NLP RESUMMATION AND THE ENDPOINT DIVERGENT CONTRIBUTION IN DIS

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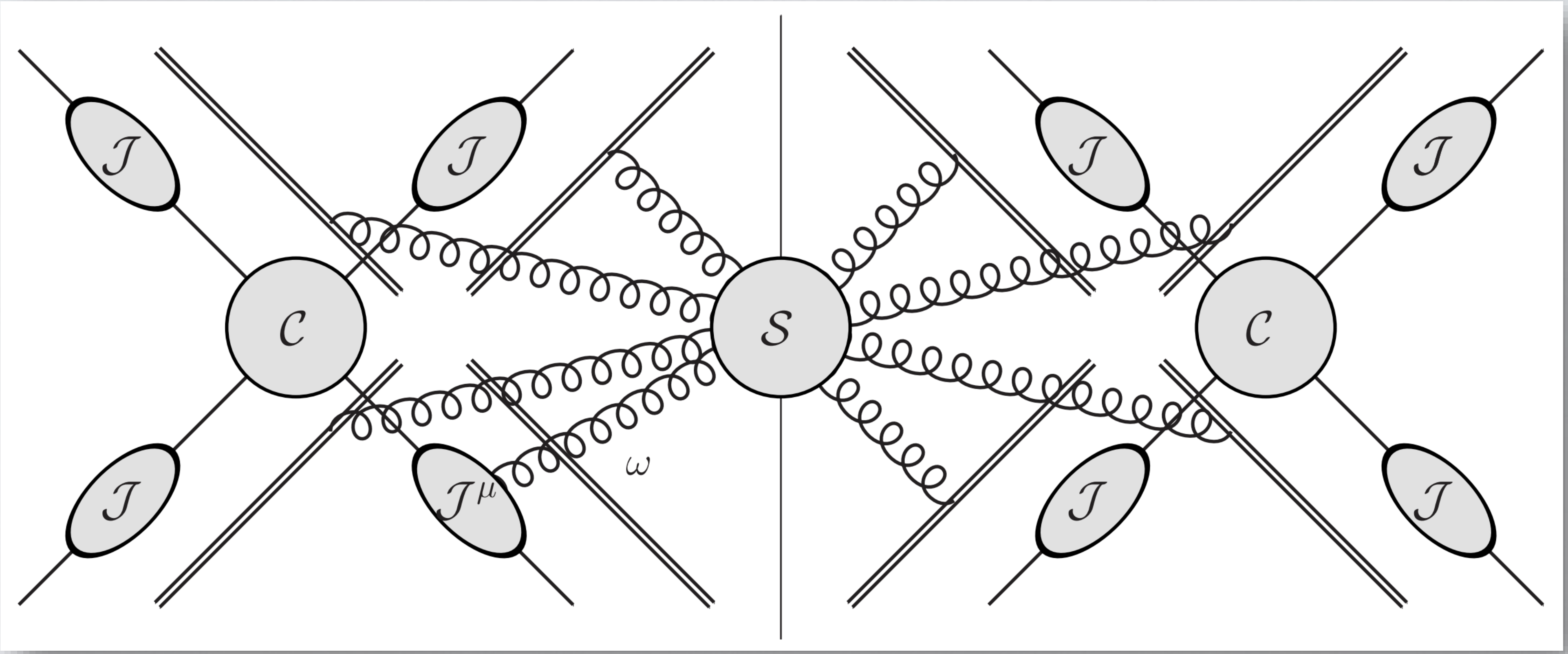
OUTLINE

- **Factorisation and resummation in SCET: LP vs NLP**
- **Deep Inelastic scattering: qg channel**
    - **DIS and endpoint divergences**
    - **Resummation from D-dimensional consistency conditions**
    - **Resummation from re-factorization: a glimpse**

JHEP 10 (2020), 196, [arXiv:2008.04943],

with M. Beneke, M. Garny, S. Jaskiewicz, R. Szafron and J. Wang.

FACTORIZATION AND RESUMMATION IN SCET: LP VS NLP



FACTORIZATION AND RESUMMATION AT NLP

$$\frac{d\sigma}{d\xi} \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(\xi)}{\xi}\right]_+ + d_{nm} \ln^m(\xi) \right) + \dots \right].$$

- Understanding the factorization and resummation of large logarithms at next-to-leading power (NLP) has been subject of intense work in the past few years!

- Drell-Yan, Higgs and DIS near threshold

  Del Duca, 1990; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016;
  Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019;
  van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021;
  Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
  Beneke, Broggio, Jaskiewicz, LV, 2019;
  Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019, 2020.

