PyHEP 2021 (virtual) Workshop

Gallifray
A Geometric Modelling and Parameter Estimation Framework for Black hole images using Bayesian Techniques

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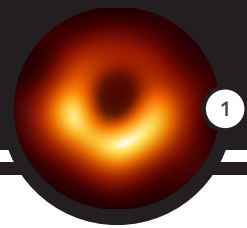
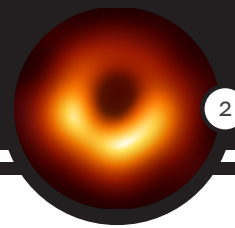
Outline



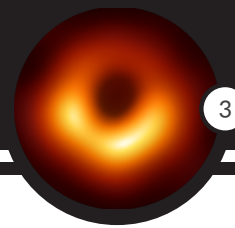
- General Relativistic Modelling
  - General solution [Modified]
  - Numerical Scheme for Shadow
  - Accretion structure
  - Shadow images
- GALLIFRAY - Geometric modelling and Parameter estimation framework using Bayesian formalism
  - Geometric Modelling
  - Syntheic VLBI [EHT]
  - Parameter estimation using MCMC



A bit of history...

Grand Master - Albert Einstein

- 1915 - General Relativity
- 1916 - Schwarzschild Black hole **(Static)**
- 1963 - Kerr Black hole **(Rotating)**
- 1965 - Kerr-Newman Black hole **(Charged and Rotating)**



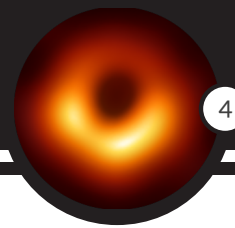
Kerr metric

- $ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\Sigma}((r^2+a^2)d\phi - adt^2)^2 + \frac{\Sigma}{\Delta}dr^+\Sigma d\theta^2$
  $\Delta = r^2 - 2Mr + a^2, \Sigma = r^2 + a^2\cos\theta^2$

No-hair Theorem

The properties of a Black hole can be characterised by only

- Mass
- Angular Momentum
- Charge



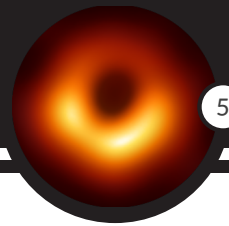
One such solution (KRZ solution[1])

- Parameterized spacetime [Konoplya, Rezzolla Zhidenko Phys. Rev. D93 (2016)]
- $ds^2 = -\frac{A^2 - W^2 \sin^2\theta}{K^2} dt^2 - 2Wr \sin^2\theta dt d\phi + (\frac{\Sigma B^2}{A^2}) dr^2 + \Sigma r^2 d\theta^2 + K^2 r^2 \sin^2\theta d\phi^2$
- Extra/deformation parameters (a_{o1}, w_{o1})

Geodesic Equation

Geodesic Equation

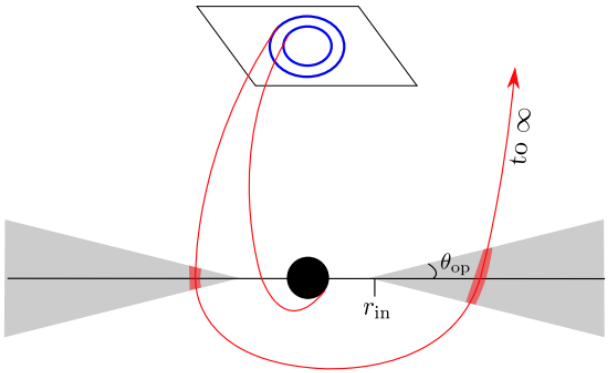
- $\frac{d^2x^\mu}{ds^2} = -\Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}$



Geometrically thick disk (Example model)

- The observed specific intensity $I_{\nu o}$ at the observed photon frequency ν_o at the point (X,Y) in the observer's sky is given by [Jaroszynski & Kurpiewski 1997; Bambi 2013],

