

Using lattice QCD to correlate pion structure and EHM

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JLab and W&M

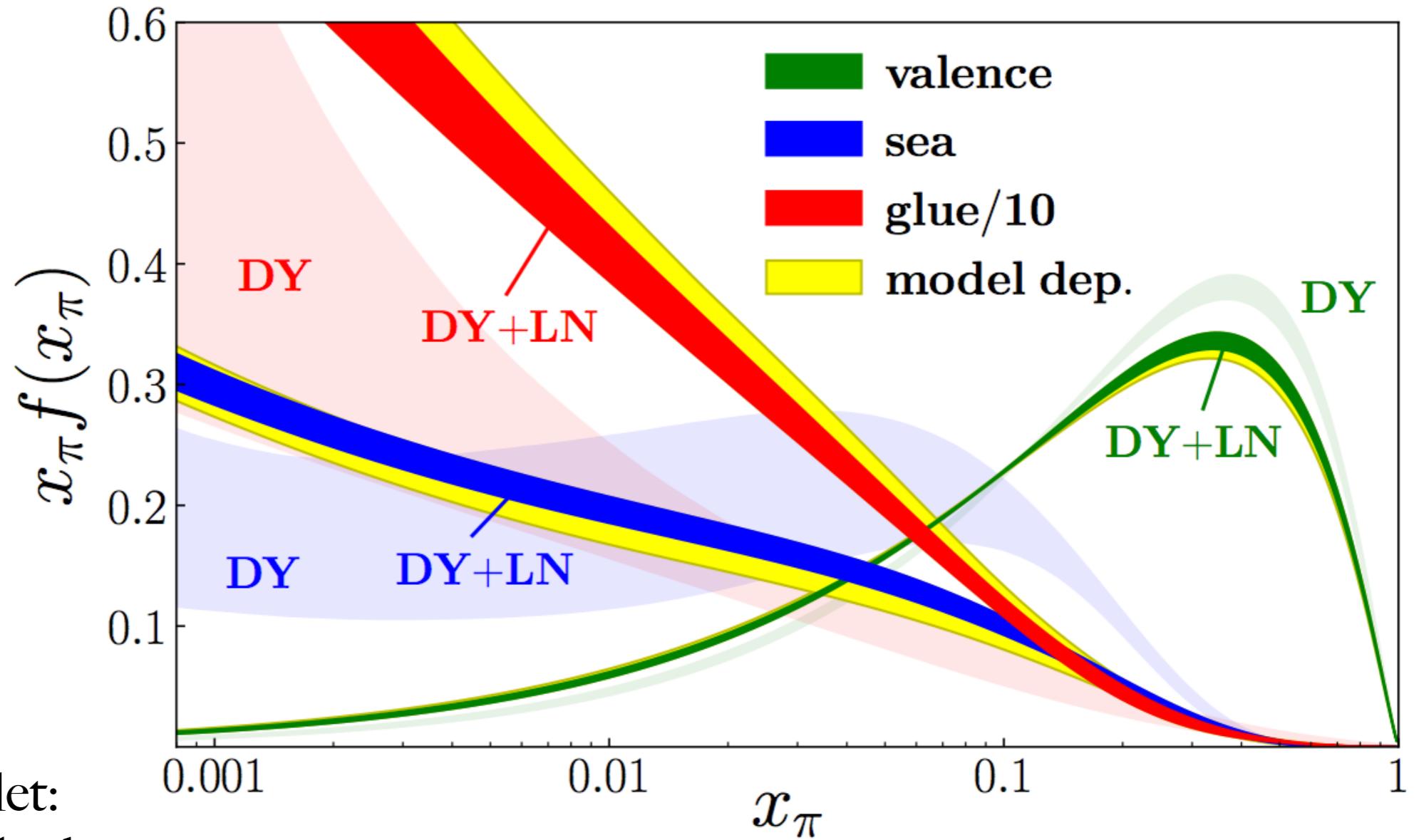
Perceiving Emergent Hadron Mass through AMBER@CERN - V

In collaboration with X. Gao, L. Jin, C. Kallidonis, S. Mukherjee, P. Petreczky,
C. Shugert, S. Syritsyn, Y. Zhao

Valence PDF of $\pi^+ (u\bar{d})$

$$f_v^\pi(x) = f_u(x) - f_{\bar{u}}(x) = f_u(x) - f_d(x), \quad 0 < x < 1$$

(sea+valence) (sea) (Isospin symmetry)



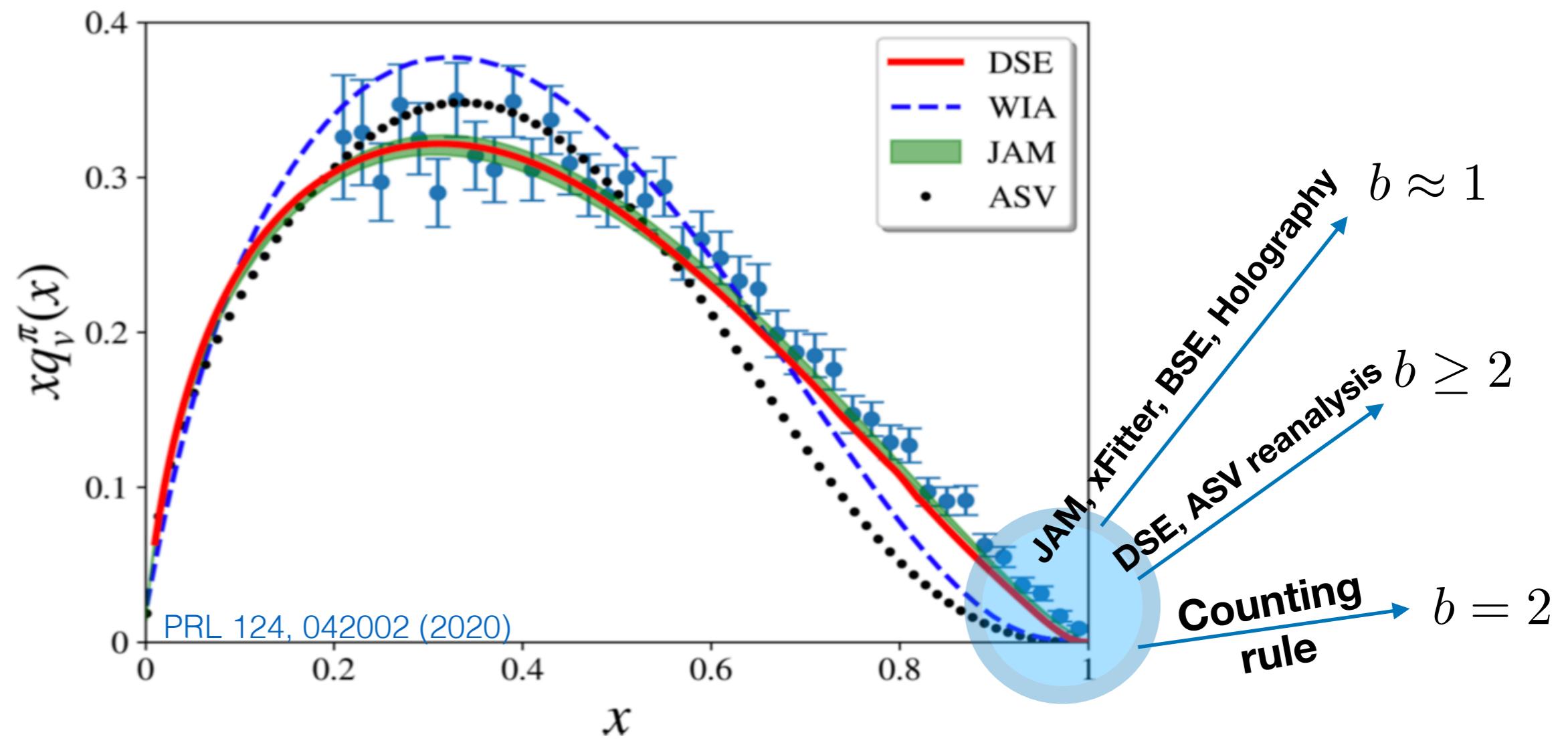
Iso-triplet:

- No mixing with gluon
- No fermion disconnected lines

JAM18, P. C. Barry et al, 2018

Key physics issue is $x=1$ behavior: $\lim_{x \rightarrow 1} f_v^\pi(x) \sim (1 - x)^b$

Review in “Insights into the Emergence of Mass from Studies of Pion and Kaon Structure”,
C. D. Roberts et al, 2102.01765



**First principle calculation
essential!**

Can we use lattice QCD as a complementary “theoretical collider” to learn about mass-gap in QCD and the non-perturbative origin of PDFs?

pion PDF in QCD and QCD-like theories



Chiral symmetry-broken QCD vacuum physics

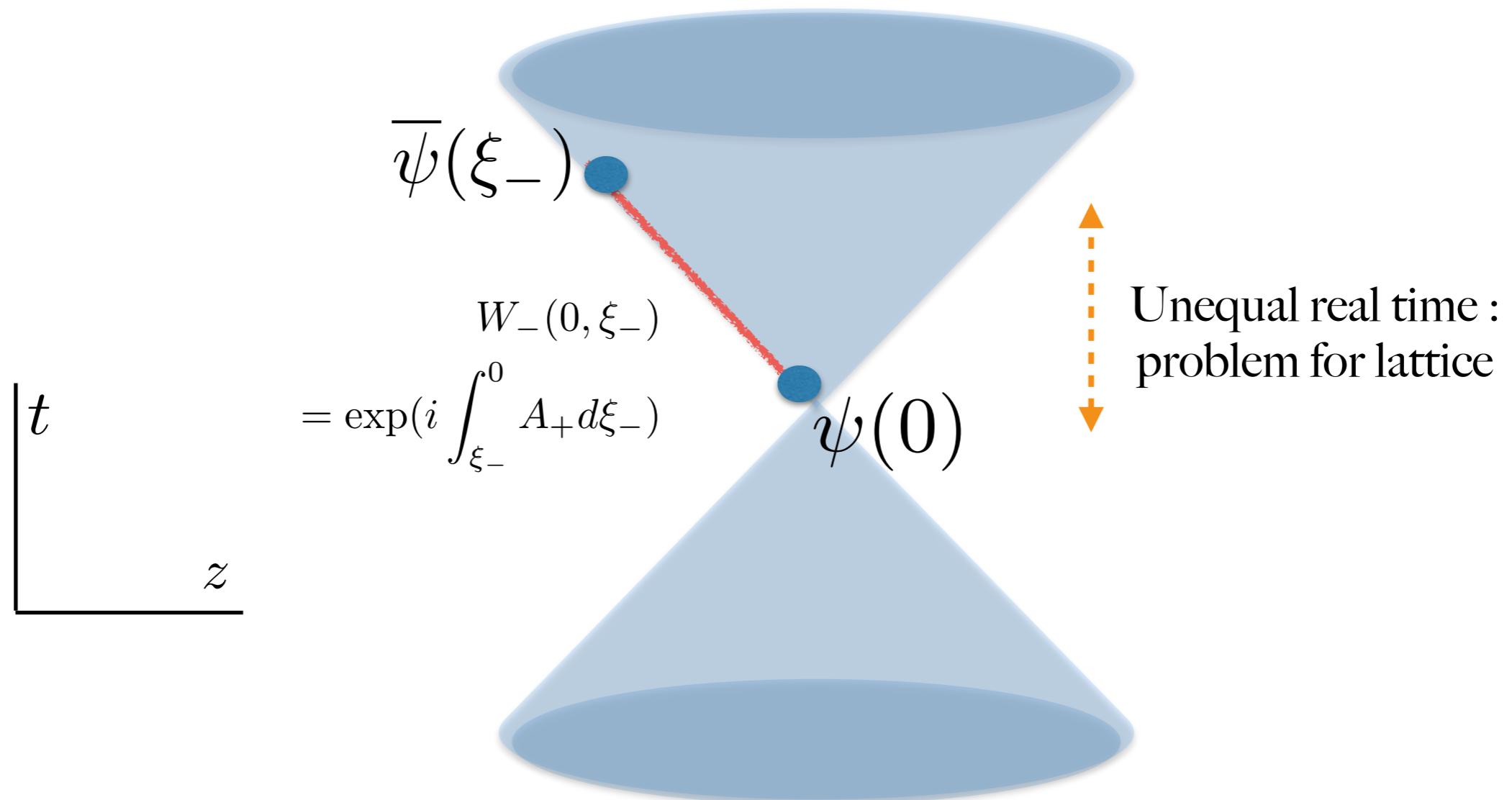
Previous approaches
DSE, BSE, Holography, ...

Questions I want to address in this talk:

- ✳ What is the nature of the large- x $(1-x)^b$ behavior of pion valence PDF?
- ✳ What happens to the quark-structure when pion is radially excited?
- ✳ How is the quark structure of the Goldstone pion sensitive to the symmetry-broken vacuum structure?

