False vacuum decay *in* strongly-interacting dark sectors

Eleanor Hall
UC Berkeley

Forthcoming work with Djuna Croon and Hitoshi Murayama

Aspen Winter Conference – A Rainbow of Dark Sectors – 3/25/21

Gravitational waves + dark sectors

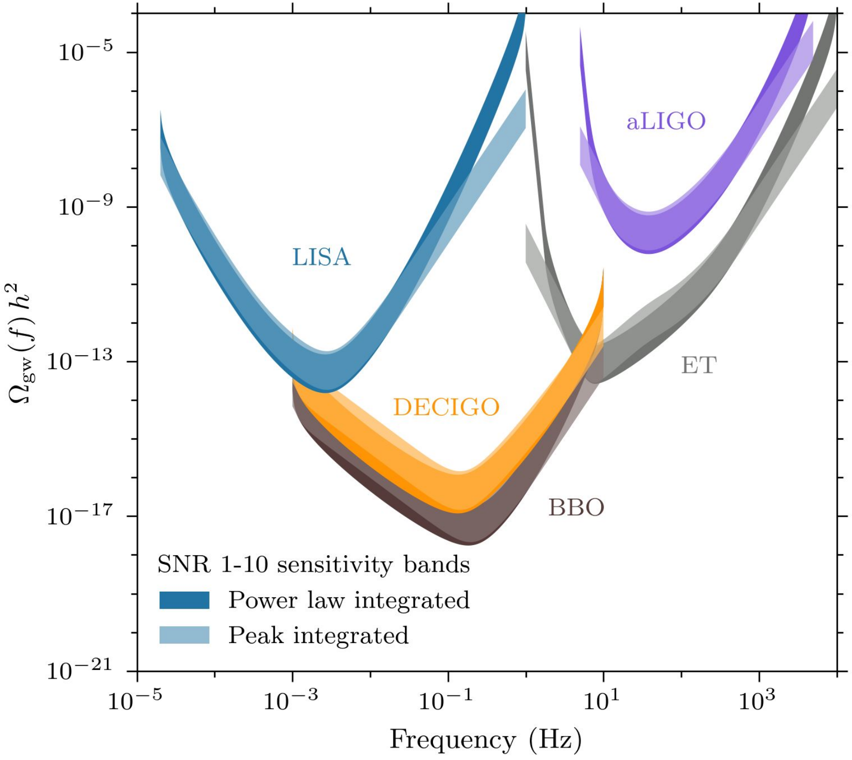
GWs: one of our most promising signal channels of dark matter

➔ Pedro Schwaller's talk this past Monday!

Big developments underway in GW experiment

First order phase transitions in dark sectors produce GW signature: sound waves, turbulence

Probes the **full richness of dark sectors**



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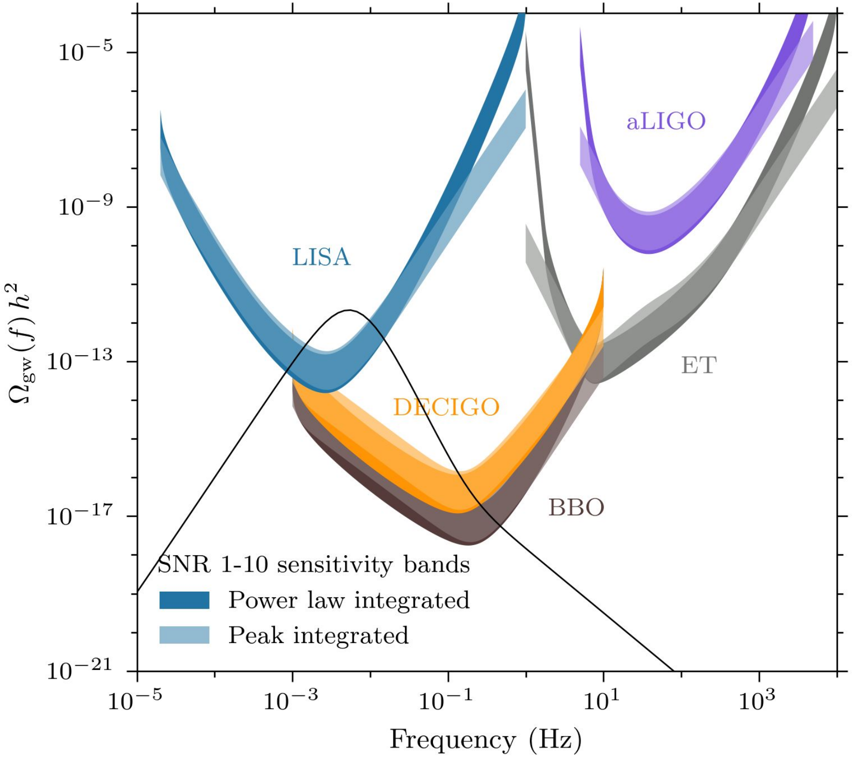