$$I_{\nu o}(X, Y) = \int_{\gamma} g^3 j(\nu_e) dl_{\text{prop}} \tag{1}$$

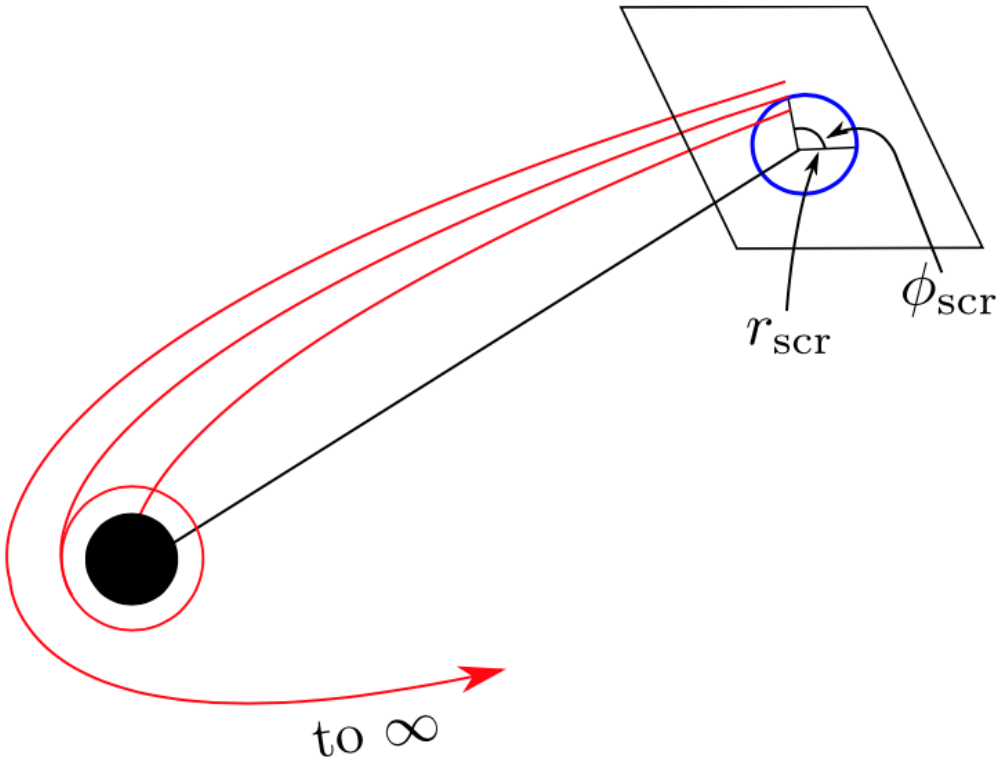


SN, Suvorov Kokkotas Phys. Rev. D102 (2020)

General Relativistic Modelling

Numerical Scheme

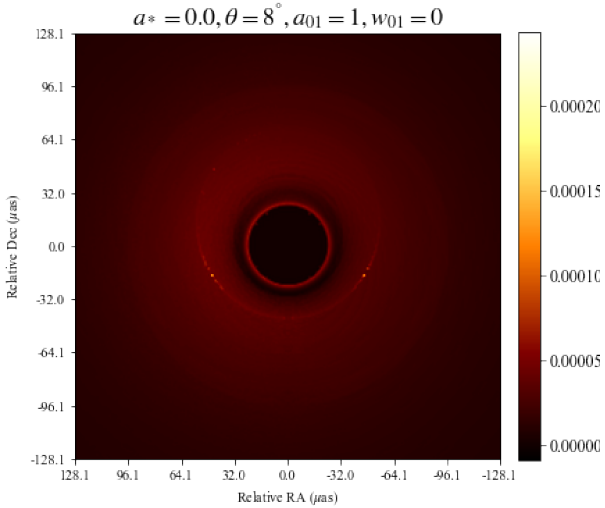
- Numerical Algorithm - Backward Raytracing
  - Trace photons backwards in time from an observer located far away towards the black hole

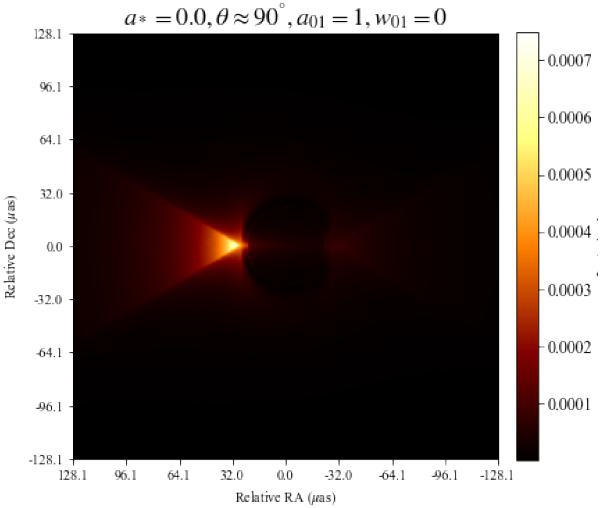


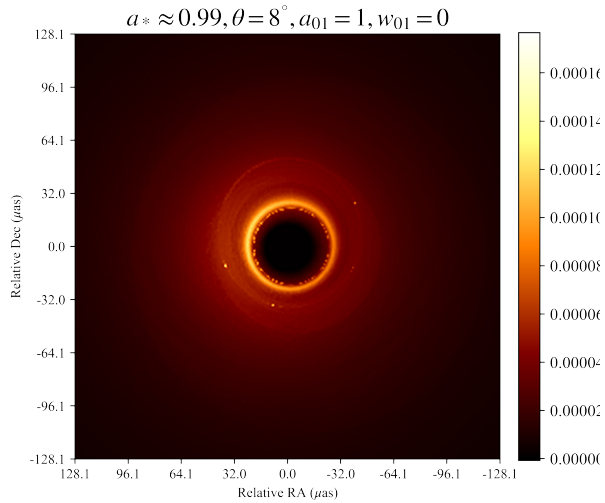
SN, Suvorov Kokkotas Phys. Rev. D102 (2020)

