# Exercise 1

**Exercise 1:** **FCNC in a model with two Higgs doublets**

**a)** Write down the Yukawa Lagrangian of the Standard Model (SM) and the resulting fermion mass matrices after the Higgs field $\Phi$ acquires a vaccum expectation value (vev). Rotate the fermion fields to the mass basis, and derive the interactions between the gauge bosons and the quark fields in this basis.

**b)** Let us consider now a model with two Higgs doublets $\Phi_{1,2}$, which transform as $SU(2)_L$ doublets with hypercharge $Y = 1/2$. Assume that the two fields can be expressed as

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}, \qquad (a = 1, 2) \tag{1}$$

where $v_a$ denotes the Higgs vev's, and $\rho_a$, $\eta_a$ and $\phi_a$ are scalar fields. How many physical scalars exist in this model? Show that the masses of gauge bosons are the same as in the SM,

$$m_W = g_2 \frac{v}{2}, \qquad m_Z = \sqrt{g_1^2 + g_2^2}\,\frac{v}{2}, \tag{2}$$

where $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV.

**c)** Write down the most general Yukawa interactions for up- and down-type quarks in this model. Rotate the fermion fields to the mass basis and show that the $Z$ couplings are flavor diagonal.

**d)** Show that FCNC can be mediated at tree-level by the scalar bosons.

**e)** **[Extra]** Can you devise a symmetry to avoid these dangerous FCNC effects at tree-level?

@ SM $\qquad Y_{u,d,e} \equiv M_{3,3}(\mathbb{C})$

$$\mathcal{L}_Y = -\bar{Q} Y_d\, d \cdot \Phi - \bar{Q} Y_u\, u_R \cdot \tilde{\Phi} - \bar{L} Y_e\, e_R \cdot \Phi + h.c$$

Notation:

$$\psi = (\psi_1, \psi_2, \psi_3), \qquad \psi \in \{u_R, d_R, e_R, Q, L\}$$

After EWSB,

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix}$$

$$\mathcal{L}_Y \to -\left(1 + \frac{h}{v}\right)\left[\bar{d}_L \cdot M_d \cdot d_R + \bar{u}_L \cdot M_u \cdot u_R + \bar{e}_L \cdot M_e \cdot e_R\right] + h.c$$

$$\boxed{M_f = \frac{v}{\sqrt{2}} Y_f}, \qquad f = u, d, e$$

Mass matrices can be diagonalized by bi-unitary transformations:

$$\psi_{L,R} \longrightarrow \mathcal{U}_{\psi_{L(R)}} * \psi_{L(R)}$$

interaction basis $\qquad$ mass basis

with

$$\mathcal{U}^\dagger_{\psi_L} \cdot M_\psi \cdot \mathcal{U}_{\psi_R} \equiv \hat{M}_\psi = \mathrm{diag}(m_{\psi_1}, m_{\psi_2}, m_{\psi_3})$$

$$\mathcal{L}_Y \to -\left(1 + \frac{h}{v}\right)\left[\bar{d} \cdot \hat{M}_d \cdot d + \bar{u} \cdot \hat{M}_u \cdot u + \bar{e} \cdot \hat{M}_e \cdot e\right]$$

NB. Higgs couplings are diagonal too!

- NC interactions:

Matrix notation: $\psi = (\psi_1, \psi_2, \psi_3)$

$\mathbb{1}_{3\times 3}$

$$\mathcal{L}_{NC} = \sum_\psi \bar{\psi} \not{D}_{NC} \mathbb{1} \psi \to \sum_\psi \bar{\psi} \not{D}_{NC} \mathbb{1} \psi$$

$\psi = Q, L, u_R, d_R, e_R$

... remain diagonal!

$$\mathcal{U}^\dagger_{\psi_L} * \mathbb{1} * \mathcal{U}_{\psi_L} = \mathbb{1}$$
$$\mathcal{U}^\dagger_{\psi_R} * \mathbb{1} * \mathcal{U}_{\psi_R} = \mathbb{1}$$

