DARK MATTER INVISIBLES '21 SCHOOL:

[Yonit Hochberg, HUJI]

Hi!! Rough plan: 1.5h + 1.5h + 1.5 + 1.5h + 1h

Ballpark: Intro + mechanisms + models + detection

I Outline:

- Intro
- cheat sheet - early universe cosmology
- mechanism - $2 \to 2$ (WIMP)

⋮

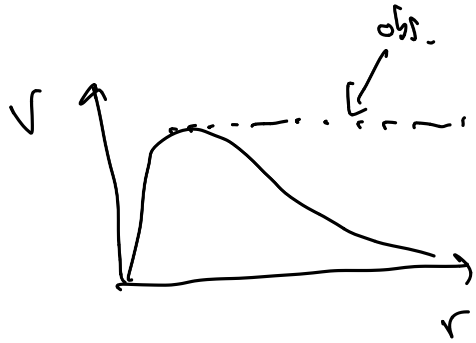
What we know & don't know:

Dark Matter (DM) = 27% of the energy content of the universe. $\Rightarrow$ $\rho_{DM} \simeq 5\, \rho_{baryons}$

How do we know?

* Motion of stars:

look vs. motion.



Galaxy rotation curve:

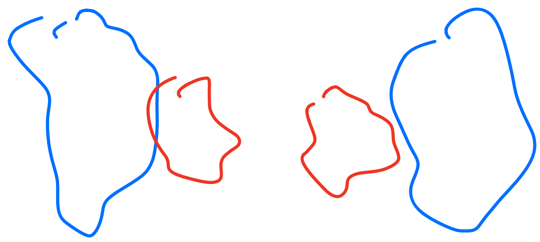
Something is out there, permeating the galaxy, extending to the halo.

* **lensing:**

Cheshire Cat group.

Something bending & twisting light.

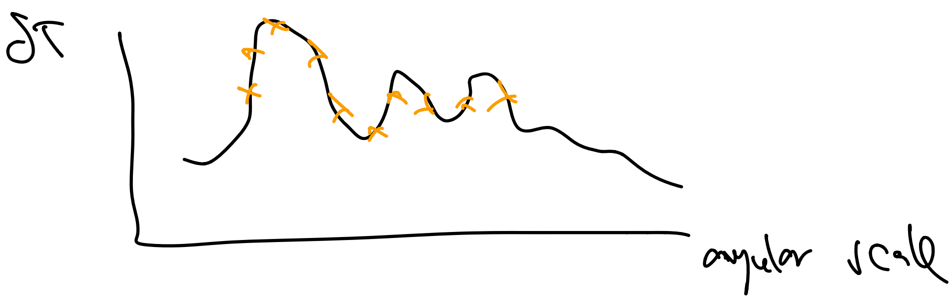
* **Colliding clusters (Bullet cluster):**



gravitational center

visible center

* **CMB:** Power spectrum - Temp fluctuations vs. angular scales -



[Roni Harnik, MITP school, DM - lesson #1 - evidence for DM]

Properties:

- universe is dark - $\Omega_{DM} = \frac{\rho_{DM}}{\rho_c} = 0.27$

$\longleftrightarrow \quad \rho_{DM} \sim 5\,\rho_{baryons}$

locally, $\rho_{DM} = 0.3 \frac{GeV}{cm^3}$

- massive - ($m = ????$)

too fluffy: wavelength exceeds galaxy size

- Shouldn't interact too strongly w/ QED & QCD
- Not too strongly w/ itself [$\leftrightarrow$ signal? see later]
- Wouldn't be here without DM!

Picture:

rotating pita with a falafel ball.

DM wind - $v \sim 10^{-3} c \sim 10^{-3}$
natural units

What to know: what is it? mechanism in early universe to get abundance? Model building / how to detect? (+ what are constraints)

Dark matter cheat-sheet?

Universe is expanding $\ell \nearrow \Rightarrow \tilde{\ell} = a \ell \nearrow$ a = scale factor

volume expanding as a^3

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2$$

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{a}\frac{\partial a}{\partial t}$$ Hubble. [convention: $a(t_0) = 1$ today]

1st Friedman eq: $H^2 = \frac{\rho}{3M_{pl}^2}$, $\rho \propto T^4$ black body

$$\Rightarrow H \sim \frac{T^2}{M_{pl}}$$

Early universe = thermal environment. For species in th. equilibrium : phase space distribution

$$f_{eq}(p) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$\oplus$: fermions

$\ominus$: bosons

$\Rightarrow$ number density : $n = g \int \frac{d^3p}{(2\pi)^3} f(p)$

energy density : $\rho = g \int \frac{d^3p}{(2\pi)^3} E \, f(p)$

g = # internal dof (spin, polarization, color, etc)