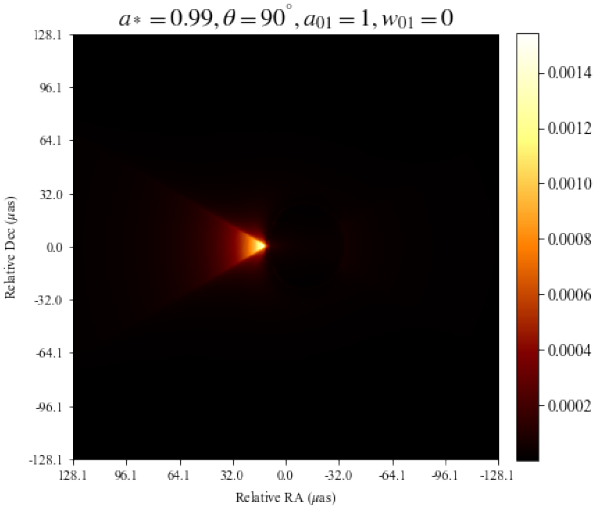
General Relativistic Modelling

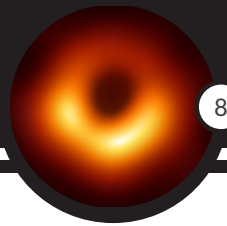
Shadow images





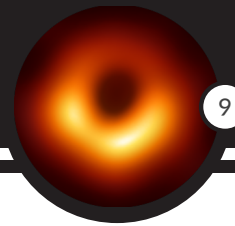




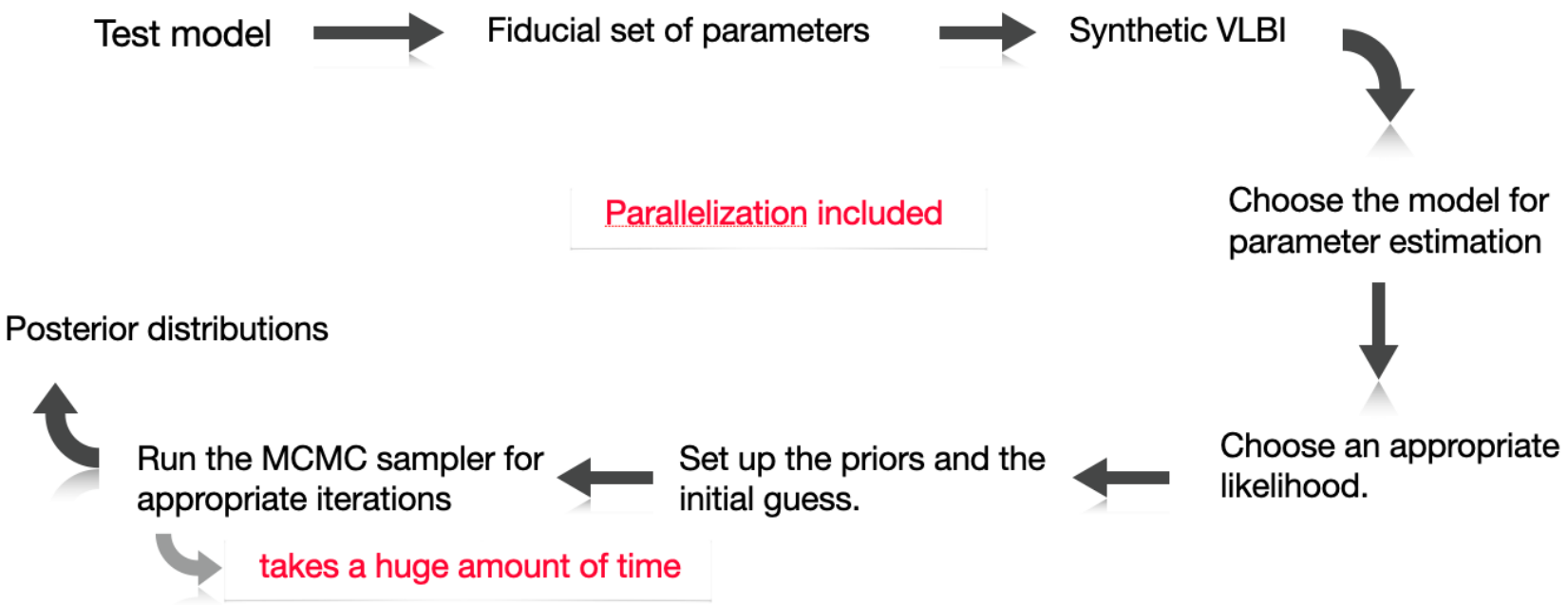


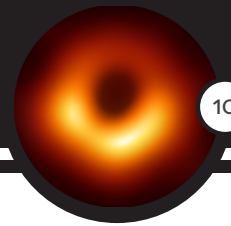
What is it?

- Modelling tool [Geometric + General Relativistic]
- Extracting basic parameters (Diameter, width etc.)
- Bayesian formalism and parameter extraction
- Written purely in Python

