Large logarithm summation by parton showers

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work with Zoltan Nagy, DESY

(This version has a couple of small corrections. 26 May)

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Prequel

- Sometimes parton shower event generators can sum large logarithms.

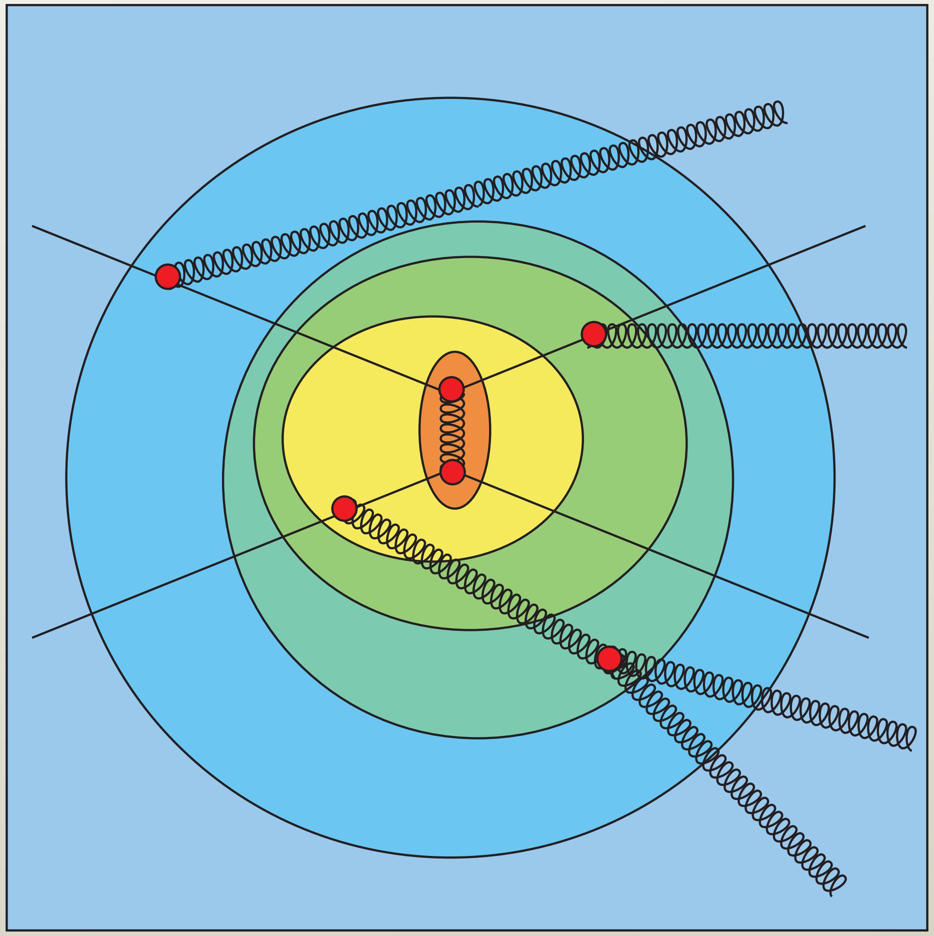
- Zoltan Nagy (DESY) and I have studied this in general and, in particular, for the thrust distribution in electron-positron annihilation in arXiv:2011.04773 and arXiv:2011.04773.

- We use the formulation for parton showers that forms the basis for our shower generator DEDUCTOR.

- I will devote most this talk to a pedagogical review of our view of the theoretical basis for summing logs with parton showers.

Quantum mechanics in parton showers

Renormalization group



- Start at hardest interaction and move to interactions with smaller scales μ^2.

Statistical space (omitting spin and color)

- Momenta and flavors

$$\{p, f\}_m = \{p_{\rm a}, f_{\rm a}, p_{\rm b}, f_{\rm b}, p_1, f_1, \dots, p_m, f_m\}.$$

- Probability density $\rho(\{p, f\}_m)$.

- The functions ρ form a vector space with vectors $\big|\rho\big)$.

- Use basis vectors $\big|\{p, f\}_m\big)$.

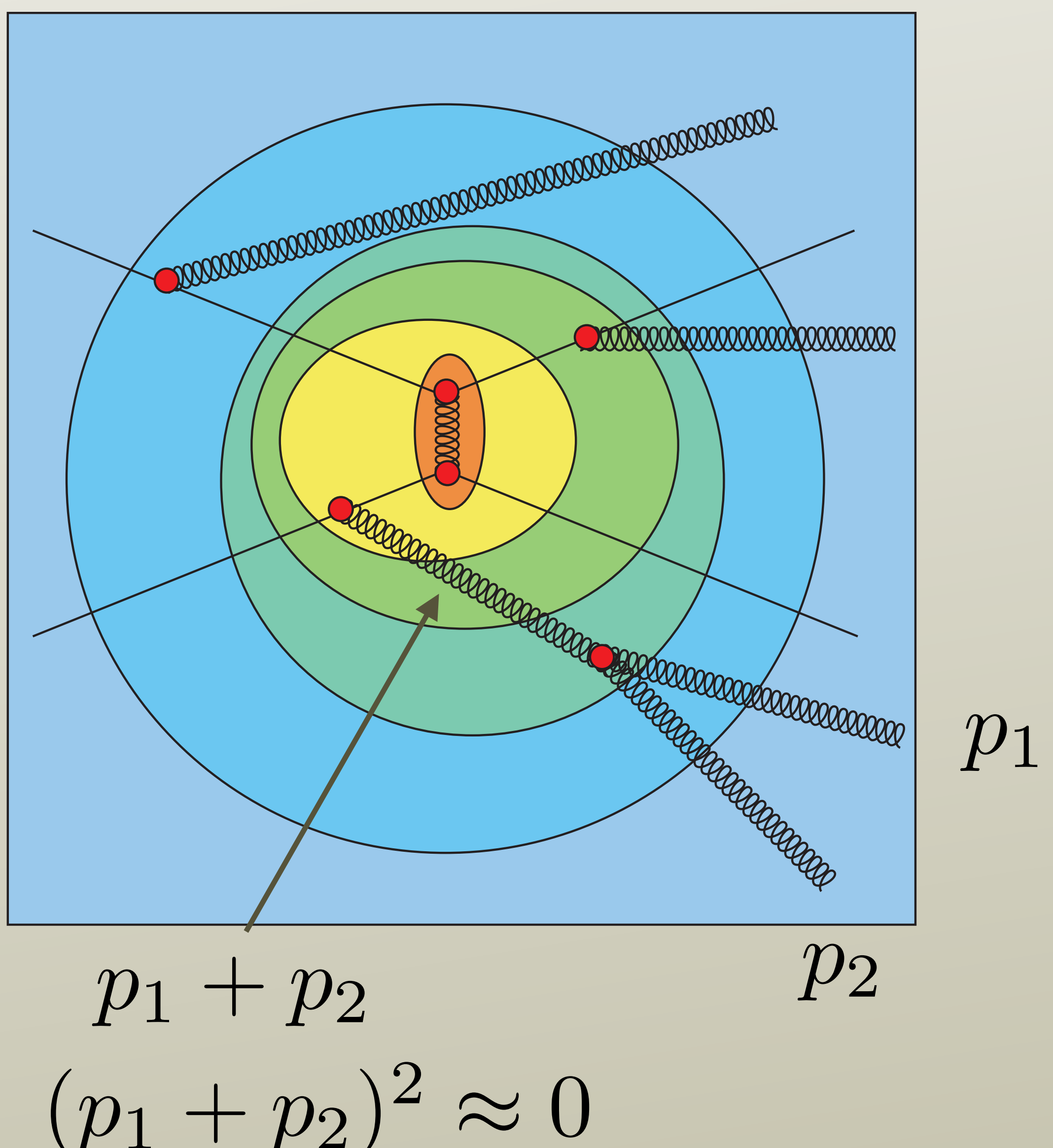
- Renormalization group equation

$$\mu \frac{d}{d\mu} \big|\rho(\mu)\big) = \mathcal{S}^{[1]}(\mu)\big|\rho(\mu)\big)$$

