

Transverse momentum resummation in the leptonic azimuthal spectra of Drell-Yan processes

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in collaboration with Marek Schönherr

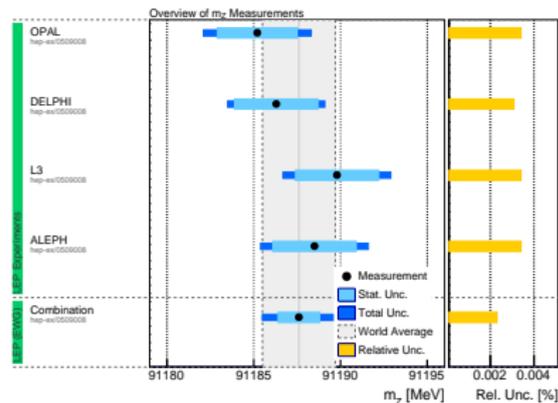
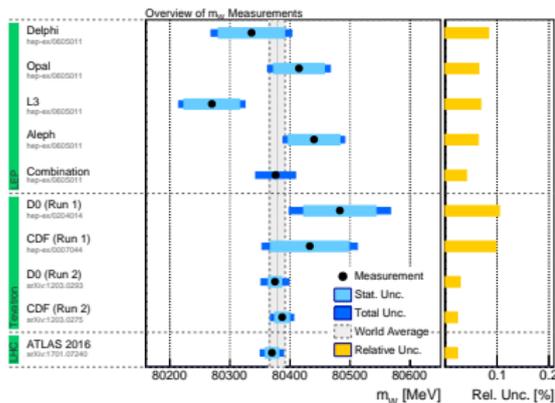
IPPP, Durham University

based on "A novel approach to correlate W and Z production spectra", to appear

Parton Showers and Resummation, 27 May 2021

Experimental measurements

- Precise measurement on the masses of the electroweak boson W^\pm/Z
 - $M_Z = 91.1876 \pm 0.0021$ GeV [PDG]
 - $M_W = 80.379 \pm 0.012$ GeV [PDG]



[Erlar:2019hds]

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Experimental measurements

- Precise measurement on the M_W at LHC
 - PDF uncertainties
 - electroweak corrections
 - modelling effects in the small q_T regime

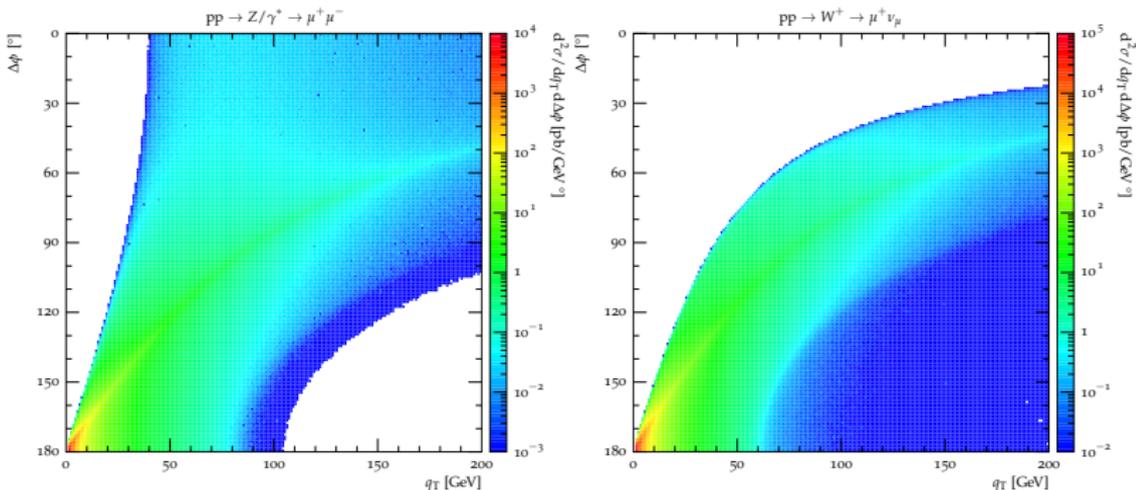
$$R_{W/Z}(p_T) = \left(\frac{1}{\sigma_W} \cdot \frac{d\sigma_W(p_T)}{dp_T} \right) \left(\frac{1}{\sigma_Z} \cdot \frac{d\sigma_Z(p_T)}{dp_T} \right)^{-1}$$

1. Tailoring PS with Z data (percent level) [[ATLAS/Eur. Phys. J. C 78 \(2018\) 110](#)]
 2. Direct measurements (percent level)
[[CMS/CMS-SMP-14-012;ATLAS/ATL-PHYS-PROC-2020-014](#)]
 3. QCD resummation method → This Work
 4.
-

Experimental measurements

- Correlation with $\Delta\phi$

$$\Delta\phi = \arccos \left[\frac{\vec{p}_{1,T} \cdot \vec{p}_{2,T}}{|\vec{p}_{1,T}| |\vec{p}_{2,T}|} \right]. \quad (1)$$



- Singularity $q_T \rightarrow 0$; $\Delta\phi \rightarrow \pi$

Development of the transverse momentum resummation

- The first all-order proof of $\log[q_T/M_V]$ exponentiation
[\[Collins&Soper&Sterman1984\]](#)
- The direct momentum-space resummation
[\[Monni:2016ktx,Bizon:2017rah,Bizon:2019zgf,Bizon:2018foh\]](#)
- Soft-collinear effective theory (SCET) → This Work
[\[Bauer:2000yr,Bauer:2001yt,Beneke:2002ph\]](#).

$$\text{QCD}_6 \rightarrow \underbrace{\text{QCD}_5 \rightarrow \text{SCET}}_{\text{rapidity singularities}}$$

