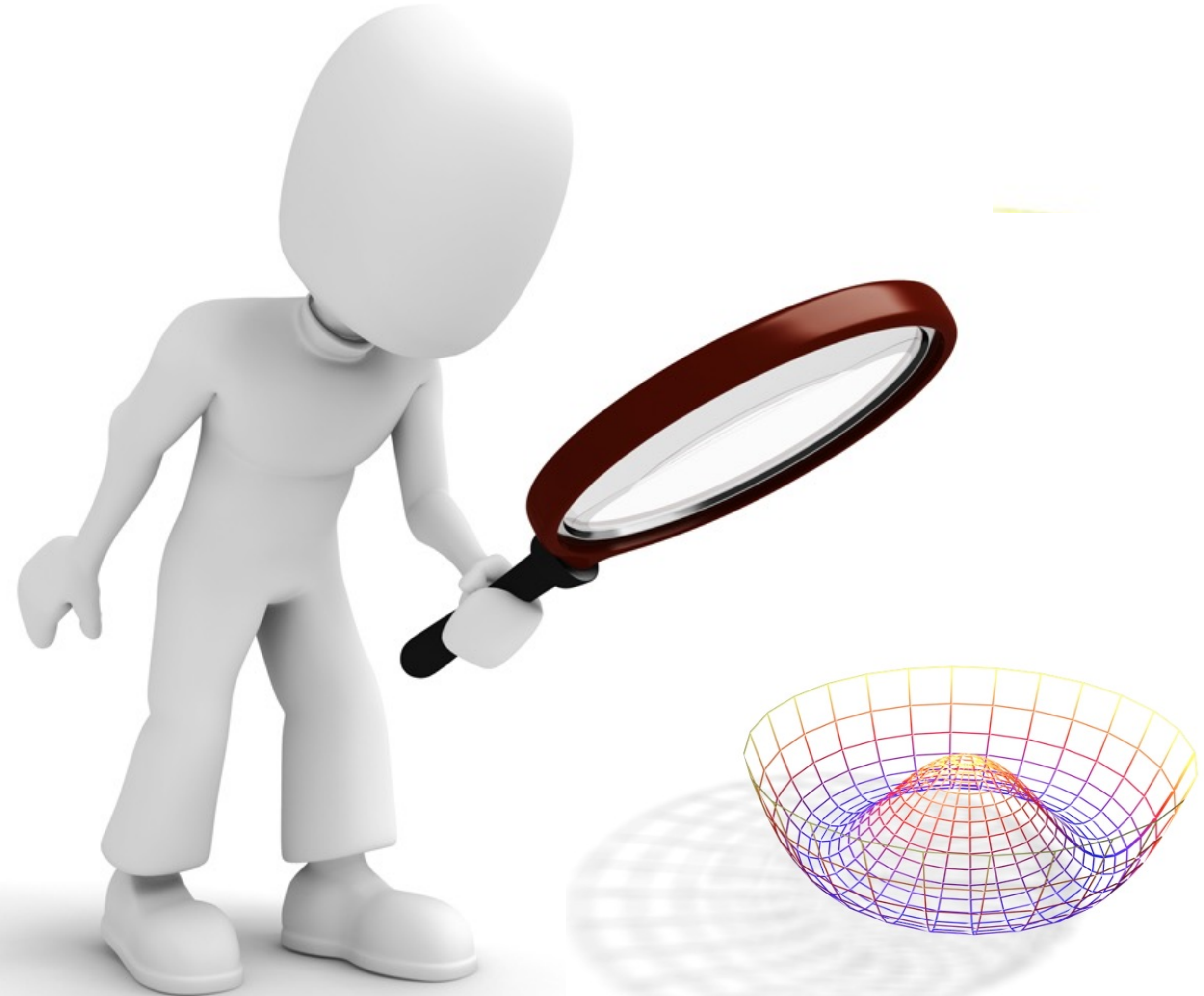




# MADMINER: A PYTHON BASED TOOL FOR SIMULATION-BASED INFERENCE IN HEP



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New York University  
Department of Physics  
Center for Data Science  
CILVR Lab

# Acknowledgements



Johann Brehmer



Gilles Louppe



Juan Pavez



Markus Stoye



Felix Kling



Irina Espejo

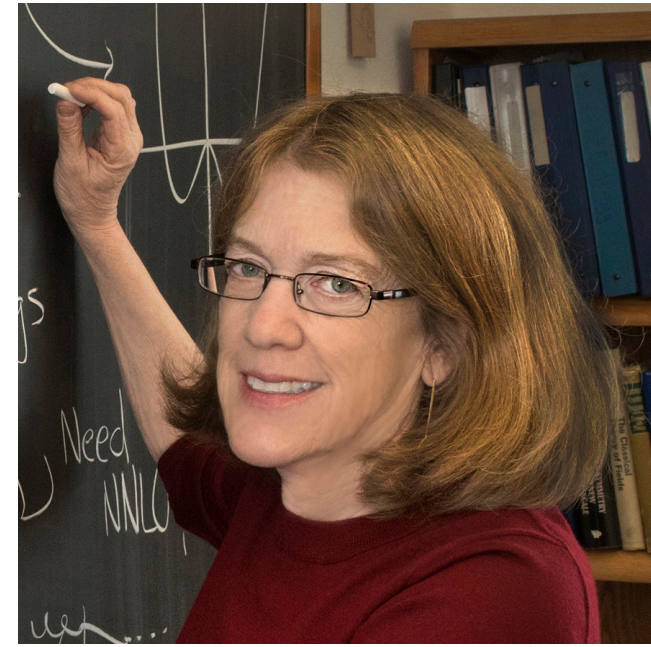


Sinclert Perez

Special thanks  
to Johann  
for slides I  
borrowed



Tilman Plehn



Sally Dawson



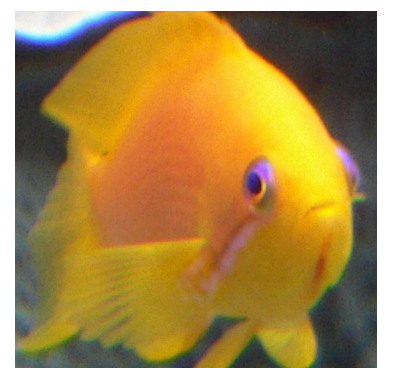
Sam Homiller



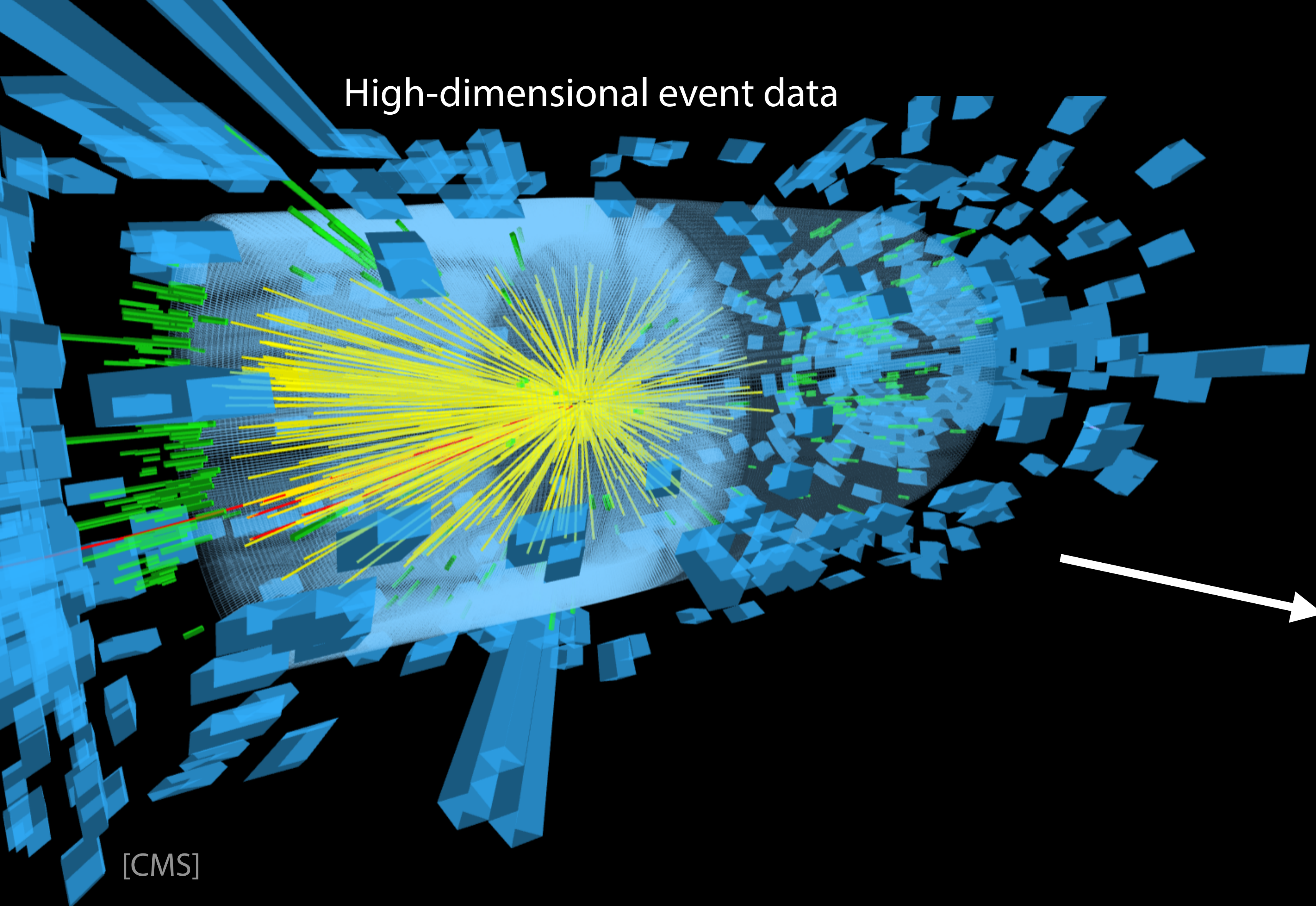
Matthew Feickert



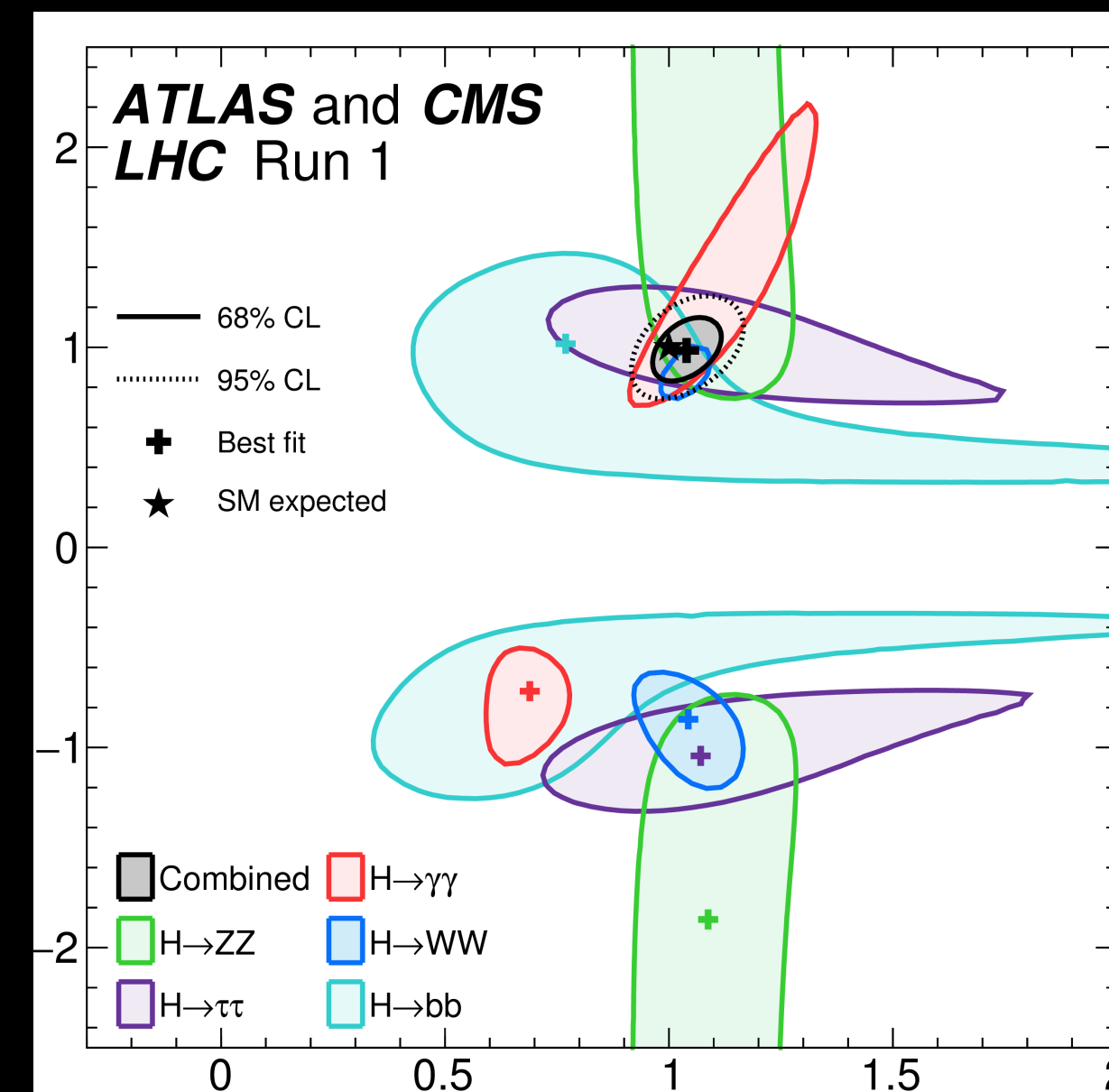
The SCALFIN Project  
[scailfin.github.io](https://scailfin.github.io)



High-dimensional event data



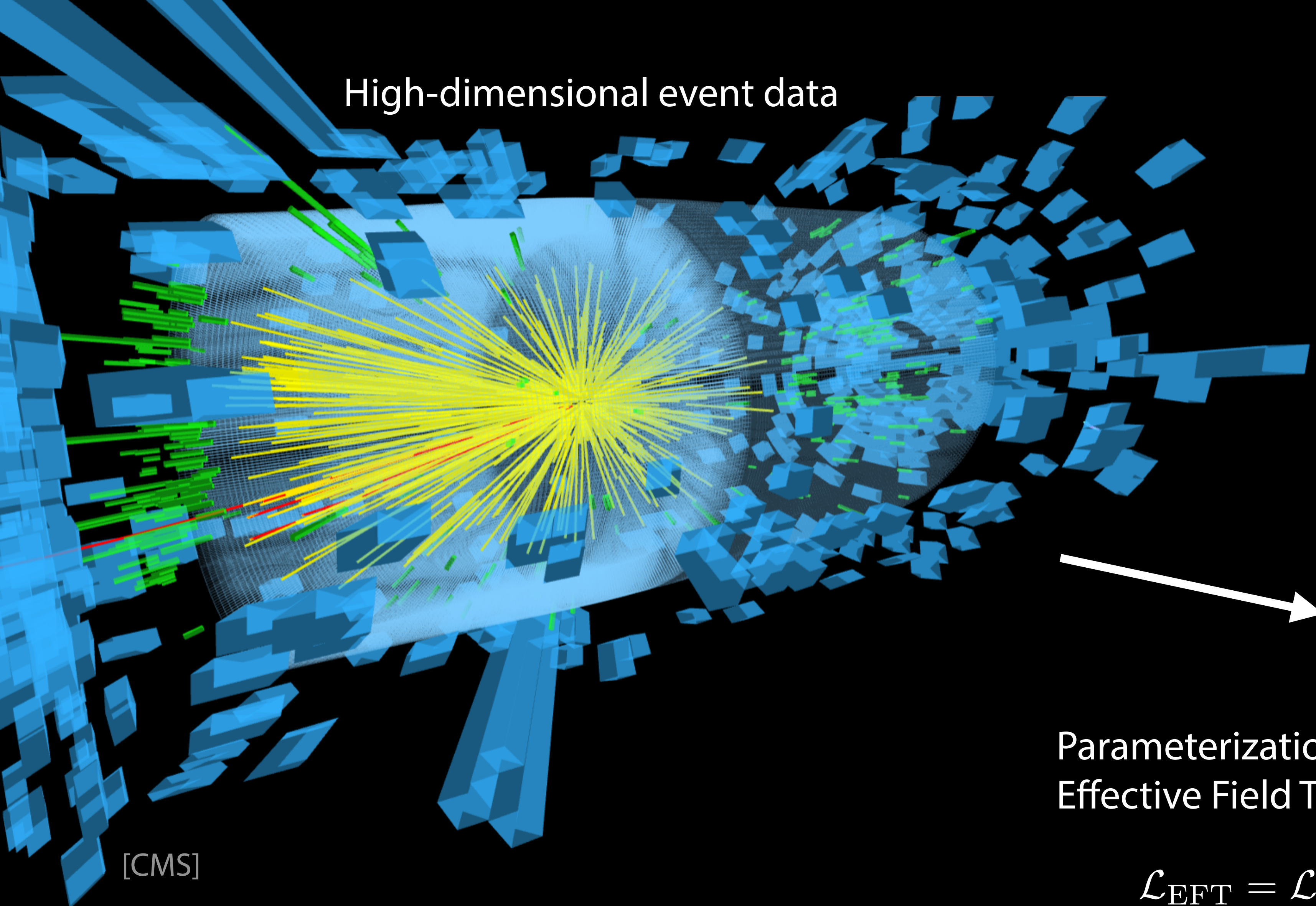
[CMS]



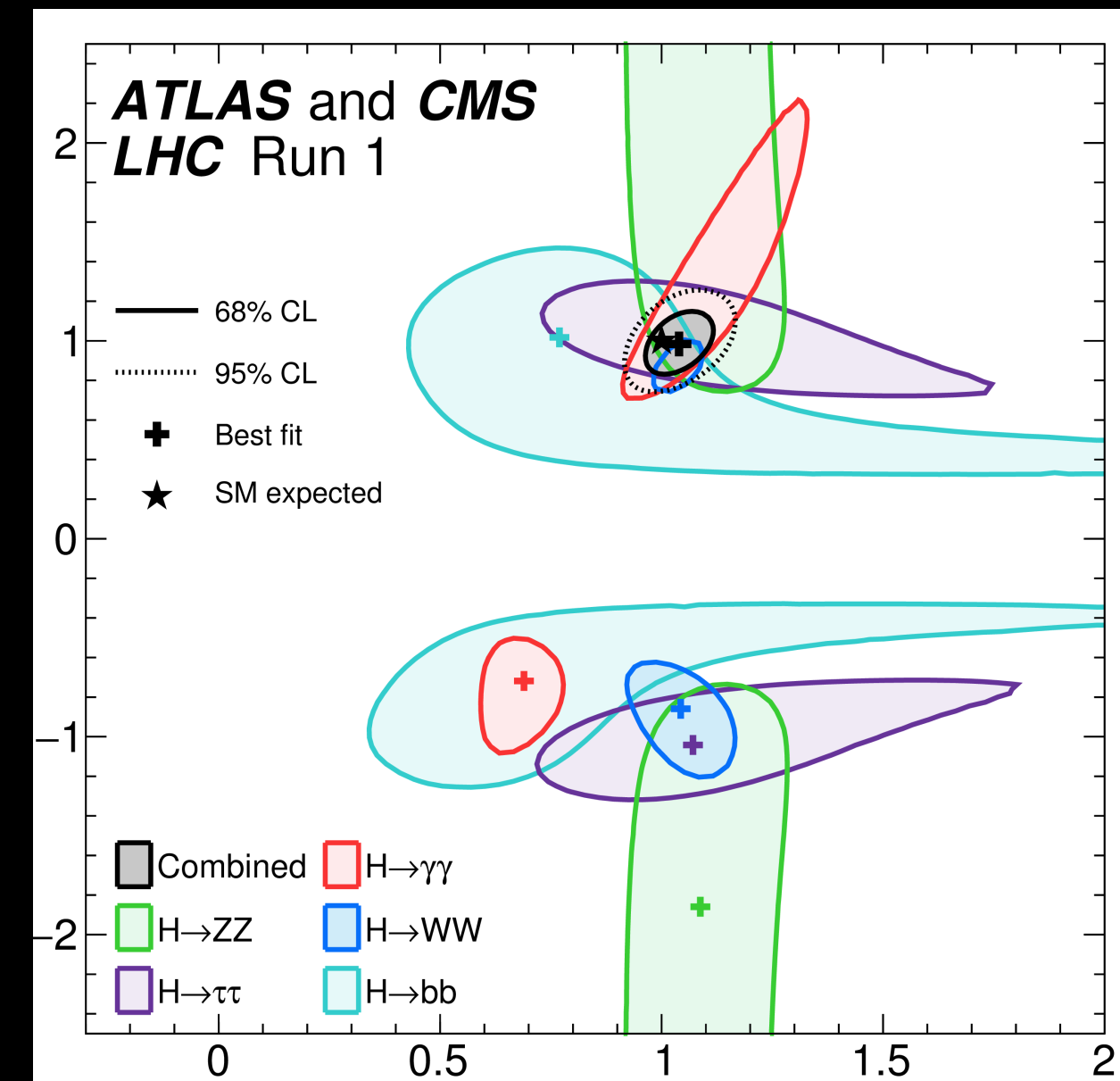
[ATLAS, CMS 1606.02266]

Precision constraints on new physics

High-dimensional event data



[CMS]



Precision constraints on new physics

Parameterization e.g. in Effective Field Theory:

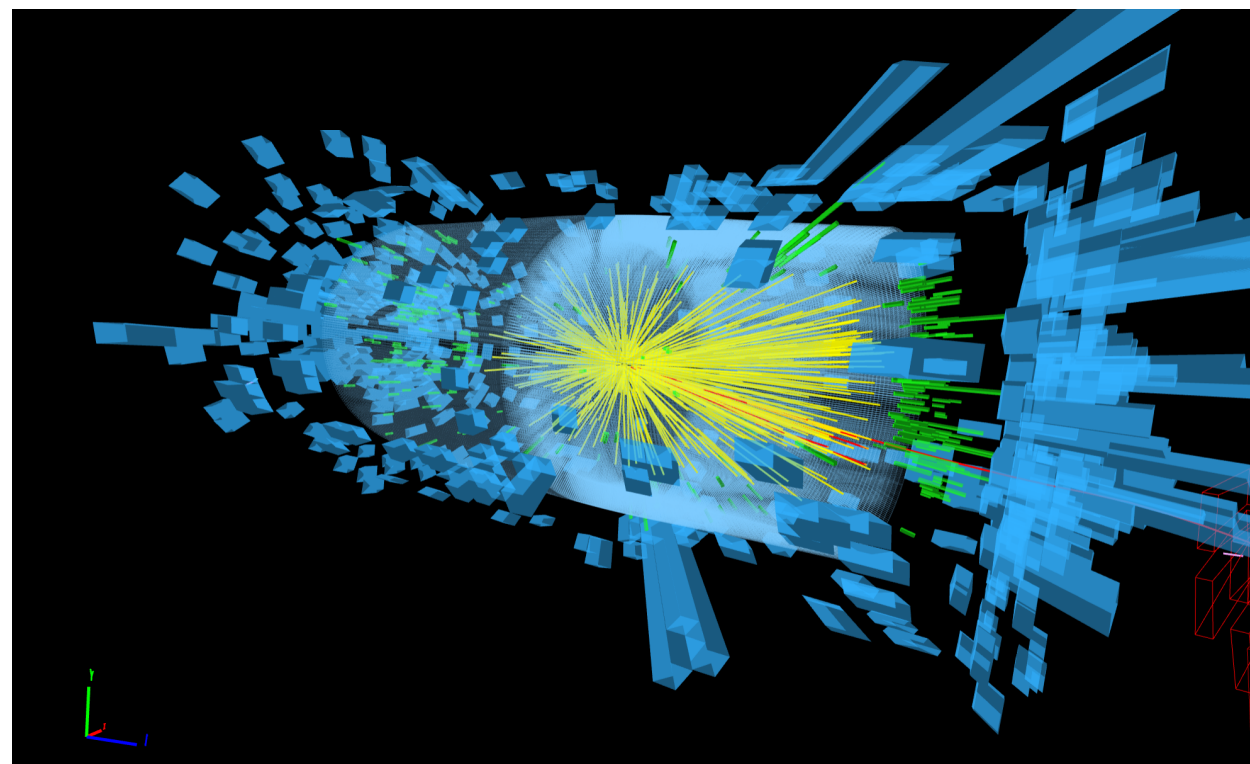
systematic expansion of new physics around Standard Model

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i + \dots$$

10s to 100s "universal" parameters to measure

# The likelihood is a key object

Let  $\theta$  denote the coefficients of higher dimensional operators in the Lagrangian,  $x$  be high-dimensional data associated to an event, and  $p(x | \theta) = \frac{1}{\sigma(\theta)} \frac{d\sigma}{dx}$  be the distribution for the data



High-dimensional event data  $x$

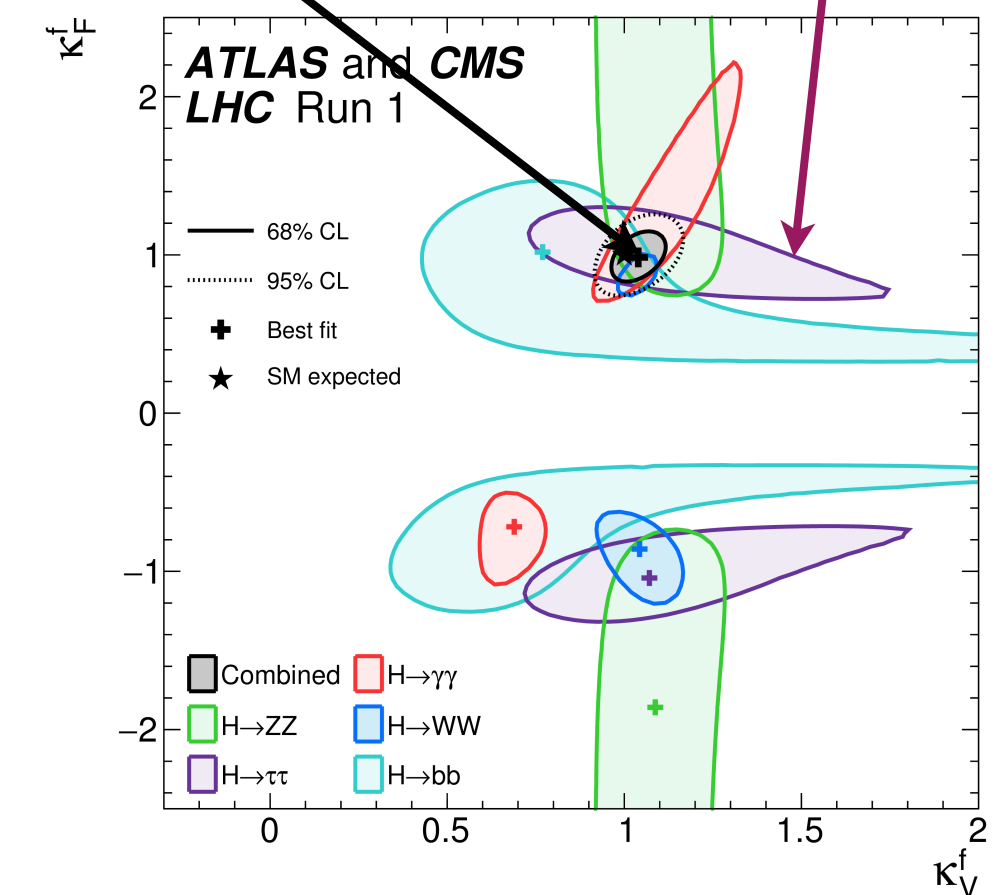


Likelihood function  
 $p(x|\theta)$



Maximum-likelihood estimator

Confidence limits based on likelihood ratio tests



Constraints on parameters  $\theta$

Now for some bad news....

Particle physics processes do not have a tractable likelihood function.

# Modeling particle physics processes

Theory  
parameters  
 $\theta$





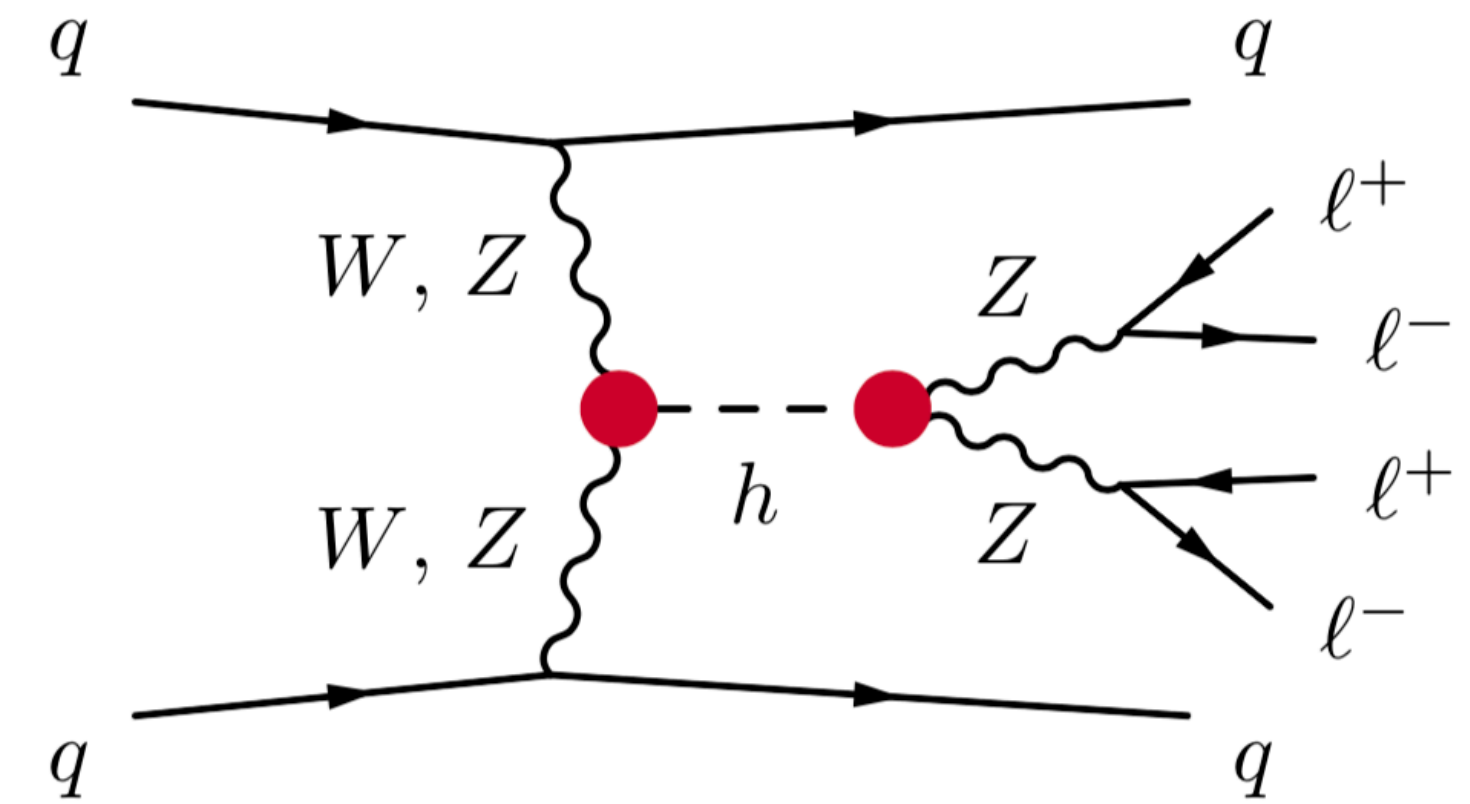
# Modeling particle physics processes

Latent variables

Parton-level  
momenta

Theory  
parameters

$z_p$  ←  $\theta$



← Evolution

# Modeling particle physics processes

Latent variables

Shower  
splittings

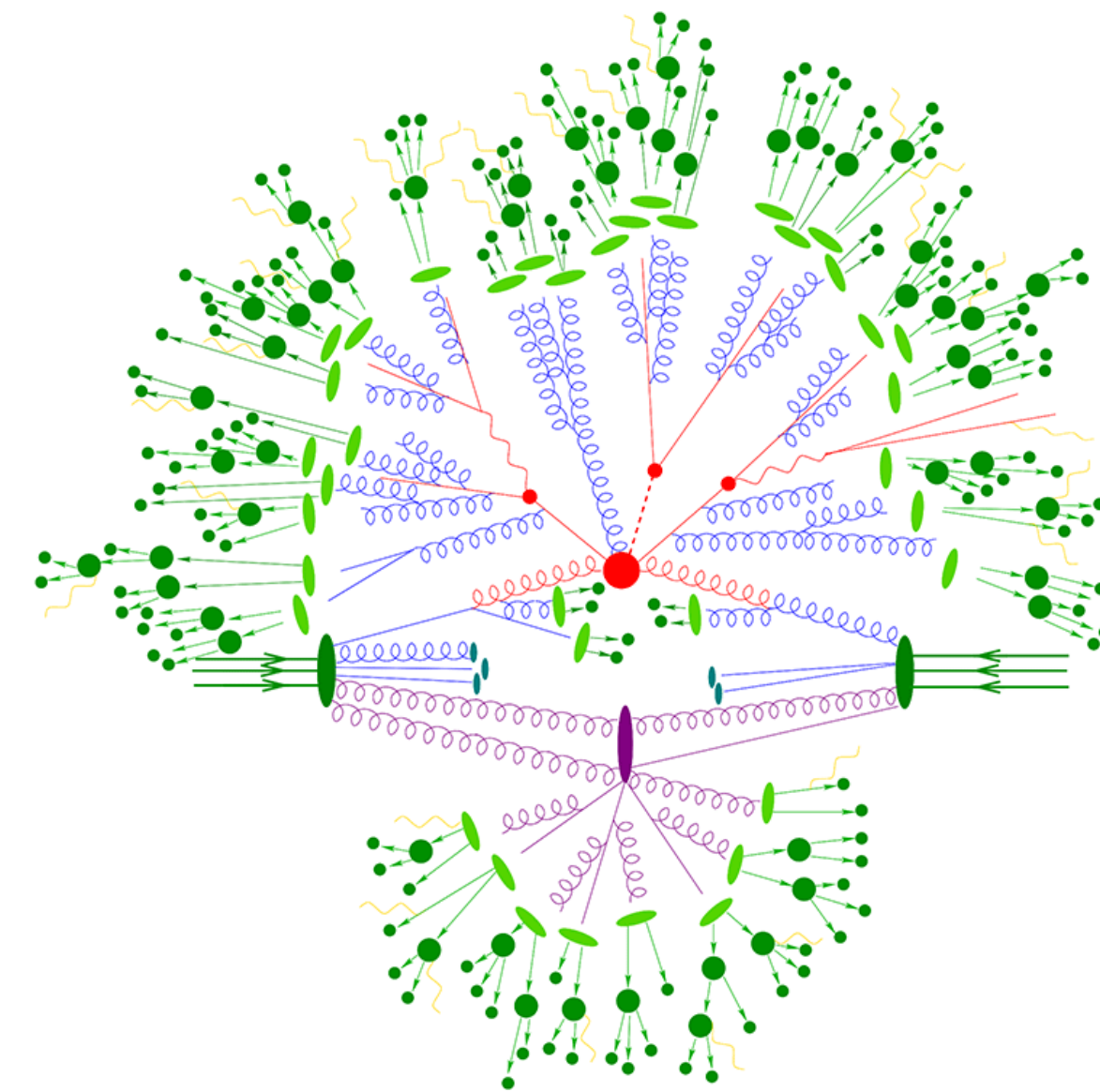
Parton-level  
momenta

Theory  
parameters

$z_s$

$z_p$

$\theta$

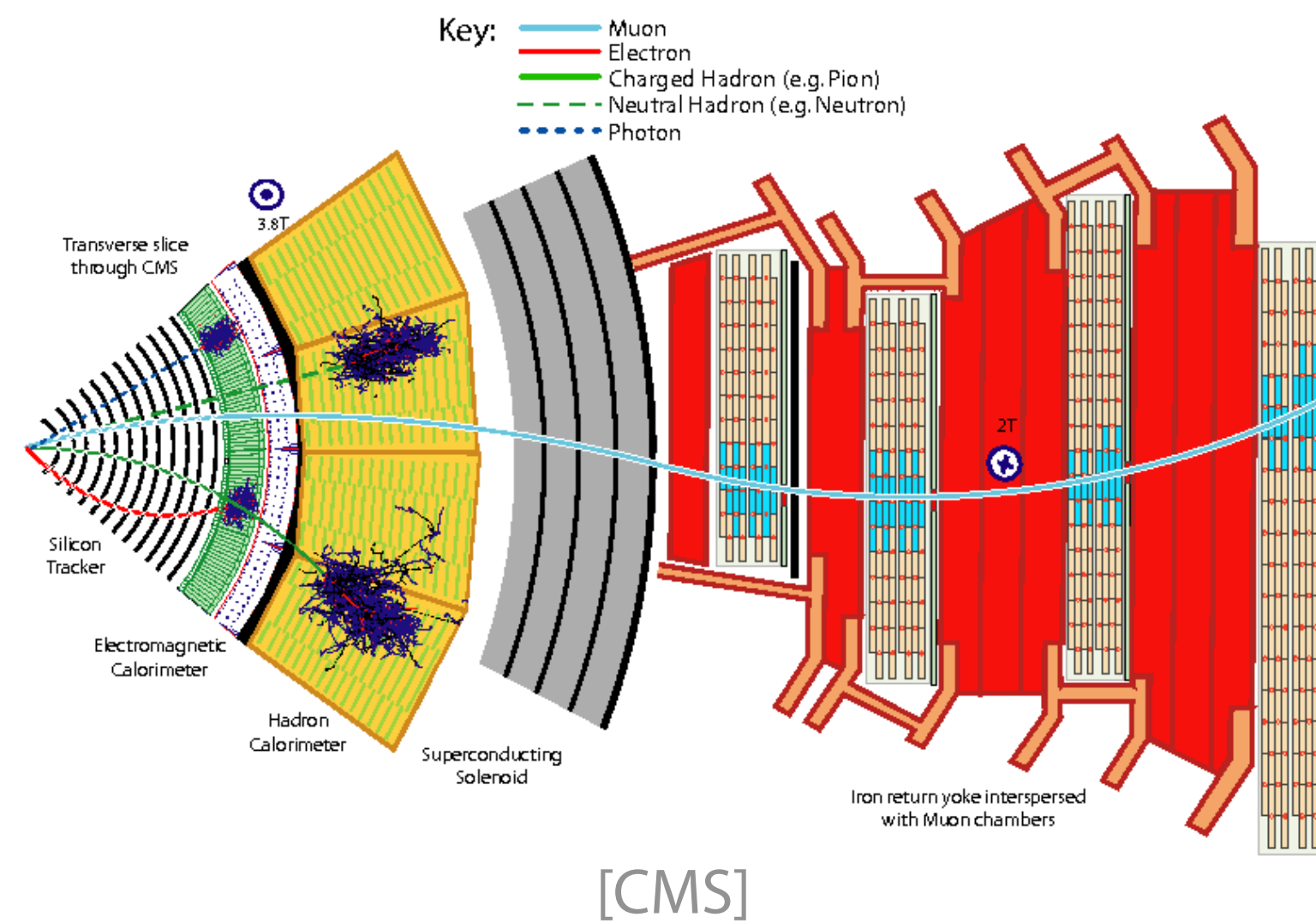
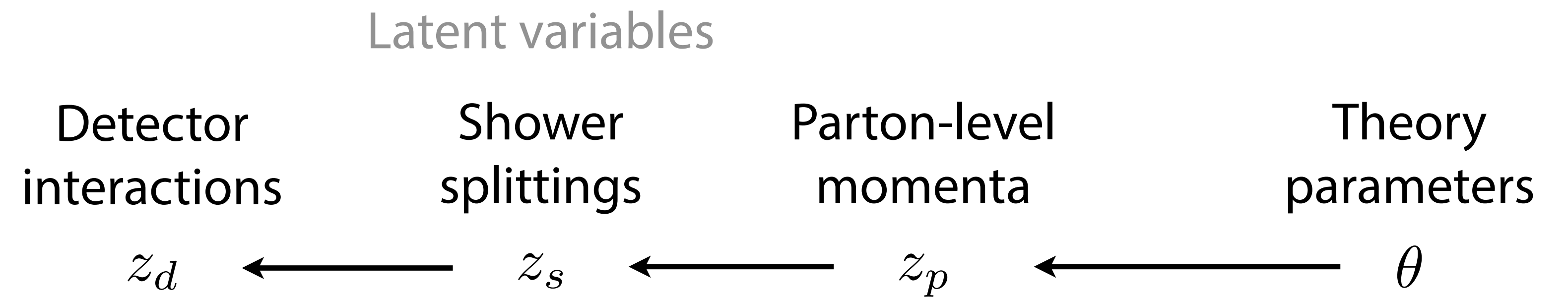


[F. Krauss]