“Classical” momenta

- Quantum statistical mechanics would use a density matrix

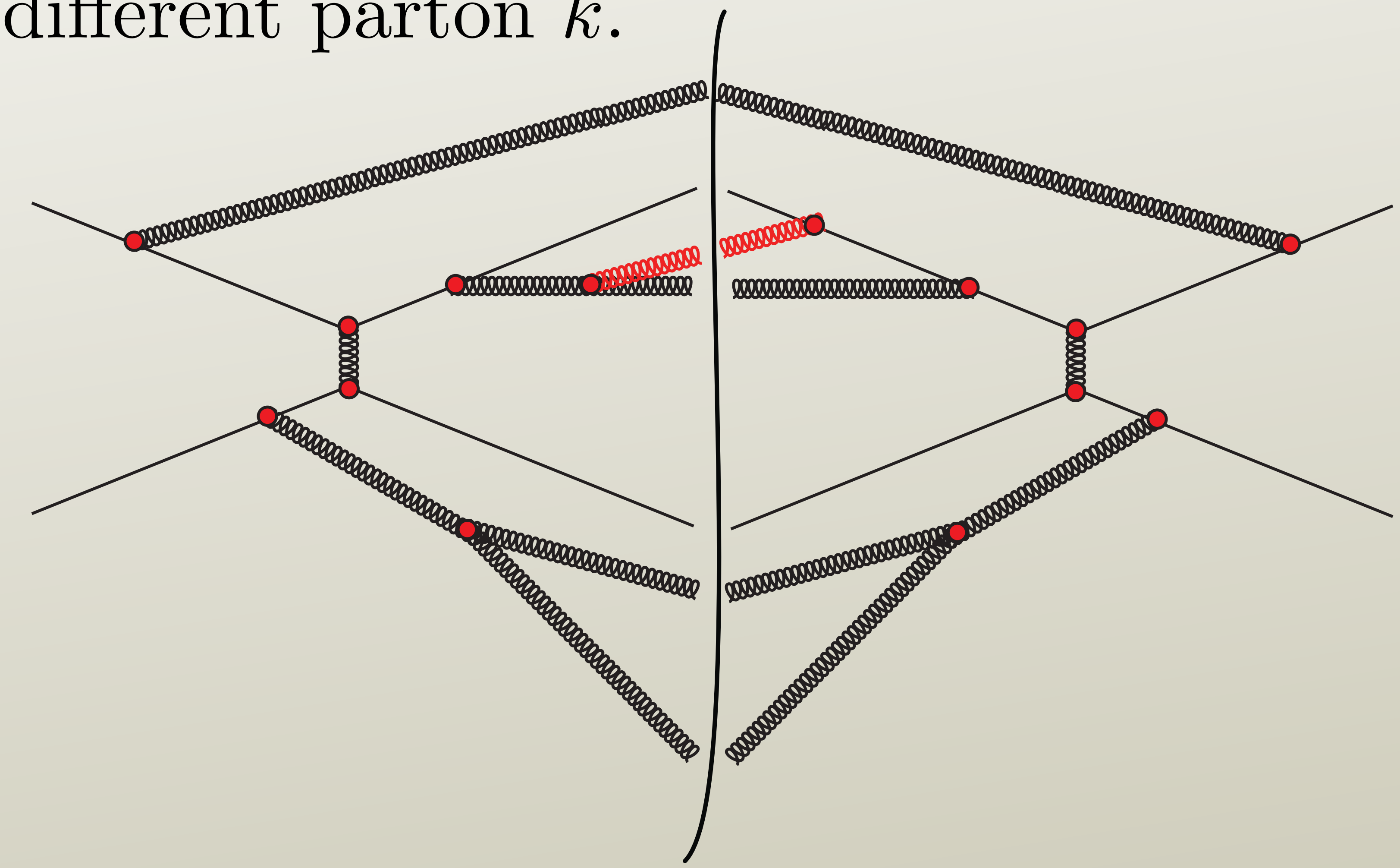
$$\big|\{p, f\}_m\big\rangle\big\langle\{p', f'\}_m\big|$$



- But $\{f'\}_m \approx \{f\}_m$ and it is a good approximation to use $\{p'\}_m \approx \{p\}_m$.

Interference diagrams

- The parton shower needs to include the interference between emitting a gluon from parton l and emitting the same gluon from a different parton k.



- With physical polarizations, the gluon must be soft.

- A dipole shower includes this.

Spin and color

- Each parton in a shower carries spin and color.
- We need to describe quantum field theory.
- For a statistical treatment, use quantum statistical mechanics.
- Use the spin-color density matrix, with basis elements

$$\big|\{c, c', s, s'\}_m\big) \Leftrightarrow \big|\{c, s\}_m\big\rangle\big\langle\{c', s'\}_m\big|$$

Color

- For this talk, just consider color:

$$\big|\{c, c'\}_m\big) \Leftrightarrow \big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$

- Splittings involve operators on the color space. *E.g.*

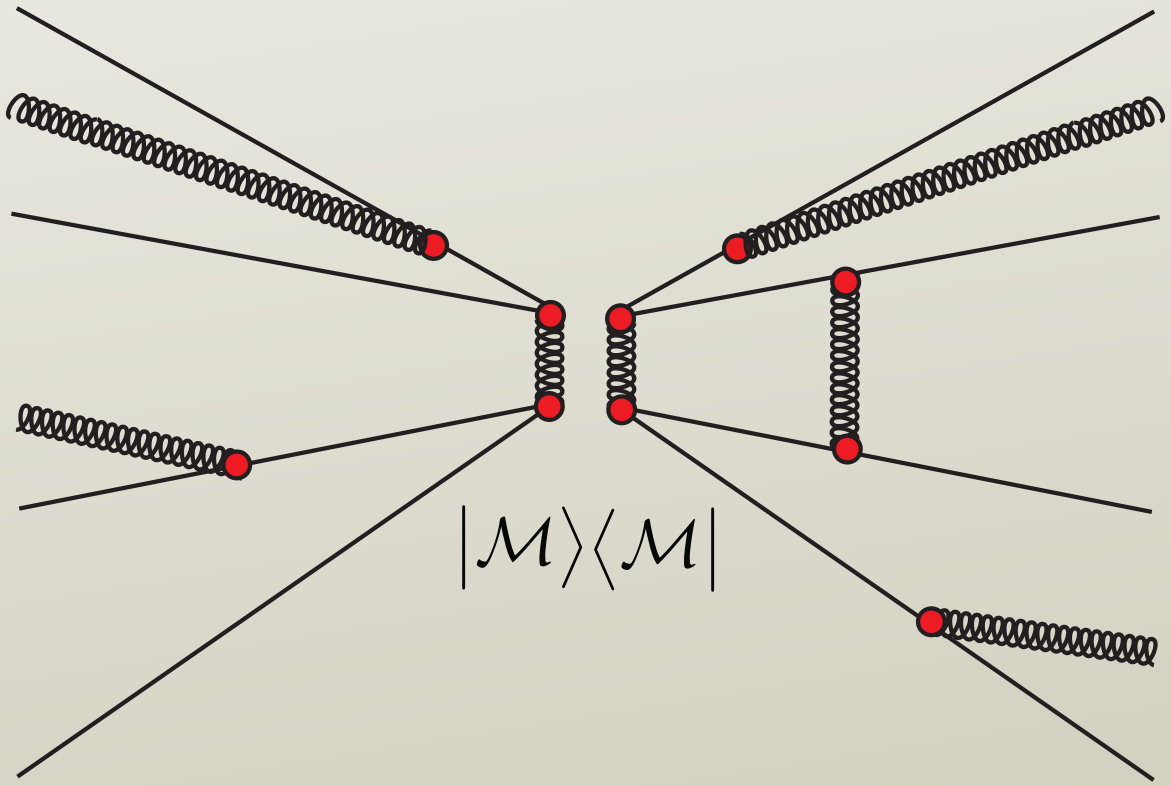
$$t_l^\dagger(f_l \to \hat{f}_l + \hat{f}_{m+1}) \otimes t_k(f_k \to \hat{f}_k + \hat{f}_{m+1})$$