- Analytic regulator [\[Becher:2010tm\]](#).
- η regulator [\[Chiu:2011qc,Chiu:2012ir\]](#)
- Exponential regulator [\[Li:2016axz\]](#) → This Work
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- The distributional space proposal [\[Ebert:2016gcn\]](#)
-

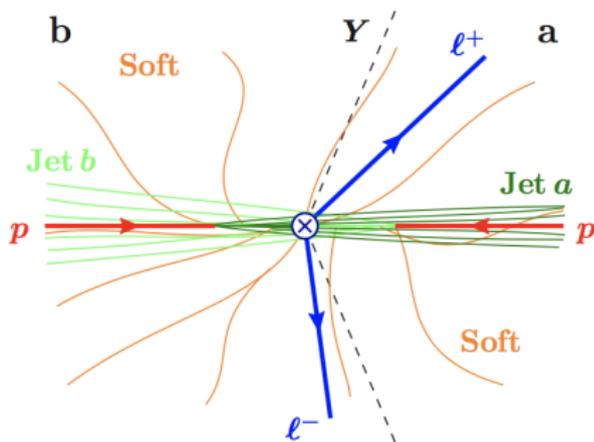
Factorization and resummation

- Factorization in the RaGE&RGE scheme

[Chiu:2011qc,Chiu:2012ir,Li:2016axz]

$$\frac{d^4\sigma}{d^2\vec{q}_T dY_L dM_L^2 d\Omega} \sim \mathcal{H}_{ij}^V \otimes \mathcal{B}_n^i \otimes \mathcal{B}_{\bar{n}}^j \otimes \mathcal{S}_{ij} \quad (2)$$

- q_T TM of LP
- M_L IM of LP
- Y_L Rapidity of LP
- Ω solid angles of final lepton



[1004.2489]

Factorization and resummation

- Factorization in the RaGE&RGE scheme

[Chiu:2011qc,Chiu:2012ir,Li:2016axz]

$$\frac{d^4\sigma}{d^2\vec{q}_T dY_L dM_L^2 d\Omega} \sim \mathcal{H}_{ij}^V \otimes \mathcal{B}_n^i \otimes \mathcal{B}_{\bar{n}}^j \otimes \mathcal{S}_{ij} \quad (3)$$

- Soft function @N³LO [Li:2014afw,Li:2016ctv]
- Beam functions @N³LO [Luo:2019szz,Luo:2020epw,Ebert:2020yqt]
- Hard function @N³LO
 - $pp \rightarrow W^{\pm*} \rightarrow \mu^{\pm}\nu(\bar{\nu})$: non-singlet diagrams
 - $pp \rightarrow Z^*/\gamma^* \rightarrow \mu^+\mu^-$: singlet&non-singlet diagrams

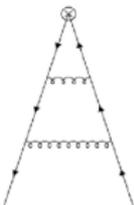
Factorization and resummation

- Fixed-order ingredients in the DY process

1. Hard function @N³LO

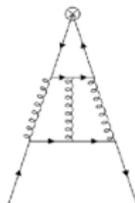
- Non Singlet: external quark lines are connected to the electric-weak (EW) vertex
- Singlet: external quark lines are **NOT** connected to the electric-weak (EW) vertex

Vector&Axial-Vector



(a) Non-singlet amplitude

Vector



(b) Singlet amplitude induced by vector current

Axial-Vector



(c) Singlet amplitude induced by axial-vector current

Hard function of $pp \rightarrow Z^*/\gamma^* \rightarrow \mu^+\mu^-$: $\text{QCD}_6 \rightarrow \text{QCD}_5$

- Effective vector current: $V_{\text{EFT}} \rightarrow V_{\text{SM}}$ up to the redefinition of α_s and fields

[Chetyrkin:1997un,Appelquist:1974tg]

$$\begin{aligned} V_\gamma^\mu &= \sum_{q_i=u,d,c,s,b} g_\gamma^{q_i} \bar{q}_i \gamma^\mu q_i, \\ V_Z^\mu &= \sum_{q_i=u,d,c,s,b} g_V^{q_i} \bar{q}_i \gamma^\mu q_i. \end{aligned} \tag{4}$$

Hard function of $pp \rightarrow Z^*/\gamma^* \rightarrow \mu^+\mu^-$: $\text{QCD}_6 \rightarrow \text{QCD}_5$

- Effective axial-vector current:

[Chetyrkin:1993ug,Chetyrkin:1993jm,Larin:1993tq,Chetyrkin:1993hk]

$$A_Z^\mu = g_A \left[\sum_{i=1,2,3} \Delta_i^{\text{ns}} + C_t \mathcal{O}_s \right], \quad (5)$$

where

$$\begin{aligned} \Delta_1^{\text{ns}} &= \bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d, \\ \Delta_2^{\text{ns}} &= \bar{c} \gamma^\mu \gamma_5 c - \bar{s} \gamma^\mu \gamma_5 s, \\ \Delta_3^{\text{ns}} &= \frac{\mathcal{O}_s}{N_f} - \bar{b} \gamma^\mu \gamma_5 b. \end{aligned} \quad (6)$$

Hard function of $pp \rightarrow Z^*/\gamma^* \rightarrow \mu^+\mu^-$: $\text{QCD}_6 \rightarrow \text{QCD}_5$

- Effective axial-vector current:

[Chetyrkin:1993ug,Chetyrkin:1993jm,Larin:1993tq,Chetyrkin:1993hk]

$$A_Z^\mu = g_A \left[\sum_{i=1,2,3} \Delta_i^{\text{ns}} + C_t \mathcal{O}_s \right], \quad (7)$$

where

$$\mathcal{O}_s = \sum_{q_i=u,d,c,s,b} \bar{q}_i \gamma^\mu \gamma_5 q_i. \quad (8)$$