PDF as light-like separated q- \bar{q} correlation

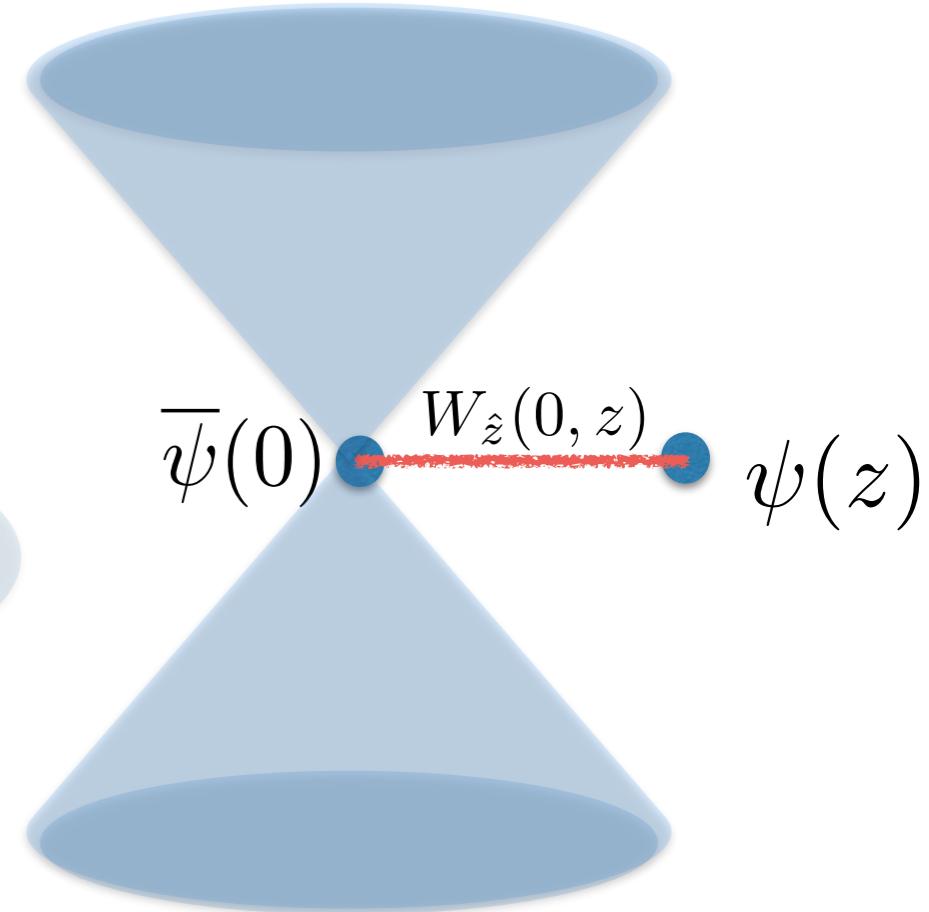
$$f(x) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle H(P) | \bar{\psi}(\xi_-) \gamma_+ W_-(0, \xi_-) \tau \psi(0) | H(P) \rangle$$



LaMET and SDF methodologies

X. Ji '13
A. Radyushkin '17
V. Braun, D. Muller '08

Distill leading-twist terms from Euclidean matrix elements at large-hadron momentum and/or short-distances



(Renormalized quantity from LQCD)

$$\left\langle P_z, E \middle| O_R(z) \middle| P_z, E \right\rangle$$

(pert. coeff)

$$= \sum_n c_n(z^2 \mu^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle(\mu)$$

(PDF moments)

+ twist-2 trace terms $M^k P_z^{n-k} z^n$

+ Higher twist terms $\Lambda_{\text{QCD}}^{2k} P_z^{2n-2k} z^{2n}$

Computing the pion valence PDF

Computational details

HotQCD HISQ ensembles at two fine lattice spacings:

$$a = 0.06 \text{ fm}, \quad 48^3 \times 64$$
$$a = 0.04 \text{ fm}, \quad 64^3 \times 64$$

Pion momenta:

$$P_z \in [0, 2.15] \text{ GeV}$$
$$P_z \in [0, 2.42] \text{ GeV}$$

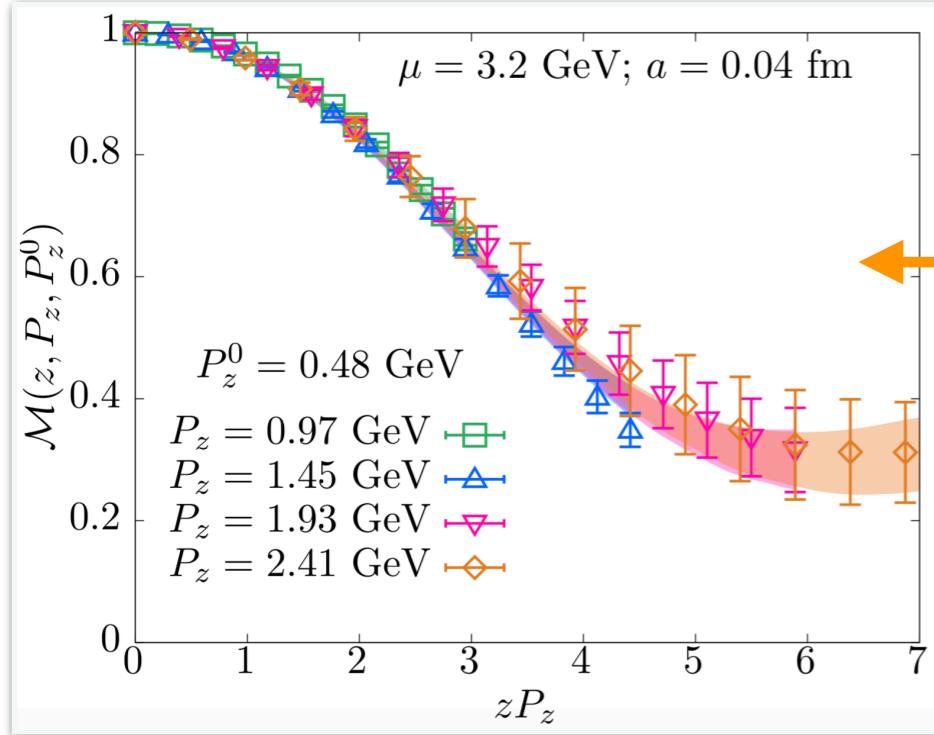
Tadpole improved Wilson-clover valence fermion

Valence pion mass = 300 MeV

Source-sink separations $t_s \in [0.36, 0.72] \text{ fm}$

Statistics $\sim 500 (\times 32 \text{ AMA})$

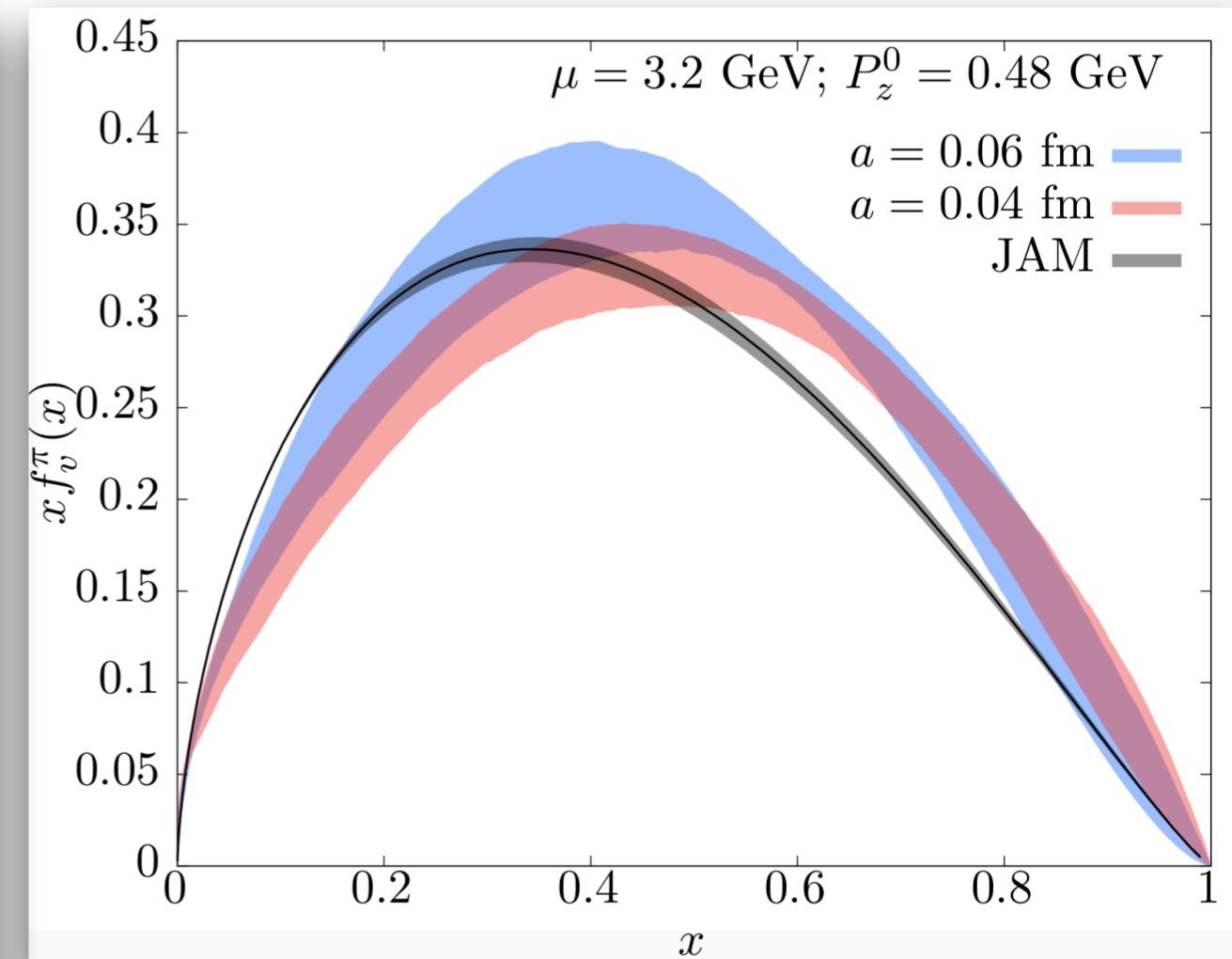
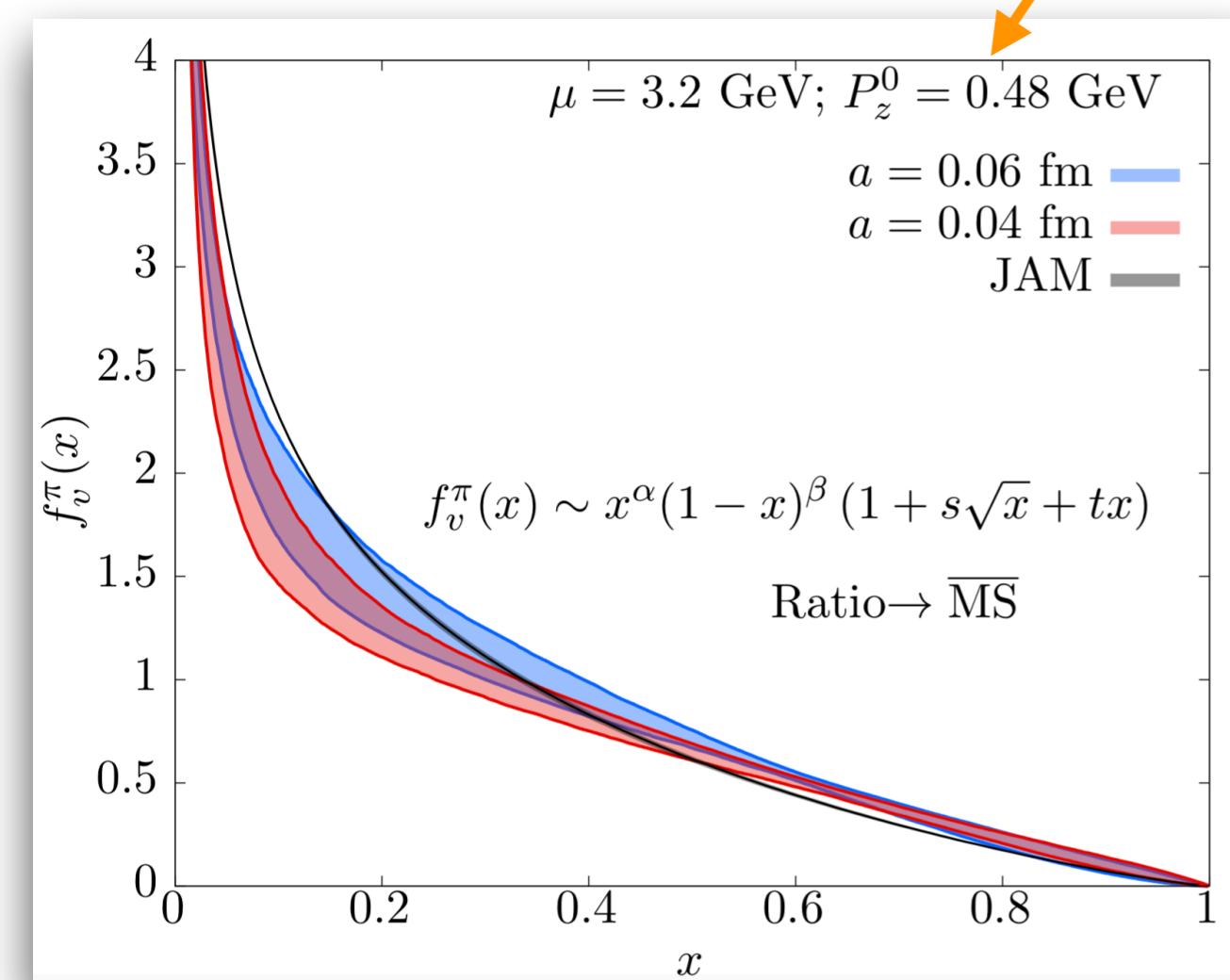
Reconstructing PDFs from lattice