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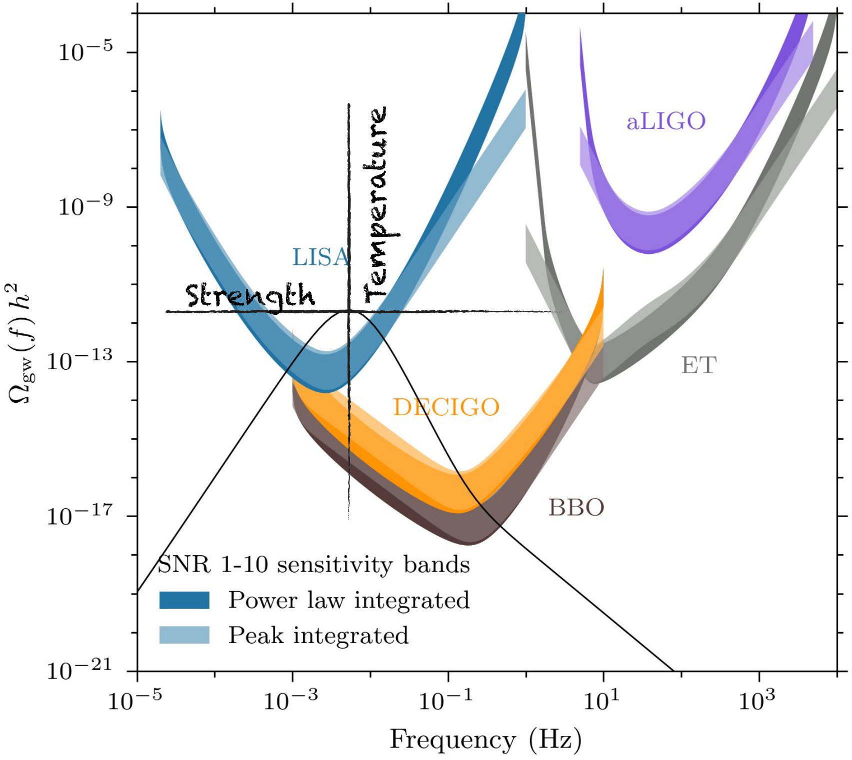
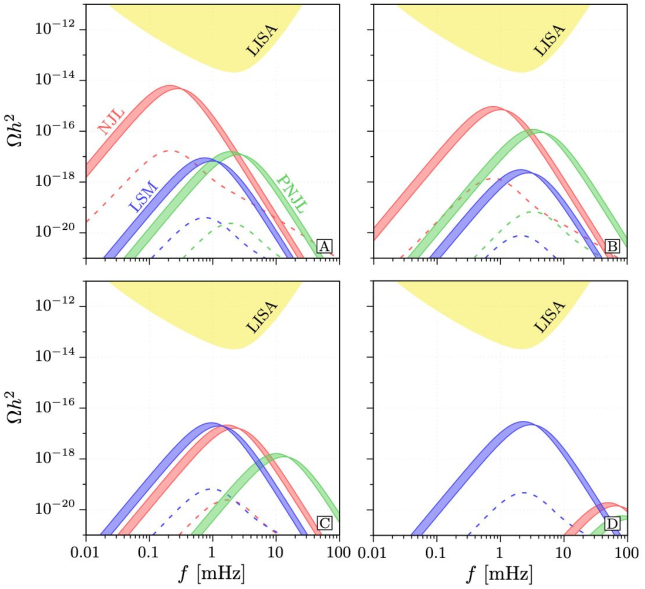
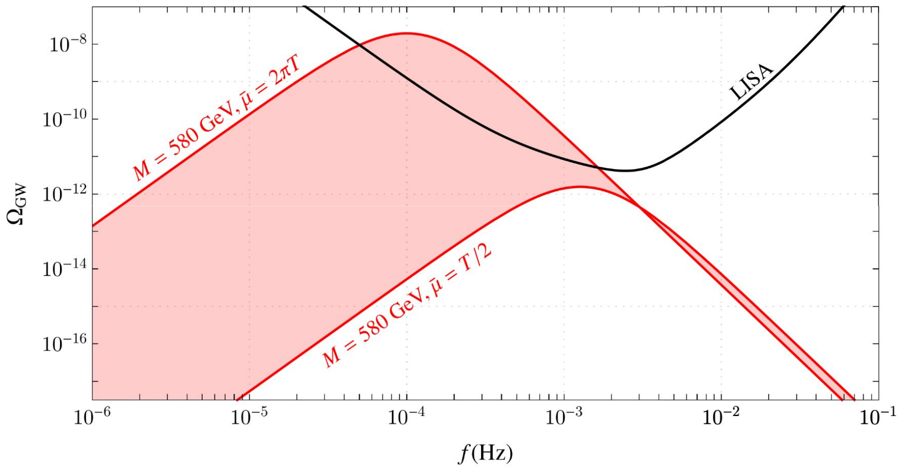


Problem: theoretical predictions aren't robust!

Perturbative analysis **fails for strong couplings** [Helmboldt et al., arxiv:1904.07891]



Unphysical scale dependence: 4d approach without RGE-running



O(10^4) uncertainty even from perturbative models [Croon, Gould, Schicho, Tenkanen, White, arxiv:2009.10080]

Need new robust, versatile techniques for calculating GW signal!

Calculating the GW signal

DM Model

Effective potential

$$V_{\text{eff}} = V + V_{\text{CW}} + V_{\text{th}}$$

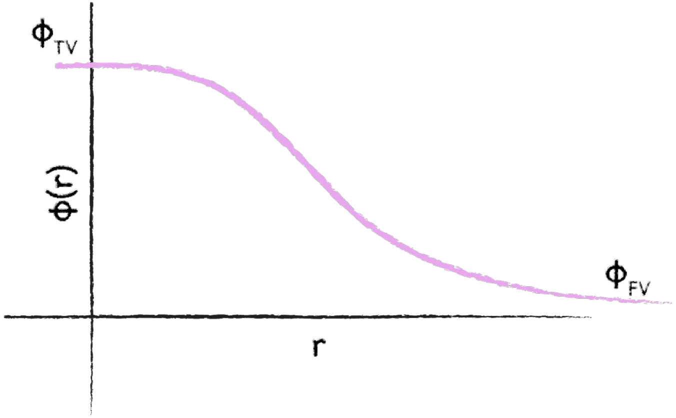
Bounce equations

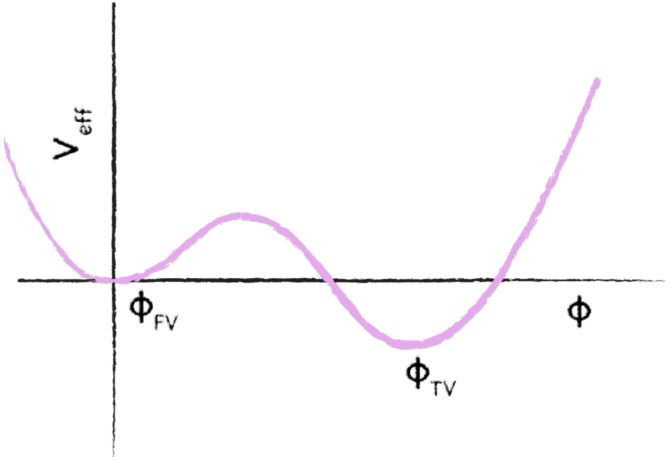
$$\partial_r^2 \phi + \frac{2}{r}\partial_r \phi = \frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}\phi}$$

Parameters

$$T^*, \gamma \leftrightarrow \alpha, \beta$$

GW signal!





