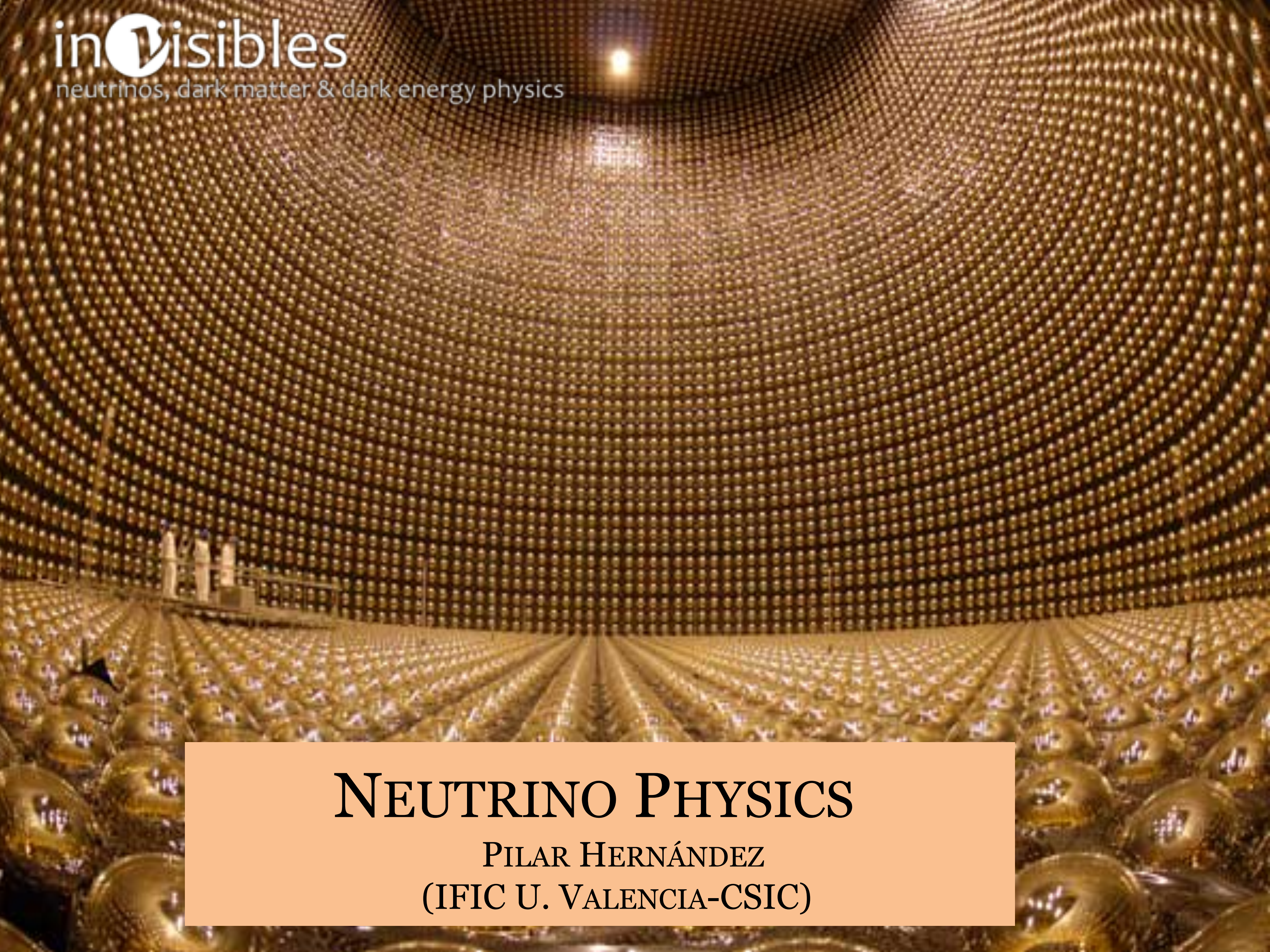

NEUTRINO PHYSICS

PILAR HERNÁNDEZ
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LECTURE II

- Neutrino oscillations in vacuum and in matter
- Experimental evidence for neutrino masses & mixings