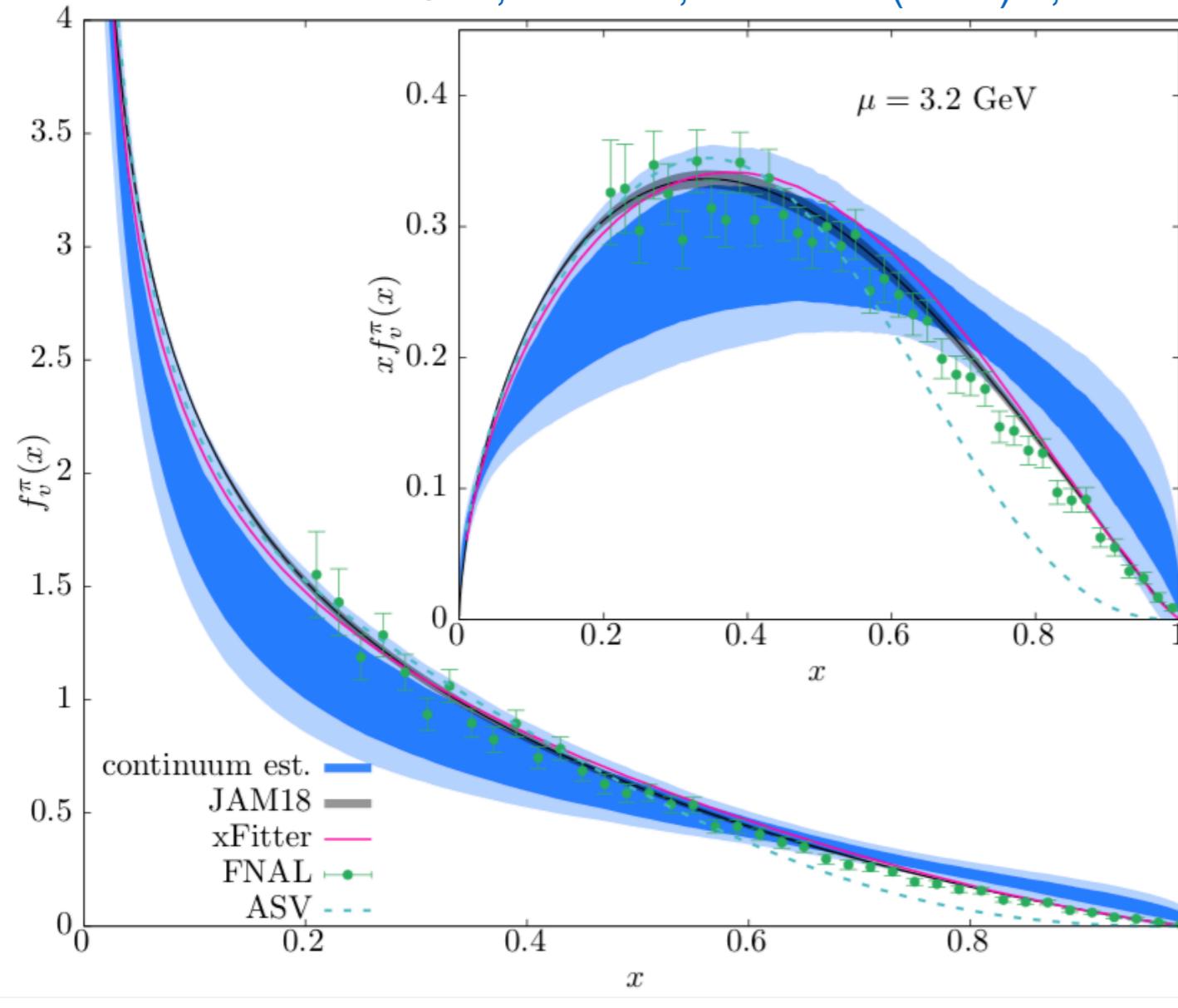


Fit ITD to model PDFs

$$f_v^\pi(x, \mu) = \mathcal{N} x^\alpha (1-x)^\beta (1 + s\sqrt{x} + tx)$$

Corresponding PDF





Towards continuum limit

- Combined fits to two fine lattice spacing with a^2 corrections
- Systematic error from fit range, scheme dependence, PDF ansatz

$$\langle x \rangle_v = \begin{cases} 0.2491^{+(77)(61)}_{-(81)(61)}, & a = 0.06 \text{ fm} \\ 0.2296^{+(79)(57)}_{-(87)(57)}, & a = 0.04 \text{ fm.} \end{cases}$$

$$\langle x^2 \rangle_v = \begin{cases} 0.1174^{+(50)(71)}_{-(44)(71)}, & a = 0.06 \text{ fm} \\ 0.1122^{+(45)(57)}_{-(52)(57)}, & a = 0.04 \text{ fm.} \end{cases}$$

$$\langle x^3 \rangle_v = \begin{cases} 0.0698^{+(52)(80)}_{-(48)(80)}, & a = 0.06 \text{ fm} \\ 0.0690^{+(52)(60)}_{-(52)(60)}, & a = 0.04 \text{ fm.} \end{cases}$$

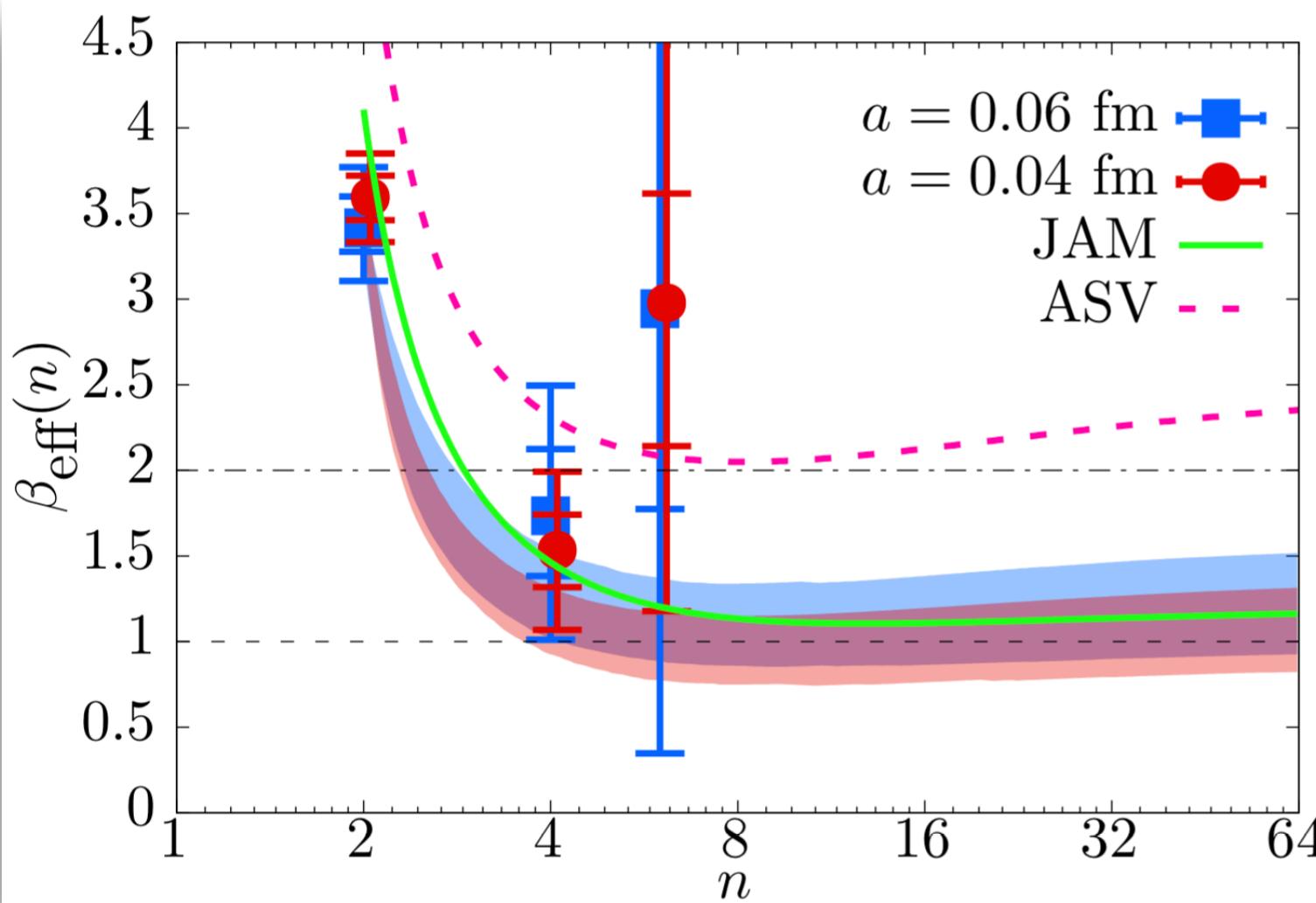
$$\langle x^4 \rangle_v = \begin{cases} 0.0470^{+(52)(76)}_{-(47)(76)}, & a = 0.06 \text{ fm} \\ 0.0478^{+(44)(58)}_{-(51)(58)}, & a = 0.04 \text{ fm.} \end{cases}$$

$a \rightarrow 0$	JAM
0.213(19)(08)	0.223
0.1009(68)(42)	0.095
0.0607(40)(47)	0.052
0.0410(40)(47)	0.032

Large-x $(1 - x)^\beta$ behavior and lattice QCD

Infer from fits

★ Downside: dependence on the complexity of ansatz



Model independent approach:

★ Note the universal exponent of large moments

$$\langle x^n \rangle \propto n^{-\beta-1} (1 + O(1/n)).$$

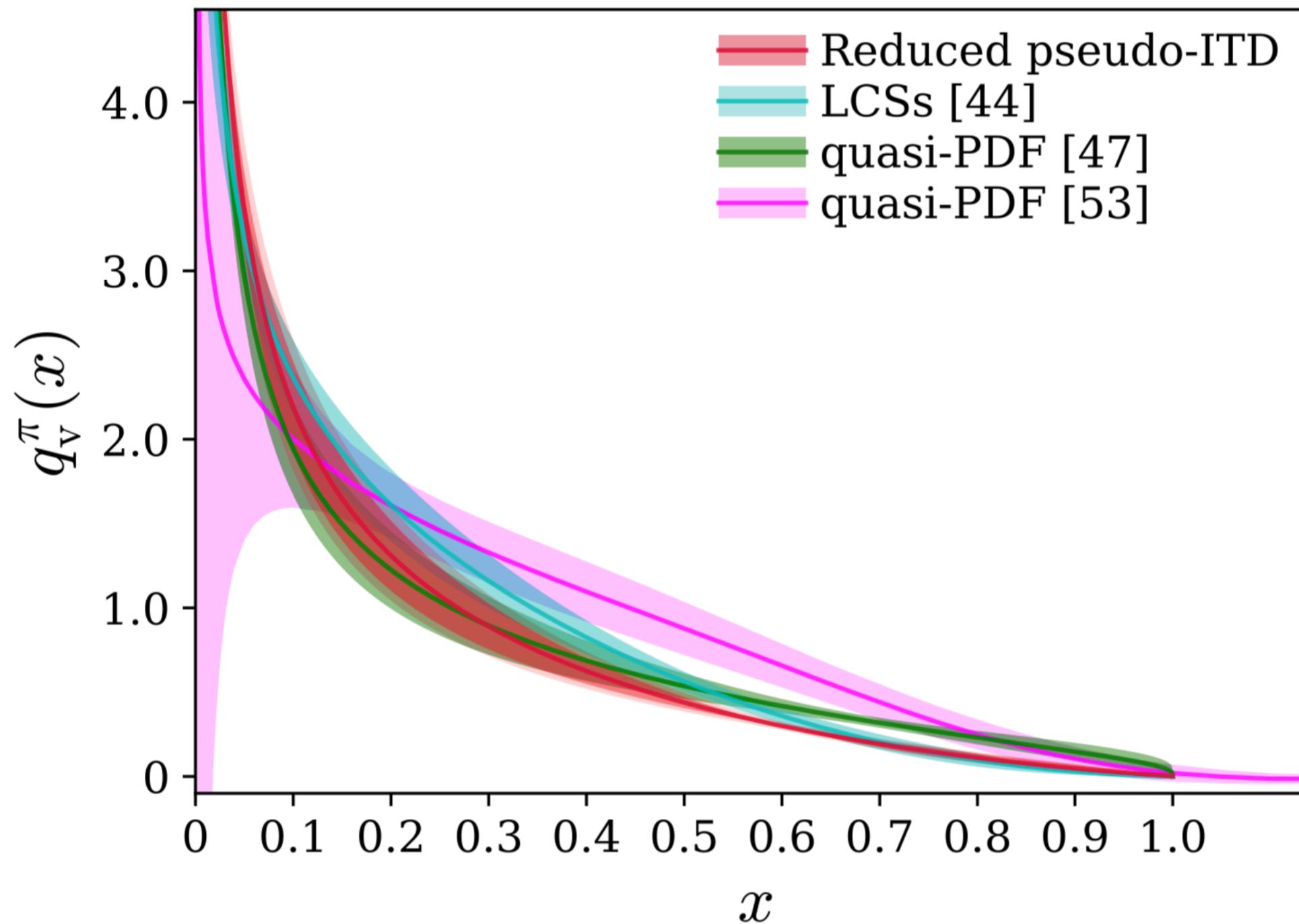
$$\rightarrow \beta + 1 = -\frac{d \log (\langle x^n \rangle)}{d \log (n)}$$