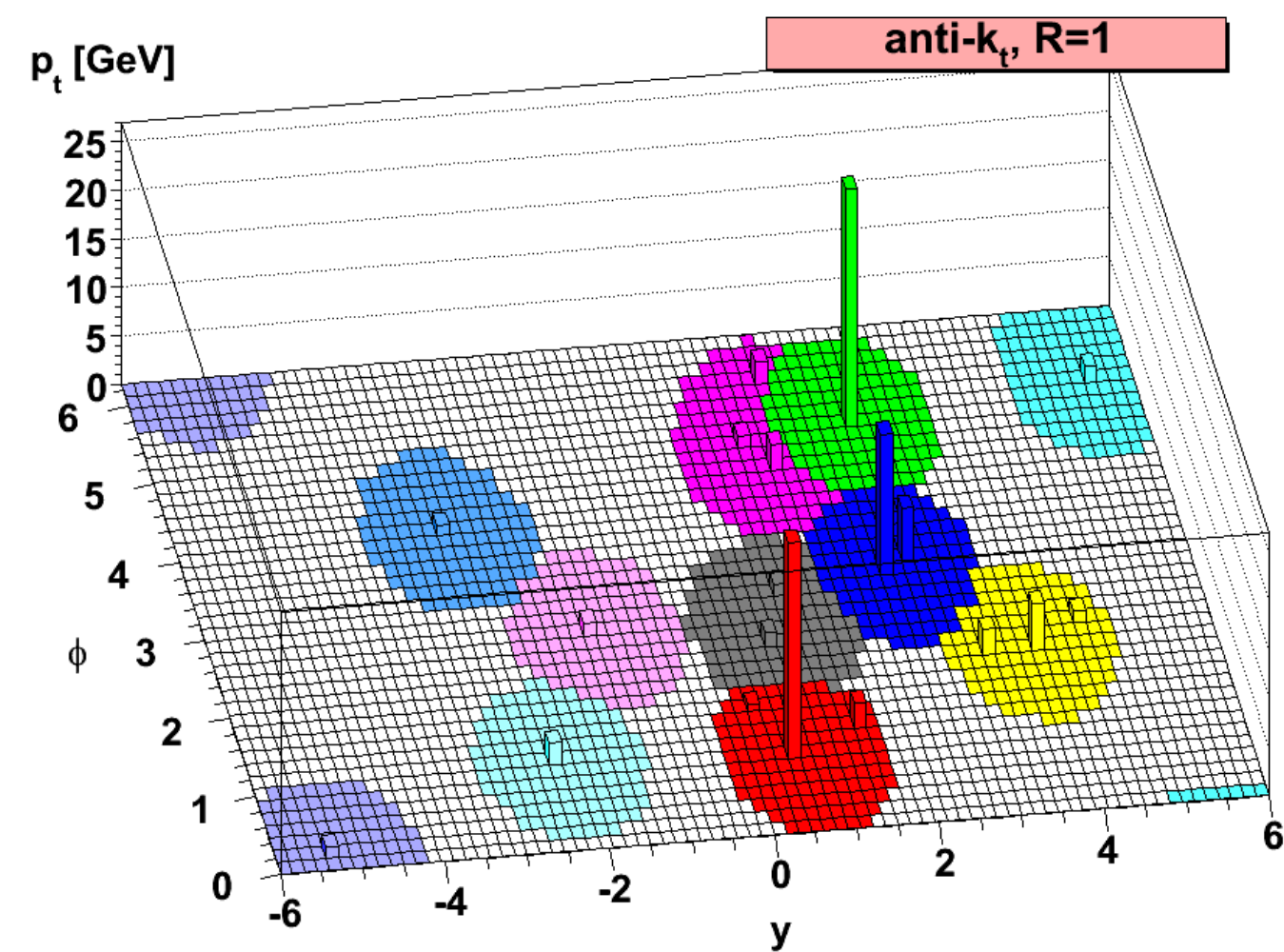
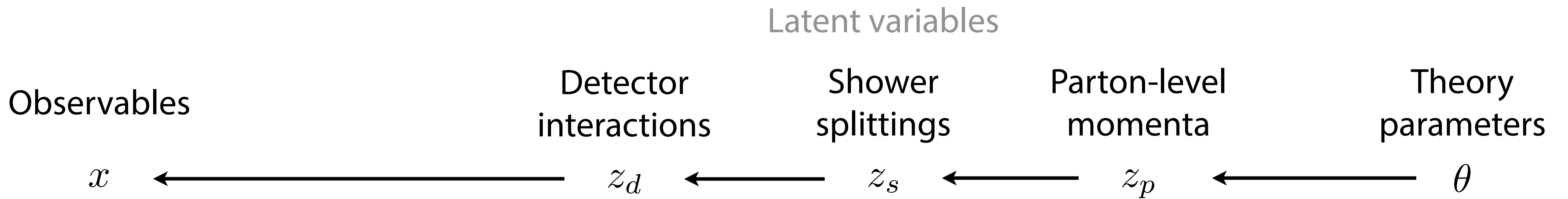


Evolution

# Modeling particle physics processes



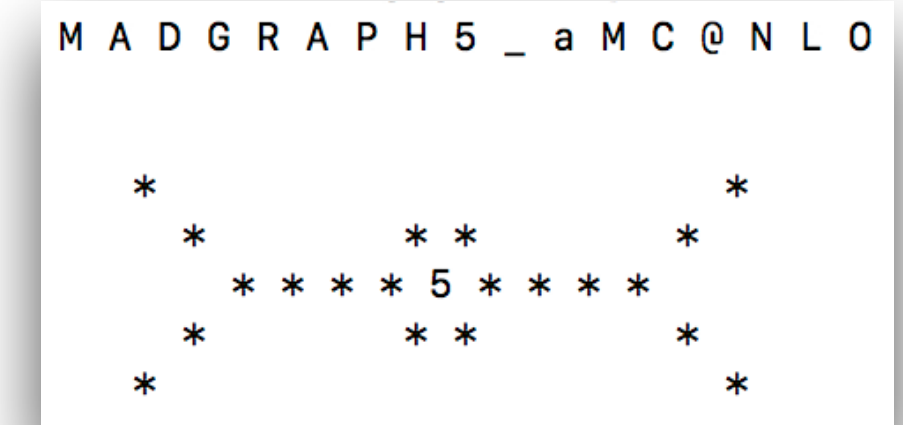
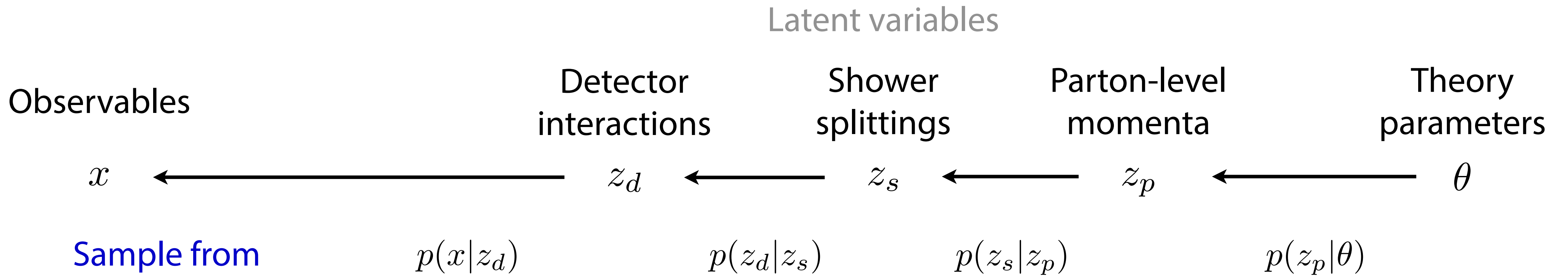
# Modeling particle physics processes



[M. Cacciari, G. Salam, G. Soyez 0802.1189]

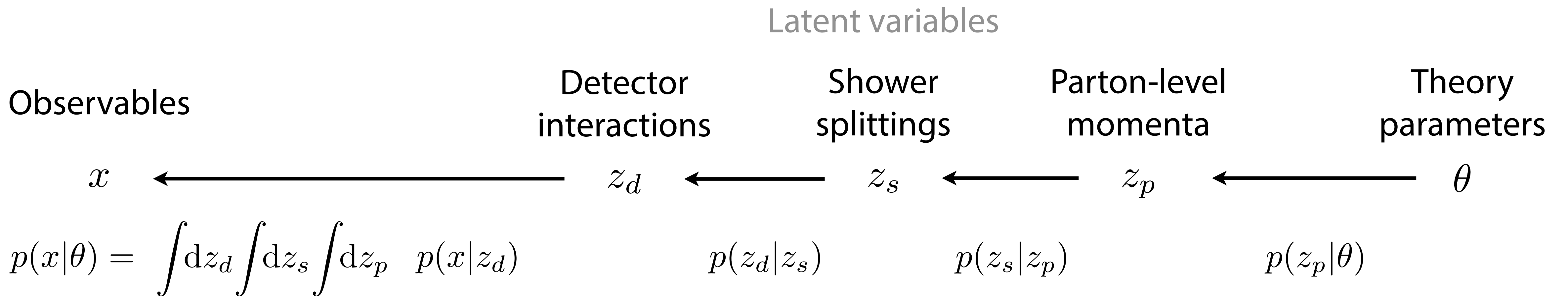


# Modeling particle physics processes



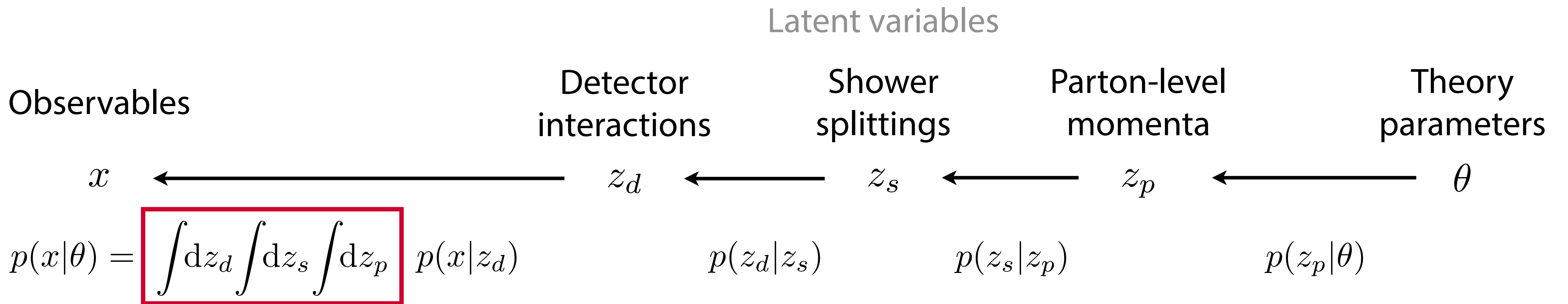
←————— Prediction (simulation)

# Modeling particle physics processes



Inference

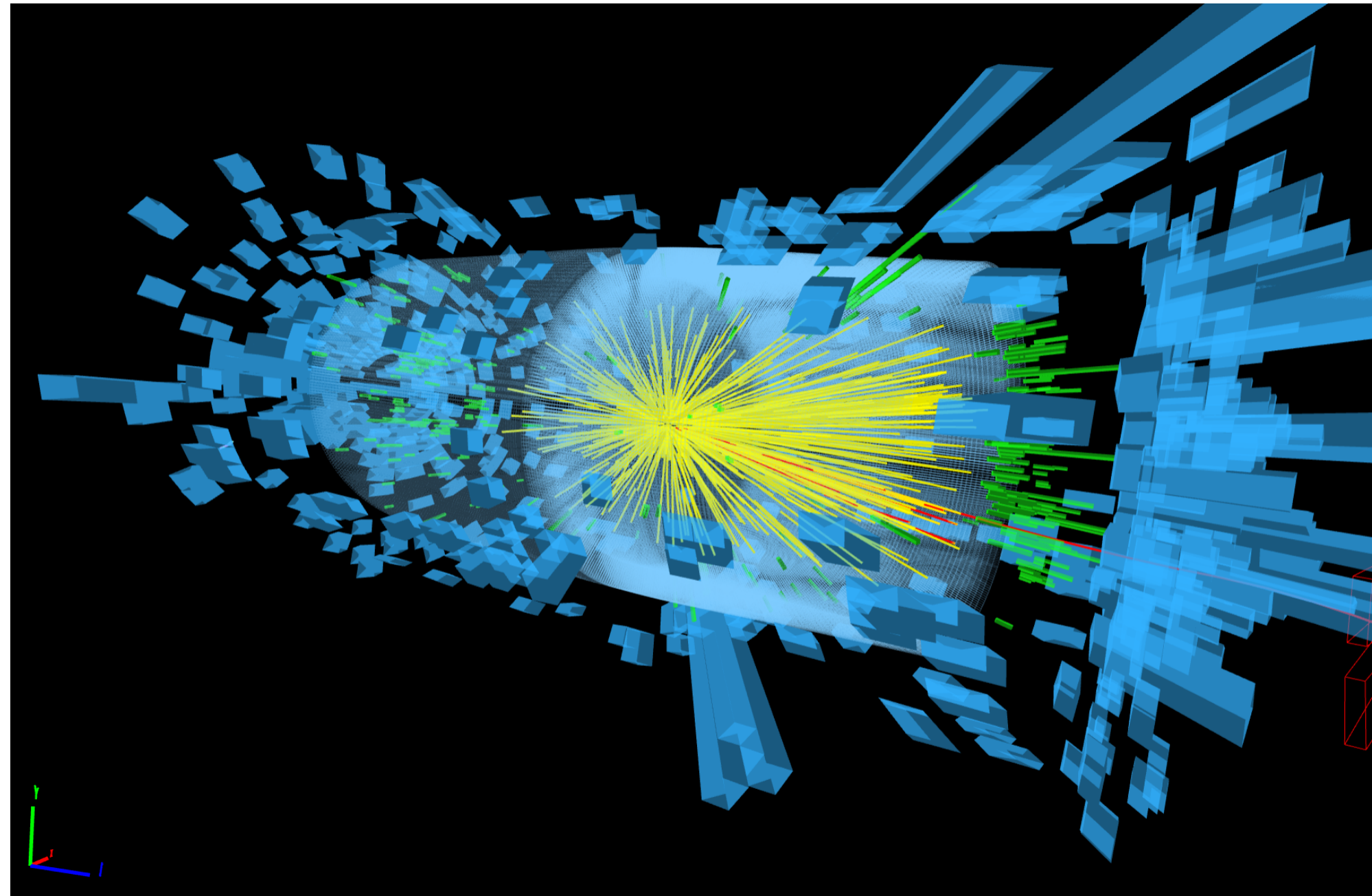
# Modeling particle physics processes



It's infeasible to calculate the integral over this enormous space!

Inference

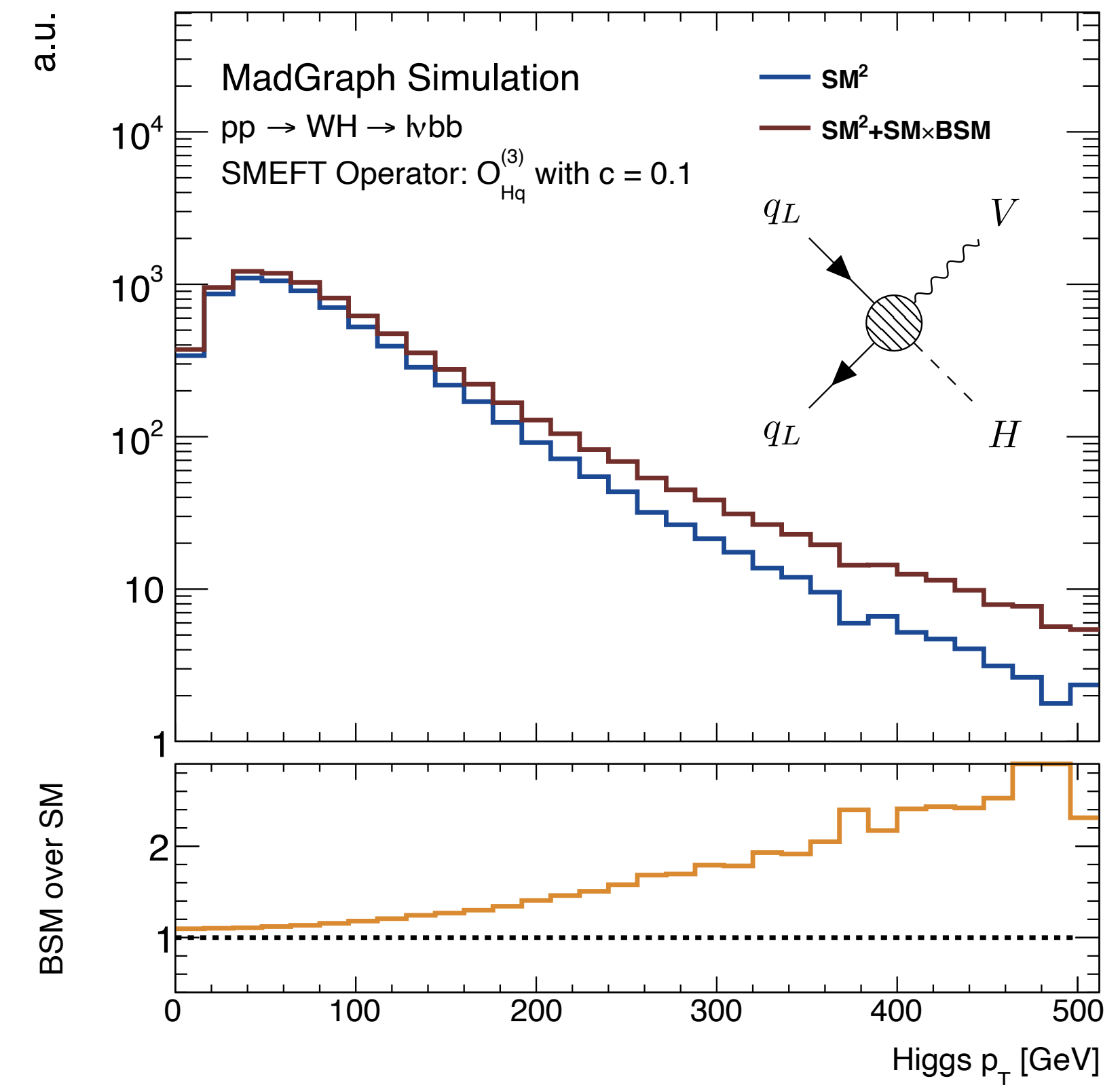
# What we usually do: number counting or singly differential



High-dimensional event data  $x$

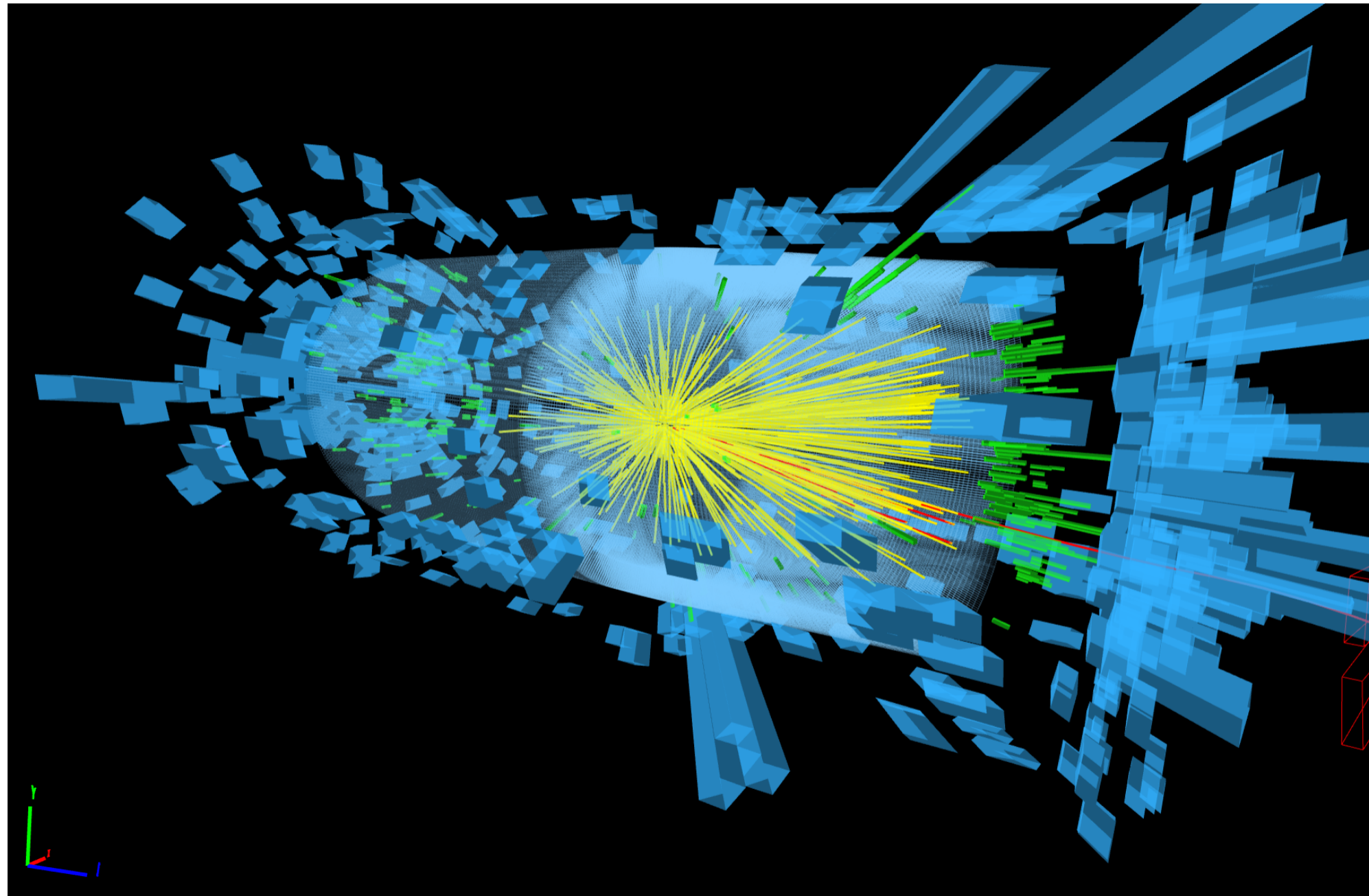
$p(x|\theta)$  cannot be calculated

SMEFT:  $O_{Hq}^{(3)} = 0.1$





# What we usually do: number counting or singly differential

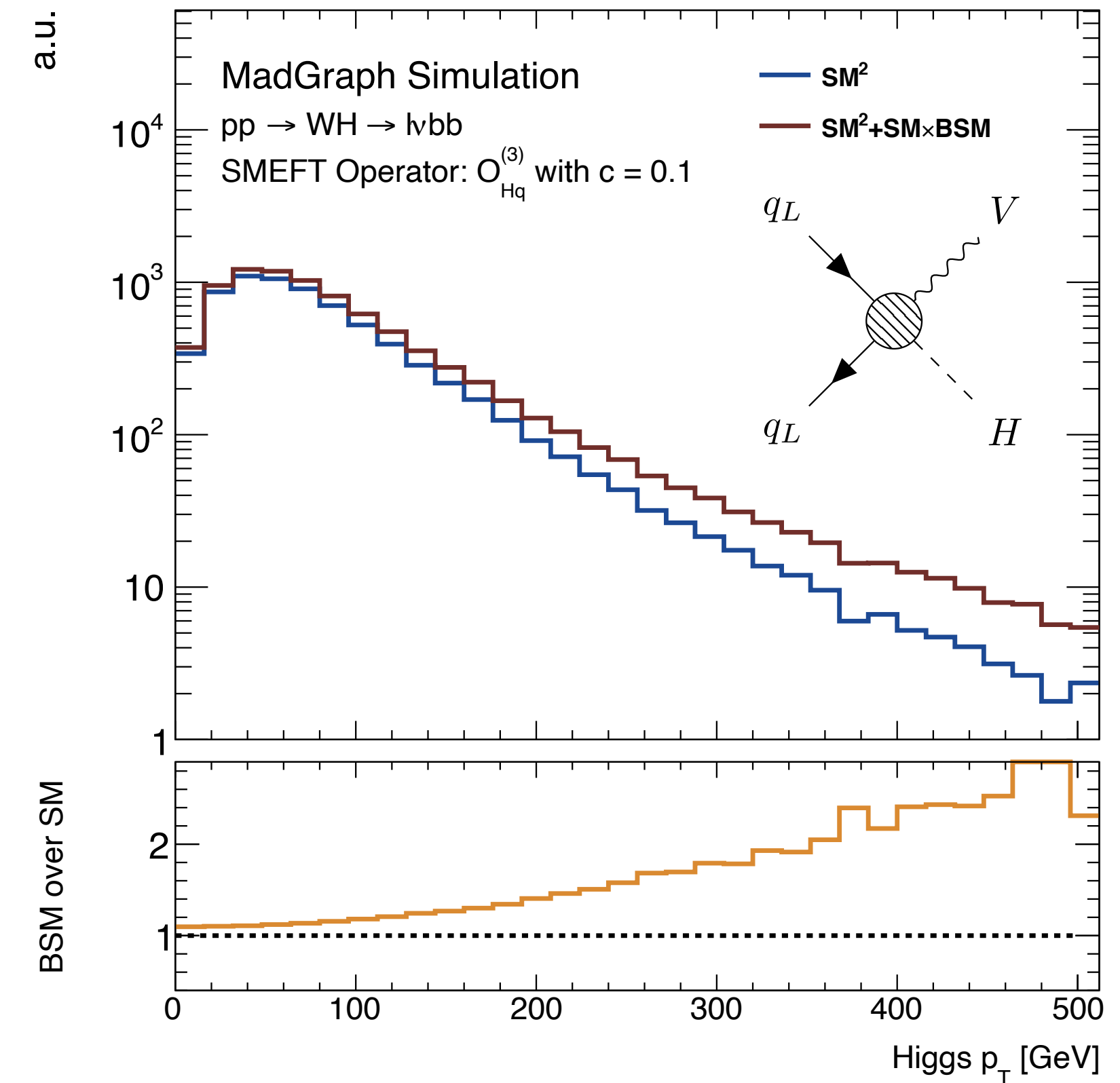


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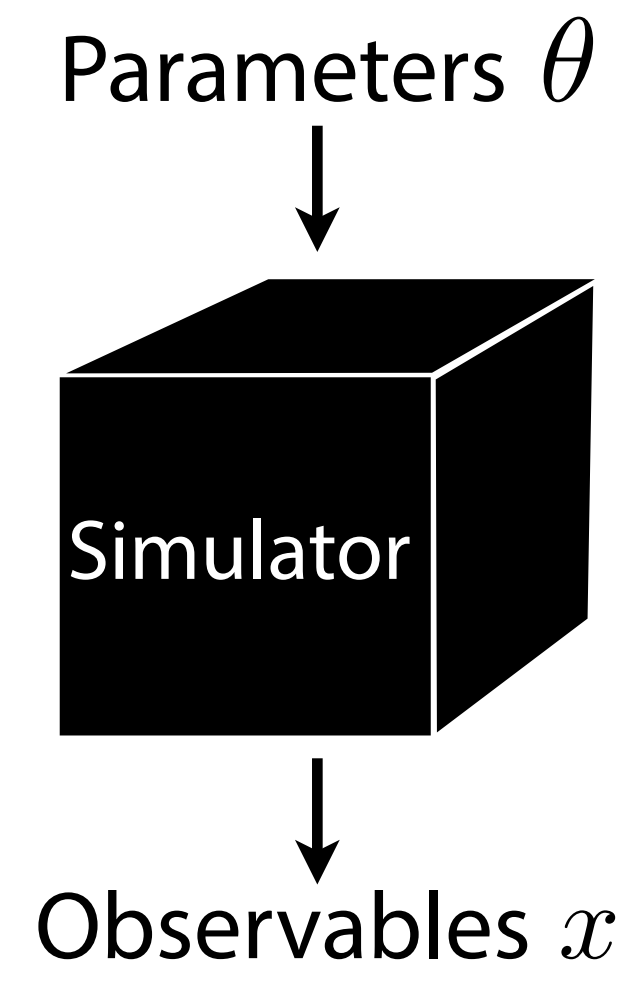
One or two summary statistics  $x'$

$p(x'|\theta)$  can be estimated  
with histograms

n.b. "summary statistic" = a sensitive observable

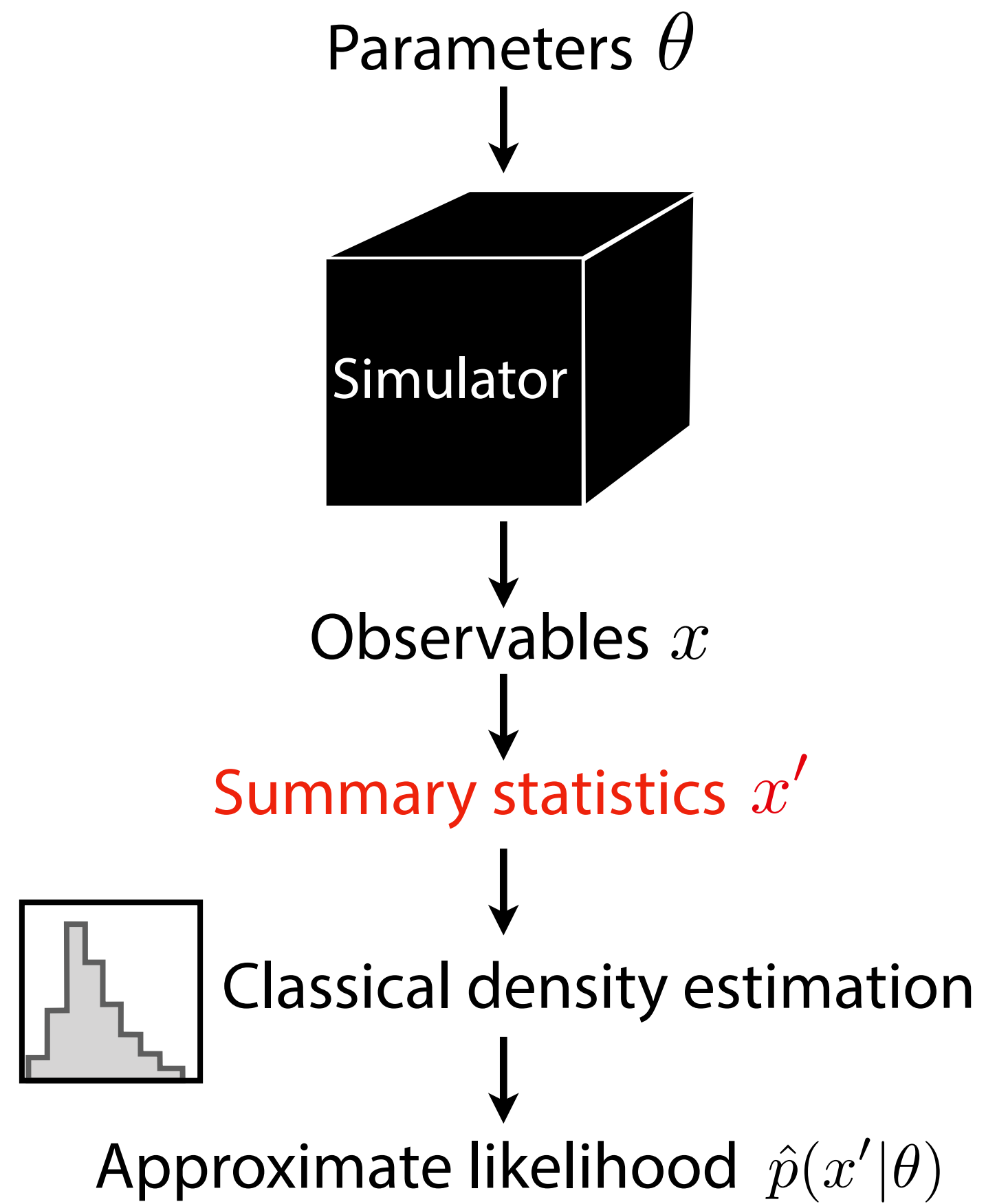
# Inference by estimating the likelihood

[e.g. P. Diggle, R. Gratton 1984]



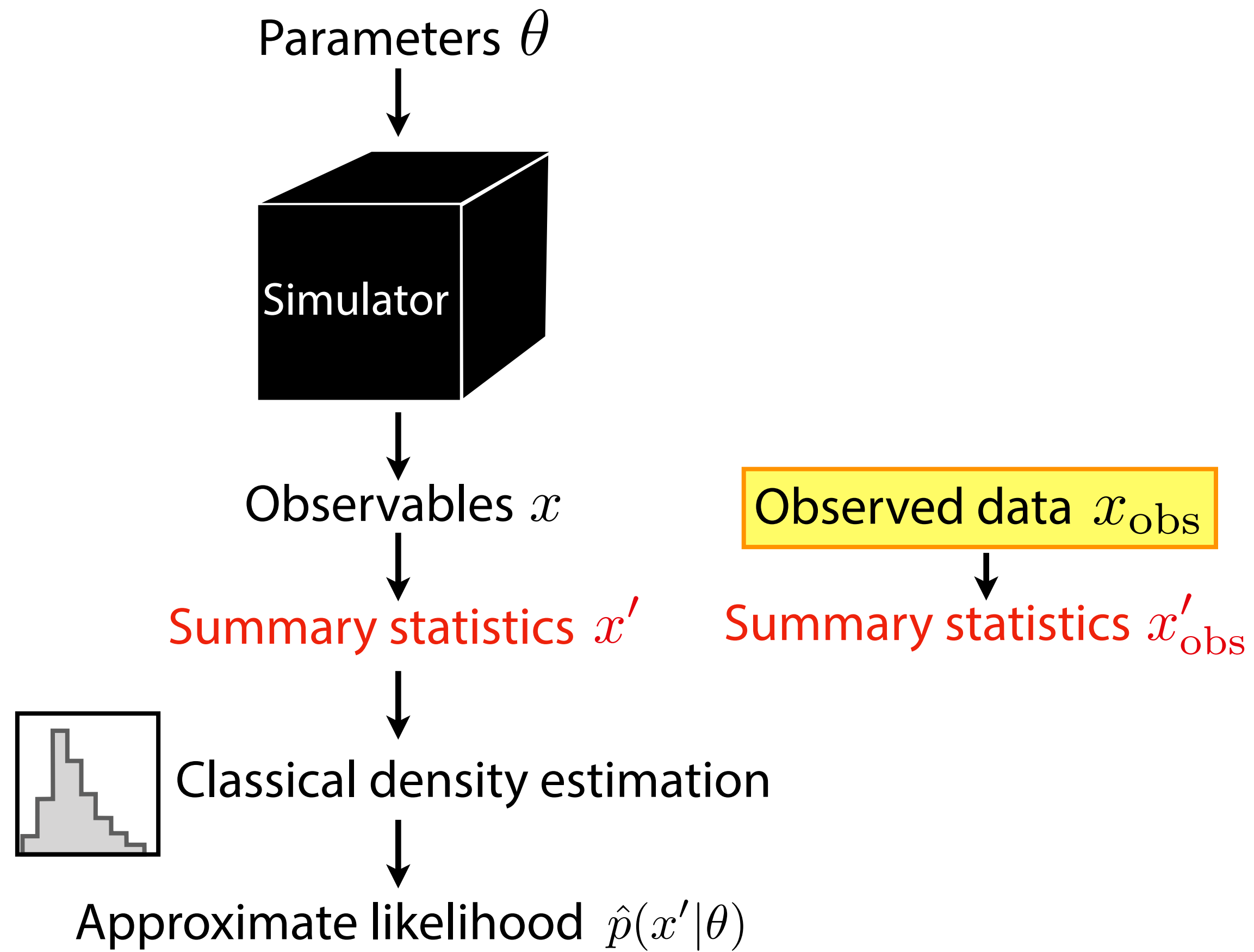
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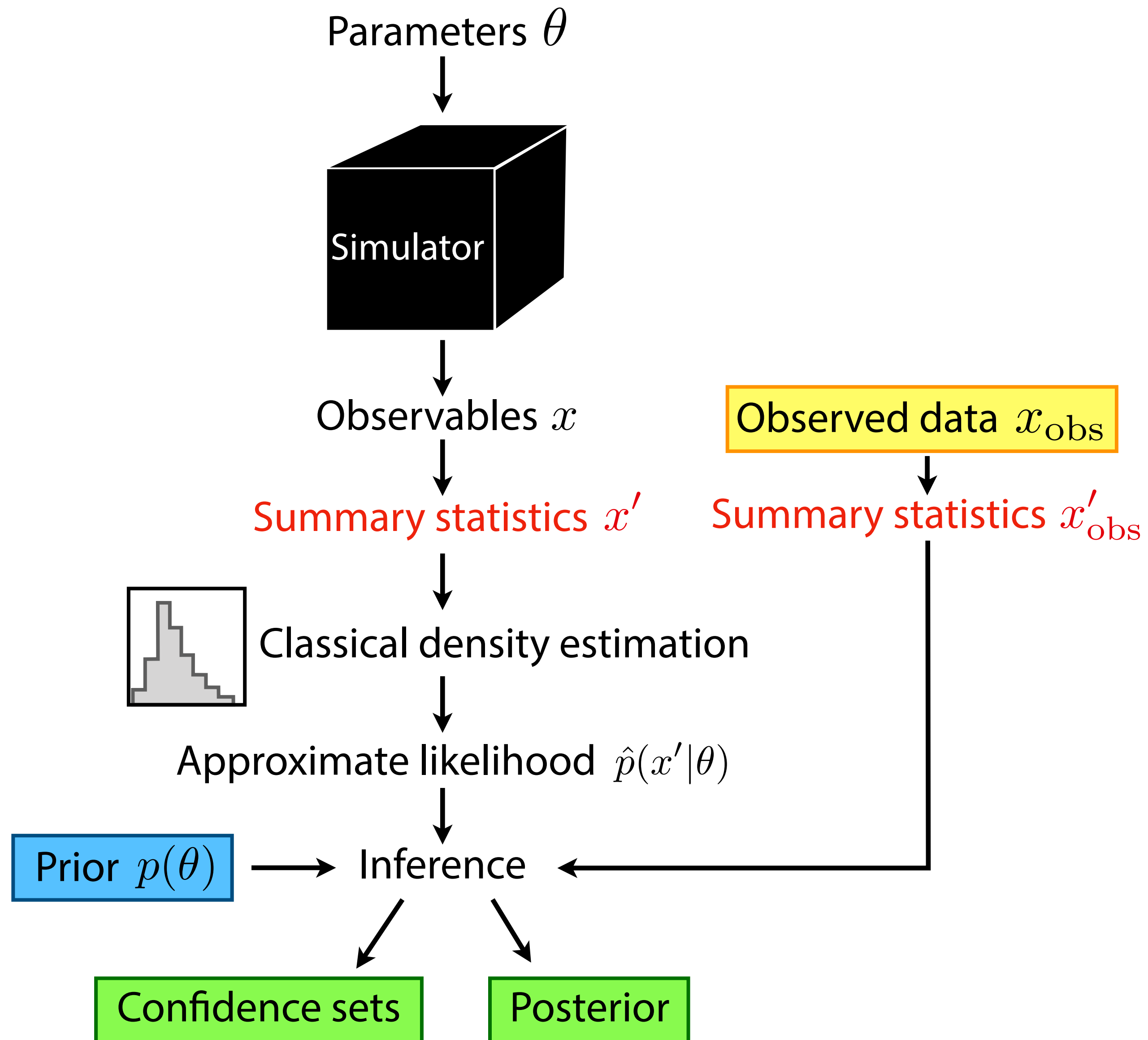
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[e.g. P. Diggle, R. Gratton 1984]



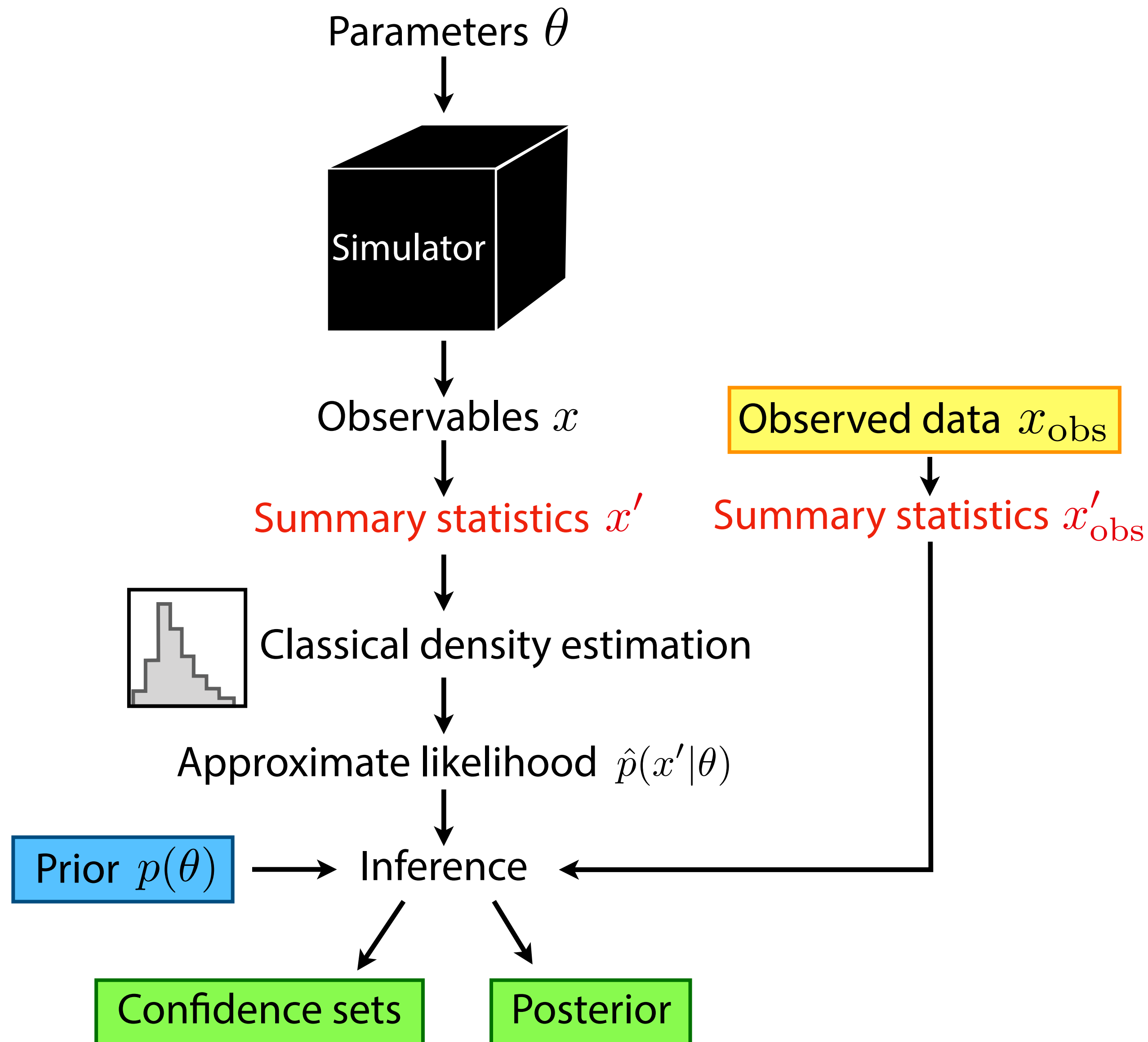
# Inference by estimating the likelihood

[e.g. P. Diggle, R. Gratton 1984]