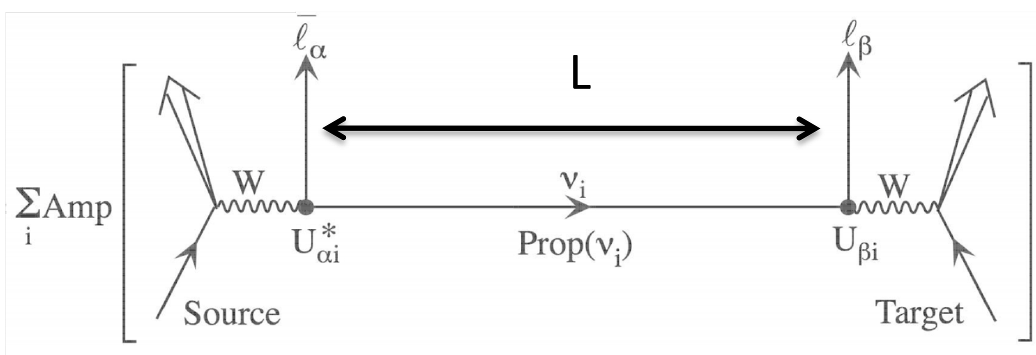
Neutrino oscillations

1968 Pontecorvo

If neutrinos are massive

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \ldots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

A neutrino experiment is an interferometer in flavour space, because neutrinos are so weakly interacting that can keep coherence over very long distances !



ν_i pick up different phases when travelling in vacuum

Neutrino oscillations in QM (plane waves)

$$|\nu_\alpha(t_0)\rangle = \sum_i U^*_{\alpha i}|\nu_i(\mathbf{p})\rangle, \qquad \hat{H}|\nu_i(\mathbf{p})\rangle = E_i(\mathbf{p})|\nu_i(\mathbf{p})\rangle, \quad \mathbf{p}^2 + m_i^2 = E_i^2(\mathbf{p})$$

$\downarrow$ time evolution

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i} e^{-iE_i(\mathbf{p})(t-t_0)}|\nu_i(\mathbf{p})\rangle$$

$$P(\nu_\alpha \to \nu_\beta)(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |\sum_i U_{\beta i}U^*_{\alpha i}e^{-iE_i(t-t_0)}|^2$$

$$= \sum_{i,j} e^{-i(E_i-E_j)(t-t_0)} U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}$$

$$E_i(\mathbf{p}) - E_j(\mathbf{p}) \simeq \frac{1}{2}\frac{m_i^2 - m_j^2}{|\mathbf{p}|} + \mathcal{O}(m^4) \qquad L \simeq t - t_0, v_i \simeq c$$

$$P(\nu_\alpha \to \nu_\beta)(L) \simeq \sum_{i,j} e^{i\frac{\Delta m^2_{ji} L}{2E}} U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}$$

Neutrino oscillations in QM (plane waves)

Well founded criticism to this derivation

Why same p for the i-th states ?

Why plane waves if the neutrino source is localized ?

Why t <-> L conversion ?

Neutrino oscillations

Two basic ingredients:

✓ Uncertainty in momentum at production & detection (they must be better localized than baseline)

✓ Coherence of mass eigenstates over macroscopic distances

Quantum mechanics with neutrinos as wave packets

Quantum Field Theory <-> neutrinos as intermediate states

Neutrino oscillations in QM (wavepackets)

B. Kayser ‘81,... many more authors...

Wave packet created at source @ $(t_0, \mathbf{x}_0) = (0, \mathbf{0})$

$$|\nu_\alpha(t,\mathbf{x})\rangle = \sum_i U^*_{\alpha i} \int_{\mathbf{p}} \underbrace{f_i^S(\mathbf{p}-\mathbf{Q}_i)}_{\text{Wave packet at source}} \; e^{-iE_i(\mathbf{p})t} \; e^{i\mathbf{p}\cdot\mathbf{x}} \; |\nu_i\rangle$$

$$E_i(\mathbf{p}) \equiv \sqrt{\mathbf{p}^2 + m_i^2}$$

For example: $f_i^S(\mathbf{p}-\mathbf{Q}_i) \simeq e^{-(\mathbf{p}-\mathbf{Q}_i)^2/2\sigma_S^2}$

$\sigma_S \leftrightarrow$ momentum uncertainty

$\mathbf{Q}_i \leftrightarrow$ average momentum of $\mathrm{i-th}$ wavepacket

Wave packet created at detector @ $(t_0, \mathbf{x}_0) = (t, \mathbf{L})$

$$|\nu_\beta(t,\mathbf{x})\rangle = \sum_j U^*_{\beta j} \int_{\mathbf{p}} f_j^D(\mathbf{p}-\mathbf{Q}'_j) \; e^{-iE_j(\mathbf{p})(t-T)} \; e^{i\mathbf{p}(\mathbf{x}-\mathbf{L})} \; |\nu_j\rangle$$

Neutrino oscillations in QM (wavepackets)