Path-forward: sensitivity to large moments

Large P_{zZ} , but small z

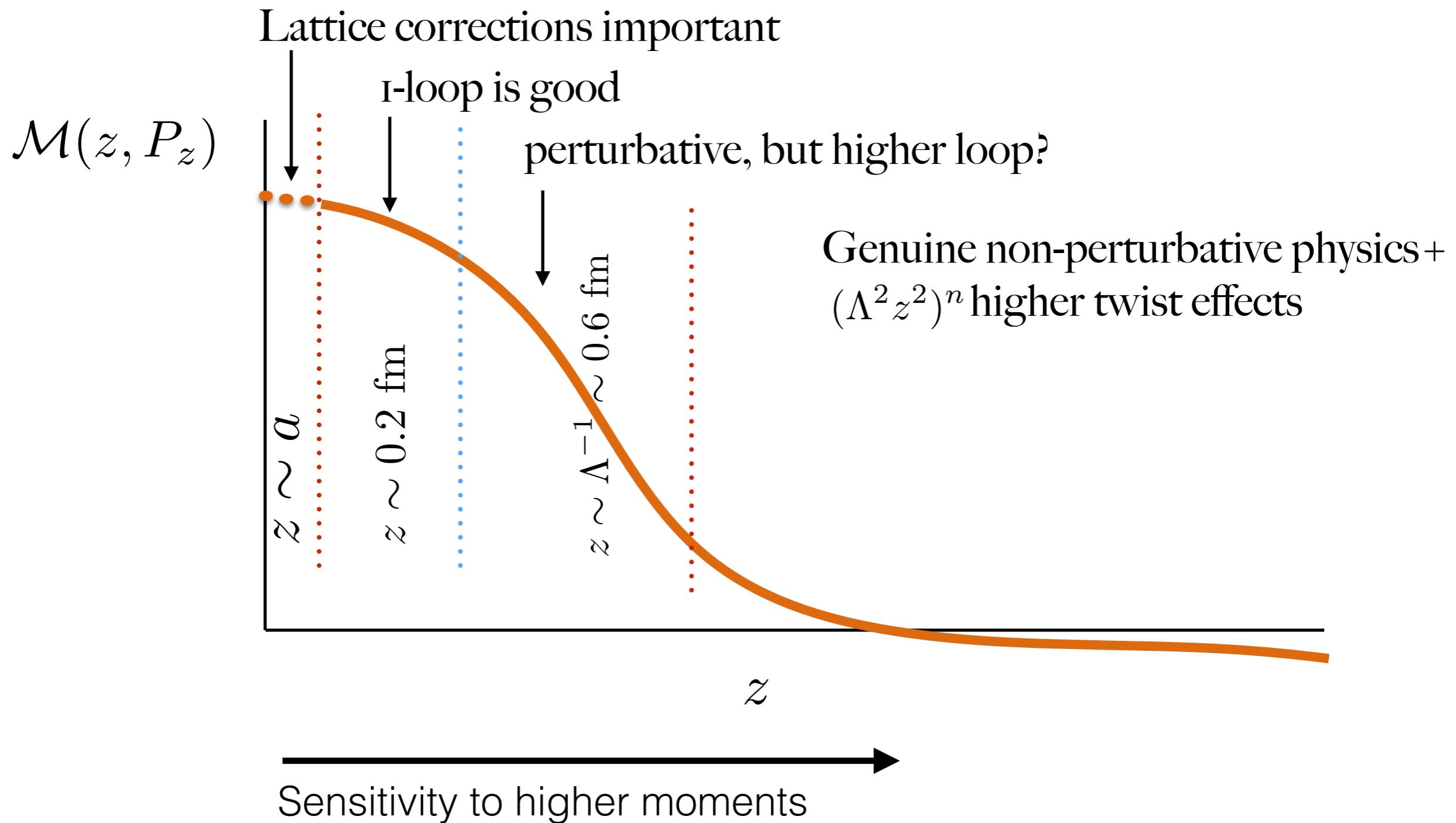
Precision to compete with $n!$ suppression

Other recent lattice computations of pion quark PDF



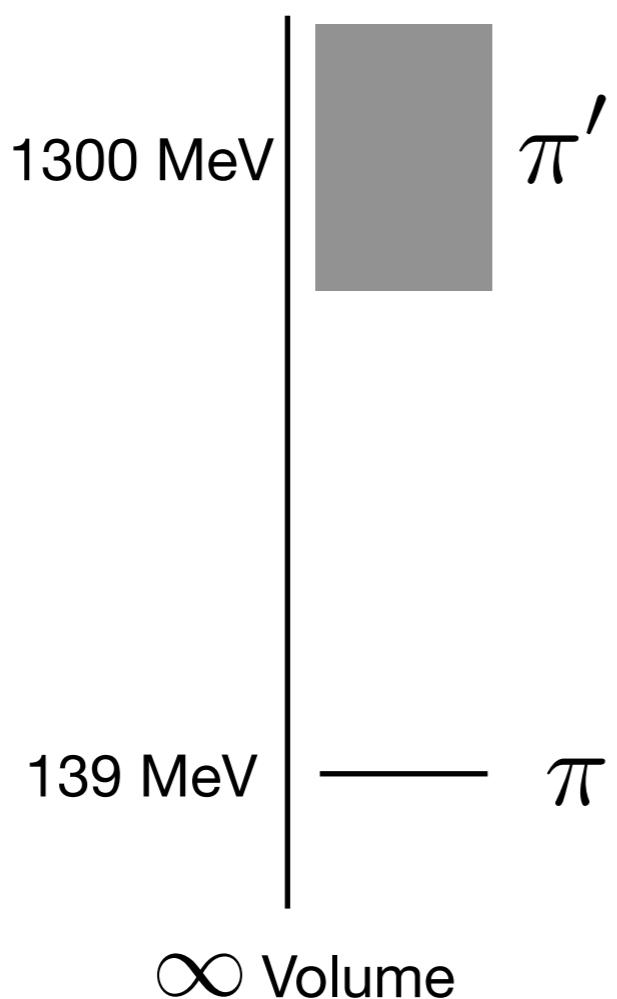
Taken from Joo et al (HadStruc), PRD 100 (2019) 11, 114512

Roadmap for future precision lattice studies of pion PDF



How does quark structure change when pion is radially excited?

arXiv:2101.11632, X. Gao, NK et al

 $\pi(1300)$ $I^G(J^{PC}) = 1^-(0^-+)$ Mass $m = 1300 \pm 100 \text{ MeV}$ [1]Full width $\Gamma = 200 \text{ to } 600 \text{ MeV}$ **$\pi(1300)$ DECAY MODES**

	Fraction (Γ_i/Γ)	p (MeV/c)
$\rho\pi$	seen	404
$\pi(\pi\pi)_{S\text{-wave}}$	seen	-

$\Pi(1300)$: non-Goldstone 0^- meson to compare and contrast with the Goldstone 0^-

Theoretical expectations in the chiral limit : $m_q \rightarrow 0$

$$M_\pi \rightarrow 0$$

$$M_{\pi'} \rightarrow \#\Lambda_{\text{QCD}}$$

$$F_\pi \rightarrow \#\Lambda_{\text{QCD}}$$

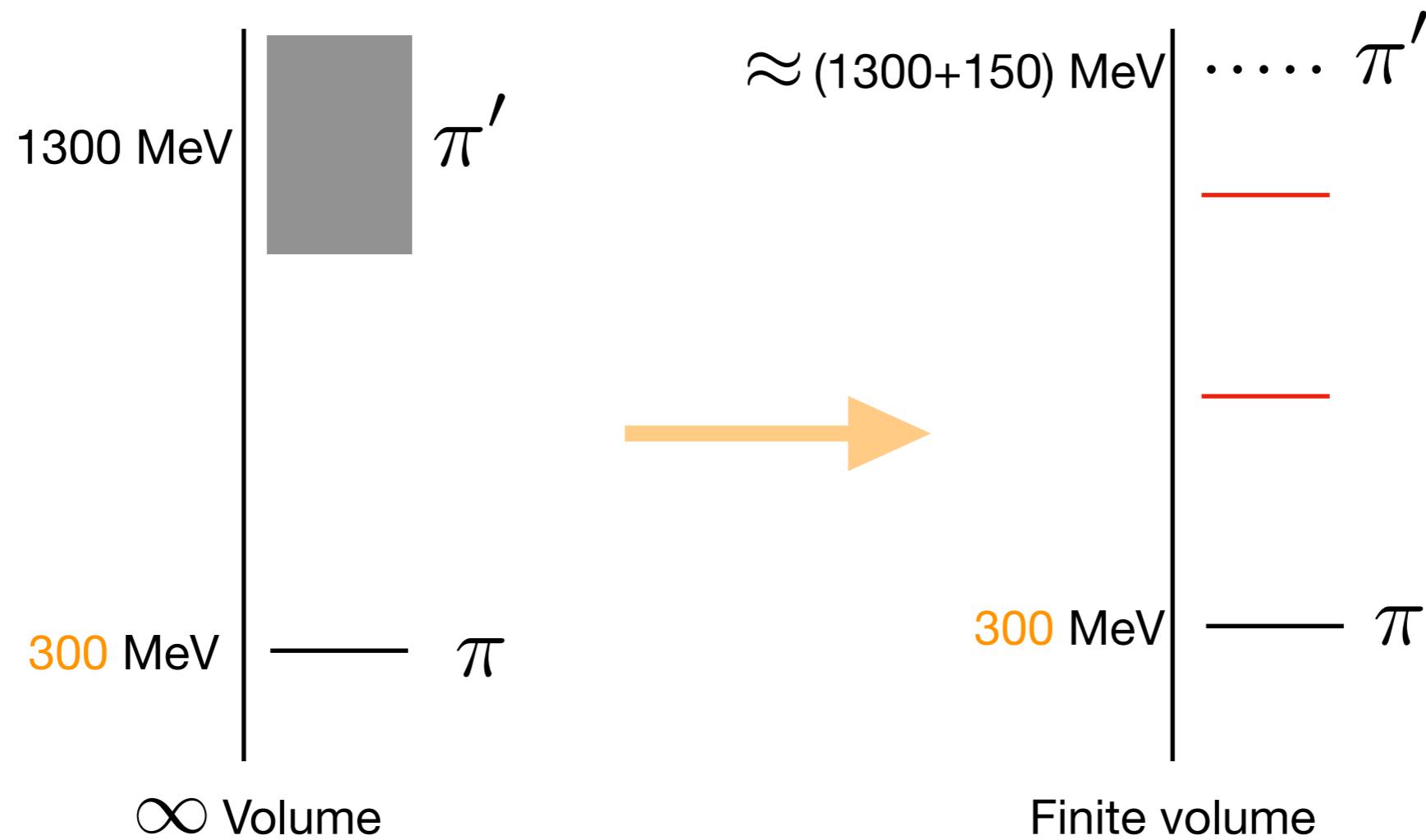
$$F_{\pi'} \rightarrow 0$$

Previous lattice works:
 Mastropas, Richards '14
 McNeile, Michael '06

Are there tell-tale differences in their partonic structures?