and

$$1 \otimes t_k(f_k \to \hat{f}_k + \hat{f}_{m+1}) t_l^\dagger(f_l \to \hat{f}_l + \hat{f}_{m+1})$$

Other authors

- Forshaw, Holguin and Plätzer (2020) use a color density matrix formulation, dubbed *amplitude evolution*.
- They use diagrams like this



- *Cf.* Forshaw, Holguin and Plätzer (2019) and Martinez, De Angelis, Forshaw, Plätzer and Seymour (2018).

Leading color approximation

- Throw away contributions that are missing if you use U(3) instead of SU(3).
- Never generate

$$\big|\{c,c'\}_m\big) = \big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big| \quad \text{with} \quad \{c\}_m \neq \{c'\}_m$$

- The terms thrown away are suppressed by $1/N_\mathrm{c}^2$.
- But some terms thrown away are enhanced by large logarithms.

LC+ approximation

- Allow color contributions of the form

$$\big|\{c,c'\}_m\big) = \big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$

with

$$\{c\}_m \neq \{c'\}_m$$

- Throw away some parts of color operators

$$t_l^\dagger(f_l \to \hat{f}_l + \hat{f}_{m+1}) \otimes t_k(f_k \to \hat{f}_k + \hat{f}_{m+1})$$

according to a simple rule. Nagy, Soper; JHEP (2012)

- This approximates the first order splitting operator $\mathcal{S}^{[1]}$ by a simpler operator $\mathcal{S}^{[1]}_{\rm LC+}$.

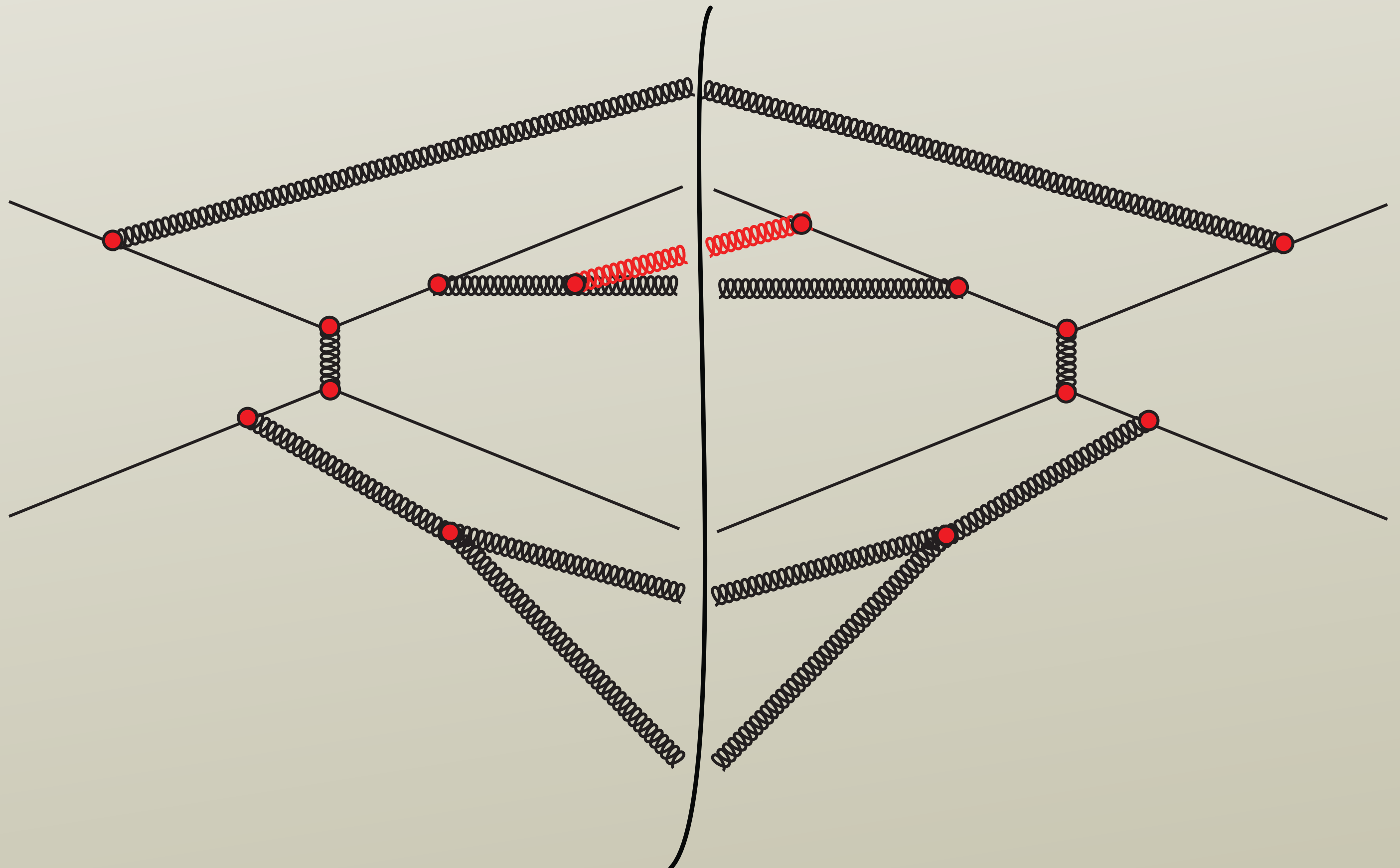
- Let

$$\Delta\mathcal{S}^{[1]} = \mathcal{S}^{[1]} - \mathcal{S}^{[1]}_{\rm LC+}$$

- When $\Delta\mathcal{S}^{[1]}$ is applied one or more times, the result is suppressed by at least one factor $1/N_{\rm c}^2$.

