QCD jet production at a high energy muon collider

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In collaboration with **Tao Han** and **Keping Xie**

[T. Han, Y. Ma, K.Xie 2007.14300]

[T. Han, Y. Ma, K.Xie upcoming]

A little bit backaground

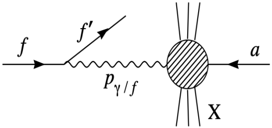
"Equivalent photon approximation (EPA)"

[C. F. von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the electron [E. J. Williams, Phys. Rev. 45, 729 (1934)]

$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2}{m_\ell^2}$$



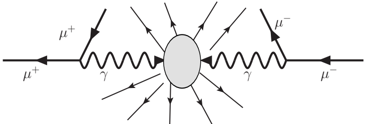
Applications at muon collider

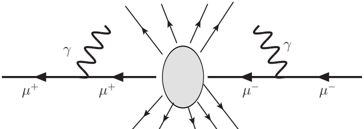
- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

- Partonic luminosities

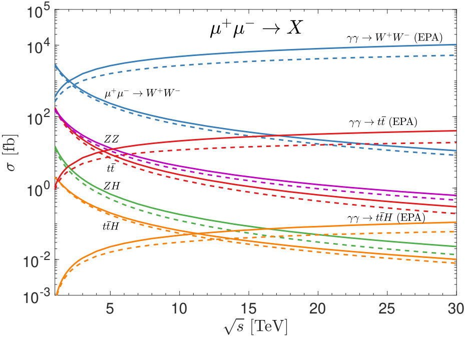
$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}}\int_\tau^1 \frac{d\xi}{\xi}\left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j)\right]$$





A high-energy muon collider at first glance (I)

What do people expect from a muon collider?



[T. Han, Y. Ma, K.Xie 2007.14300]

Some "commonsense":

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln\left(s/m_\mu^2\right)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$,
- $\gamma\gamma \to W^+W^-$ production has the largest cross section.

A high-energy muon collider at first glance (II)

What are the dominant processes at a high-energy muon collider?

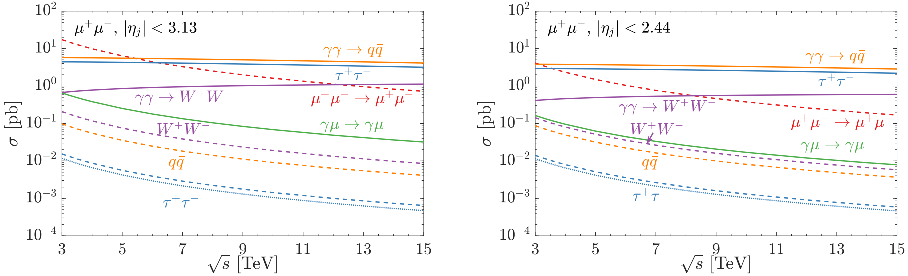
- Leading-order: $\mu^+\mu^- \to \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, W^+W^-$, and $\gamma\mu \to \gamma\mu$
- $\gamma\gamma$ scatterings: $\gamma\gamma \to \tau^+\tau^-, q\bar{q}, W^+W^-$

Need some cuts:

- Detector angle: $\theta > 5^\circ\,(10^\circ) \Longleftrightarrow |\eta| < 3.13(2.44)$
- Threshold: $m_{ij} > 20$ GeV
- Need a p_T cut to get rid of the nonperturbative hadronic production

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

$$p_T > \left(4 + \sqrt{s}/3\,\text{TeV}\right)\,\text{GeV}$$

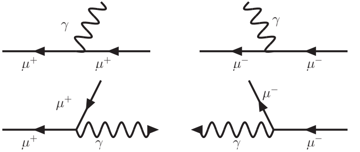


Go beyond the EPA at a high-energy muon collider

We have been doing:

- $\mu^+\mu^-$ annihilation

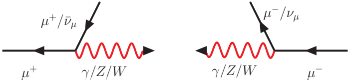
- EPA and ISR



- "Effective W Approx." (EWA)