- CC interactions:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \cdot \left[\bar{u}_L \gamma^\mu \mathbb{1} d_L + \bar{\nu}_L \gamma^\mu \mathbb{1} e_L\right] W_\mu + h.c$$

$$\to \frac{g}{\sqrt{2}}\left[\bar{u}_L \gamma^\mu \underbrace{\mathcal{U}^\dagger_{u_L} \mathcal{U}_{d_L}}_{\equiv V_{CKM}} \cdot d_L + \bar{\nu}_L \gamma^\mu \mathbb{1} e_L\right] W_\mu + h.c$$

[assuming $m_\nu = 0$]

- Summary:

$$\mathcal{L}_{flavor} = \sum_\psi \bar{\psi} \not{D}_{NC} \mathbb{1} \psi + \frac{g}{\sqrt{2}}\left[\bar{u}_L V_{CKM} \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu \mathbb{1} e_L + h.c.\right] - \left(1 + \frac{h}{v}\right) \cdot \left[\bar{d} \hat{M}_d d + \bar{u} \hat{M}_u u + \bar{e} \hat{M}_e e\right]$$

## ② 2HDM

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix} \qquad (a=1,2)$$

\# of Higgs bosons? $4\times 2 - 3 = 5$.

$\hookrightarrow \underset{\text{SM-like}}{\underline{h}}, H^0, A^0, H^\pm$

- Gauge boson masses:

$$\mathcal{L}_\Phi \supset \sum_a |D_\mu \phi_a|^2$$

$$D_\mu = \partial_\mu + i g \frac{\tau^a}{2} W_a^\mu + i g' \frac{B_\mu}{2}$$

$$= \sum_a |\partial_\mu \Phi_a|^2$$

$$+ \sum_a \left(0, \frac{v_a}{\sqrt{2}}\right) \cdot \left(\frac{g}{2}\vec{\tau}\cdot\vec{W}^\mu + \frac{g'}{2} B^\mu \mathbb{1}\right)\left(\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu + \frac{g'}{2} B_\mu \mathbb{1}\right) \cdot \begin{pmatrix} 0 \\ v_a/\sqrt{2} \end{pmatrix} + \dots$$

$$= \sum_a |\partial_\mu \Phi_a|^2$$

$$+ \frac{v_1^2 + v_2^2}{2} \underbrace{\left(\frac{g}{2}\vec{\tau}\cdot\vec{W}^\mu + \frac{g'B^\mu}{2}\mathbb{1}\right)\left(\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu + \frac{g'}{2}B_\mu \mathbb{1}\right)}_{\text{Same as in the SM!}} + \dots$$

$$= m_W^2 W^\mu W_\mu + \frac{m_Z^2}{2} Z^\mu Z_\mu + \dots$$

with $W_\mu^\pm = \dfrac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}$ and $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$

and $\rho = \dfrac{m_W^2}{m_Z^2 c_W^2} = 1$ with $v^2 = v_1^2 + v_2^2$.

(c) FCNC ?

$$\mathcal{L}_Y = -\sum_a \left[ \bar{Q} Y_d^{(a)} d \cdot \Phi^{(a)} + \bar{Q} Y_u^{(a)} u_R \cdot \tilde{\Phi}^{(a)} + \bar{L} Y_e^{(a)} e_R \cdot \Phi^{(a)} \right] + h.c.$$

$$\rightarrow - \bar{d}_L M_d d_R - \bar{u}_L M_u u_R + \bar{e}_L M_e e_R + \ldots + h.c$$

with

$$\boxed{M_\psi = Y_\psi^{(1)} \frac{v_1}{\sqrt{2}} + Y_\psi^{(2)} \frac{v_2}{\sqrt{2}}}$$

Bi-unitary transformation:

$$U_{\psi_L} \cdot M_\psi \cdot U_{\psi_R}^\dagger = \hat{M}_\psi = \text{diag}(m_{\psi_1}, m_{\psi_2}, m_{\psi_3})$$

Z-boson couplings:

$$\mathcal{L}_{NC} = \sum_\psi \bar{\psi} \not{J}_{NC} \mathbb{1} \psi \longrightarrow \mathcal{L}_{NC}$$

Nothing changes!