# Inference by estimating the likelihood

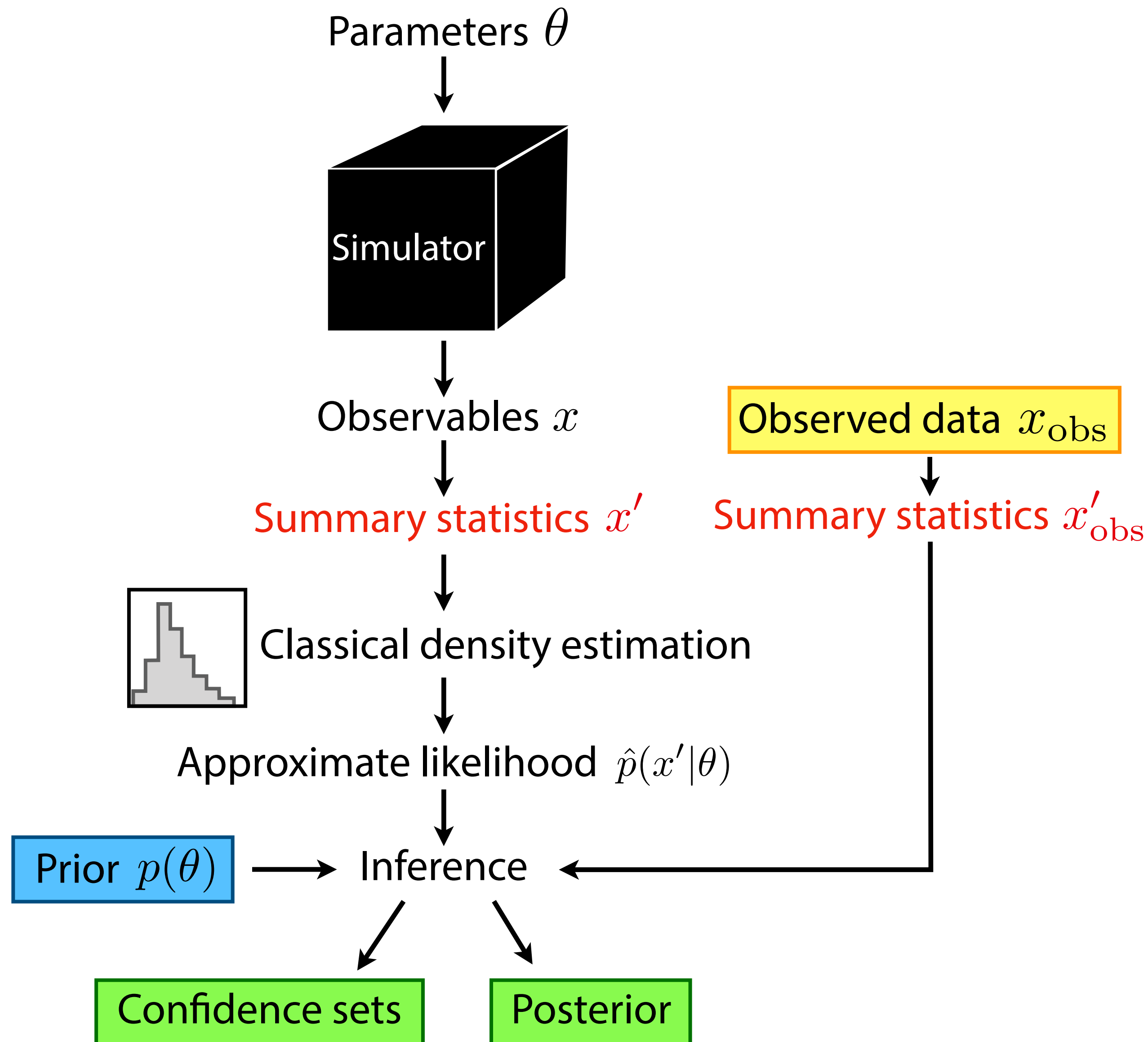
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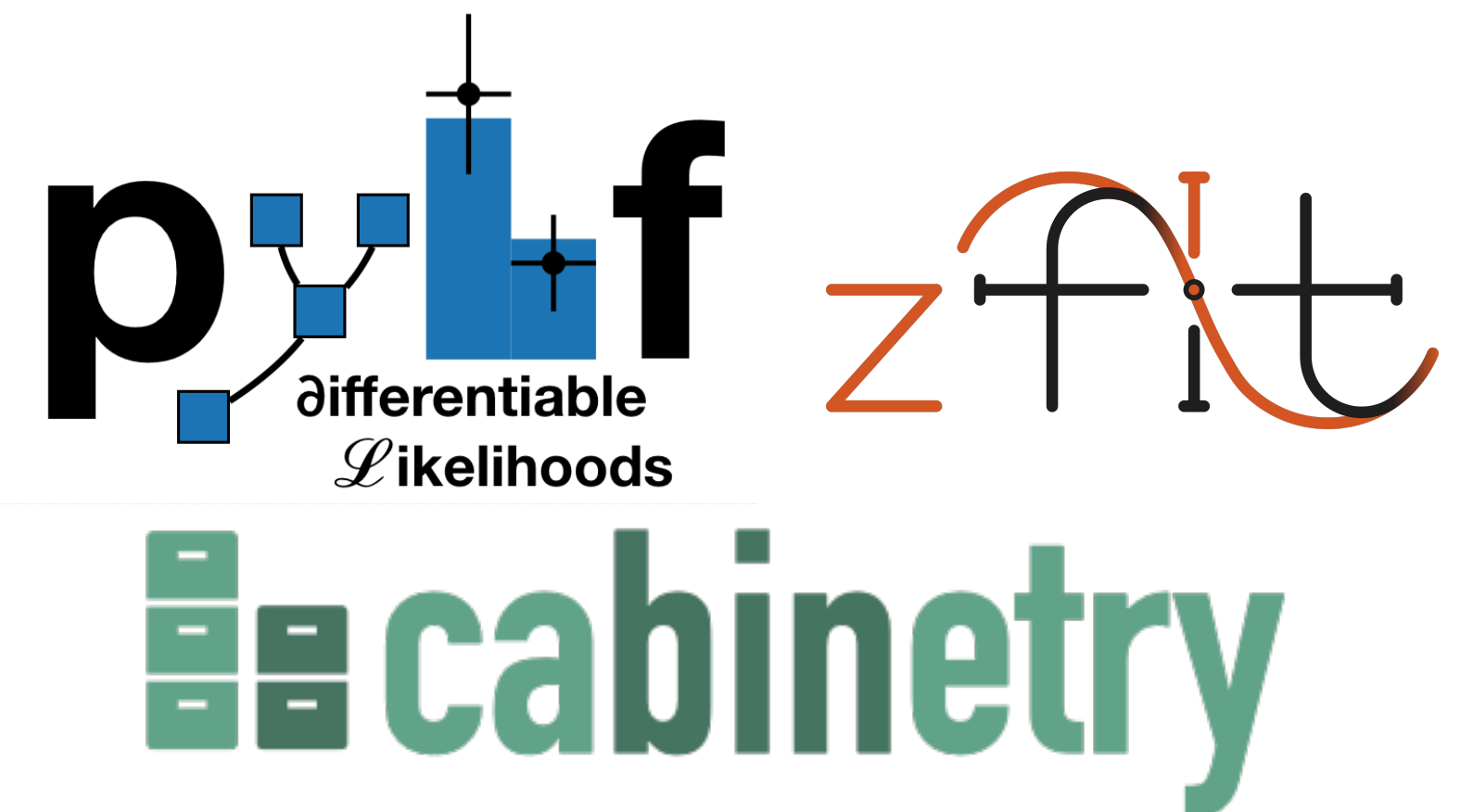
- Compression to summary statistics loses information & reduces quality of inference
- Curse of dimensionality: does not scale to more than a few summary statistics

# Inference by estimating the likelihood

[e.g. P. Diggle, R. Gratton 1984]



- Compression to summary statistics loses information & reduces quality of inference
- Curse of dimensionality: does not scale to more than a few summary statistics



# What if we could estimate the likelihood...

- for high-dimensional observables, including correlations?

like MEM: no need to pick summary statistics

- including state-of-the-art shower and detector models?

allowing for extra radiation, no need for transfer functions

- in microseconds?

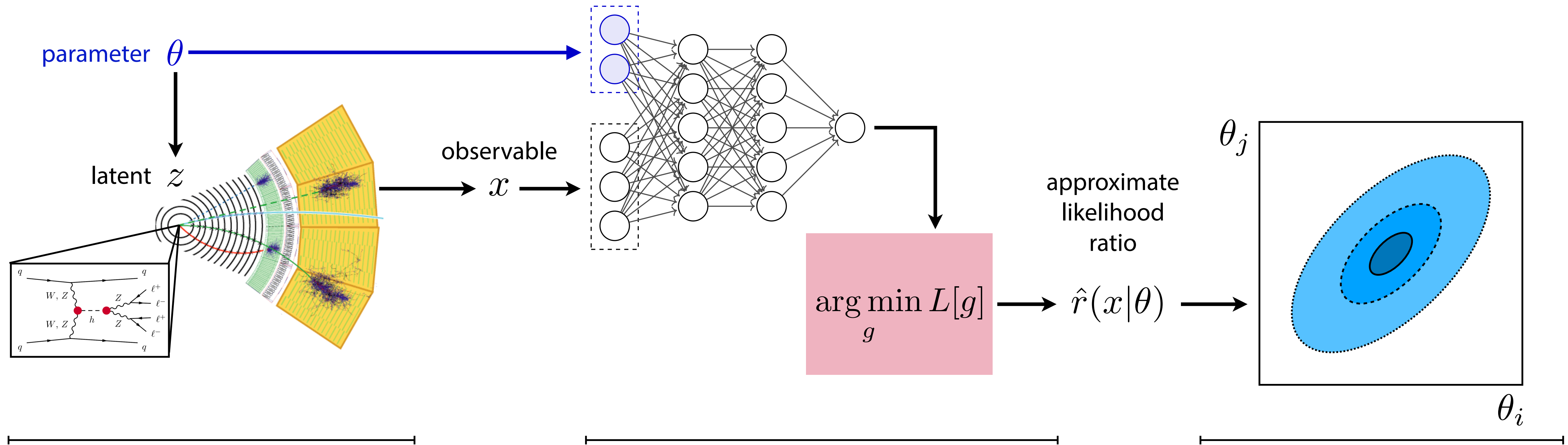
amortized inference: train once, then always evaluate fast

- requiring less training examples than established machine learning methods?

using matrix element information: "ML version of MEM"



# Learning with Simulated Data



Simulation

Machine Learning

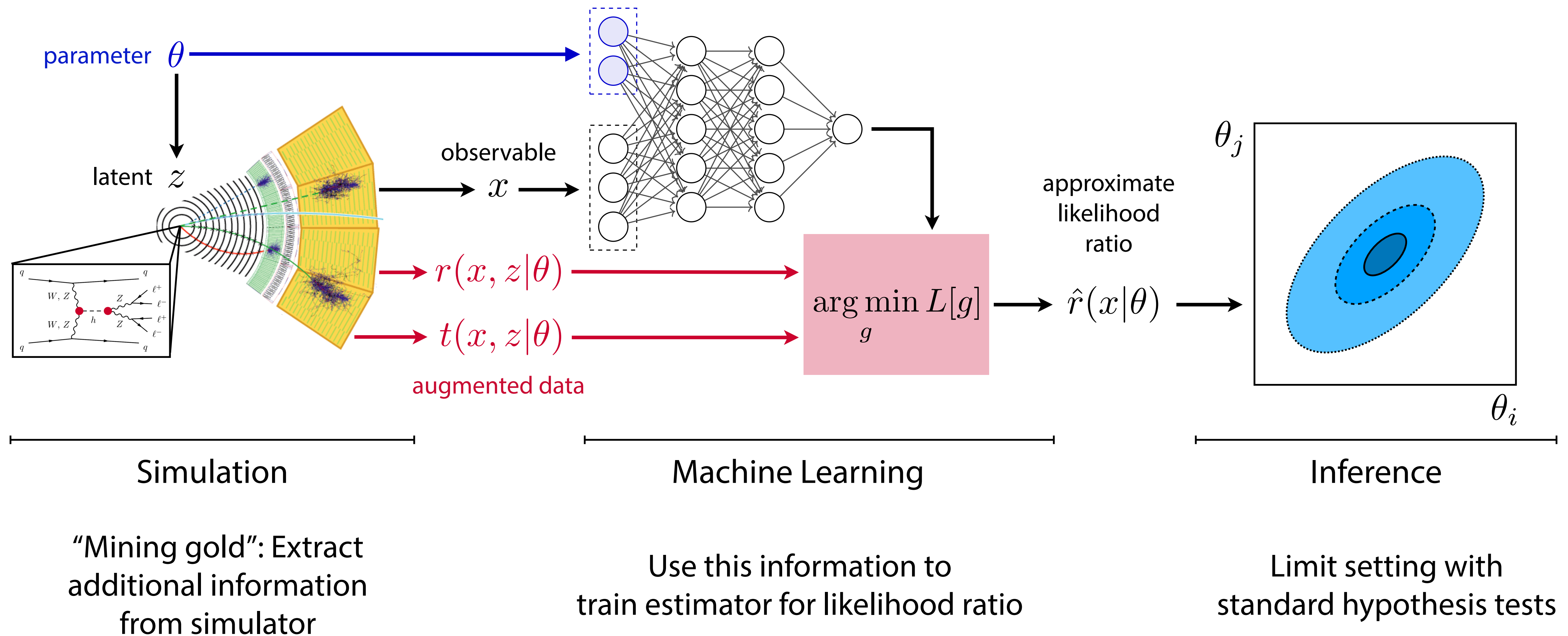
Inference

“Mining gold”: Extract additional information from simulator

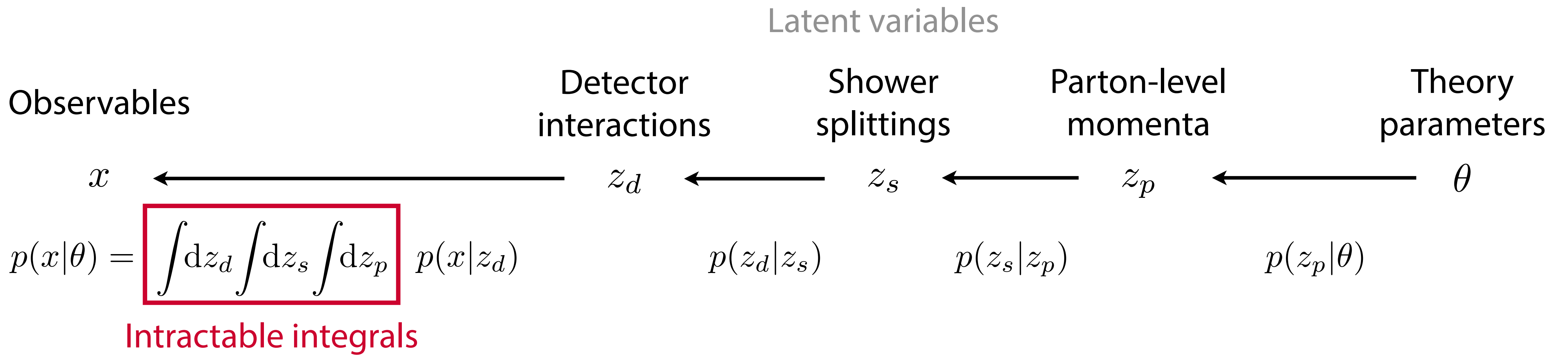
Use this information to train estimator for likelihood ratio

Limit setting with standard hypothesis tests

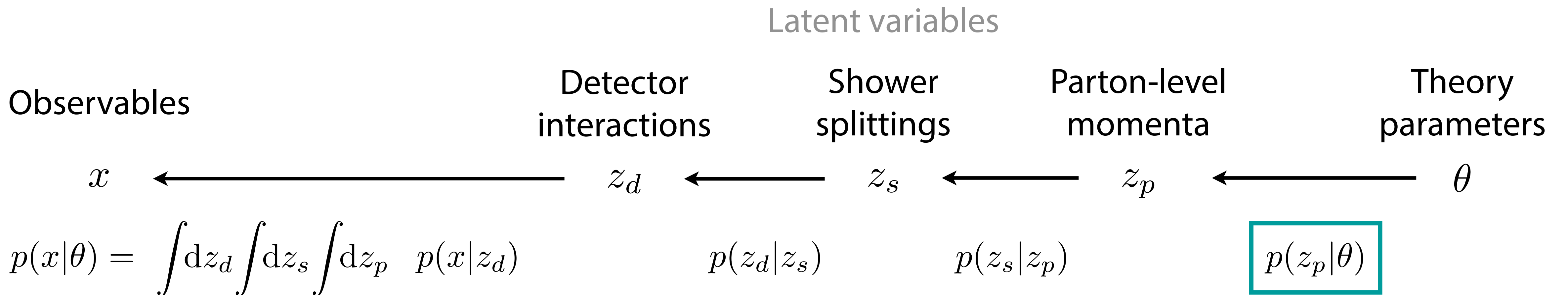
# Learning with Augmented Data



# Mining gold from the simulator



# Mining gold from the simulator



Parton-level likelihood is given by matrix element and can be evaluated!

⇒ For each simulated event, we can calculate the **joint likelihood ratio** which depends on the specific evolution of the simulation:

$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)} = \frac{p(x|z_d)}{p(x|z_d)} \frac{p(z_d|z_s)}{p(z_d|z_s)} \frac{p(z_s|z_p)}{p(z_s|z_p)} \boxed{\frac{p(z_p|\theta_0)}{p(z_p|\theta_1)} \sim \frac{|\mathcal{M}(z_p|\theta_0)|^2}{|\mathcal{M}(z_p|\theta_1)|^2}}$$

# The value of gold

We can calculate the **joint likelihood ratio**

$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)}$$



We want the **likelihood ratio function**

$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

(“How much more likely is this simulated event, including all intermediate states, for  $\theta_0$  compared to  $\theta_1$ ?”)

(“How much more likely is the observation  $x$  for  $\theta_0$  compared to  $\theta_1$ ?”)

# The value of gold

We can calculate the **joint likelihood ratio**

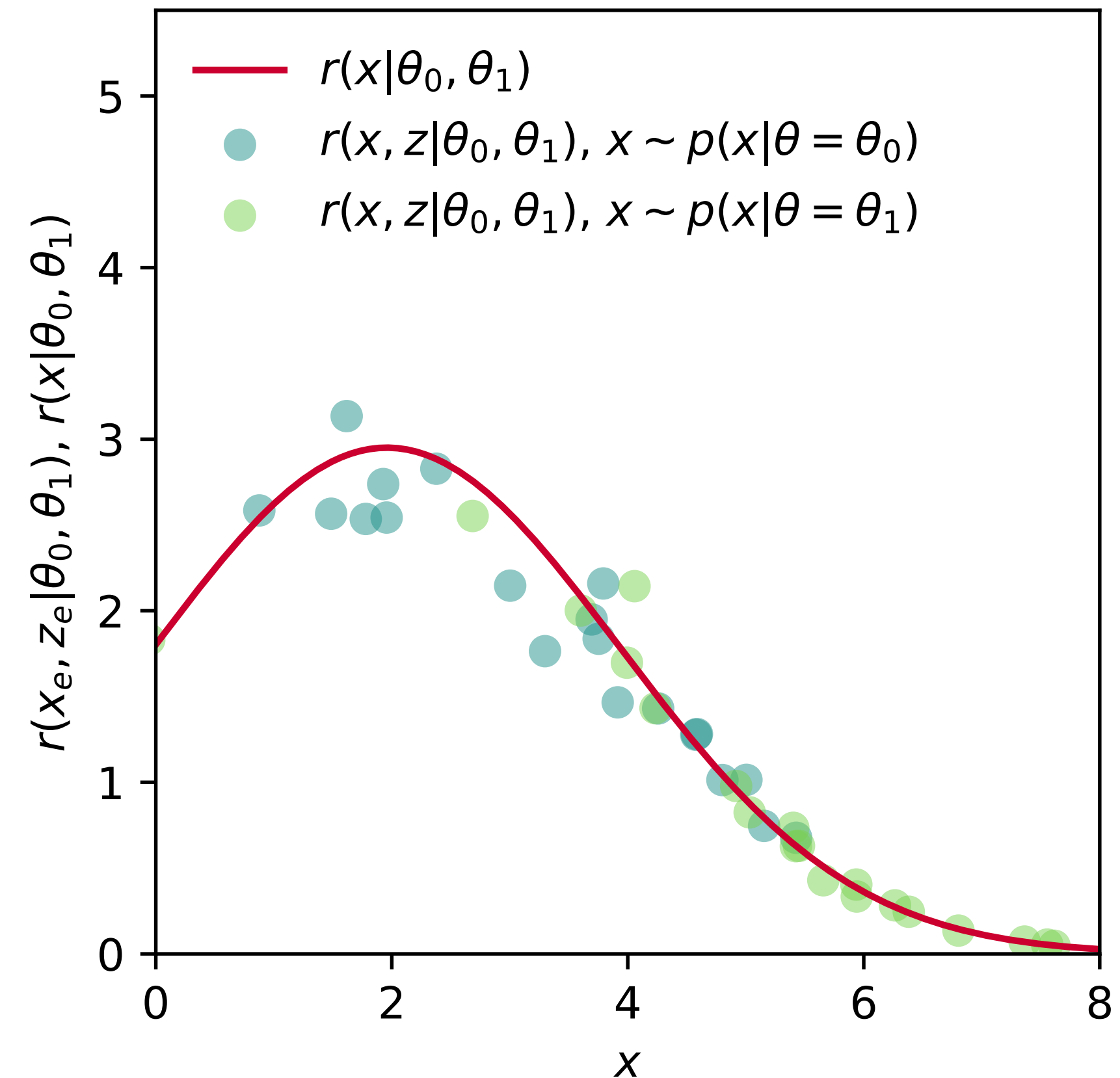
$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)}$$



$r(x, z|\theta_0, \theta_1)$  are scattered around  $r(x|\theta_0, \theta_1)$

We want the **likelihood ratio function**

$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$



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We want the **likelihood ratio function**

$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

With  $r(x, z|\theta_0, \theta_1)$ , we define a functional like

$$L_r[\hat{r}(x|\theta_0, \theta_1)] = \int dx \int dz p(x, z|\theta_1) \left[ (\hat{r}(x|\theta_0, \theta_1) - r(x, z|\theta_0, \theta_1))^2 \right]$$

It is minimized by

$$\mathbb{E}_{z \sim p(z|x, \theta_1)} [r(x, z|\theta_0, \theta_1)] = \arg \min_{\hat{r}(x|\theta_0, \theta_1)} L_r[\hat{r}(x|\theta_0, \theta_1)]!$$

(And we can sample from  $p(x, z|\theta)$  by running the simulator.)

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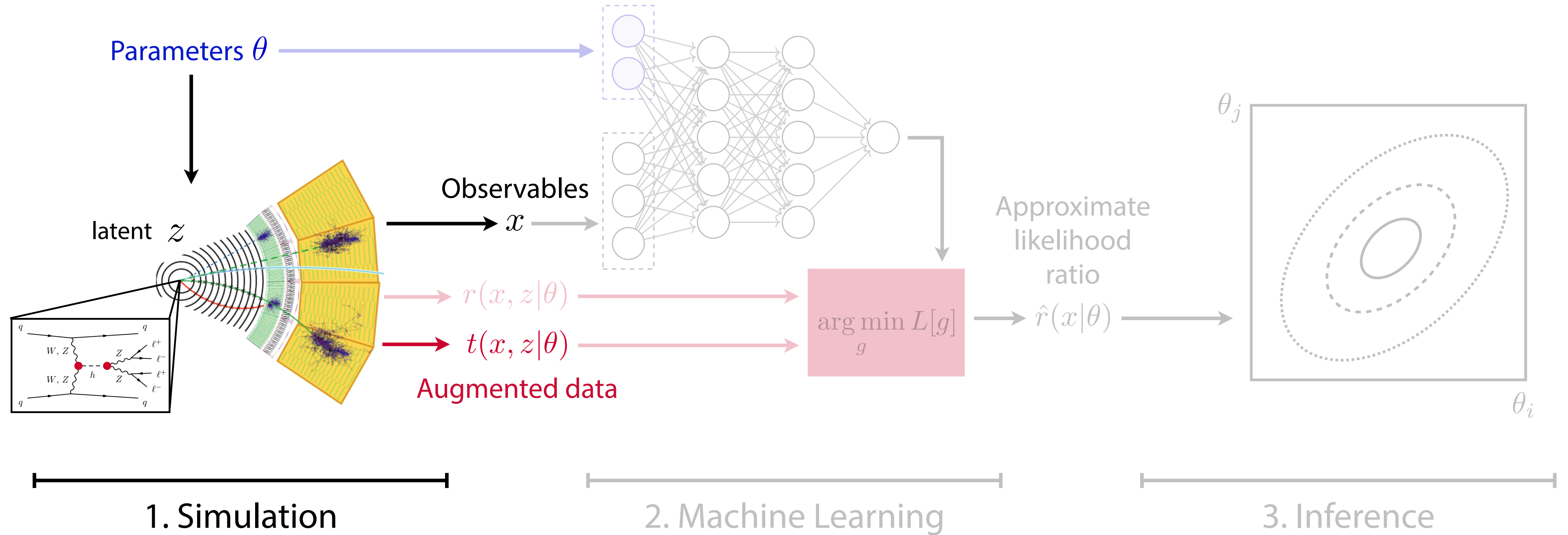
(And we can sample from  $p(x, z|\theta)$  by running the simulator.)

**.... and then magic ...**

$$\begin{aligned} \mathbb{E}_{z \sim p(z|x, \theta_1)} [r(x, z|\theta_0, \theta_1)] &= \int dz p(z|x, \theta_1) \frac{p(x, z|\theta_0)}{p(x, z|\theta_1)} \\ &= \int dz \frac{p(x, z|\theta_1)}{p(x|\theta_1)} \frac{p(x, z|\theta_0)}{p(x, z|\theta_1)} \\ &= r(x|\theta_0, \theta_1) ! \end{aligned}$$



# Learning with Augmented Data



# Learning the score (related to optimal observables)

Similar to the joint likelihood ratio, from the simulator we can extract the **joint score**

$$t(x, z|\theta_0) \equiv \nabla_{\theta} \log p(x, z_d, z_s, z_p|\theta) \Big|_{\theta_0}$$



We want the **score**

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Given  $t(x, z|\theta_0)$ ,  
we define the functional

$$L_t[\hat{t}(x|\theta_0)] = \int dx \int dz p(x, z|\theta_0) \left[ (\hat{t}(x|\theta_0) - t(x, z|\theta_0))^2 \right].$$

One can show it is minimized by

$$t(x|\theta_0) = \arg \min_{\hat{t}(x|\theta_0)} L_t[\hat{t}(x|\theta_0)].$$

Again, we implement this minimization through machine learning.

**MadMiner automates all of these methods.**

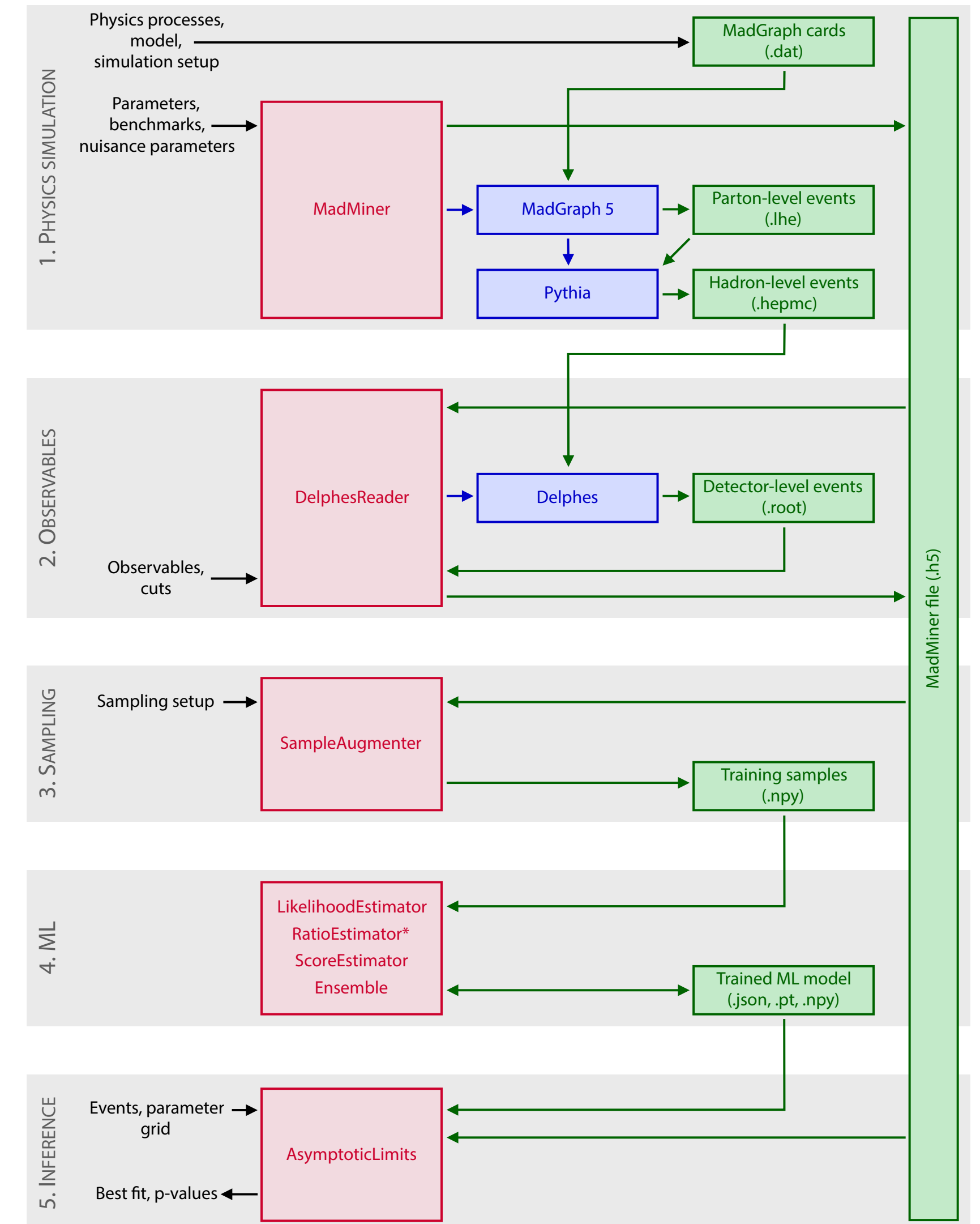
[JB, F. Kling, I. Espejo, K. Cranmer 1907.10621]

# MadMiner

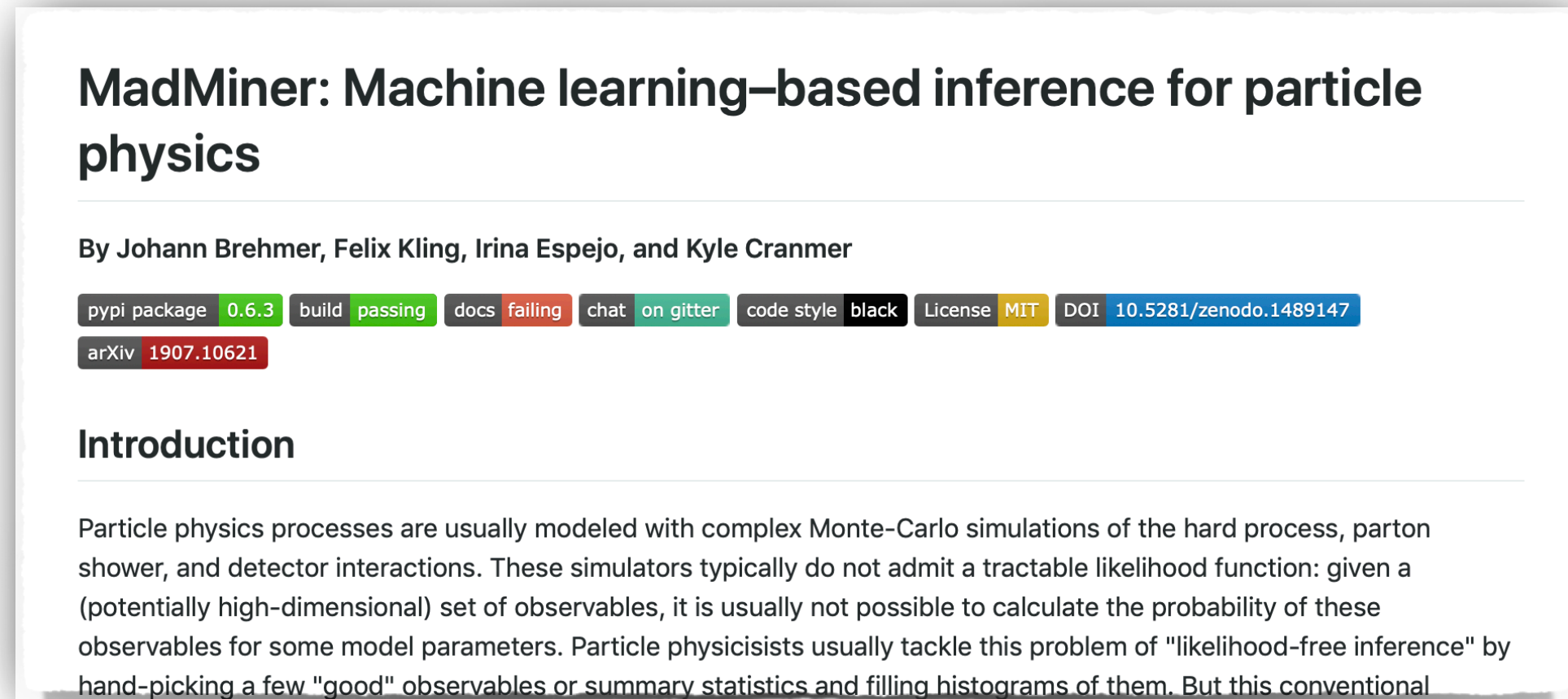
[JB, F. Kling, I. Espejo, K. Cranmer 1907.10621]

New Python package **MadMiner** makes it straightforward to apply the new techniques to LHC problems

- Out of the box: Pheno-level analyses
  - MadGraph, Pythia, Delphes, (could be GEANT4)
  - Systematic uncertainties from PDF / scale variation
- Scalable to state-of-the-art experimental tools
  - Mostly requires bookkeeping of fully differential cross sections
- Modular interface
  - Extensive documentation
  - Embedded into Python / ML ecosystem



# MadMiner resources



**MadMiner: Machine learning–based inference for particle physics**

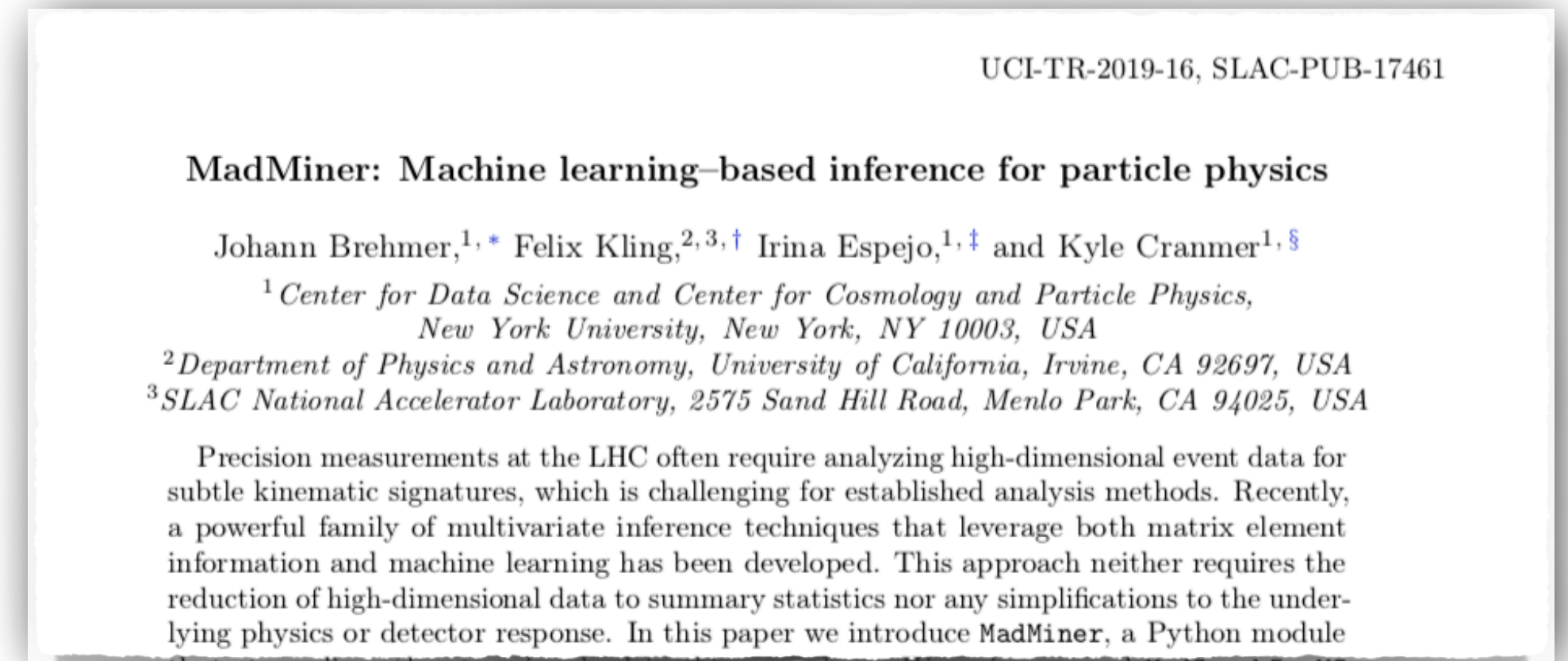
By Johann Brehmer, Felix Kling, Irina Espejo, and Kyle Cranmer

pypi package 0.6.3 build passing docs falling chat on gitter code style black License MIT DOI 10.5281/zenodo.1489147  
arXiv 1907.10621

### Introduction

Particle physics processes are usually modeled with complex Monte-Carlo simulations of the hard process, parton shower, and detector interactions. These simulators typically do not admit a tractable likelihood function: given a (potentially high-dimensional) set of observables, it is usually not possible to calculate the probability of these observables for some model parameters. Particle physicists usually tackle this problem of "likelihood-free inference" by hand-picking a few "good" observables or summary statistics and filling histograms of them. But this conventional

Repository and tutorials:  
[github.com/johannbrehmer/madminer](https://github.com/johannbrehmer/madminer)



UCI-TR-2019-16, SLAC-PUB-17461

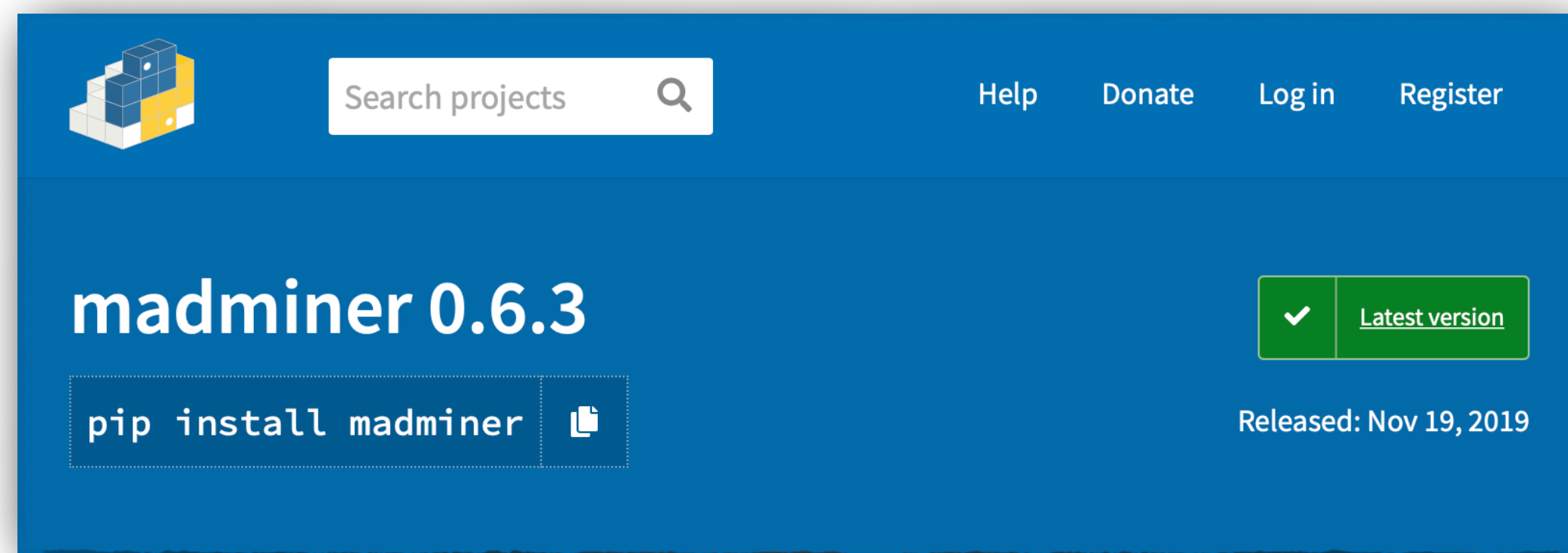
### MadMiner: Machine learning–based inference for particle physics

Johann Brehmer,<sup>1,\*</sup> Felix Kling,<sup>2,3,†</sup> Irina Espejo,<sup>1,‡</sup> and Kyle Cranmer<sup>1,§</sup>

<sup>1</sup>Center for Data Science and Center for Cosmology and Particle Physics,  
New York University, New York, NY 10003, USA  
<sup>2</sup>Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA  
<sup>3</sup>SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA

Precision measurements at the LHC often require analyzing high-dimensional event data for subtle kinematic signatures, which is challenging for established analysis methods. Recently, a powerful family of multivariate inference techniques that leverage both matrix element information and machine learning has been developed. This approach neither requires the reduction of high-dimensional data to summary statistics nor any simplifications to the underlying physics or detector response. In this paper we introduce **MadMiner**, a Python module

Paper with detailed explanations:  
[1907.10621](https://arxiv.org/abs/1907.10621)



Search projects

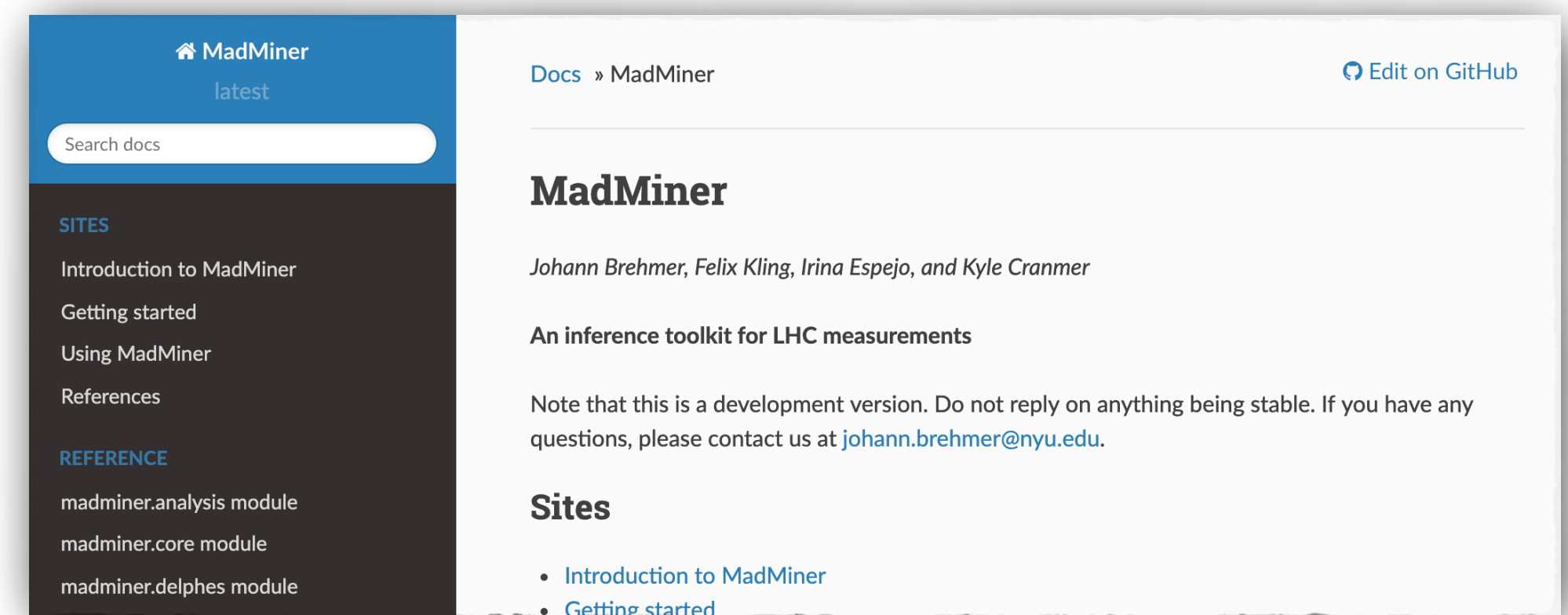
Help Donate Log in Register

## madminer 0.6.3

pip install madminer

Released: Nov 19, 2019

Installation:  
`pip install madminer`



MadMiner latest

Search docs

## MadMiner

Johann Brehmer, Felix Kling, Irina Espejo, and Kyle Cranmer

An inference toolkit for LHC measurements

Note that this is a development version. Do not rely on anything being stable. If you have any questions, please contact us at [johann.brehmer@nyu.edu](mailto:johann.brehmer@nyu.edu).

