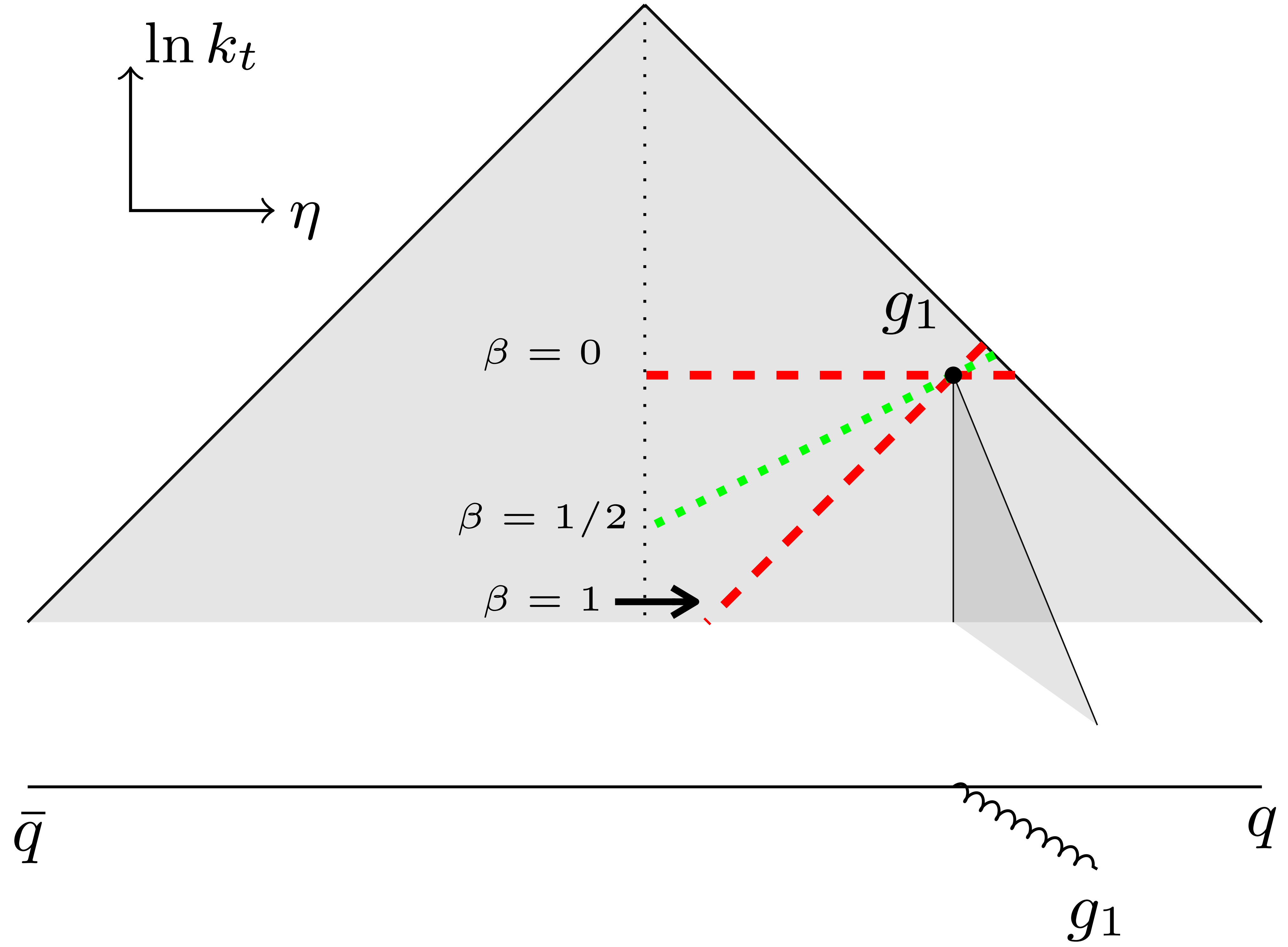
$$\text{dipole } \{\tilde{p}_i, \tilde{p}_j\} \longrightarrow \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp \,, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp \,, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1-f) k_\perp \end{aligned}$$

- Key element #2: use an evolution variable v defined as $(0 < \beta < 1)$

$$k_t = \rho v e^{\beta|\bar{\eta}|} \sim v e^{\beta|\eta^{\text{w.r.t. emitter}}|}$$

$$\rho = \left(\frac{s_{\tilde{\imath}} s_{\tilde{\jmath}}}{Q^2 s_{\tilde{\imath}\tilde{\jmath}}}\right)^{\frac{\beta}{2}}$$



k_T ordering corresponds to $\beta=0$

The PanLocal shower (local recoil)