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = \int_{\mathbf{x}} \langle \nu_\beta(t,\mathbf{x}) | \nu_\alpha(t,\mathbf{x}) \rangle$$
$$= \sum_i U^*_{\alpha i} U_{\beta i} \int_{\mathbf{p}} e^{iE_i(\mathbf{p})T} \; e^{-i\mathbf{p}\mathbf{L}} \underbrace{f_i^{D*}(\mathbf{p}-\mathbf{Q}'_i) f_i^S(\mathbf{p}-\mathbf{Q}_i)}_{overlap}$$

For Gaussian wave packets overlap is also gaussian:

$$f_i^{D*} f_i^S = f_i^{ov}(\mathbf{p} - \langle \mathbf{Q} \rangle_i) \; e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2/4/(\sigma_S^2+\sigma_D^2)}$$

$$\langle \mathbf{Q} \rangle_i \equiv \left(\frac{\mathbf{Q}_i}{\sigma_S^2} + \frac{\mathbf{Q}'_i}{\sigma_D^2} \right) \sigma_{ov}^2$$
$$\sigma_{ov}^2 \equiv \frac{1}{1/\sigma_S^2 + 1/\sigma_D^2}$$

$\mathbf{v}_i$ group velocity

$$E_i(\mathbf{p}) \simeq E_i(\langle \mathbf{Q} \rangle_i) + \overbrace{\left.\frac{\partial E}{\partial p_k}\right|_{\langle \mathbf{Q} \rangle_i}}^{\mathbf{v}_i} (p_k - \langle Q_k \rangle_i) + \mathcal{O}(p_k - \langle Q_k \rangle_i)^2$$

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) \propto \sum_i U^*_{\alpha i} U_{\beta i} \; e^{iE_i(\langle \mathbf{Q} \rangle_i)T} \; e^{-i\langle \mathbf{Q} \rangle_i \mathbf{L}} \; e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2/4/(\sigma_S^2+\sigma_D^2)} \; e^{-(\mathbf{L}-\mathbf{v}_i T)^2 \sigma_{ov}^2/2}$$

Neutrino oscillations in QM (wavepackets)

$$\langle \mathbf{Q} \rangle_i \simeq \langle \mathbf{Q}' \rangle_i, \quad \mathbf{L} || \langle \mathbf{Q} \rangle_i$$

$$L_{coh}^{-1}(i,j) \sim \sigma_{ov} \frac{|\mathbf{v}_i - \mathbf{v}_j|}{\sqrt{\mathbf{v}_i^2 + \mathbf{v}_j^2}} \simeq \frac{|m_j^2 - m_i^2|}{2\langle Q \rangle} \frac{\sigma_{ov}}{\langle Q \rangle}$$

$$P(\nu_\alpha \to \nu_\beta) \propto \int_{-\infty}^{\infty} dT \, |\mathcal{A}(\nu_\alpha \to \nu_\beta)|^2$$

$$\propto \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \, e^{i \frac{m_j^2 - m_i^2}{2E} L} \times e^{-L^2 / L_{coh}(i,j)^2} \times e^{-\left(\frac{\Delta_{ij} E(\langle Q \rangle)}{2 \sigma_{ov} \langle v \rangle} \right)^2},$$

$L > L_{coh}$ coherence is lost

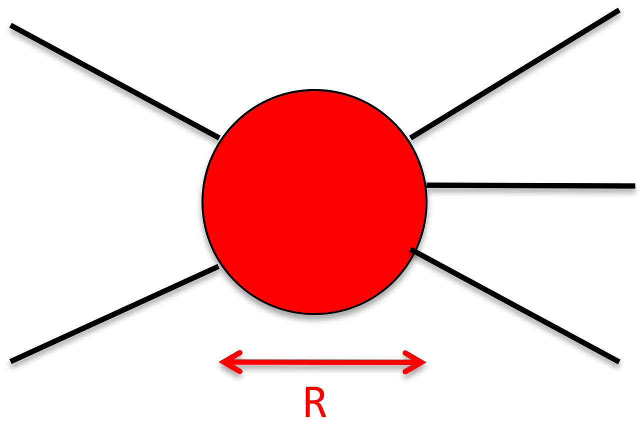
There must be sufficient uncertainty in production & detection so that wave packets include all mass eigenstates: $\Delta E \ll \sigma$

Problems: normalization is arbitrary, needs to be imposed a posteriori

$$\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$$

Can be cured in QFT...