- Operators and Anomalous dimensions

  Larkoski, Neill, Stewart 2014;
  Moult, Stewart, Vita 2017; Feige, Kolodrubetz, Moult, Stewart 2017;
  Beneke, Garny, Szafron, Wang, 2017, 2018, 2019.

- Thrust

  Moult, Stewart, Vita, Zhu 2018, 2019.

- pT and Rapidity logarithms

  Ebert, Moult, Stewart, Tackmann, Vita, 2018,
  Moult, Vita Yan 2019;
  Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020.

- Mass effects

  Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang, Fleming, 2020;
  Liu, Mecaj, Neubert, Wang, 2020;
  Anastasiou, Penin, 2020.

And many more!
[O(50 publications) and counting]

FACTORIZATION AND RESUMMATION IN SCET AT LP

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\rm SCET} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \ldots dt_n \, \mathcal{C}(t_1, \ldots, t_n) \, \phi_1(t_1 n_{1+}) \ldots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \ldots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

FACTORIZATION AND RESUMMATION IN SCET

- Decompose momenta on the light-cone:

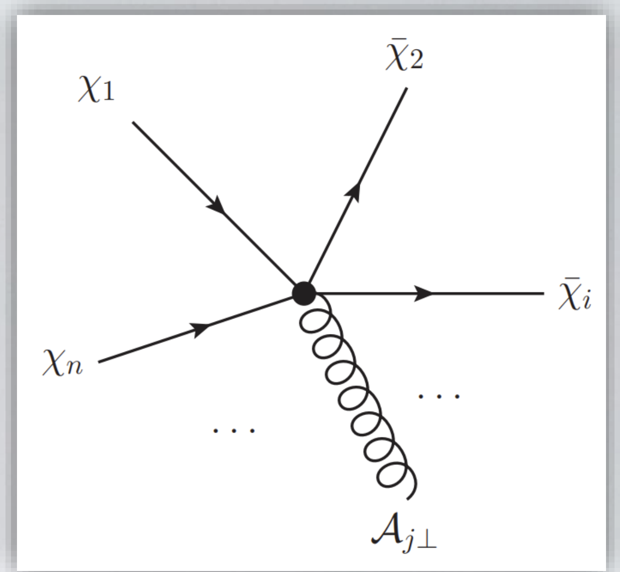
$$l^\mu = n_{i+}l\,\frac{n_{i-}^\mu}{2} + l_\perp^\mu + n_{i-}l\,\frac{n_{i+}^\mu}{2}, \qquad n_{i\pm}^2 = 0, \quad n_{i+}\cdot n_{i-} = 2.$$

- Systematic expansion in $\lambda \sim \dfrac{l_\perp}{n_+ l} \ll 1.$
  - Hard-collinear momenta: $n_+ l \sim Q, \qquad l_\perp \sim \lambda Q, \qquad n_- l \sim \lambda^2 Q,$
  - Soft momenta: $n_+ l \sim \lambda^2 Q, \qquad l_\perp \sim \lambda^2 Q, \qquad n_- l \sim \lambda^2 Q.$
- Every field has a well-defined scaling.
  - Hard-collinear quark: $\chi_c \sim \lambda,$
  - Hard-collinear gluon: $A_{c\perp} \sim \lambda,$
  - Soft quark: $q_s \sim \lambda^3.$
- A physical observables has the general expansion

$$\frac{d\sigma}{d\lambda} = \sum_{n=0}^{\infty}\sum_{m=0}^{2n+1}\left(\frac{c_{nm}^{\rm LP}}{\lambda^2} + d_{nm}^{\rm NLP}\right)\left(\frac{\alpha_s}{\pi}\right)^n ln^m \lambda.$$