$\Rightarrow$ Ballpark : $\begin{cases} n \sim T^3 & \text{(Rel)} \\ \rho \sim T^4 & \text{(Rel)} \end{cases}$

$$\begin{cases} n = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T} & \text{NR } (m \gg T) \\ \rho = \left(m + \frac{3}{2}T\right) n \sim mn & \text{NR} \end{cases}$$

In particular, when $\mu = 0$ — $n \propto e^{-m/T}$

exp suppression of n!

entropy density : $S = \frac{2\pi^2}{45} g_{*s} T^3 \sim T^3$

$$g_{*s} = \sum_{\text{boson}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

$$\rho = g_* \frac{\pi^2}{30} T^4 , \qquad g_* = \sum_b g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_f g_i \left(\frac{T_i}{T}\right)^4$$

Boltzmann Equation:

Consider a system w/o collisions - free particles:

$$\frac{\partial N}{\partial t} = 0 \qquad\qquad N = n \cdot V$$

$$\frac{\partial (nV)}{\partial t} = 0 = V \frac{\partial n}{\partial t} + n \frac{\partial V}{\partial t}$$

$$\Rightarrow \quad \frac{\partial n}{\partial t} + \frac{n}{V} \frac{\partial V}{\partial t} = 0$$

$$\left(V \propto a^3 \ : \ \frac{1}{V} \frac{\partial V}{\partial t} = \frac{3}{a} \frac{\partial a}{\partial t} \right)$$

$$\Rightarrow \quad \frac{\partial n}{\partial t} + 3n \underbrace{\left(\frac{\dot a}{a}\right)}_{\equiv H} = 0$$

$$\boxed{\frac{\partial n}{\partial t} + 3Hn = 0} \ \text{(free)}$$

If have collisions - RHS:

collision term

$$\frac{\partial n}{\partial t} + 3Hn = -C[n]$$

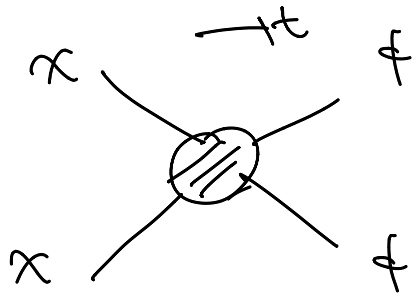
MECHANISMS:

Types of processes in early universe that set the relic abundance of DM.

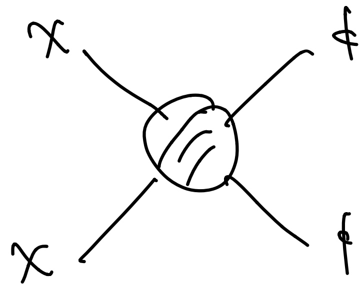
The 2→2 zoo:

DM = χ $\longrightarrow$ time

[starting point - beyond later]

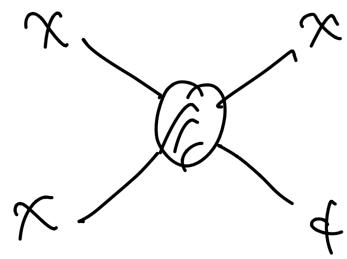


WIMP (ann.)

