Subleading colour effects in the PanScales showers and beyond

K. Hamilton, R. Medves, G. Salam, G. Soyez, **L. Scyboz**

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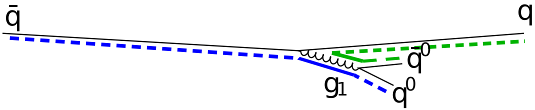

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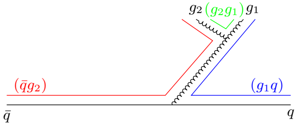
PSR 21

May 26th 2021

- Event-shape-type observable V: probability $P\left(V < e^{-|L|}\right)$

$$P(\alpha_s, L) = P(\alpha_s, 0)\exp\left(\underbrace{\frac{1}{\alpha_s}g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \cdots\right)$$

- Dipole showers reproduce leading logarithmic (LL) terms in the large-N_C limit



Accuracy criteria?

- Minimal set of criteria for defining/validating the accuracy of parton showers in [Dasgupta et al. '18]
- see P. Monni's talk

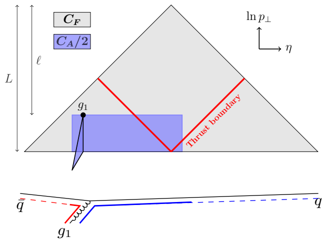
The "naive" colour factor assignment

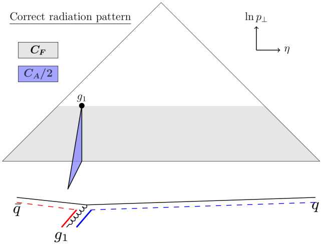
- Colour factor from the identified emitting dipole end

$$P_{gq}(z) \sim \frac{2C_F}{z}, \qquad P_{gg}(z) \sim \frac{C_A}{z}$$

- $\rightsquigarrow$ incorrect DL-NLC contributions for certain observables, like the thrust $1-T$ [Dasgupta et al., '18]

$$\delta\Sigma(L) = -\frac{1}{64}\left(\frac{\alpha_s C_F}{\pi}\right)^2 L^4 \left(\frac{C_A}{2C_F} - 1\right)$$





CFFE

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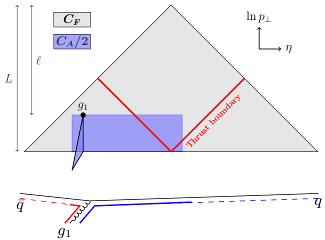
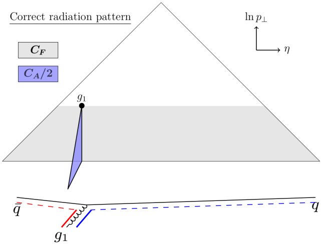
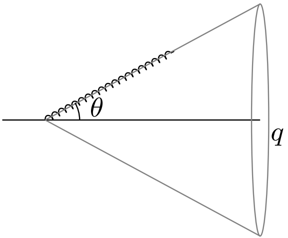




- At LL: angular ordering → colour factors



- Count the (net) number of quarks: $n = |n_q - n_{\bar{q}}|$
- Colour factor for the emission of g

$$C = \begin{cases} C_F, & \text{if } n = 1 \\ C_A/2, & \text{if } n = 0 \end{cases}$$

See also [Gustafson '93], [Holguin et al. '20]

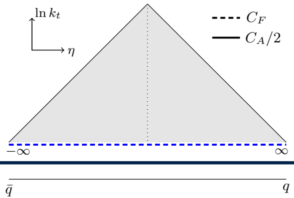
Solution #1: A segment approach

- **Idea**:
  1. keep track of the angle (rapidity) of successive splittings
  2. identify intervals in rapidity with net colour charge

- Introduce **segments** associated with a colour factor

$$C \in \{\frac{C_A}{2}, C_F\}$$

- Notation for a C_F-segment spanning rapidities $-\infty$ (antitriplet) to $+\infty$ (triplet end):



$$[-\infty, C_F, \infty]_{\bar{q}q}$$

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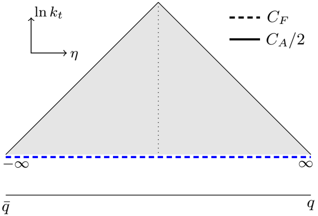