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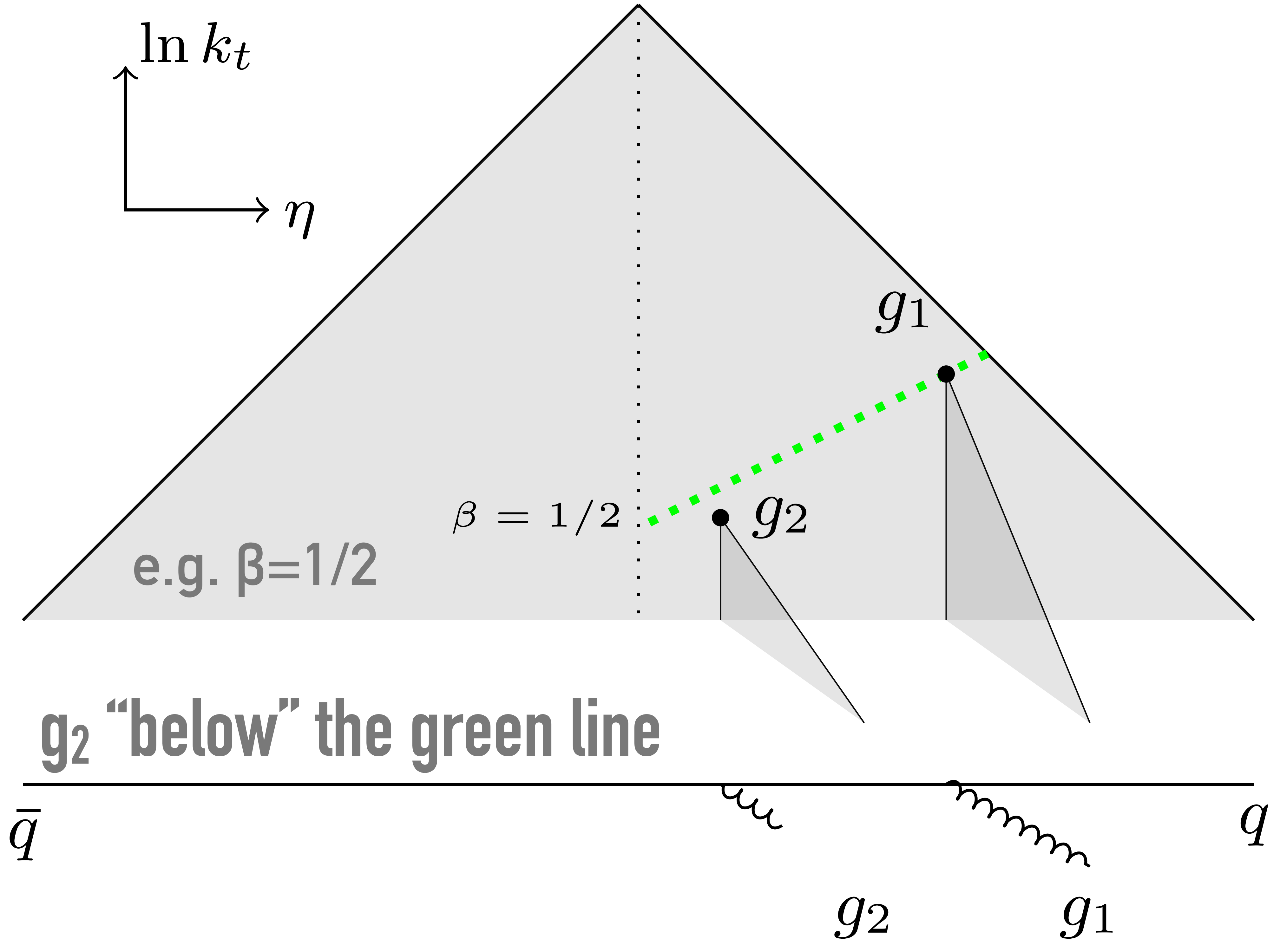
- Key element #2: use an evolution variable v defined as ($0 < \beta < 1$)

  - Ordering in v now implies that $k_{t2} << k_{t1}$ [i.e. no recoil]

  - The combination of partition⊕ordering creates a mechanism in which the recoil is always taken from the hard extremities of the dipole chain [correct at NLL] ✓