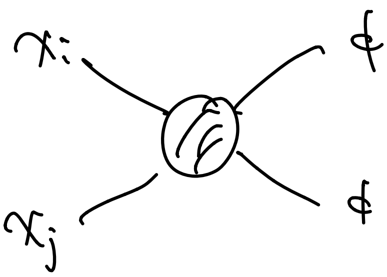


$m_\chi < m_f$

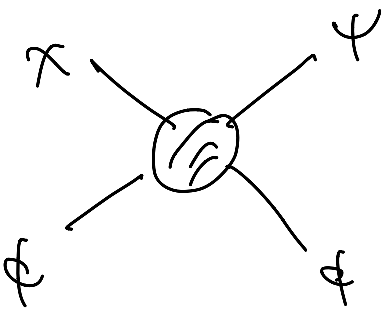
Forbidden



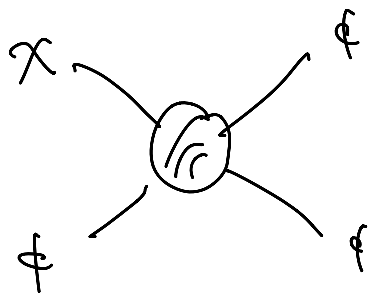
semi-annihilation



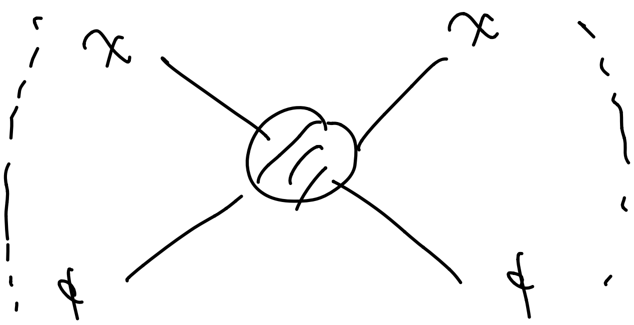
coannihilations



co-scattering



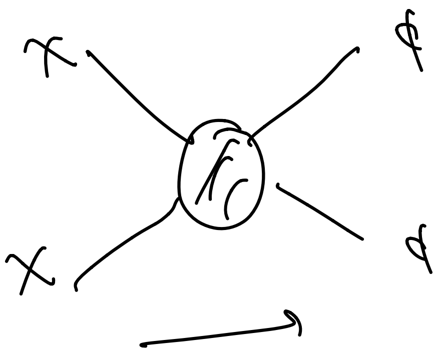
zombie



ELDER

WIMP: Star of the show for 40^+ years.

$\chi\chi \to \phi\phi$



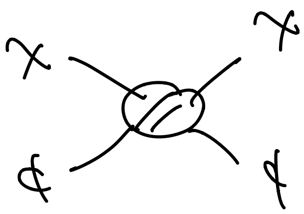
time

if fast, $\mu_\chi = \mu_\phi$

$$\frac{\partial n_\chi}{\partial t} + 3 n_\chi H = \cancel{0}\ C[n_\chi]$$

Let's write out collision terms:

(Sidenote: also elastically scatter $\chi\phi \to \chi\phi$



& assume ϕ = some bath particle (SM, or thermalized w/ SM).

$\mu_\phi = 0$ & has T.

$\Rightarrow \mu_\chi = \mu_\phi = 0$)

$$C[n] = \int d\Pi_{\chi_1}\, d\Pi_{\chi_2}\, d\Pi_{\phi_1}\, d\Pi_{\phi_2}\, (2\pi)^4 \delta^{(4)}(p_i - p_f) \cdot \overline{|M|^2} \left(f_{\chi_1} f_{\chi_2} - f_{\phi_1} f_{\phi_2} \right)$$

(averaged over all initial & final dof)

where $d\Pi_a = g_a \dfrac{d^3 p_a}{(2\pi)^3\, 2E_a}$ = Lorentz inv phase space.

Makes sense? [M] ✓ ff ✓

Roughly : thermally averaged cross section × number density.

write in following way :

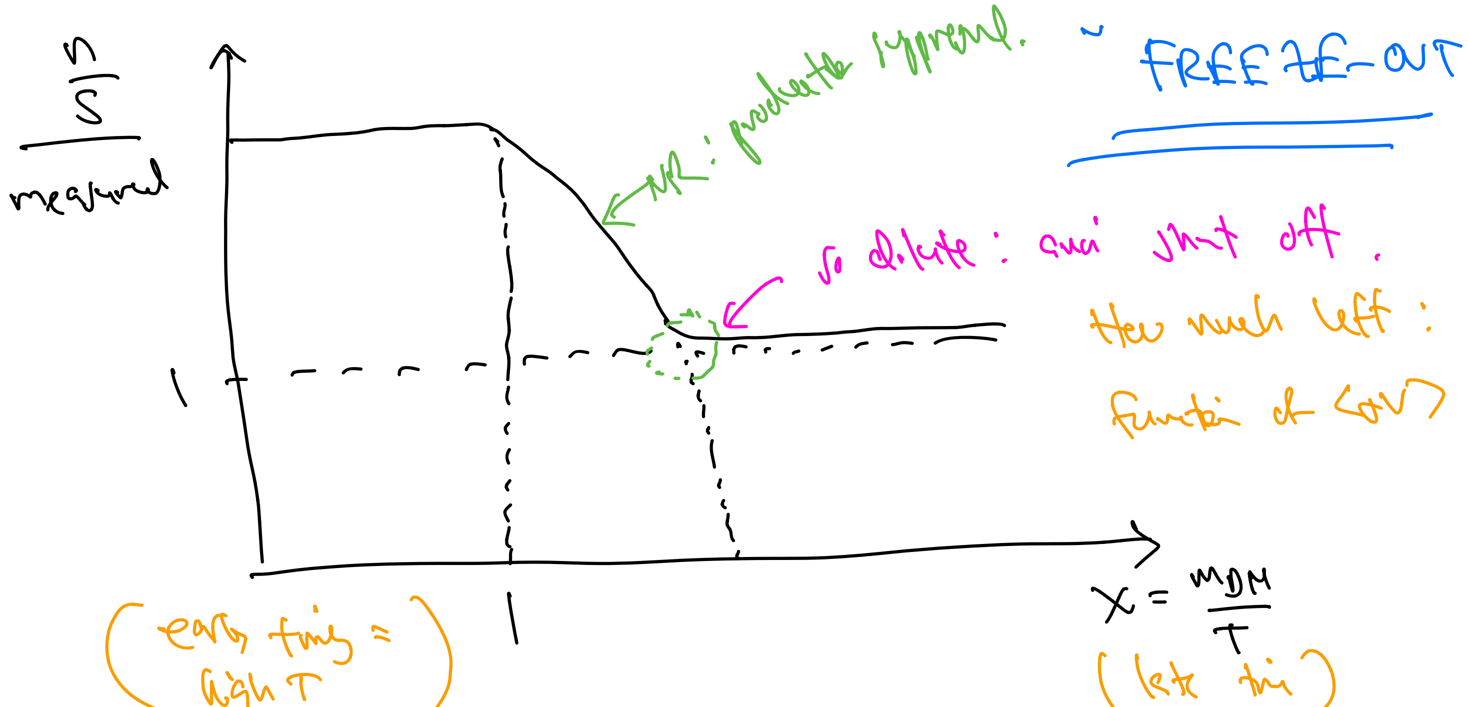
$$\frac{\partial n_\chi}{\partial t} + 3 n_\chi H = -\langle \sigma v \rangle_{\chi\chi \to f\bar{f}} \left(n_\chi^2 - n_\chi^{eq\,2} \right)$$