C_t extraction:

Larin Scheme: [Bernreuther:2005rw] NNLO

KC scheme: [Chetyrkin:1993jm,Chetyrkin:1993ug] NNNLO

- Amplitudes induced by V_Z and V_γ
 1. non-singlet: $\gamma^* q\bar{q}$ form factor [Moch:2005id, Baikov:2009bg, Gehrmann:2010ue]
 2. singlet: [Baikov:2009bg, Gehrmann:2010ue]
- Amplitudes induced by A_Z
 1. non-singlet: $\gamma^* q\bar{q}$ form factor [Moch:2005id, Baikov:2009bg, Gehrmann:2010ue]
 2. (pure-)singlet: [Bernreuther:2005rw]NNLO& [Larin:1993tq]NNLO_[log]

Hard function of $pp \rightarrow W^\pm \rightarrow \mu^\pm \nu(\bar{\nu})$: $\text{QCD}_5 \rightarrow \text{SCET}_{\text{II}}$

- only non-singlet amplitudes

1. non-singlet: $\gamma^* q \bar{q}$ form factor [[Moch:2005id](#), [Baikov:2009bg](#), [Gehrmann:2010ue](#)]
2.

Resummation

- Resummation: RaGE&RGE.

$$\frac{d^4 \sigma_{\text{res}}}{d^2 \vec{q}_T dY_L dM_L^2 d\Omega_L} \sim \sum_{i,j=q,\bar{q},g} \int d^2 \vec{b}_T e^{i\vec{b}_T \cdot \vec{q}_T} \mathcal{U}_V(\mu_h, \mu_{b_n}, \mu_{b_{\bar{n}}}, \mu_s) \mathcal{U}_R(\nu_{b_n}, \nu_{b_{\bar{n}}}, \nu_s) \quad (9)$$

$$\times \tilde{H}_{ij}^{V,\text{res}} \mathcal{B}_n^i(x_n, \vec{b}_T, \mu_{b_n}, \nu_{b_n}) \mathcal{B}_{\bar{n}}^j(x_{\bar{n}}, \vec{b}_T, \mu_{b_{\bar{n}}}, \nu_{b_{\bar{n}}}) \mathcal{S}_{ij}(\vec{b}_T, \mu_s, \nu_s),$$

where the kernels $\mathcal{U}_{V,R}$ respectively take care of the virtuality and rapidity evolutions. Their explicit expressions read,

$$\mathcal{U}_V = \exp \left\{ \int_{\mu_h^2}^{\mu_{b_n}^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[-\Gamma_{\text{cusp}} \ln \left(\frac{\bar{\mu}^2}{M^2} \right) + \gamma_h \right] + \int_{\mu_s^2}^{\mu_{b_n}^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\Gamma_{\text{cusp}} \ln \left(\frac{\bar{\mu}^2}{\nu_{b_n}^2} \right) - \gamma_s \right] \right\},$$

$$\mathcal{U}_R = \exp \left\{ \ln \left[\frac{\nu_{b_n}^2}{\nu_s^2} \right] \left[\gamma_r(\alpha_s(b_0/|\vec{b}_T|)) + \int_{\mu_s^2}^{b_0^2/\vec{b}_T^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Gamma_{\text{cusp}}[\alpha_s(\bar{\mu})] \right] \right\}. \quad (10)$$

Numerical Results

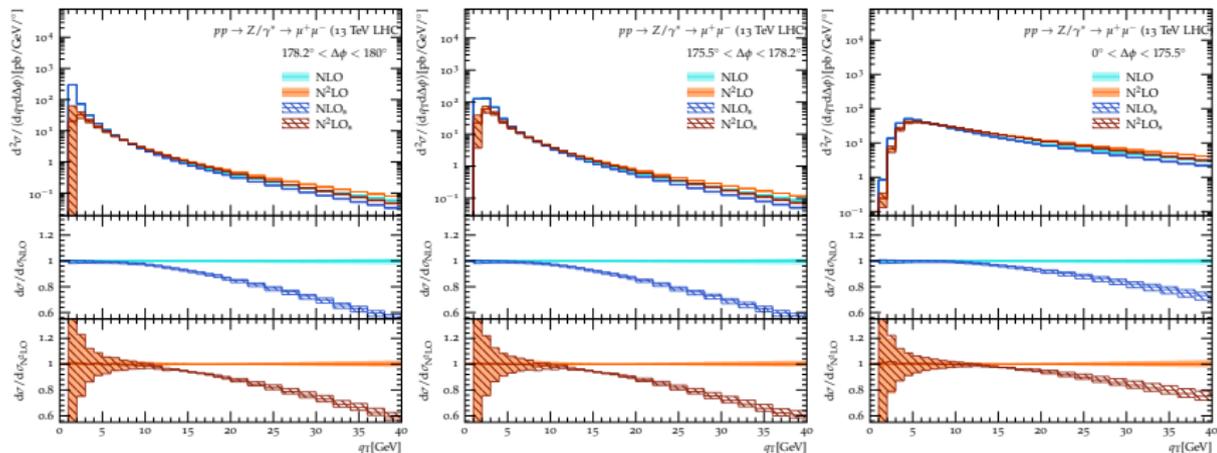
- Numerical Inputs and fiducial cuts

	W^+	W^-	Z
$p_T(\ell^+)$	$[20, \infty]$ GeV	–	$[20, \infty]$ GeV
$ \eta(\ell^+) $	$[-2.4, 2.4]$	–	$[-2.4, 2.4]$
$p_T(\ell^-)$	–	$[20, \infty]$ GeV	$[20, \infty]$ GeV
$ \eta(\ell^-) $	–	$[-2.4, 2.4]$	$[-2.4, 2.4]$
\cancel{p}_T	$[20, \infty]$ GeV	$[20, \infty]$ GeV	–
m_T	$[40, \infty]$ GeV	$[40, \infty]$ GeV	–
$m_{\ell\ell}$	–	–	$[80, 100]$ GeV

Table 1: Fiducial phase space.