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Hydrodynamical simulations
[arxiv:1910.13125]

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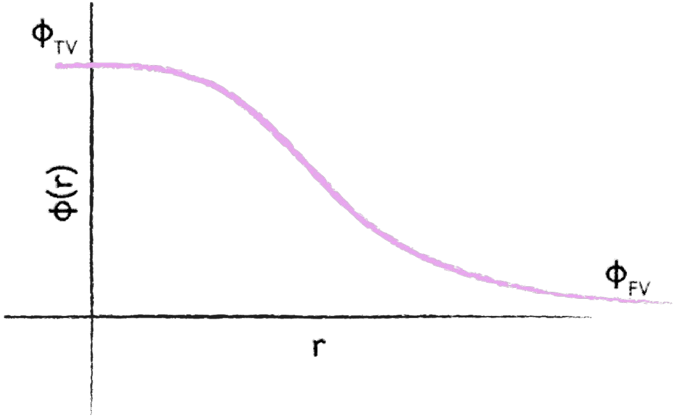

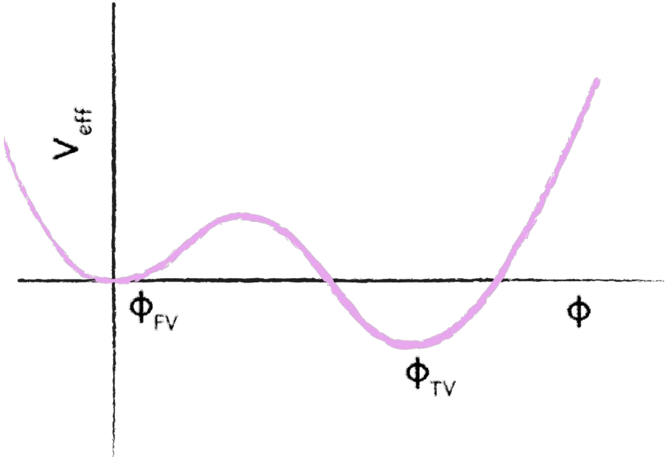

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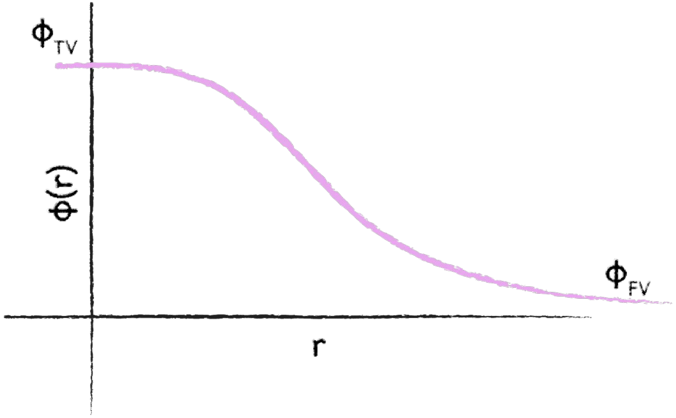
Origin of uncertainty: Work needed!



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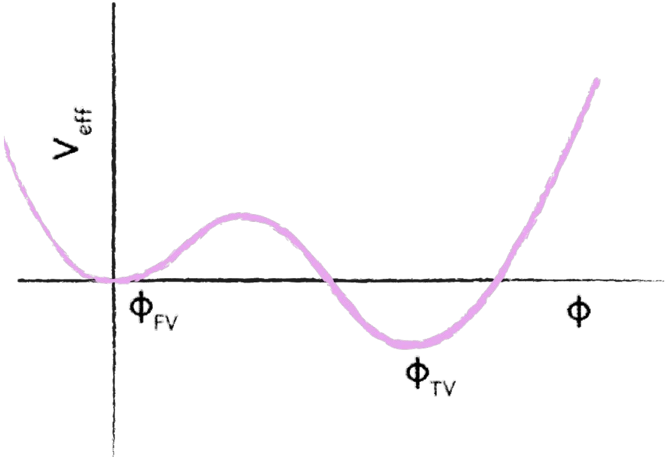

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GW signal!

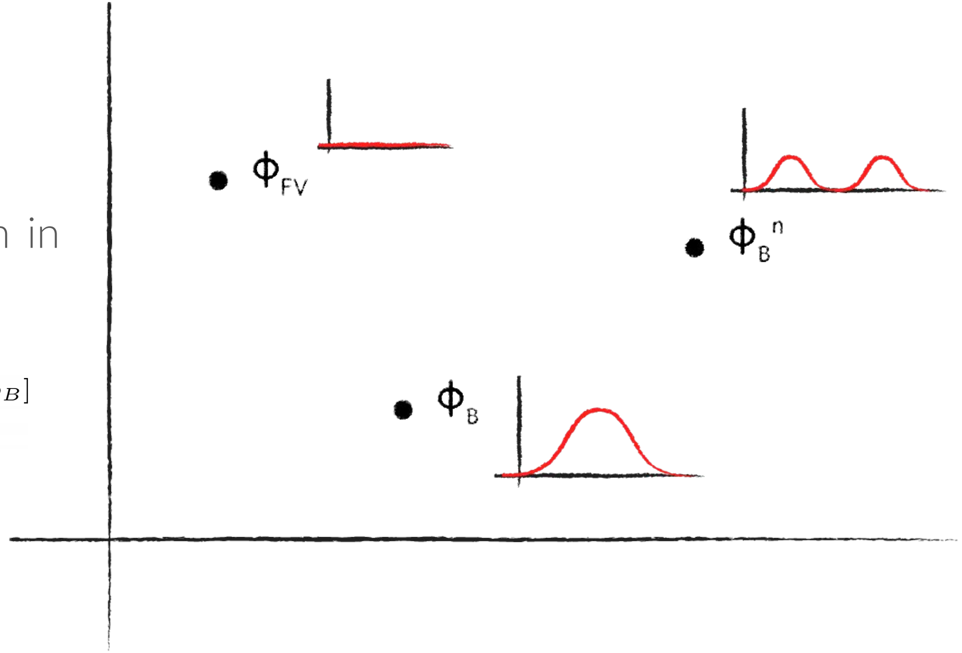
Bounce formalism = stationary phase

FV decay rate ↔ imaginary part of FV energy

$$\gamma = -2\,\mathrm{Im}\,\mathcal{E} \simeq \frac{2}{\mathcal{V}}\mathrm{Im}\ln\int_{\phi_F}^{\phi_F}\mathcal{D}\phi\, e^{-S[\phi]}$$

Stationary phase / saddle point approximation in terms of bounce solutions

$$Z \simeq \sum_n \mathcal{N}[\det S''[\bar\phi_n]]^{-1/2} e^{-S[\bar\phi_n]} \implies \gamma = Ae^{-S[\phi_B]}$$



