MAJORANA FERMION: LORENTZ INVARIANCE

$$x_\mu \longrightarrow x'_\mu = x_\mu + w_{\mu\nu} x^\nu \qquad S^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\psi(x) \longrightarrow \psi'(x') = e^{-\frac{i}{4} w_{\mu\nu} S^{\mu\nu}} \psi(x)$$

Assume $\psi = \psi^c$, $\psi'^c = \psi'$?

$$\psi'^c(x') = C\gamma_0 \psi'^* = C\gamma_0 e^{\frac{i}{4} w_{\mu\nu} S^{\mu\nu *}} \psi^*(x)$$

Since $\psi = \psi^c = C\gamma_0 \psi^* \Rightarrow \psi^* = \gamma_0 C^{-1} \psi$

$$\psi'^c = C\gamma_0 e^{\frac{i}{4} w_{\mu\nu} S^{\mu\nu *}} \gamma_0 C^{-1} \psi$$

Now, $C\gamma_0 S^{\mu\nu *} \gamma_0 C^{-1} = -S^{\mu\nu}$; $C\gamma_0 S^{\mu\nu *} S^{\mu\nu *} \gamma_0 C^{-1} = C\gamma_0 S^{\mu\nu *} \gamma_0 C^{-1} C\gamma_0 S^{\mu\nu *} \gamma_0 C^{-1} =$

$= S^{\mu\nu} S^{\mu\nu}$

$$C\gamma_0 (S^{\mu\nu *})^N \gamma_0 C^{-1} = (-S^{\mu\nu})^N$$

$$C\gamma_0 e^{\frac{i}{4} w_{\mu\nu} S^{\mu\nu *}} \gamma_0 C^{-1} = e^{-\frac{i}{4} w_{\mu\nu} S^{\mu\nu}}$$

$$\psi'^c = e^{-\frac{i}{4} w_{\mu\nu} S^{\mu\nu}} \psi = \psi'$$

MAJORANA MASS TERM

$$\psi = L\psi_L + R\psi_R \quad [\psi_{L(R)} \equiv L(R)\psi]$$

$$C\,L \equiv i\gamma_2\gamma_0 L = i\gamma_2 R\gamma_0 = i L\gamma_2\gamma_0 = LC$$

$$CR \equiv i\gamma_2\gamma_0 R = i\gamma_2 L\gamma_0 = iR\gamma_2\gamma_0 = RC$$

$$-\mathcal{L}_m = \frac{1}{2}m\left[(\psi_L^T L + \psi_R^T R)C(L\psi_L + R\psi_R) + \frac{1}{2}m(\overline{\psi}_L R + \overline{\psi}_R L)C(R\overline{\psi}_L^T + L\overline{\psi}_R^T)\right.$$

Crossed terms do not survive

$$-\mathcal{L}_m = \frac{1}{2}m\left[\psi_L^T C\psi_L + \psi_R^T C\psi_R + \overline{\psi}_L C\overline{\psi}_L^T + \overline{\psi}_R C\overline{\psi}_R^T\right] =$$

$$= \frac{1}{2}m\left[\overline{\psi_L^c}\psi_L + \overline{\psi}_L\psi_L^c + \overline{\psi_R^c}\psi_R + \overline{\psi}_R\psi_R^c\right]$$

$$\left.\begin{aligned} \psi_1 &\equiv \psi_L + \psi_L^c \\ \psi_2 &\equiv \psi_R + \psi_R^c \end{aligned}\right\} \quad -\mathcal{L}_m = \frac{1}{2}m\overline{\psi}_1\psi_1 + \frac{1}{2}m\overline{\psi}_2\psi_2$$

$$(\overline{\psi}_L\psi_L = \overline{\psi}RL\psi = 0 \,;\; \overline{\psi_L^c}\psi_L^c = 0)$$

ψ_1 and ψ_2 are the fields that propagate

PHASES IN U^{lep}

3×3 matrix $\Rightarrow$ 9 phases

Unitary: $U^{\dagger}U = \mathbb{1}$; $\sum_{\alpha} U^{*}_{i\alpha} U_{\alpha j} = \delta_{ij}$

$i=j \quad \sum_{\alpha} |U_{i\alpha}|^2 = 1$

$i\neq j \quad \sum_{\alpha} U^{*}_{i\alpha} U_{\alpha j} = 0 \Rightarrow$ 3 conditions