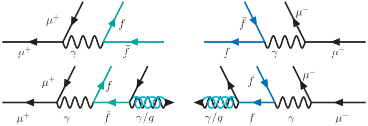
[G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

[S. Dawson, NPB 249 (1985) 42]



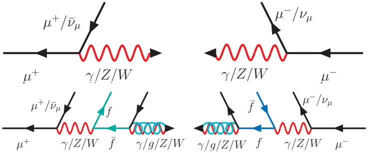
We will add:

- Above $\mu_{\rm QCD}$: QED$\otimes$QCD
  q/g emerge



- Above $\mu_{\rm EW} = M_Z$: EW$\otimes$QCD
  EW partons emerge

[T. Han, Y. Ma, K.Xie 2007.14300]



In the end, everything is parton, i.e. the full SM PDFs.

The PDF evolution

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\mathrm{QCD}}$: QED
  - $Q = \mu_{\mathrm{QCD}} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\mathrm{QCD}} < Q < \mu_{\mathrm{EW}}$: QED⊗QCD
  - $Q = \mu_{\mathrm{EW}} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\mathrm{EW}} < Q$: EW⊗QCD.

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

Decomposition into singlet and non-singlet PDFs

The singlets

$$f_L = \sum_{i=e,\mu,\tau}(f_{\ell_i}+f_{\bar{\ell}_i}),\ f_U = \sum_{i=u,c}(f_{u_i}+f_{\bar{u}_i}),\ f_D = \sum_{i=d,s,b}(f_{d_i}+f_{\bar{d}_i})$$

The non-singlets

- The only non-trivial singlet $f_{e,NS} = f_e - f_{\bar{e}}$
- the leptons

$$f_{\ell_i,NS} = f_{\ell_i} - f_{\bar{\ell}_i}(i=2,3),\ f_{\ell,12} = f_{\bar{e}} - f_{\bar{\mu}},\ f_{\ell,13} = f_{\bar{e}} - f_{\bar{\tau}};$$

- the up-type quarks

$$f_{u_i,NS} = f_{u_i} - f_{\bar{u}_i},\ f_{u,12} = f_u - f_c;$$

- and the down-type quarks

$$f_{d_i,NS} = f_{d_i} - f_{\bar{d}_i},\ f_{d,12} = f_d - f_s,\ f_{d,13} = f_d - f_b.$$

Reconstruction:

$$f_e = \frac{f_L + (2N_\ell - 1)f_{e,NS}}{2N_\ell},\ f_{\bar{e}} = f_\mu = f_{\bar{\mu}} = f_\tau = f_{\bar{\tau}} = \frac{f_L - f_{e,NS}}{2N_\ell}.$$

$$f_u = f_{\bar{u}} = f_c = f_{\bar{c}} = \frac{f_U}{2N_u},\ f_d = f_{\bar{d}} = f_s = f_{\bar{s}} = f_b = f_{\bar{b}} = \frac{f_D}{2N_d}.$$

The QED⊗QCD DGLAP evolution

- The singlets and gauge bosons

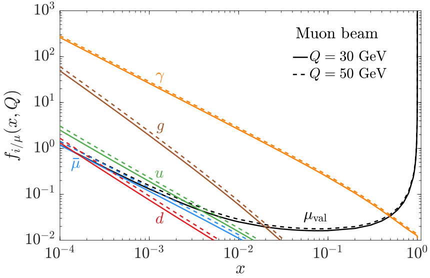
$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

- The non-singlets

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.$$

- The PDFs: $f_{\mu_{\mathrm{val}}}, f_\gamma, f_{\ell_{\mathrm{sea}}}, f_q, f_g$
- Scale uncertainty: 20% for f_g
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x)\mathrm{d}x$, $\sum_i \langle x_i \rangle = 1$