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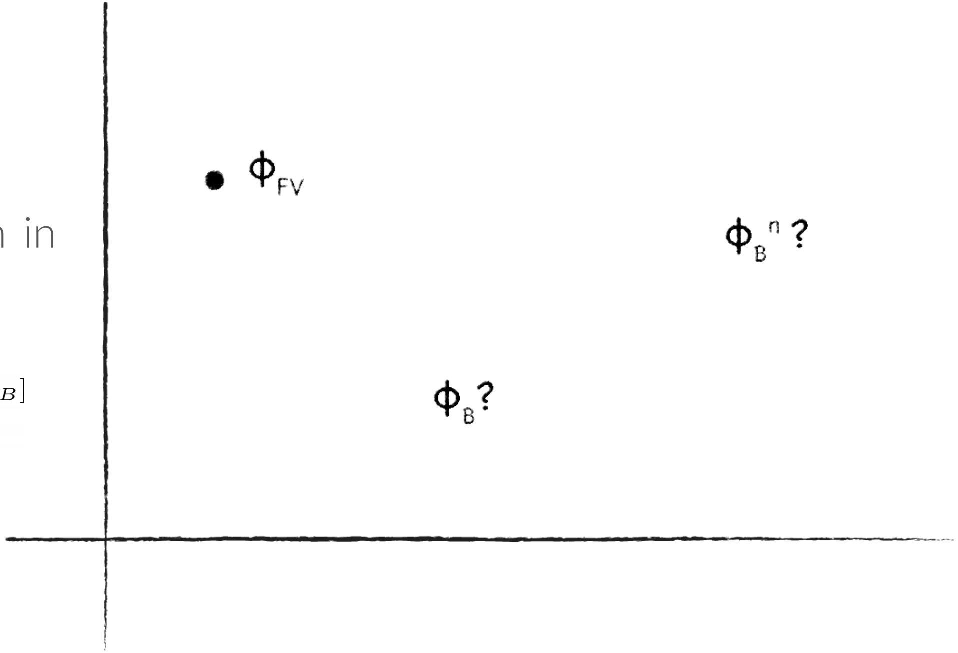
But what about radiative corrections?



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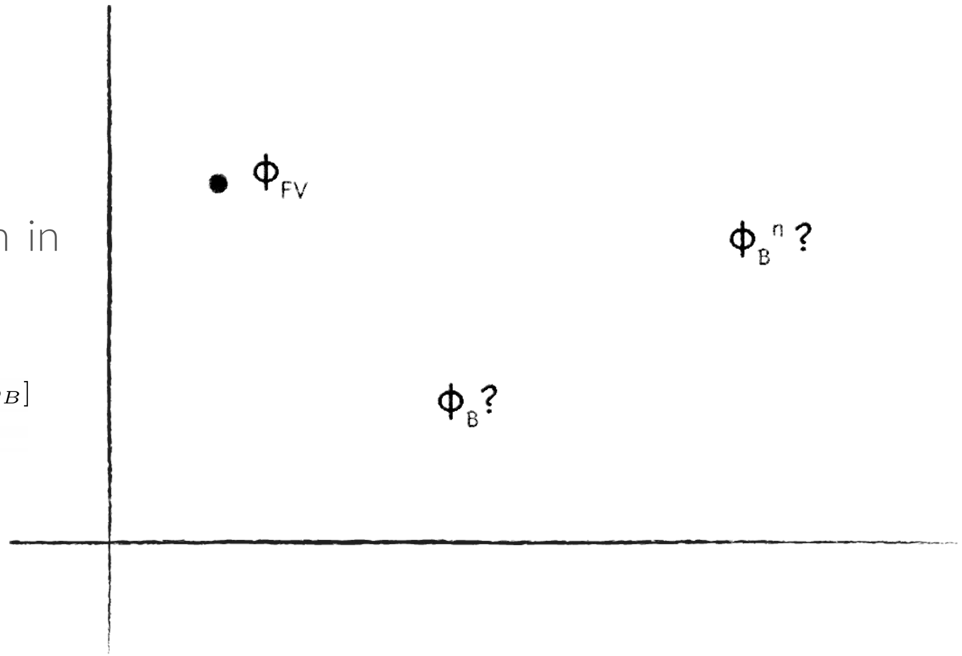

But what about radiative corrections?

Usual answer: use effective action

$$\gamma \simeq Ae^{-S_{\mathrm{eff}}[\phi_B]}$$

What is the correct effective action for FV decay?

Perturbative one-loop effective action?
- ✅ non-convex
- ❌ perturbative
- ❌ scale-dependent

Exact effective action?
- ❌ convex
- ✅ non-perturbative
- ✅ scale-independent

Coarse-grained effective action?
- ✅ non-convex
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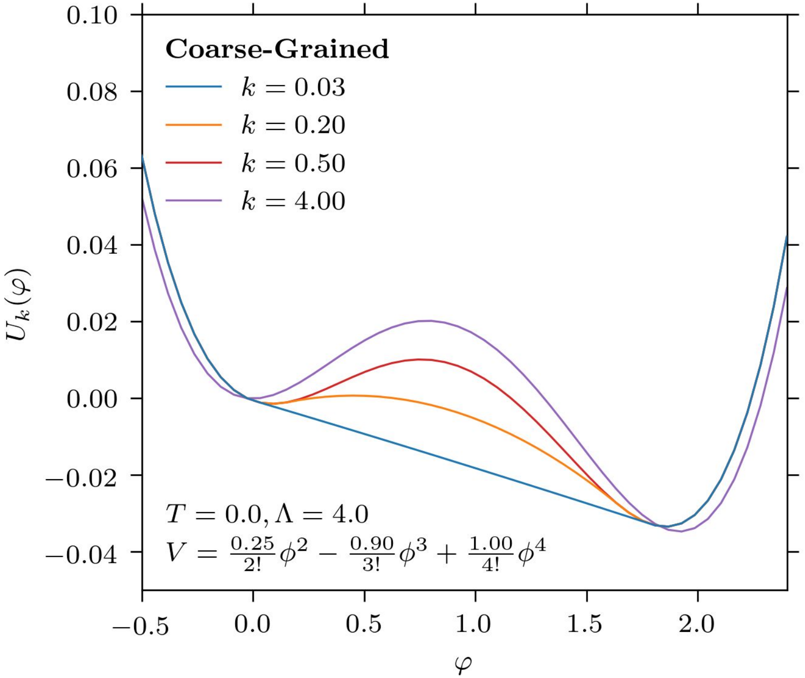