## d) FCNC via scalars?

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_Y = -\sum_a \left[ \bar{Q} Y_d^{(a)} d \cdot \Phi^{(a)} + \bar{Q} Y_u^{(a)} u_R \cdot \tilde{\Phi}^{(a)} + \bar{L} Y_e^{(a)} e_R \cdot \Phi^{(a)} \right] + h.c.$$

e.g., for down-type quarks

$$\mathcal{L}_Y \supset - \sum_a \bar{Q} Y_d^{(a)} d_R \Phi^{(a)}$$

$$\xrightarrow{SSB} -\bar{d}_L \left( Y_d^{(1)} \frac{v_1}{\sqrt{2}} + Y_d^{(2)} \frac{v_2}{\sqrt{2}} \right) d_R$$

$$- \frac{(\rho_1 + i\eta_1)}{\sqrt{2}} \cdot \bar{d}_L Y_d^{(1)} d_R - \frac{(\rho_2 + i\eta_2)}{\sqrt{2}} \cdot \bar{d}_L Y_d^{(2)} d_R + h.c.$$

Quark masses: $\quad M_d = Y_d^{(1)} \frac{v_1}{\sqrt{2}} + Y_d^{(2)} \frac{v_2}{\sqrt{2}}$

$\exists\ U_{d_L}, U_{d_R}$ such that

$$U_{d_L}^{\dagger} M_d U_{d_R} = \hat{M}_d = (m_d, m_s, m_b)$$

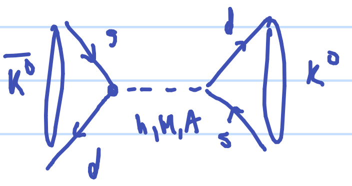
But $\{ M_d, Y_d^{(1)}, Y_d^{(2)} \}$ are <u>NOT</u> simultaneously diagonalizable in general ...

Physical scalars (CP-conserving potential):

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \qquad \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$\tan\beta = \frac{v_2}{v_1}$ $\qquad$ depends on scalar potential

$\Rightarrow$ Neutral Higgs can have FCNC couplings @ <u>tree-level</u>:




## e) Way-out?

$$\mathcal{L}_Y = -\sum_a \left[ \bar{Q} Y_d^{(a)} d \cdot \Phi^{(a)} + \bar{Q} Y_u^{(a)} u_R \cdot \tilde{\Phi}^{(a)} + \bar{L} Y_e^{(a)} e_R \cdot \Phi^{(a)} \right] + h.c.$$

e.g. $\mathbb{Z}_2$ symmetry!

Exercise II

**Exercise 2: CKM matrix and unitarity**

The Unitarity Triangle (UT) is defined in the complex plane $z = \bar{\rho} + i\,\bar{\eta}$ by the equation,

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0\,, \tag{3}$$

as illustrated in Fig. 1. The three internal angles are denoted by $\alpha$, $\beta$ and $\gamma$.

**a)** Show that

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \qquad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{4}$$

**b)** Show that these angles are invariant under phase redefinitions of quark fields: $q_i \to e^{i\alpha_i} q_i$.

**c)** Show that the area of the UT is proportional to the *Jarlskog invariant*,

$$J = \mathrm{Im}(V_{ud}V_{cd}^* V_{cb} V_{ub}^*)\,. \tag{5}$$

**d)** Which are the simplest tree-level mesons decays that can be used to extract $|V_{ij}|$ for different flavor indices $i, j$?