$9-3=6$

$$\left.\begin{aligned} l_\alpha &\to l_\alpha e^{i\varphi_\alpha} \\ \nu_i &\to \nu_i e^{i\varphi_i} \end{aligned}\right\} U_{\alpha i} \to e^{i(\varphi_i - \varphi_\alpha)} U_{\alpha i} : 6\ \varphi,\ 5 \text{ non-independent } \Delta\varphi$$

$6-5=1_m$

Majorana: $\cancel{\nu_i \to \nu_i e^{i\varphi_i}}$ $\quad \nu_i = C\gamma_0 \nu_i^{*}$

$U_{\alpha i} \to e^{-i\varphi_\alpha} U_{\alpha i}$: $6-3=3_m$

$U_{\alpha i} = U^{D}_{\alpha i} e^{i\eta_i}$

Neutrino oscillations depend on $\sum_{ij} \underbrace{U_{\beta i} U^{*}_{\alpha i} U^{*}_{\beta j} U_{\alpha j}}_{U^{D}_{\beta i} U^{*D}_{\alpha i} U^{*D}_{\beta j} U^{D}_{\alpha j} \cancel{e^{i\eta_i}} \cancel{e^{-i\eta_i}} \cancel{e^{-i\eta_j}} \cancel{e^{i\eta_j}}} e^{i\frac{\Delta m^2_{ji} L}{2E}} = P_{\alpha\beta}$

Majorana phases do not enter! (Alternatively $H_{osc} = \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^{\dagger}$)

OSCILLATIONS

$$P_{\alpha\beta} \sim \sin^2 \frac{\Delta m^2 L}{4E} \sim \sin^2\left(1'27 \frac{\Delta m^2 (eV^2) L(K)}{E(GeV)}\right)$$

E_ν? Nuclear reactor: $E \sim MeV = 10^{-3}\,GeV$

$\left(L \sim \frac{4E}{\Delta m^2}\right)$

$$\boxed{L \sim Km}$$

$$N_{evts} = \phi \cdot \sigma \cdot N_{targets} \cdot \Delta t \sim 10\,\nu; \quad N_{targets} \sim \frac{M}{10^{-3} kg / 10^{24}} = 10^{27} \frac{M}{kg}$$

$$\Delta t \sim 1\,day \sim 24\,hours \sim 10^5\,s$$

$$M \sim \frac{10}{\frac{\phi_0}{4\pi L^2} \sigma \Delta t} m_{NUCLEON}$$

$\left(\frac{E}{G_F}\right)^2$

$$\phi \sim \frac{10^{20}\,\nu/s}{4\pi (10^3 m)^2} \sim 10^{13} \frac{\nu}{m^2 s}$$

$$\sigma \sim 10^{-44}\,cm^2 \sim 10^{-48}\,m^2$$

$$10^{13} \frac{\nu}{\cancel{m^2} \cdot \cancel{s}} 10^{-48} \cancel{m^2}\, 10^{27} \frac{M}{kg}\, 10^5 \cancel{s} \sim 10$$

$$10^{-3} \frac{M}{kg} \sim 10 \Rightarrow \boxed{M \sim 10^4\,kg}$$

$$N^i_{evts} = \int_{E_0^i}^{E_1^i} dE_\nu^{rec} \int_0^\infty dE_\nu \frac{d\phi}{dE_\nu}(E_\nu)\, \sigma(E_\nu)\, N_{targets}\, \Delta t\;\; P(\nu_e \to \nu_e)\; \underbrace{R_{ENERGY}(E_\nu, E_\nu^{rec})}_{\propto e^{-\frac{E_\nu - E_\nu^{rec}}{2\sigma_E^2}}} + N^i_{bkg}$$

$\Rightarrow$ Relevant for Δm^2

Normalization: $\frac{d\phi}{dE_\nu}(E_\nu) \longrightarrow (1+\eta)\frac{d\phi}{dE_\nu}(E_\nu) \Rightarrow$ Relevant for θ

$N^i_{bkg} \longrightarrow (1+\eta^B) N^i_{bkg} \Rightarrow$ Relevant for θ

Energy scale: $E_0^i \longrightarrow (1+\xi) E_0^i$, $E_1^i \longrightarrow (1+\xi) E_1^i$ } Relevant for Δm^2

MSW & STERILES

$$E_{res} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F N_e} \sim \mathrm{TeV} \left(\frac{\Delta m^2}{\mathrm{eV}}\right)^2 \left(\frac{g/cm^3}{\rho}\right) \cos 2\theta$$

$$N_e = N_p \sim \frac{\rho}{2 m_p}$$

Atmospheric flux at TeV energies $\sim 10^{-8} \frac{\nu}{\mathrm{TeV\, cm^2\, s\, sr}}$