Method for the first go at this problem:

- Perform the study In fixed finite volume: convert resonance into a finite volume eigenstate



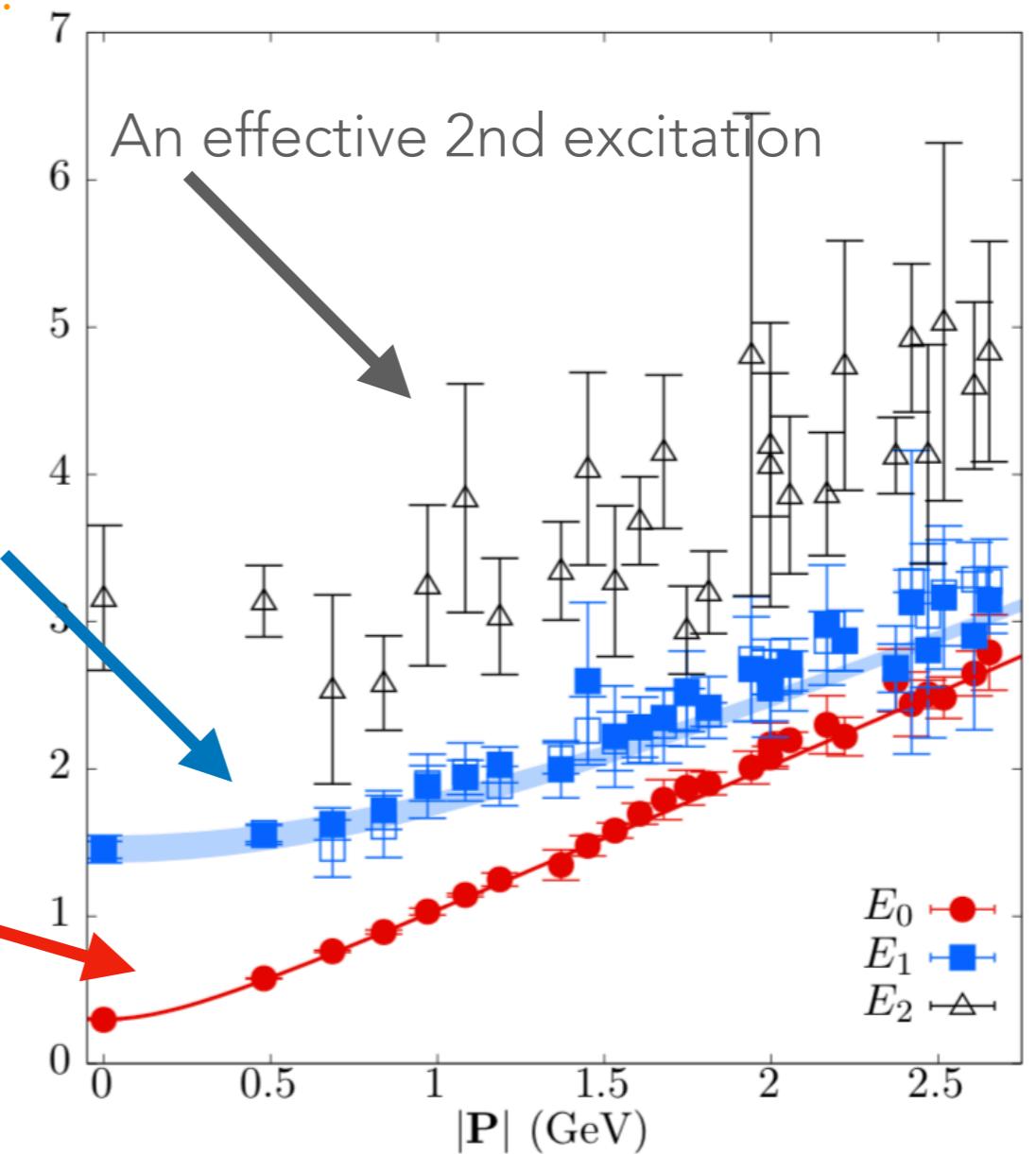
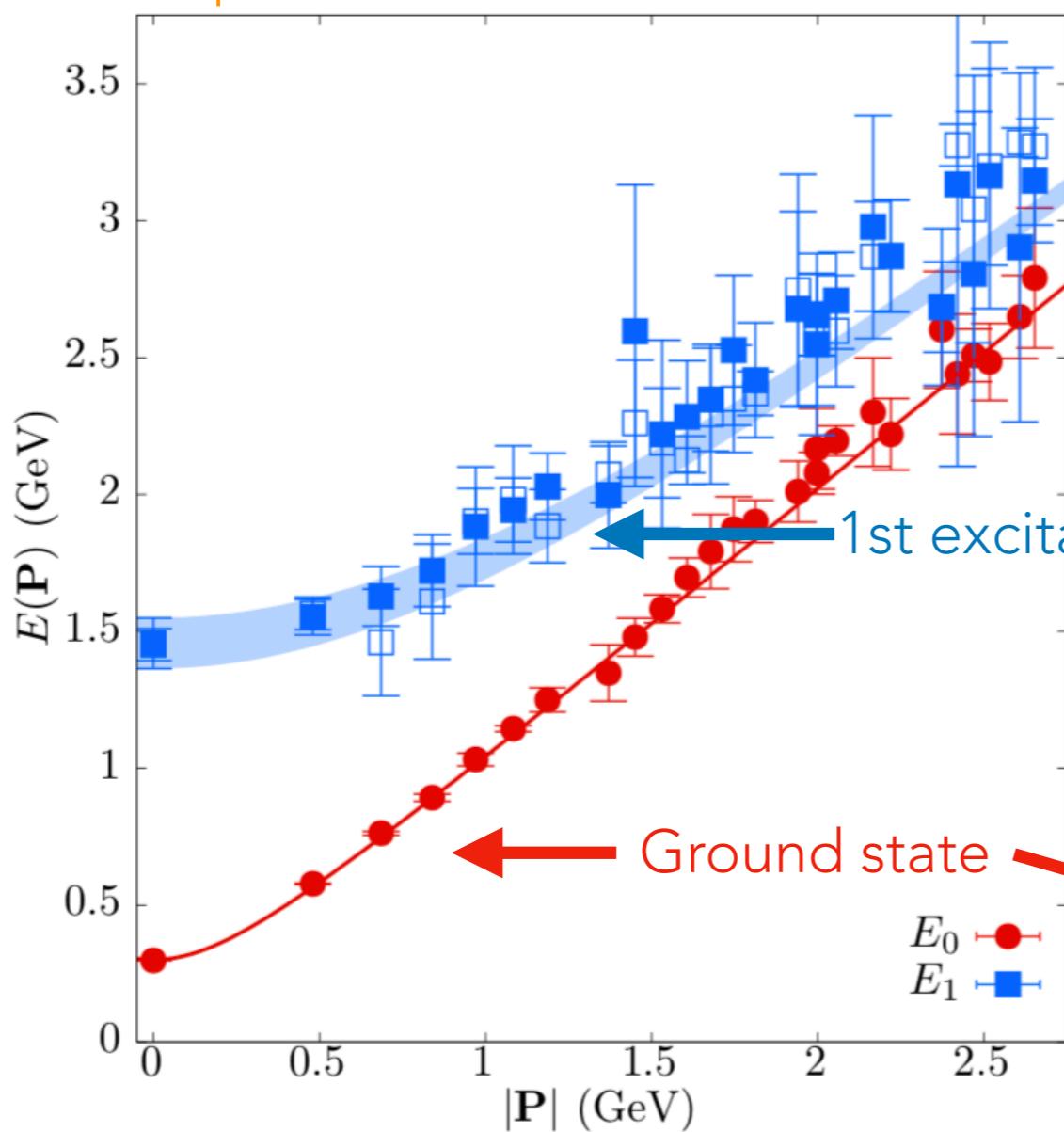
- Use local pion creation operators to suppress contributions from multi-particle states

$$\bar{u}d|0\rangle = c_0|\pi\rangle + c_1|\pi'\rangle + c_2|\pi, \rho\rangle + c_3|\pi, \pi, \pi\rangle + \dots$$

- Room for lot of improvements: Luscher method, GEVP with extended operator basis,...

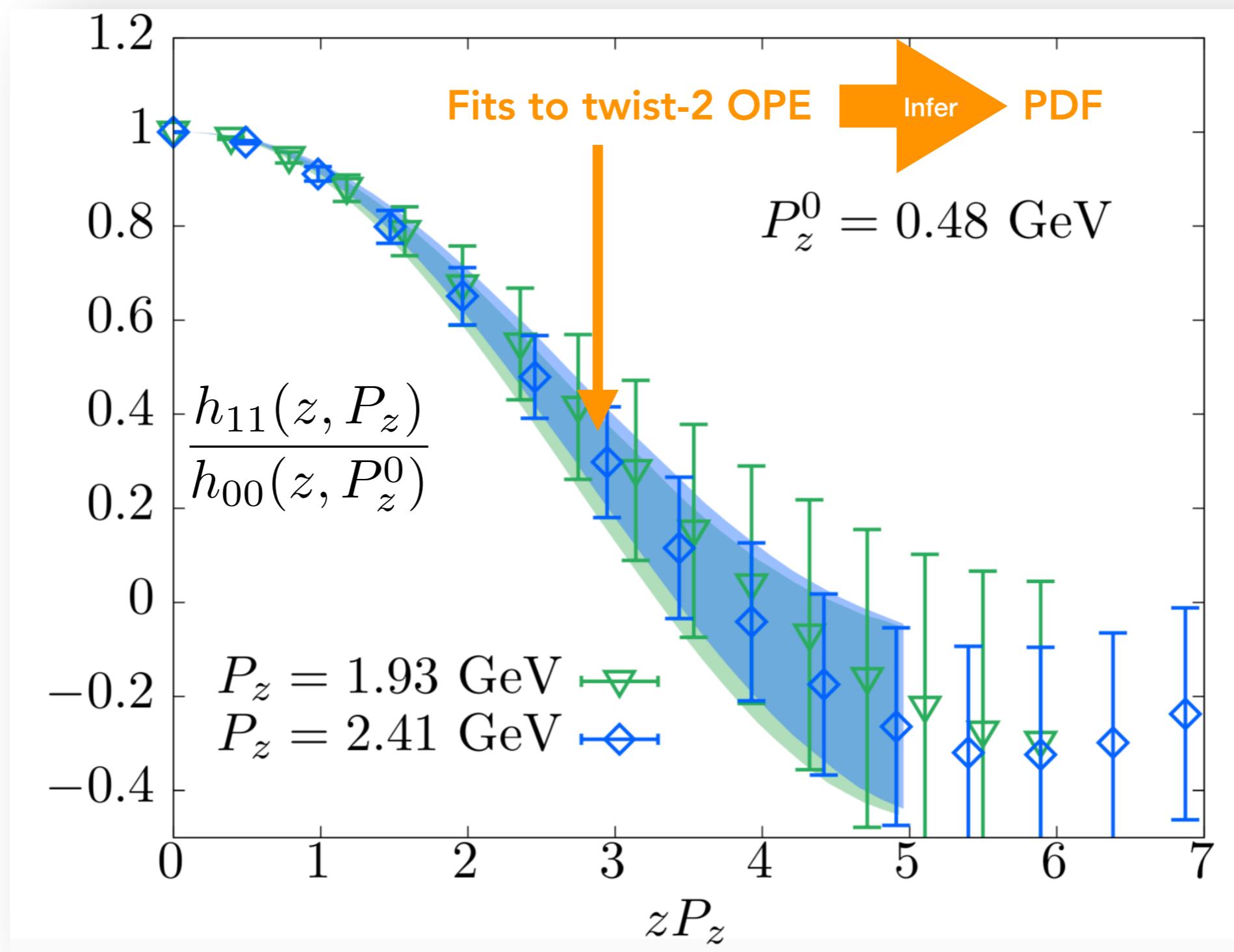
The 1st excited state in pion 2-pt function follows
single-particle dispersion with mass close to that of $\pi(1300)$