### Sites

- Introduction to MadMiner
- Getting started

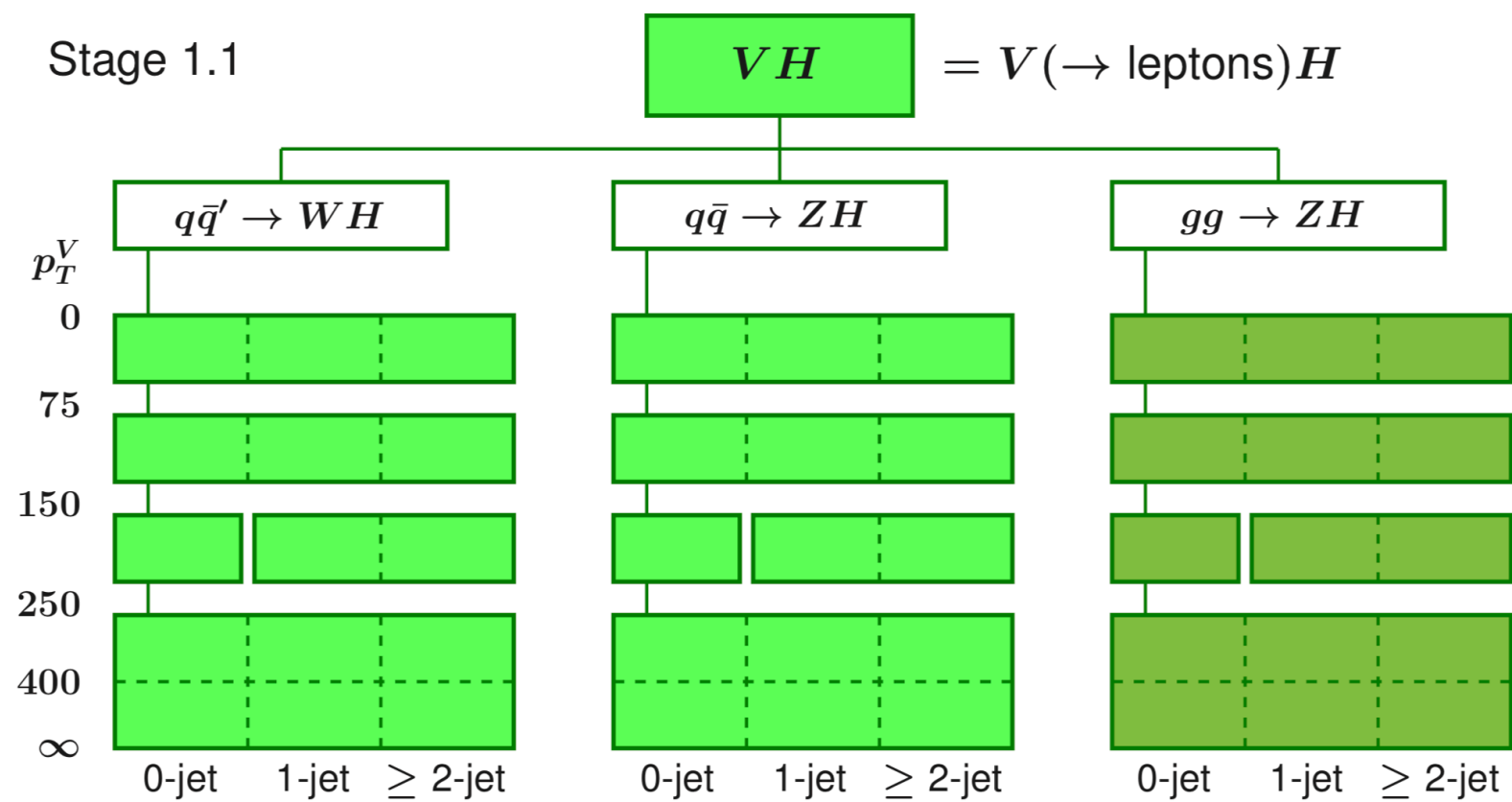
API documentation:  
[madminer.readthedocs.io](https://madminer.readthedocs.io)

# An example: STXS vs. MadMiner in WH

[JB, S. Dawson, S. Homiller, F. Kling, T. Plehn 1908.06980]

- Simplified Template Cross-Sections (STXS) define observable bins that are supposed to capture as much information on NP as possible

[N. Berger et al. 1906.02754; HXSWG YR4]



- Let's check! How much information on

$$\tilde{\mathcal{O}}_{HD} = \mathcal{O}_{H\Box} - \frac{\mathcal{O}_{HD}}{4} = (\phi^\dagger \phi) \Box (\phi^\dagger \phi) - \frac{1}{4} (\phi^\dagger D^\mu \phi)^* (\phi^\dagger D_\mu \phi)$$

$$\mathcal{O}_{HW} = \phi^\dagger \phi W_{\mu\nu}^a W^{\mu\nu a}$$

$$\mathcal{O}_{Hq}^{(3)} = (\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi) (\bar{Q}_L \sigma^a \gamma^\mu Q_L),$$

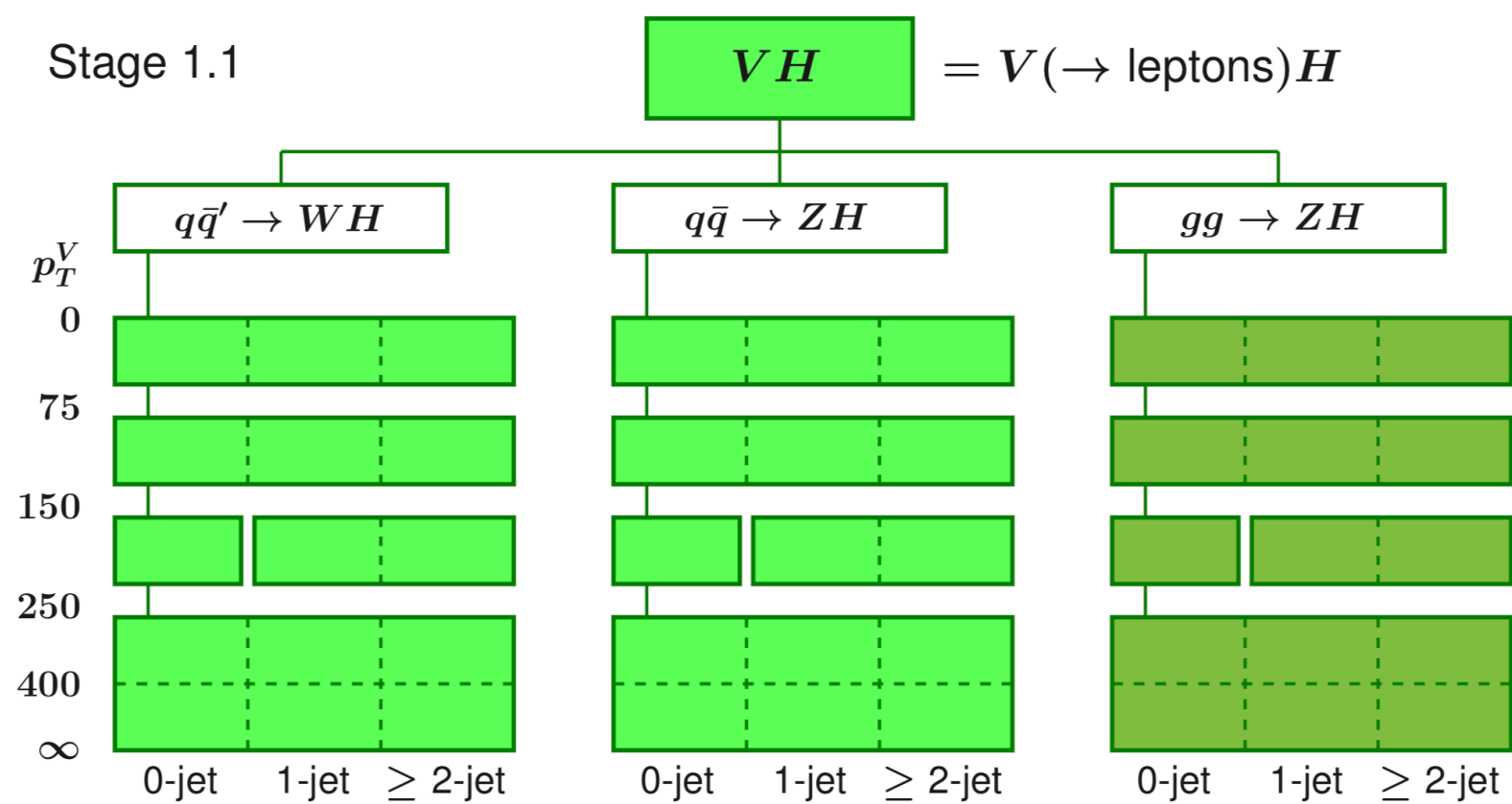
can we extract from  $pp \rightarrow WH \rightarrow \ell\nu b\bar{b}$  ?

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[N. Berger et al. 1906.02754; HXSWG YR4]



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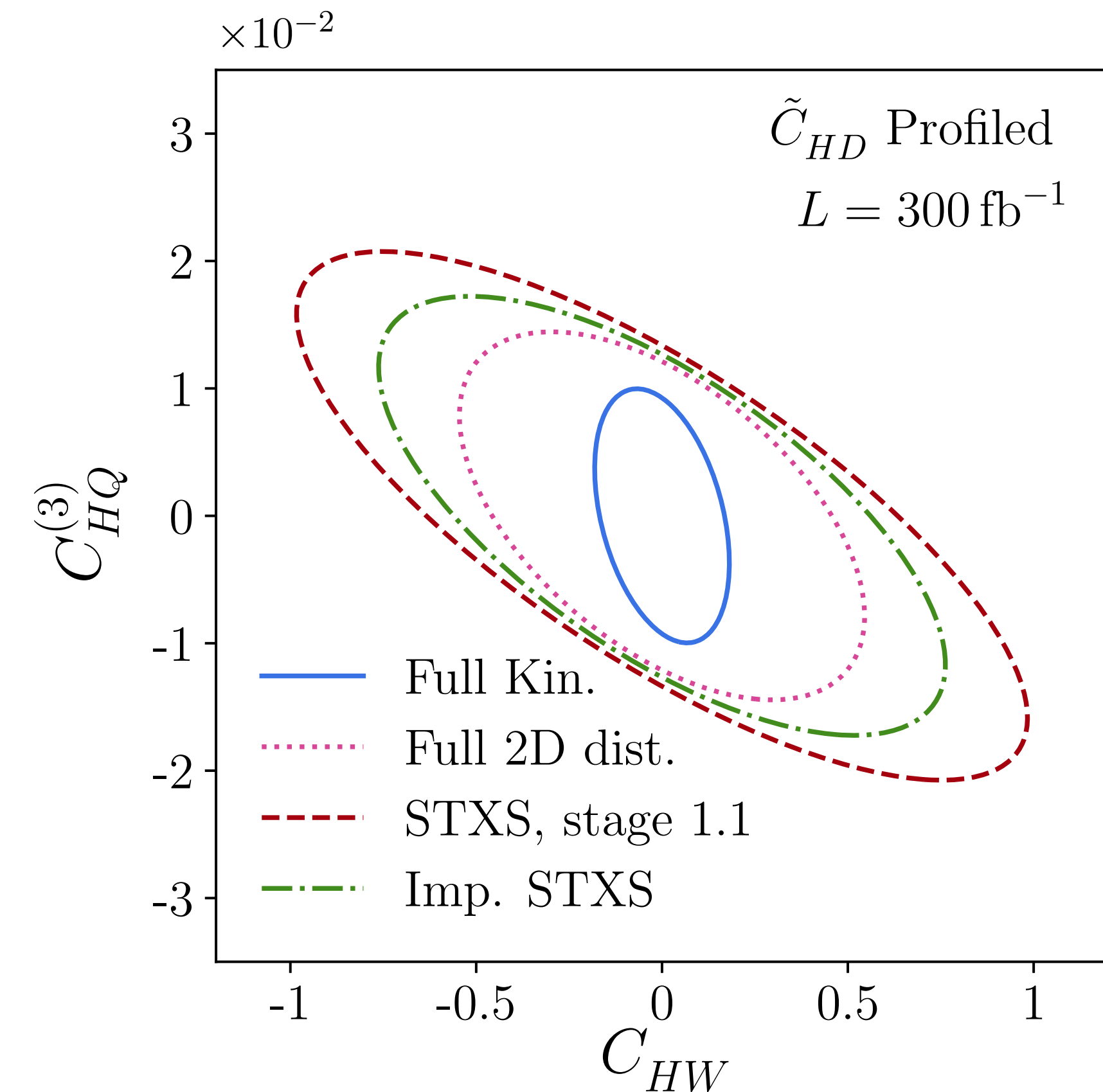
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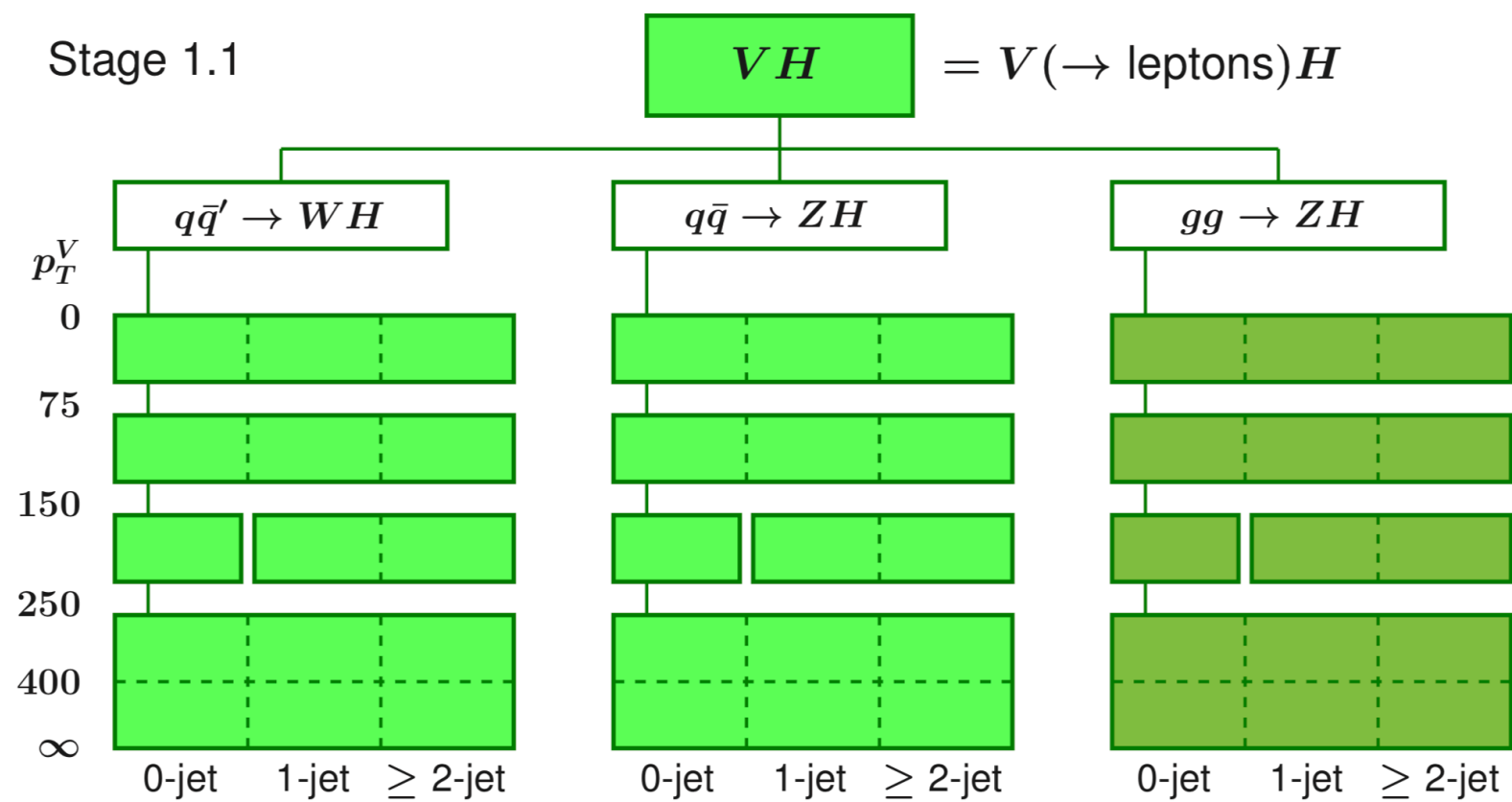


# An example: STXS vs. MadMiner in WH

[JB, S. Dawson, S. Homiller, F. Kling, T. Plehn 1908.06980]

- Simplified Template Cross-Sections (STXS) define observable bins that are supposed to capture as much information on NP as possible

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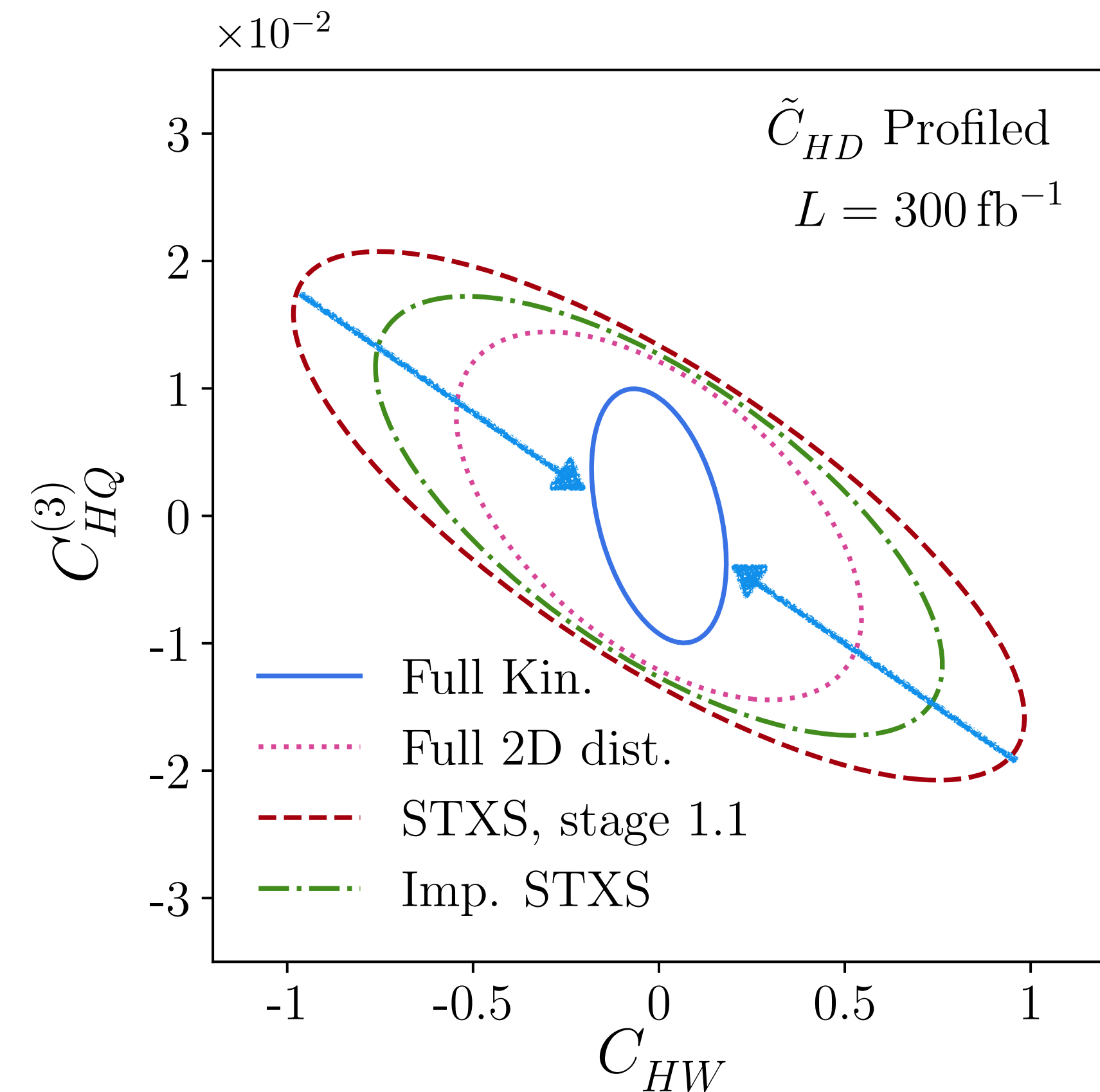
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# Video tutorials

1h15m video tutorial through

<https://www.youtube.com/watch?v=3kERPu5Wmgk>

- Step-by-step instructions
- Do Preliminaries
  - Setup REANA Client
  - Get the workflow
  - Run on REANA
  - Run locally
- There's also a video

<https://www.youtube.com/watch?v=jzU5j-FKy6g>

30m video running on REANA

<https://www.youtube.com/watch?v=lccGgEdUQJ8>

Below Kyle Cranmer gives an overview of the MadMiner workflows and how to set up REANA and yadage.

Watch on YouTube

Below Irina Espejo gives a demo of running MadMiner on the REANA instance at Brookhaven National Lab.

Watch on YouTube

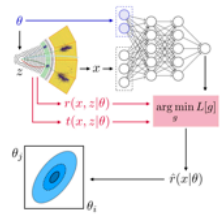
# reana

Reproducible research data analysis platform

<b>Flexible</b> Run many computational workflow engines.	<b>Scalable</b> Support for remote compute clouds.	<b>Reusable</b> Containerise once, reuse elsewhere. Cloud-native.	<b>Free</b> Free Software. MIT licence. Made with ❤️ at CERN.

# MadMiner Tutorial JupyterBook

<http://theoryandpractice.org/madminer-tutorial/intro>



## MadMiner Tutorial

Introduction

Demo video

## MadMiner Tutorial

Preliminaries

Overview

Define process to study \*

Morphing

Interactive Morphing Demo

Create training data

Set MadGraph Directory

Parton Level \*

With Delphes

Train model

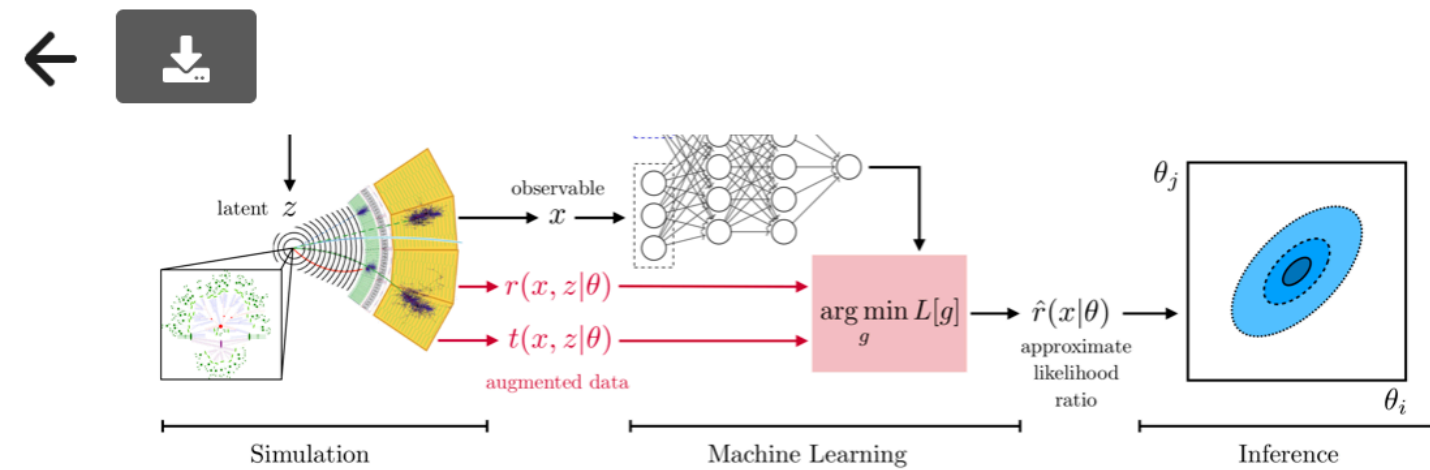
Likelihood Ratio \*

Score \*

Likelihood

Statistical Analysis

Limits on EFT parameters \*



## Introduction to MadMiner

Particle physics processes are usually modelled with complex Monte-Carlo simulations of the hard process, parton shower, and detector interactions. These simulators typically do not admit a tractable likelihood function: given a (potentially high-dimensional) set of observables, it is usually not possible to calculate the probability of these observables for some model parameters. Particle physicists usually tackle this problem of “likelihood-free inference” by hand-picking a few “good” observables or summary statistics and filling histograms of them. But this conventional approach discards the information in all other observables and often does not scale well to high-dimensional problems.

In the three publications “[Constraining Effective Field Theories With Machine Learning](#)”, “[A Guide to Constraining Effective Field Theories With Machine Learning](#)”, and “[Mining gold from implicit models to improve likelihood-free inference](#)”, a new approach has been developed. In a nut shell, additional information is extracted from the simulations that is closely related to the matrix elements that determine the hard process. This “augmented data” can be used to train neural networks to efficiently approximate arbitrary likelihood ratios. We playfully call this process “mining gold” from the simulator, since this information may be hard to get, but turns out to be very valuable for inference.

But the gold does not have to be hard to mine. This package automates these inference strategies. It wraps around the simulators MadGraph and Pythia, with different options for the detector simulation. All steps in the analysis chain from the simulation to the extraction of the augmented data, their processing, and the training and evaluation of the neural estimators are implemented.



### ON THIS PAGE

MADMINER TUTORIAL

[INTRODUCTION TO MADMINER](#)

[ABOUT THE TUTORIAL](#)

[THE PAPER DESCRIBING THE MADMINER TOOL:](#)

[DOCUMENTATION](#)

Special thanks to help with docker, binder, etc. points 📌



Irina Espejo



Sinclert Perez

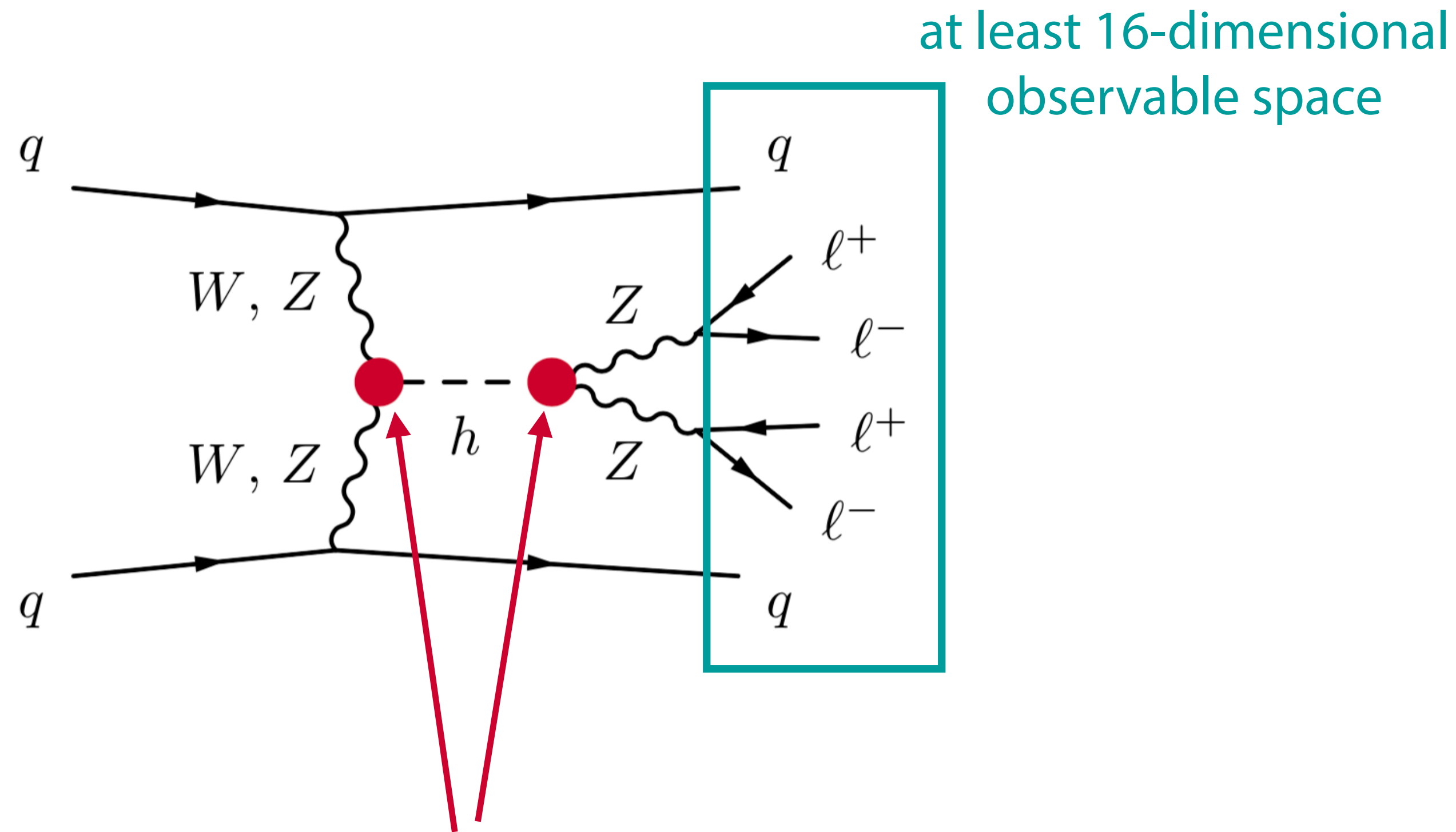


Matthew Feickert

More examples & Backup

# Proof of concept: Higgs production in weak boson fusion

[JB, K. Cranmer, G. Louppe, J. Pavez  
1805.00013, 1805.00020]



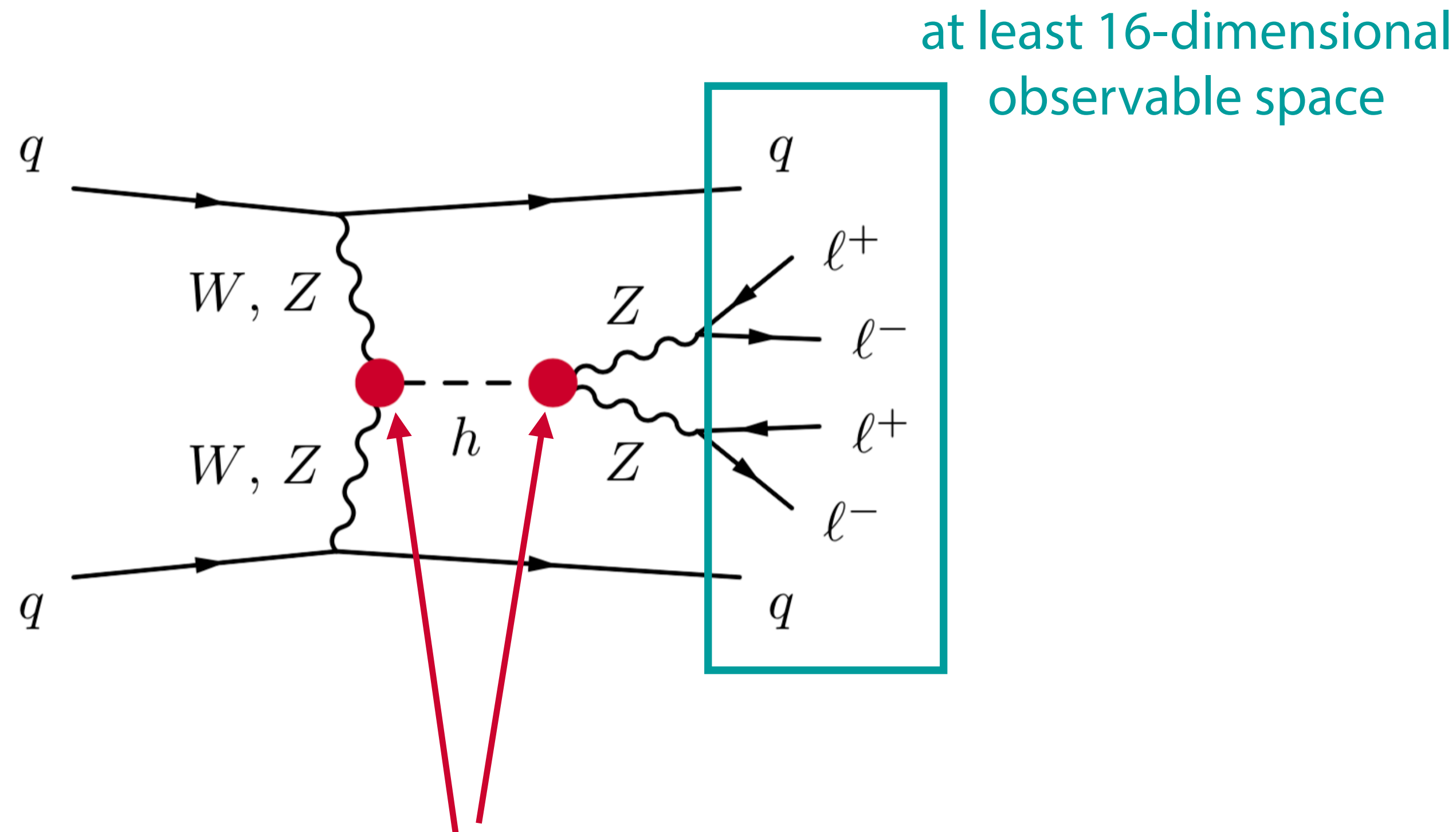
Exciting new physics might hide here!

We parameterize it with two EFT coefficients:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \boxed{\frac{f_W}{\Lambda^2}} \underbrace{\frac{ig}{2} (D^\mu \phi)^\dagger \sigma^a D^\nu \phi W_{\mu\nu}^a}_{\mathcal{O}_W} - \boxed{\frac{f_{WW}}{\Lambda^2}} \underbrace{\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^a W^{\mu\nu a}}_{\mathcal{O}_{WW}}$$

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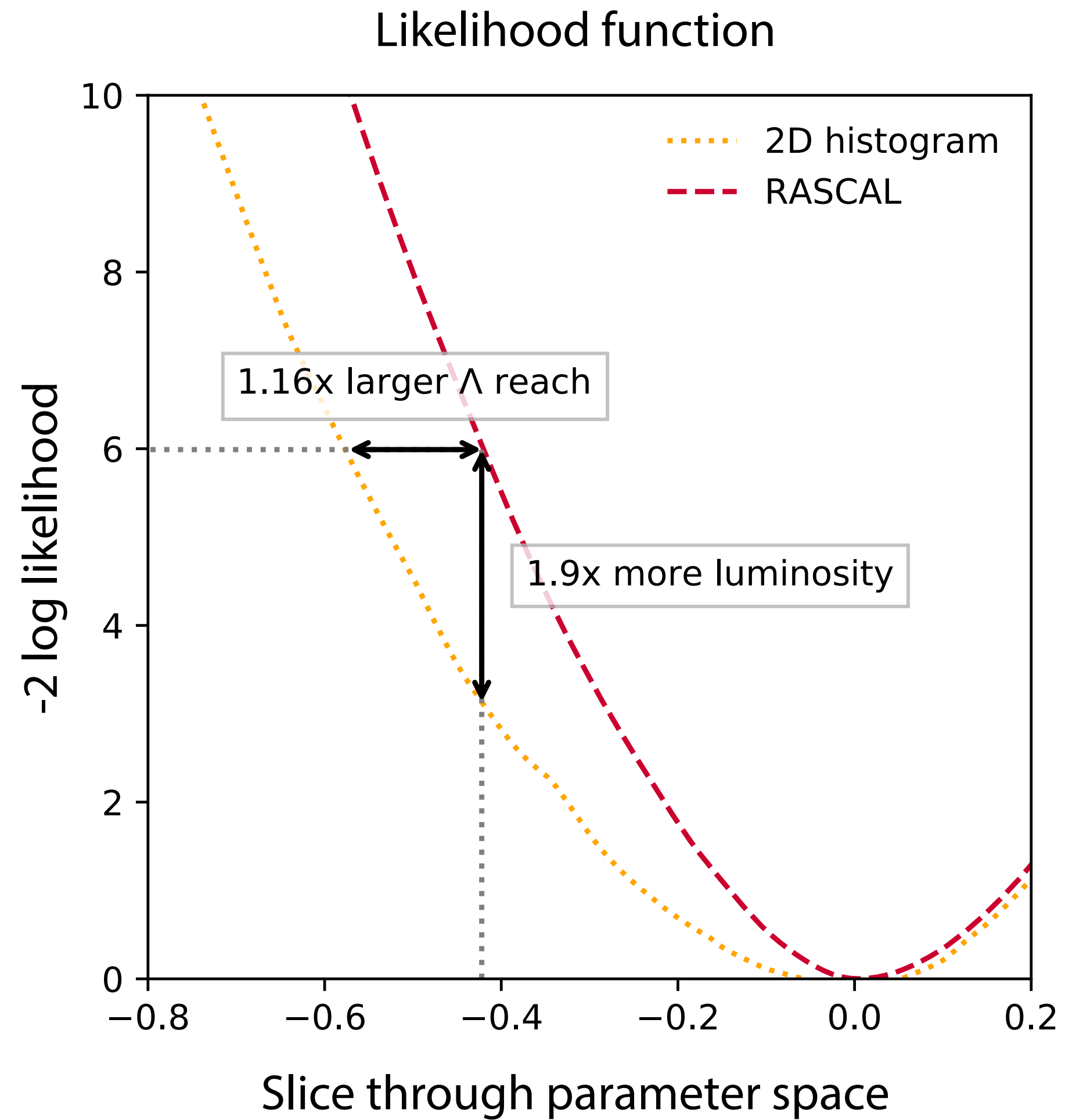
Goal: constrain the **two EFT parameters**

- new inference methods
- baseline: 2d histogram analysis of **jet momenta & angular correlations**

Two scenarios:

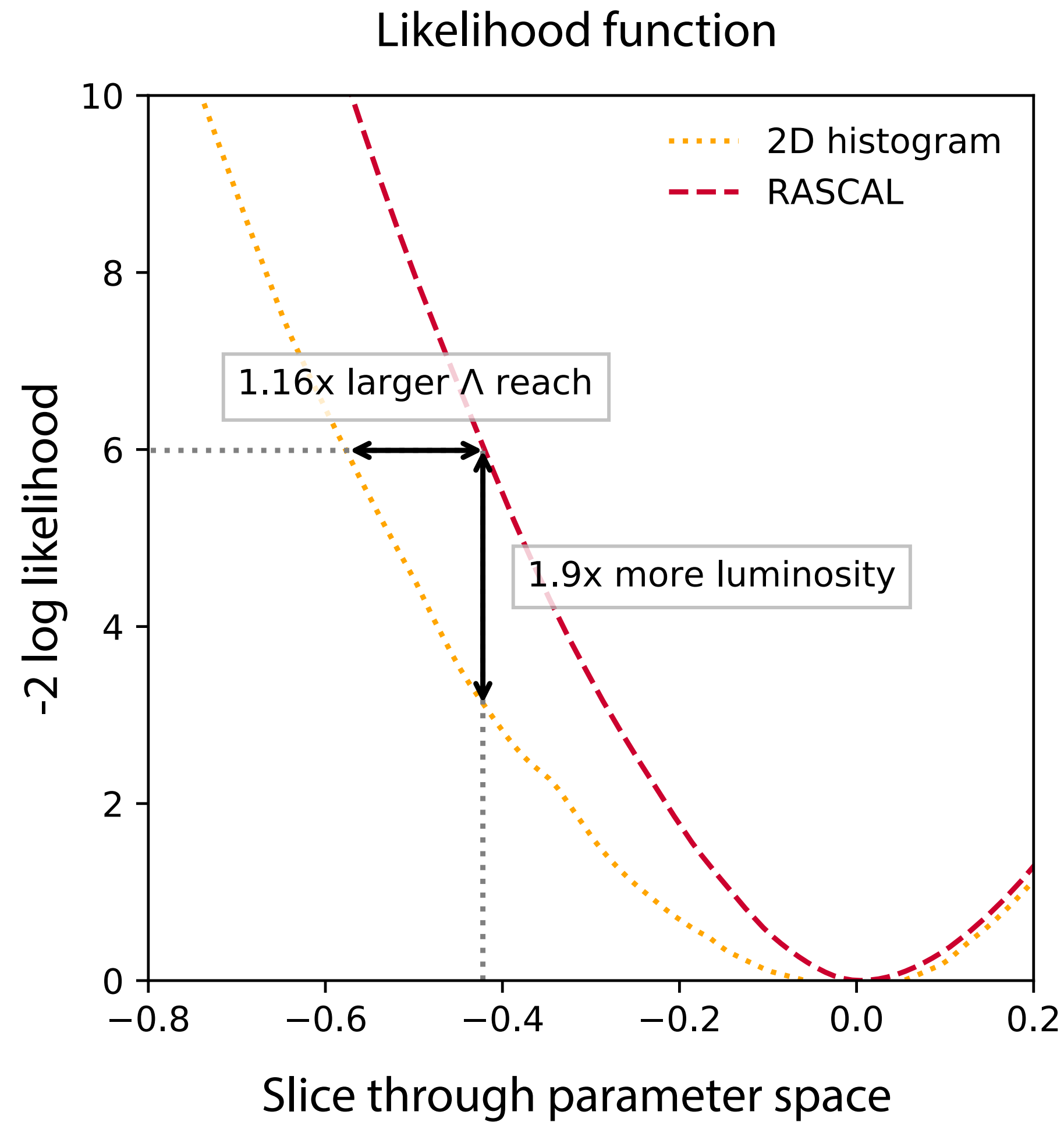
- Simplified setup in which we can compare to true likelihood
- "Realistic" simulation with approximate detector effects