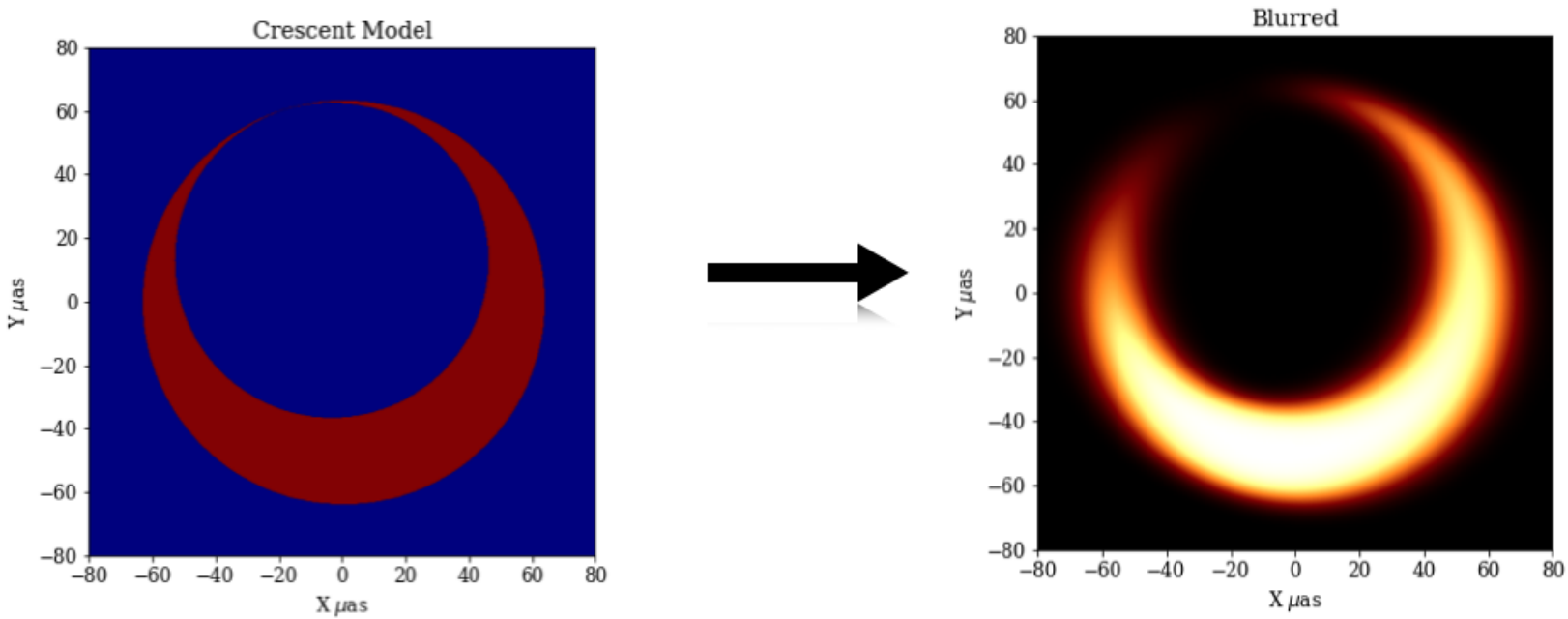


- Workflow


Test model
Fiducial set of parameters
Synthetic VLBI
Parallelization included
Choose the model for parameter estimation
Choose an appropriate likelihood.
Set up the priors and the initial guess.
Run the MCMC sampler for appropriate iterations
takes a huge amount of time
Posterior distributions

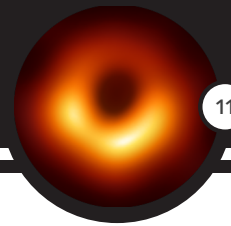


- Geometric Models
- Simplification of a black hole accretion image
- Eg: Crescent Model [Kamruddin, A. B., & Dexter, J. ,2013, MNRAS, 434, 765]



GALLIFRAY

Geometric modelling and Parameter estimation framework using Bayesian formalism



*** to be implemented** Generalised Crescent Models

S.No	Model	No. of Parameters	Parameters	$I(x,y)$	$\mathscr{V}(u,v)$
1	**Symmetric Gaussian**	3	I_0, σ, ϕ	Implemented	In-process
2	**Asymmetric Gaussian**	4	I_0, σ, A, ϕ	Implemented	In-process
3	**Filled Disk**	2	I_0, R_{ex}	Implemented	Implemented
4	**Crescent [2]**	5	$I_0, R_{ex}, R_{in}, \epsilon, f, \phi$	Implemented	Implemented
5	**xs-ring [3]**	6	$I_0, R_{ex}, \psi, \tau, \phi$	Implemented	Implemented
6	**xs-ringauss [3]**	9	$I_0, R_{ex}, R_{in}, \epsilon, f, g_{ax}, a_q, g_q, \phi$	Implemented	Implemented
7	***EHT xs-ring + [4] emission floor**	10	-	For 2nd release	For 2nd release
8	***EHT xs-ringauss + [4] emission floor**	14	-	For 2nd release	For 2nd release

- Example notebook

In [1]:
```
import numpy as np
import gallifray as gr
import matplotlib.pyplot as plt
```

```
/Users/geodesix/opt/anaconda3/lib/python3.8/site-packages/pyfits/__init__.py:21: PyFITSDeprecationWarning: PyFITS is deprecate
d, please use astropy.io.fits
  warnings.warn('PyFITS is deprecated, please use astropy.io.fits',
```

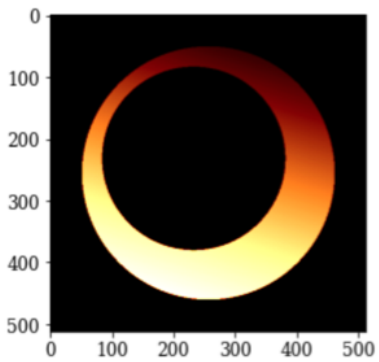
Setting up parameters for the model and initialising the image object

In [2]:
```
I0 = 1
Rp = 40
Rn = 29
ecn = -0.42
f = 0.1
phi = 17*np.pi/180
gax=0.2
aq=1
gq=0
fov = Rp*2 + 20
xs1 = gr.models.xsringauss(I0,Rp,Rn,ecn,f,gax=gax,aq=aq,gq=gq,phi=phi,fov=fov)
```

Initiating and creating the image object

In [3]:
```
im = xs1.sky_map() # Returns the 2-D array image
plt.imshow(im, cmap='afmhot')
```