$Q(\mu^\pm)$	μ_{val}	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062



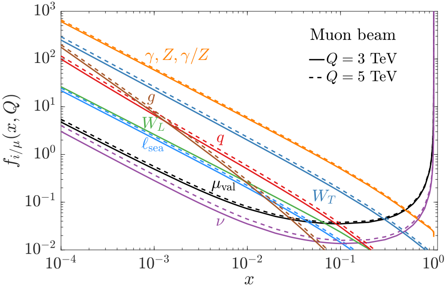
EWPDFs at a muon collider

- The sea leptonic and quark PDFs

$$\nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ \ell\text{sea} = \bar{\mu} + \sum_{i \neq \mu} (\ell_i + \bar{\ell}_i), \ \ q = \sum_{i=d}^{t} (q_i + \bar{q}_i)$$

  There is flavor change due to the EW sector
- The averaged momentum fractions $\langle x_i \rangle$ [percentage]

Q	μ	$\gamma, Z, \gamma Z$	$W^\pm$	ν	ℓsea	q	g
M_Z	97.9	2.06	0	0	0.028	0.035	0.0062
3 TeV	91.5	3.61	1.10	3.59	0.069	0.13	0.019
5 TeV	89.9	3.82	1.24	4.82	0.077	0.16	0.022



- **All SM particles are partons**

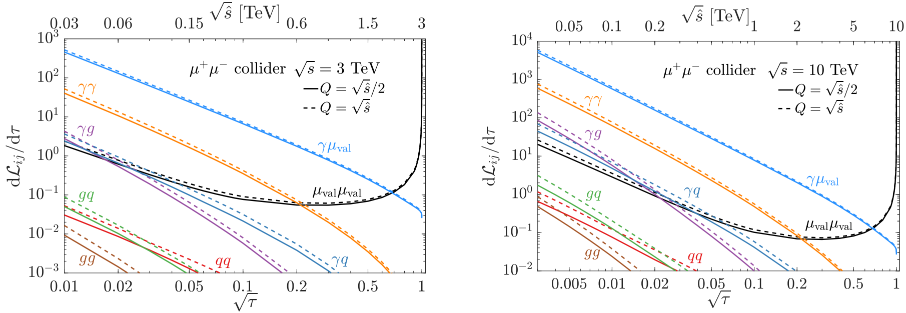
  [T. Han, Y. Ma, K.Xie 2007.14300]
- W_L does not evolve: **Bjorken-scaling restoration**.
- The EW correction is not small: $\sim 100\%$ for $f_{d/\mu}$ due to **relatively large SU(2) gauge coupling**.
- Scale uncertainty: $\sim 20\%$ between $Q = 3$ TeV and $Q = 5$ TeV

Parton luminosities at a possible muon collider

Consider a 3 TeV and a 10 TeV machine

- Partonic luminosities for

$$\mu^+\mu^-, \gamma\mu, \gamma\gamma, qq, \gamma q, \gamma g, gq, \text{ and } gg$$



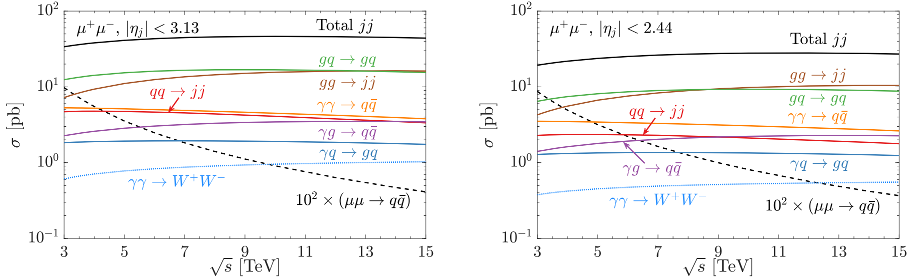
- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 20\%$ of the $\gamma\gamma$ one
- The partonic luminosities of qq, gq, and gg are $\sim 0.5\%$ of the $\gamma\gamma$ one
- Given the stronger QCD coupling, **sizable QCD cross sections are expected**.
- Scale uncertainty is $\sim 20\%(\sim 50\%)$ for photon (gluon) initiated processes.