# Better sensitivity to new physics

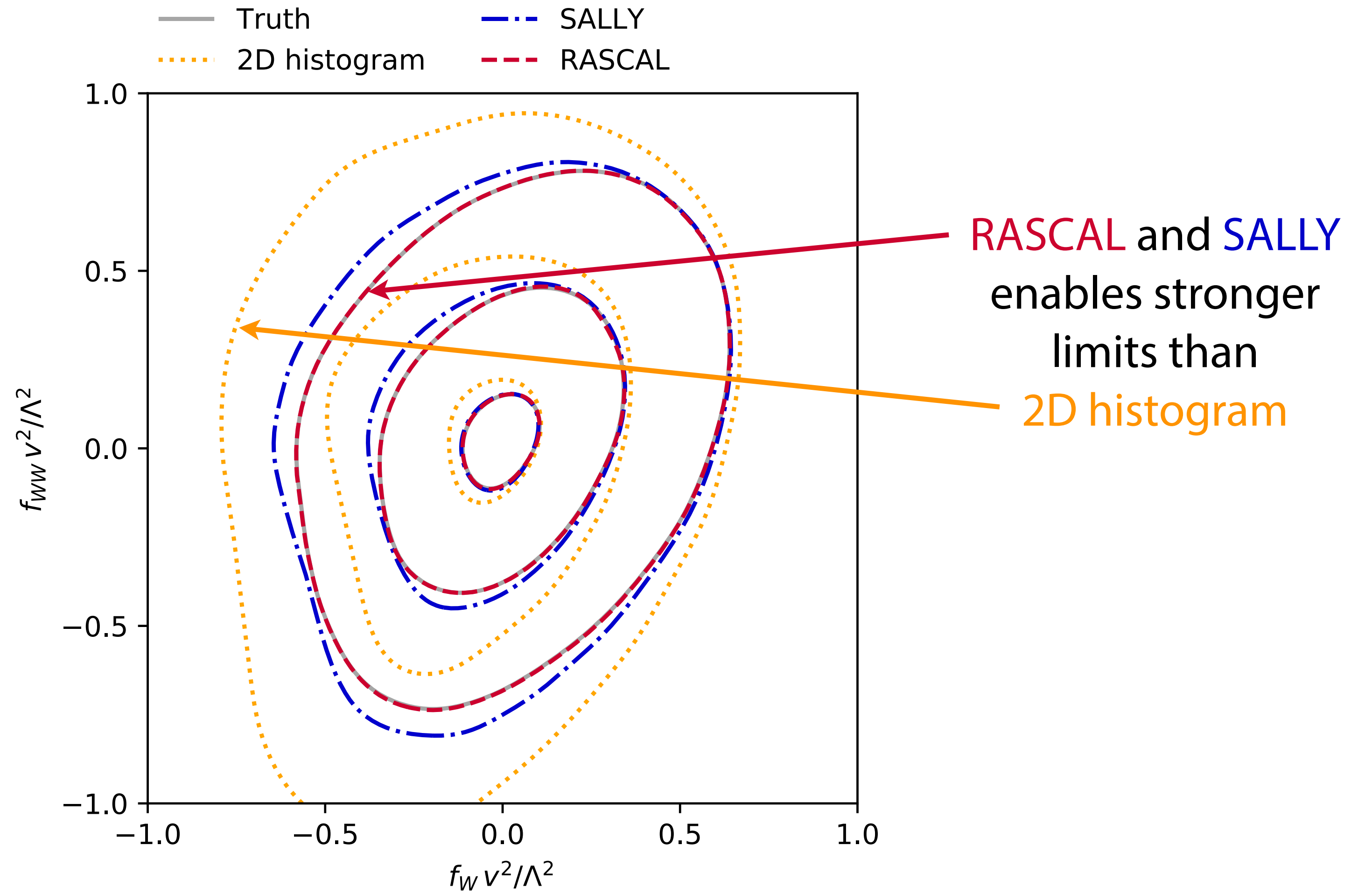


Results are based on 36 observed events, assuming SM

# Better sensitivity to new physics



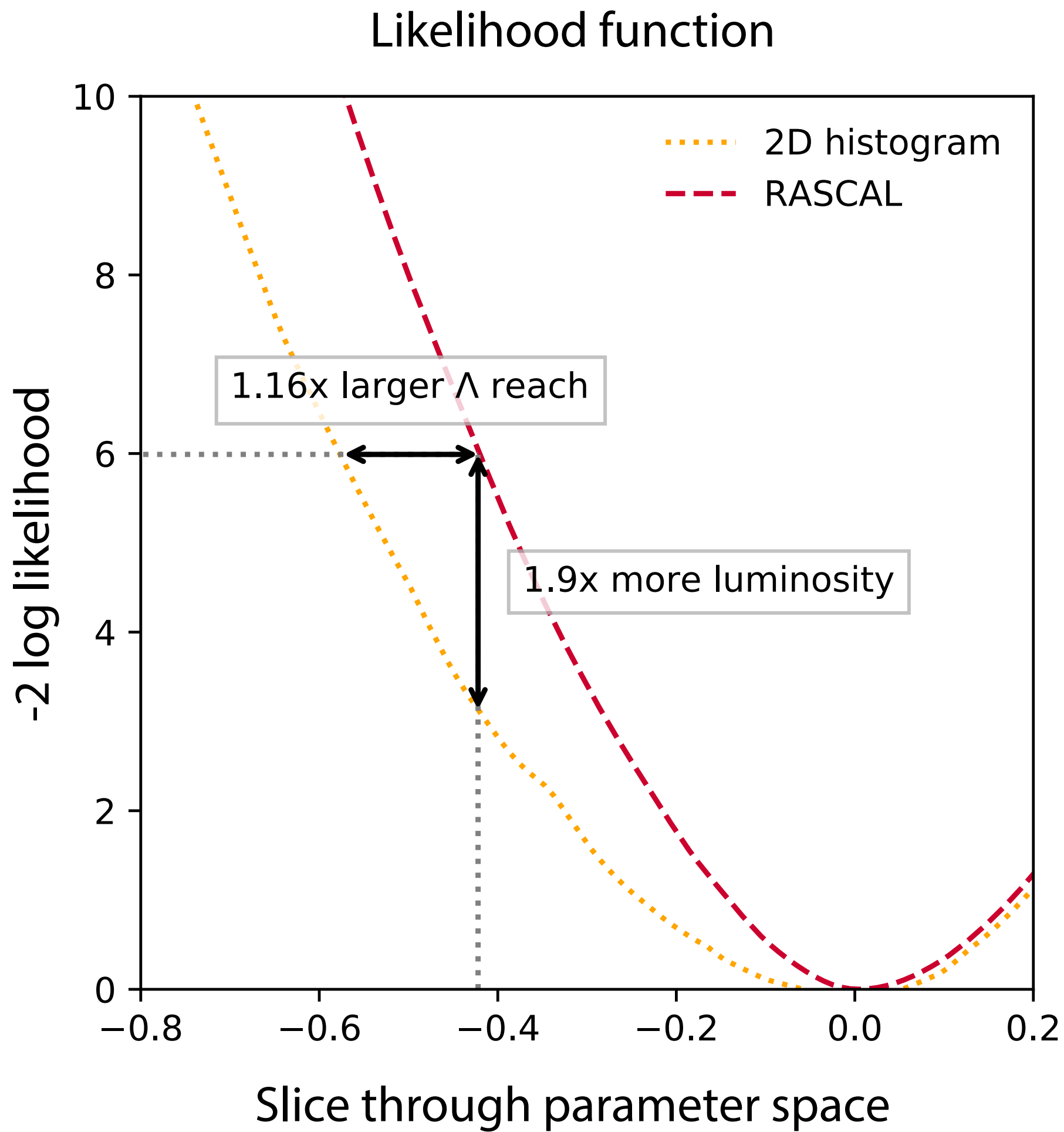
Expected exclusion limits at 68%, 95%, 99.7% CL



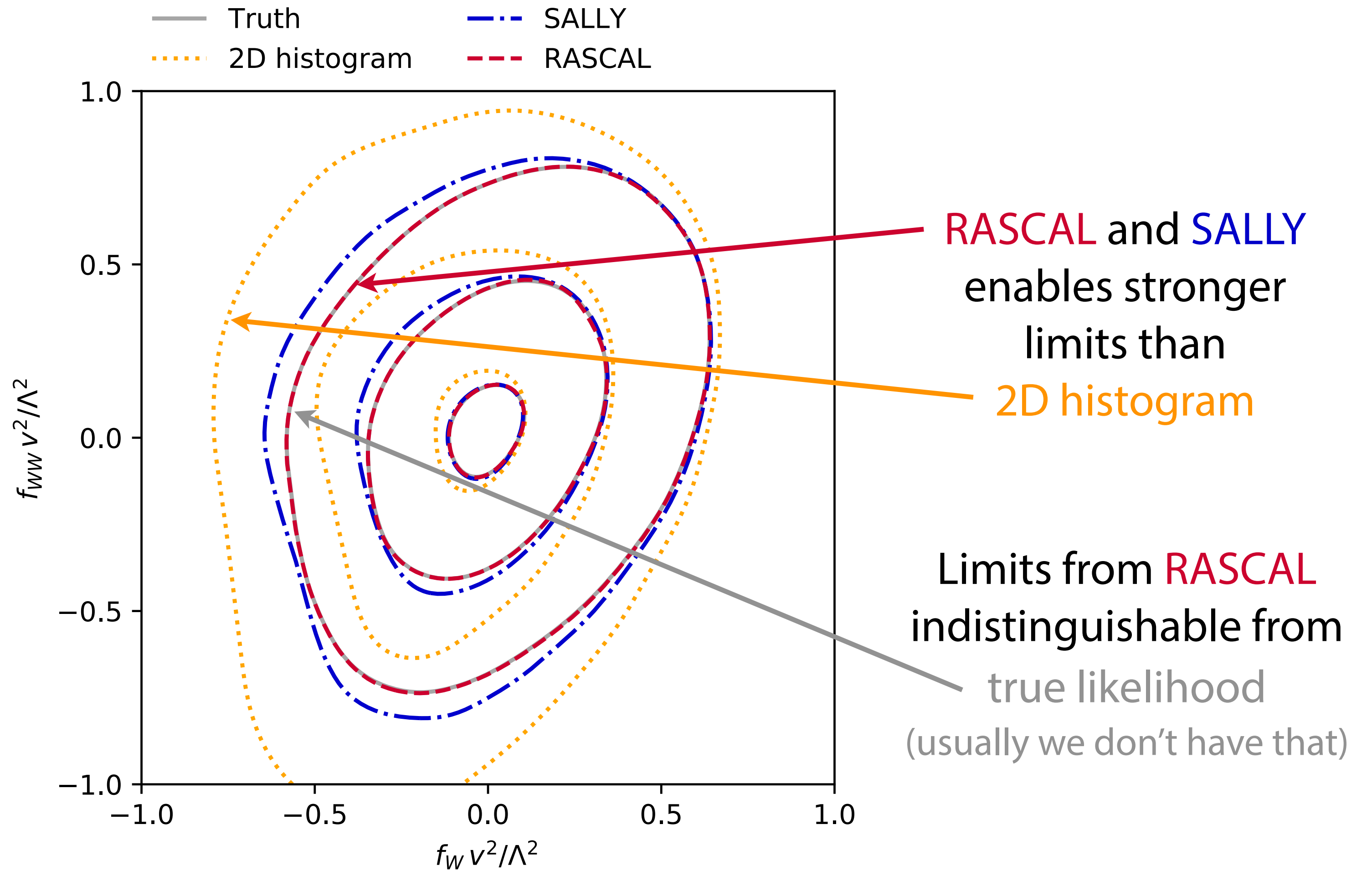
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# Constraining operators in ttH effectively

[JB, F. Kling, I. Espejo, K. Cranmer 1907.10621]

- Pheno-level analysis of

$$pp \rightarrow t\bar{t}h \rightarrow (bl^+) (\bar{b}l^-) (\gamma\gamma) E_T^{\text{miss}}$$

with MadGraph + Pythia + Delphes

- Inference on three EFT operators:

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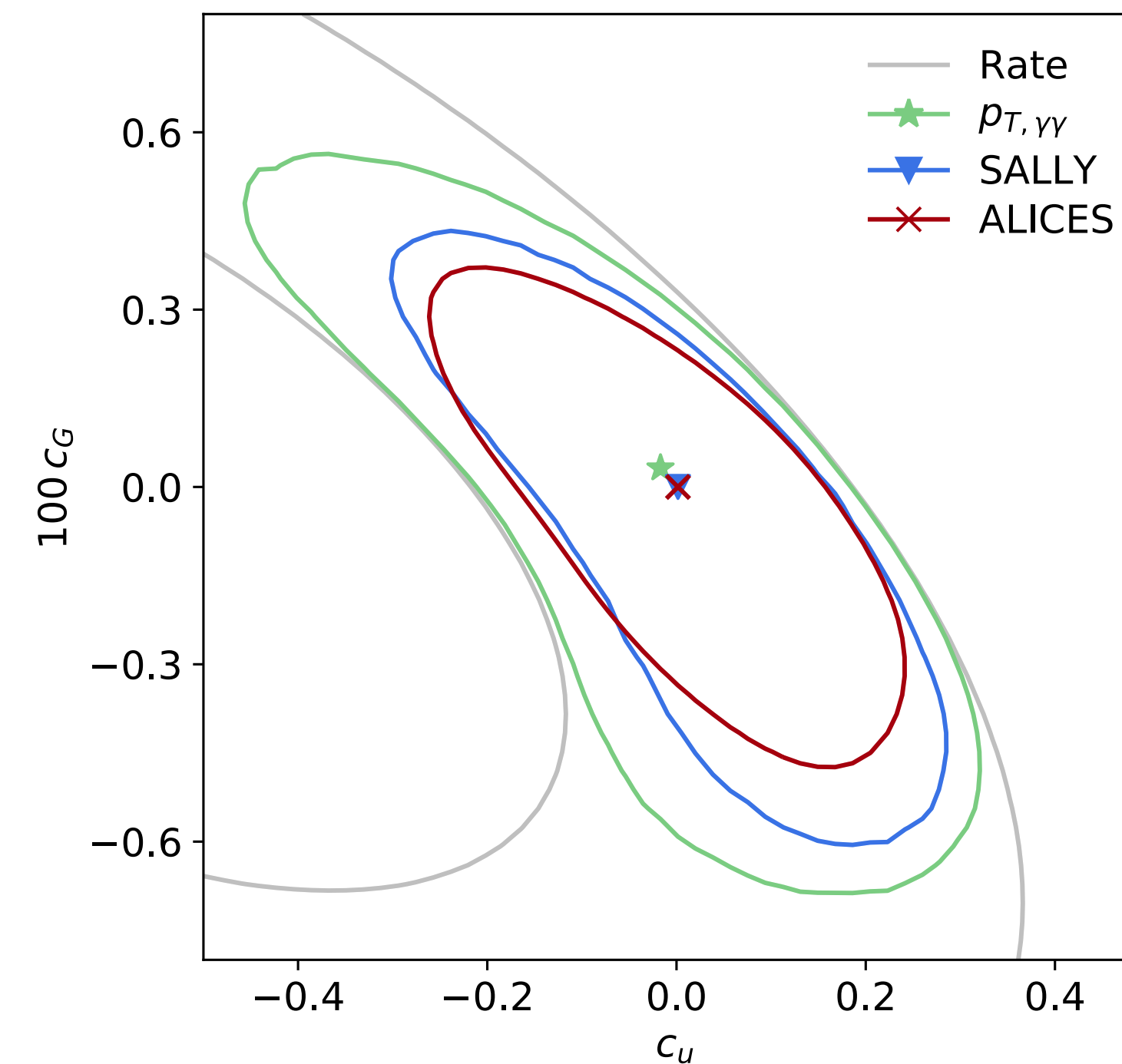
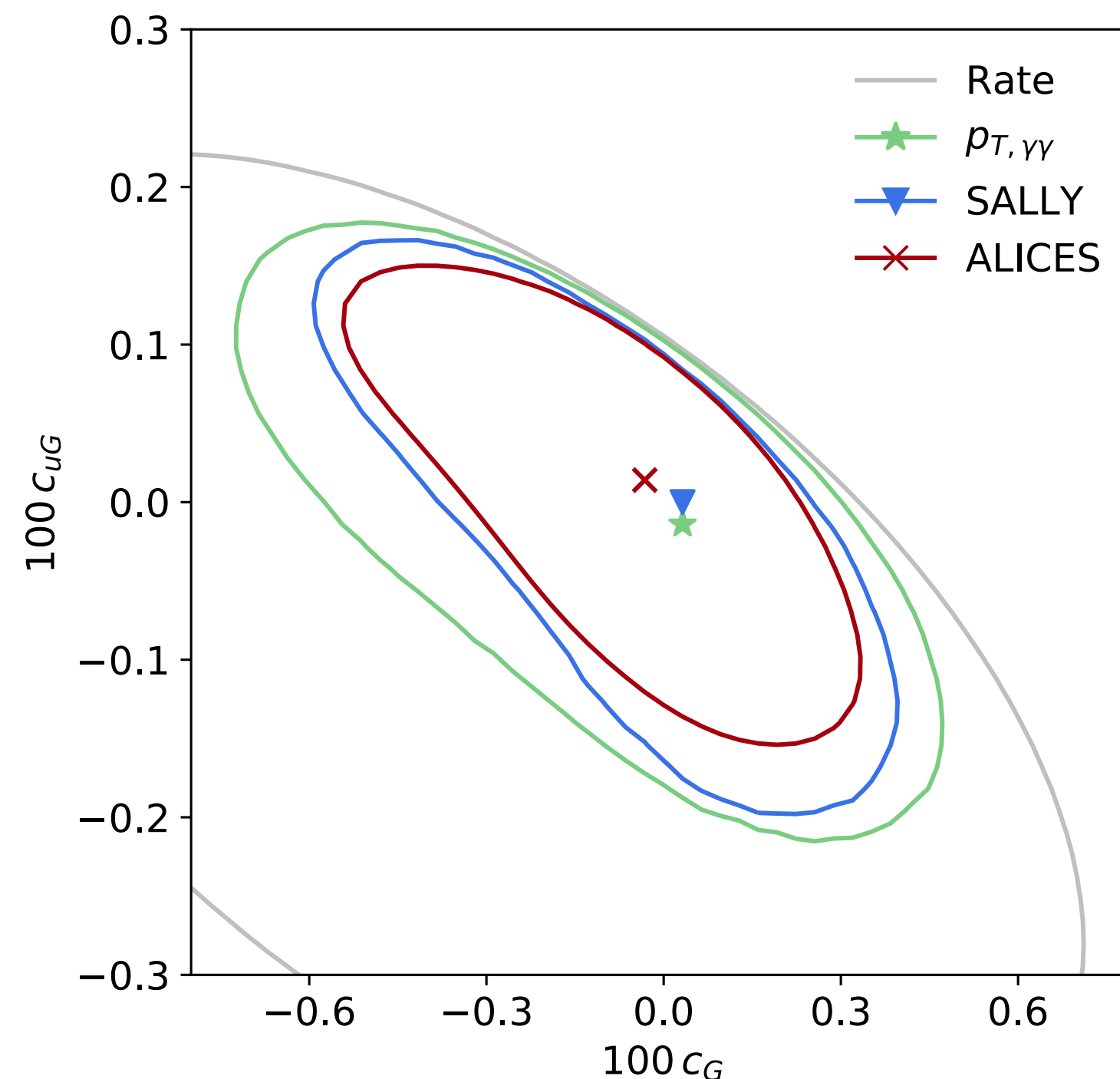
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- New **inference techniques** improve expected HL-LHC limits compared to **histogram baseline**:

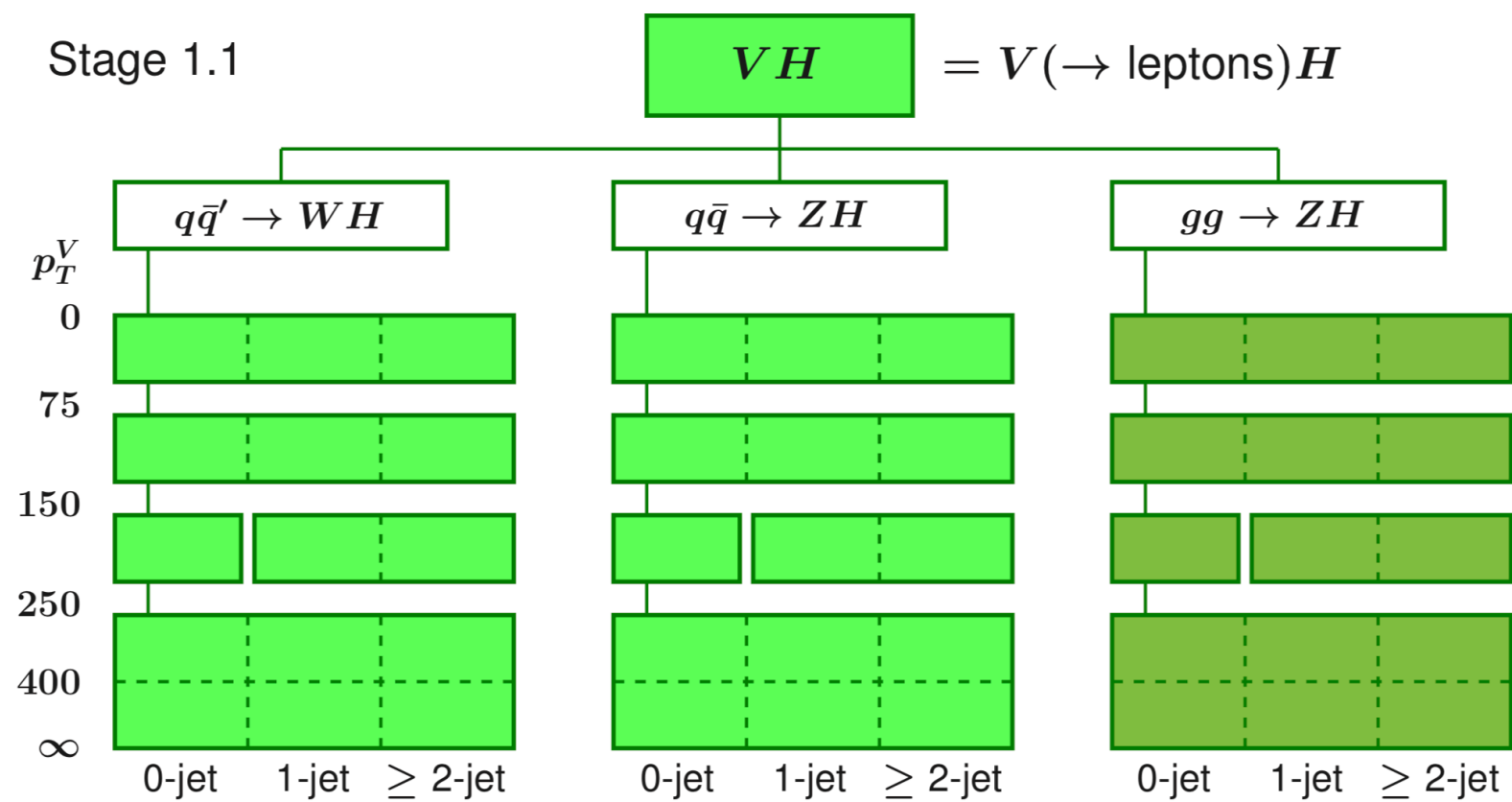


# Benchmarking STXS in WH

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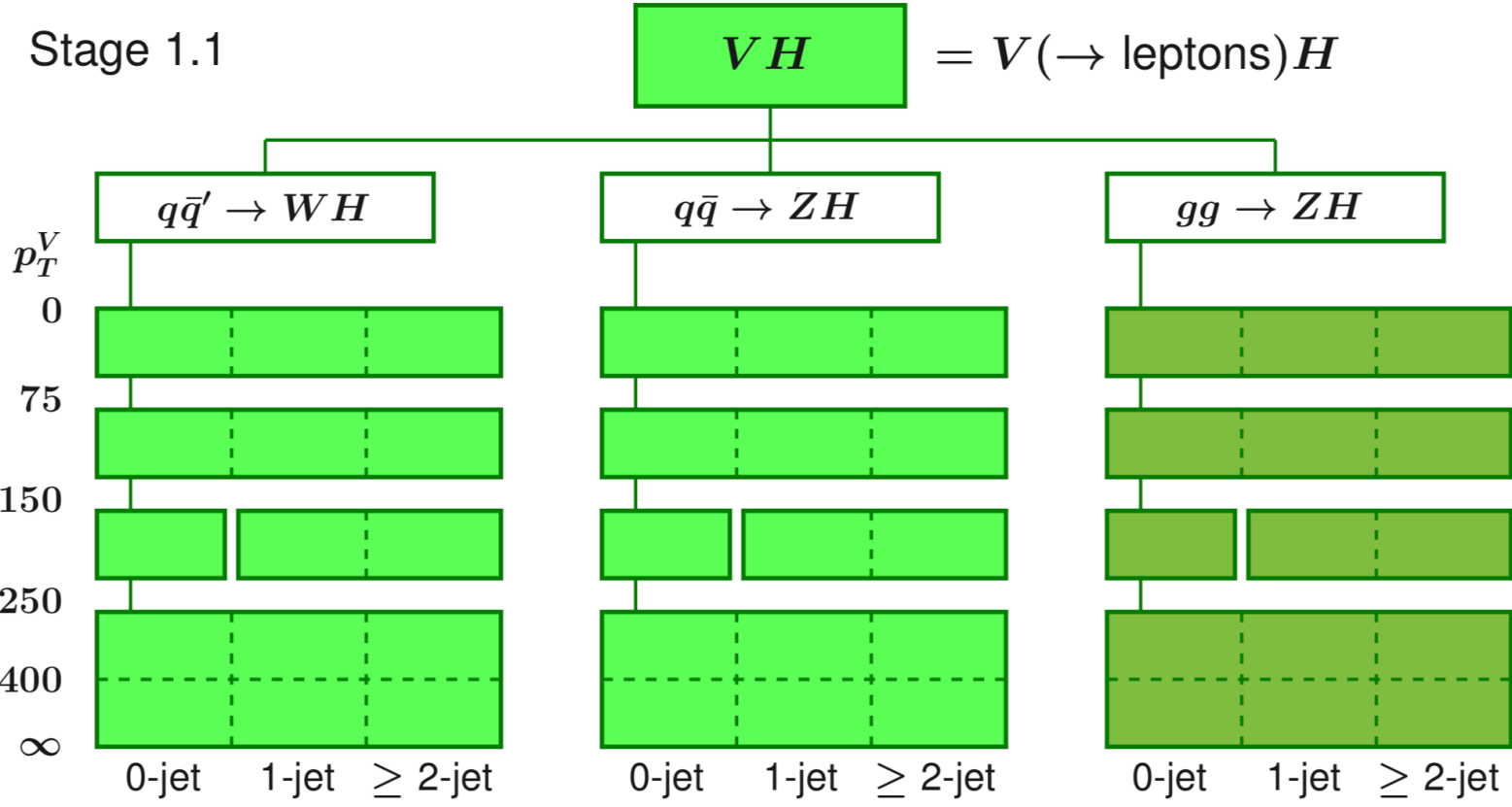
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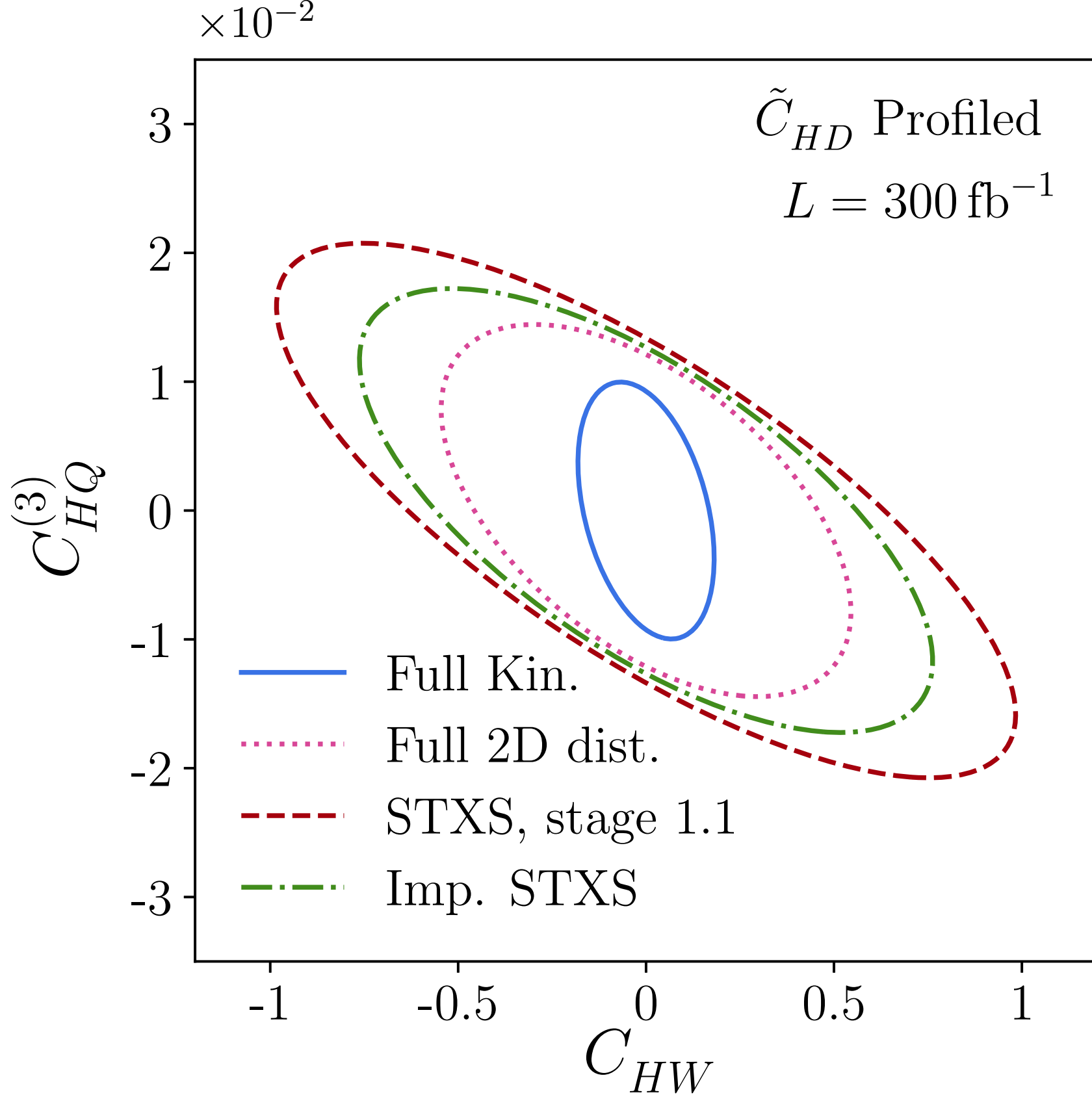
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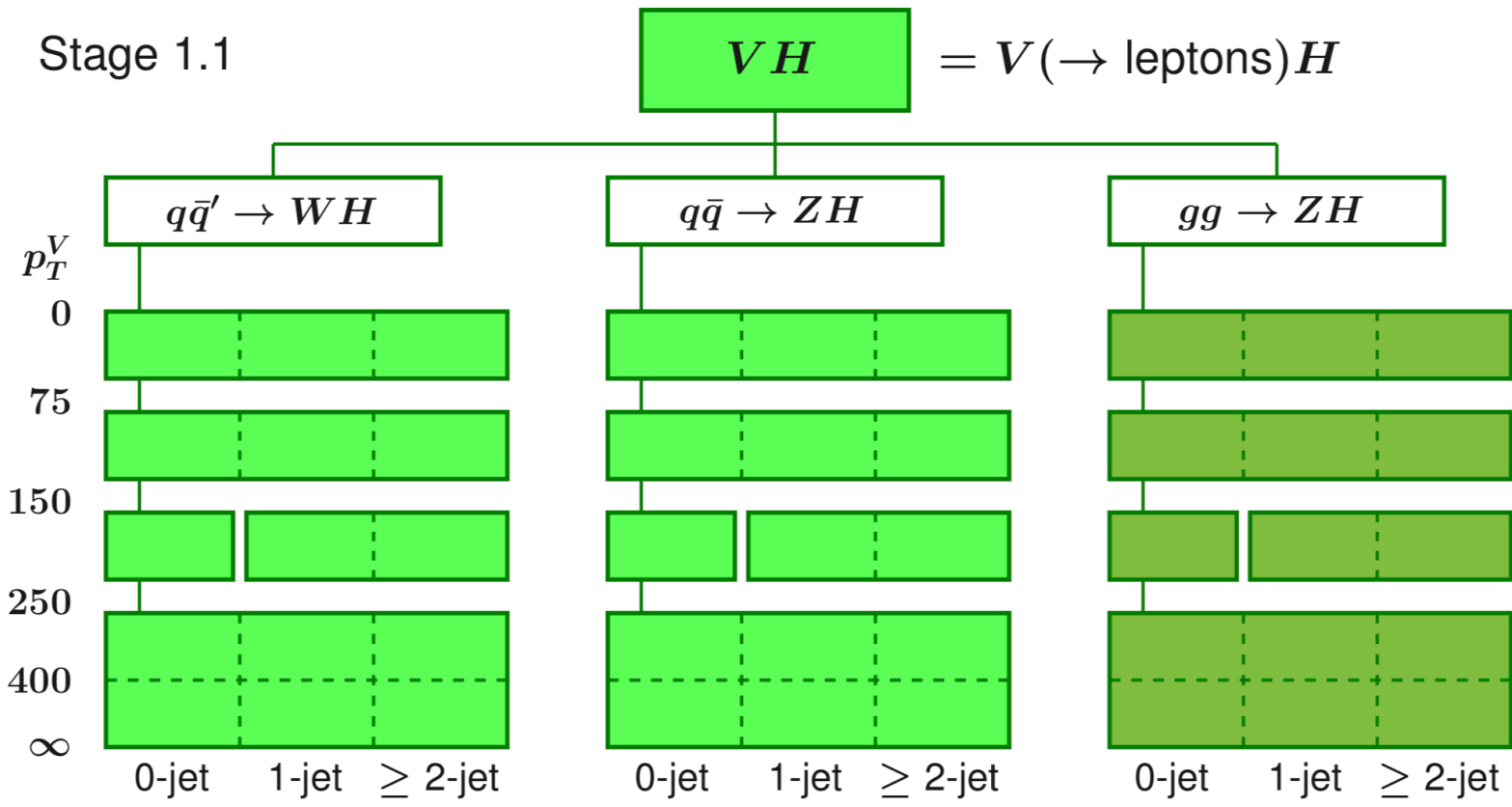


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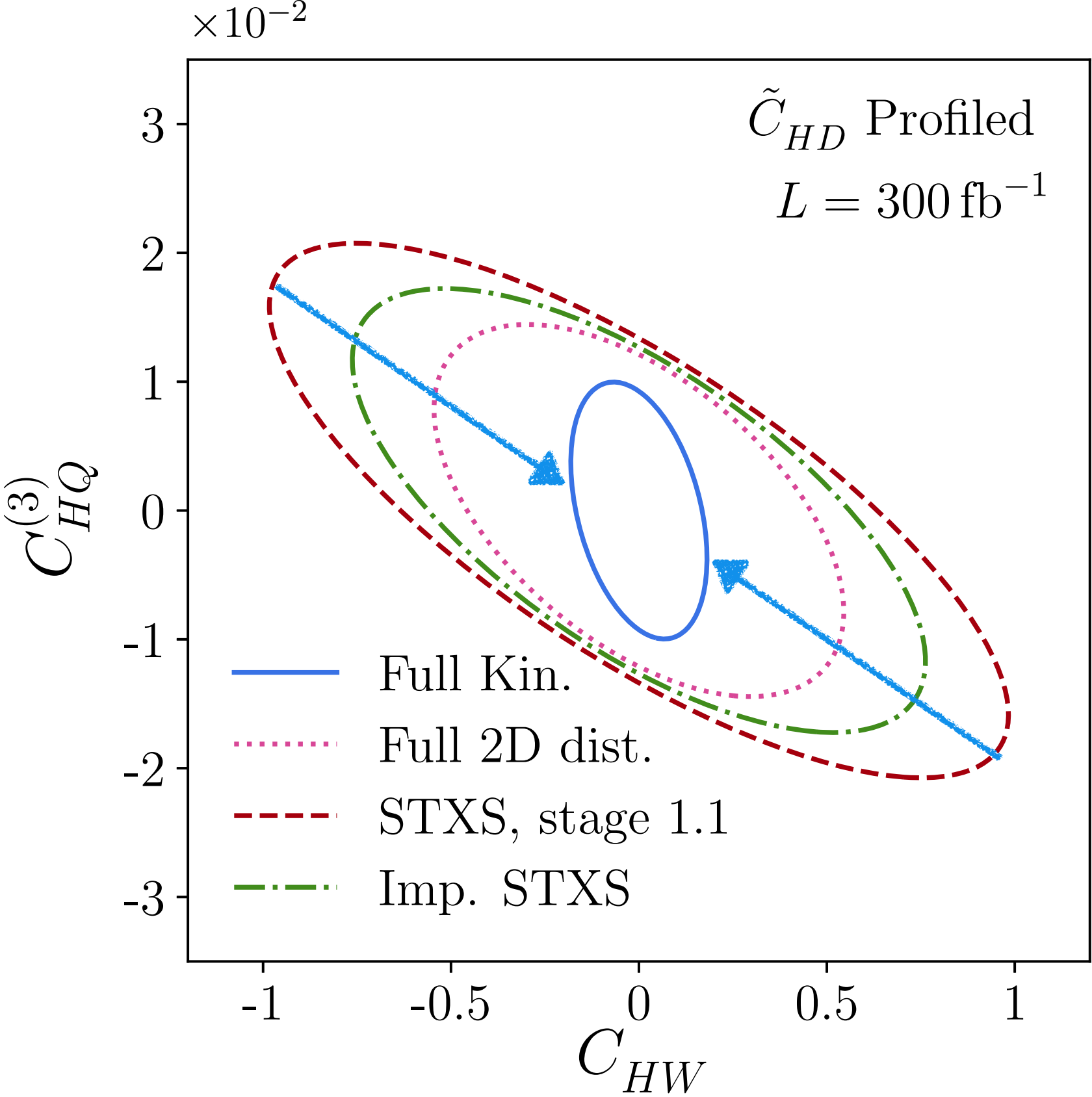
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# Diboson production

- In inclusive observables, the interference between SM and new physics amplitudes vanishes

⇒ Reduced sensitivity to new physics

- “Diboson interference resurrection”:  
an **angular variable**  $\varphi$  can be constructed to be sensitive to this interference

[G. Panico, F. Riva, A. Wulzer 1708.07823;

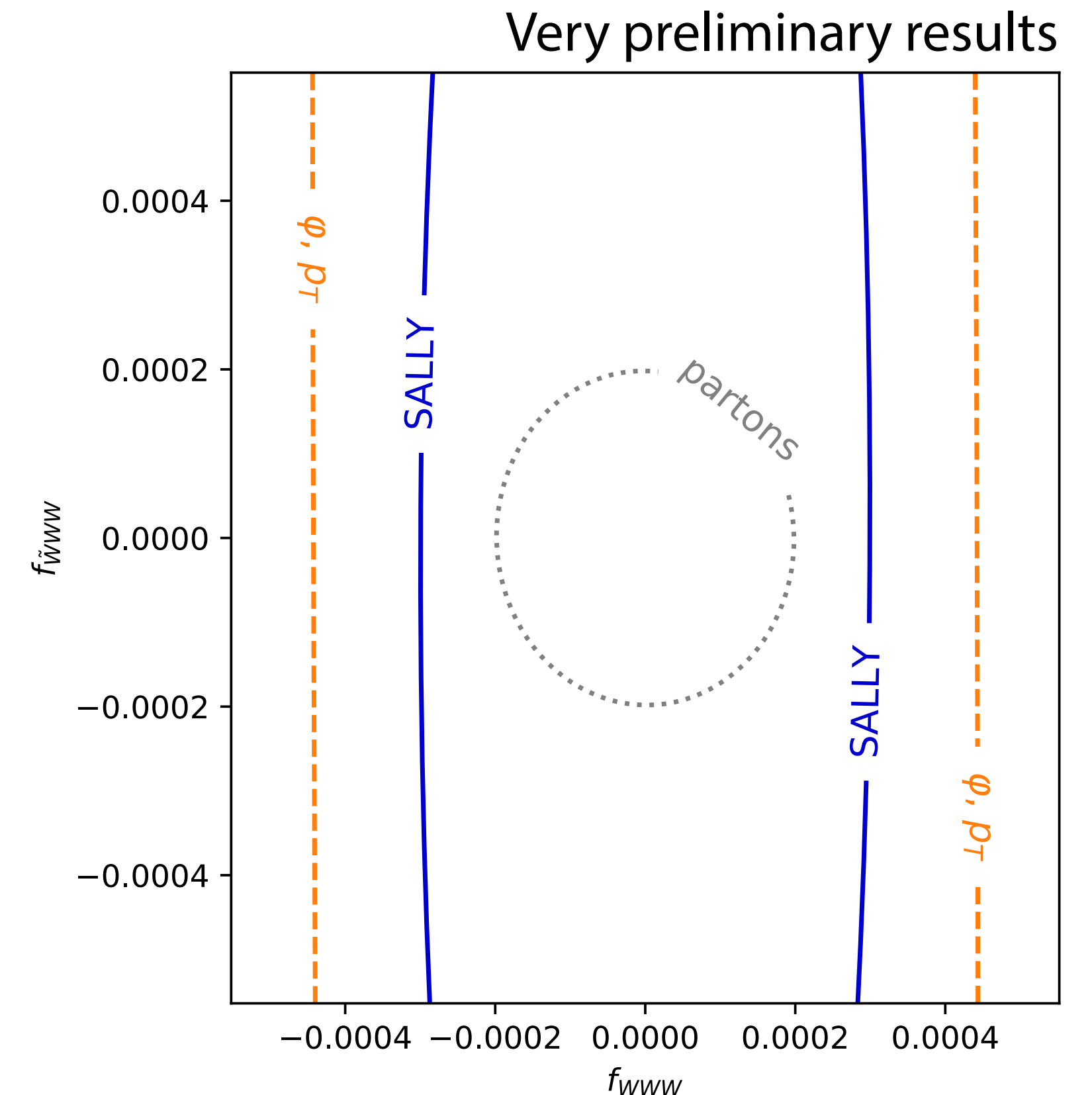
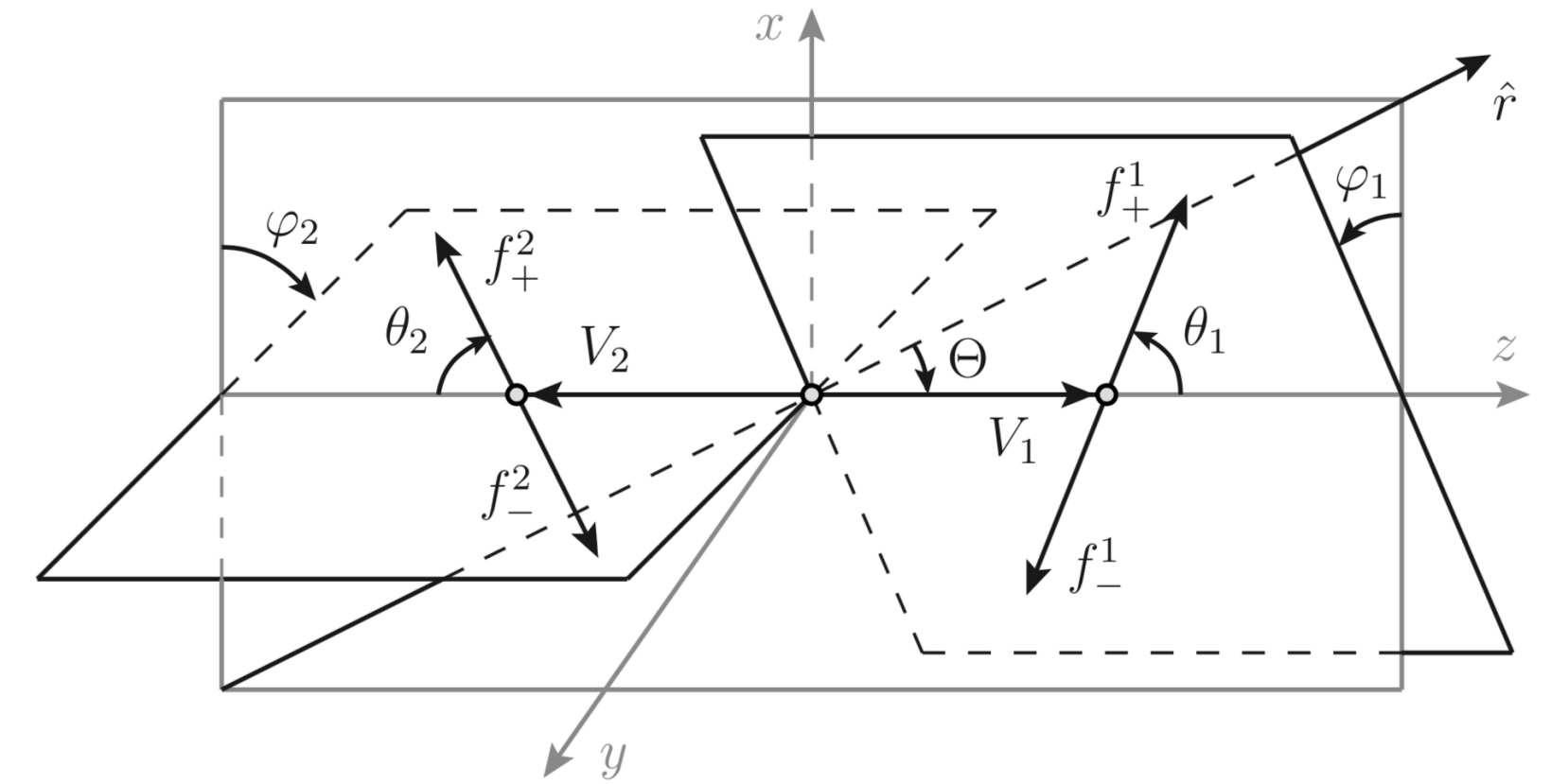
A. Azatov, D. Barducci, E. Venturini 1901.04821]