**Hint:** *The valence content of the lighest charged pseudoscalar mesons are $\pi^- = \bar{u}d$, $K^- = \bar{u}s$, $D^- = \bar{c}d$, $D_s^- = \bar{c}s$ and $B^- = \bar{u}b$.*

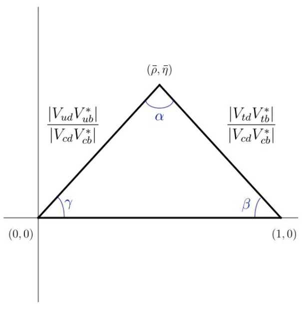


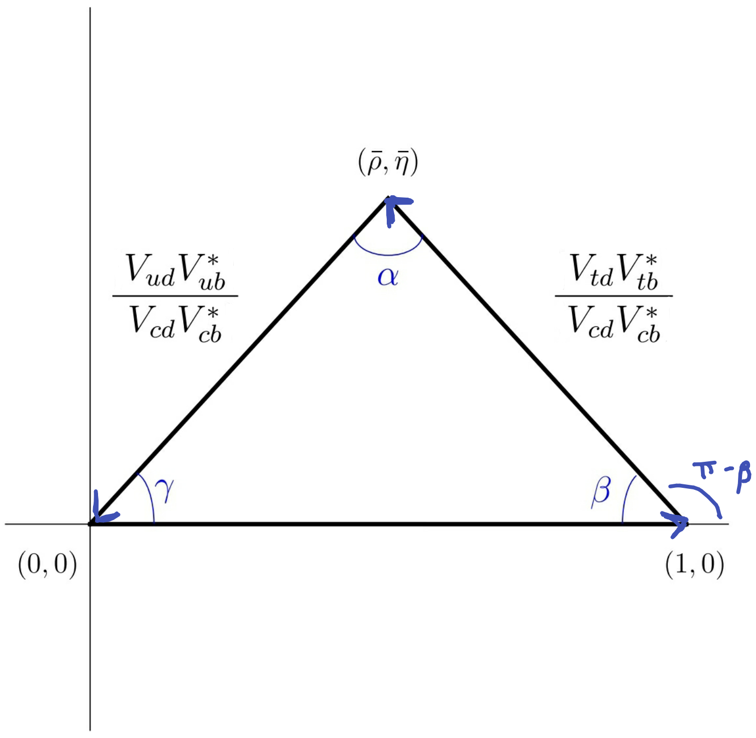

Figure 1: *Unitarity triangle.*

ⓐ

$$V^\dagger V = V V^\dagger = \mathbb{1} \Rightarrow \sum_k V_{ik} V_{jk}^* = \sum_k V_{ki} V_{kj}^* = \delta_{ij}$$

$$(i,j) = (1,3) \Rightarrow V_{ud} \cdot V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$




- $\gamma = \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ //

- $\pi - \beta = \arg\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right) = -\arg\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$

$\Rightarrow \beta = \pi + \arg\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$ //

- $\alpha = \pi - \beta - \gamma = \pi - \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) - \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$

$= \pi - \arg\left(\frac{V_{ud}V_{ub}^*}{V_{td}V_{tb}^*}\right) = -\arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$ //

(b)

Rephasing of up-quarks:

$$\begin{aligned} V_{td}V_{tb}^* &\to e^{i\phi} V_{td}V_{tb}^* e^{-i\phi} = V_{td}V_{tb}^* \\ V_{cd}V_{cb}^* &\to \dots = V_{cd}V_{cb}^* \\ V_{ud}V_{ub}^* &\to \dots = V_{ud}V_{ub}^* \end{aligned}$$

Rephasing of down-quarks:

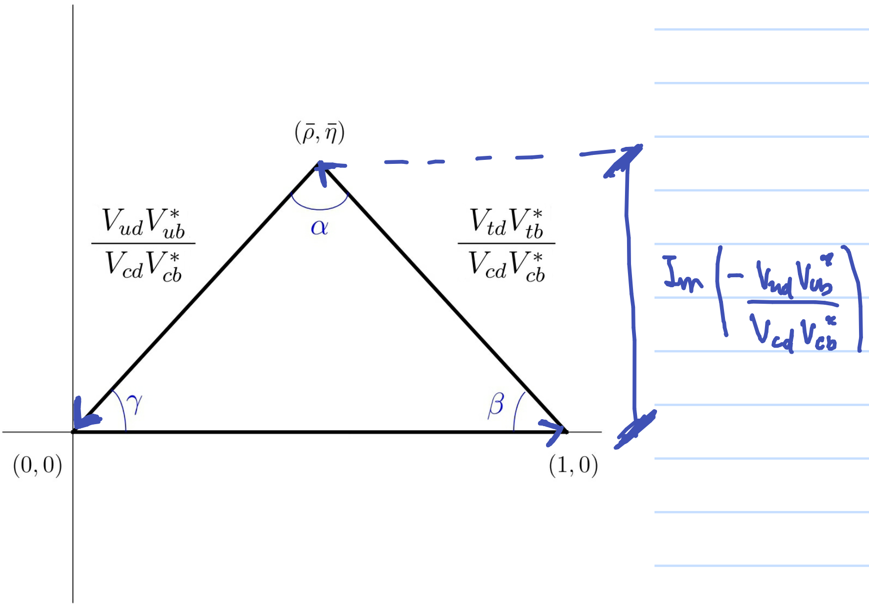
$$\frac{V_{td}}{V_{ud}} \to \frac{e^{i\phi}}{e^{i\phi}}\frac{V_{td}}{V_{ud}} = \frac{V_{td}}{V_{ud}}$$

$\vdots$

always cancel in the denominator!

$\Rightarrow$ $\alpha, \beta, \gamma$ are reparametrization invariant! (thus, physical)

(c)




$$\text{Area} = \frac{1}{2} \times 1 \times \mathrm{Im}\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$= -\frac{1}{2}\,\frac{1}{|V_{cd}V_{cb}|^2}\cdot \underbrace{\mathrm{Im}\left(V_{ud}V_{ub}^* \cdot V_{cd}^* \cdot V_{cb}\right)}_{\equiv J}$$

## Exercise III

**Exercise 3: Accidental symmetries in the SM**

**a)** Write down the lowest-dimension operators with the SM field content that violate lepton ($L$) and baryon ($B$) number.

**b)** Show that the decay $p \to \pi^0 e^+$ can be mediated by the $B$-violating operators. Use naive dimensional analysis to estimate the proton lifetime as a function of the EFT cutoff $\Lambda$.

**c)** The Super-Kamiokande experiment has searched for the $p \to \pi^0 e^+$ decays, obtaining the lower limit $\tau(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years on the proton lifetime. Provide a rough estimate for the lowest scale $\Lambda$ allowed by this experimental constraint by setting the Wilson coefficients to unity.

**c)** **[Extra]** What is the lowest dimension $B$ violating operator which is invariant under the $SU(3)^5 \equiv SU(3)_Q \times SU(3)_L \times SU(3)_U \times SU(3)_D \times SU(3)_E$ flavor symmetry?

**a)**

$U(1)_L \Rightarrow \quad O^{(5)} = (\overline{L^c}\,\tilde{H}^*)(\tilde{H}^\dagger L) \qquad (\dim = 5)$

$U(1)_B \Rightarrow \quad$ Need 3 quarks and 1 lepton! $\quad (\dim = 6)$

$\alpha, \beta, \gamma \equiv$ color indices
$i, j, k, l \equiv SU(2)_L$ indices
$A, B, C, D \equiv$ flavor indices

$$O_1 = \varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}\cdot\varepsilon_{kl}\cdot(\overline{q_i^{c\alpha}}\,q_j^\beta)(\overline{q_k^{c\gamma}}\cdot l_l) \qquad Q^3L$$

$$O_2 = \varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}\cdot(\overline{q_i^{c\alpha}}\,q_j^\beta)\cdot(\overline{u^{c\gamma}}\,e) \qquad Q^2UE$$