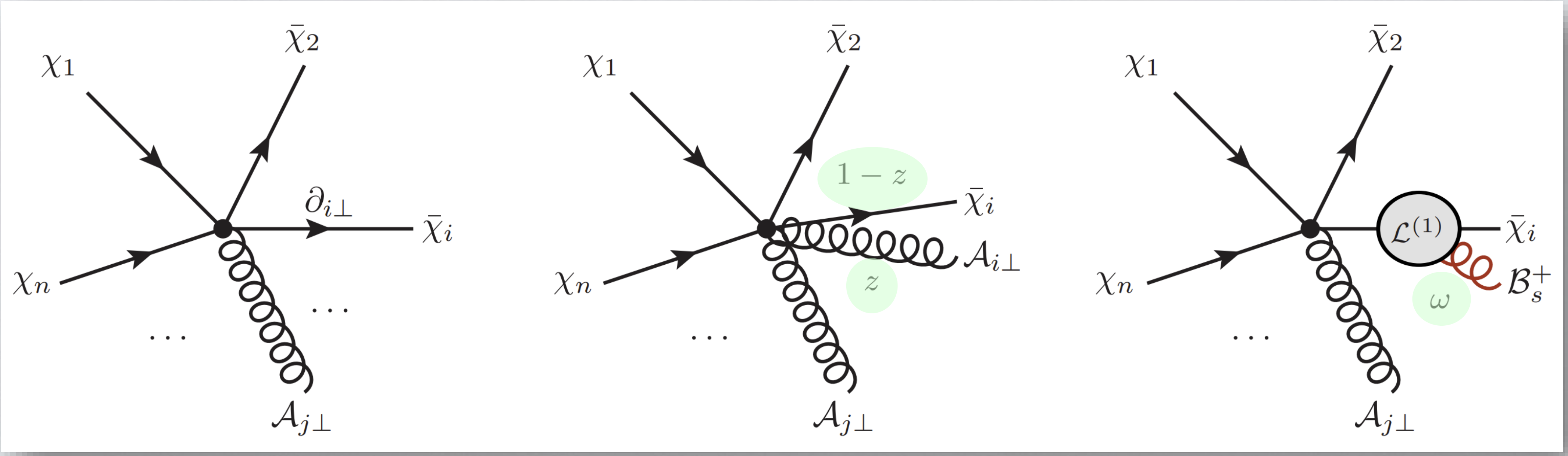
FACTORIZATION AND RESUMMATION IN SCET: LP VS NLP

- Leading power (LP):
  - N-jet operators;
  - Soft-collinear decoupling.

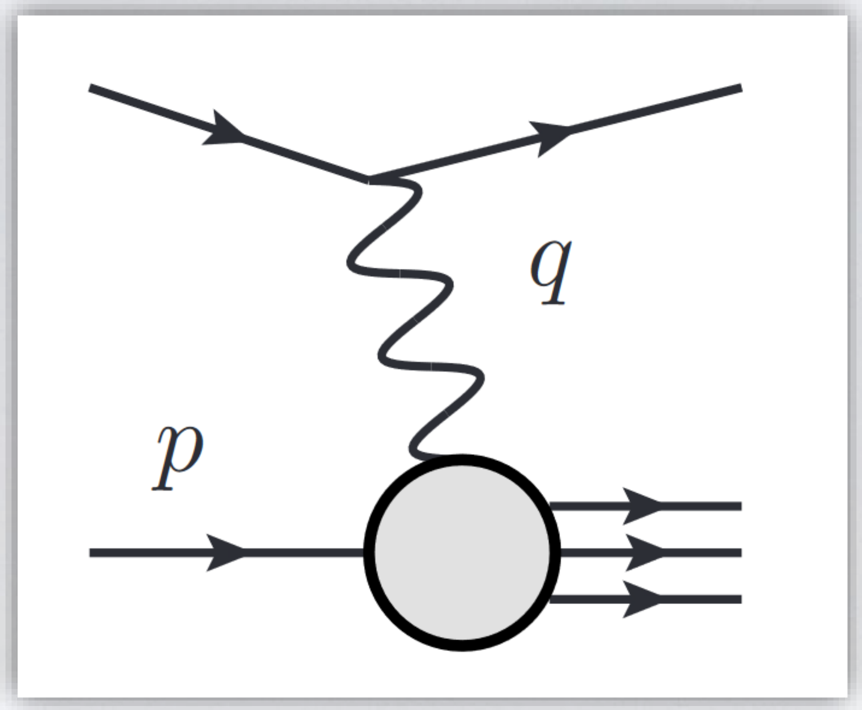


- Next-to-leading power (NLP):
  - Kinematic suppression;
  - Multi-particle emission along the same collinear direction;
  - No soft-collinear decoupling.