- We test the ML approach in EFT measurements in  $W\gamma \rightarrow \ell\nu\gamma$

[JB, K. Cranmer, M. Farina, F. Kling, D. Pappadopulo, J. Ruderman in progress]

**New:**  $WZ \rightarrow \ell\ell \ell\nu$  by Chen, Glioti, Panico, Wulzer [arXiv:2007.10356](https://arxiv.org/abs/2007.10356)

- Preliminary results: we can extract more information when we **analyze events with SALLY** than with **histograms of  $\varphi$  and standard observables**



# Conclusion

Likelihood fits in the data space are the gold standard for statistical inference

- RECAST and likelihood publishing are technical solutions that address model dependence and the theory-experiment interface
- STXS a good step, but more differential information can lead to large gain in sensitivity

Properties we want

- Ability to be fully differential
- Exploit highest fidelity simulation (QCD, detector simulation) without approximations that introduce additional systematic errors
- Clear statistical motivation and compatibility with traditional combined analyses
- Scalability in terms of channels and parameters

The approach I presented (implemented in MadMiner) achieves these goals



# References

## Opinionated review

K. Cranmer, JB, G. Louppe:  
“The frontier of simulation-based inference”  
[1911.01429]

## Do It Yourself (for LHC physics)

JB, F. Kling, I. Espejo, K. Cranmer:  
“MadMiner: Machine learning—based inference for particle physics”  
[CSBS, 1907.10621, <https://github.com/diana-hep/madminer>]

## LHC HXSWG YR4 STXS

JB, S. Dawson, S. Homiller, F. Kling, T. Plehn:  
“Benchmarking simplified template cross sections in WH production”  
[JHEP, 1908.06980]

## Use in Astro: Strong lensing

JB, S. Mishra-Sharma, J. Hermans, G. Louppe, K. Cranmer  
“Mining for Dark Matter Substructure: Inferring subhalo population properties from strong lenses with machine learning”  
[ApJ, 1909.02005]

## Original works

JB, K. Cranmer, G. Louppe, J. Pavez:  
“A guide to constraining Effective Field Theories with machine learning”  
[PRD, 1805.00020]

JB, G. Louppe, J. Pavez, K. Cranmer:  
“Mining gold from implicit models to improve likelihood-free inference”  
[PNAS, 1805.12244]

## Follow-up with incremental improvements

M. Stoye, JB, K. Cranmer, G. Louppe, J. Pavez:  
“Likelihood-free inference with an improved cross-entropy estimator”  
[NeurIPS workshop, 1808.00973]

# An incomplete wrap-up of simulation-based inference methods

Method	Approximations	Upfront cost	Eval
Summary statistics:			
Likelihood for summary stats (standard histograms)	Reduction to summary stats	Fast	Fast
Approximate Bayesian Computation	Reduction to summary stats	Depends	Depends
Matrix elements:			
Matrix Element Method	Transfer fns	Fast	Slow
Optimal Observables	Transfer fns, optimal only locally	Fast	Slow
Neural networks:			
Neural likelihood	NN	Needs many samples	Fast
Neural posterior	NN	Needs many samples	Fast
Neural likelihood ratio	NN	Needs many samples	Fast
Neural networks + matrix elements:			
Neural likelihood (ratio) + gold mining (RASCAL etc)	NN	Needs less samples	Fast
Neural optimal observables (SALLY)	NN, optimal only locally	Needs less samples	Fast

# Mining gold: A family of new inference techniques

Method	Simulate	Extract		NN estimates	Asympt. exact	Generative
		$r(x, z)$	$t(x, z)$			
ROLR	$\theta_0 \sim \pi(\theta), \theta_1$	✓		$\hat{r}(x \theta_0, \theta_1)$	✓	
CASCAL	$\theta_0 \sim \pi(\theta), \theta_1$		✓	$\hat{r}(x \theta_0, \theta_1)$	✓	
ALICE	$\theta_0 \sim \pi(\theta), \theta_1$		✓	$\hat{r}(x \theta_0, \theta_1)$	✓	
RASCAL	$\theta_0 \sim \pi(\theta), \theta_1$	✓	✓	$\hat{r}(x \theta_0, \theta_1)$	✓	
ALICES	$\theta_0 \sim \pi(\theta), \theta_1$	✓	✓	$\hat{r}(x \theta_0, \theta_1)$	✓	
SCANDAL	$\theta \sim \pi(\theta)$		✓	$\hat{p}(x \theta)$	✓	✓
SALLY	$\theta_{\text{ref}}$		✓	$\hat{t}(x \theta_{\text{ref}})$	in local approx.	
SALLINO	$\theta_{\text{ref}}$		✓	$\hat{t}(x \theta_{\text{ref}})$	in local approx.	

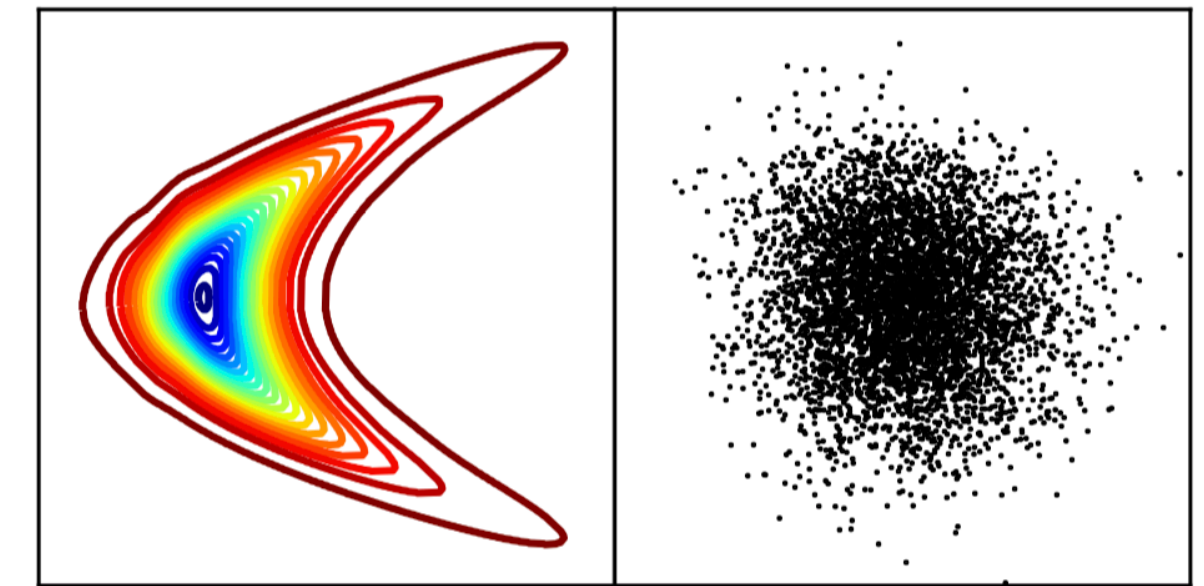
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Performance gains with cross-entropy-based loss  
 [M. Stoye, JB, K. Cranmer, G. Louppe, J. Pavez 1808.00973]

# Mining gold: A family of new inference techniques

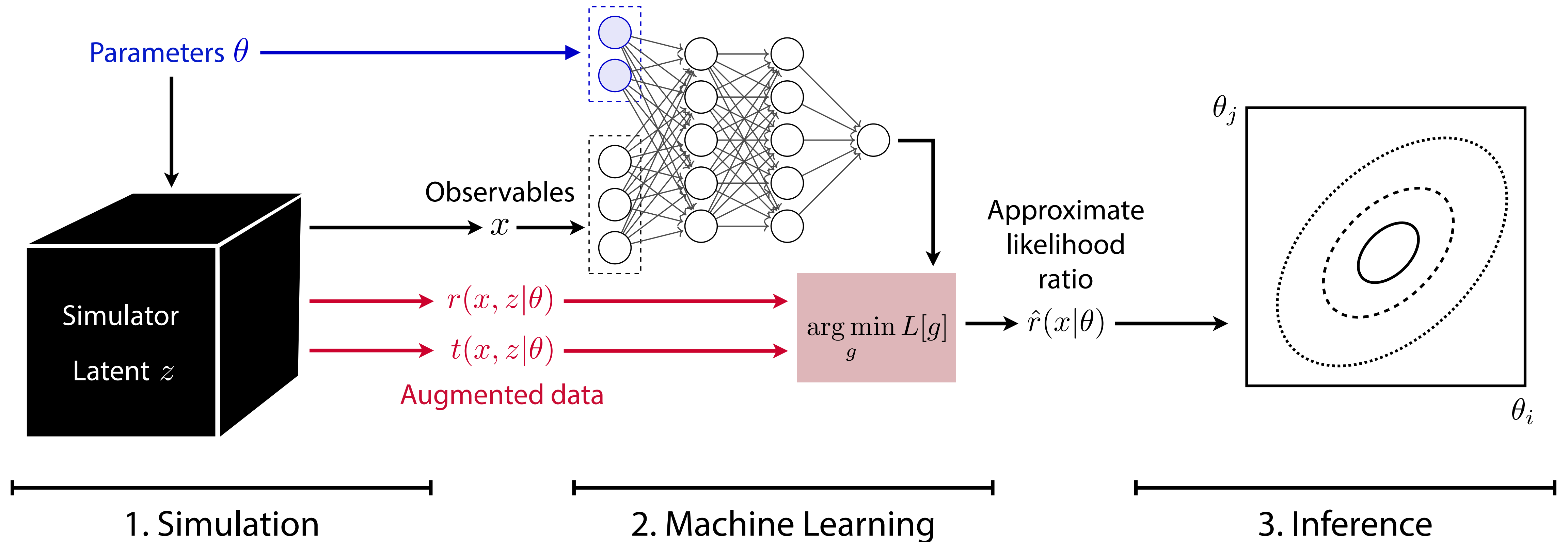
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Combination with state-of-the-art conditional neural density estimators, e.g. normalizing flows

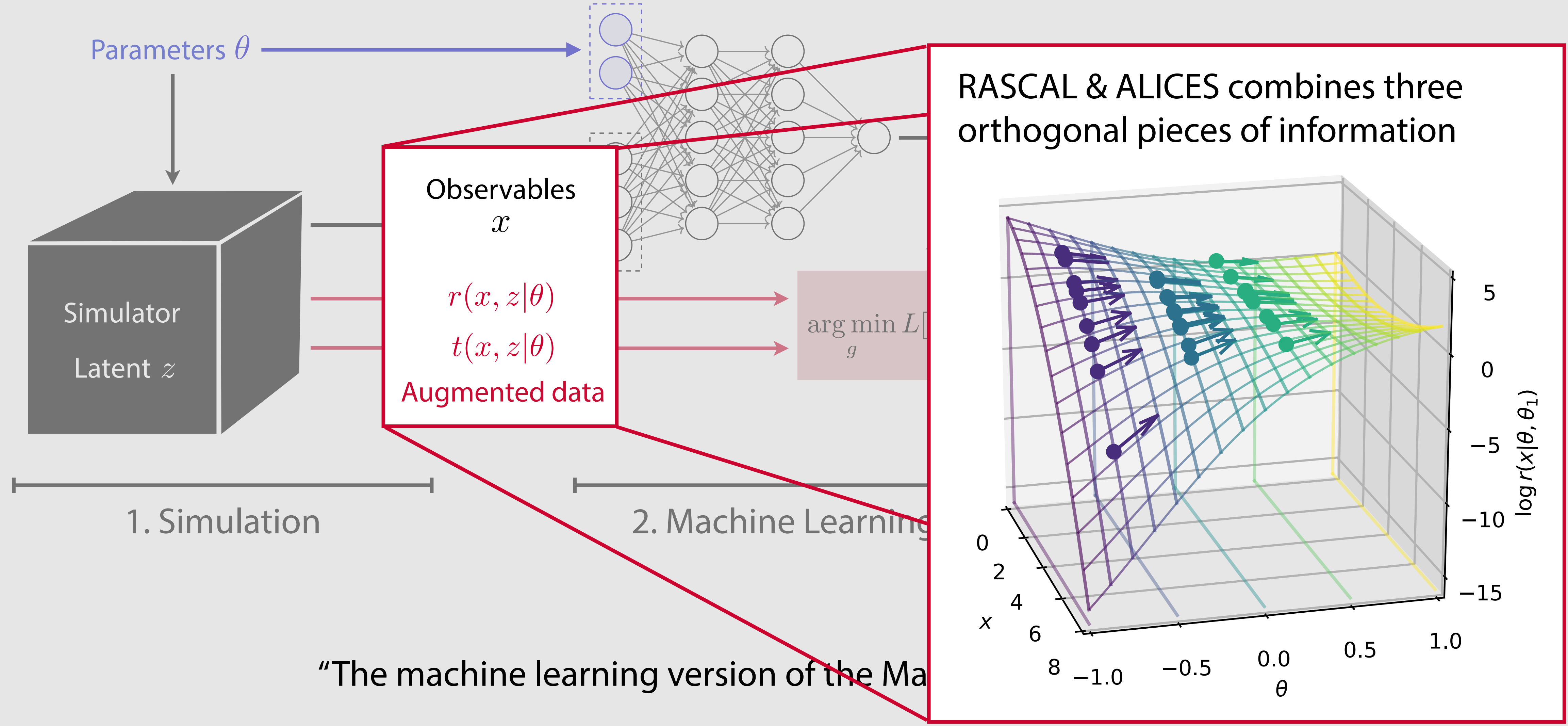
[everything by G. Papamakarios:  
G. Papamakarios, T. Pavlakou, I. Murray 1705.07057;  
G. Papamakarios, D. Sterratt, I. Murray 1805.07226; ...]

# Putting the pieces together: RASCAL & ALICES



"The machine learning version of the Matrix Element Method"

# Putting the pieces together: RASCAL & ALICES

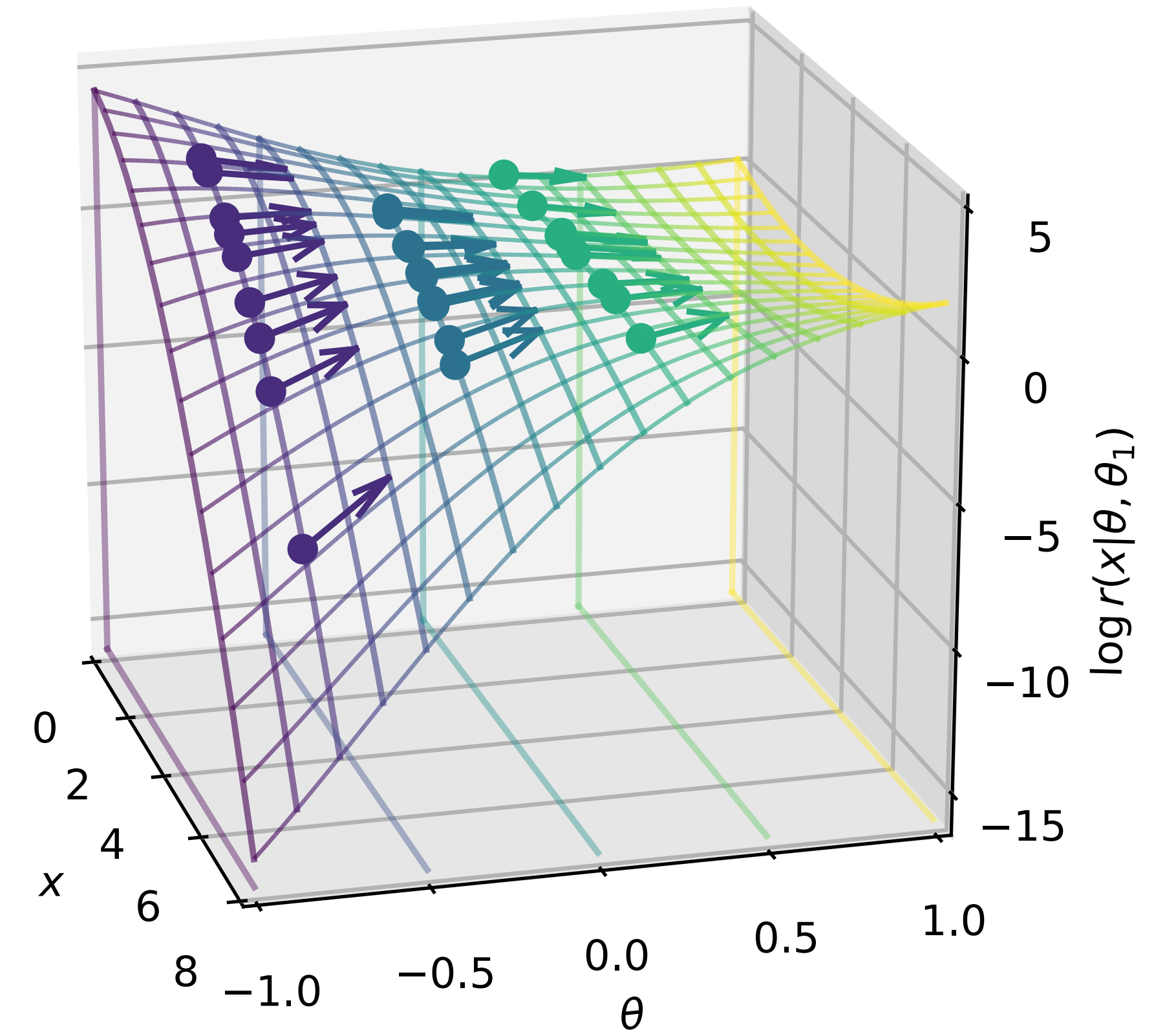
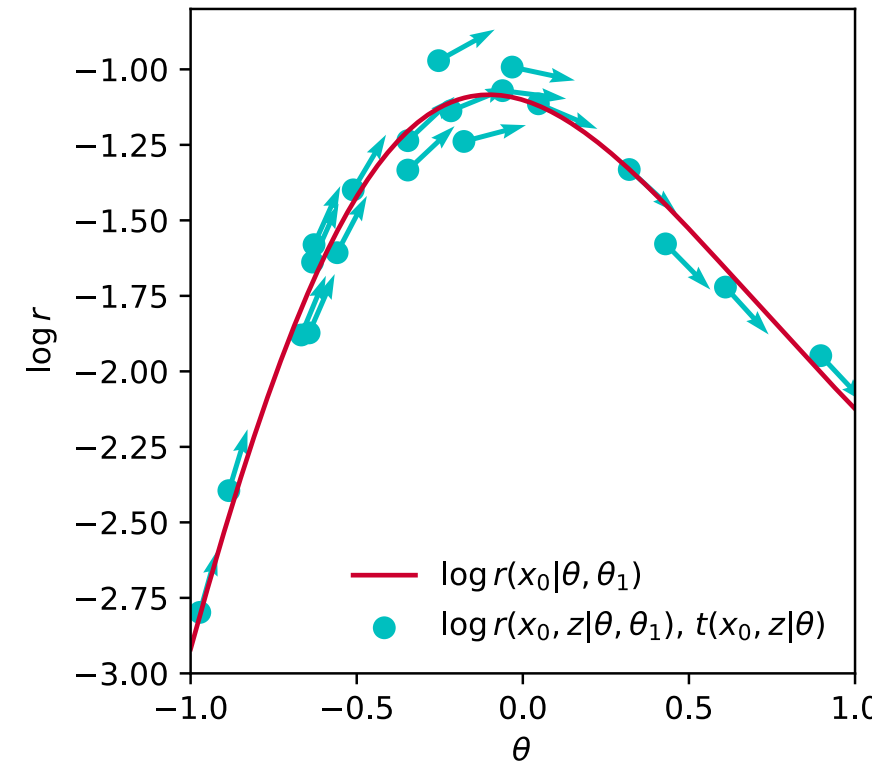
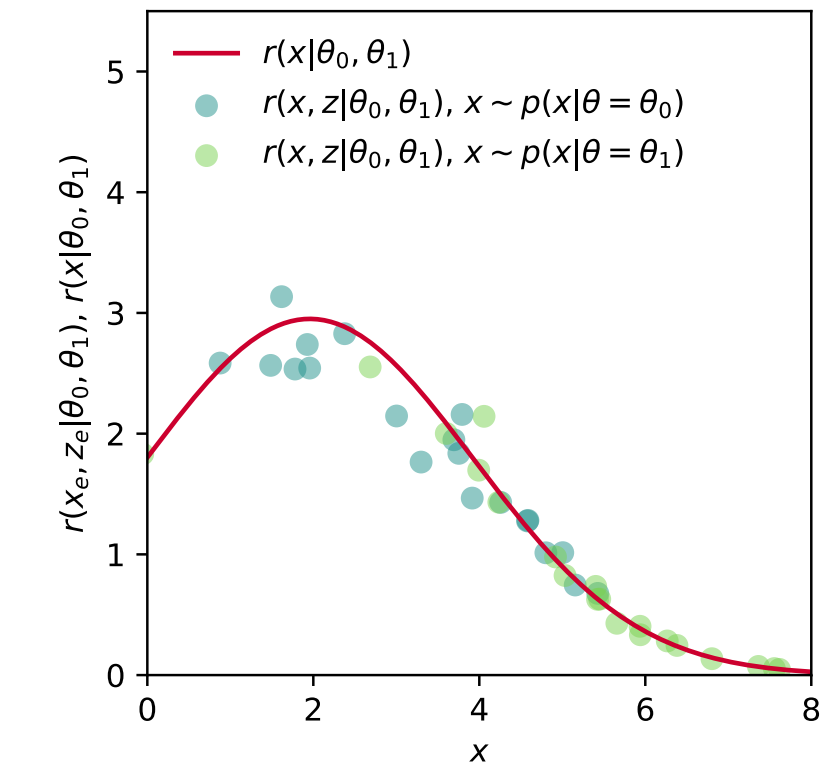
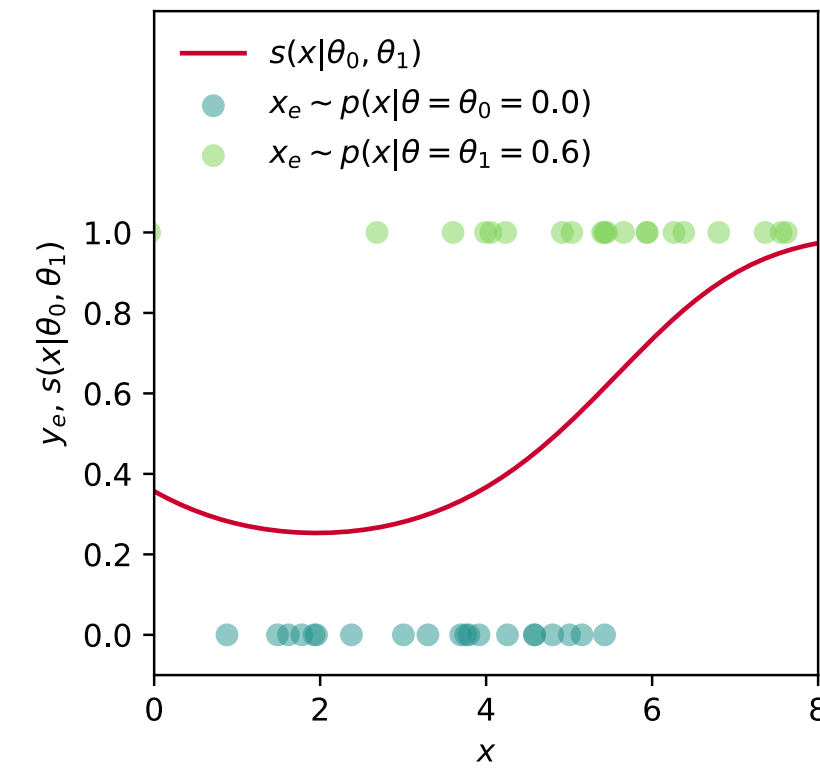


"The machine learning version of the Ma

# Gold mining: augmenting the training data

The augmented training data converts supervised classification into supervised regression with lower variance

- improvement in training efficiency

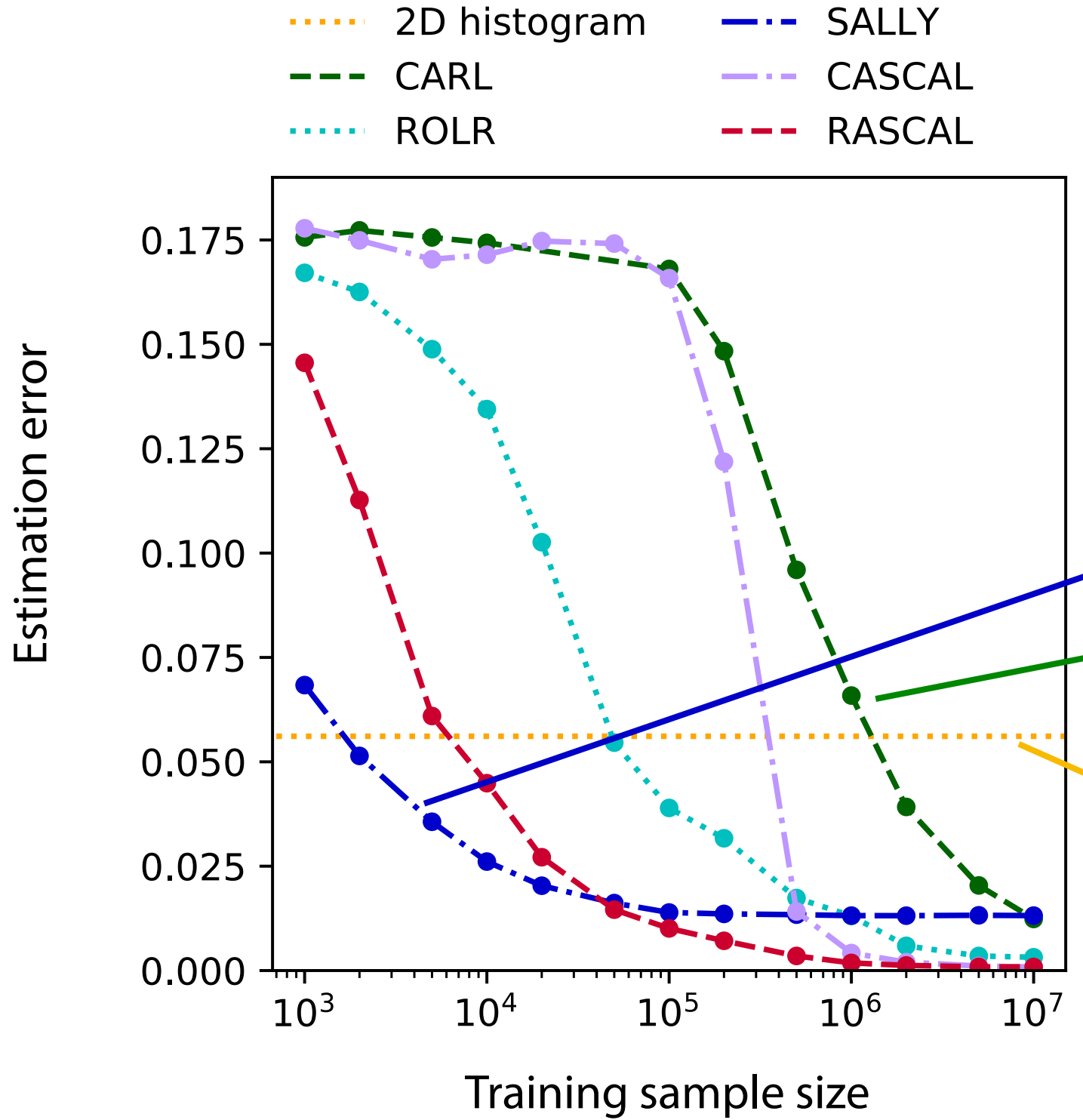
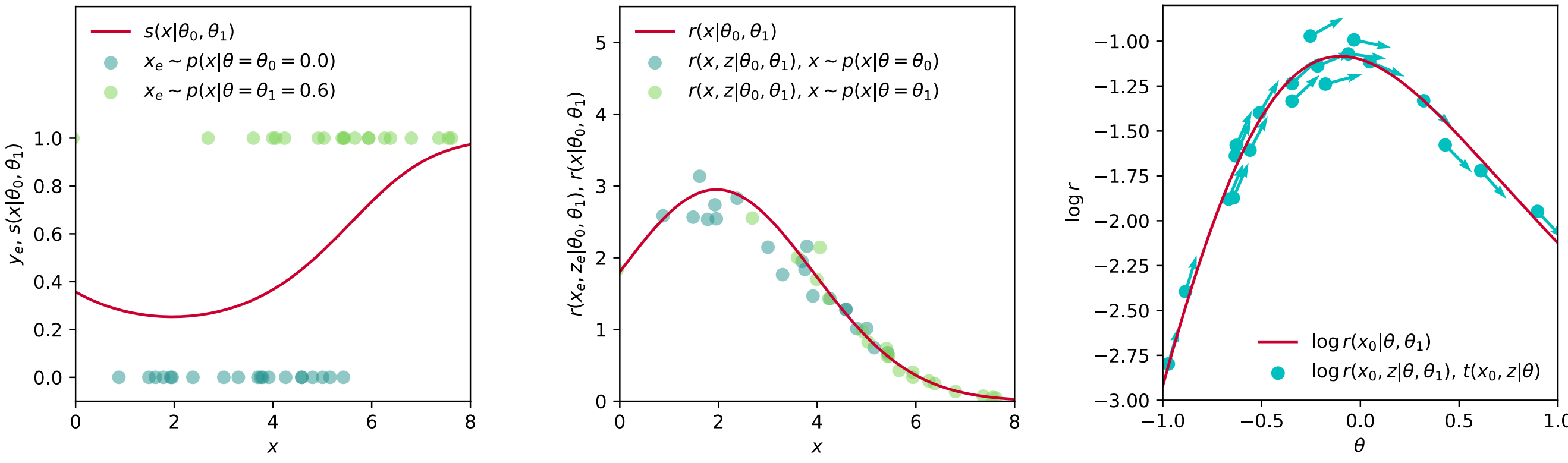




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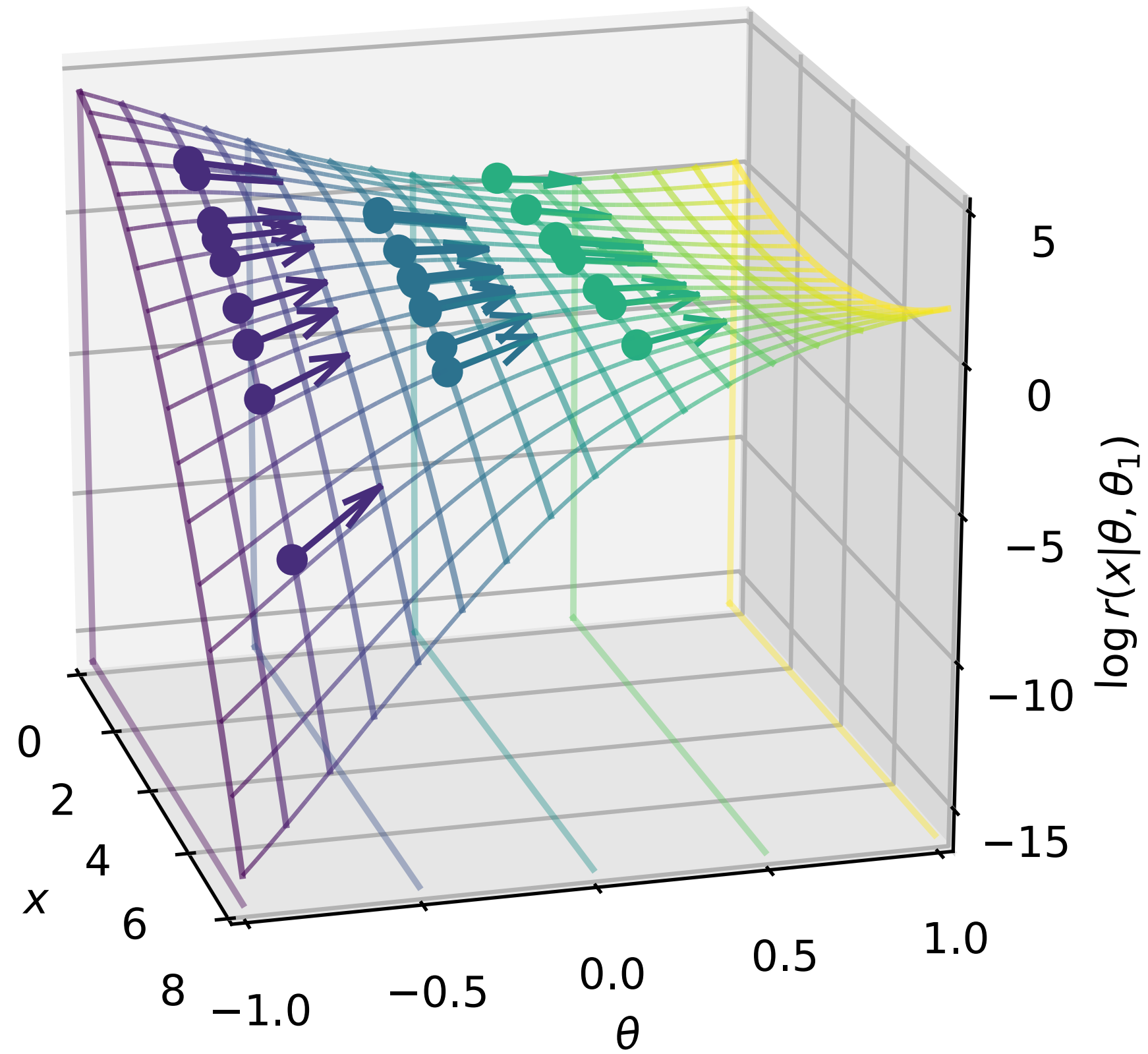
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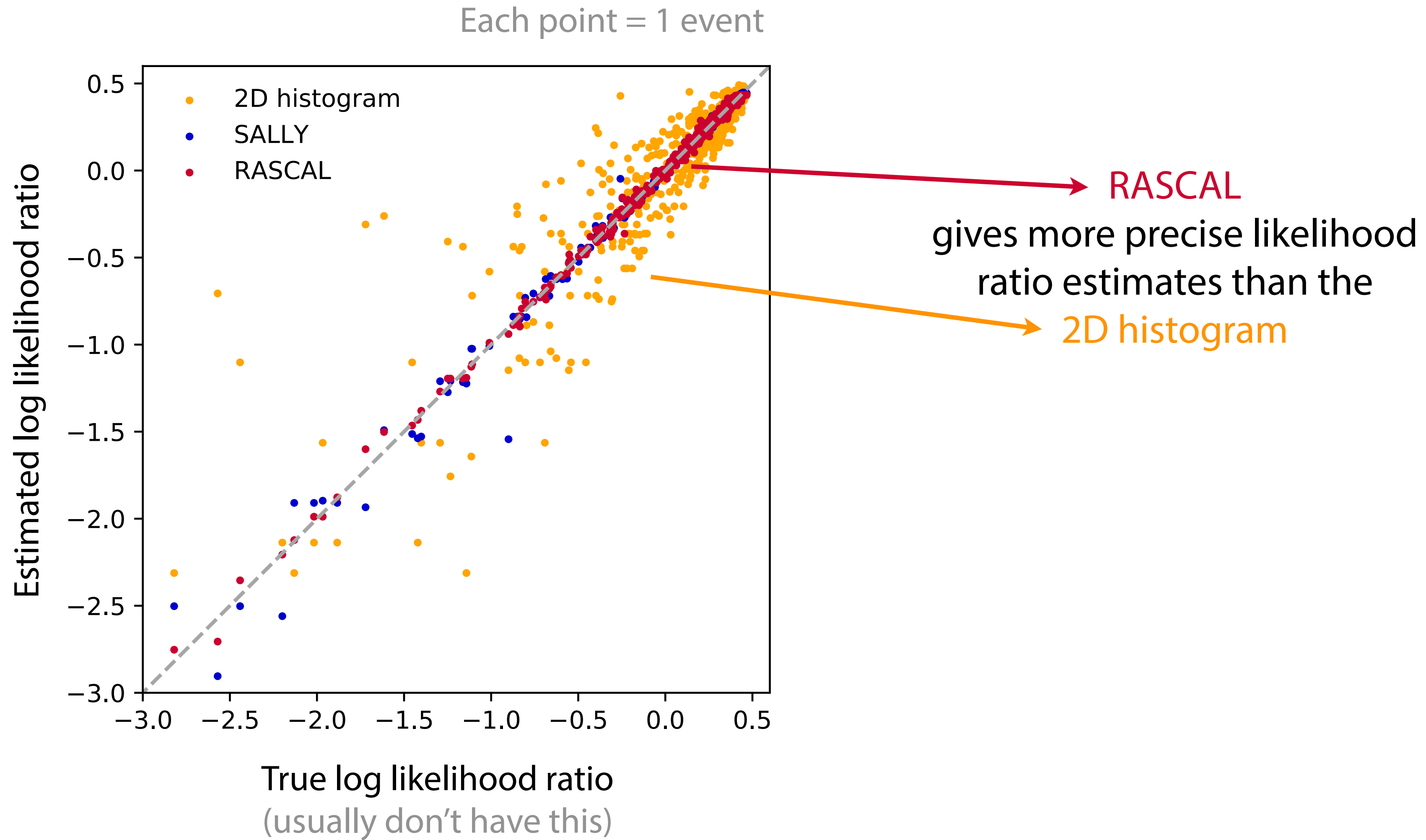