Neutrino oscillations in QFT



in-states

out-states

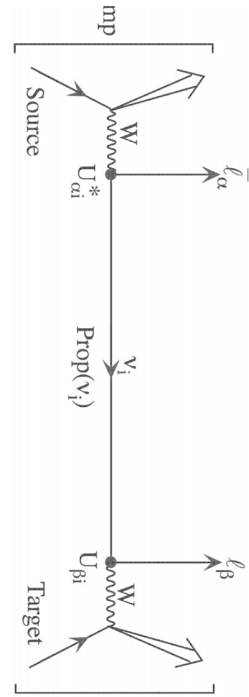
Idealization: asymptotic states are plane waves if R << Compton wavelength, in reality in-states are wave packets

$$\mathcal{A} = \langle \mathrm{out}; p'_1, ..., p'_n | \mathrm{in}; p_1, p_2 \rangle$$

Neutrino oscillations in QFT

Neutrinos are not the asymptotic states…

in-states



out-states

$$\mathcal{A} \sim \sum_i \mathcal{A}_S \; U^*_{\beta i} \; \frac{i}{\not p - m_i} U_{i\alpha} \mathcal{A}_D$$

Neutrino propagator: intermediate state

Neutrino oscillations in QFT

Necessary to adapt standard formalism:

1) macroscopic separation of Source and Detector L (eg. localized wave packets of in-states + static approximation)

2) oscillation probability from factorization:

decay x propagation x ν cross-section

$$\frac{dW(\pi n \to p\mu l_\beta)}{dt dp_\mu dp_p dp_l} = \int d|q| \underbrace{\frac{dW(\pi \to \mu\nu)}{L^2 dt d\Omega_\nu d|q| dp_\mu}}_{\text{Flux per unit neutrino momentum}} \times P(\nu_\mu \to \nu_\beta) \times \underbrace{\frac{1}{2|q|}\frac{dW(\nu n \to pl)}{dt dp_p dp_l}}_{\text{interaction probability per unit flux}}$$

Oscillation probability is indeed properly normalized!

Neutrino Oscillation

$$P(\nu_\alpha \to \nu_\beta) = \sum_{ij} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i\frac{(m_i^2 - m_j^2)L}{2E}}$$

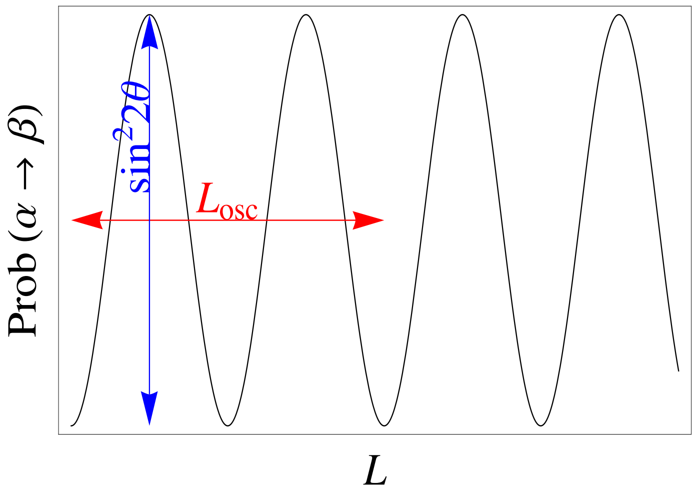
$\alpha \neq \beta$ appearance probability
$\alpha = \beta$ disappearance or survival probability

$$L_{osc} \sim \frac{E}{m_i^2 - m_j^2}$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(1.27 \frac{\Delta m^2(eV^2) L(km)}{E(GeV)}\right)$$



$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

$$L_{osc}(km) = \frac{\pi}{1.27} \frac{E(GeV)}{\Delta m^2(eV^2)}$$

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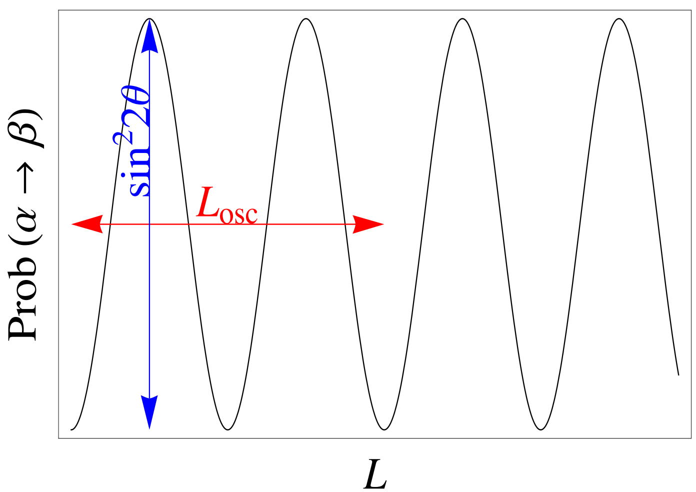