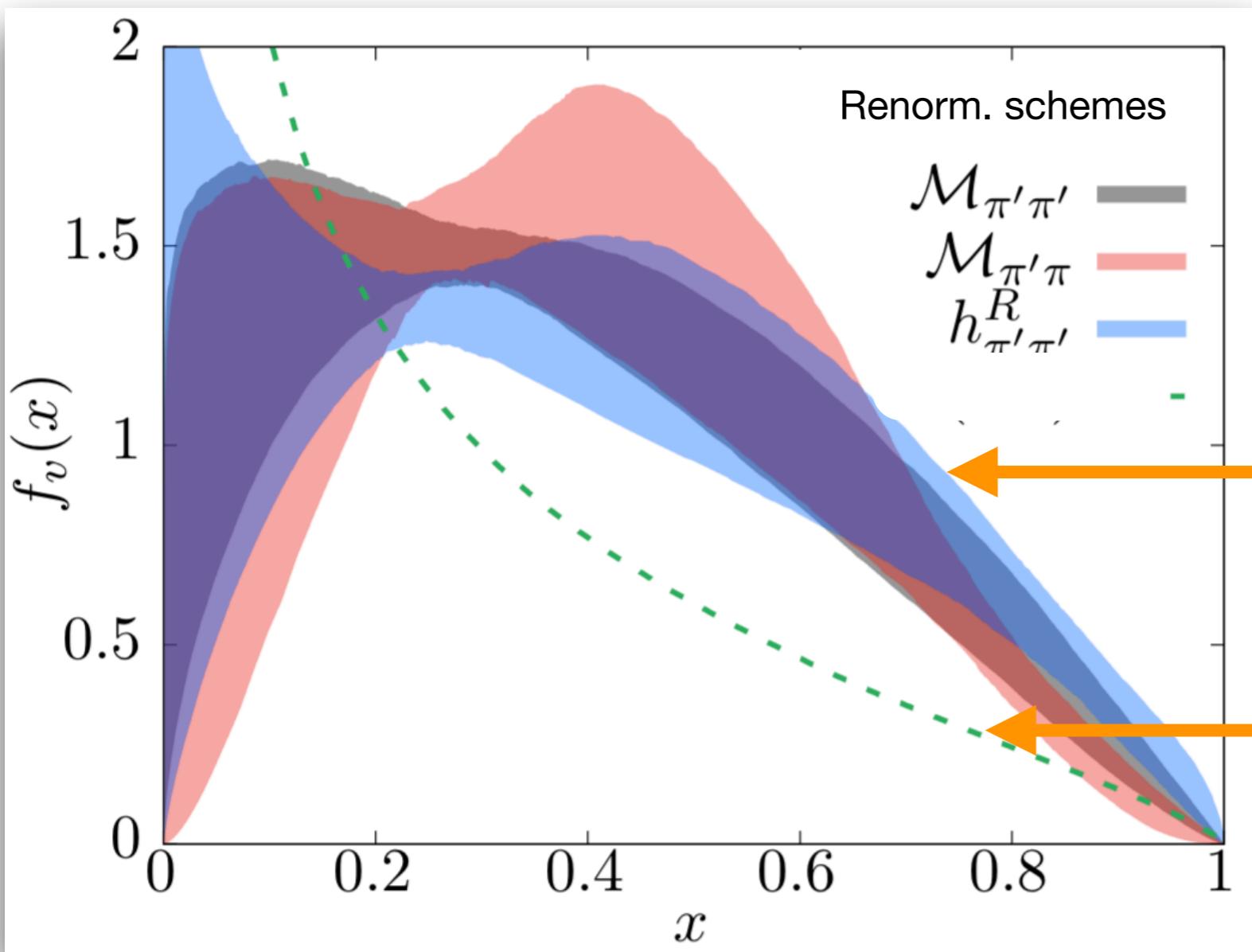
Dispersion relation obtained numerically:



$$\left\langle \hat{\pi}(t_s) \bar{\psi}^{(0)}_{W(0,z)} \psi^{(z)} \hat{\pi}^\dagger(0) \right\rangle = \sum_{i,j} A_i^* A_j h_{ij}(z, P_z) e^{-E_i(t_s - \tau) - E_j \tau}$$

Fit ground state h_{00} and excited state h_{11}





PDF reconstructed based
on $x^a(1-x)^b$ Ansatz

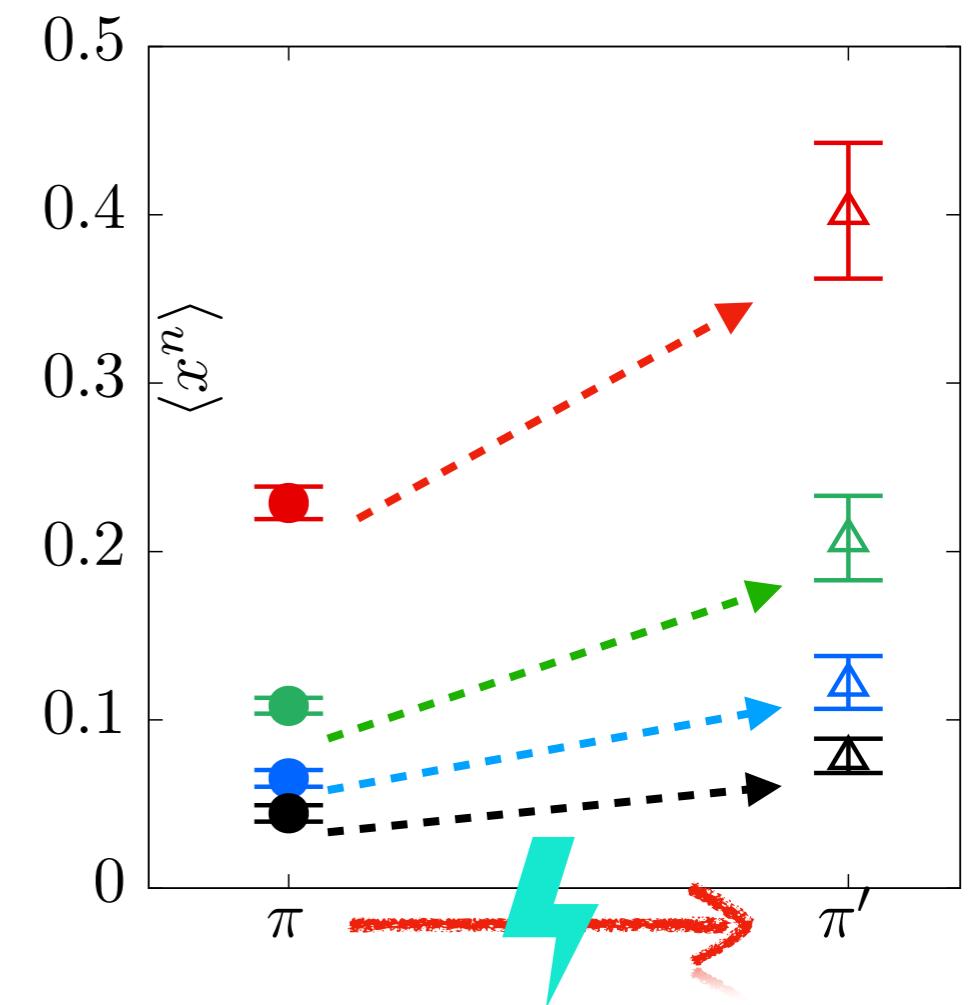
Pi(1300) valence
PDF

pion valence PDF in the
same ensemble

A ratio of momentum differentials:

$$\zeta = \frac{2M_{\pi'} \langle x \rangle_{\pi'} - 2M_\pi \langle x \rangle_\pi}{M_{\pi'} - M_\pi}$$

$$\approx 80 - 99\%$$



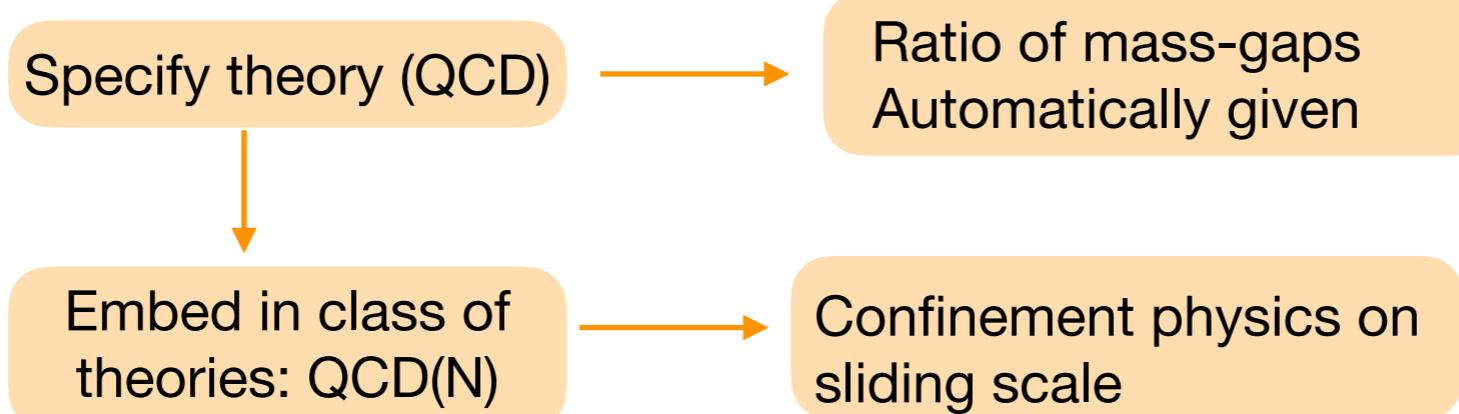
**How is the internal structure of Nambu-Goldstone
mode affected by the long-distance vacuum structure?**

arXiv:2101.02224, NK

What aspects of pion ITD and PDF are determined by
(or sensitive to) the symmetry-breaking physics?



Slowly change strength of symmetry-breaking and record how
PDFs change?



R. Pisarski '84

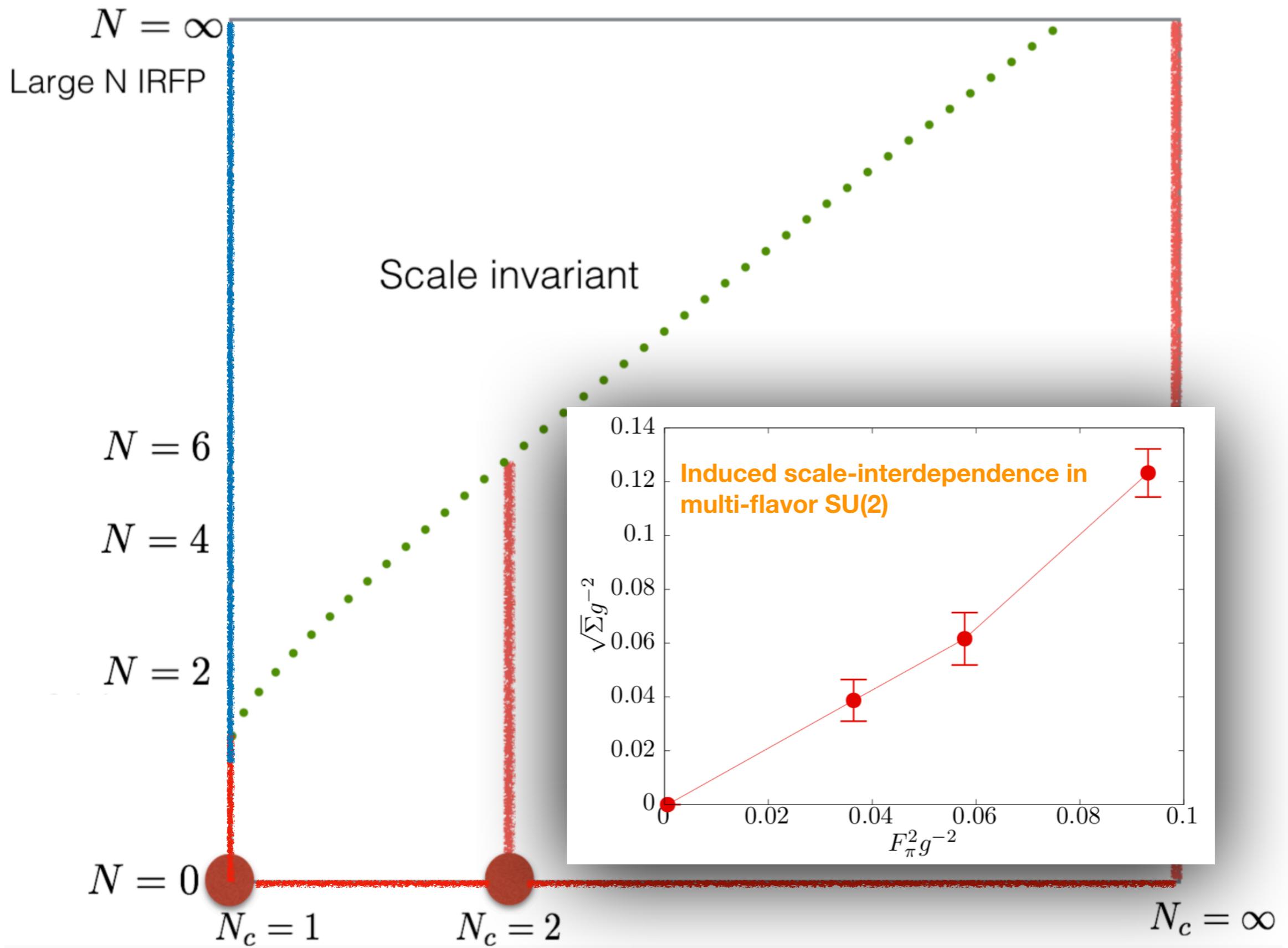
T. Appelquist et al, '86

Lot of literature till now

A model QFT: 2+1d SU(2) gauge theory coupled to massless N fermion flavors

- Computationally cheaper to do.
- Most important physics reason: parallel theoretical developments in 2+1d in understanding confinement and mass-gap via classification of RG flows; e.g., conformal bootstrap.