Jet production at a possible muon collider

- Low-p_T range: photon induced non-perturbative hadronic production $\sim 10^4$ pb. [T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]
- High-p_T range $[p_T > (4 + \sqrt{s}/3\,\text{TeV})\ \text{GeV}]$: perturbatively computable

$$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$$
$$qq \to qq\ (gg),\ gq \to gq \text{ and } gg \to gg\ (q\bar{q}).$$

- $Q = \sqrt{\hat{s}}/2$, due to large $\alpha_s \ln(Q^2)$, a $30 \sim 40\%$ enhancement if $Q = \sqrt{\hat{s}}$

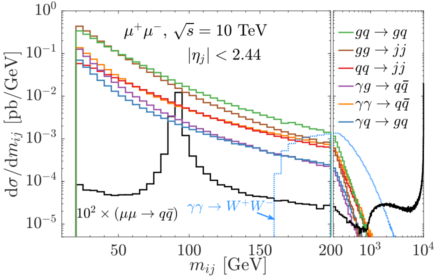


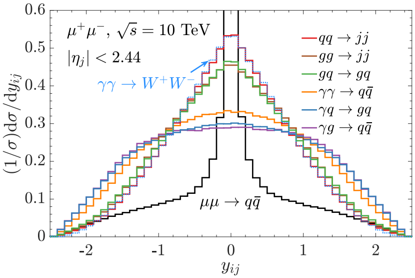
- Including the QCD contribution leads to much larger total cross section.
- gg initiated cross sections are large for its large multiplicity;
- gq initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections here are smaller than the EPA results.

Di-jet distributions at a muon collider

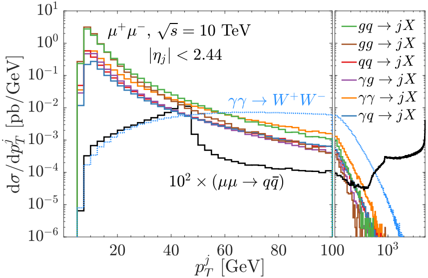
Rather a conservative set up: $\theta = 10^\circ$

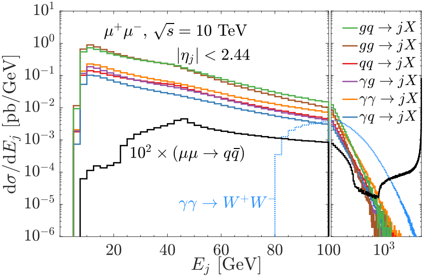
- Some physics:
  Two different mechanisms: $\mu^+\mu^-$ **annihilation** VS **Fusion processes**
  - Annihilation is more than 2 orders of magnitude smaller than fusion process.
  - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
  - Fusion processes peak near m_{ij} threshold.
  - Annihilation is very central, spread out due to ISR;
  - Fusion processes spread out, especially for γq and γg initiated ones.

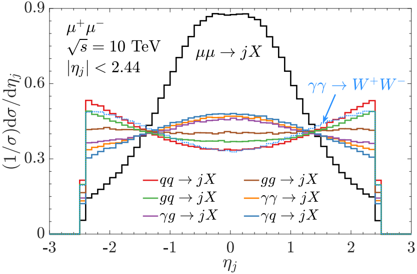




Inclusive jet distributions at a muon collider







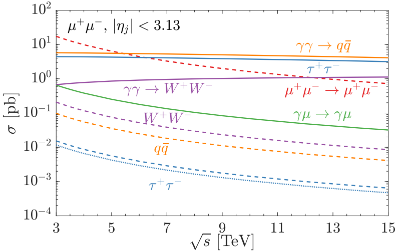
- Jet production dominates over WW production until $p_T > 60$ GeV;
- WW production takes over around energy ~ 200 GeV.
- QCD contributions are mostly forward-backward; $\gamma\gamma$, γq, and γg initiated processes are more isotropic.