[“Convexity, gauge-dependence and tunneling rates”, **Plascencia** + Tamarit, arxiv:1510.07613]
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Our proposal: Quasi-stationary effective action
- ✅ non-convex
- ✅ non-perturbative
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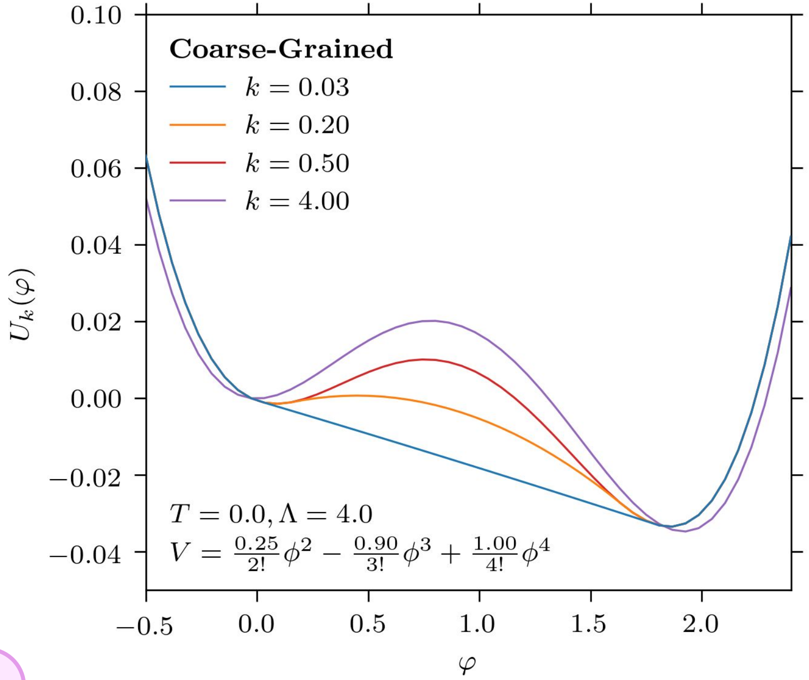

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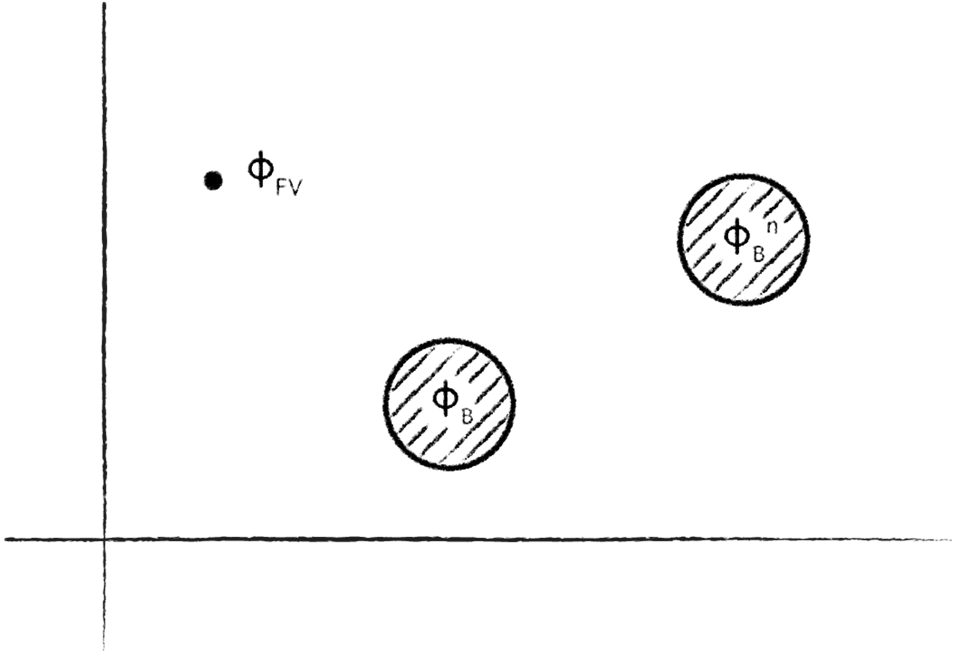
Radiative corrections = quasi-stationary patches

Regions of quasi-stationary bounce-like field configurations still dominate integral

$$Z \simeq \sum_n \int_{U_n} \mathcal{D}\phi \, e^{-S[\phi]}$$



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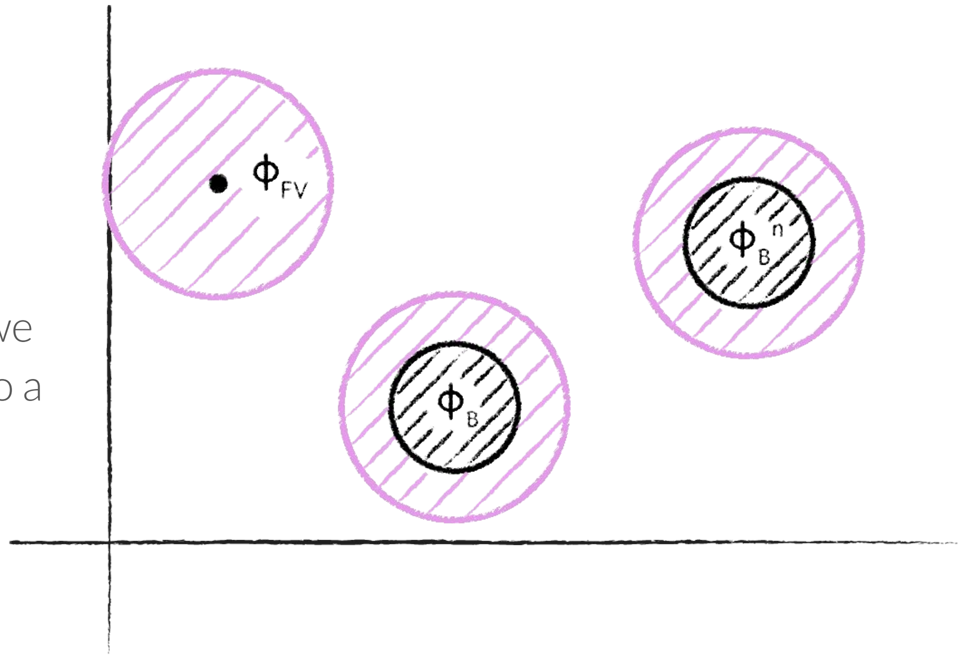
Rather than integrating over high momenta, we want to integrate over **local fluctuations** up to a scale which encloses all quasi-stationary patches

$$\frac{1}{\mathcal{V}} \langle |\phi - \bar{\phi}|^2 \rangle = G[\bar{\phi}; p] \lesssim \mu$$