Beneke, Garny, Szafron, Wang, 2017,2018



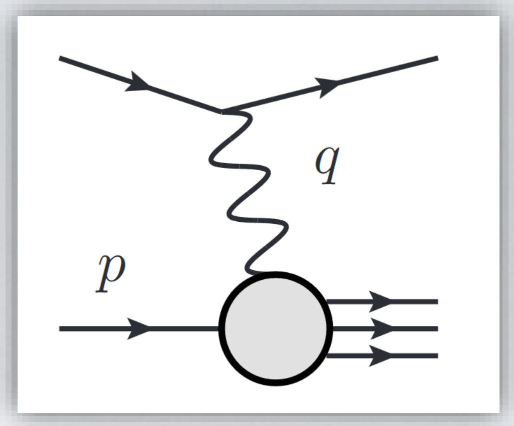
DEEP INELASTIC SCATTERING



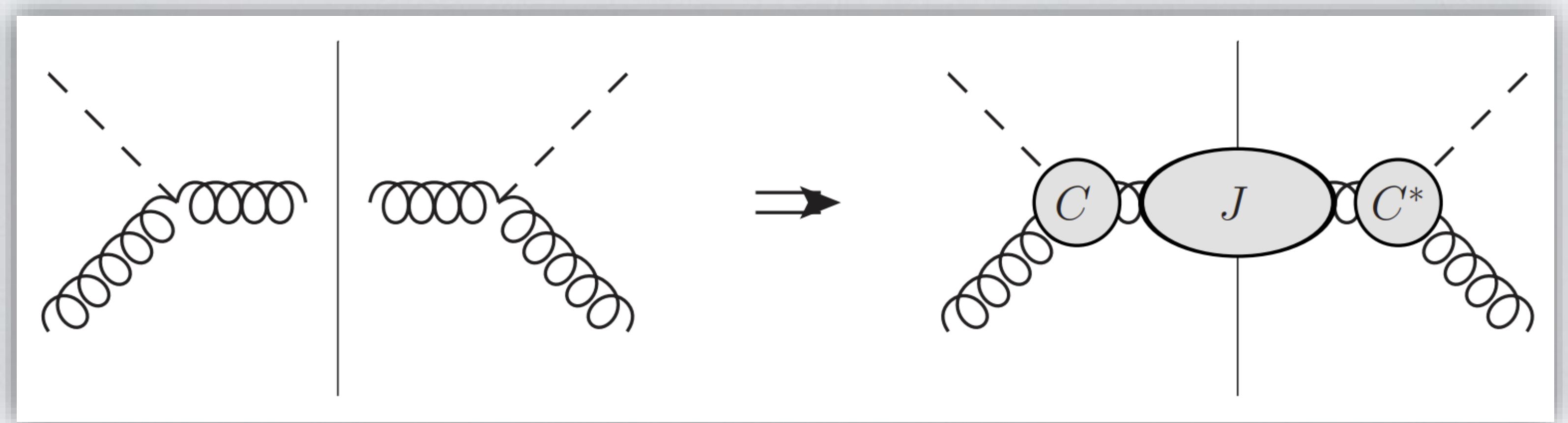
DEEP INELASTIC SCATTERING

- Deep inelastic scattering (DIS) near threshold develops a hierarchy of scales:

$$Q^2 \gg P_X^2 \sim Q^2(1-x), \qquad \text{with} \qquad x \equiv \frac{Q^2}{2p\cdot q} \to 1.$$



- Factorization and resummation well understood at LP:



Sterman 1987; Catani, Trentadue 1989; Korchemsky, Marchesini, 1993; Moch, Vermaseren, Vogt 2005; Becher, Neubert, Pecjak, 2007

$$W_\phi = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\,\langle N(P)\big|\big[G^A_{\mu\nu}G^{\mu\nu A}\big](x)\big[G^B_{\rho\sigma}G^{\rho\sigma B}\big](0)\big|N(P)\rangle$$

$$= |C(Q^2,\mu)|^2\int_x^1 \frac{d\xi}{\xi}\, J\left(Q^2\frac{1-\xi}{\xi},\mu\right)\frac{x}{\xi}f_g\left(\frac{x}{\xi},\mu\right).$$

Short-distance coefficient and jet function are single scale object – resummation obtained by solving the corresponding RGE.

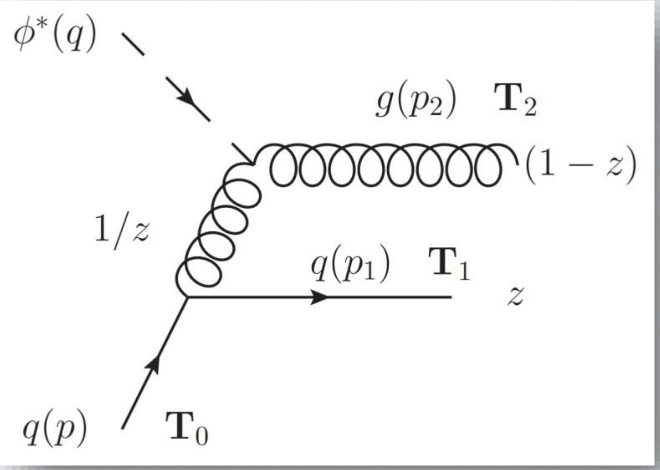
DIS: OFF-DIAGONAL CHANNEL

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- The off-diagonal channel $q(p) + \phi^*(q) \to X(p_X)$ contributes to DIS at NLP. Consider the partonic structure function

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg},z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}, \quad \mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)} \frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2}.$$

with momentum fraction $z \equiv \dfrac{n_-p_1}{n_-p_1 + n_-p_2}$, and $\bar{z} = 1 - z$.