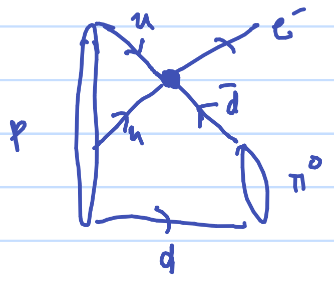
$$O_3 = \varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}\,(\overline{u^{c\alpha}}\,d^\beta)(\overline{q_i^{c\gamma}}\,l_j) \qquad QUDL$$

$$O_4 = \varepsilon_{\alpha\beta\gamma}\cdot(\overline{d^{c\alpha}}\,u^\beta)(\overline{u^{c\gamma}}\,e)$$

$$O_5 = \varepsilon_{\alpha\beta\gamma}\cdot(\overline{u^{c\alpha}}\,u^\beta)(\overline{d^{c\gamma}}\,e)$$

$\Big\} \; U^2DE$

**b)**




$$\Gamma(p \to \pi^0 e^+) \simeq \frac{1}{8\pi}\frac{c^2}{\Lambda^4}m_p^5$$

$$\tau_p \simeq \frac{1}{\Gamma(p\to\pi^0 e^+)} \qquad ; \quad c \simeq 1$$

**c)**

Putting numbers:
$$\frac{1}{8\pi}\frac{m_p^5}{\Lambda^4} = \tau_p^{-1} \leq (1.6\times 10^{34}\ \text{years})^{-1}$$

$$\Rightarrow \boxed{\Lambda \gtrsim 10^{16}\ \text{GeV}}$$

**d)**

$SU(3)_f$ contractions: $\underline{\varepsilon^{IJK}\,\psi_I\,\psi_J\,\psi_K}$

$\Rightarrow$ We need an even # of fermions

$\Rightarrow$ We need at least $\underline{\text{three}}$ lepton fields

$q^3 l\cdot l^2 \;\to\;$ Can't be made $\underline{SU(2)_L \times U(1)_Y\ \text{invariant}}$!

$\Rightarrow$ Next options is 9 quarks and 3 leptons:

e.g.: $\mathcal{L} \supset \dfrac{1}{\Lambda^{12}}(q^3 l)^3 \qquad \Rightarrow$ dimension 12!
(thus, suppressed p-decay)

Indices to be contracted as follows:

$$O = \left(\varepsilon_{\alpha\beta\gamma}\cdot\varepsilon_{ij}\,\varepsilon_{kl}\cdot q^\alpha_{iA}\,q^\beta_{jB}\,q^\gamma_{kC}\cdot l_{lD}\right)$$
$$\times\left(\varepsilon_{\alpha\beta\gamma}\cdot\varepsilon_{ij}\,\varepsilon_{kl}\cdot q^\alpha_{iA'}\,q^\beta_{jB'}\,q^\gamma_{kC'}\cdot l_{lD'}\right)$$
$$\times\left(\varepsilon_{\alpha\beta\gamma}\cdot\varepsilon_{ij}\,\varepsilon_{kl}\cdot q^\alpha_{iA''}\,q^\beta_{jB''}\,q^\gamma_{kC''}\cdot l_{lD''}\right)\times\varepsilon_{AA'A''}\,\varepsilon_{BB'B''}$$
$$\times\,\varepsilon_{CC'C''}\times\varepsilon_{DD'D''}$$

# Exercise 4

$$R_{K^{(*)}} = \frac{B(B\to K^{(*)}\mu\mu)}{B(B\to K^{(*)}ee)} \overset{SM}{\simeq} 1 + O\left(\frac{m_\mu^2}{m_b^2}\right) \quad \text{vs} \quad R_{K^{(*)}}^{exp} \approx 0.85 \qquad (!)$$

$$\mathcal{L}_{eff}^{NP} = \frac{4G_F}{\sqrt{2}}\,\lambda_t\,\frac{\alpha_{em}}{2\pi}\,\delta C_L\,(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu P_L\mu) + h.c \qquad (*)$$