Like coarse graining, but in field space rather than position space

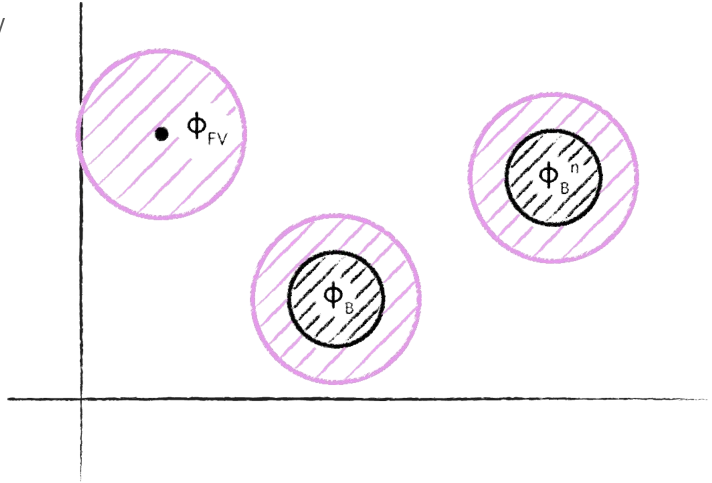


The quasi-stationary effective action

Effective action which integrates over all fluctuations below some characteristic scale

$$\Gamma_k[\bar\phi] = -W[J] + \int J\bar\phi \quad \text{s.t. } \bar\phi = \langle\phi\rangle \text{ and } \langle|\phi-\bar\phi|^2\rangle \lesssim \mu(k)$$

where $W[J] = \ln \int \mathcal{D}\phi\, e^{-S[\phi]+\int J\phi}$



Allows us to write the path integral and decay rate as

$$Z \simeq \sum_n e^{-\Gamma_k[\bar\phi_n]} \implies \gamma \simeq A e^{-\Gamma_k[\phi_B]}$$

Self-consistently takes into account fluctuations around *quantum* bounce solution

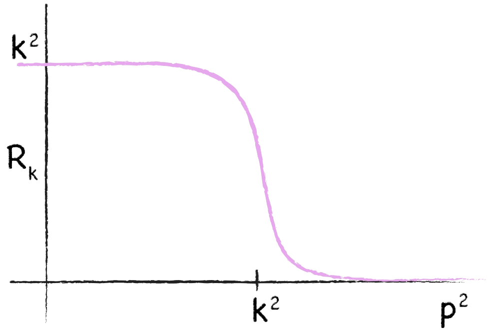
By construction insensitive to scale!

Non-perturbative implementation in the **functional renormalization group**

The functional renormalization group

Regulator function makes modes with $p \lesssim k$ massive

$$S_k[\phi] = S[\phi] + \Delta S_k[\phi], \text{ where } \Delta S_k[\phi] = \frac{1}{2}\int R_k(p)\phi(-p)\phi(p)$$



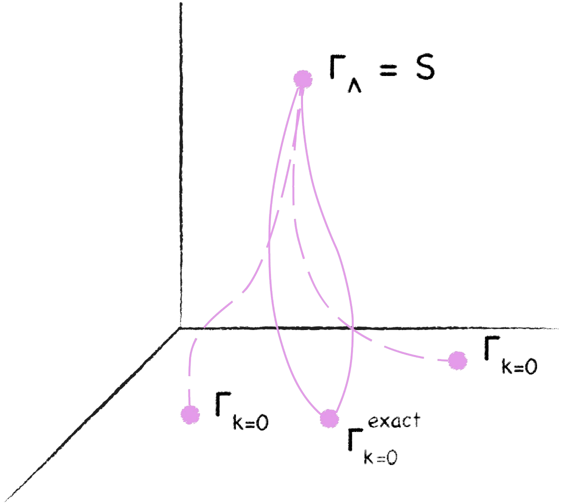
Scale-dependent effective action for the theory at a scale k

$$\Gamma_k[\bar{\phi}] = -W_k[J] + \int J\bar{\phi} - \Delta S_k[\bar{\phi}]$$

Exact flow equation that flows the effective action across different scales

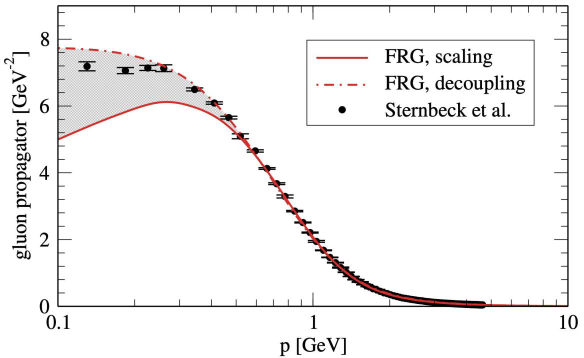
$$\partial_k \Gamma_k[\bar{\phi}] = \frac{1}{2}\int (\partial_k R_k) G_k[\bar{\phi}; p] = \frac{1}{2}\int \frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}$$

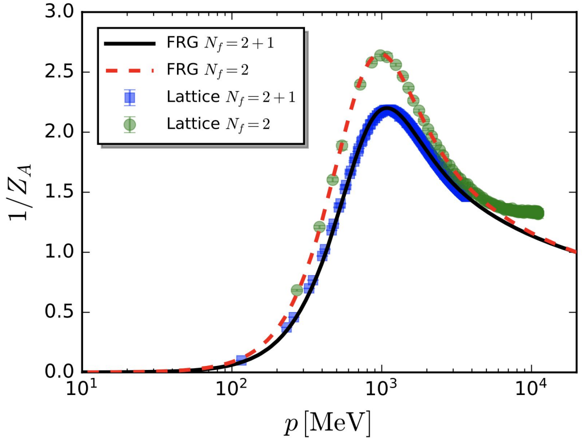
Approximation schemes like the derivative expansion and vertex expansion that don't spoil the non-perturbativity