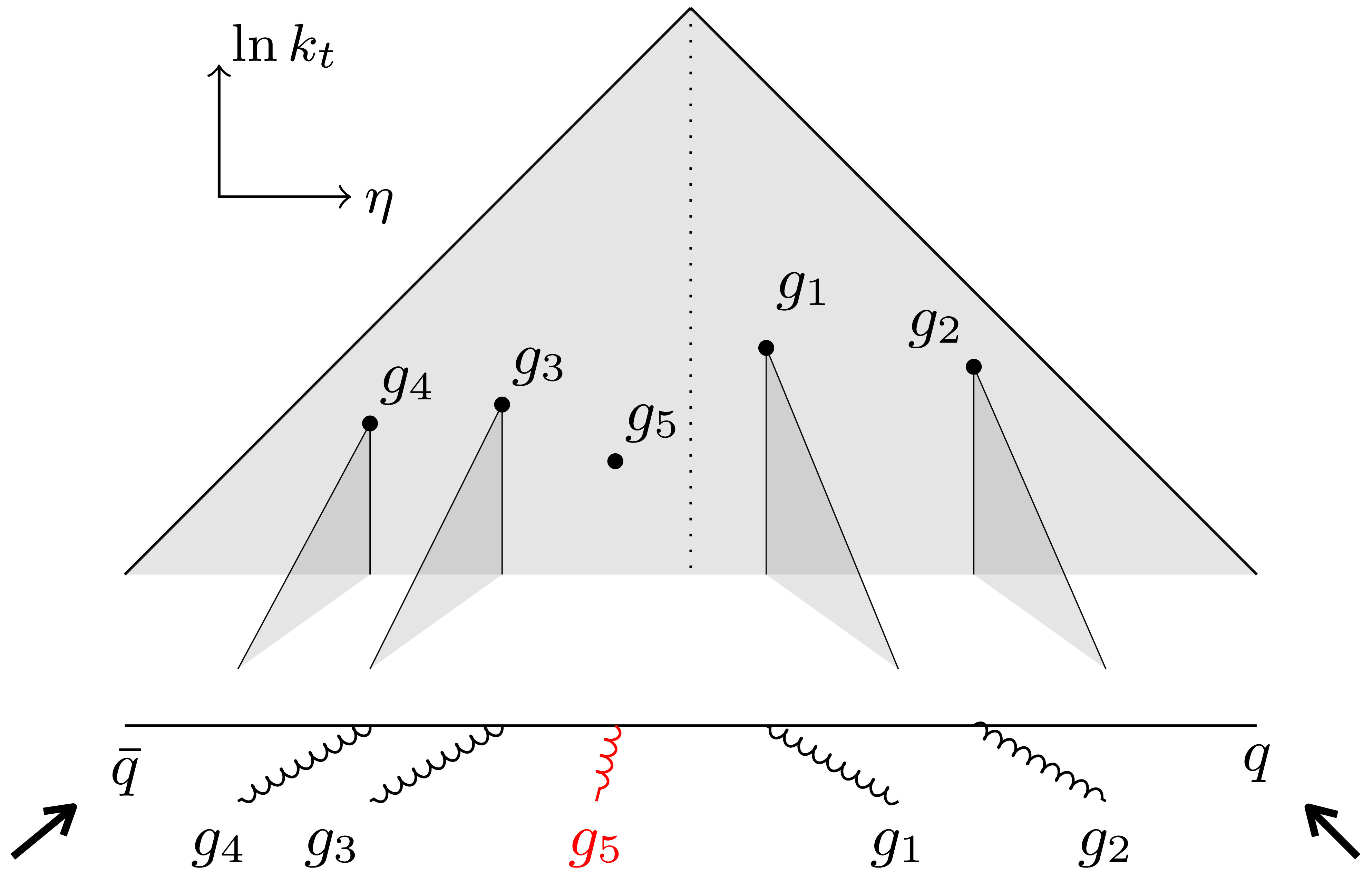
The PanGlobal shower (global recoil)

- Longitudinal recoil is kept dipole local

$$\text{dipole } \{\widetilde{p}_i, \widetilde{p}_j\} \longrightarrow \begin{aligned} \bar{p}_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp \,, \\ \bar{p}_i &= (1 - a_k)\tilde{p}_i \,, \\ \bar{p}_j &= (1 - b_k)\tilde{p}_j \,. \end{aligned}$$