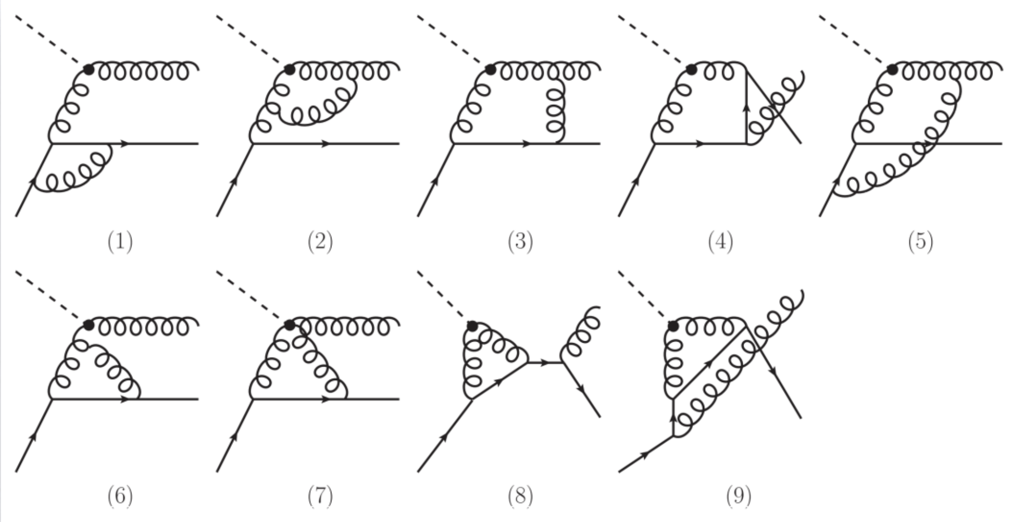
- At LO one has

$$\mathcal{P}_{qg}(s_{qg})|_{\text{tree}} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}, \quad \Rightarrow \quad W_{\phi,q}\Big|^{\text{NLP}}_{\mathcal{O}(\alpha_s),\,\text{leading pole}} = -\frac{1}{\epsilon}\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{Q^2(1-x)}\right)^{\epsilon}.$$

The single pole originate from *z → 0*, due to the *1/z* of the momentum distribution function.

- At NLO:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}$$
$$\cdot\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$



ON THE ENDPOINT DIVERGENCES

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

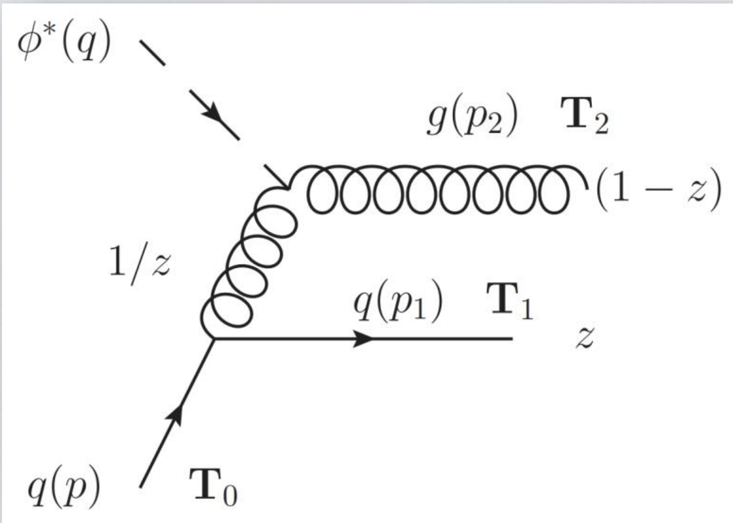
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots\,.$$

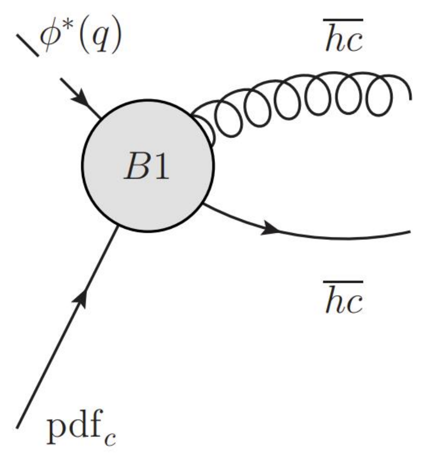
- Expansion in ε not possible before integration!
- The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