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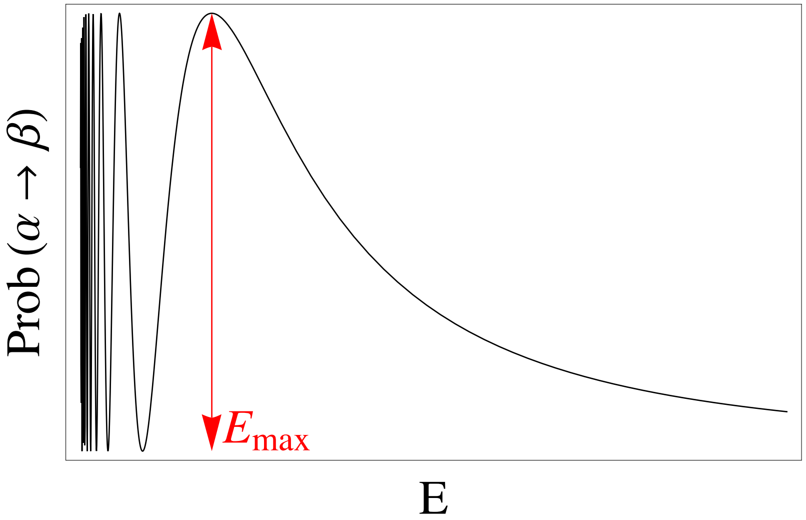


$$E_{max}(GeV) = 1.27\frac{\Delta m^2(eV^2)L(km)}{\pi/2}$$

L, E dependence give Δm^2 amplitude of oscillation gives θ

Optimal experiment: $\frac{E}{L} \sim \Delta m^2$

$\frac{E}{L} \gg \Delta m^2$ Oscillation suppressed

$$P(\nu_\alpha \to \nu_\beta) \propto \sin^2 2\theta \left(\Delta m^2\right)^2$$

$\frac{E}{L} \ll \Delta m^2$ Fast oscillation regime

$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2 2\theta \, \langle \sin^2 \frac{\Delta m^2 L}{4E} \rangle \simeq \frac{1}{2} \sin^2 2\theta = |U^*_{\alpha 1} U_{\beta 1}|^2 + |U^*_{\alpha 2} U_{\beta 2}|^2$$

Equivalent to incoherent propagation: sensitivity to mass splitting is lost

Neutrino vs Antineutrino: CP

$$P(\nu_\alpha \to \nu_\beta) = \underbrace{2\sum_{i<j} Re[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}] + \sum_{i=j}|U_{\alpha i}|^2|U_{\beta i}|^2}_{\delta_{\alpha\beta}}$$

CP-even

$$- 4\sum_{i<j}\mathrm{Re}[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}]\sin^2\left[\frac{\Delta m^2_{ji}L}{4E}\right]$$

CP-odd

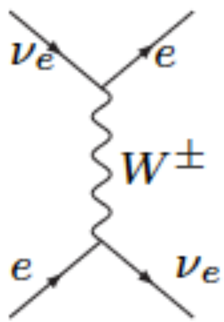
$$- 2\sum_{i<j}\mathrm{Im}[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}]\sin\left[\frac{\Delta m^2_{ji}L}{2E}\right]$$

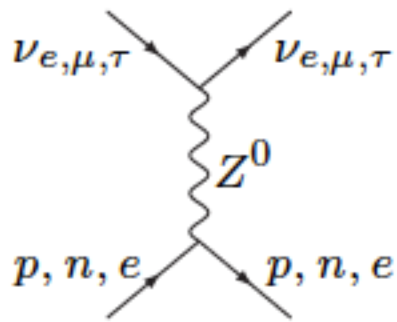
Exercise: check that Majorana phases do not contribute to this.

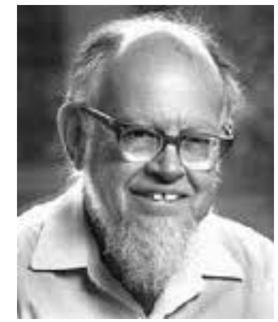
Exercise: do leptons oscillate?
(hint: be precise about what you mean)

Neutrino Oscillations in matter

Many neutrino oscillation experiments involve neutrinos propagating in matter (Earth for atmospheric neutrinos or accelerator experiments, Sun for solar neutrinos)







Wolfenstein

Index of refraction (coherent forward scattering) can strongly affect the oscillation probability