- Transverse recoil is distributed globally across the event via a Lorentz boost + rescaling
  i.e. recoil is taken from (and shared among) the hard extremities of the dipole string

  - With this scheme, also k_t ordering is a viable option



Similar scheme also in [Forhsaw, Holguin, Plaetzer '20]

Testing the logarithmic accuracy

- Tests of logarithmic accuracy in the full shower* against NLL resummations:
  - Consider cumulative distributions for an observable (e.g. jet rate, event shapes, …) in the limit $\alpha_s |L| \sim 1$, and $|L| \gg 1$

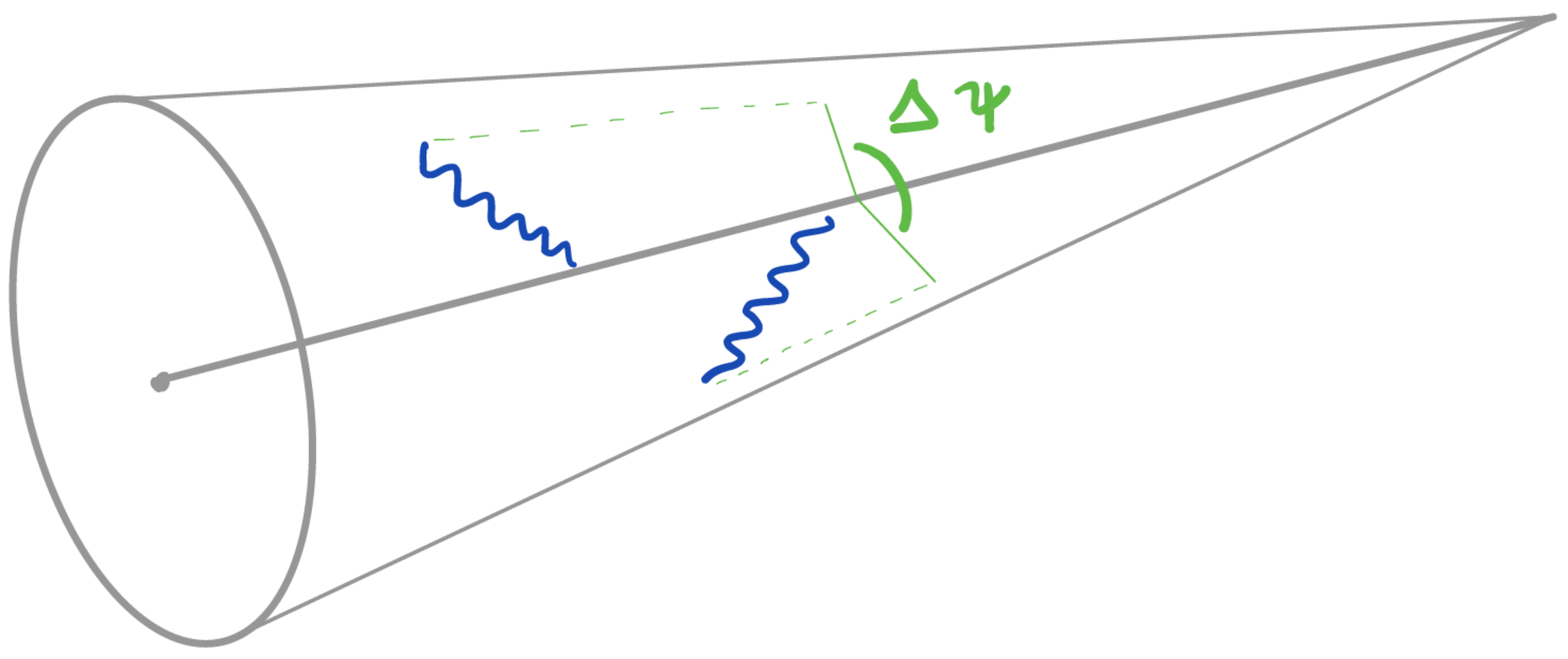
LL (=0 sometimes) NLL

$$\Sigma(\alpha_s, \alpha_s L) = \exp\left[\alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \mathcal{O}\left(\alpha_s^n L^{n-1}\right)\right]$$