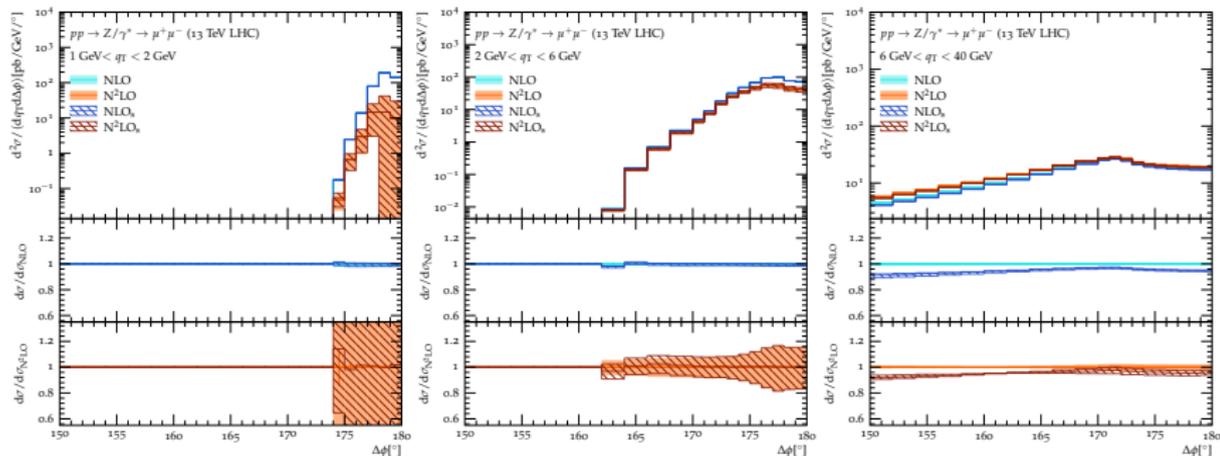
- Throughout this paper, we work in the complex-mass scheme and take all the input parameters (including electroweak coupling as well as the involved masses and widths) [PDG].
- The PDFs utilized in this work is NNPDF3.1 from [Ball:2017nwa].

Validation



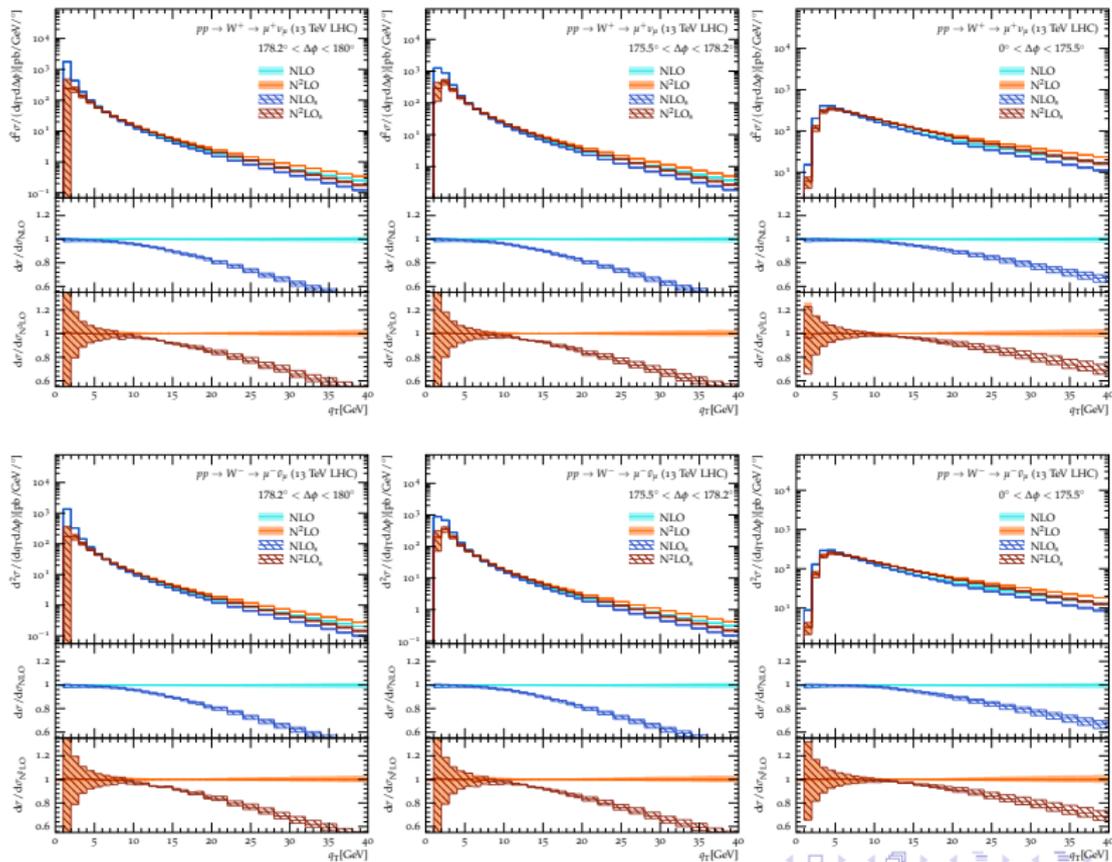
- Uncertainties: matching scale $[2\mu, \mu/2]$
- Obvious agreements between approximate and exact results in $q_T \rightarrow 0$
- Comparison at NNNLO is left to future work.