Neutrino Oscillations in matter

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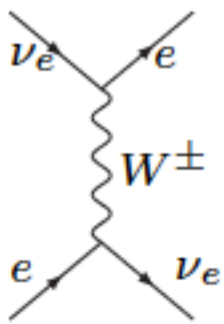
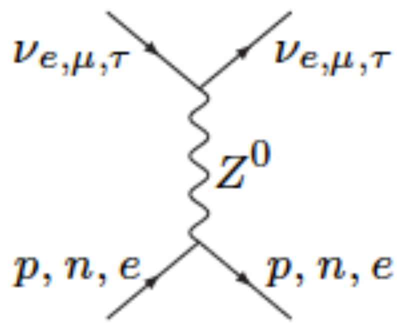




Wolfenstein

Index of refraction (coherent forward scattering) can strongly affect the oscillation probability

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)e] = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma_\mu(1-\gamma_5)e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e]$$

$$\langle\bar{e}\gamma_\mu(1-\gamma_5)e\rangle_{\text{unpol.medium}} = \delta_{\mu 0}N_e$$

Neutrino propagation in matter

$$\langle \mathcal{H}_{CC} + \mathcal{H}_{NC} \rangle_{\rm medium} = \sqrt{2}\, G_F\, \bar{\nu}\, \gamma_0 \begin{pmatrix} N_e - \frac{N_n}{2} & & \\ & -\frac{N_n}{2} & \\ & & -\frac{N_n}{2} \end{pmatrix} \nu \equiv \bar{\nu}\, \gamma_0 V_m \nu$$

$$\mathcal{L} \simeq \bar{\nu} \left(i \partial\!\!\!/ - M_\nu - \gamma_0 V_m \right) \nu + \ldots$$

$$\mathcal{O}(V_m^2, M_\nu^2 V_m)$$

$$E^2 - \mathbf{p}^2 \quad = \quad \pm 2\, V_m E + M_\nu^2$$

Earth: $V_m \simeq 10^{-13} eV \rightarrow 2 V_m E \simeq 10^{-4} eV^2 \left[\frac{E}{1GeV} \right]$

Sun: $V_m \simeq 10^{-12} eV \rightarrow 2 V_m E \simeq 10^{-6} eV^2 \left[\frac{E}{1MeV} \right]$

Oscillations in constant matter density

Effective mixing angles and masses depend on energy

$$\begin{pmatrix} \tilde{m}_1^2 & 0 & 0 \\ 0 & \tilde{m}_2^2 & 0 \\ 0 & 0 & \tilde{m}_3^2 \end{pmatrix} = \tilde{U}^\dagger_{\mathrm{PMNS}} \left(M_\nu^2 \pm 2E \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix} \right) \tilde{U}_{\mathrm{PMNS}}$$

For two families

$$\sin^2 2\tilde{\theta} = \frac{(\Delta m^2 \sin 2\theta)^2}{(\Delta m^2 \cos 2\theta \mp 2\sqrt{2} G_F E N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$

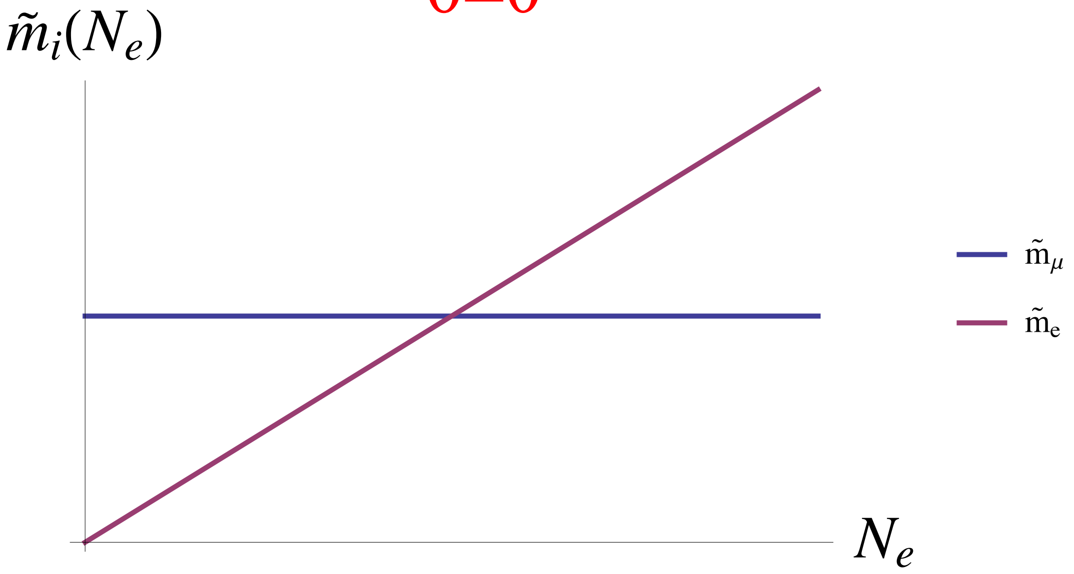
$$\Delta \tilde{m}^2 = \sqrt{(\Delta m^2 \cos 2\theta \mp 2\sqrt{2} G_F E N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\Delta m^2 \cos 2\theta \mp 2\sqrt{2} G_F E_{\mathrm{res}} N_e = 0 \qquad \sin^2 2\tilde{\theta} = 1, \quad \Delta \tilde{m}^2 = \Delta m^2 \sin 2\theta$$