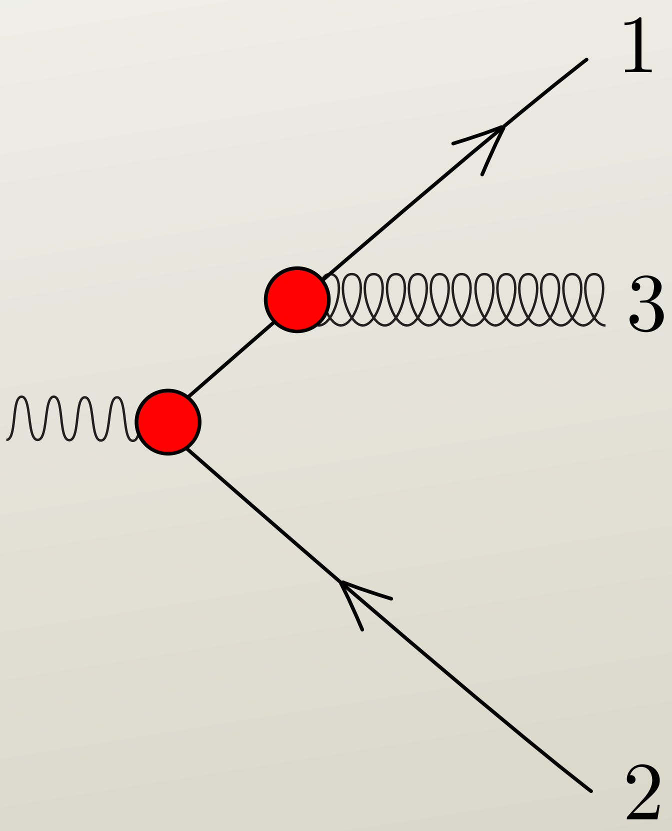
Color and large logarithms

- For observables with large logarithms, $\mathcal{S}^{[1]}$ can generate two large logarithms per loop.
- The operator $\Delta\mathcal{S}^{[1]}$ is sensitive to soft gluon singularities but not collinear singularities.
- Thus $\Delta\mathcal{S}^{[1]}$ can generate just one large logarithm per loop.

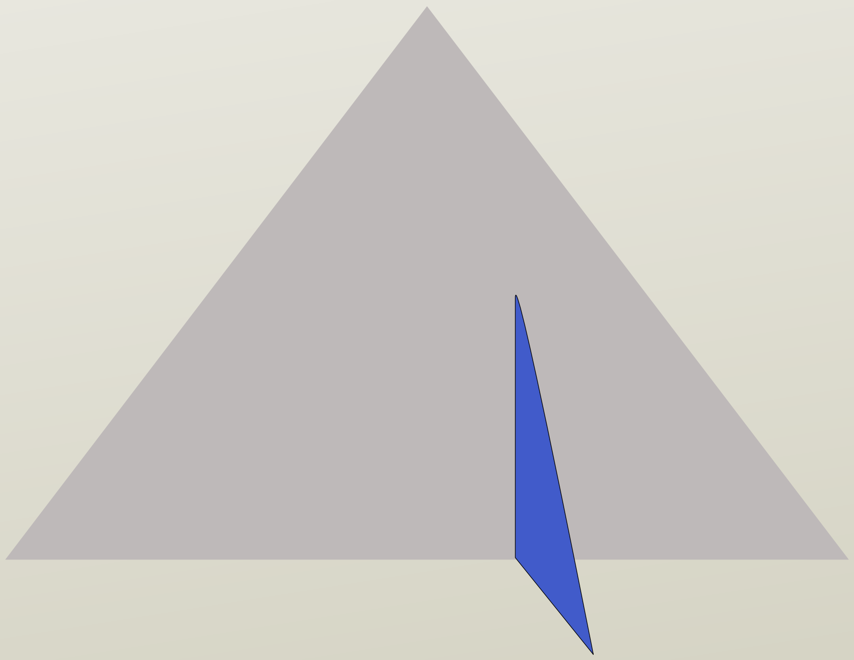


A historical example

- Consider e^+e^- annihilation, starting at $q\bar{q}$g production.