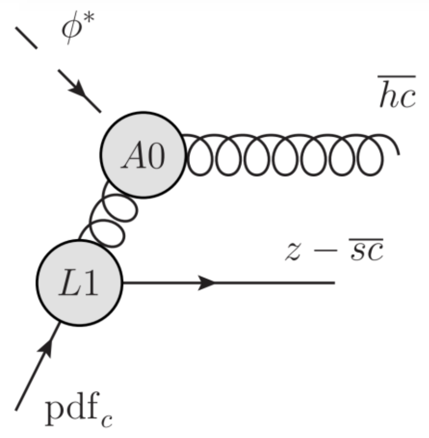
- What happens for $z \to 0$?



For $z \sim 1$ intermediate propagator is hard



For $z \ll 1$ intermediate propagator cannot be integrated out



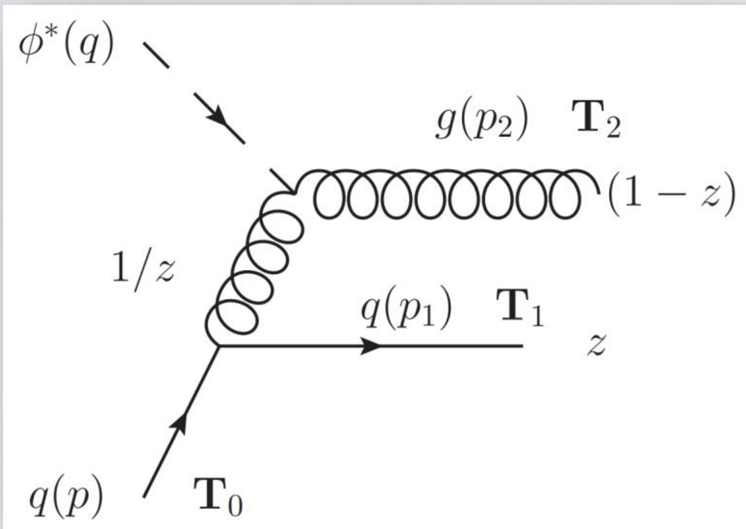
- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need "z-SCET".
- z-modes are non-physical! Not related to external scales of the problem.

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

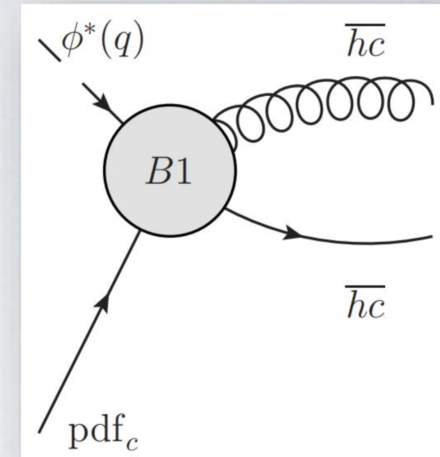
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

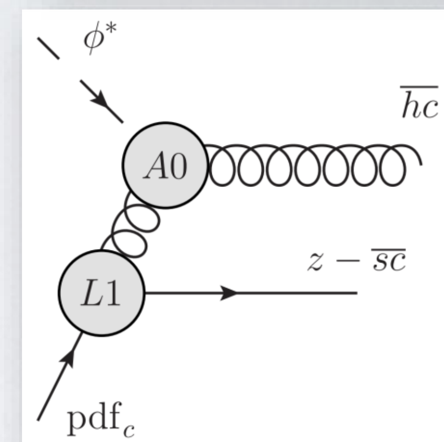
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



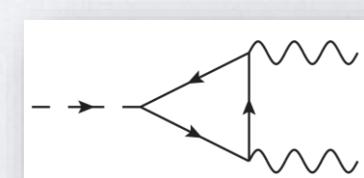
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need "z-SCET".
- z-modes are non-physical! Not related to external scales of the problem.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2)\int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2) D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



DIS FACTORIZATION

- Re-factorization is nontrivial: needs to be embedded in a full DIS EFT description:
- Physical modes:

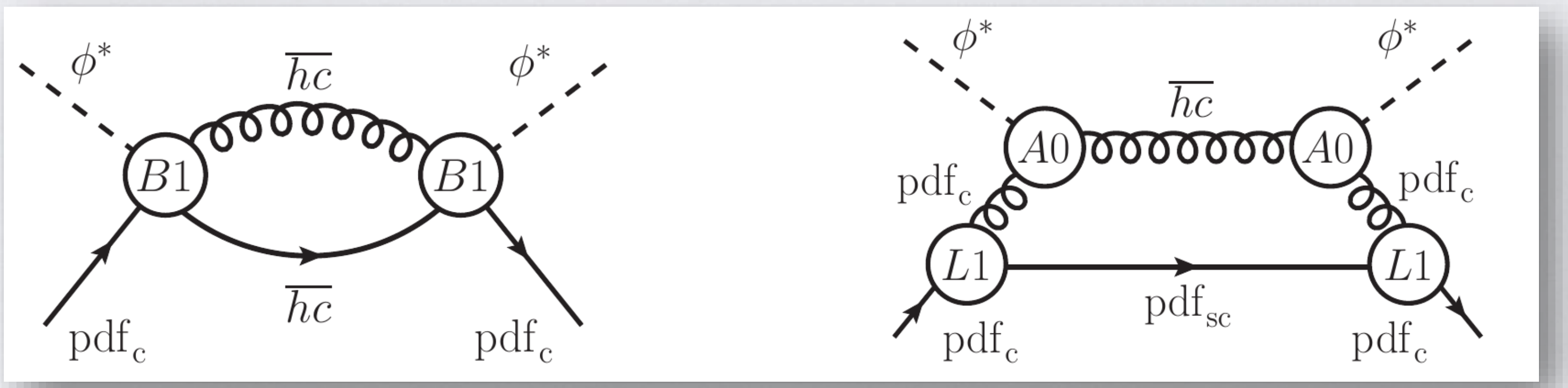
Perturbative modes
$$\left\{\begin{array}{ll} \text{Hard:} & p^2 = Q^2, \\ \text{Hard-collinear:} & p^2 = Q^2\lambda^2 = Q^2/N, \end{array}\right.$$

"z-SCET" is here

Non-perturbative modes
$$\left\{\begin{array}{ll} \text{Collinear:} & p^2 = \Lambda^2, \\ \text{Soft-collinear:} & p^2 = \Lambda^2\lambda^2 = \Lambda^2/N. \end{array}\right.$$

$$g(N) \equiv \int_0^1 dx\, x^{N-1}\, g(x),$$

$$x \to 1 \;\Leftrightarrow\; N \to \infty.$$



Time-ordered product contribution

B-type current contribution

- Both terms contain endpoint divergences in the convolution integral.
- We could reshuffle factorization theorem;
  → however, use d-dimensional consistency conditions to start with. ⟶ ***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020***

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Hadronic structure function is finite:

$$W = \sum_i W_{\phi,i} f_i = \sum_k \tilde{C}_{\phi,k} \tilde{f}_k.$$

- Focus on the bare functions: expansion to NLP gives

$$\sum_i (W_{\phi,i} f_i)^{NLP} = W_{\phi,q}^{NLP} f_q^{LP} + W_{\phi,\bar{q}}^{NLP} f_{\bar{q}}^{LP} + W_{\phi,g}^{NLP} f_g^{LP} + W_{\phi,g}^{LP} f_g^{NLP}.$$

- Work in d-dimensions: ε regularizes endpoint divergences in convolution integrals. The general expansion of the cross section reads

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon} + f_{\bar{q}}(\Lambda),\ f_g(\Lambda) \text{ terms}.$$

- In this equation:

Each hard loop gives $\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}$, each hard-collinear loop gives $\left(\frac{\mu^2}{Q^2} N\right)^{\epsilon}$,

Each collinear loop gives $\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}$, each soft-collinear loop gives $\left(\frac{\mu^2}{\Lambda^2} N\right)^{\epsilon}$.

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Invoking pole cancellations:

$$\sum_{k=0}^{n}\sum_{j=0}^{n} j^r k^s c_{kj}^{(n)} = 0 \qquad \text{for } s+r < 2n-1,\ r,s \geq 0\,,$$

  allows $(n+1)^2$ coefficients $c_{kj}^{(n)}$ to be determined from $2n^2$-n equations.

- Three unknowns: these can be reduced using information from the region expansion:
  - $c_{n0}^{(n)} = 0$ (final state cannot be purely hard)
  - $c_{00}^{(n)} = 0$ (without any hard or anti-hardcollinear loops there must be at least one soft-collinear loop).
  - Non-trivial condition is given by $c_{n1}^{(n)}$: this is the n-loop hard region!