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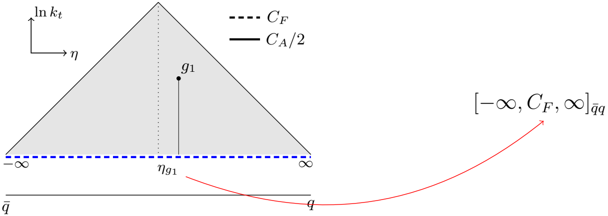

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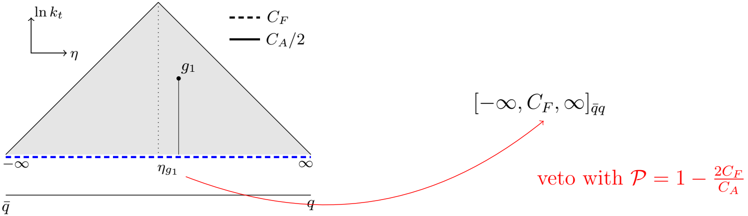

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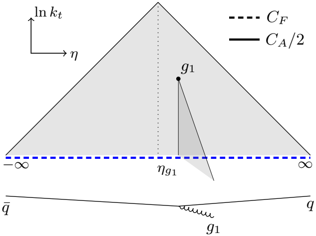

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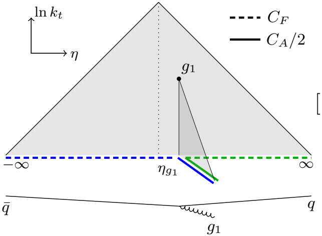

$$[-\infty, C_F, \eta^L_{g_1}, C_A, \infty]_{\bar{q}g_1} + [-\infty, C_A, \eta^R_{g_1}, C_F, \infty]_{g_1 q}$$

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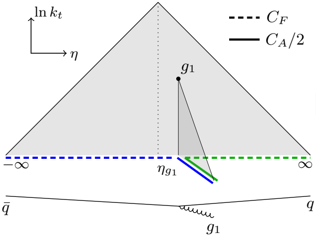

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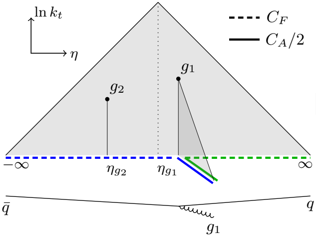

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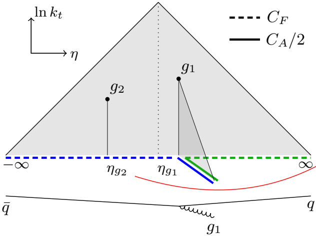

$$[-\infty, C_F, \eta_{g_1}^L, C_A, \infty]_{\bar{q}g_1} + [-\infty, C_A, \eta_{g_1}^R, C_F, \infty]_{g_1 q}$$

veto with $\mathcal{P} = 1 - \frac{2C_F}{C_A}$

Solution #1: A segment approach

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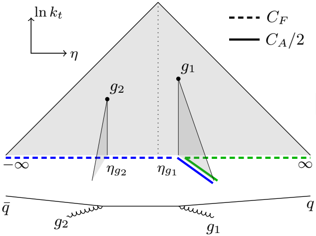

$$[-\infty, C_F, \eta^L_{g_1}, C_A, \infty]_{\bar{q}g_1} + [-\infty, C_A, \eta^R_{g_1}, C_F, \infty]_{g_1 q}$$