$SU(3)_c \times U(1)_{em}$

$\Rightarrow$ can explain deviations with $\delta C_L \approx -0.4$

ⓐ $SU(3)_c \times SU(2)_L \times U(1)_Y$:

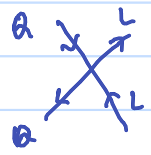
$$\mathcal{L} = \frac{C_i}{\Lambda^2} O_i^{(6)} + h.c$$

Building blocks: $\bar{Q}, L, u_R, d_R, e_R, H$

$$Q_i = \begin{pmatrix} (V^\dagger u_L)_i \\ d_{L i} \end{pmatrix}$$

$(ijkl) = (2322)$

Which operators? $(\bar{s}_L\gamma_\mu b_L)\times(\bar{\mu}_L\gamma^\mu\mu_L)$



$$[O_1]_{ijkl} = (\bar{Q}_i\gamma^\mu Q_j)\times(\bar{L}_k\gamma_\mu L_l) \checkmark \qquad (\#)$$

$$[O_3]_{ijkl} = (\bar{Q}_i\gamma^\mu\tau^I Q_j)(\bar{L}_k\gamma_\mu\tau^I L_l)$$

Completeness relation: $\sum_I \tau^I_{ab}\cdot\tau^I_{cd} = 2\,\delta_{ad}\,\delta_{bc} - \delta_{ab}\cdot\delta_{cd}$

ⓑ Matching:

$$[O_1]_{2322} = (\bar{Q}_{L2}\gamma^\mu Q_{L3})\times(\bar{L}_2\gamma_\mu L_2)$$

$$\bar{u}'_{L2}\gamma^\mu u_{L3} + \bar{d}_{L2}\gamma^\mu d_{L3} \qquad \bar{\nu}_{L\mu}\gamma^\mu\nu_{L\mu} + \bar{\mu}_L\gamma^\mu\mu_L$$

$$\sim (\bar{d}_{L2}\gamma^\mu d_{L3})(\bar{\mu}_L\gamma_\mu\mu_L) + \ldots$$

$$[O_3]_{2322} = (\bar{Q}_{L2}\gamma^\mu\tau^I Q_{L3}).(\bar{L}_2\gamma_\mu\tau^I L_2)$$

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= (\bar{d}_{L2}\gamma^\mu d_{L3})(\bar{\mu}_L\gamma_\mu\mu_L) + \ldots$$

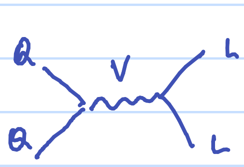
$$\mathcal{L}_{eff}^{NP} = \frac{4G_F}{\sqrt{2}}\,\lambda_t\,\frac{\alpha_{em}}{2\pi}\,\delta C_L\,(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu P_L\mu) + h.c$$

$$= \frac{C_1 + C_3}{\Lambda^2}\cdot\bar{s}_L\gamma_\mu b_L \times \bar{\mu}_L\gamma^\mu\mu_L \qquad G_F = \frac{1}{\sqrt{2}\,v^2}$$

$$\leftrightarrow \quad \frac{C_1+C_3}{\Lambda^2} = \frac{\delta C_L}{v^2}\times\frac{\alpha_{em}\,\lambda_t}{\pi}$$

$$\simeq -0.4 \times \frac{1}{(246\,\text{GeV})^2}\times\frac{1}{129}\times\frac{1\times 0.04}{\pi} = \frac{-1}{(12\,\text{TeV})^2}$$

New mediator:



e.g. $(1,1,0) \sim Z'$

$$\frac{C_1+C_3}{\Lambda^2} \sim \frac{g_{NP}^2}{m_V^2} \overset{pert.}{\Rightarrow} m_V = (12\,\text{TeV})\times g_{NP}$$

$$\Rightarrow m_V \lesssim 35\,\text{TeV} \;!$$