- Compute the ratio $\dfrac{\Sigma_{\rm PS}}{\Sigma_{\rm NLL}}$
  - PS is LL: Σ_{PS} misses O(1) corrections, i.e. $\lim_{\alpha_s \to 0} \dfrac{\Sigma_{\rm PS}}{\Sigma_{\rm NLL}} \neq 1$
  - PS is NLL: Σ_{PS} misses $O(\alpha_s)$ corrections, i.e. $\lim_{\alpha_s \to 0} \dfrac{\Sigma_{\rm PS}}{\Sigma_{\rm NLL}} = 1$

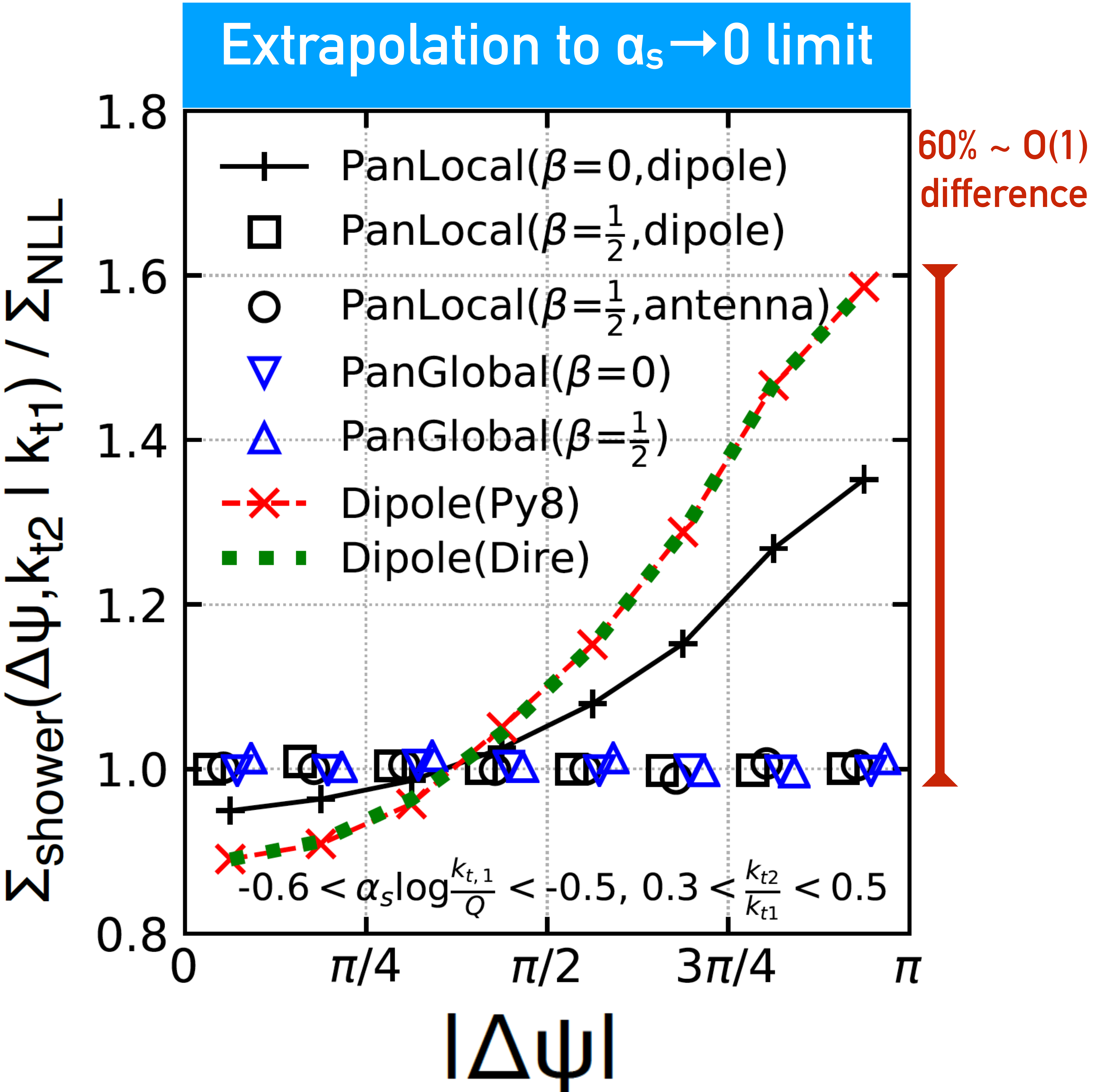
*Further tests performed at fixed (up to third and fourth) and all orders within a toy shower of only primary emissions