Solution #1: A segment approach

segment

- **Idea**:
  1. keep track of the angle (rapidity) of successive splittings
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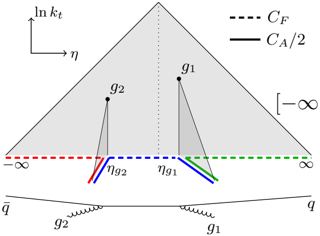
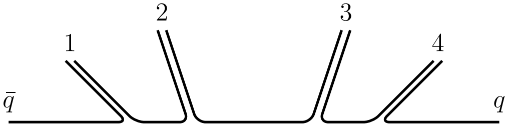


$$[-\infty, C_F, \eta^L_{g_2}, C_A, \infty]_{\bar{q}g_2} + [-\infty, C_A, \eta^R_{g_2}, C_F, \eta_{g_1}, C_A, \infty]_{g_2 g_1}$$

$$+[-\infty, C_A, \eta^R_{g_1}, C_F, \infty]_{g_1 q}$$

Solution #2: a nested matrix-element correction

NODS



$$(ab) \equiv 16\pi\alpha_s \frac{2p_a.p_b}{(2p_a.k)(2k.p_b)}$$

$$\propto \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{kb})}$$

- FC matrix element:

$$|M^2| = |M^2_{\mathrm{LC}}| + \left(C_F - \frac{C_A}{2}\right)(\bar{q}q)\,,$$

$$\text{with}\,|M^2_{\mathrm{LC}}| = \frac{C_A}{2}\left[(\bar{q}1) + (12) + (23) + \ldots + (nq)\right]$$

- Introduce an acceptance probability

$$p^{\mathrm{accept}}(\bar{q}, 1, 2, \ldots, n, q) \equiv \frac{|M|^2}{|M_{\mathrm{LC}}|^2} = 1 + \left(\frac{2C_F - C_A}{C_A}\right)\frac{(\bar{q}q)}{(\bar{q}1) + \ldots + (nq)}$$

- Make it local to the dipole neighbourhood for each C_F-segment

$$p^{\mathrm{accept}}_{(23)} \to p^{\mathrm{accept}}(1, 2, 3, 4)$$

Numerical validation

1. Matrix element tests → expected to work in angular-ordered regions
2. Subjet multiplicities → expected to work to NDL-FC
3. Global event shapes → expected to work to NLL-FC
4. Non-global observables → expected to remain only NLL (SL)-LC accurate

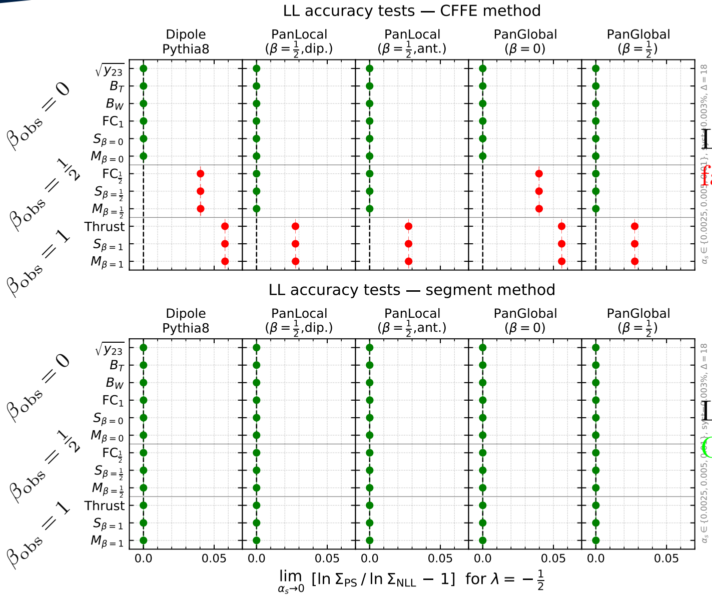
Shown below

- Isolate N^kLL terms [Dasgupta et al. '20]
- → see Pier's talk

$$P(\alpha_s, L) = P(\alpha_s, 0)\exp\left(\underbrace{\alpha_s^{-1} g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \cdots\right)$$

- Wide range of observables examined, differentiated by their
  - dependence on the η of each emission $\sim k_T\, e^{-\beta_{\text{obs}}|\eta|}$
  - sensitivity to the sum or max of individual emission contributions
- E.g. Cambridge $\sqrt{y_{23}}$, fractional EE correlation moments FC_x, thrust, ...