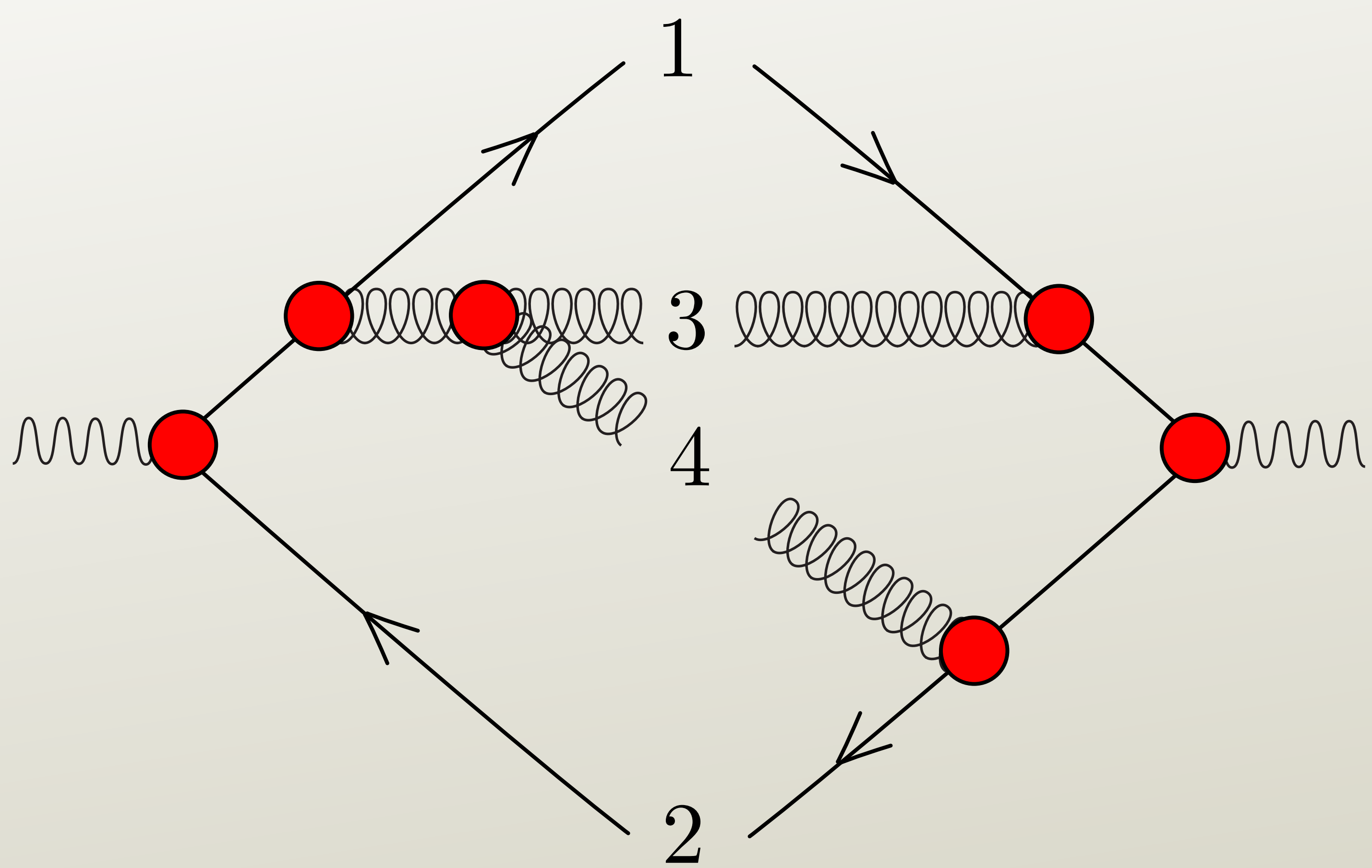
Feynman diagram



Lund diagram

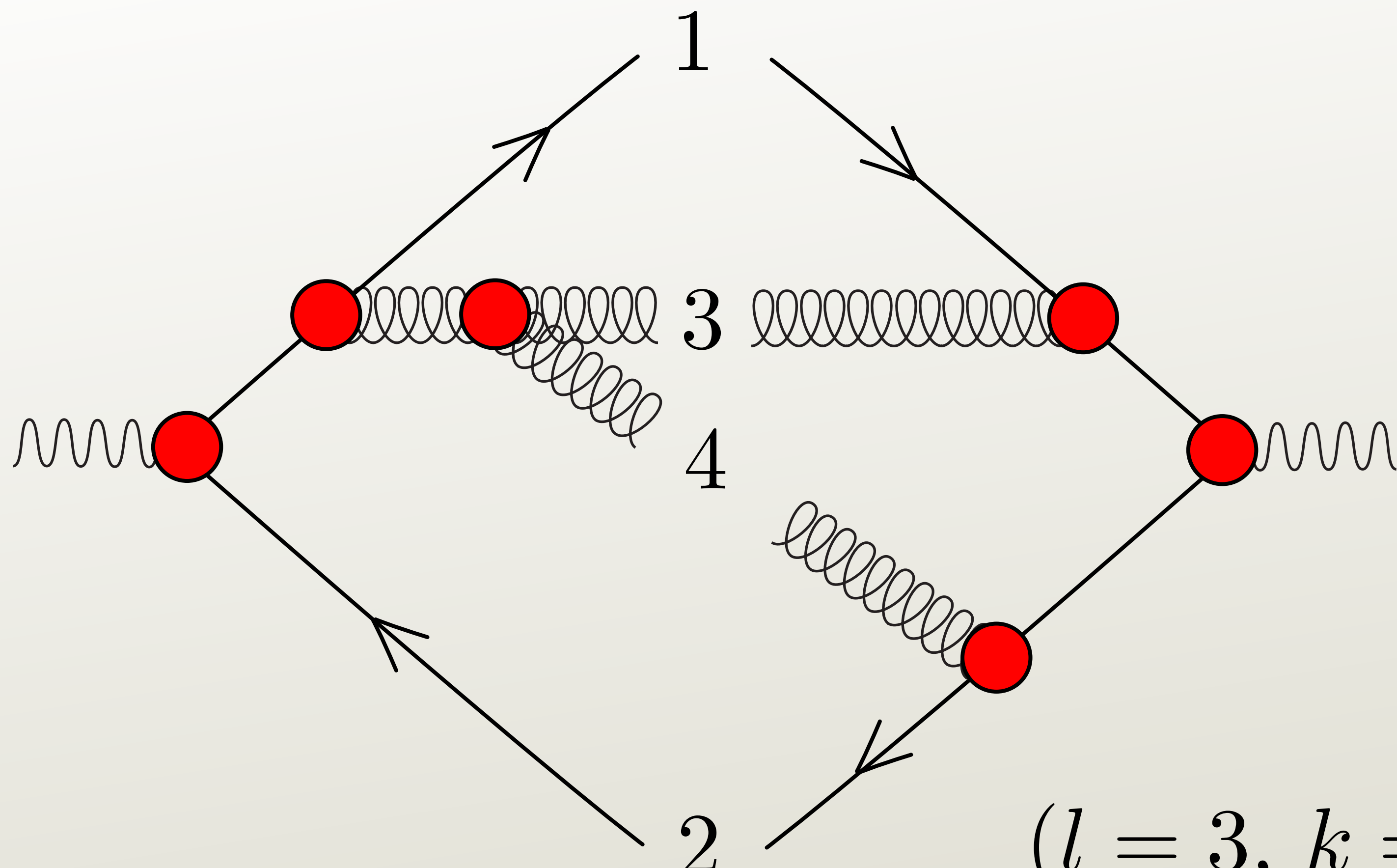
- I will use the Feynman diagram picture.

- Add one more gluon.



- Gluon 4 is very soft ($\hat{p}_4 \to 0$ at constant angle).

- It is emitted from parton l with dipole partner k. ($l = 3$, $k = 2$ here.)



$(l = 3,\ k = 2$ here.)

- Emission probability:

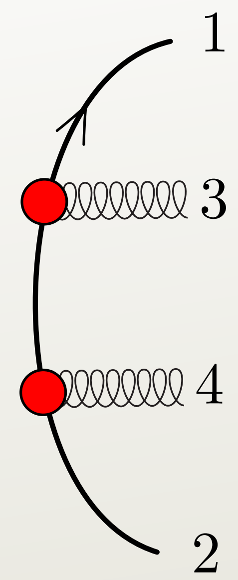
$$\Phi_{lk} = \frac{\hat{p}_4 \cdot \hat{p}_k\ \hat{p}_l \cdot Q}{\hat{p}_4 \cdot \hat{p}_k\ \hat{p}_l \cdot Q + \hat{p}_4 \cdot \hat{p}_l\ \hat{p}_k \cdot Q}\ \frac{2\hat{p}_k \cdot \hat{p}_l}{\hat{p}_4 \cdot \hat{p}_k\, \hat{p}_4 \cdot \hat{p}_l}$$

partitioning factor — dipole factor

emission from l

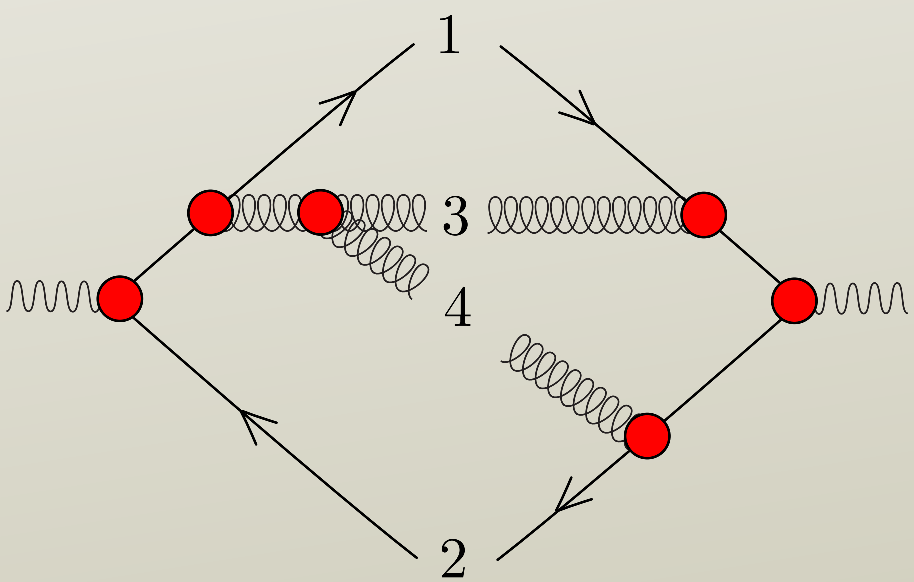
- Color states.

$$\big|[1,3,4,2]\big\rangle \propto$$



- New statistical state:

$$\rho = \left(\Phi_{31}C_{31} + \Phi_{32}C_{32} + \Phi_{13}C_{13} + \Phi_{23}C_{23} + \Phi_{12}C_{12} + \Phi_{21}C_{21}\right)\Phi_0$$



$$C_{32} = (C_{\rm F}/2)\left(\big|[1,3,4,2]\big\rangle - \big|[1,4,3,2]\big\rangle\right)\big\langle[1,3,4,2]\big| + (C_{\rm F}/2)\big|[1,3,4,2]\big\rangle\left(\big\langle[1,3,4,2]\big| - \big\langle[1,4,3,2]\big|\right)$$