Validation

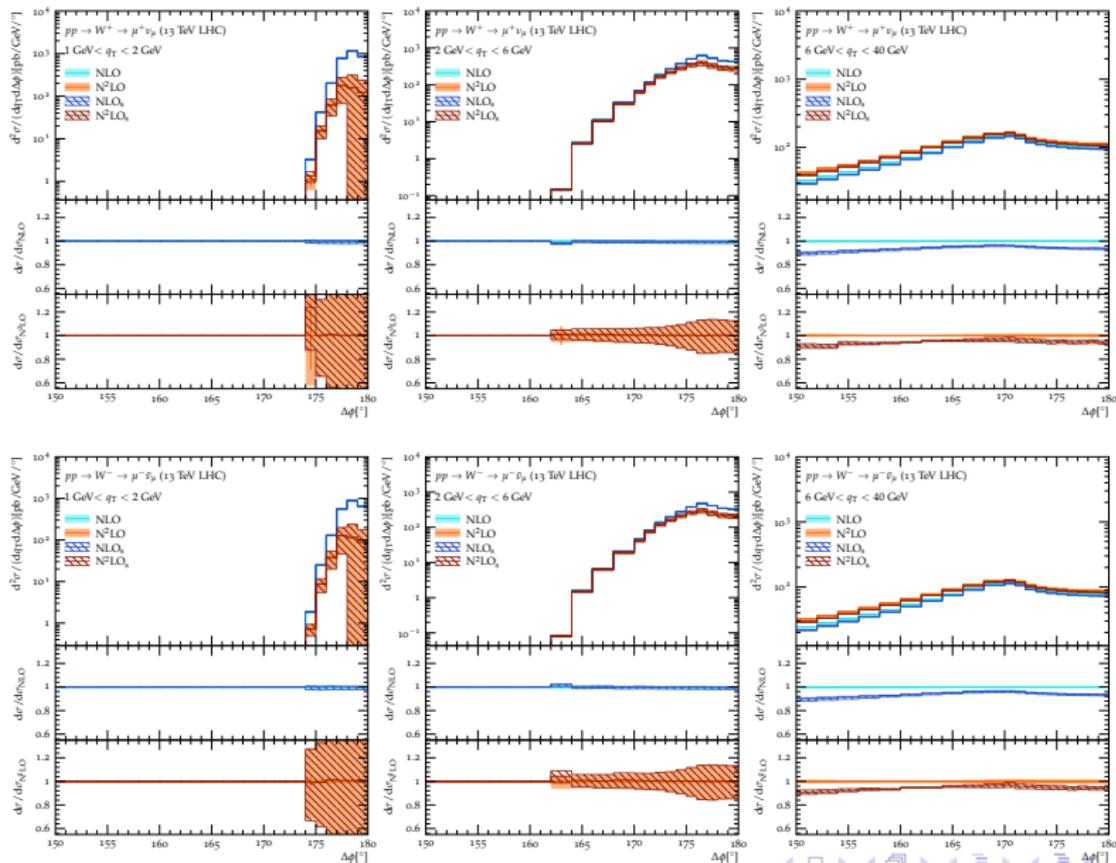


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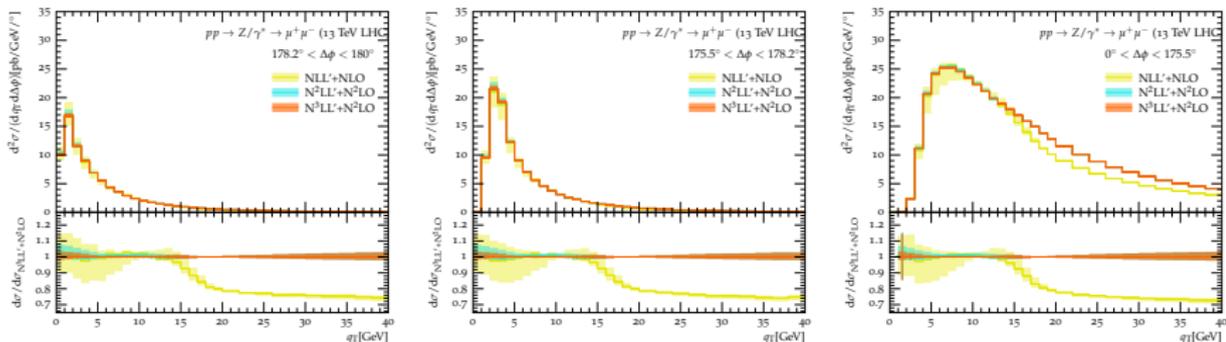
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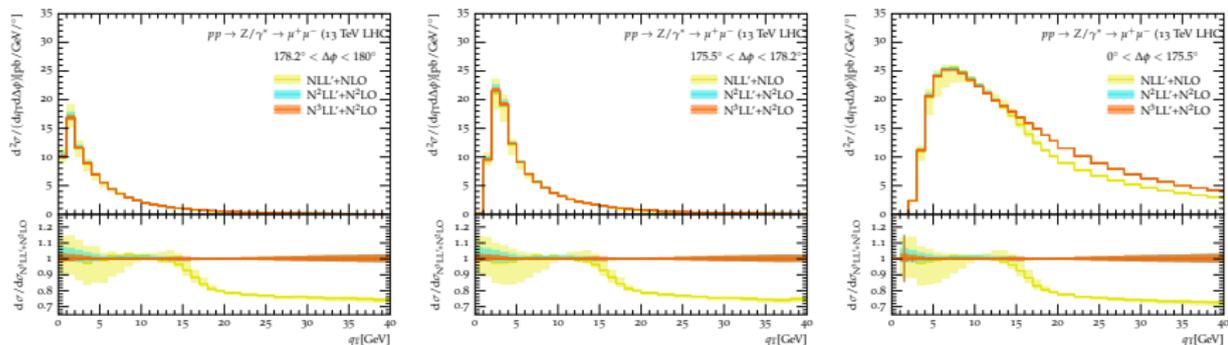