Out[3]:
```
<matplotlib.image.AxesImage at 0x7ff31df7b7c0>
```



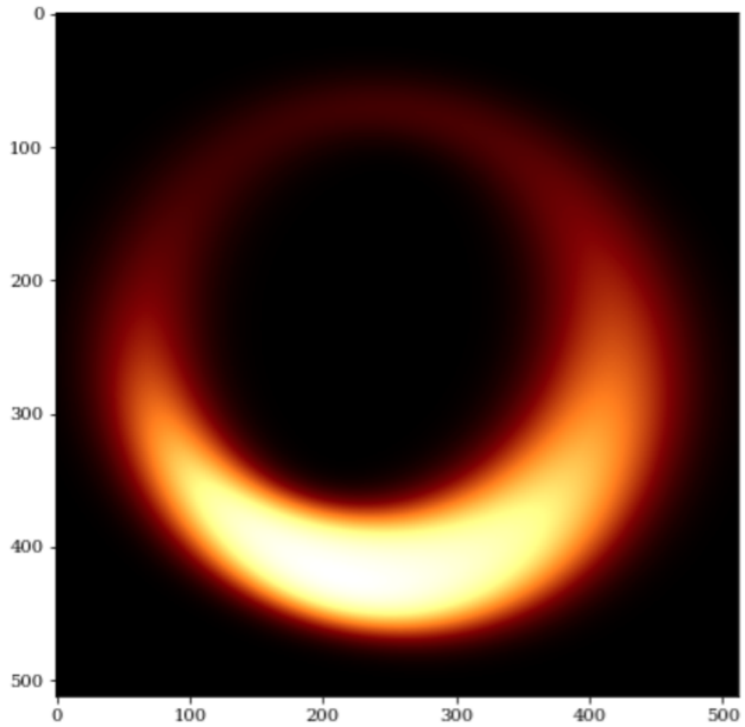
creates the sky map and returns a 2-D array image

- Example notebook

In [15]:
```
1 blur = xs1.sky_blur()
2
3 plt.figure(figsize=(7,7))
4 plt.imshow(blur,cmap='afmhot')
```

Convolves the image with Sagittarius A* interstellar Scattering kernel

Out[15]: <matplotlib.image.AxesImage at 0x7f9d8793e7f0>



Caution!

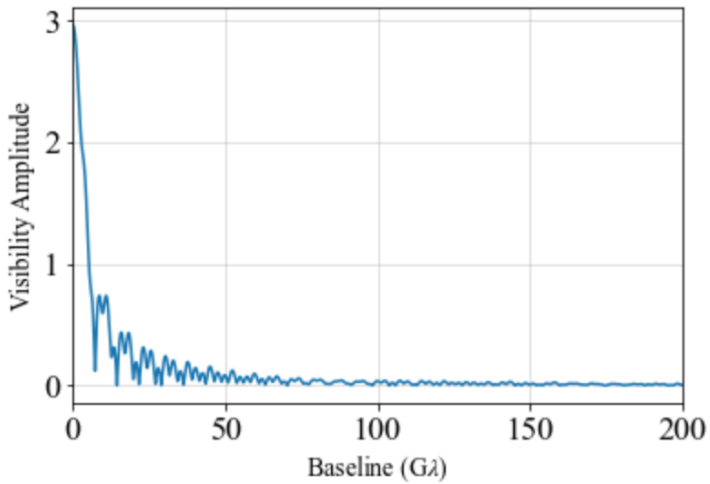
This has to be called with the initial image object not the sky image array

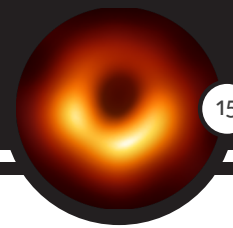
- Visibility Amplitudes

In [65]:
```
vis_amp = xsring.visamp()
uv_dis  = xsring.baseline()
```

In [69]:
```
plt.plot(dis*1e-9, vi)
plt.xlim(0,200)
plt.grid('on', alpha=0.5)
plt.ylabel('Visibility Amplitude',fontsize=14)
plt.xlabel(r'Baseline (G$\lambda$)',fontsize=14)
```

Out[69]: `Text(0.5, 0, 'Baseline (G$\\lambda$)')`



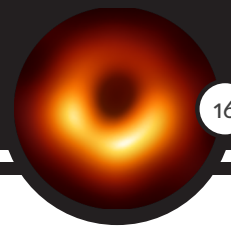


Model Fitting (MCMC)

- Likelihood Construction
- Rice Distribution (Visibility Amplitude)

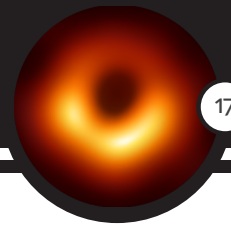
$$P^{obs}(\mathcal{V}_m; \mathcal{V}_{ij}, \sigma_{ij}) = \frac{\mathcal{V}_m}{-\sigma_{ij}^2} \left[\frac{-(\mathcal{V}_m^2 + \mathcal{V}_{ij}^2)}{2\sigma_{ij}^2} \right] I_0 \left(\frac{\mathcal{V}_m \mathcal{V}_{ij}}{\sigma_{ij}^2} \right) \tag{2}$$

- where I_0 is the modified Bessel function of the first kind.
- at high S/N (S/N2), this becomes a gaussian.