An example: azimuthal substructure of jets



Lund Jet Plane (LJP) based azimuthal angle ΔΨ between two leading primary declusterings [actual definition involves a dynamical frame]. Ratio to NLL shows a residual shape difference in the limit $\alpha_s \to 0$ for k_T-ordered & local dipole showers

➥ New classes of PanScales algorithms (PanLocal = local recoil map; PanGlobal = global recoil map) reproduce correct NLL results as expected



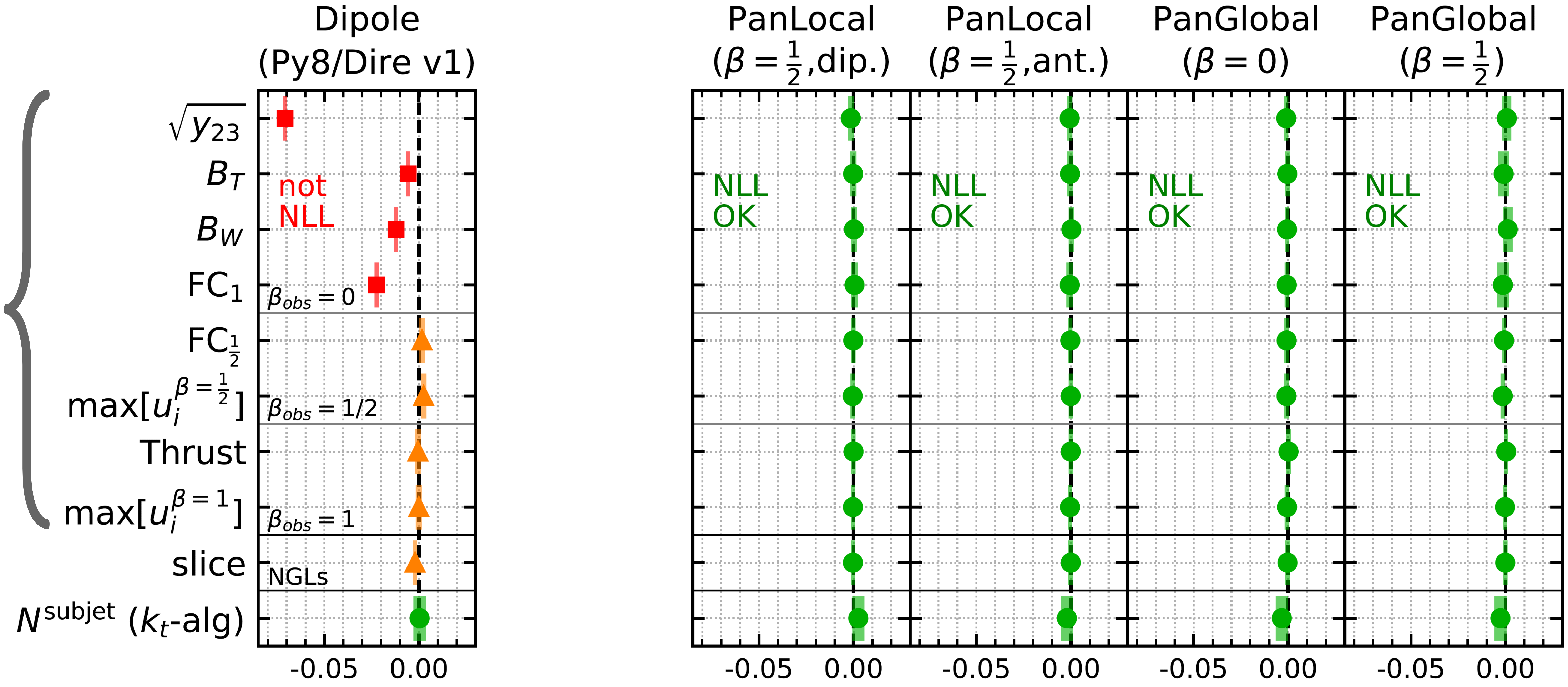
[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez ’20]
cf. [Dreyer, Salam, Soyez ’18] for construction of the Lund Jet Plane

Accuracy across many observables

Plots: relative deviation from exact NLL [in $\alpha_s \to 0$ limit at fixed $\alpha_s L$]

[Sjostrand et al. '15]
[Hoeche, Prestel '15]

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]



Global observables: sensitive to strong η (θ) separation at NLL

e.g.