Mapping the IR phase diagram of 2+1d massless QCD

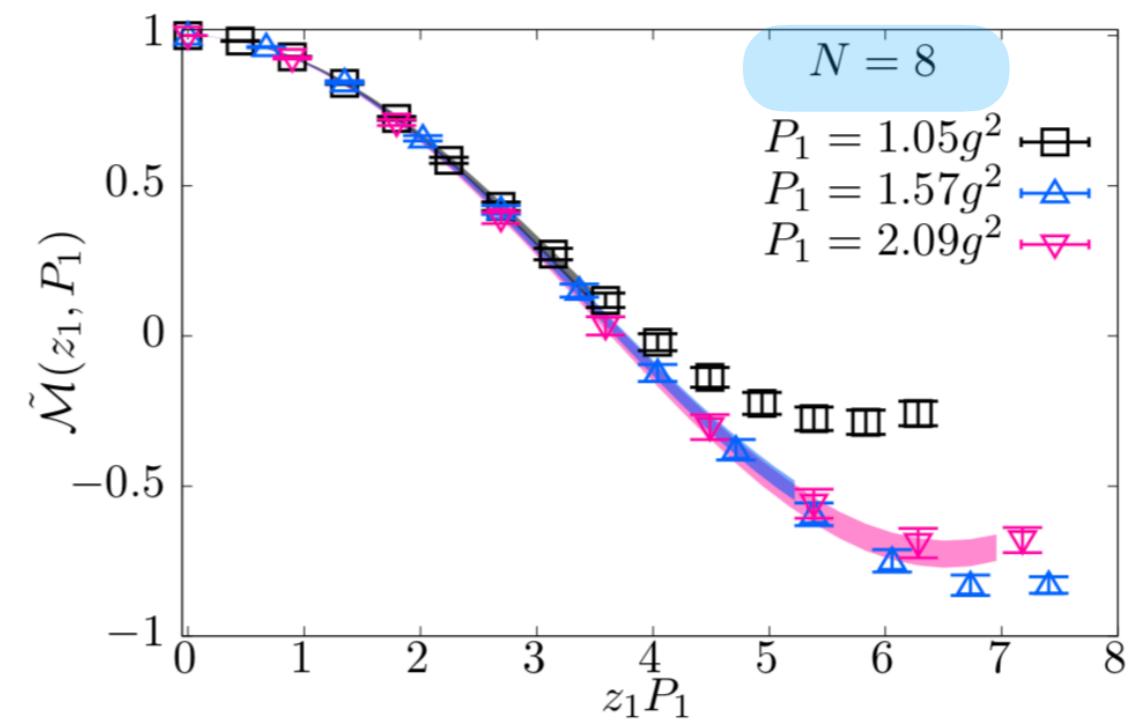
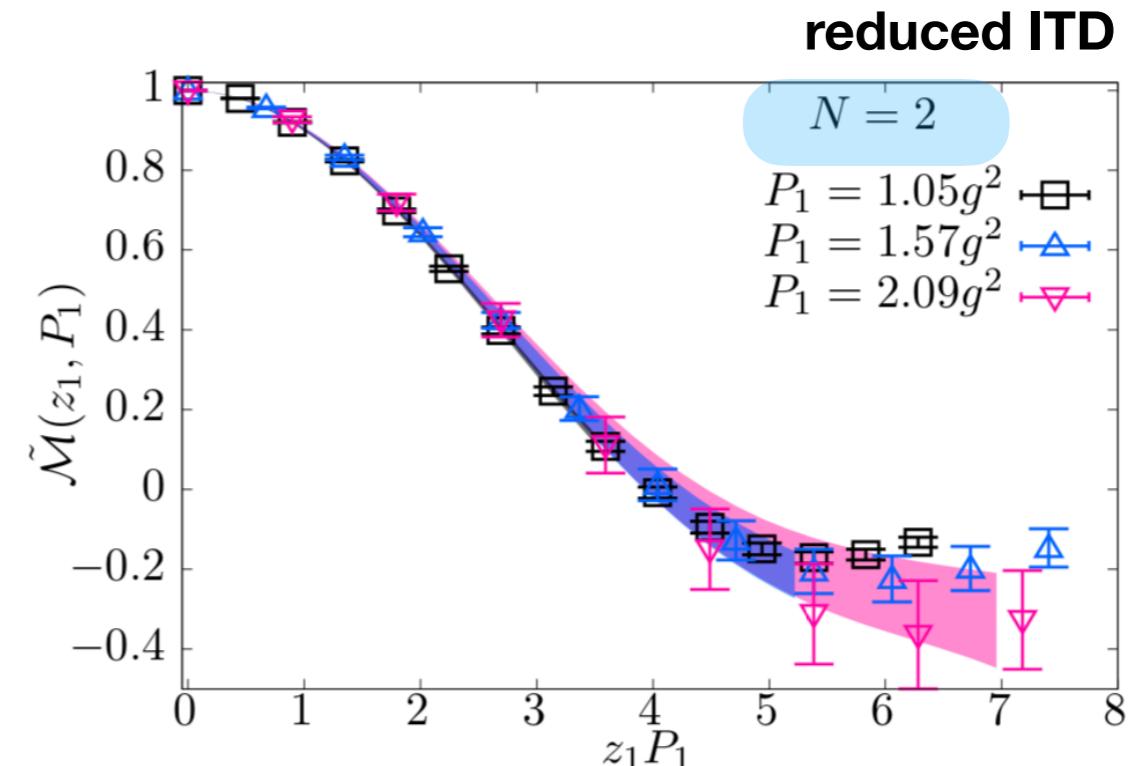
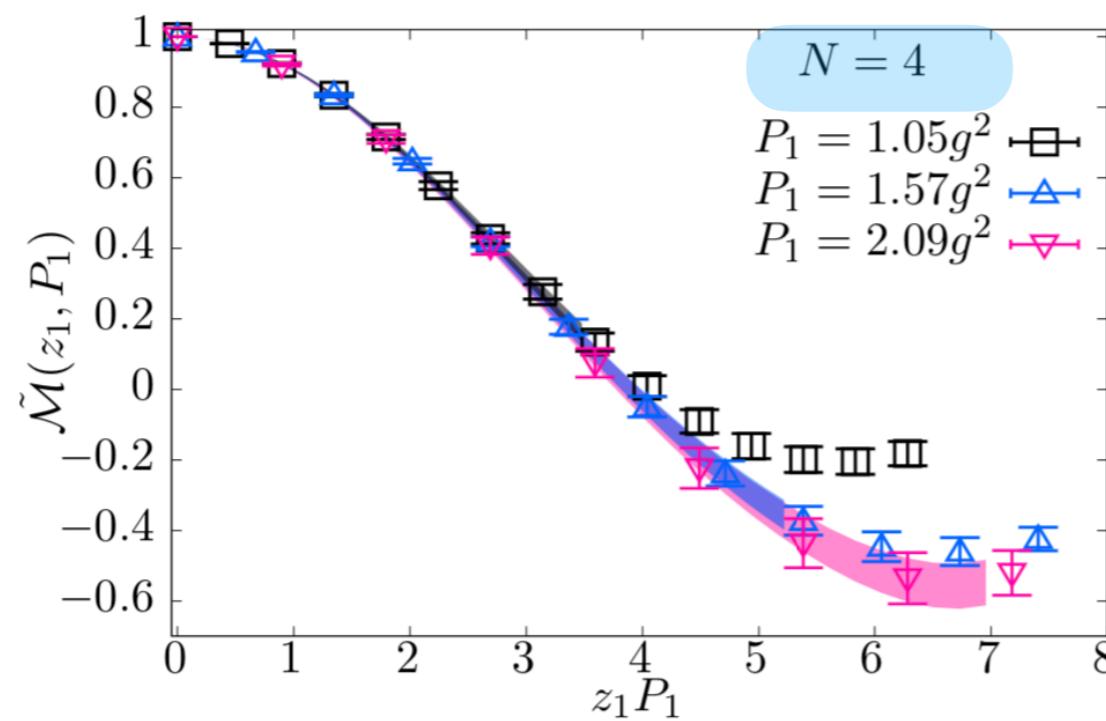
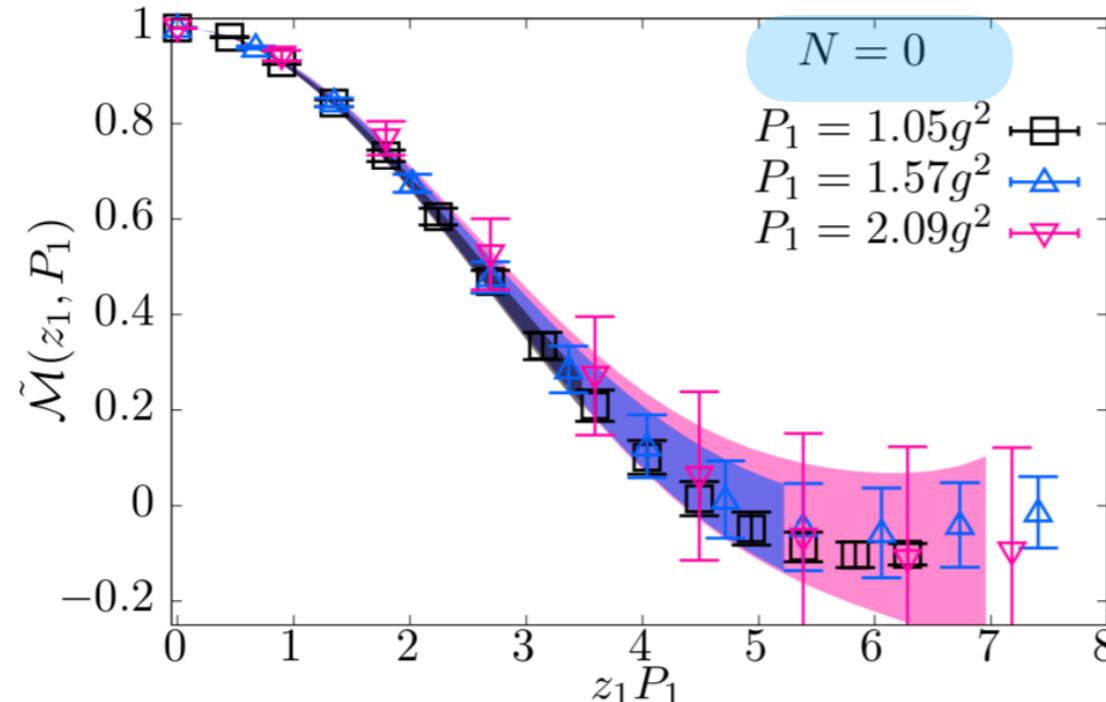