Rewrite the story

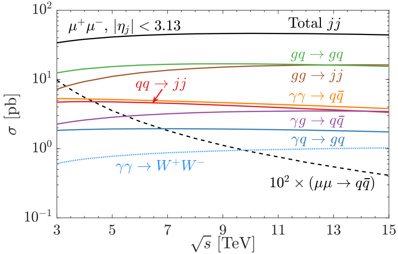
What is the dominant process at a high-energy muon collider?

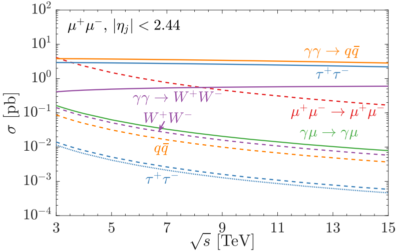
- **Quark/gluon initiated jet production dominates**

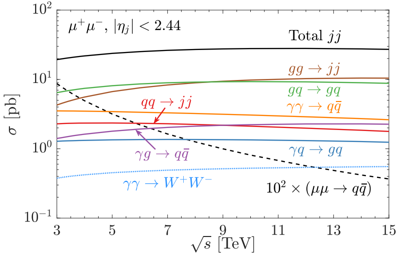
Before:



After:







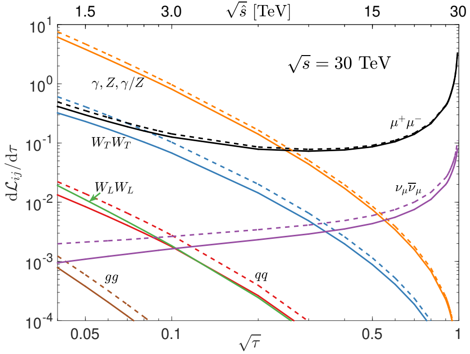
The EW parton luminosities

- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

- Partonic luminosities

$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$

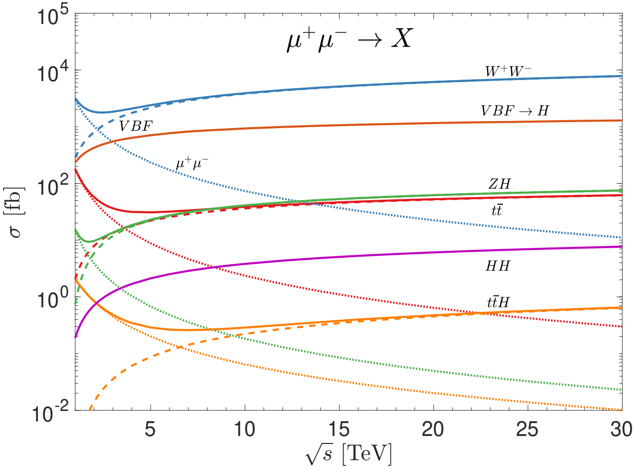


[T. Han, Y. Ma, K.Xie 2007.14300]

Semi-inclusive processes

Just like in hadronic collisions:

$$\mu^+\mu^- \to \text{exclusive particles} + \text{remnants}$$

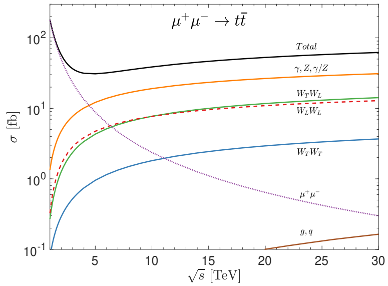


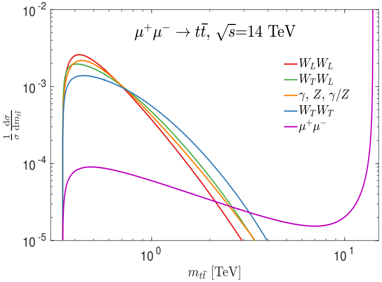
[T. Han, Y. Ma, K.Xie 2007.14300]