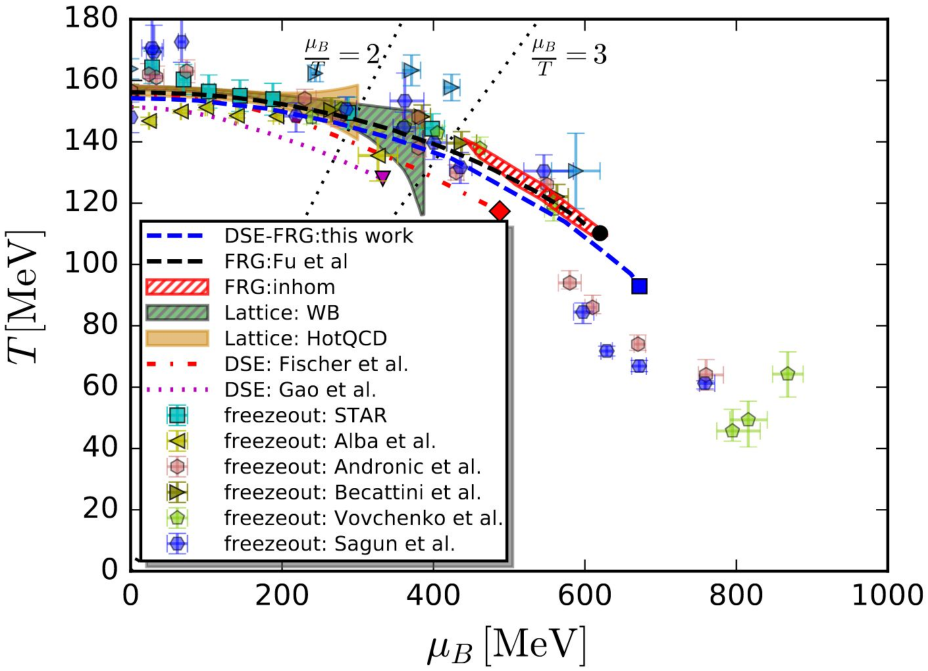


[Dupuis et al., arxiv:2006.04853]

The functional renormalization group: QCD



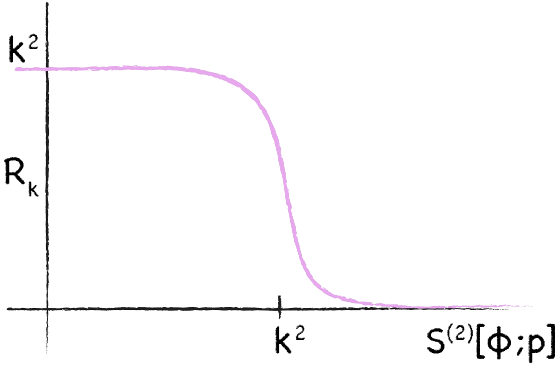




Highly successful in matching QCD lattice data
[1605.01856,1909.02991, 2002.07500]

The FRG *for fluctuations*

Modified FRG in terms of fluctuations rather than momentum describes the **flow of the QSEA**



Regulator function which is allowed to be **field-dependent** makes large fluctuations massive, so that at the scale k the theory obeys

$$\frac{1}{\mathcal{V}}\langle|\phi-\bar{\phi}|^2\rangle = G_k[\bar{\phi};p] \lesssim \frac{1}{k^2}$$

Since at leading order $G_k^{-1}[\bar{\phi};p] \simeq S_k^{(2)}[\bar{\phi};p] = S^{(2)}[\bar{\phi};p] + R_k(p)$, we will focus on the choice of regulator

$$R_k = \left(k^2 - S^{(2)}[\bar{\phi};p]\right)\Theta\left(k^2 - S^{(2)}[\bar{\phi};p]\right)$$

Which minimally enforces $S_k^{(2)} \geq k^2$. In practice, we can relax $k \to 0$

Flow of the quasi-stationary effective action

The QSEA obeys an **exact flow equation** that is **modified** by the field-dependence of the regulator

$$\partial_k \Gamma_k[\bar\phi] = \frac{1}{2}\int (\partial_k R_k) G_k[\bar\phi;p] \qquad G_k[\bar\phi;p] = \left[\Gamma_k^{(2)} + R_k + \Delta + \frac{1}{2}\int G_k[\bar\phi;q] R_k^{(2)}(q)\right]^{-1}$$

As in the usual FRG, to evaluate the flow equation we must use some **approximation scheme** that's non-perturbative in the coupling

Local potential approximation: lowest-order of DE, ansatz $\Gamma_k[\bar\phi] = \int_x \left[\frac{1}{2}(\partial\bar\phi)^2 + U_k(\bar\phi)\right]$

Together with our choice of regulator $R_k = \left(k^2 - S^{(2)}[\bar\phi;p]\right)\Theta\left(k^2 - S^{(2)}[\bar\phi;p]\right)$, we arrive at the flow equation

$$\partial_k U_k = \frac{k\Theta(k^2 - V'')}{V''''}\left[\sqrt{(k^2 + U_k'' - V'')^2 + \frac{(k^2 - V'')^2 V''''}{16\pi^2}} - (k^2 + U_k'' - V'')\right]$$

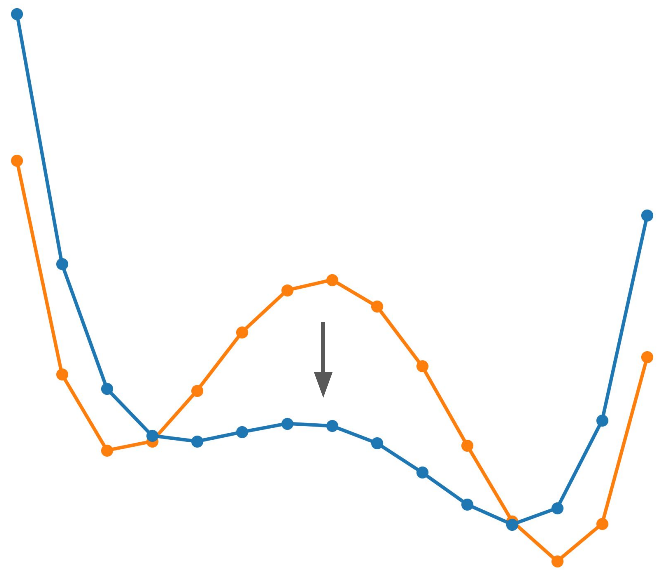
Evaluating the flow

$$\partial_k U_k = \frac{k\Theta(k^2 - V'')}{V''''}\left[\sqrt{(k^2 + U_k'' - V'')^2 + \frac{(k^2 - V'')^2 V''''}{16\pi^2}} - (k^2 + U_k'' - V'')\right]$$