Event shapes @ LL



LL CFFE tests:
failure for $\beta_{\text{obs}} > \beta_{\text{PS}}$

LL segment tests:
OK for all obs.

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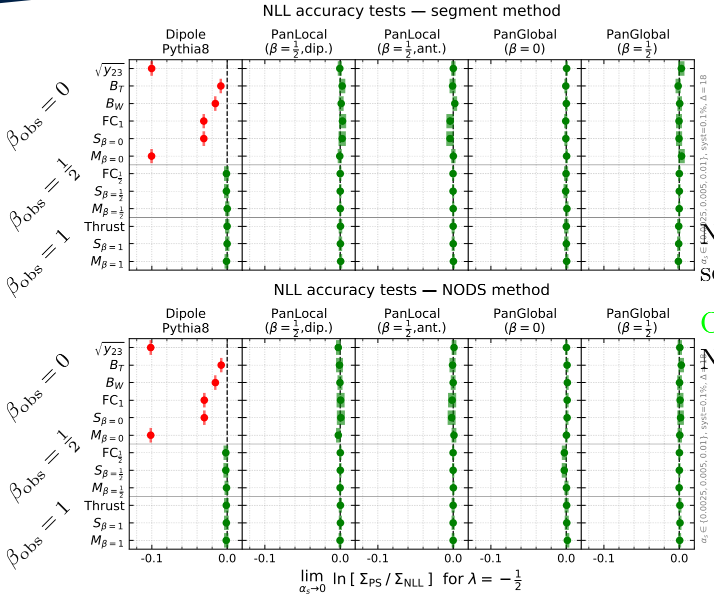
Event shapes @ NLL



NLL tests for segment and NODS:

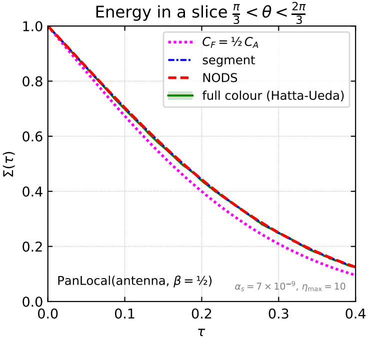
OK for all obs. and NLL-acc. showers

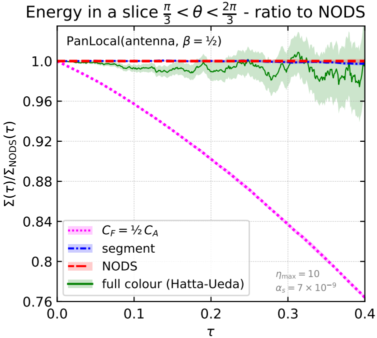
FC non-global logarithms (NGL)

- Energy flow E_{out} in a central rapidity slice

$$\Sigma(\tau(\alpha_s, L)) < e^{-L}Q, \quad \tau(\alpha_s, L) = \int_{e^{-L}Q}^{Q} \frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}$$

- FC results known numerically (reformulated in terms of a Langevin equation for Wilson lines) [Hatta, Ueda '13,'16,'21]
- FC contribution: segment up to $\mathcal{O}(\alpha_s L)$, NODS $\mathcal{O}(\alpha_s^2 L^2)$





Summary & Outlook

- Given $\alpha_s \approx 1/N_C^2$, we consider LL-NLC terms to be as important as NLL-LC terms
- Introduced two efficient prescriptions that yield full-N_C, LL accuracy
  - A simple, segment-based algorithm
  - An algorithm based on local, nested ME corrections (NODS)
- **Fast**, **adaptable** to other parton showers than PanScales

- NLL- and NDL-FC for global event shapes, and particle and jet multiplicities
- Do not expect SL-FC accuracy for NGLs: still, good agreement with numerical FC results