Resummation



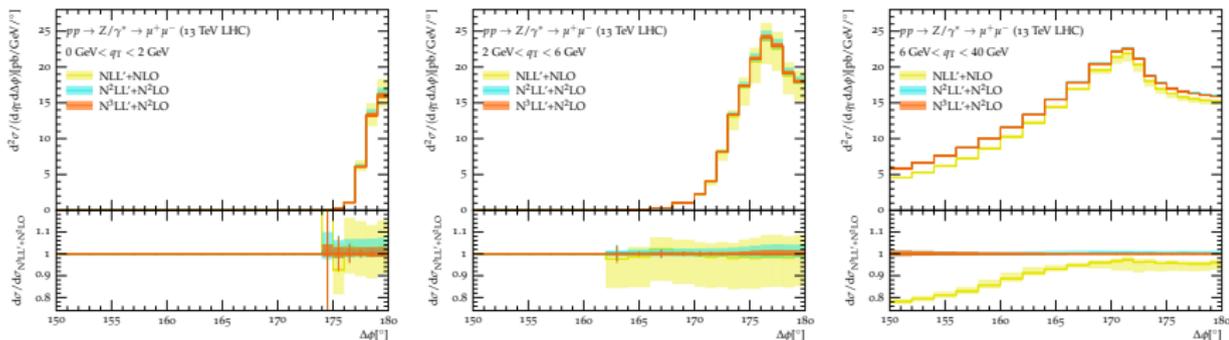
- Uncertainties: intrinsic scales $[2\mu, \mu/2]$
- Small q_T regime
 - The central values are close to each other.
 - With the accuracy growing, the uncertainties decrease considerably.
 - The error bands of higher accuracy are contained by those with lower accuracy.

Resummation



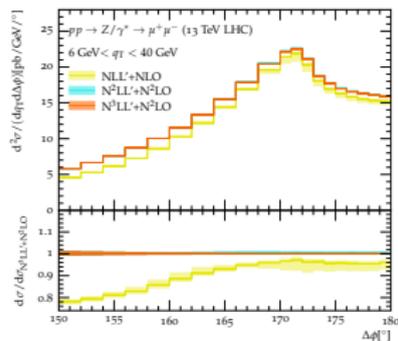
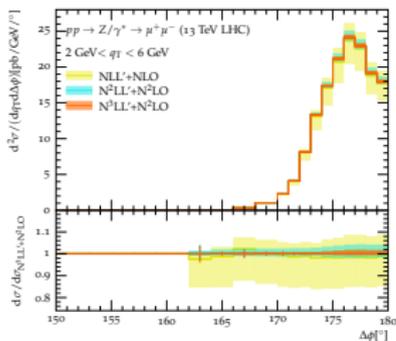
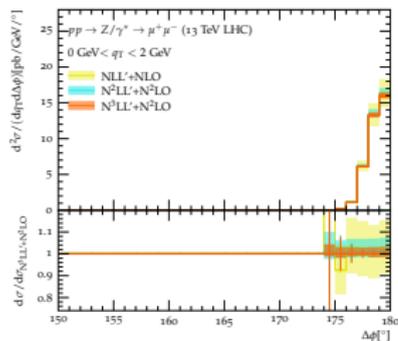
- Uncertainties: intrinsic scales $[2\mu, \mu/2]$
- Intermediate & large q_T region
 - The resummation is gradually switched off.

Resummation



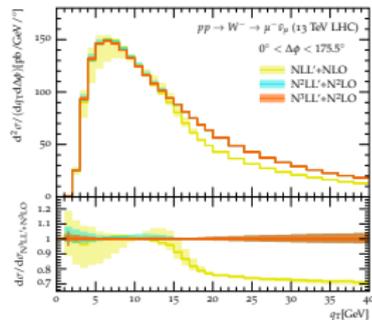
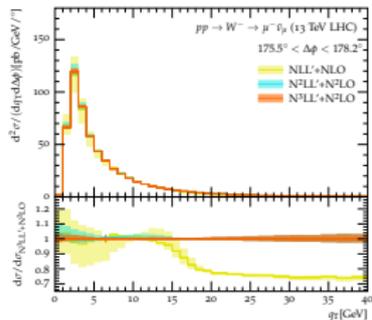
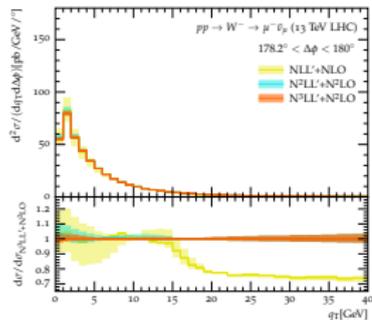
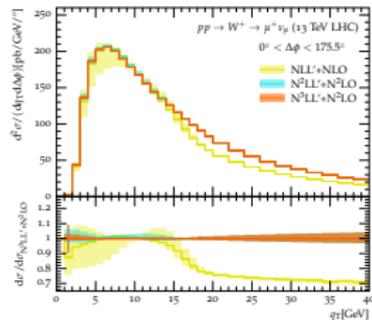
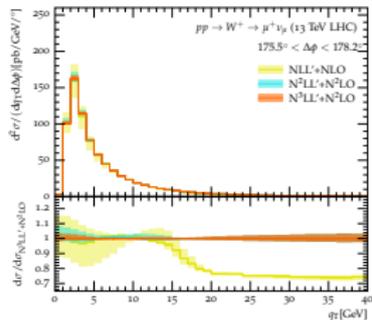
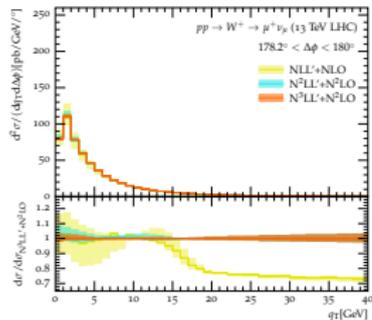
- Uncertainties: intrinsic scales $[2\mu, \mu/2]$
- First two slices
 - The central values are close to each other.
 - With the accuracy growing, the uncertainties decrease considerably.
 - The error bands of higher accuracy are contained by those with lower accuracy.