MSW resonance

Mikheyev, Smirnov '85

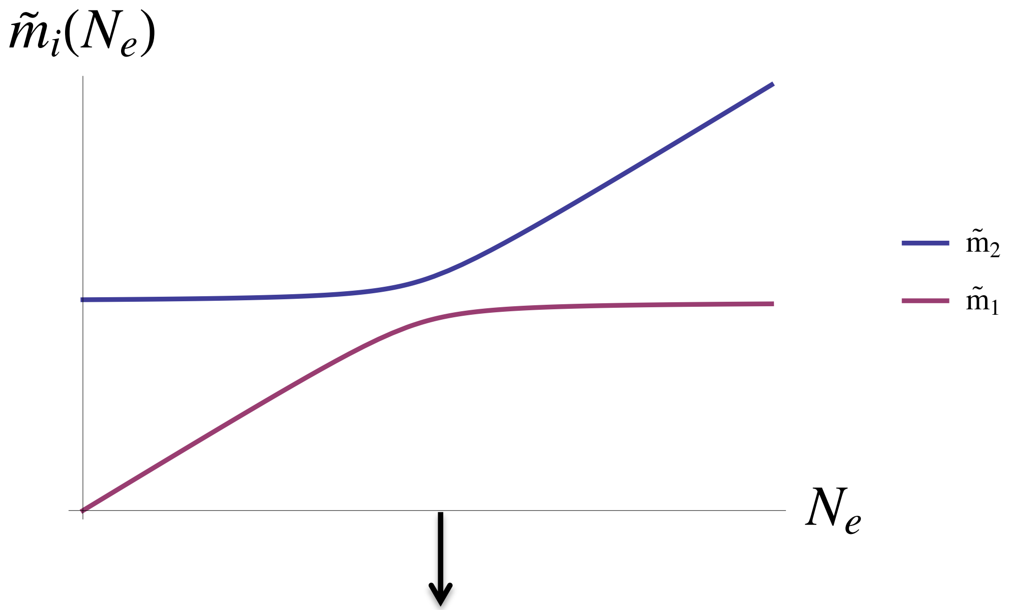
$\theta = 0$



MSW resonance

Mikheyev, Smirnov ‘85

θ≠0



$$\Delta m^2 \cos 2\theta \mp 2\sqrt{2} G_F E N_e^{\text{res}} = 0$$

MSW Resonance:

-Only for ν or $\bar{\nu}$, not both

-Only for one sign of $\Delta m^2 \cos 2\theta$

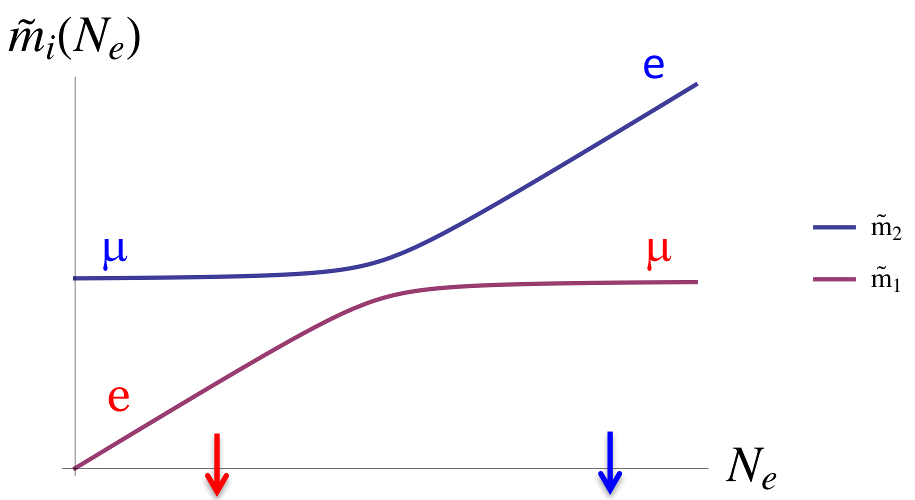
Neutrinos in variable matter

Solar neutrinos propagate in variable matter:

$$N_e(r) \propto N_e(0)e^{-r/R}$$

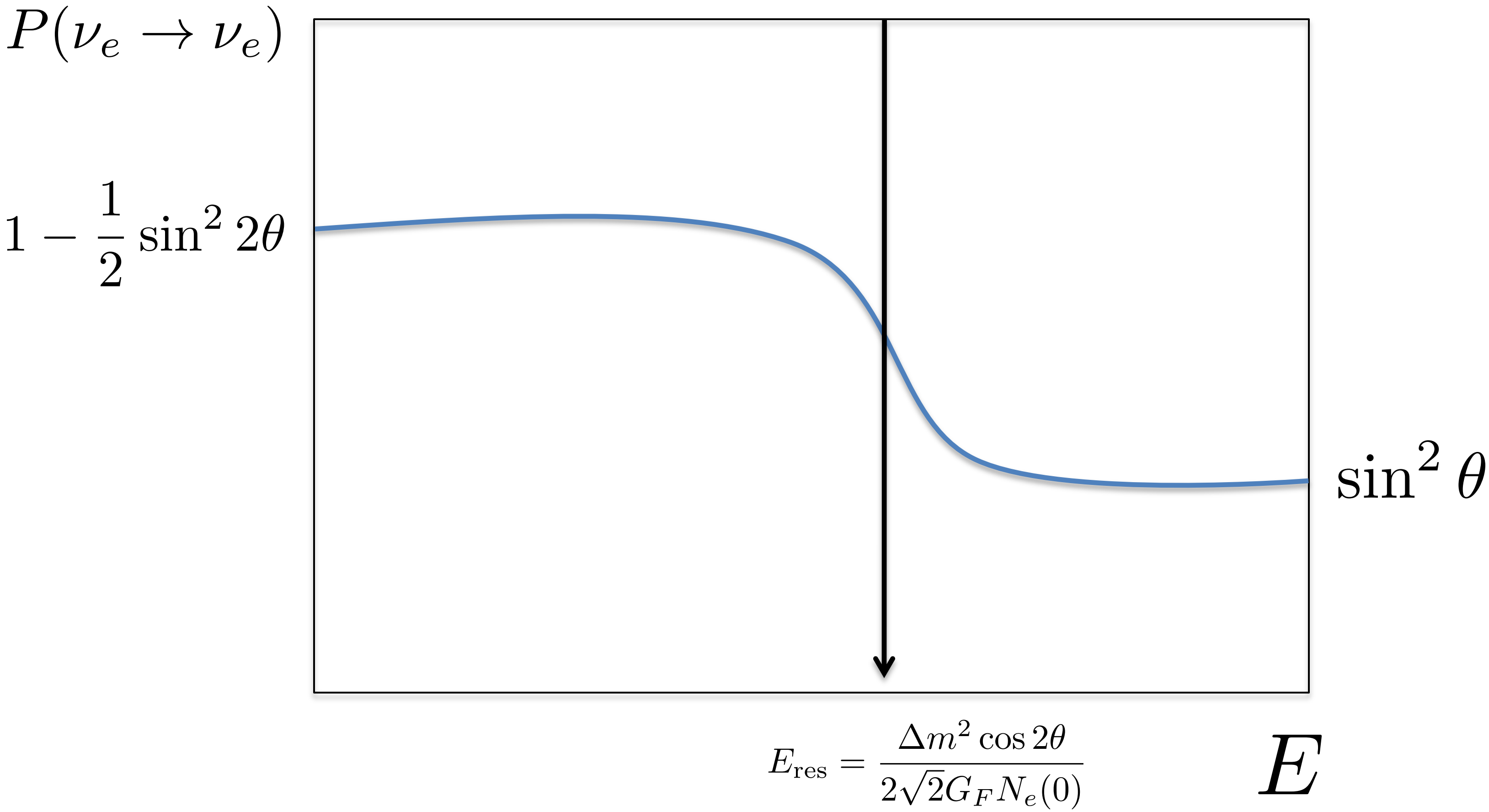
If the variation is slow enough: adiabatic approximation (if a state is at r=0 in an eigenstate $\tilde{m}_i^2(0)$ it remains in the i-th eigenstate until it exits the sun)

$$P(\nu_e \to \nu_e) = \sum_i |\langle \nu_e | \tilde{\nu}_i(\infty) \rangle|^2 |\langle \tilde{\nu}_i(0) | \nu_e \rangle|^2$$



$P(\nu_e \to \nu_e) \simeq 1 - \frac{1}{2}\sin^2 2\theta$ $\quad$ $P(\nu_e \to \nu_e) \simeq \sin^2 \theta$

Solar neutrinos



MSW resonance energy

In most physical situations: piece-wise constant matter or adiabatic approx. good enough