New techniques require less data than without augmented data

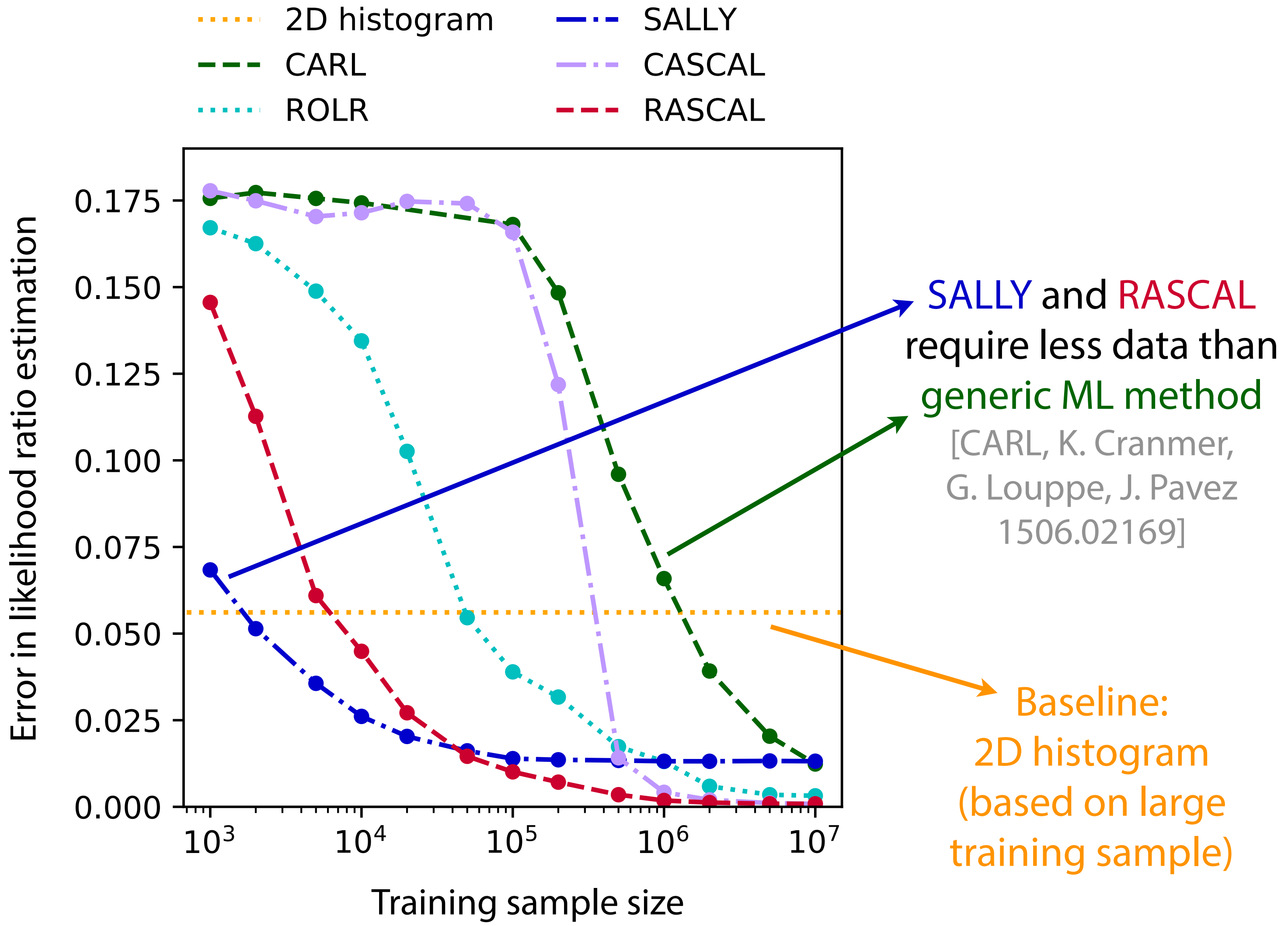
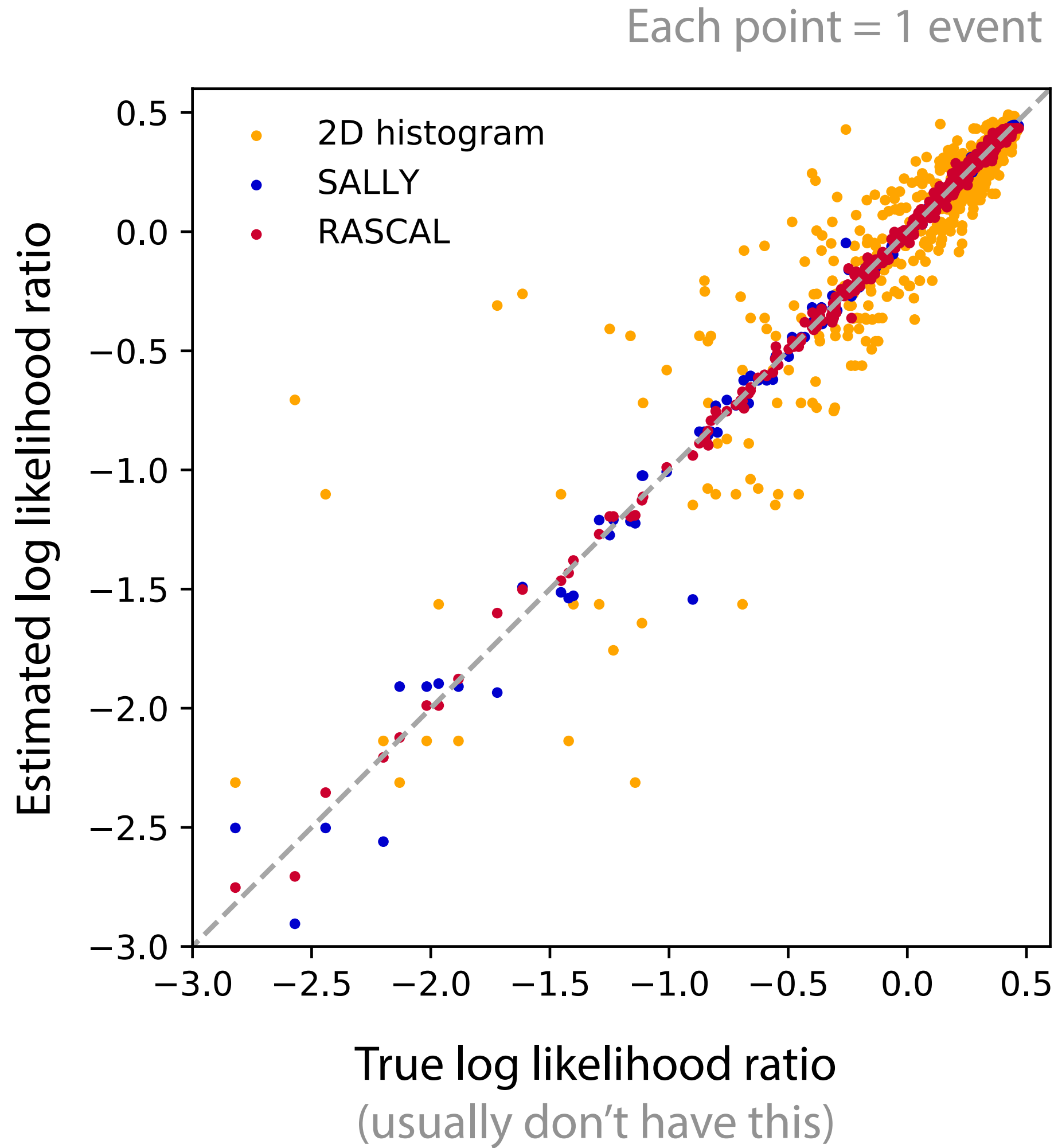
Traditional Approach no NN



# More precise likelihood ratio estimates with less training data



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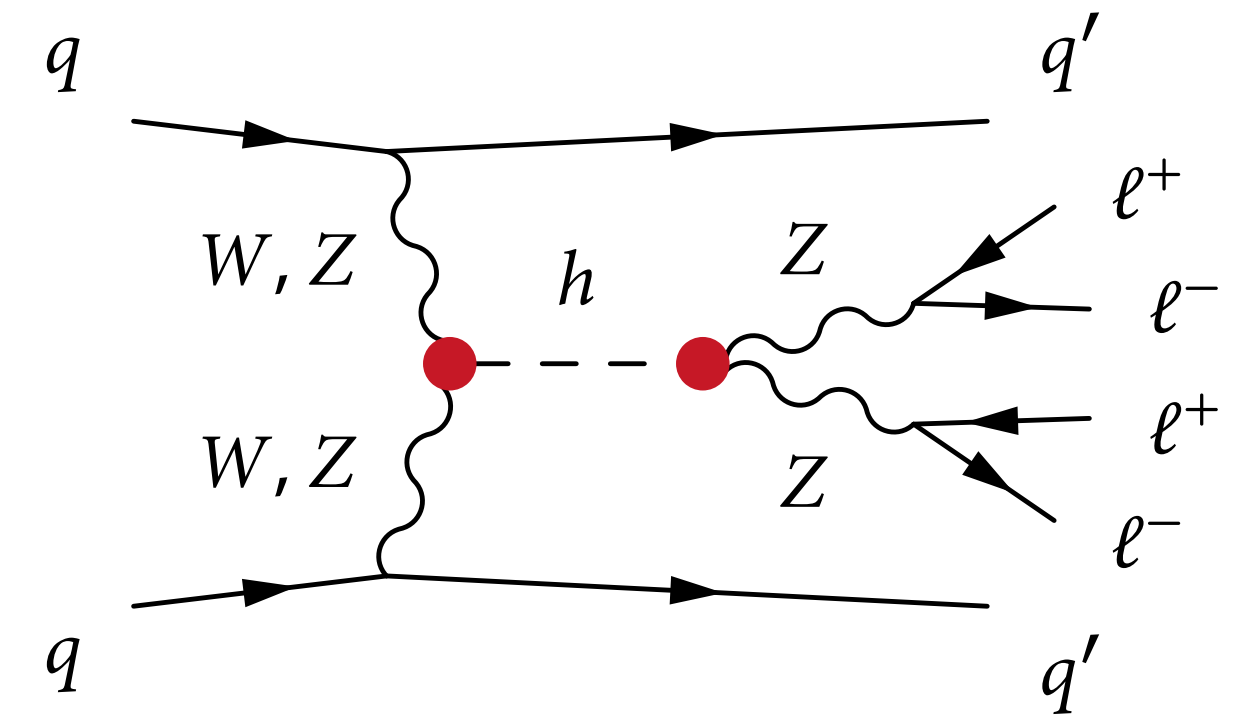


# Challenge for EFT

Let  $\theta$  denote the coefficients of higher dimensional operators in the Lagrangian,  $x$  be high-dimensional data associated to an event, and  $p(x | \theta) = \frac{1}{\sigma(\theta)} \frac{d\sigma}{d\theta}$  be the distribution for the data

- we want to compare any two points in EFT parameter space

- evaluate the **likelihood ratio**  $r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$



Difficulty is that one changes the parameters of the EFT, the distributions  $p(x|\theta)$  change due to interference.

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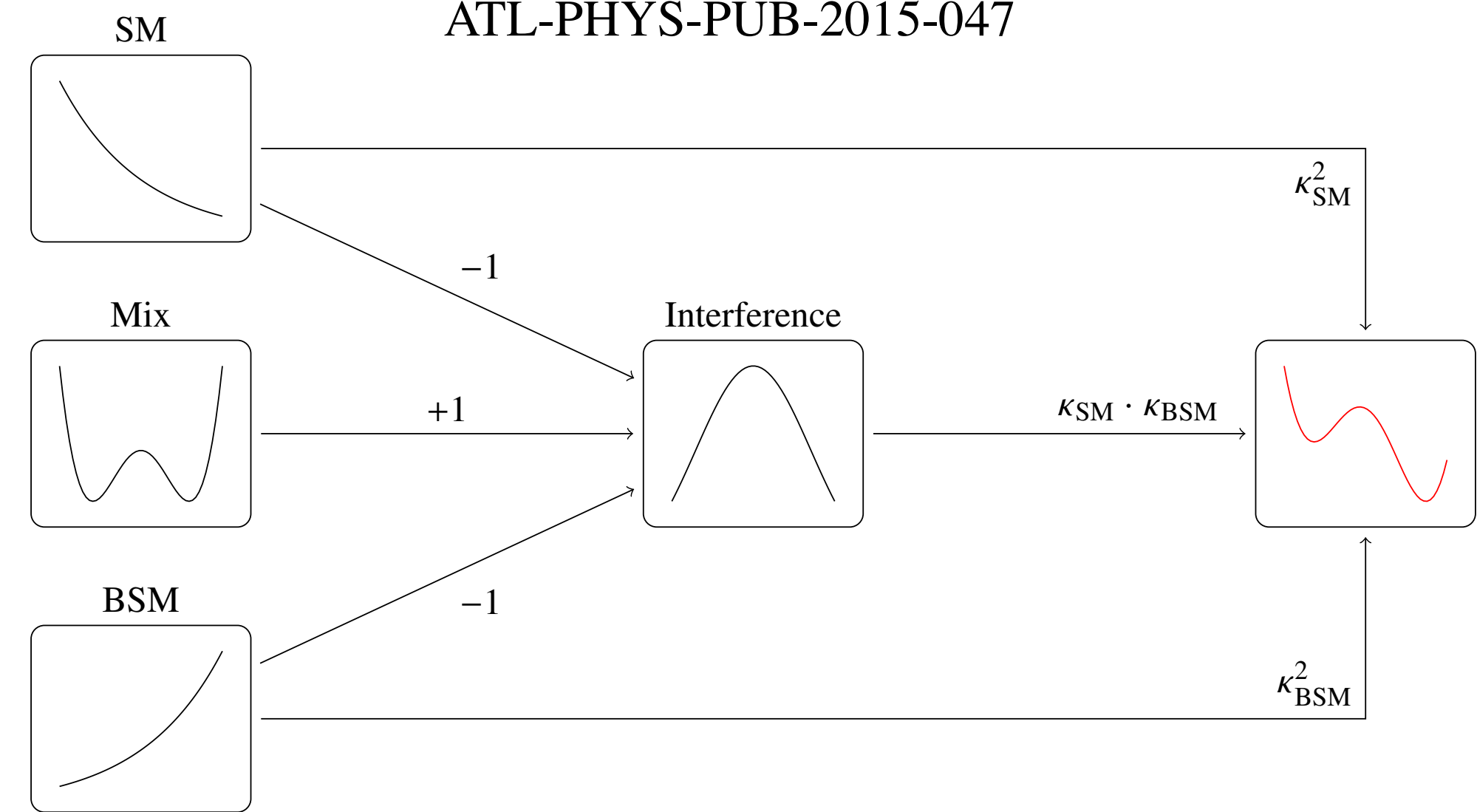
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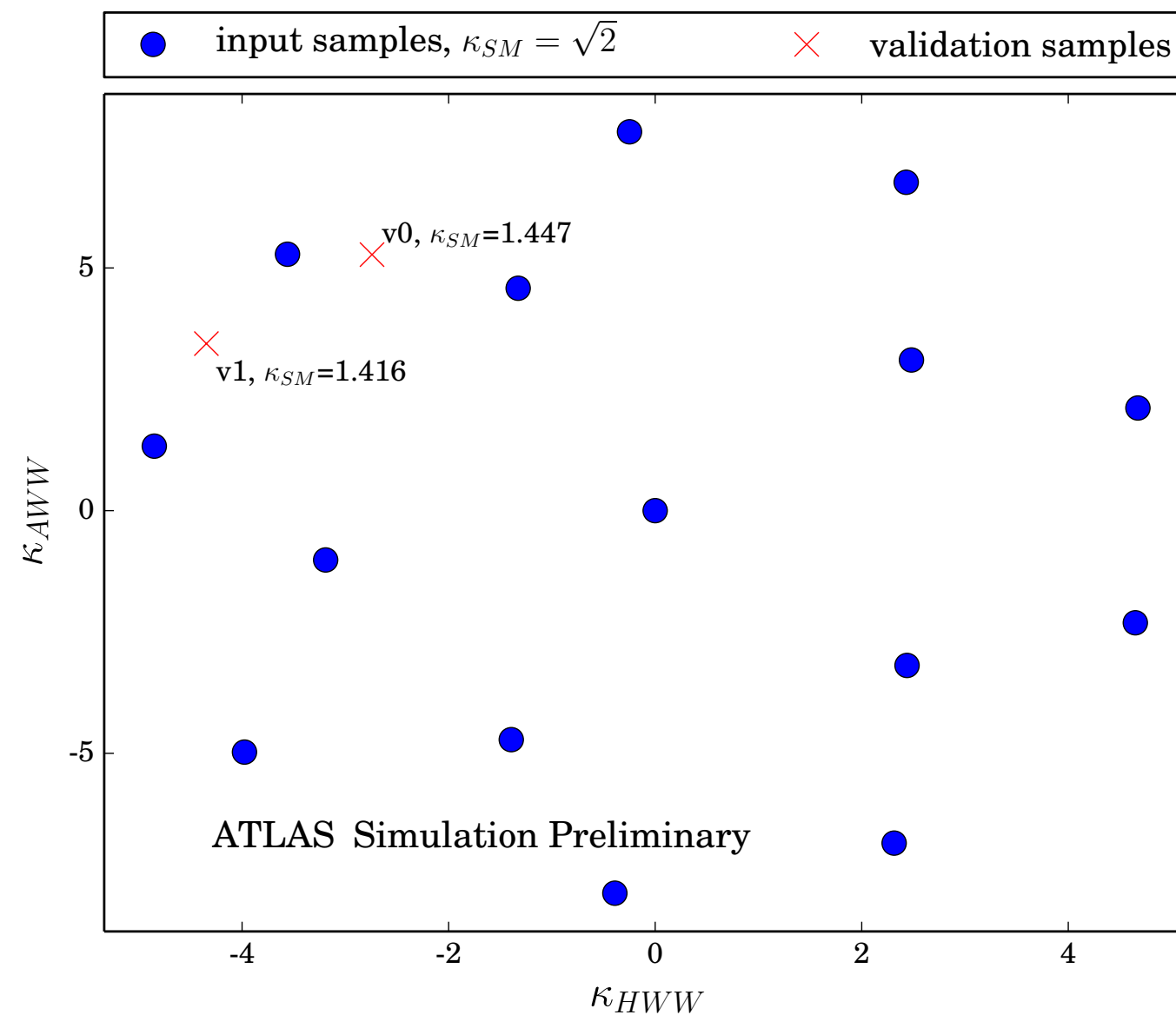
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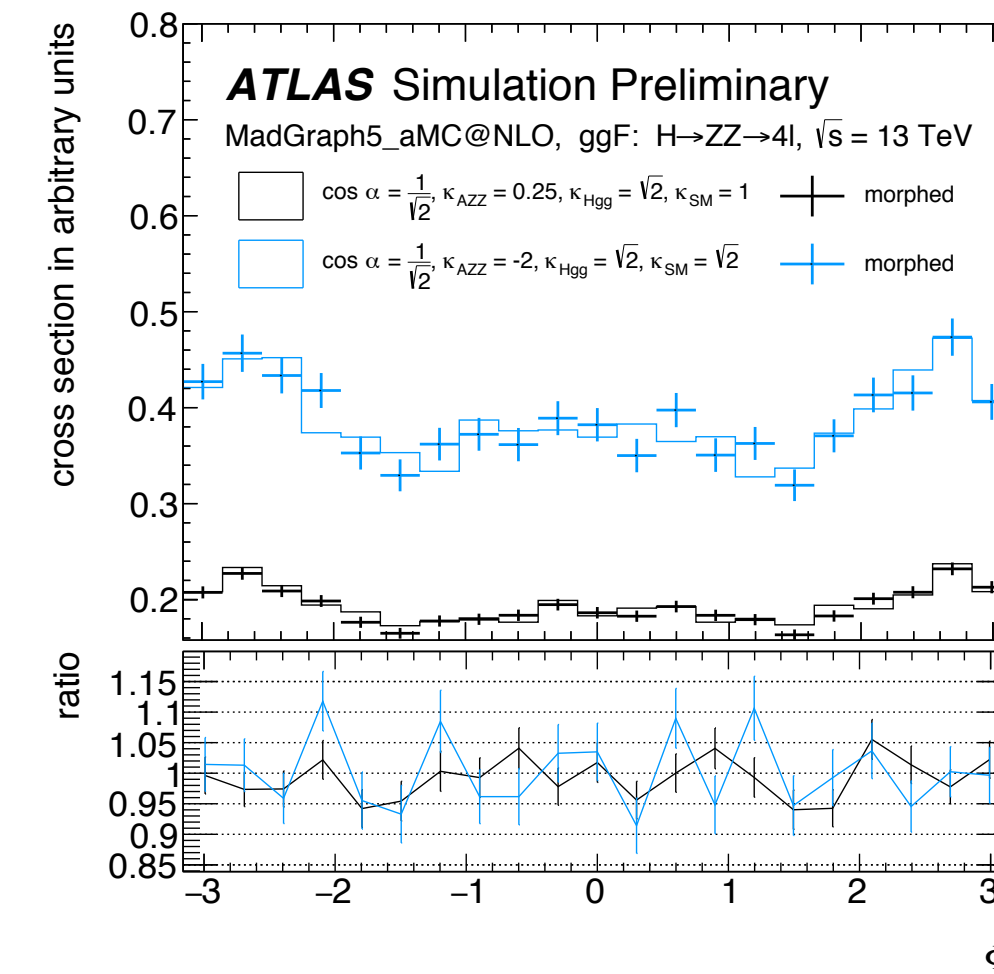
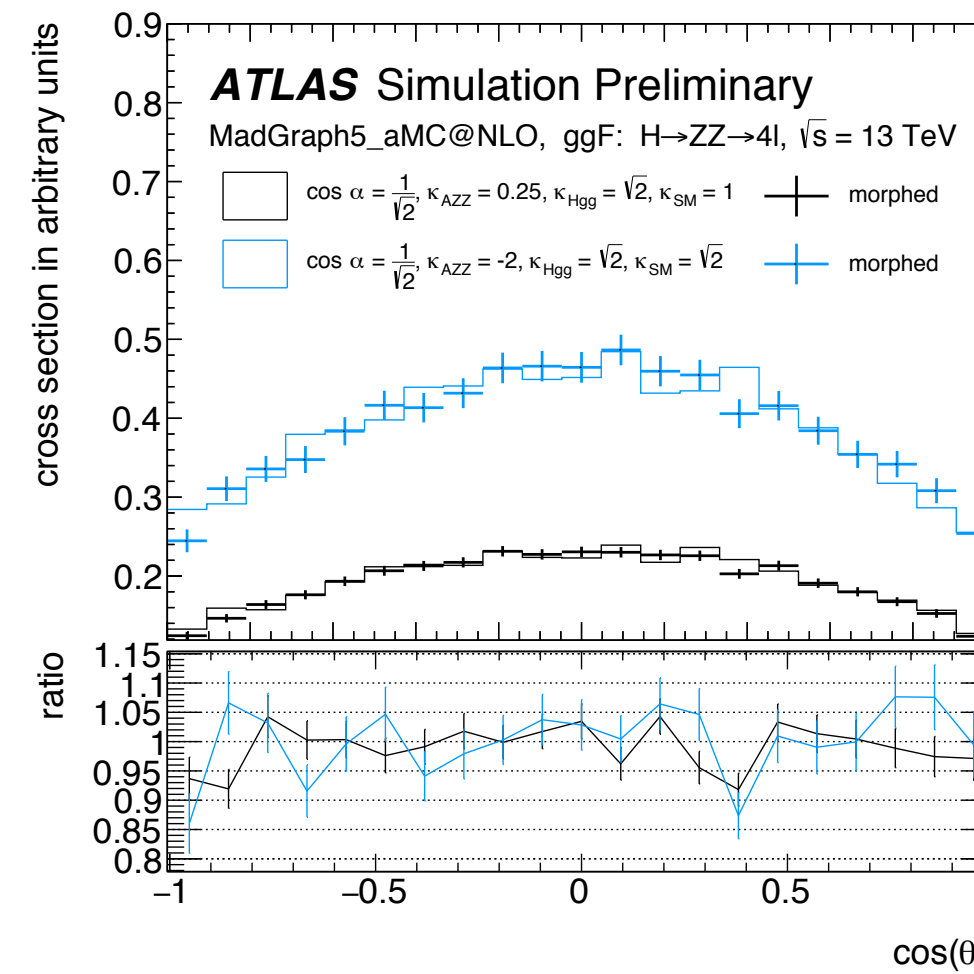
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3-d vector space, distribution for any point in this space is linear mixture of distribution for 3 basis samples!



(real examples need more basis samples)



# EFT Decomposition

$$d\sigma \propto \left| \left( \mathcal{M}_{\text{SM}}^p + \sum_i \frac{f_i}{\Lambda^2} \mathcal{M}_i^p \right) \left( \mathcal{M}_{\text{SM}}^d + \sum_j \frac{f_j}{\Lambda^2} \mathcal{M}_j^d \right) \right|^2$$

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$$p(x|\theta) = \sum_c w_c(\theta) p_c(x)$$

$w_c(\theta)$  are polynomials

$\nabla_{\theta} \log p(x|\theta)$  is now possible!

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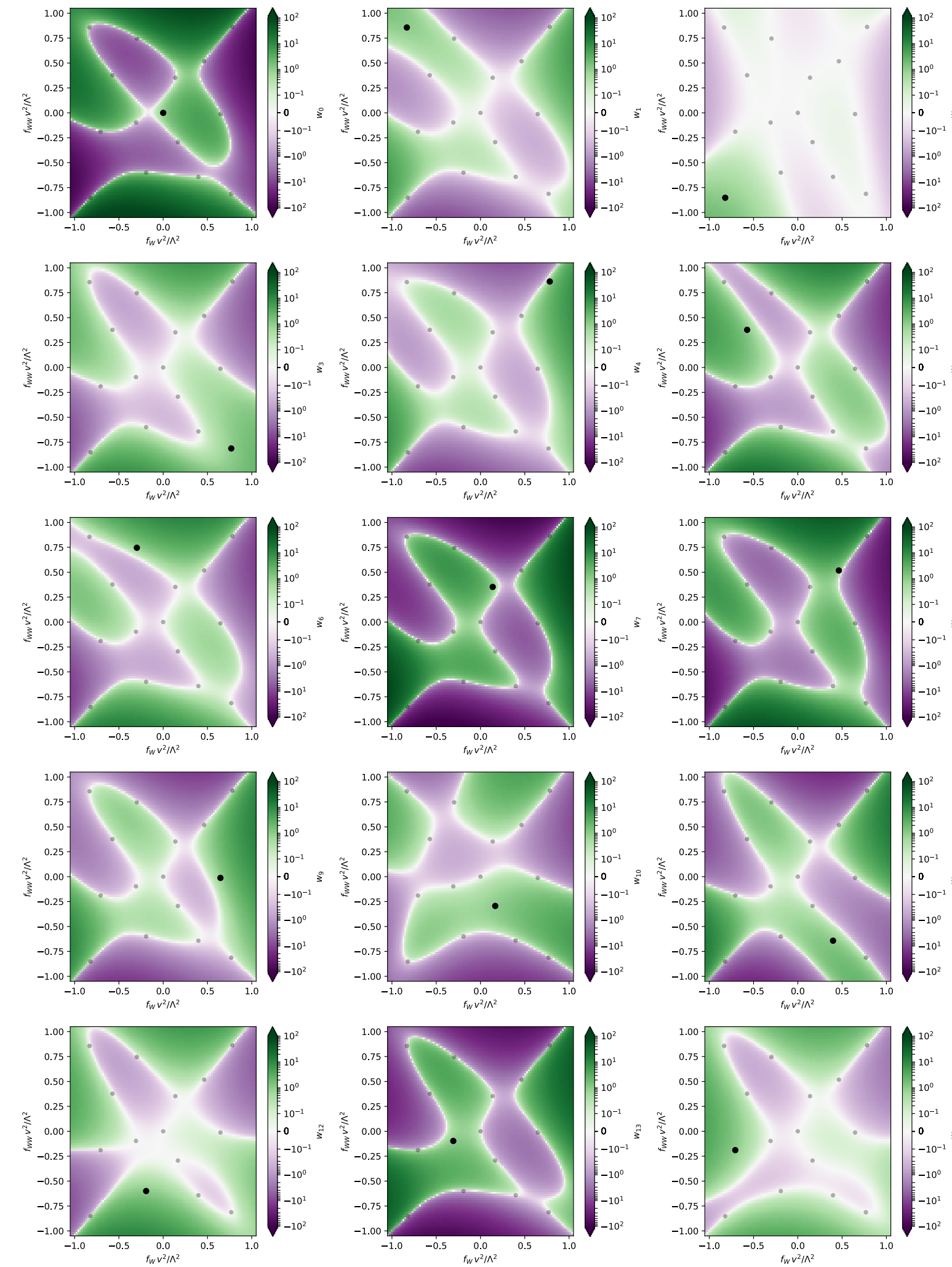
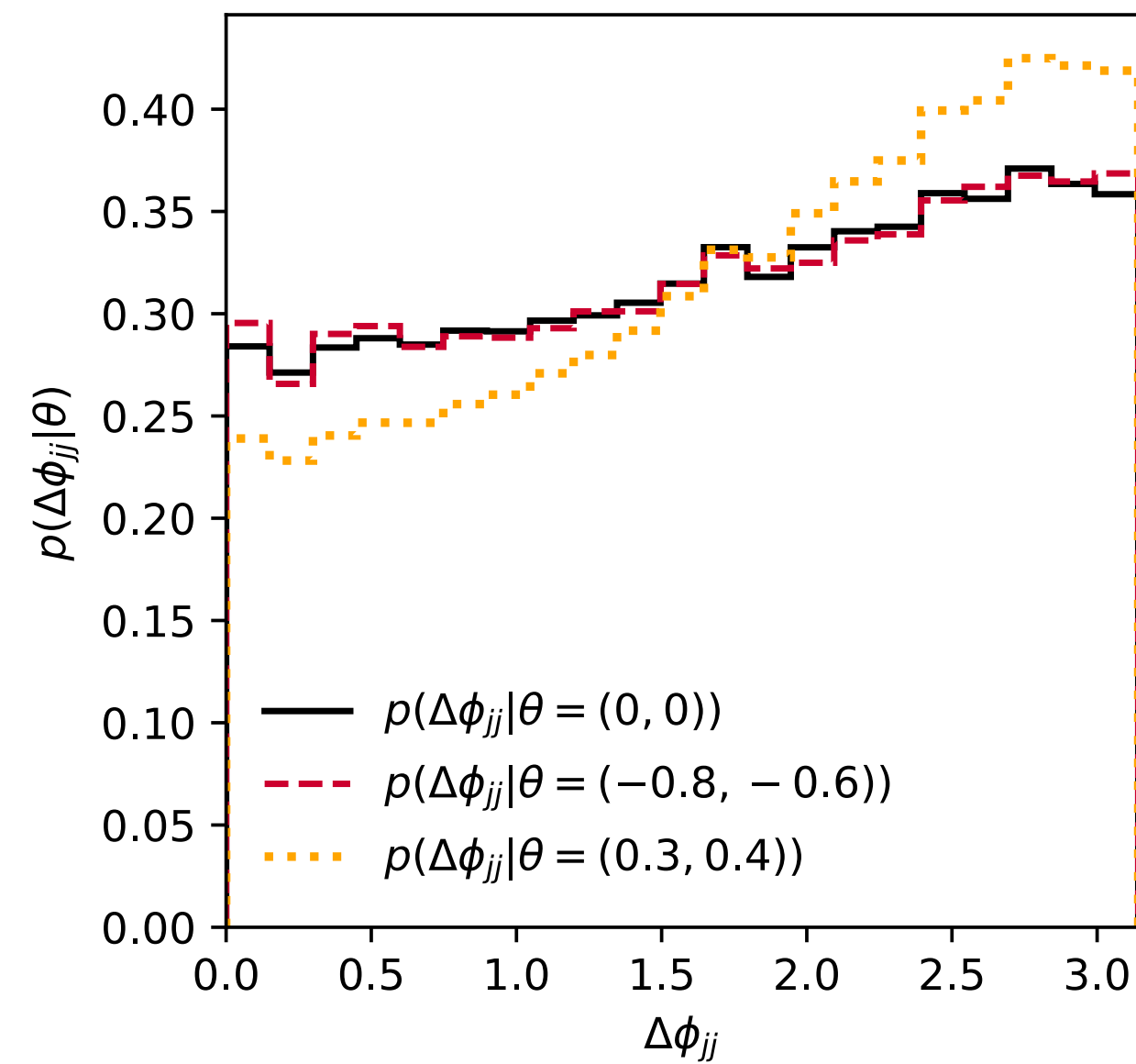


Figure 13: Morphing weights  $w_i(\theta)$  for basis points distributed over the full relevant parameter space.

For 2 BSM operators affecting VBF Higgs production and decay, we need a 15-D vector space

For 5 BSM operators we need 126-D vector space

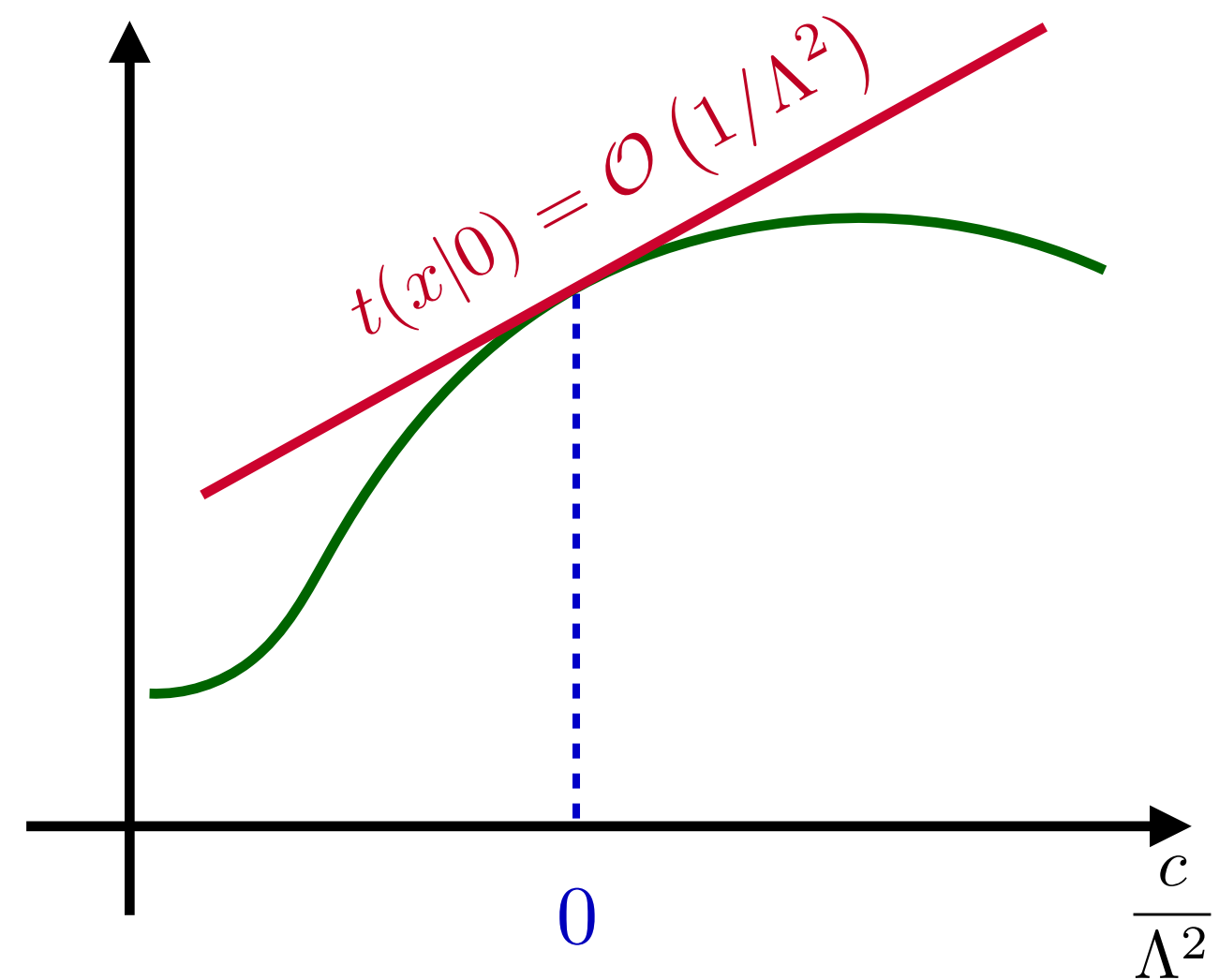
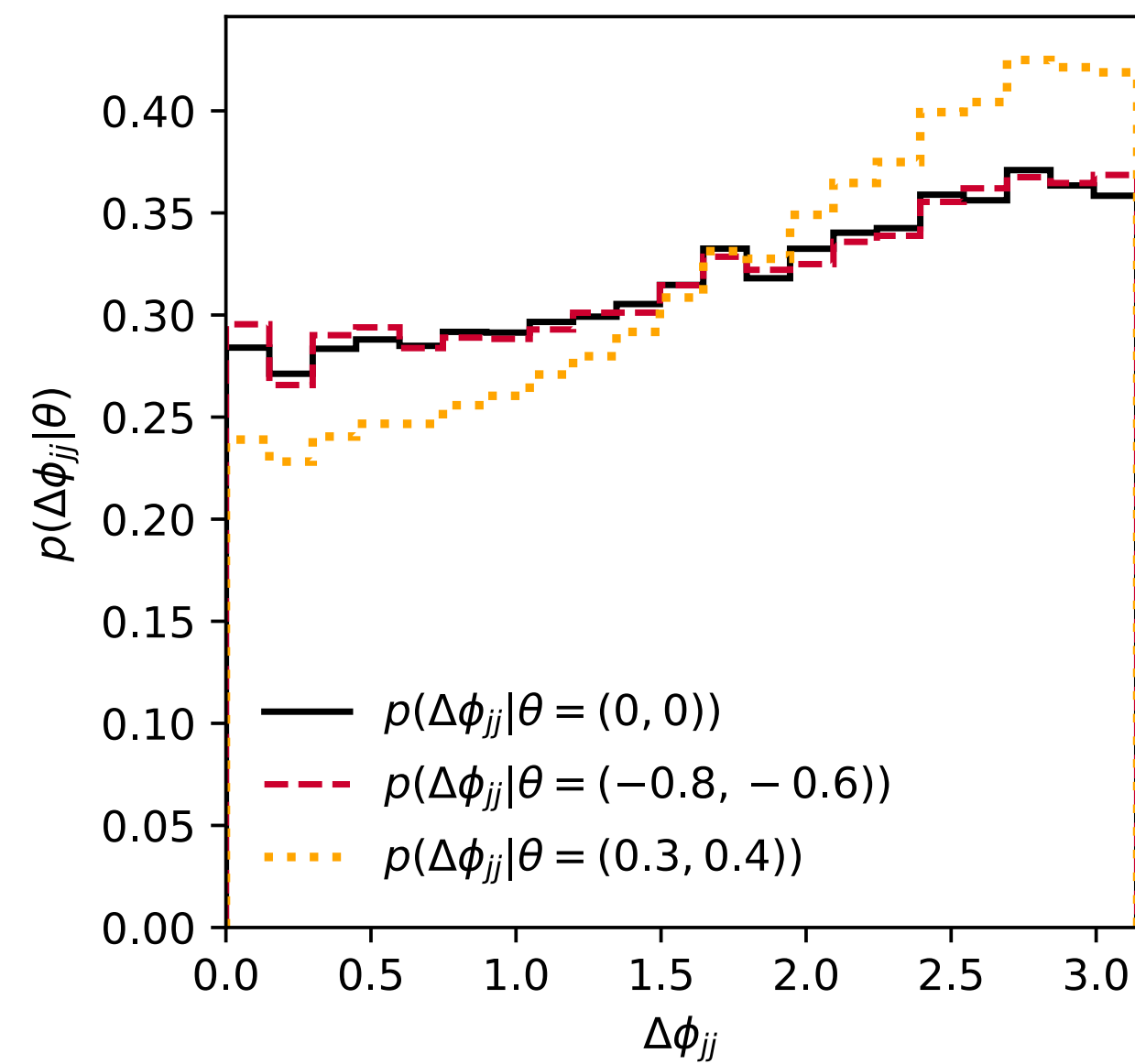
# Perfect match for EFT measurements



- Good for subtle kinematic effects

(Subtle point: Large overlap of kinematic distributions reduces variance of joint likelihood ratio / joint score)

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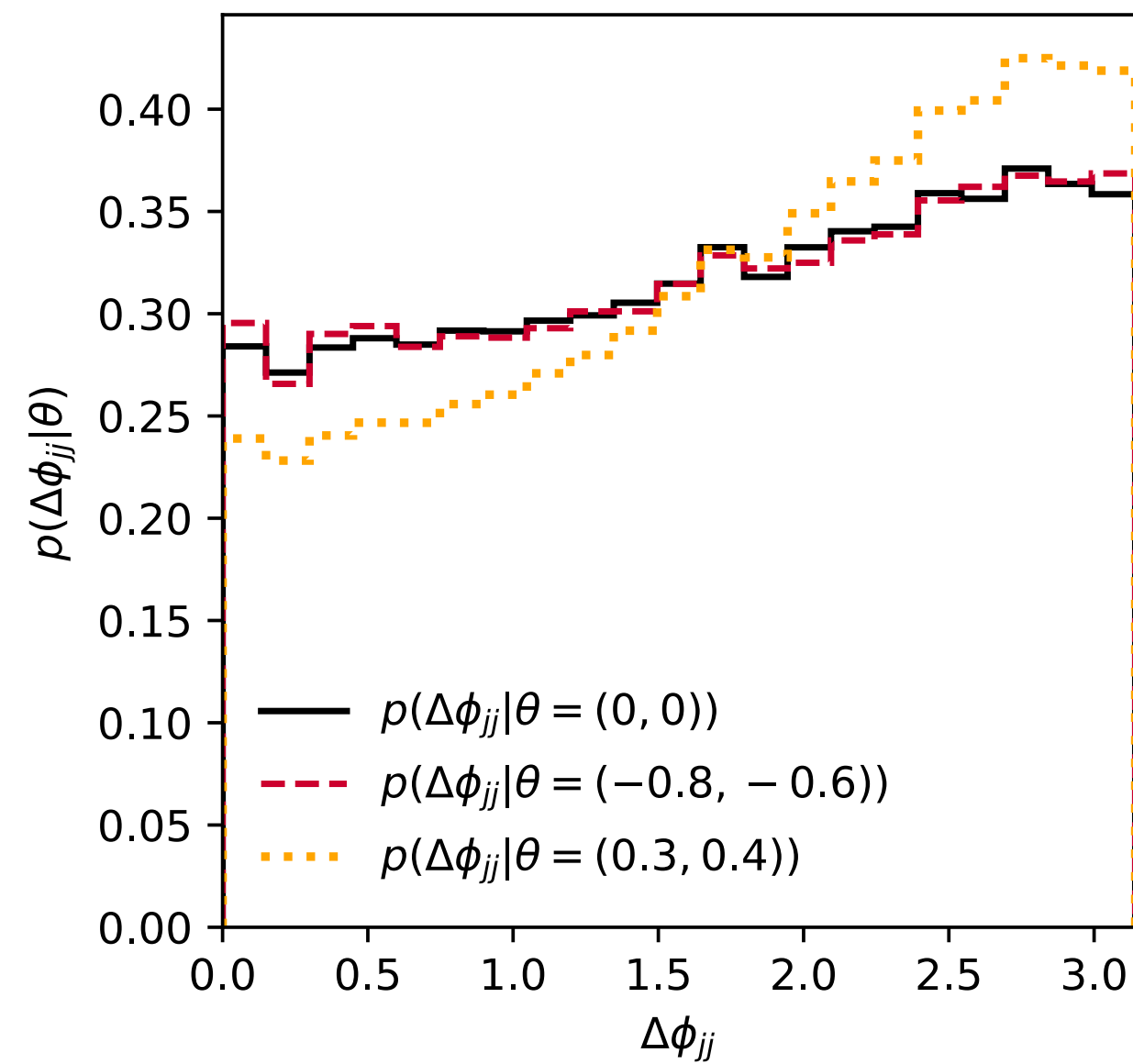
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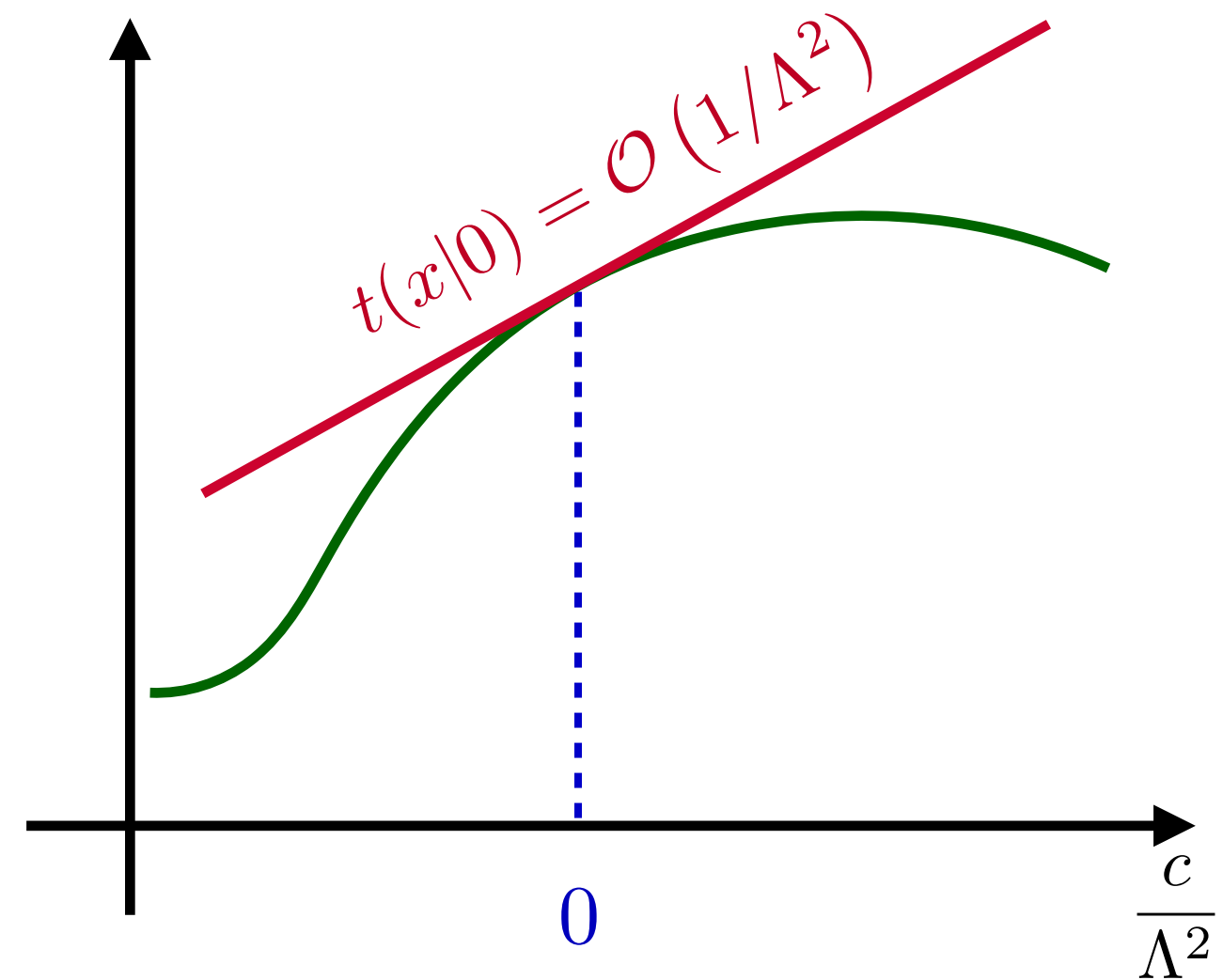