$$T = \max_{\hat{n}} \frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|}$$

$$FC_x = \sum_{i \neq j} \frac{E_i E_j}{(\sum_i E_i)^2} |\sin\theta_{ij}|^x (1 - |\cos\theta_{ij}|)^{1-x} \Theta((\mathbf{p}_i \cdot \hat{n})(\mathbf{p}_j \cdot \hat{n}))$$

$$\max\left[u_i^\beta\right] = \max_{\text{primary decl.}} \left\{k_{t1} e^{-\beta|\eta_1|}, \ldots, k_{tn} e^{-\beta|\eta_n|}\right\}$$

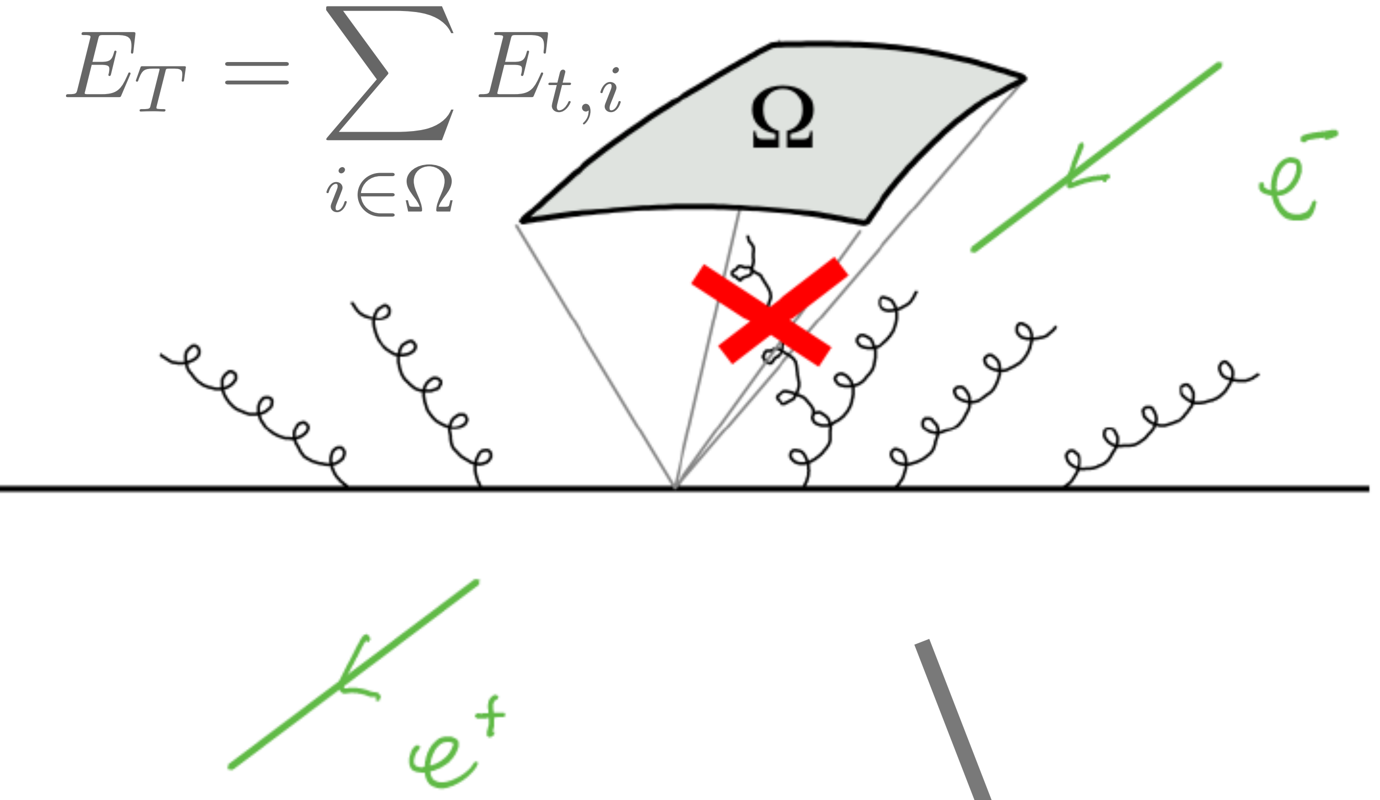
Orange triangles indicate spurious terms (either NLL or SLL) at fixed order, that become small when resummed