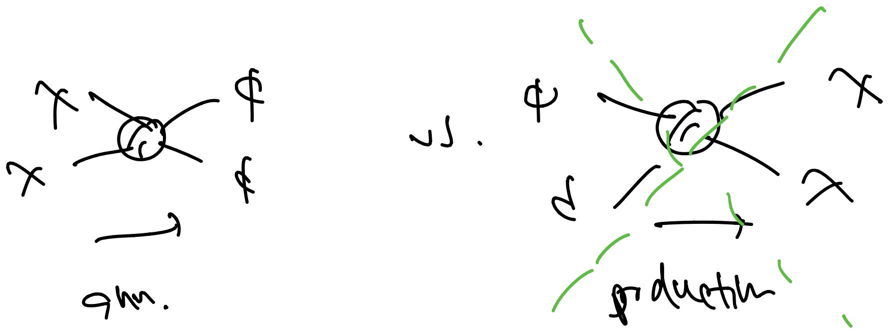
Use trick : Detailed Balance !

In eq., forward & backward process both rapid & should cancel out !

What happens? Instead of density in a box → think of density of particles in a box that's expanding w/ the universe : (yield $Y = \frac{n}{S} \sim n a^3$)

" FREEZE-OUT "





Back of envelope: F.O. @ $\Gamma_{2\to 2} \sim H$

$$(*)\quad \Gamma_{2\to 2} \sim n_\chi^F \langle \sigma v \rangle \sim H \sim \frac{T^2}{M_{pl}}$$

(think of what a DM particle needs to happen)

relate to measured quantities!

Redshifts: entropy S conserved: $S = s a^3 = \text{constant}$

use this to redshift $\quad s \sim \frac{1}{a^3} \sim T^3$

$$\left(T \propto \frac{1}{a}\right)$$

After F.O., comoving number density of DM is constant.

redshift from today to F.O.:

$$(**)\quad n_\chi(x_0) = n_\chi(x_F)\,\frac{s(x_0)}{s(x_F)} = \frac{H(x_F)}{\langle \sigma v \rangle}\,\frac{s(x_0)}{s(x_F)}$$

DM NR today, so energy density:

$$\text{today}\quad \Omega_{DM} = \Omega_\chi = \frac{\rho_\chi}{\rho_c} = \frac{m_\chi n_\chi}{\rho_c} \underset{(**)}{=} \frac{m_\chi}{\rho_c}\,\frac{H(x_F)}{\langle \sigma v \rangle}\,\frac{s(x_0)}{s(x_F)}\Big]_0 =$$

$$H \sim \frac{T^2}{M_{pl}} = \frac{m_\chi^2}{x_f^2 M_{pl}}$$

$$\uparrow \quad x = \frac{m}{T}$$

$$s(x_f) \sim T_f^3 \sim \frac{m_\chi^3}{x_f^3}$$

$$s_0 = 2.2 \cdot 10^{-38}\ \mathrm{GeV}^3$$

$$\rho_c = 1.053 \cdot 10^5\, h^2 \frac{\mathrm{GeV}}{c^2}\, \mathrm{cm}^{-3} \quad (h \sim 0.7)$$

$$\Rightarrow \Omega_{DM} = \frac{x_f}{20} \left(\frac{10.75}{g_*}\right)^{\frac{1}{2}} \frac{10^{-9}\ \mathrm{GeV}^{-2}}{\langle \sigma v \rangle}$$

What value is x_f?