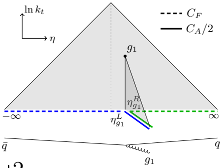
Backup

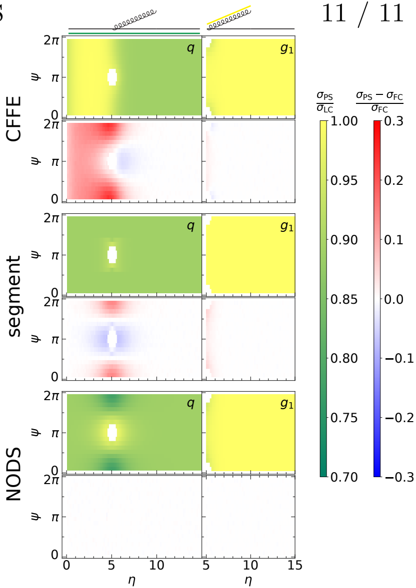
Tree-level matrix element tests

- $2 \to 3+1$ configuration: $\bar{q} g_1 q$
- Compare parton shower weight to the tree-level matrix element squared

$$\frac{d\sigma_{\mathrm{PS}}}{d\eta d\phi} \sim \frac{d\Phi_g}{d\eta d\phi} \frac{|M_{\bar{q}g_1qg}|^2}{|M_{\bar{q}g_1q}|^2}$$

in Lund declustering coordinates (η, ϕ)

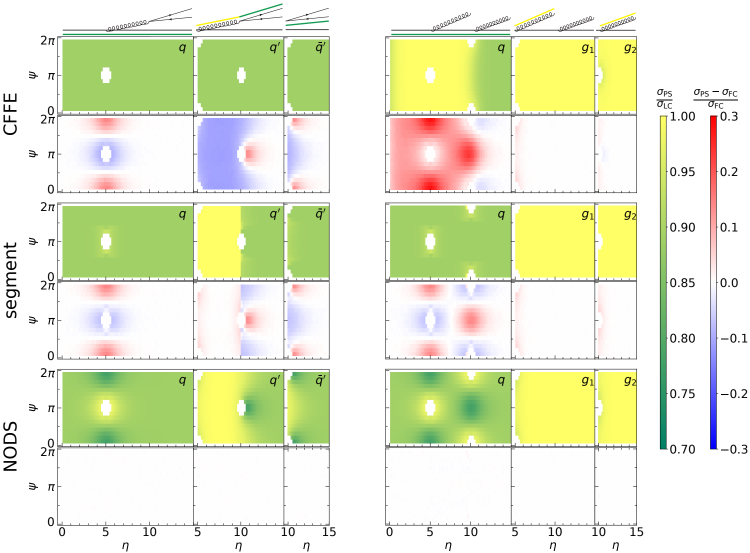


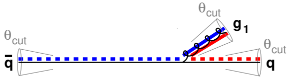


$$\frac{|M_{\bar{q}g_1qg}|^2}{|M_{\bar{q}g_1q}|^2} = (4\pi\alpha_s)\left(C_A\langle q, g_1; g\rangle + C_A\langle g_1, \bar{q}; g\rangle + (2C_A - C_F)\langle q, \bar{q}; g\rangle\right)$$

Tree-level matrix element tests

- $2 \to 4+1$ configurations: $\bar{q}g_1g_2q$ and $\bar{q}q'\bar{q}'q$





- Consider $\bar{q}g_1q$ with g_1 collinear to the quark-end, $\theta_{g_1q} \ll 1$
- For a (soft) second gluon emission g_2, the integrated rate of emission in the region

$$\{\theta_{g_2q}, \theta_{g_2\bar{q}}, \theta_{g_2g_1}\} > \theta_{\rm cut}\,.$$