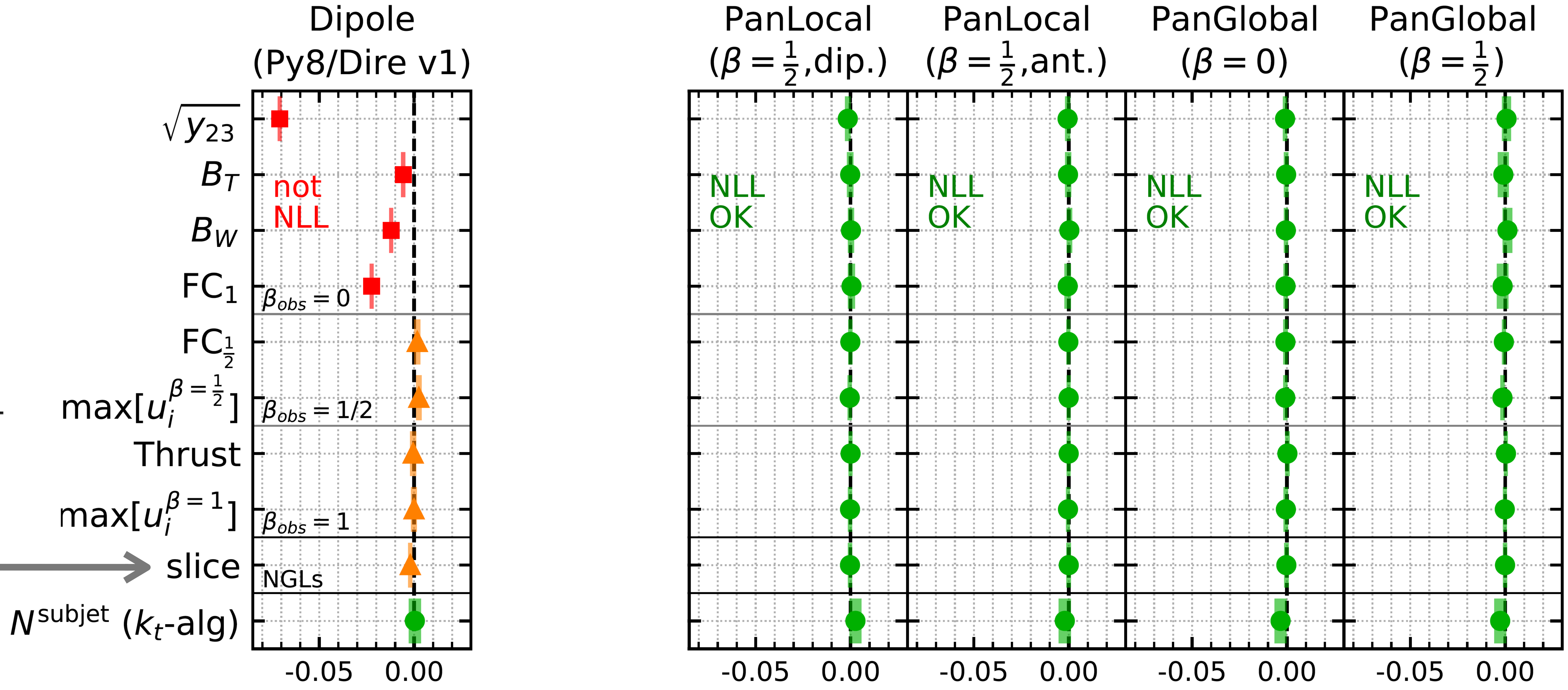
Accuracy across many observables

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez ’20]

Non-global observables: sensitive to strong kt (E) separation at NLL
e.g.

$$E_T = \sum_{i \in \Omega} E_{t,i}$$





Subjet particle multiplicity in the k_T algorithm: sensitive to full recursive shower structure

New classes of shower: NLL for all observables considered [global & non-global at once]

Conclusions and Outlook

- Formulation of accuracy criteria for parton showers guided by principles of QCD resummations
  - Testing framework for algorithms based on comparison to exact all order calculations
- With seemingly simple methods, one can engineer new PS algorithms that are NLL accurate for rIRC-safe global & non-global observables at once
  - Demonstration of NLL accuracy both at fixed order and all orders !
- Some aspects not addressed here (ISR, spin correlations, classes of subleading colour corr^{ns}) but the proposed technology can incorporate solutions to the above problems

➥ Talks by R. Verheyen, L. Scyboz

- Powerful avenue to look beyond NLL …

see also related work by
[Hoeche, Prestel, + Krauss ’17; Dulat, Hoeche, Prestel ’18; Forhsaw, Holguin, Plaetzer ’20; Nagy, Soper ’20]