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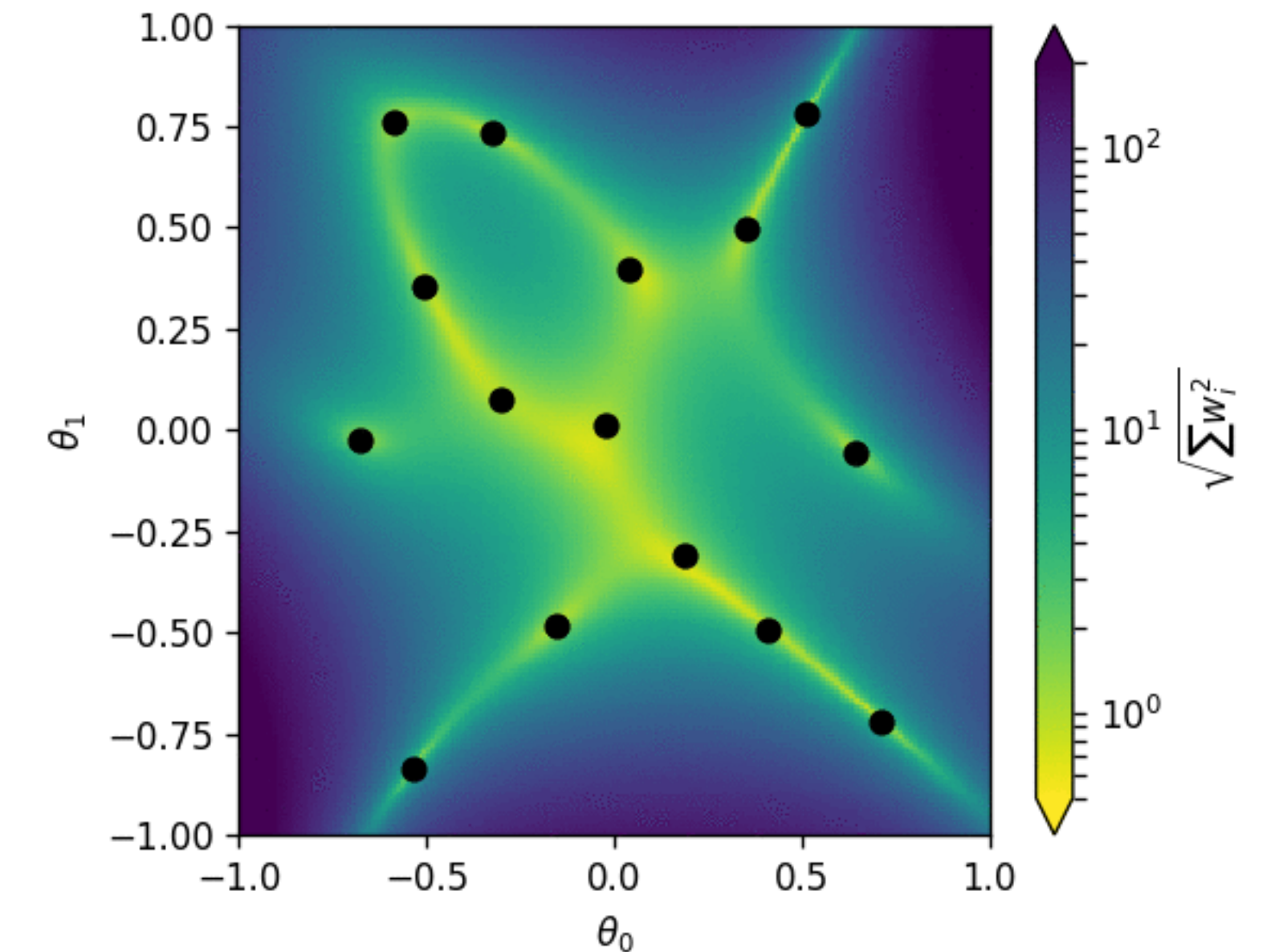


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- Morphing techniques allow fast reweighting to any parameter points

[e.g. ATL-PHYS-PUB-2015-047]

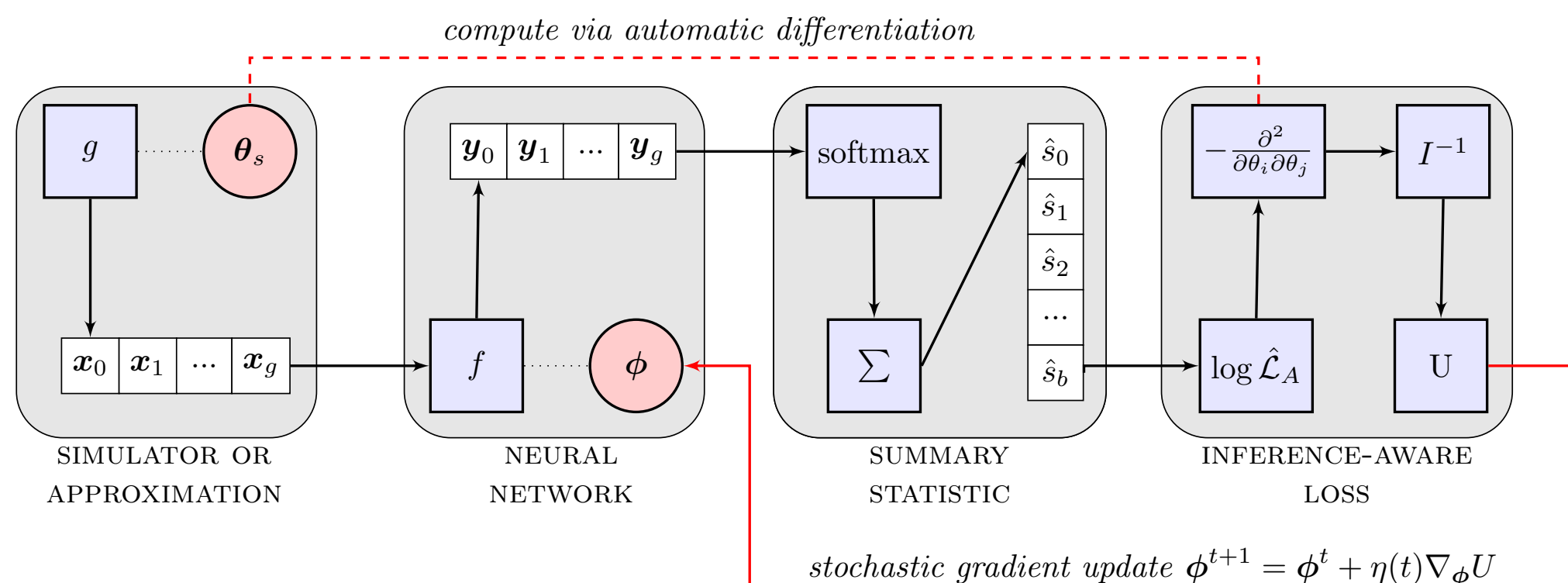
# End-to-end optimization with autodiff

- With tools like MadMiner the objective is to learn a likelihood ratio, which is known optimal properties for measurements etc.
- In INFERNO and Neo the inference objective is directly optimized

## INFERNO: Inference-Aware Neural Optimisation

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**Kyle Cranmer** @KyleCranmer · 19h

Take note! Here is a nice example of differentiable programming. It shows end-to-end optimization of a NN for event categorization wrt. final statistical analysis (using pyhf). Requires running gradients through results of maximum likelihood with fixed-point differentiation 🌞

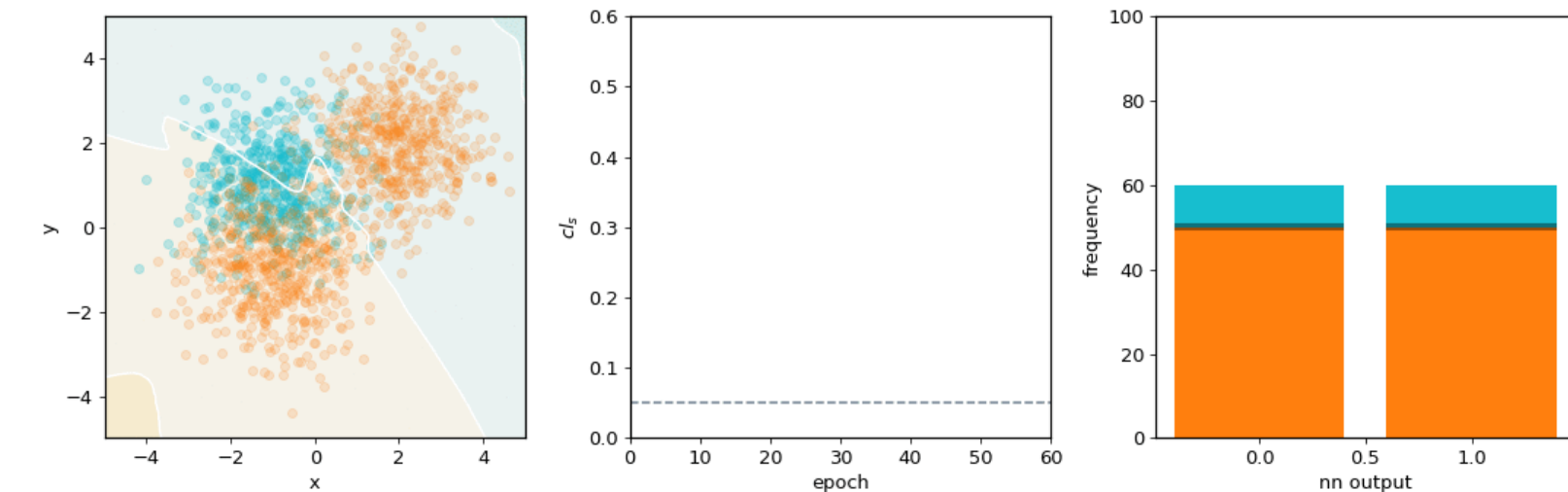


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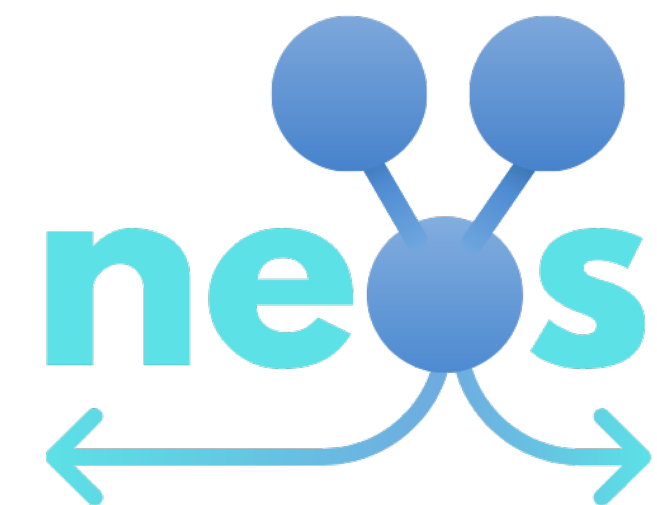
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We've developed a module that performs end-to-end learning with respect to statistical inference in particle physics.

try it yourself at [github.com/pyhf/neos](https://github.com/pyhf/neos)! :)



10:58 AM · Mar 5, 2020 · Twitter Web App



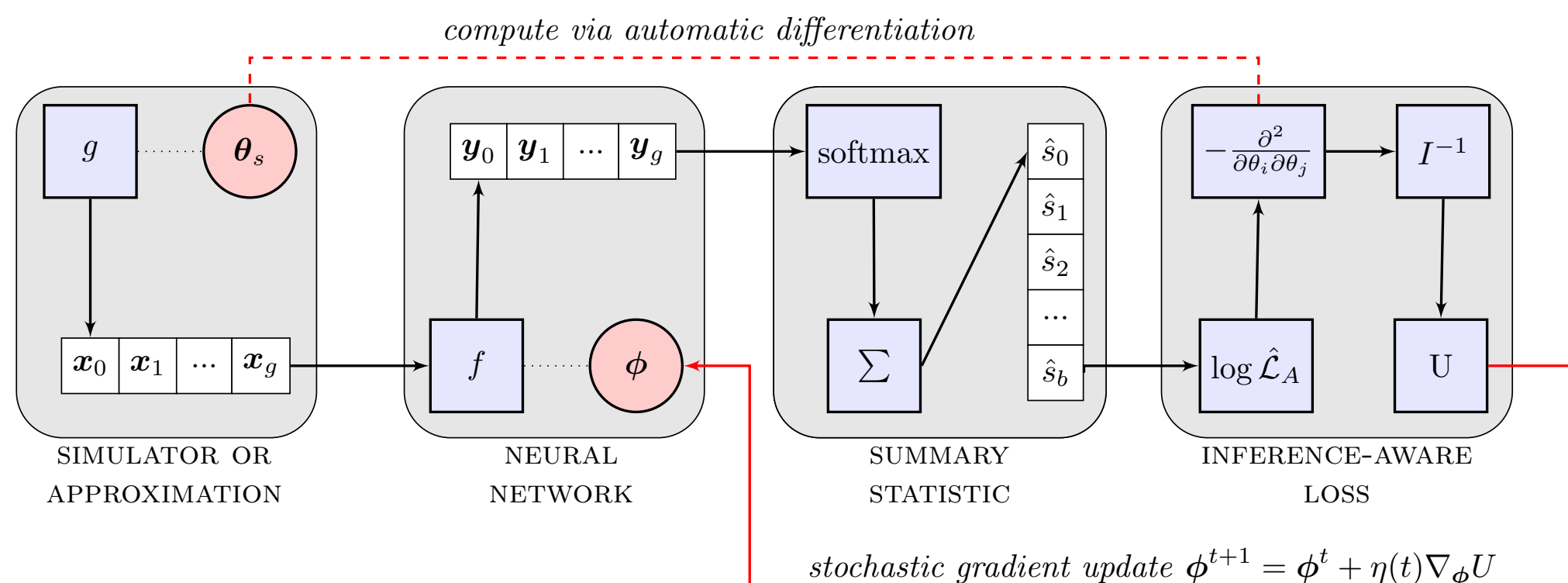
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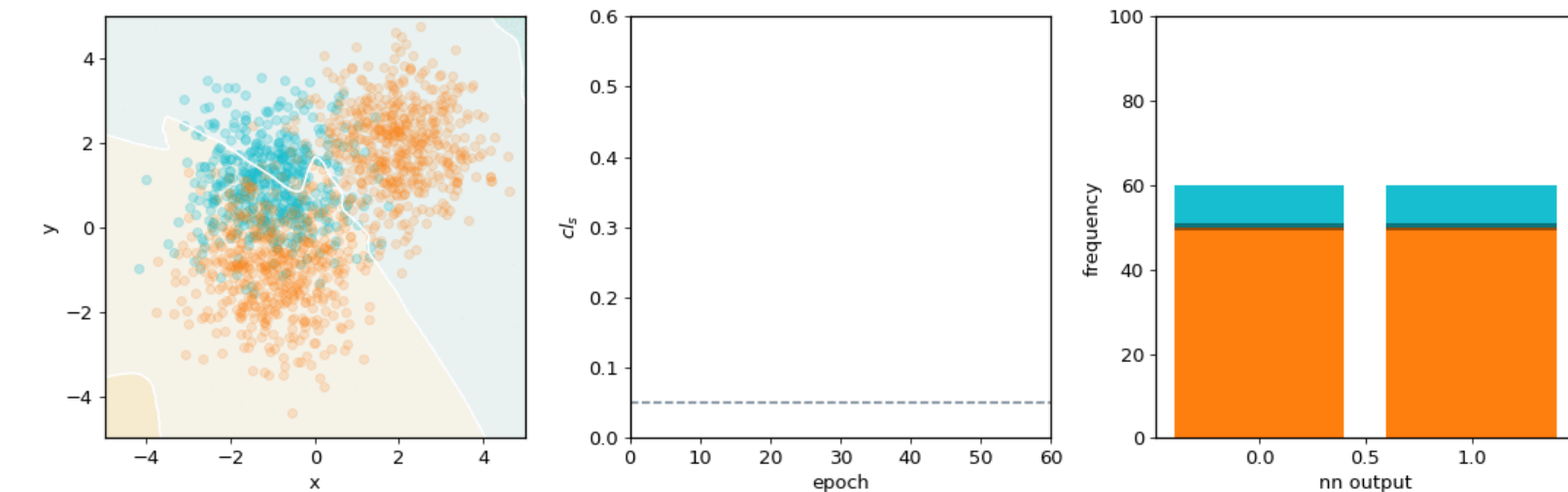


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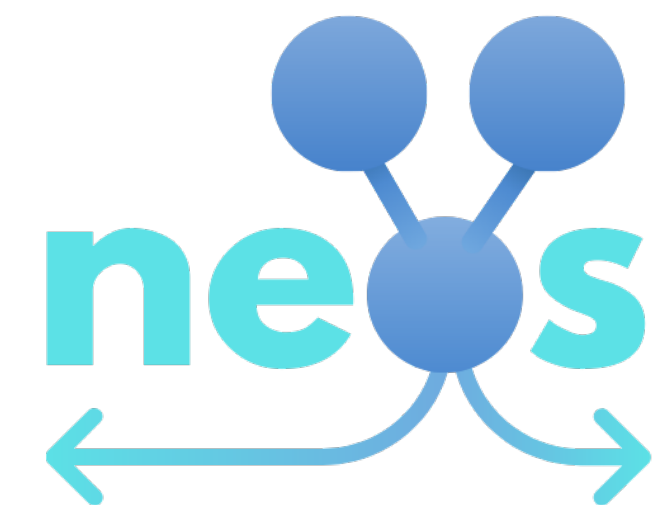
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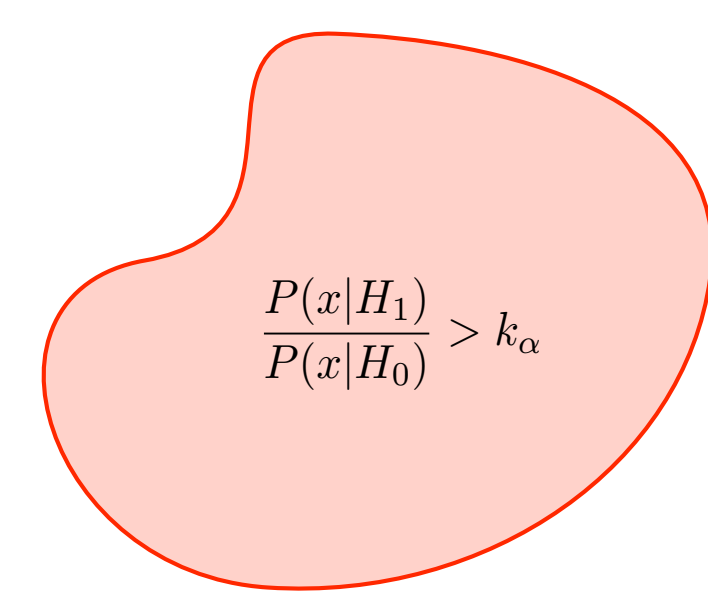
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# Recap on Likelihood Ratios


$$\frac{P(x|H_1)}{P(x|H_0)} > k_\alpha$$

For signal vs. background searches:

- **Neyman-Pearson Lemma**: optimal hypothesis test given by **likelihood ratio** (basis of Higgs search)
- Likelihood ratio  $\frac{p(x|\theta_0)}{p(x|\theta_1)}$  also used for exclusion contours

For estimates of parameters  $\hat{\theta}$

- **Cramér-Rao bound** states  $\text{cov}[\hat{\theta}|\theta_0]_{ij} \geq I_{ij}^{-1}(\theta_0)$  where  $I_{ij}$  is the **Fisher-information matrix** (Hessian of log-likelihood)
- Motivates **Information Geometry** as a phenomenological tool
- Maximum-likelihood (asymptotically) saturates the bound

Note:  $\nabla_\theta \log p(x|\theta)$  acts like a likelihood ratio locally

# Cramér-Rao Bound

The minimum variance bound on an unbiased estimator is given by the Cramér-Rao bound:

$$\text{cov}[\hat{\theta}|\theta_0]_{ij} \geq I_{ij}^{-1}(\theta_0)$$

Expected error of best-fit parameter      Inverse of Fisher information

Fisher information matrix (is also a Riemannian metric! )

$$I_{ij}[\theta] = -\mathbb{E} \left[ \frac{\partial^2 \log p(x|\theta)}{\partial \theta_i \partial \theta_j} \middle| \theta \right]$$

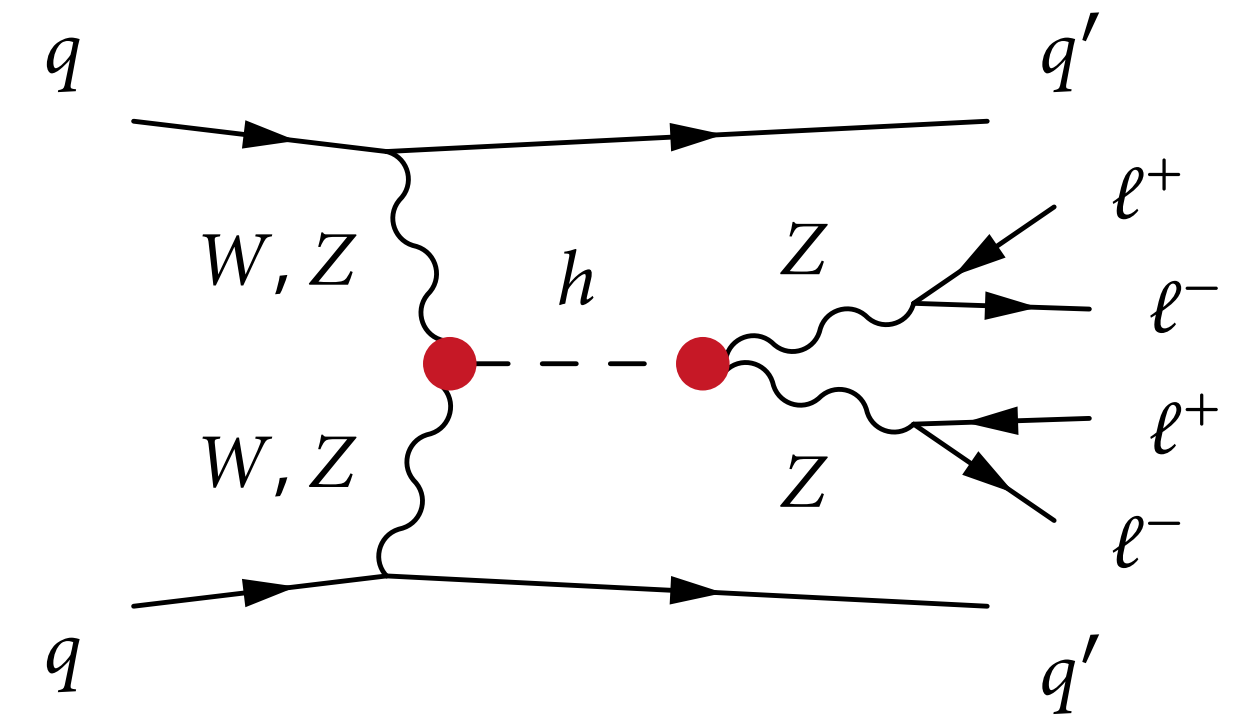
Maximum Likelihood Estimators *asymptotically* reach this bound

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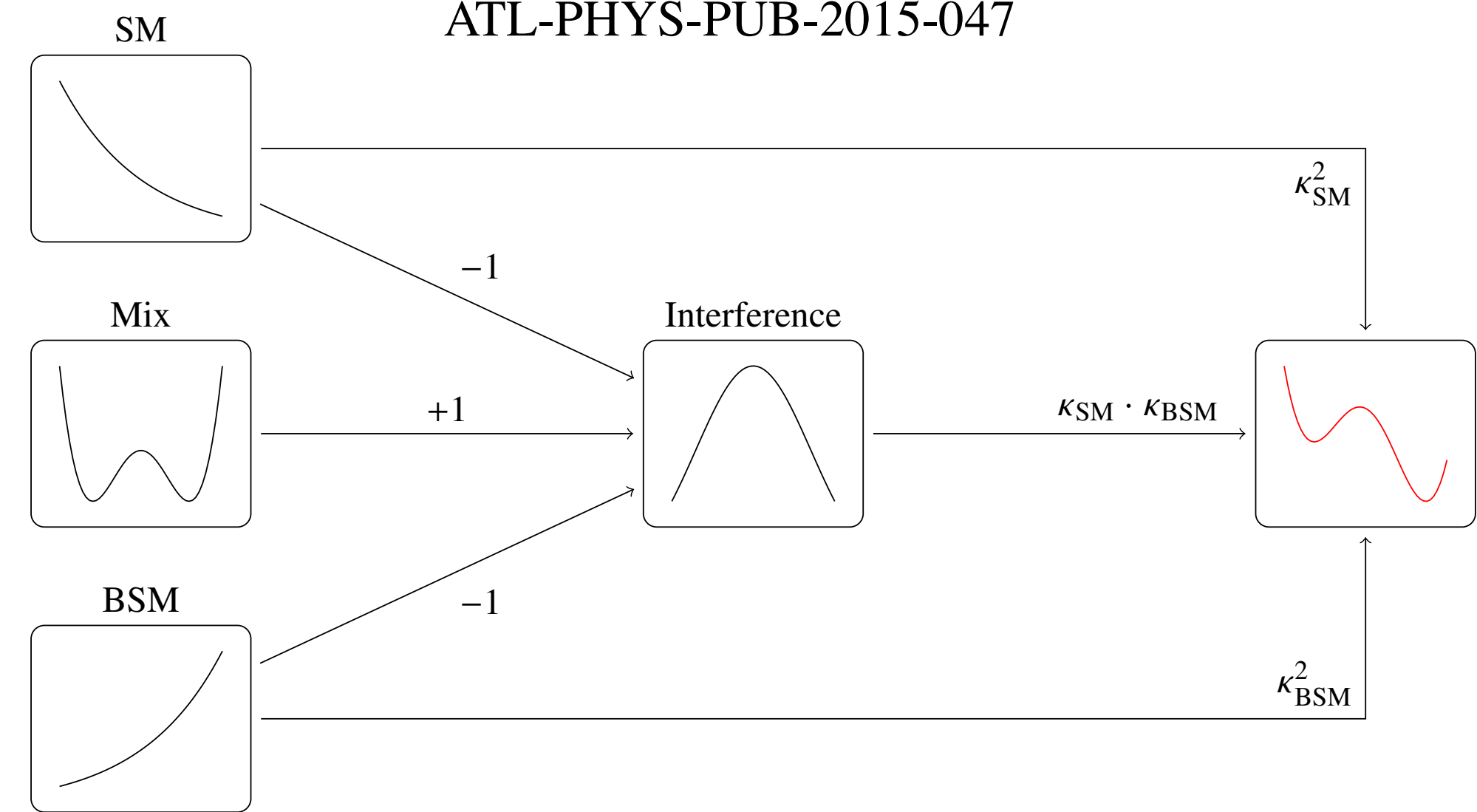
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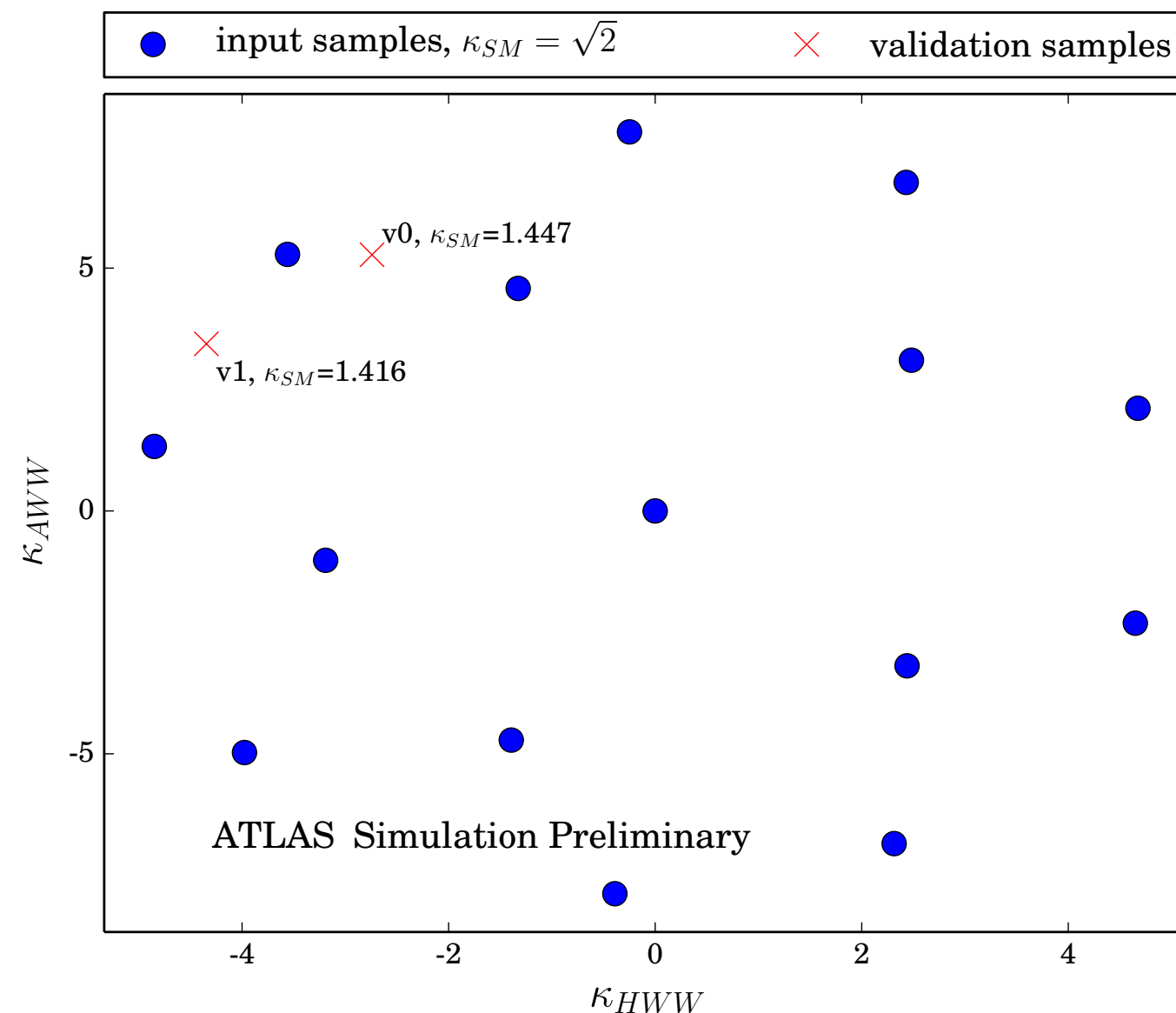
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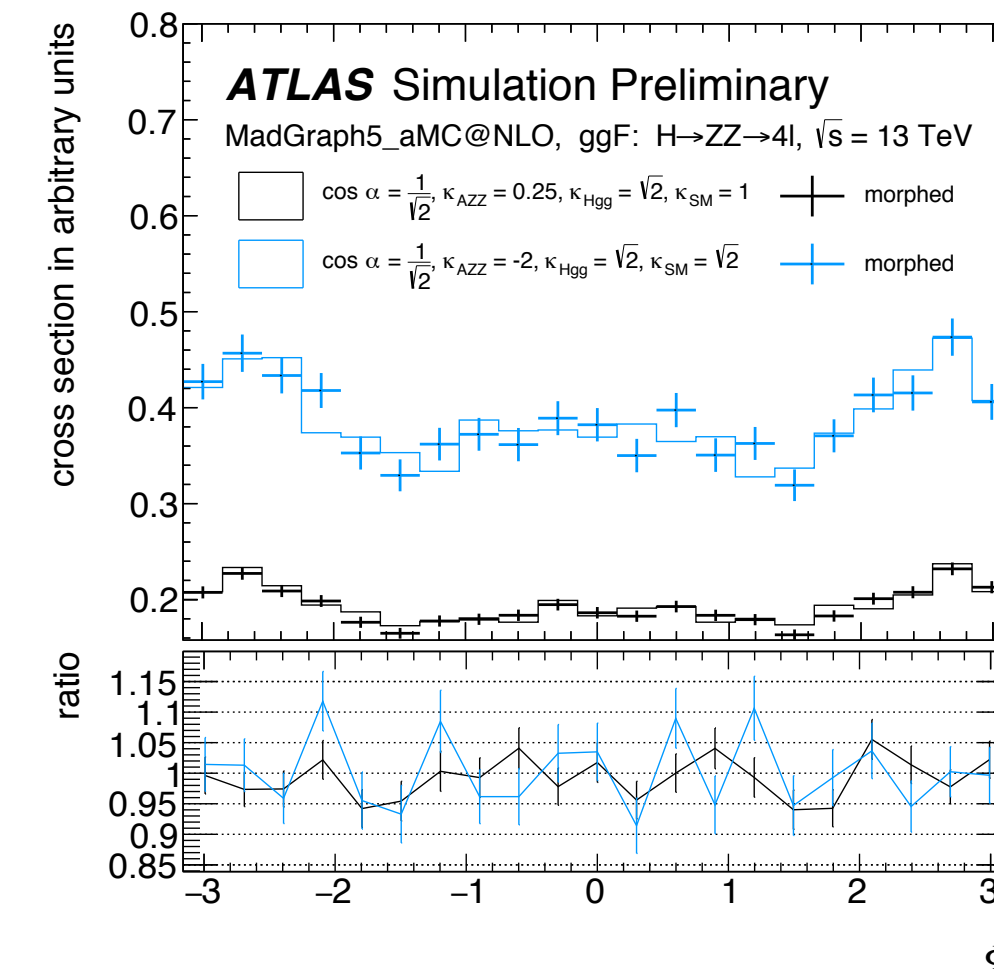
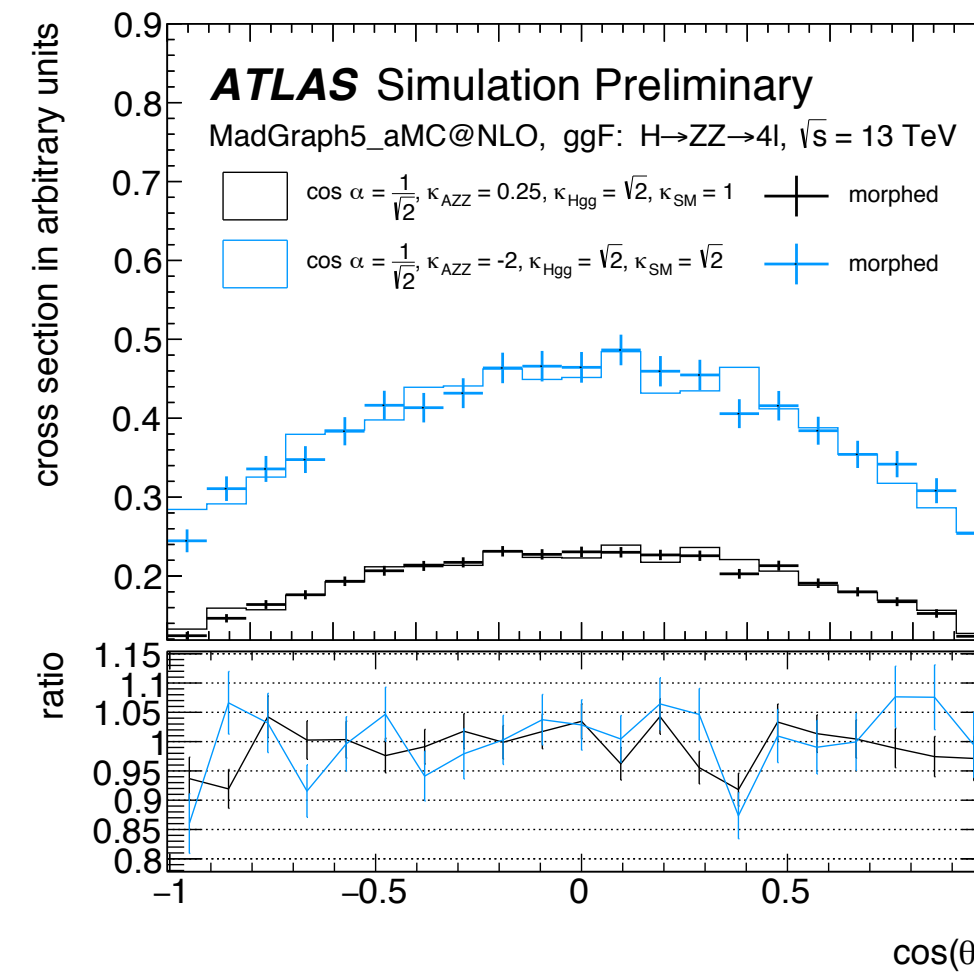
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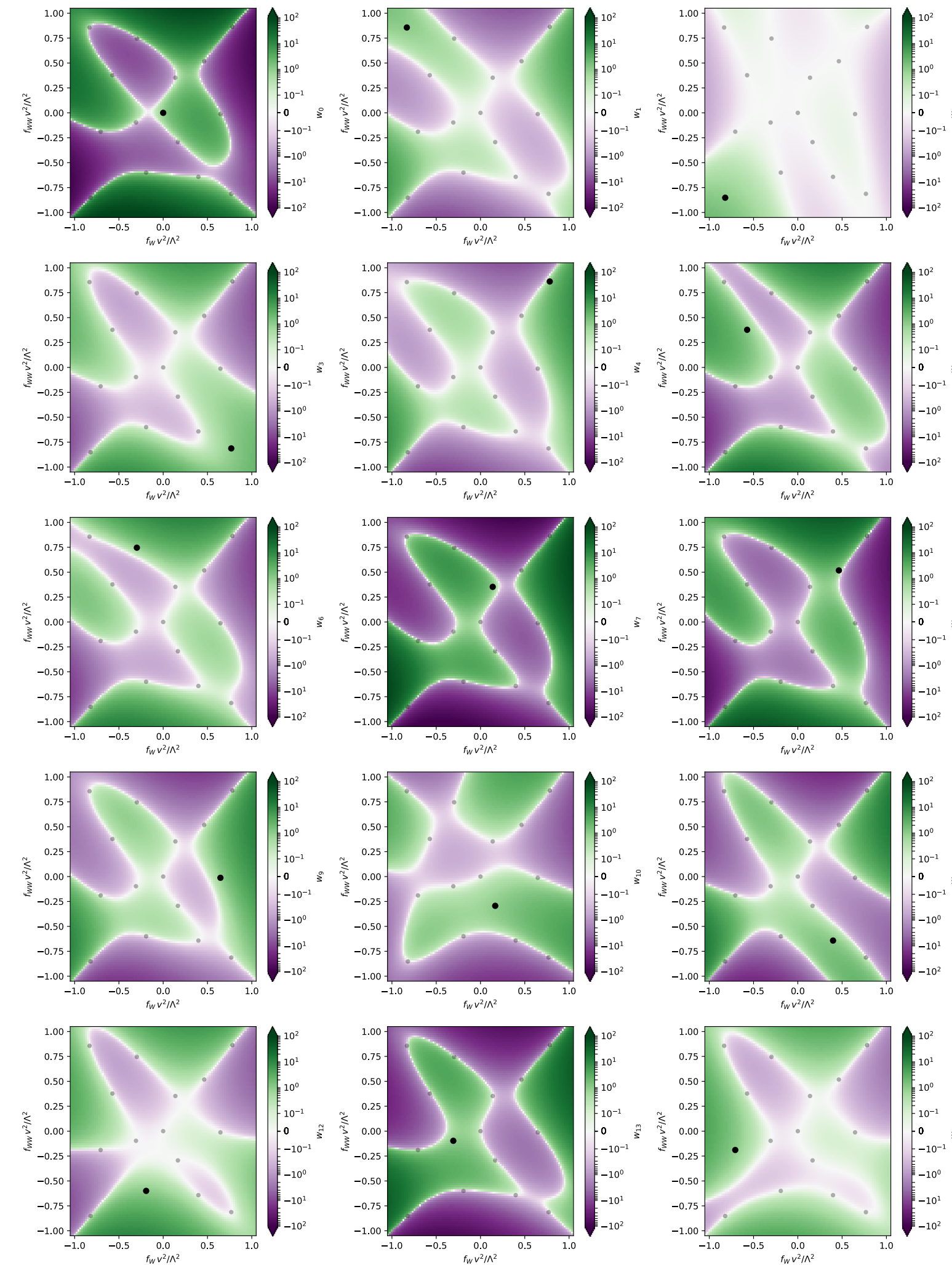


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