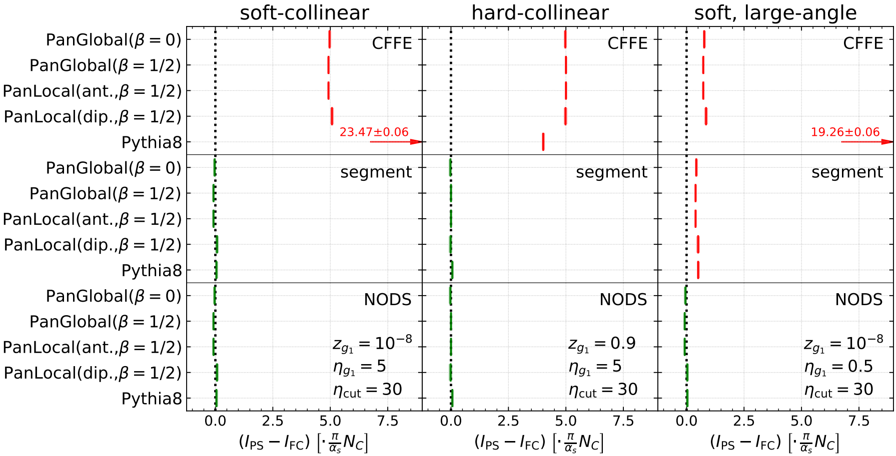
is given by

$$\begin{aligned} I_{\rm FC} &= \frac{2\alpha_s\,\Delta\ln E}{\pi}\left[2C_F\ln\frac{2}{\theta_{\rm cut}} + C_A\ln\frac{\theta_{g_1q}}{\theta_{\rm cut}}\right], \\ &= \frac{2\alpha_s\,\Delta\ln E}{\pi}\left[2C_F\eta_{\rm cut} + C_A(\eta_{\rm cut} - \eta_{g_1})\right] \end{aligned}$$

- Necessary condition to get multiplicity at NDL-FC

Integral tests

- $I_{\text{shower}} - I_{\text{correct}}$ (with $\eta_{\max} = -30,\ \eta_{\min} = 30$)



- Non-exponentiating observables
  - e.g. particle and subjet multiplicities, ...

$$P(\alpha_s, L) = P(\alpha_s, 0)\left(\underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\alpha_s^{1/2} h_2(\alpha_s L^2)}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL}} + \cdots\right)$$

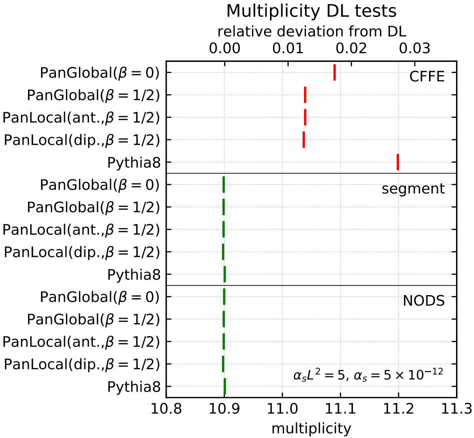
Procedure:

- Fix $\xi = \alpha_s L^2$ and take the limit $\alpha_s \to 0$

$$\delta P_{\mathrm{N}^k\mathrm{DL}} = \lim_{\alpha_s \to 0}\left(\frac{P_{\text{shower}}(\alpha_s, -\sqrt{\xi/\alpha_s}) - P_{\mathrm{N}^k\mathrm{DL}}(\alpha_s, -\sqrt{\xi/\alpha_s})}{\alpha_s^{k/2}}\right)$$

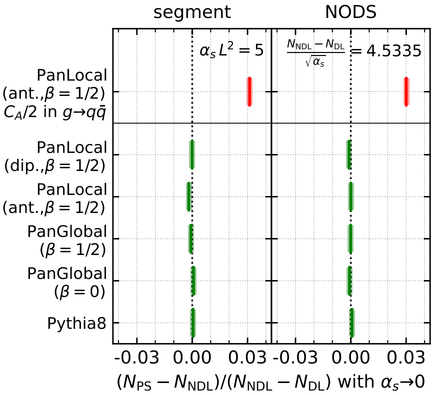
Particle multiplicity @ DL

- Exp.: $N_{\mathrm{DL}} = 10.9008$
- CFFE larger by $\sim 3\%$ at DL for Pythia8