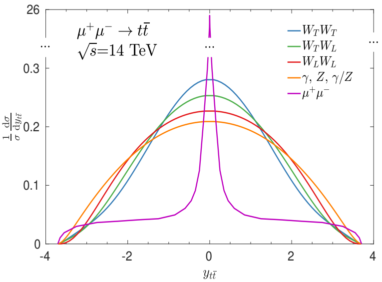
Summary and prospects

- At very high energies, the collinear splittings dominate. All SM particles should be treated as partons that described by proper PDFs.
- The large collinear logarithm needs to be resummed via solving the DGLAP equations. QCD partons (quarks and gluons) enter the picture via the DGLAP evolution.
- When $Q > M_Z$, the EW partons are activated. The light QED$\otimes$QCD partons may receive big corrections due to the large SU(2) gauge coupling.
- At a high energy muon collider, the fusion processes exceed the $\mu^+\mu^-$ annihilation. The photon-photon collision processes take over the leading order processes, e.g. Bhabha and Compton scatterings.
- The main background at a high energy lepton collider is the jet production:
  - In the low p_T range, the nonperturbative photon-photon initiated hadronic production dominates, which is extensively studied at CLIC. This is a few 10^4 pb level.
  - In the high p_T range, the quark and gluon initiated jet production dominates, which is around one order of magnitude larger than the $\gamma\gamma$ initiated processes.

The decomposition and distributions for $t\bar{t}$ production

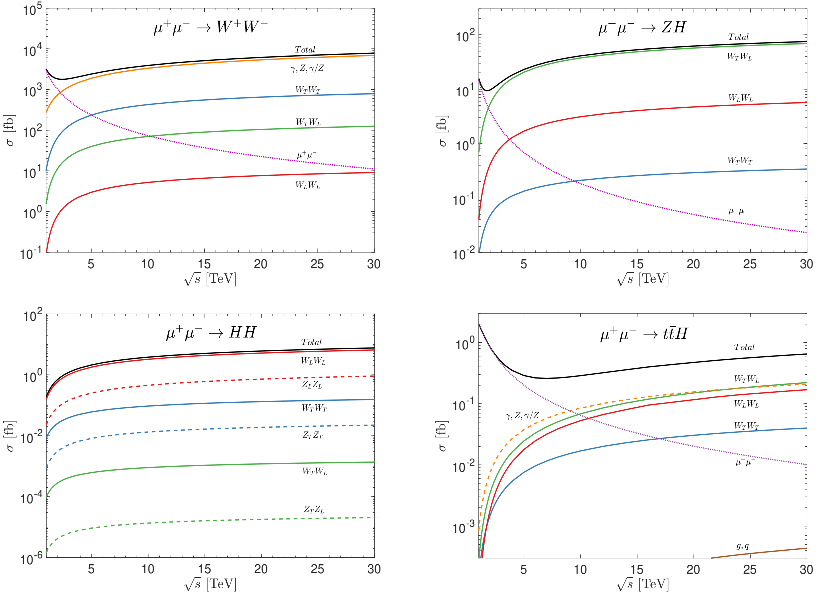






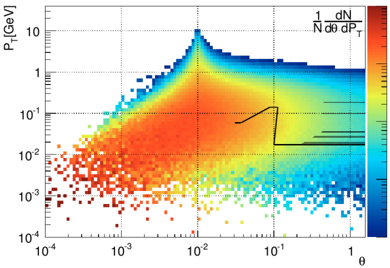
[T. Han, Y. Ma, K.Xie 2007.14300]

Other processes: $W^+W^-, ZH, HH, t\bar{t}H$

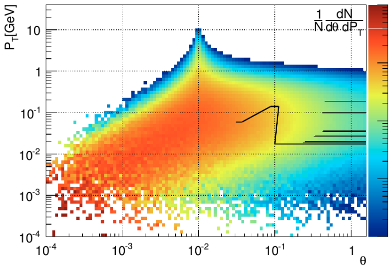


Nonperturbative hadronic production

The nonperturbative hadronic production events populate at low p_T regime.



(a) Pythia sample



(b) SLAC sample

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]