- Assume exponentiation of 1-loop result:

Similar conjecture "soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019.

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\Bigg(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}$$
$$+\,\mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\Bigg) + \mathcal{O}(\epsilon^{-1}).$$

- Restricting to the hard region and substituting color operators one has

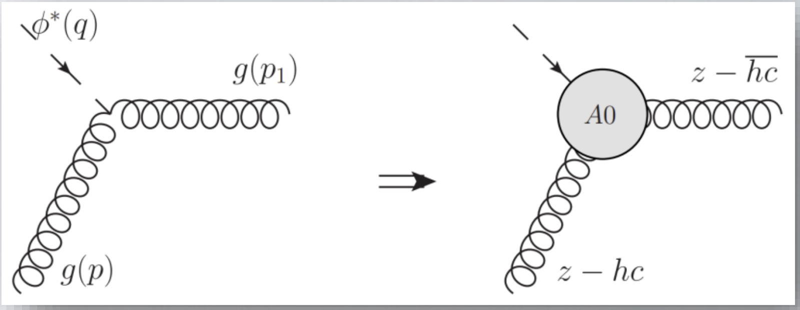
$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\,\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

RESUMMATION FROM RE-FACTORIZATION: A GLIMPSE

- Is it possible to achieve this in SCET? Another look at re-factorisation:

$$C^{B1}(Q,z)J^{B1}(z) \overset{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)J^{B1}_{z-\overline{sc}}.$$

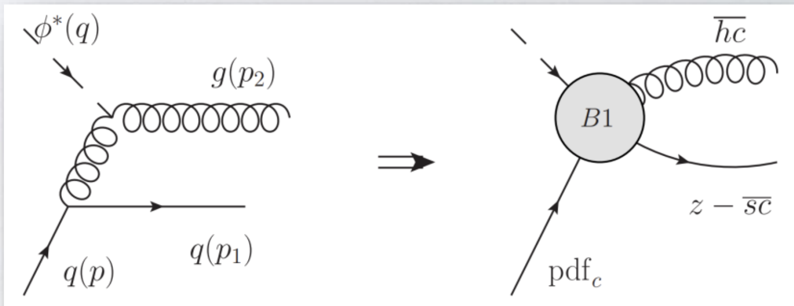
- Integrate out hard modes (solve RGEs in d-dimensions)



$$\frac{d}{d\ln\mu}C^{A0}(Q^2,\mu^2) = \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\,C^{A0}(Q^2,\mu^2)\,.$$

$$\Rightarrow \qquad \left[C^{A0}\left(Q^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right].$$

- Integrate out z-hardcollinear modes



$$\frac{d}{d\ln\mu}D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha_s}{\pi}\left(C_F-C_A\right)\ln\frac{zQ^2}{\mu^2}\,D^{B1}\left(zQ^2,\mu^2\right).$$

$$\Rightarrow \qquad \left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right].$$

- This reproduces

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + \left(C_A-C_F\right)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

OFF-DIAGONAL DIS: FINITE STRUCTURE FUNCTION

- Integrating P_{qg} we get W:

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg}, z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}},$$

- Furthermore, recall

$$W = \sum_i W_{\phi,i} f_i = \sum_k \tilde{C}_{\phi,k} \tilde{f}_k, \quad \Rightarrow \quad W_{\phi,q}^{NLP} = \tilde{C}_{\phi,q}^{NLP} Z_{qq}^{LP} + \tilde{C}_{\phi,g}^{LP} Z_{gq}^{NLP}.$$

- We obtain a solution for $\tilde{C}$:

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

$$\tilde{C}_{\phi,q}^{NLP,LL}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{C_F}{C_F - C_A}\Bigg(\mathcal{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N \ln\frac{\mu^2}{Q^2}\right)\right] - \exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N\ln\frac{\mu^2}{Q^2}\right)\right]\Bigg), \qquad \text{with} \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N,$$

and

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d\ln\mu}(Z^{-1})_{kj}, \qquad \gamma_{gq}^{NLP,LL}(N) = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a), \qquad \mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n,$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, …

- Reproduces earlier conjecture by *Vogt, 2010*.

CONCLUSION

- Factorisation theorems beyond LP involves divergences in convolution integrals:

  → require additional refactorization.

- New “internal” modes appear due to endpoint divergences;

  → rigorous factorization and resummation still possible, but highly non-trivial.

- Relationships between bare and renormalized objects need to be better understood.

- We provided an explicit example (off-diagonal DIS *qg* channel) where these problems can be studied, obtaining new insight.

- Solving this problem would open up the way for interesting applications in collider and flavour physics.