Particle multiplicity @ NDL

- Good agreement for segment and NODS methods at NDL-FC



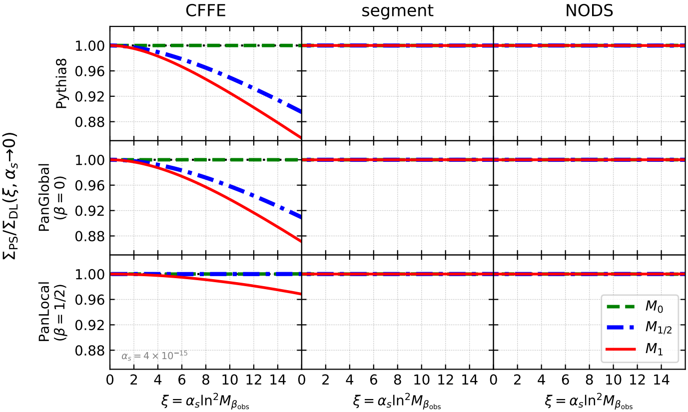
← Without $2C_F/C_A$ correction for emissions from $g \to q\bar{q}$

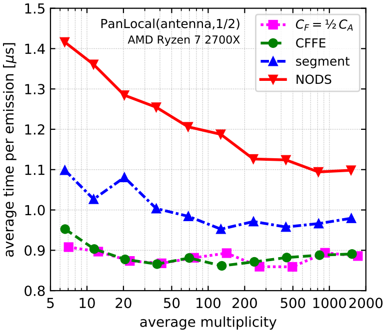
Event shapes @ DL

- Definition: $u_i^{\beta_{\rm obs}} = \frac{k_t}{Q} e^{-\beta_{\rm obs}|\eta_i|}$

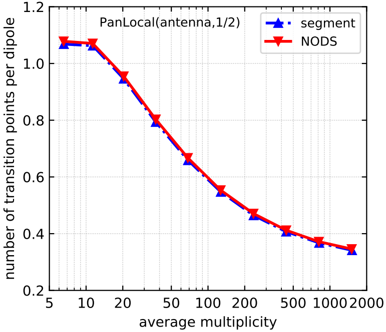
$$M_{\beta_{\rm obs}} \equiv \max_i \left\{ u_i^{\beta_{\rm obs}} \right\}, \qquad S_{\beta_{\rm obs}} \equiv \sum_i u_i^{\beta_{\rm obs}}$$

- At $\xi \sim 5$, differences of $\mathcal{O}(\text{few}\,\%)$ for CFFE



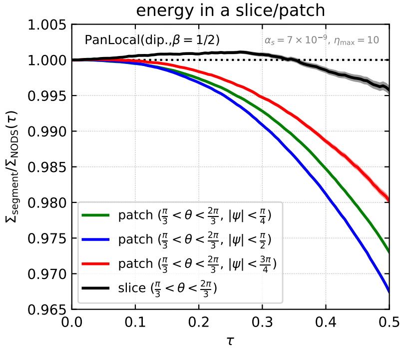


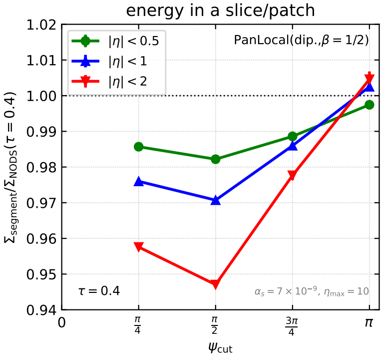
PanLocal(antenna,1/2)
AMD Ryzen 7 2700X
$C_F = ½ C_A$
CFFE
segment
NODS
average time per emission [μs]
average multiplicity
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
5
10
20
50
100
200
500
1000
2000



PanLocal(antenna,1/2)
segment
NODS
number of transition points per dipole
average multiplicity
1.2
1.0
0.8
0.6
0.4
0.2
5
10
20
50
100
200
500
1000
2000

NGLs: patch v. slice





NODS: 2 emissions at commensurate angle

- NODS reproduces the tree-level matrix element for any configuration with at most two emissions at commensurate angles (i.e. all *pairs* of emissions well-separated in rapidity)
- Look at emission rate from $\bar{q} g_1 g_2 q$, with $\eta_{g_1} \sim \eta_{g_2}$
  - $\Delta\eta_{g_1 g_2} = 0, 1, 2$, with $\Delta\psi_{g_1 g_2} = \pi$