LC+ approximation

- The LC+ approximation omits some parts of the color operator

$$\rho^{\mathrm{LC+}} = \left(\sum_{lk} \Phi_{lk}\, C_{lk}^{\mathrm{LC+}} \right) \Phi_0$$

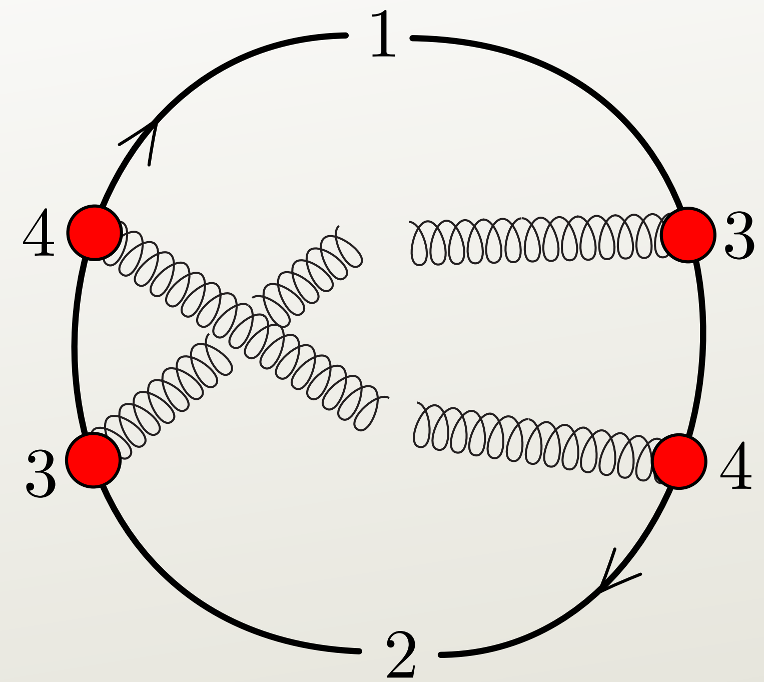
- Define omitted terms

$$\Delta\rho = \rho - \rho^{\mathrm{LC+}} \qquad \Delta C_{lk} = C_{lk} - C_{lk}^{\mathrm{LC+}}$$

so

$$\Delta\rho = \left(\sum_{lk} \Phi_{lk}\, \Delta C_{lk} \right) \Phi_0$$

- The omitted color states



$\big|[1,4,3,2]\big\rangle\big\langle[1,3,4,2]\big|$

$$\Delta C_{31} = 0$$

$$\Delta C_{32} = 0$$

$$\Delta C_{13} = -\frac{C_\mathrm{F}}{2}\big|[1,4,3,2]\big\rangle\big\langle[1,3,4,2]\big| - \frac{C_\mathrm{F}}{2}\big|[1,3,4,2]\big\rangle\big\langle[1,4,3,2]\big|$$

$$\Delta C_{23} = -\frac{C_\mathrm{F}}{2}\big|[1,3,4,2]\big\rangle\big\langle[1,4,3,2]\big| - \frac{C_\mathrm{F}}{2}\big|[1,4,3,2]\big\rangle\big\langle[1,3,4,2]\big|$$

$$\Delta C_{12} = \frac{C_\mathrm{F}}{2}\big|[1,4,3,2]\big\rangle\big\langle[1,3,4,2]\big| + \frac{C_\mathrm{F}}{2}\big|[1,3,4,2]\big\rangle\big\langle[1,4,3,2]\big|$$

$$\Delta C_{21} = \frac{C_\mathrm{F}}{2}\big|[1,3,4,2]\big\rangle\big\langle[1,4,3,2]\big| + \frac{C_\mathrm{F}}{2}\big|[1,4,3,2]\big\rangle\big\langle[1,3,4,2]\big|$$

Omitted probabilities

- Calculate probabilities by taking the trace of the color density matrix.
- In a parton shower, this calculation is at the end.

$$\mathrm{Tr}\Delta\rho = \left(\sum_{l,k} \Phi_{lk} \mathrm{Tr}\, \Delta C_{lk} \right) \Phi_0$$

- This given

$$\mathrm{Tr}\,\Delta\rho = \left((\Phi_{12} - \Phi_{13})\frac{-1}{2N_\mathrm{c}} + (\Phi_{21} - \Phi_{23})\frac{-1}{2N_\mathrm{c}}\right)\Phi_0$$

$$\Phi_{12} = \frac{\hat{p}_4\cdot\hat{p}_2\ \hat{p}_1\cdot Q}{\hat{p}_4\cdot\hat{p}_2\ \hat{p}_1\cdot Q + \hat{p}_4\cdot\hat{p}_1\ \hat{p}_2\cdot Q}\ \frac{2\hat{p}_2\cdot\hat{p}_1}{\hat{p}_4\cdot\hat{p}_2\,\hat{p}_4\cdot\hat{p}_1}$$

$$\Phi_{13} = \frac{\hat{p}_4\cdot\hat{p}_3\ \hat{p}_1\cdot Q}{\hat{p}_4\cdot\hat{p}_3\ \hat{p}_1\cdot Q + \hat{p}_4\cdot\hat{p}_1\ \hat{p}_3\cdot Q}\ \frac{2\hat{p}_3\cdot\hat{p}_1}{\hat{p}_4\cdot\hat{p}_3\,\hat{p}_4\cdot\hat{p}_1}$$

- $\Phi_{12} - \Phi_{13}$ is not singular when $\hat{p}_4$ becomes collinear with $\hat{p}_1$ or $\hat{p}_2$ or $\hat{p}_3$.

- $\Phi_{21} - \Phi_{23}$ also has no collinear singularities.

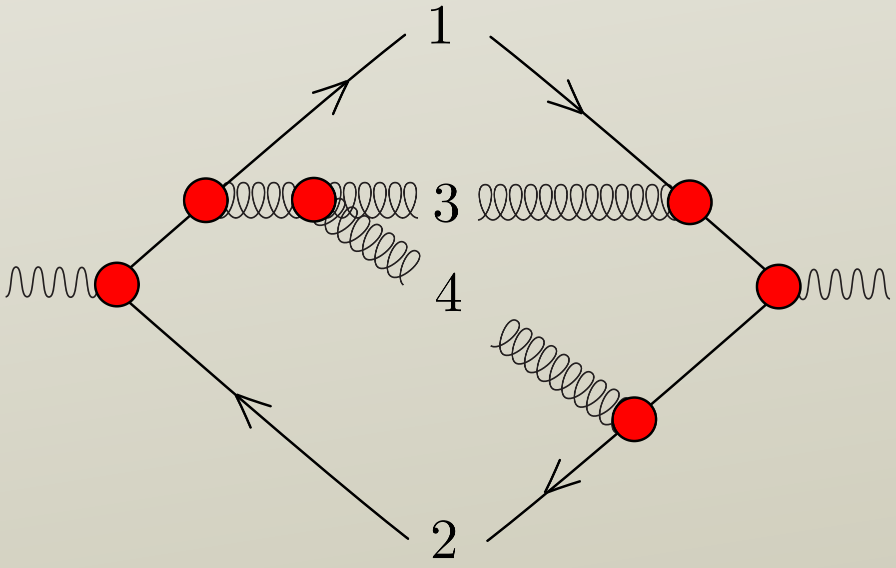
- So $\mathrm{Tr}\,\Delta\rho$ is singular only when $\hat{p}_4 \to 0$ at constant angle.