Assume instantaneous F.O. – plug in eq. density for χ's @ F.O.:

$$\Rightarrow n_\chi^{eq}(x_f) \langle \sigma v \rangle \sim H(x_f)$$

$$g_\chi \left(\frac{m_\chi^2}{x_f 2\pi}\right)^{3/2} e^{-x_f} \langle \sigma v \rangle \sim \frac{T_f^2}{M_{pl}} \sim \frac{m_\chi^2}{x_f^2 M_{pl}}$$

$$\Rightarrow x_f \sim \ln(\text{parameters})$$

F.O. temperature (x_f) roughly the same over broad range of parameters.

$\Rightarrow x_F \sim 20-30$ for $m_\chi \sim$ MeV - TeV

DM F.O. NR, $T \sim m/20$.

Correct relic abundance, $\Omega_\chi = 0.27$

$$\Rightarrow \boxed{\langle \sigma v \rangle \sim 10^{-8}\ \text{GeV}^{-2}}$$

Roughly what get for weak force interaction!

"WIMP miracle"

Alternative back-of-envelope: Redshift to T_{eq} ($\sim$ eV)

matter-radiation equality

parameterize $\langle \sigma v \rangle \equiv \dfrac{\alpha_{eff}^2}{m_\chi^2}$

@ matter-rad. equality: $\rho_{matter}^{eq.} = \rho_\chi^{eq} + \rho_b^{eq} = \rho_\gamma^{eq}$.

$$\rho_\chi \simeq 5 \rho_b \Rightarrow \rho_\chi^{eq} \sim \rho_\gamma^{eq}$$

$$\Rightarrow n_\chi^{FO} \sim n_\chi(T_{eq}) \left(\frac{T_F}{T_{eq}}\right)^3 \sim \frac{\rho_\chi(T_{eq})}{m_\chi} \frac{T_F^3}{T_{eq}^3} \sim$$

$$\sim \frac{\rho_\gamma^{eq}}{m_\chi} \frac{T_F^3}{T_{eq}^3} \underset{\rho_\gamma \sim T^4}{\sim} \frac{T_F^3\, T_{eq}}{m_\chi} \sim \frac{m_\chi^2}{x_F^3} T_{eq}$$

plug into F.O. condition (*):

$$\Gamma_{\chi\to 2} = n_\chi^{FO}\cdot\frac{\alpha_{eff}^2}{m_\chi^2} \sim \frac{\cancel{m_\chi}}{x_F^3}\,T_{eq}\cdot\frac{\alpha_{eff}^2}{\cancel{m_\chi^2}} \overset{!}{\sim} H_F \sim$$

$$\sim \frac{T_F^2}{M_{pl}} \sim \frac{m_\chi^2}{x_F^2 M_{pl}}$$

$$\Rightarrow \boxed{m_\chi \sim \alpha_{eff}\sqrt{T_{eq}M_{pl}}} \boxed{\sim \alpha_{eff}\cdot(30\ \text{TeV})}$$

If $\alpha_{eff}\sim 10^{-2}$ (weak int), weak scale emerges!

Coincidence of scales $M_{pl} \leftrightarrow T_{eq}$

If $\alpha_{eff} \ll 10^{-2}$, $m_\chi \ll$ weak scale.

Another way to write: $\langle\sigma v\rangle = \dfrac{\alpha_{eff}^2}{m_\chi^2} \sim \dfrac{1}{T_{eq}M_{pl}}$

unitarity bound: $\alpha_{eff} \lesssim 4\pi \to m_\chi \lesssim 300$ TeV

[Griest & Kamionkowski 1989]

ways to evade — thermal relics w/ mass that exceeds this by $\lesssim 10$ orders of magnitude!

[Superheavy thermal DM, Kuflik & Kim, PRL]

1906.00981

[zombies – Kramer, Kuflik et al, PRL, 2003.04900]

$m_{DM} \gtrsim$ keV for above ($\lesssim$ 300 TeV + caveats)