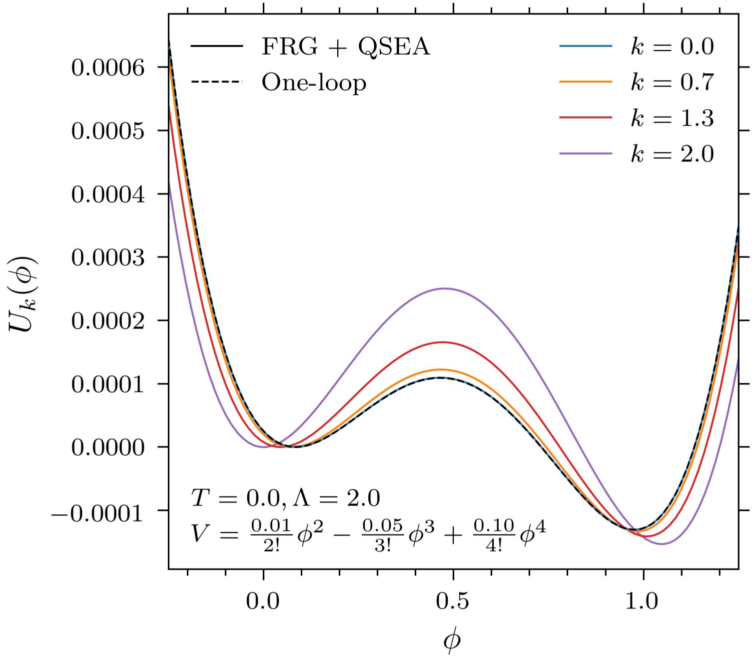
Just a differential equation!

Can be straightforwardly solved in a **few lines of Mathematica** or in scipy using the out of the box diffeq solvers

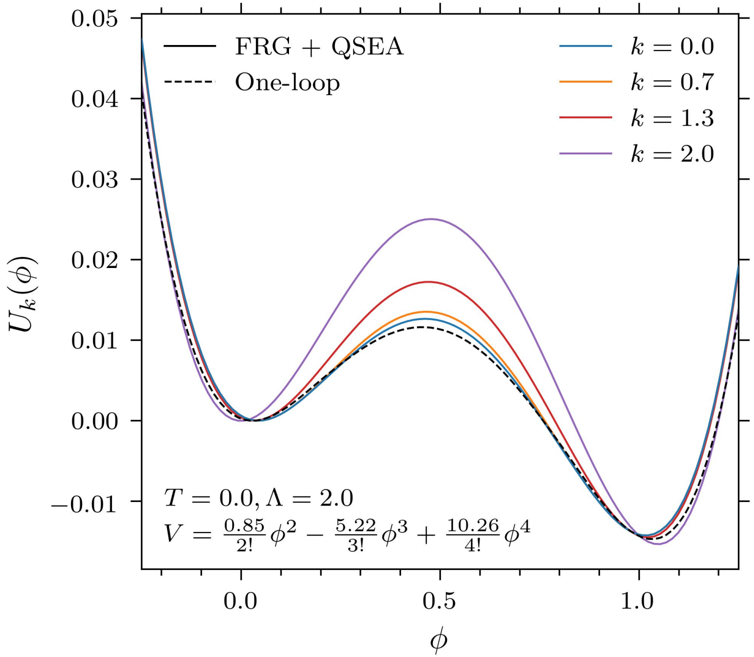
As easy (and maybe even easier) to evaluate compared to the one-loop effective potential!



Results and comparison with perturbation theory



Agrees almost exactly in perturbative regime



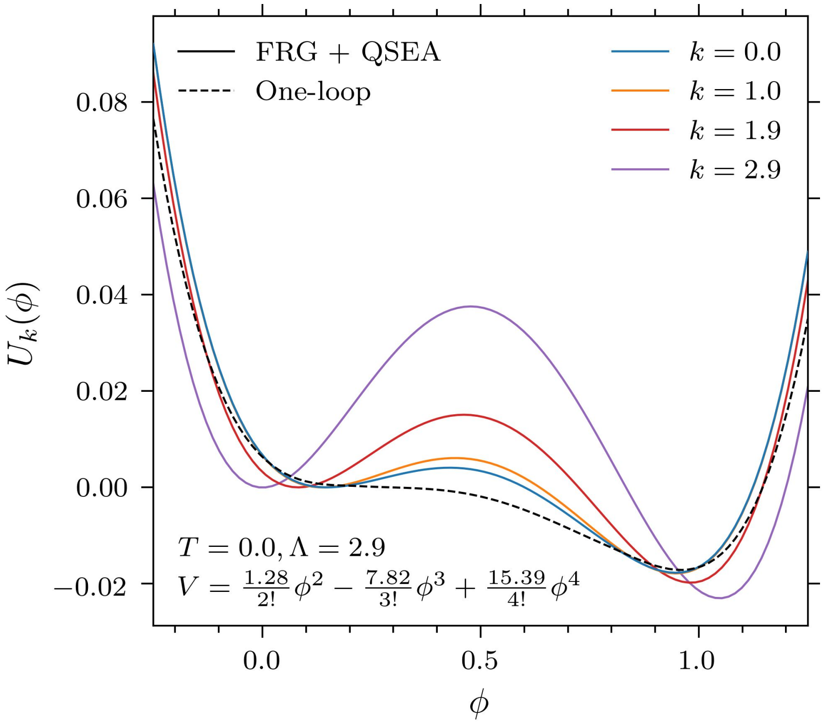
Starts to disagree as you approach larger couplings

The big point: strong interactions

Qualitatively different behavior than perturbation theory: e.g. at right QSEA + FRG predicts first-order phase transition but perturbation theory does not!

Different effective potentials => different predictions for GW signals

FRG + QSEA is robust to strong couplings: opens the door for GW analyses in a broad swath of strongly-interacting dark sectors!



Outlook and next steps

New quasi-stationary effective action based on integrating out local fluctuations implemented in a **modified FRG for fluctuations** that is **robust to strong couplings** and is **versatile + easy to use!**

Straightforwardly generalizes to finite-temperature, less minimal particle content

Opens up new range of DM theories for which we can calculate decay rate + GW signal

Work in progress with Djuna Croon [arXiv:21XX.XXXX]:
GW signals from chiral phase transitions in QCD-like dark sectors

More broadly, the **FRG is a powerful, non-perturbative tool that is underused in particle theory** and could open up broad new areas of research

Work in progress with Djuna Croon, Rachel Houtz [arXiv:21XX.XXXX]:
Improving warm DM constraints on axions with the FRG