- $\mathrm{Tr}\,\Delta\rho$ is not singular in the regions that give leading logarithms.

Probabilities with LC+

$$\sum_l \sum_{k\neq l} \Phi_{lk} \,\mathrm{Tr}\, C_{lk}^{\mathrm{LC+}} = (\Phi_{31}+\Phi_{32})\frac{C_\mathrm{A}}{2} + (\Phi_{13}+\Phi_{23})C_\mathrm{F}$$

- This result was obtained by Gustafson (1993) based on Lund diagrams and color coherence.



- This method has been extended by Hamilton, Medves, Salam, Scyboz and Soyez (2020).

Some results

- Choose the thrust distribution as an example.
- Look at the Laplace transform of the thrust distribution.

$$\tilde{g}(\nu) = \frac{1}{\sigma_{\rm H}} \big(1\big| e^{-\nu(1-T)}\, \mathcal{U}(\mu_f^2, Q^2)\, \big|\rho_{\rm H}\big)$$

- Manipulate this to the form

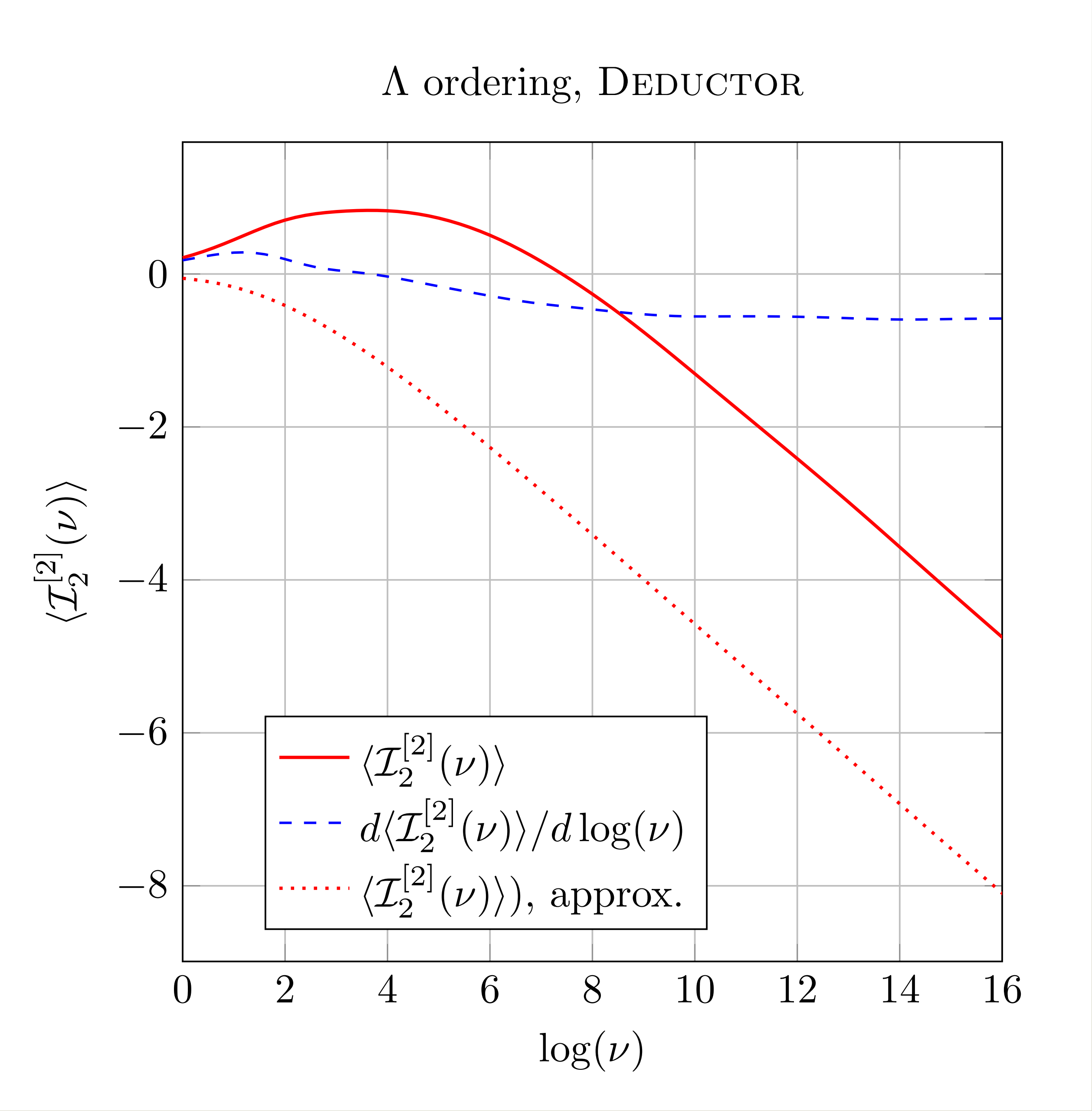
$$\tilde{g}(\nu) = \exp\left(\sum_{k=1}^{\infty} \sum_{n=k}^{\infty} \left[\frac{\alpha_{\rm s}(Q^2/\nu)}{2\pi} \right]^n \left\langle \mathcal{I}_n^{[k]}(\nu) \right\rangle \right)$$

- $\langle \mathcal{I}_n^{[k]}(\nu) \rangle$ are integrals with k factors of the shower splitting operator.

$$\tilde{g}(\nu) = \exp\left(\sum_{k=1}^{\infty}\sum_{n=k}^{\infty}\left[\frac{\alpha_s(Q^2/\nu)}{2\pi}\right]^n \left\langle \mathcal{I}_n^{[k]}(\nu)\right\rangle\right)$$