Resummation

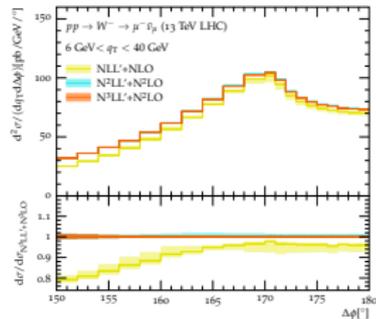
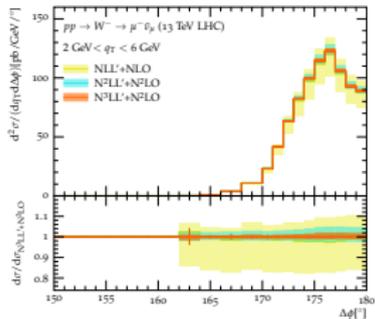
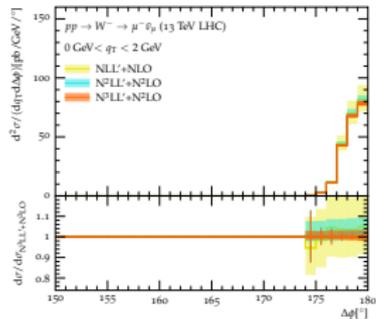
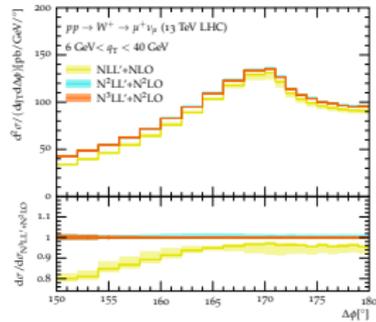
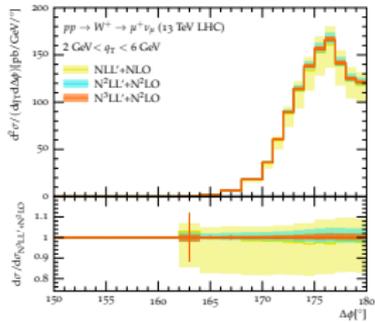
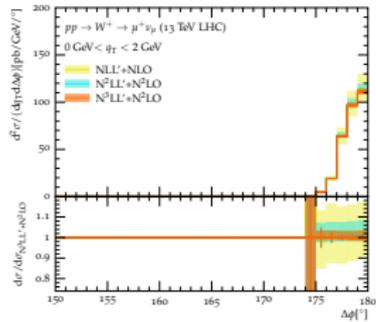


- Uncertainties: intrinsic scales $[2\mu, \mu/2]$
- Third slice
 - Dominated by the fixed-order contribution.