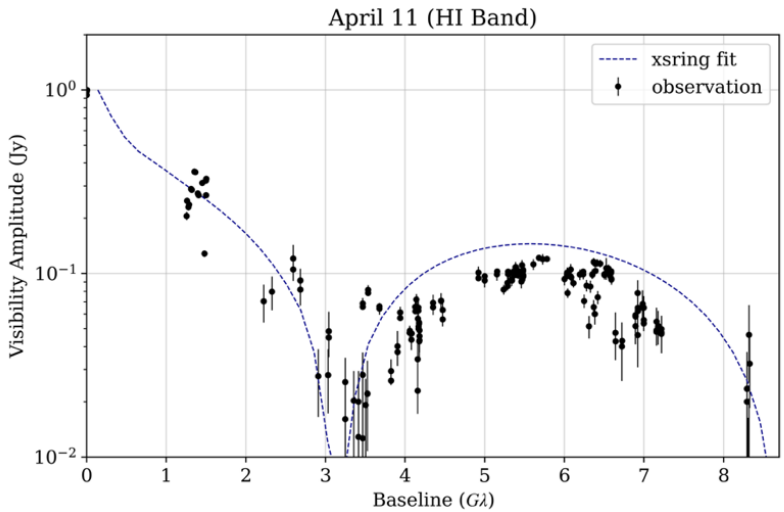
Quantities

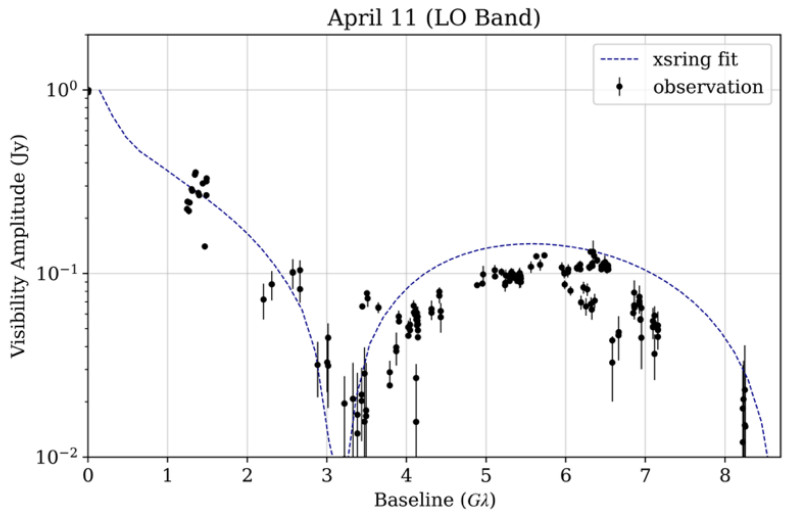
- Visibility Amplitude
- Closure Amplitude
- Closure Phases
- Log Closure Amplitudes (under development)



Sample analysis (Best fits for xsring model)

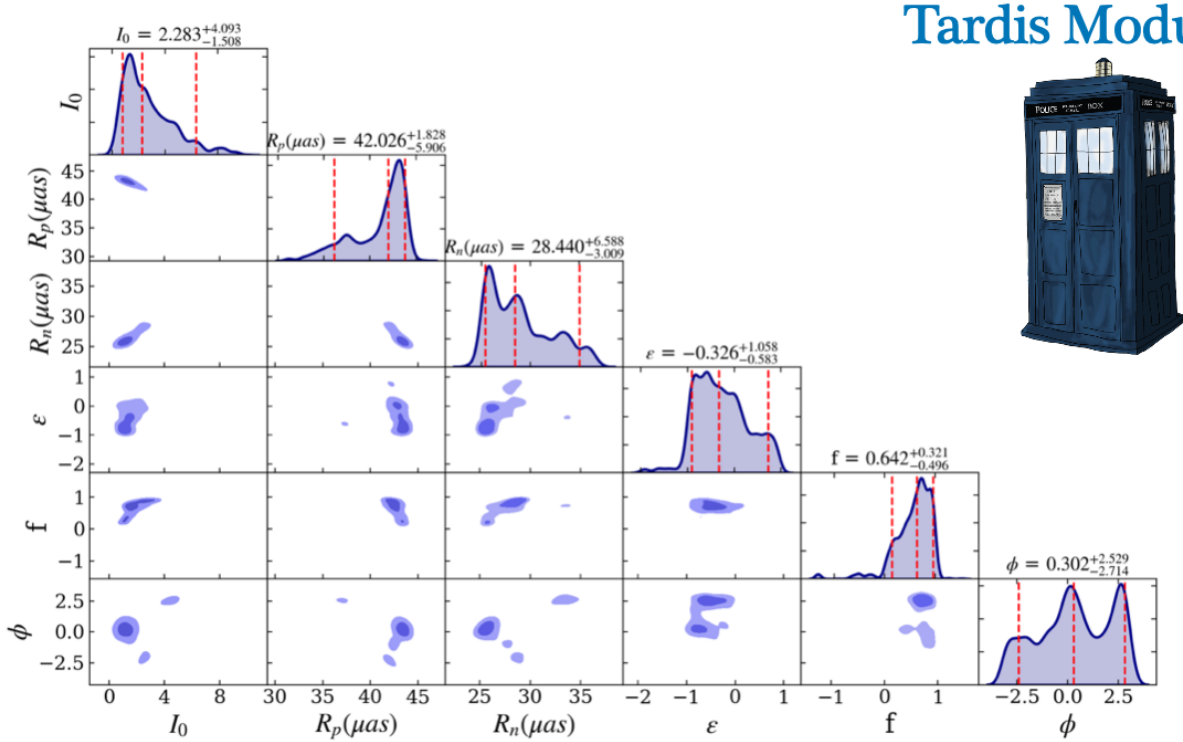
- Model fitting for 2017 M87 results (Visibility Amplitudes)





Sample analysis (Best fits for xsring model)

- Model fitting for 2017 M87 results (Visibility Amplitudes)
- Posterior distribution