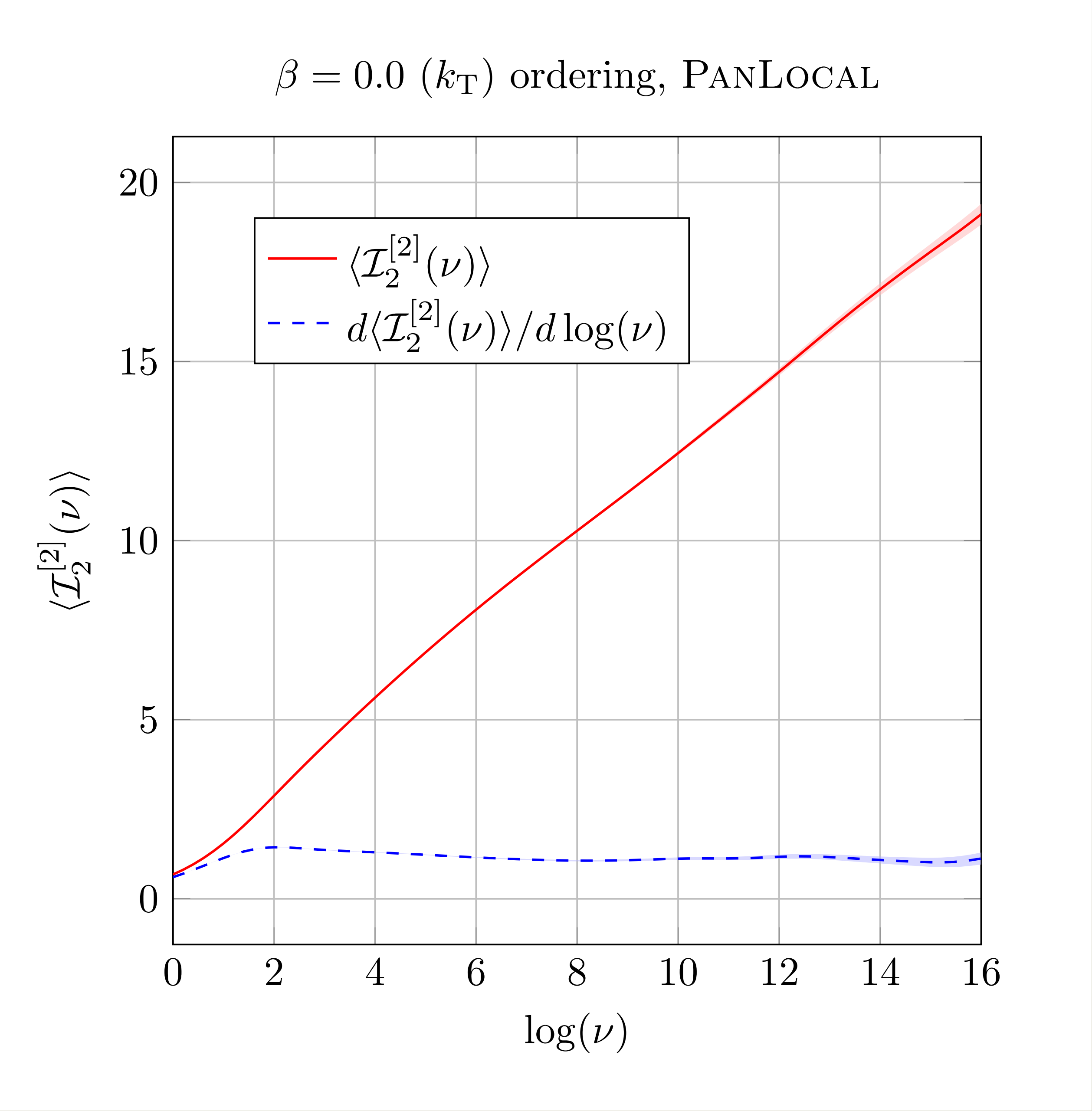
- $\langle \mathcal{I}_n^{[k]}(\nu)\rangle$ are integrals with k factors of the shower splitting operator.
- $\langle \mathcal{I}_n^{[1]}(\nu)\rangle$ gives the NLL summation of thrust logarithms.
- Check that $\langle \mathcal{I}_n^{[k]}(\nu)\rangle$ for $k > 1$ does not contribute NLL terms.
- Need $\langle \mathcal{I}_n^{[k]}(\nu)\rangle$ to contain no higher power of $\log(\nu)$ than $[\log(\nu)]^{n-1}$.
- Sometimes one can show this analytically for all k and n.
- Otherwise, use numerical checks for particular k and n.

- $\langle \mathcal{I}_2^{[2]}(\nu) \rangle$ should have no higher power than $[\log(\nu)]^1$ for large ν.



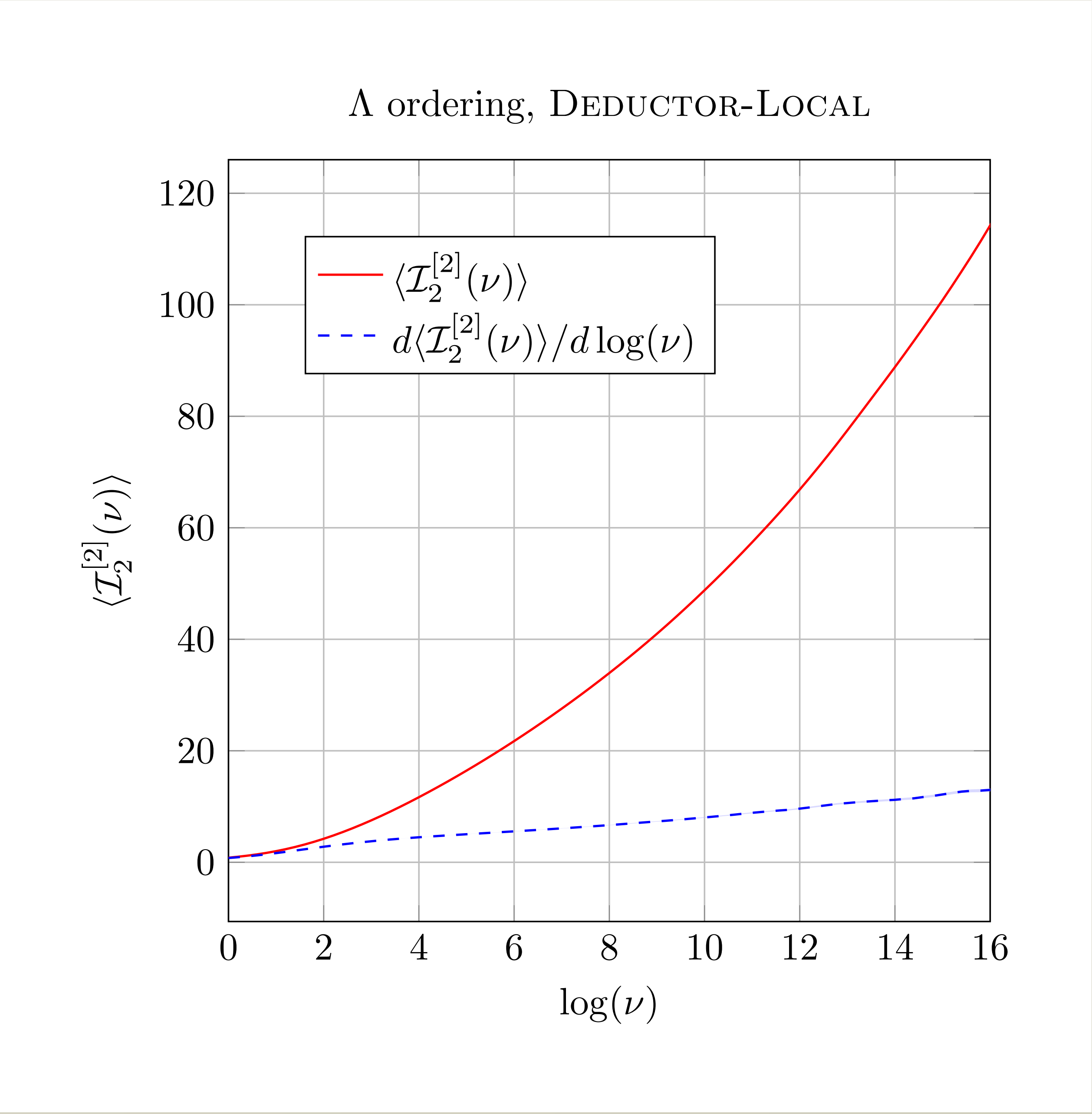
- For DEDUCTOR with its default Λ ordering, this works.

- $\langle \mathcal{I}_2^{[2]}(\nu) \rangle$ should have no higher power than $[\log(\nu)]^1$ for large ν.



- For the PanLocal shower (but with full color) this works.

- $\langle \mathcal{I}_2^{[2]}(\nu) \rangle$ should have no higher power than $[\log(\nu)]^1$ for large ν.



- For the Deductor shower with Λ ordering but with a local momentum mapping this fails.