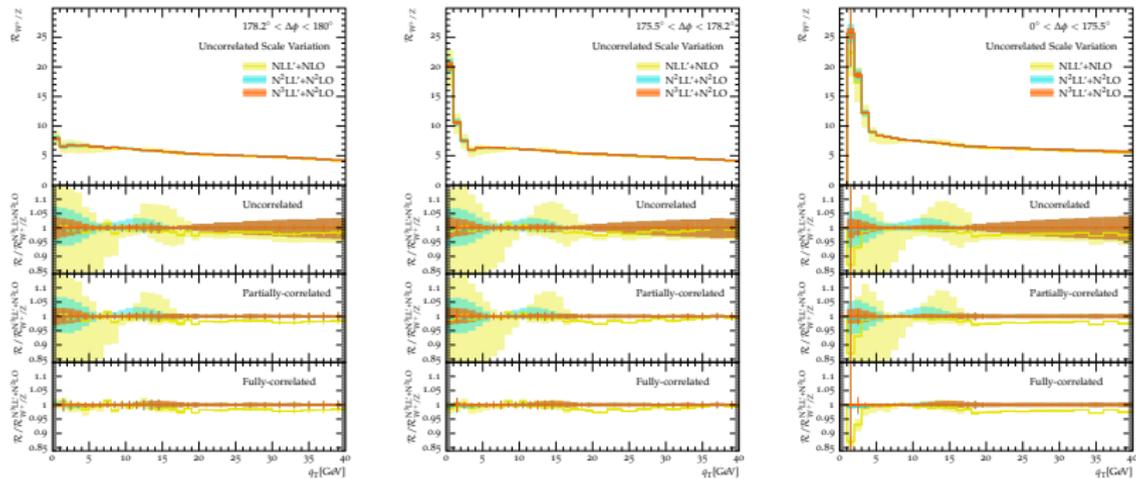
Resummation



Resummation

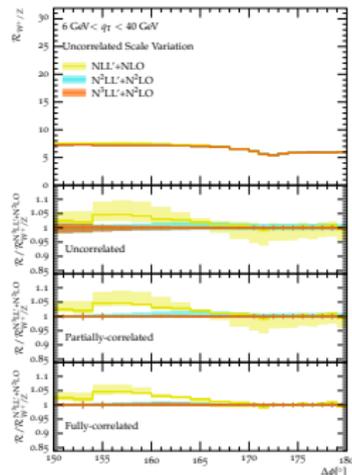
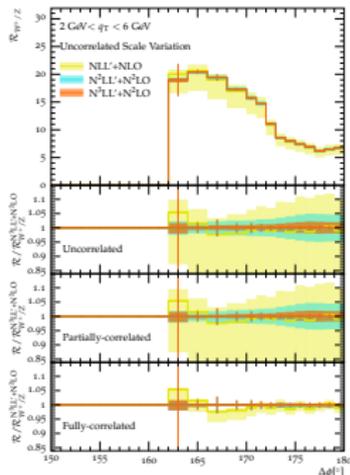
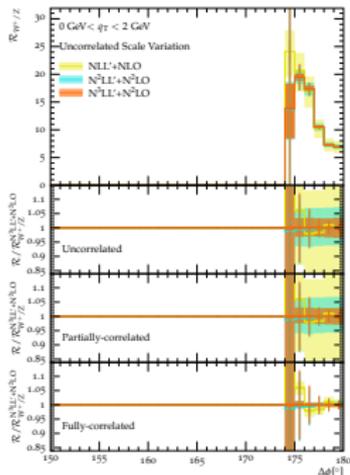


W/Z correlations



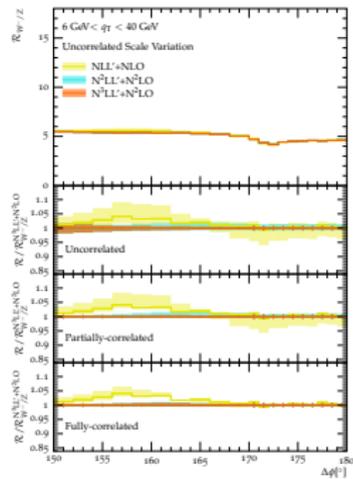
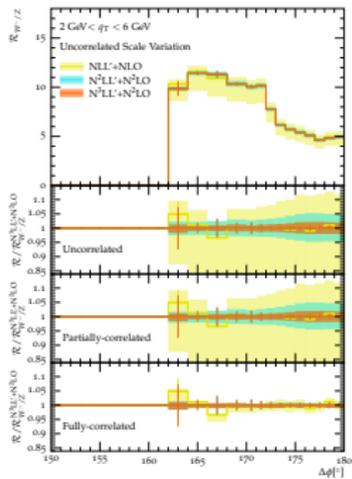
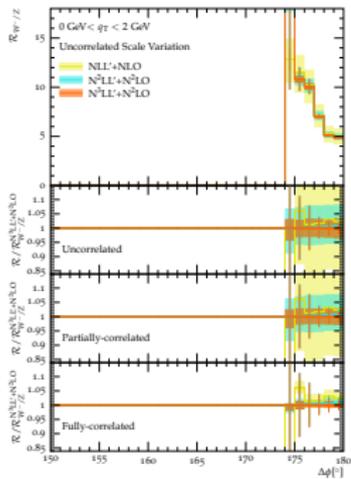
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W/Z correlations

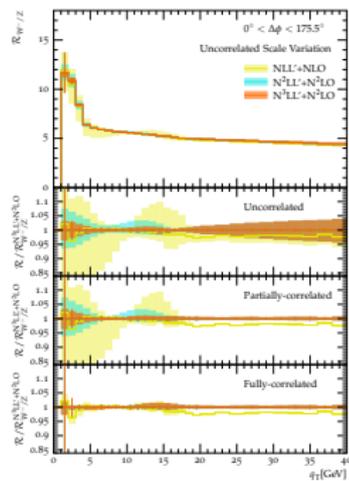
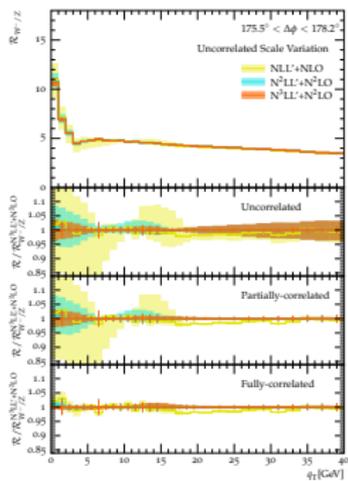
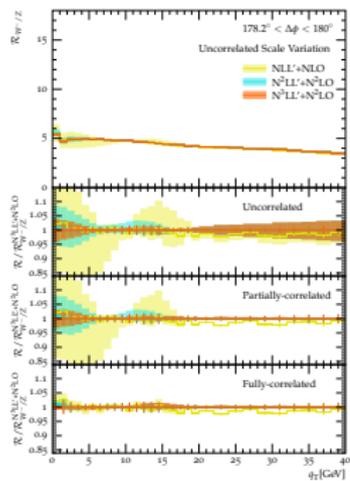


- The central values are close to each other.
- With the accuracy growing, the uncertainties decrease considerably.
- The error bands of higher accuracy are contained by those with lower accuracy.

W/Z correlations



W/Z correlations



Conclusion

- Within SCET, the $\Delta\phi$ & q_T spectra in the DY processes are investigated from $\text{NLL}' \rightarrow \text{N}^3\text{LL}'$.
- Singlet & non-singlet contributions induced (axial-)vector operators.
 1. S: thousandth-level impacts to the differential cross section.
 2. $S < \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3) > \sim \text{NS } \mathcal{O}(\alpha_s^3)$
- FO: Manifest agreements App v.s. QCD
- Resummation:
 1. Obvious convergence with the increase in the accuracy
 2. $\text{N}^3\text{LL}' \rightarrow$ percent level theoretical uncertainty

Thanks for your attention