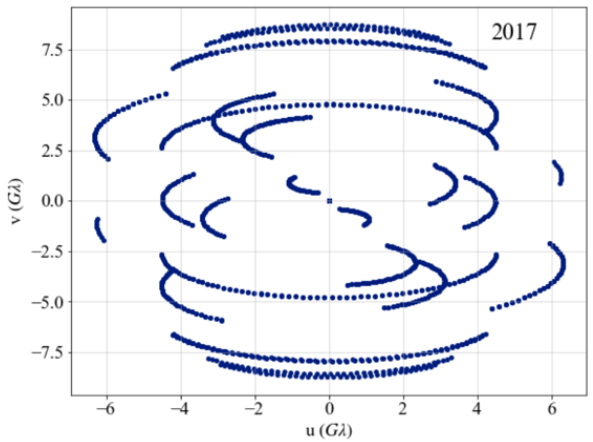
Tardis Module

Image reconstruction

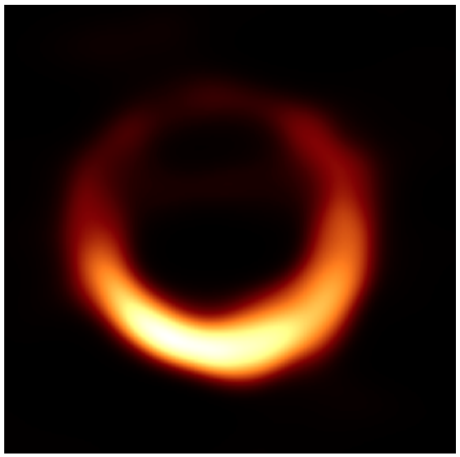
- Ex: Model image reconstruction (using eht-imaging) [MEM]

Model image*

Baseline Coverage (EHT-2017 config)



Reconstructed



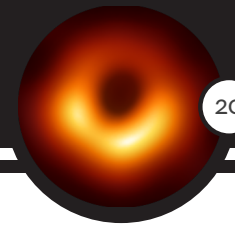
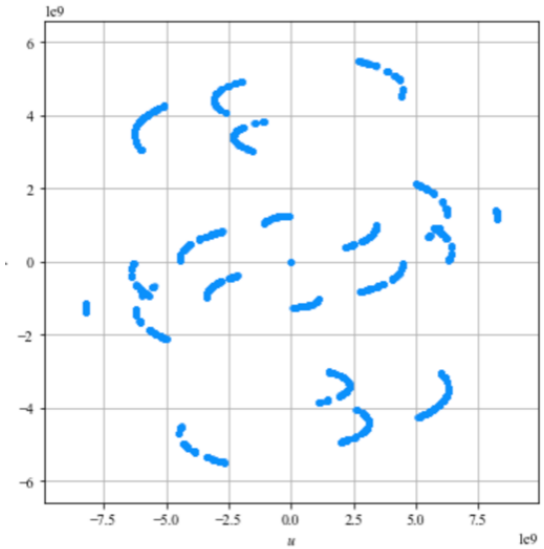
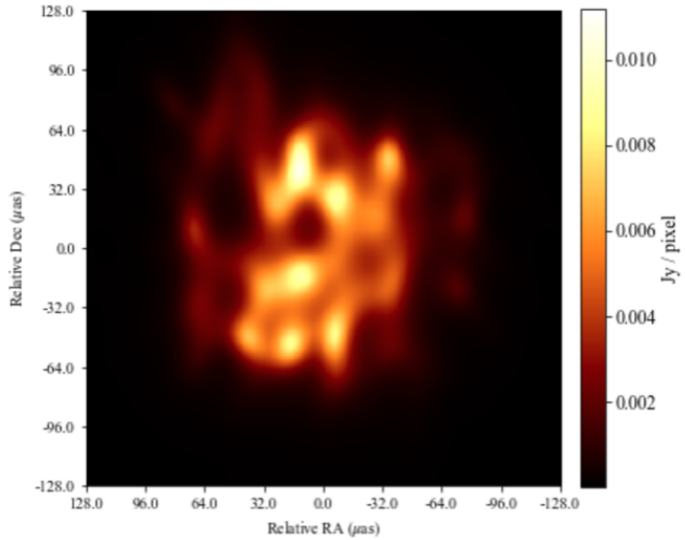


Image reconstruction

- Ex: Raytraced model image reconstruction (using eht-imaging)
- Setup : EHT-2017 M87





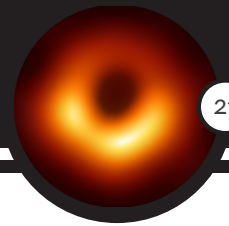
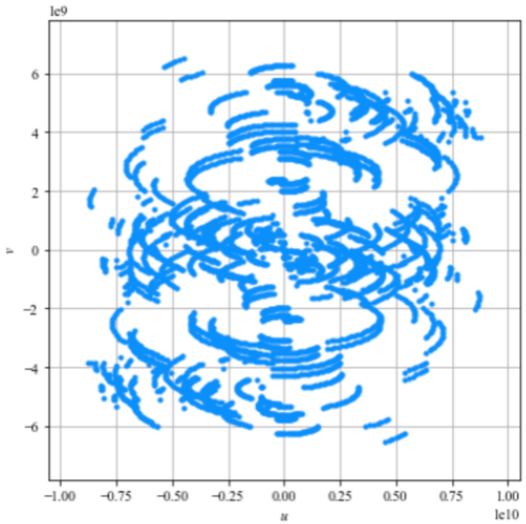
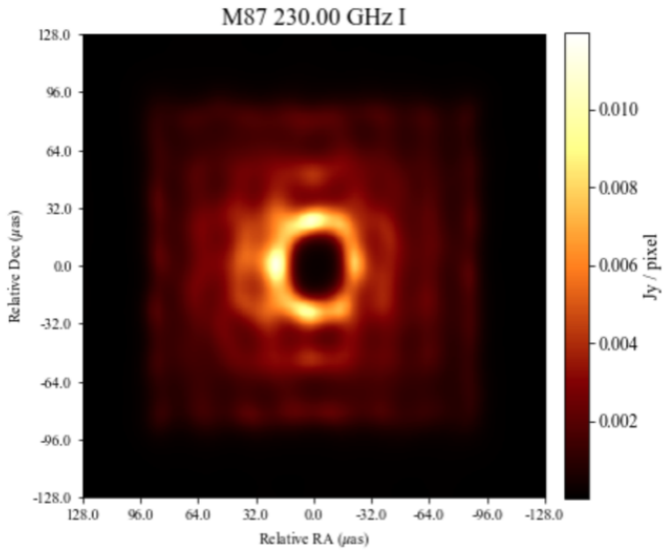
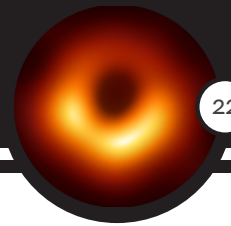


Image reconstruction

- Ex: Raytraced model image reconstruction (using eht-imaging)
- Setup : next-generation Event Horizon Telescope (ngEHT) [A. W. Raymond et. al., The Astrophysical Journal Supplement Series, 253 (2021).]

