Invisibles School – 2021
Flavor Physics

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Exercise Sheet 13

Exercise 1: FCNC in a model with two Higgs doublets

a) Write down the Yukawa Lagrangian of the Standard Model (SM) and the resulting fermion mass matrices after the Higgs field Φ acquires a vaccum expectation value (vev). Rotate the fermion fields to the mass basis, and derive the interactions between the gauge bosons and the quark fields in this basis.

b) Let us consider now a model with two Higgs doublets $\Phi_{1,2}$, which transform as $SU(2)_L$ doublets with hypercharge $Y = 1/2$. Assume that the two fields can be expressed as

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}, \qquad (a = 1, 2) \tag{1}$$

where v_a denotes the Higgs vev's, and ρ_a, η_a and ϕ_a are scalar fields. How many physical scalars exist in this model? Show that the masses of gauge bosons are the same as in the SM,

$$m_W = g_2 \frac{v}{2}, \qquad m_Z = \sqrt{g_1^2 + g_2^2}\, \frac{v}{2}, \tag{2}$$

where $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV.

c) Write down the most general Yukawa interactions for up- and down-type quarks in this model. Rotate the fermion fields to the mass basis and show that the Z couplings are flavor diagonal.

d) Show that FCNC can be mediated at tree-level by the scalar bosons.

e) [**Extra**] Can you devise a symmetry to avoid these dangerous FCNC effects at tree-level?

Exercise 2: CKM matrix and unitarity

The Unitarity Triangle (UT) is defined in the complex plane $z = \bar{\rho} + i\,\bar{\eta}$ by the equation,

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0, \tag{3}$$

as illustrated in Fig. 1. The three internal angles are denoted by α, β and γ.

a) Show that

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \qquad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{4}$$

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b) Show that these angles are invariant under phase redefinitions of quark fields: $q_i \to e^{i\alpha_i} q_i$.

c) Show that the area of the UT is proportional to the *Jarlskog invariant*,

$$J = \mathrm{Im}(V_{ud} V_{cd}^* V_{cb} V_{ub}^*) \,. \tag{5}$$

d) Which are the simplest tree-level mesons decays that can be used to extract $|V_{ij}|$ for different flavor indices i, j?

<u>Hint</u>: *The valence content of the lighest charged pseudoscalar mesons are* $\pi^- = \bar{u}d$, $K^- = \bar{u}s$, $D^- = \bar{c}d$, $D_s^- = \bar{c}s$ *and* $B^- = \bar{u}b$.

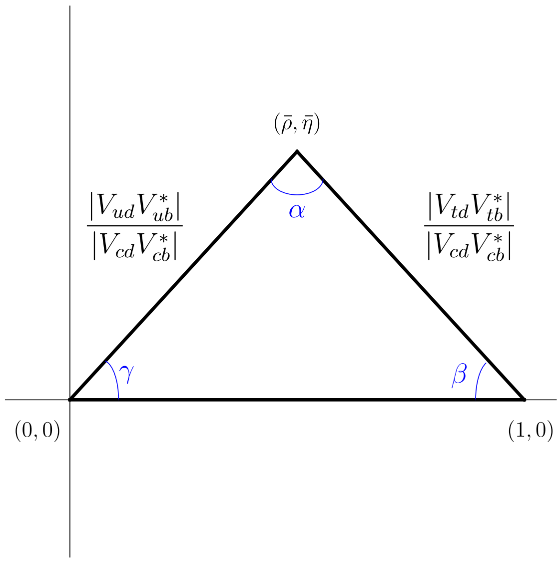

Figure 1: *Unitarity triangle.*

<u>Exercise 3</u>: Accidental symmetries in the SM

a) Write down the lowest-dimension operators with the SM field content that violate lepton (L) and baryon (B) number.

b) Show that the decay $p \to \pi^0 e^+$ can be mediated by the B-violating operators. Use naive dimensional analysis to estimate the proton lifetime as a function of the EFT cutoff Λ.

c) The Super-Kamiokande experiment has searched for the $p \to \pi^0 e^+$ decays, obtaining the lower limit $\tau(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years on the proton lifetime. Provide a rough estimate for the lowest scale Λ allowed by this experimental constraint by setting the Wilson coefficients to unity.

c) **[Extra]** What is the lowest dimension B violating operator which is invariant under the $SU(3)^5 \equiv SU(3)_Q \times SU(3)_L \times SU(3)_U \times SU(3)_D \times SU(3)_E$ flavor symmetry?

Exercise 4: ***B*-physics anomalies**

Recently, the LHCb experiment has observed discrepancies from the SM predictions in measurements of the Lepton Flavor Universality (LFU) ratios $R_{K^{(*)}} = \mathcal{B}(B \to K^{(*)}\mu\mu)/\mathcal{B}(B \to K^{(*)}ee)$ in specific bins of dilepton invariant-mass squared (q^2). These discrepancies can be explained by the following low-energy effective Lagrangian,

$$\mathcal{L}_{\text{eff}} \supset \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha_{\text{em}}}{2\pi}\, \delta C_L\, (\bar{s}\gamma^\mu P_L b)\, (\bar{\mu}\gamma_\mu P_L \mu) \; + \text{h.c.} \tag{6}$$

which should be added in addition to the SM contributions. Current data favors the coefficients $\delta C_L \approx -0.4$, with vanishing new physics couplings to electrons.

a) Write down the $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant operators at dimension-6 that can generate the low-energy Lagrangian given in Eq. (6) at tree-level. Assume that down-quark Yukawas are diagonal.

b) Let us assume that these operators are generated by the tree-level exchange of a new boson V with mass m_V and with perturbative couplings, i.e. $|g_{\text{NP}}| \lesssim \sqrt{4\pi}$. What is the maximal value of m_V which is compatible with these assumptions? What if this operator is induced at loop-level?

c) [**Extra**] Can you predict new physics contributions to other B-meson decays by using the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge-invariant operators?