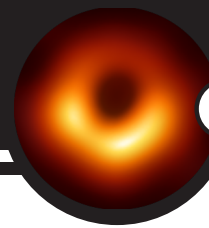






Availability

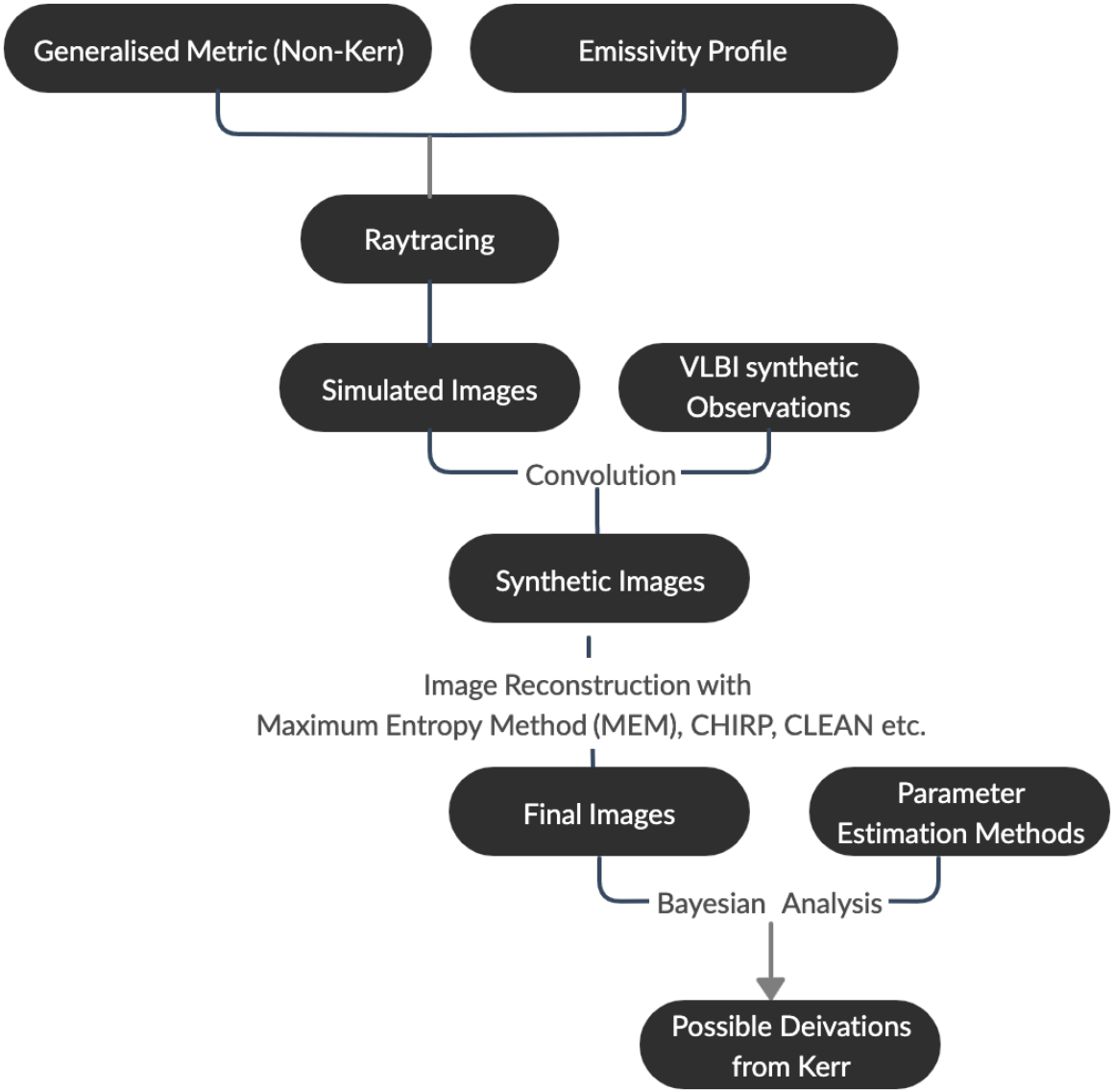
- Code available at GitHub :
- **https://github.com/Relativist1/Gallifray**
- Second Release soon!
- Feel Free to ‘Pull request’ and create an ‘issue’



Future Additions (Under development)

- Ray-Tracing Capability (2nd Release)
  - Static and Spherically Black hole surrounded with spherical accretion
- Nesting Sampling for MCMC
- Additions of more models (modelling jets etc.)
- Embedded ray-tracing routines with different accretion models.
- Space-VLBI

Summary


Generalised Metric (Non-Kerr)
Emissivity Profile
Raytracing
Simulated Images
VLBI synthetic Observations
Convolution
Synthetic Images
Image Reconstruction with
Maximum Entropy Method (MEM), CHIRP, CLEAN etc.
Final Images
Parameter Estimation Methods
Bayesian Analysis
Possible Deivations from Kerr

Thank You!