Analysis

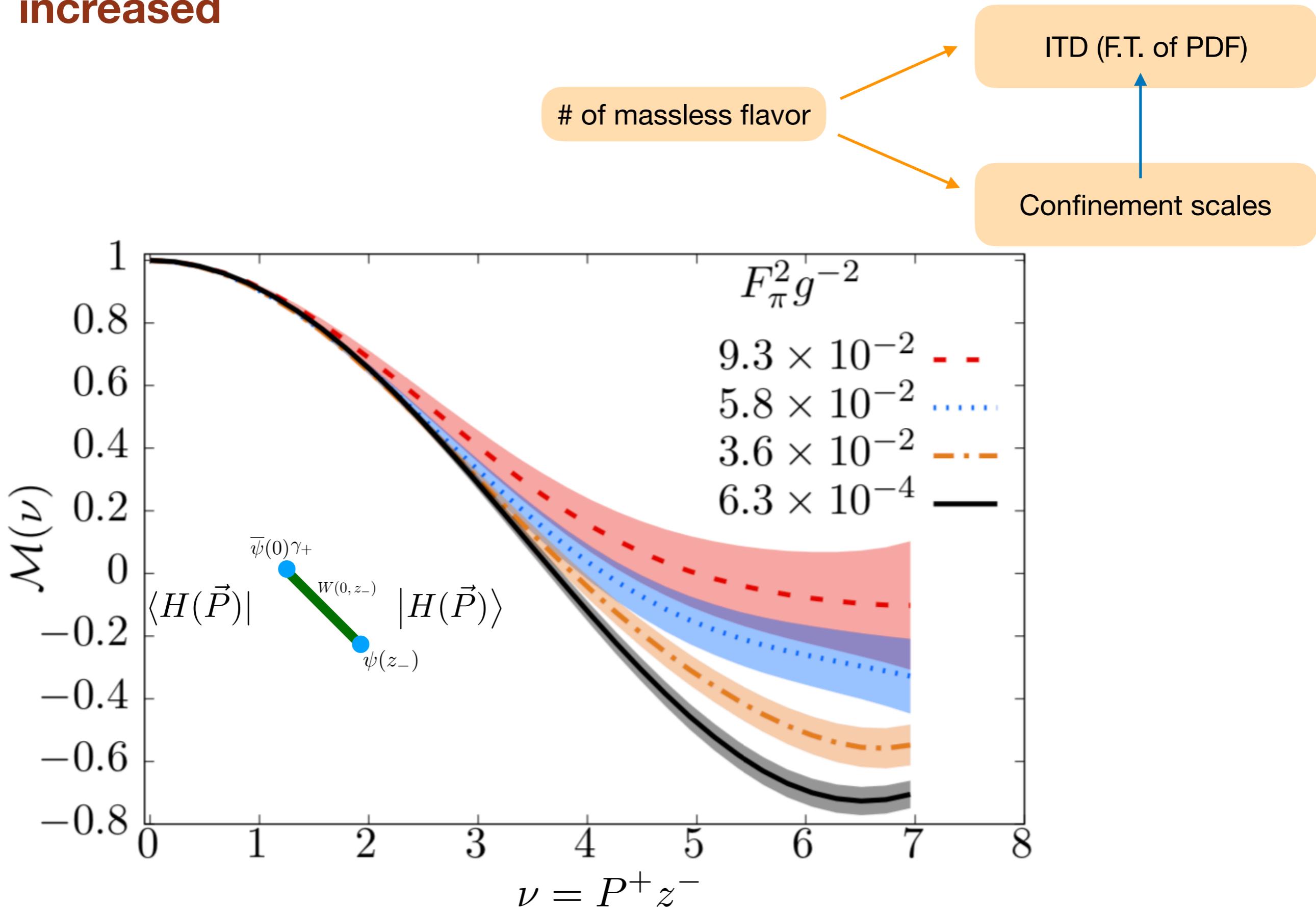
- Fit moments as fit parameters

- PDF Ansatz fit assuming $f(x) \sim x^\alpha(1-x)^\beta$

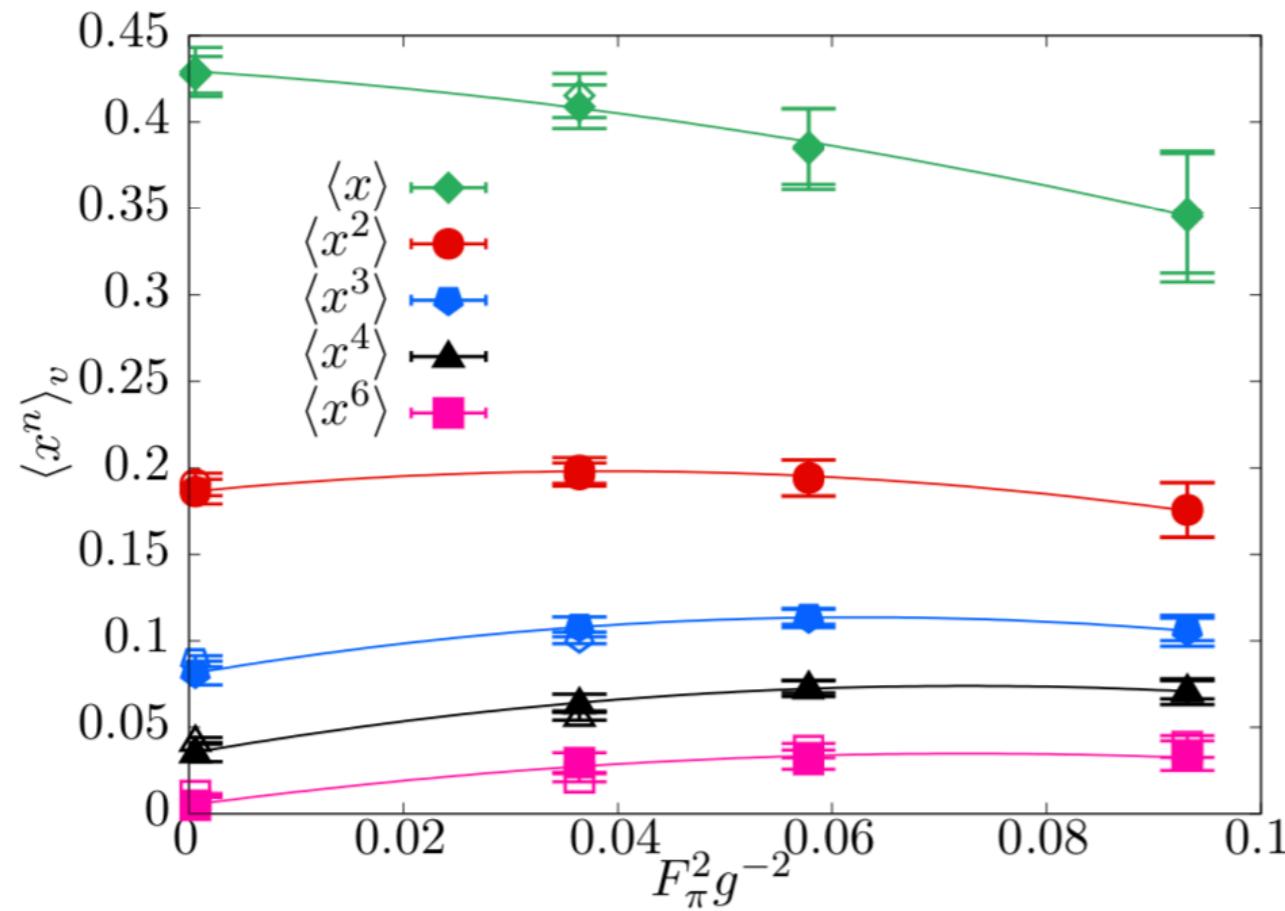
$$\tilde{\mathcal{M}}(z_1, P_1) = 1 + \left[\sum_{k=1}^{N_{\max}} (-1)^k \frac{(z_1 P_1)^{2k}}{(2k)!} \langle x^{2k} \rangle_v \right] + \text{H.T.}$$



How ITD/LF-correlation changes when decay-constant is increased



Effect on moments



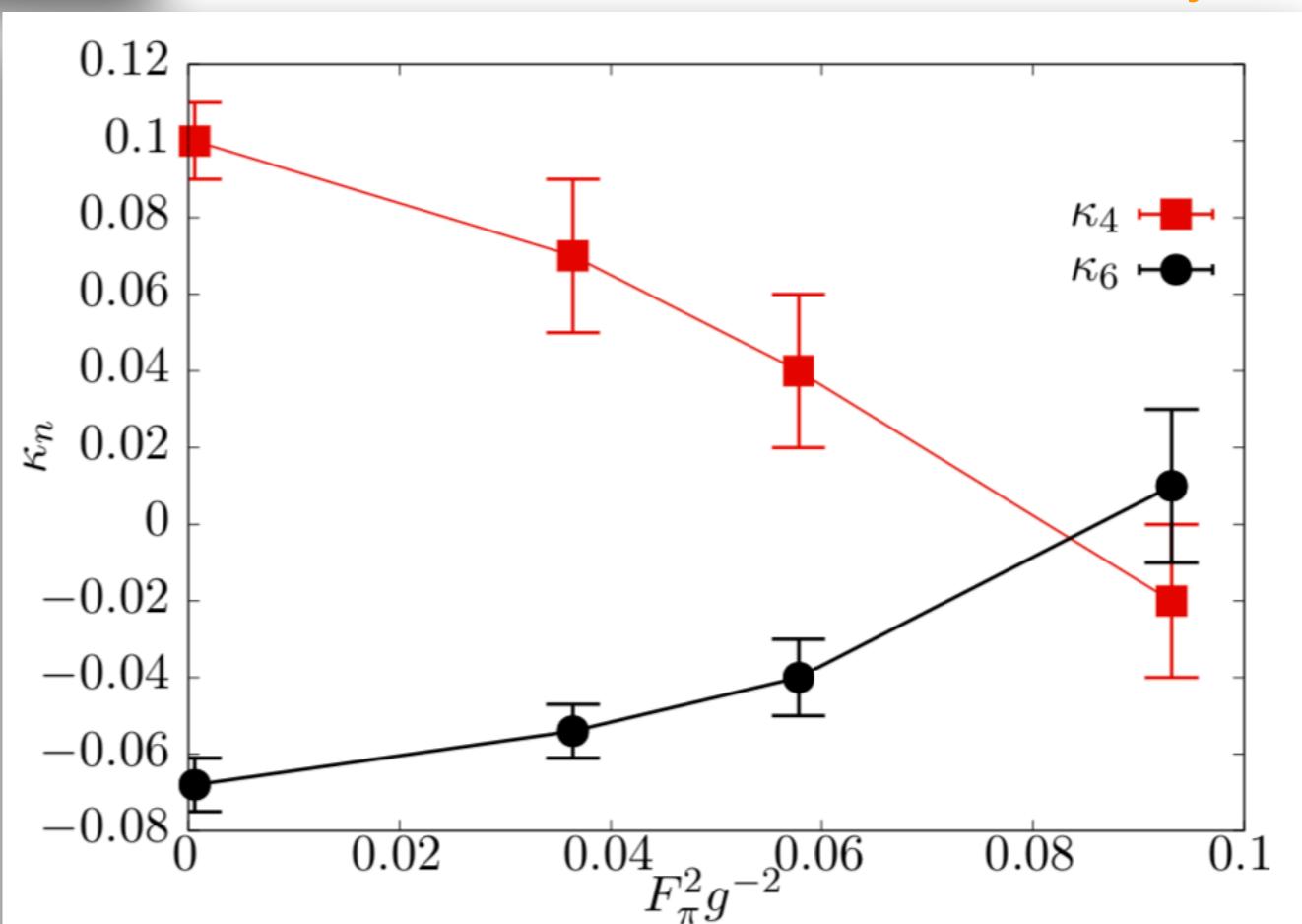
Increased IR sensitivity of PDF shape observables

Cumulants of PDF more sensitivity to IR

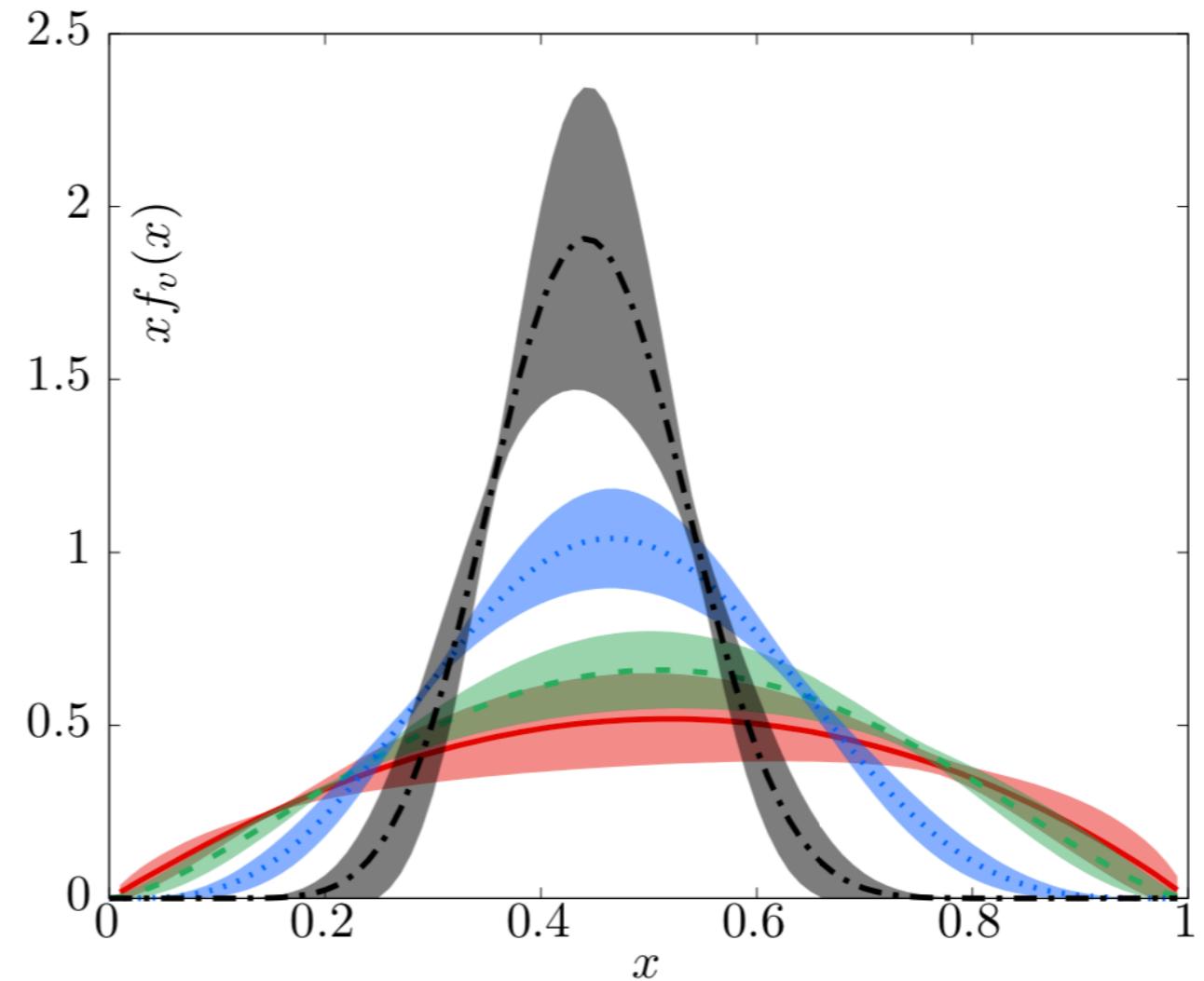
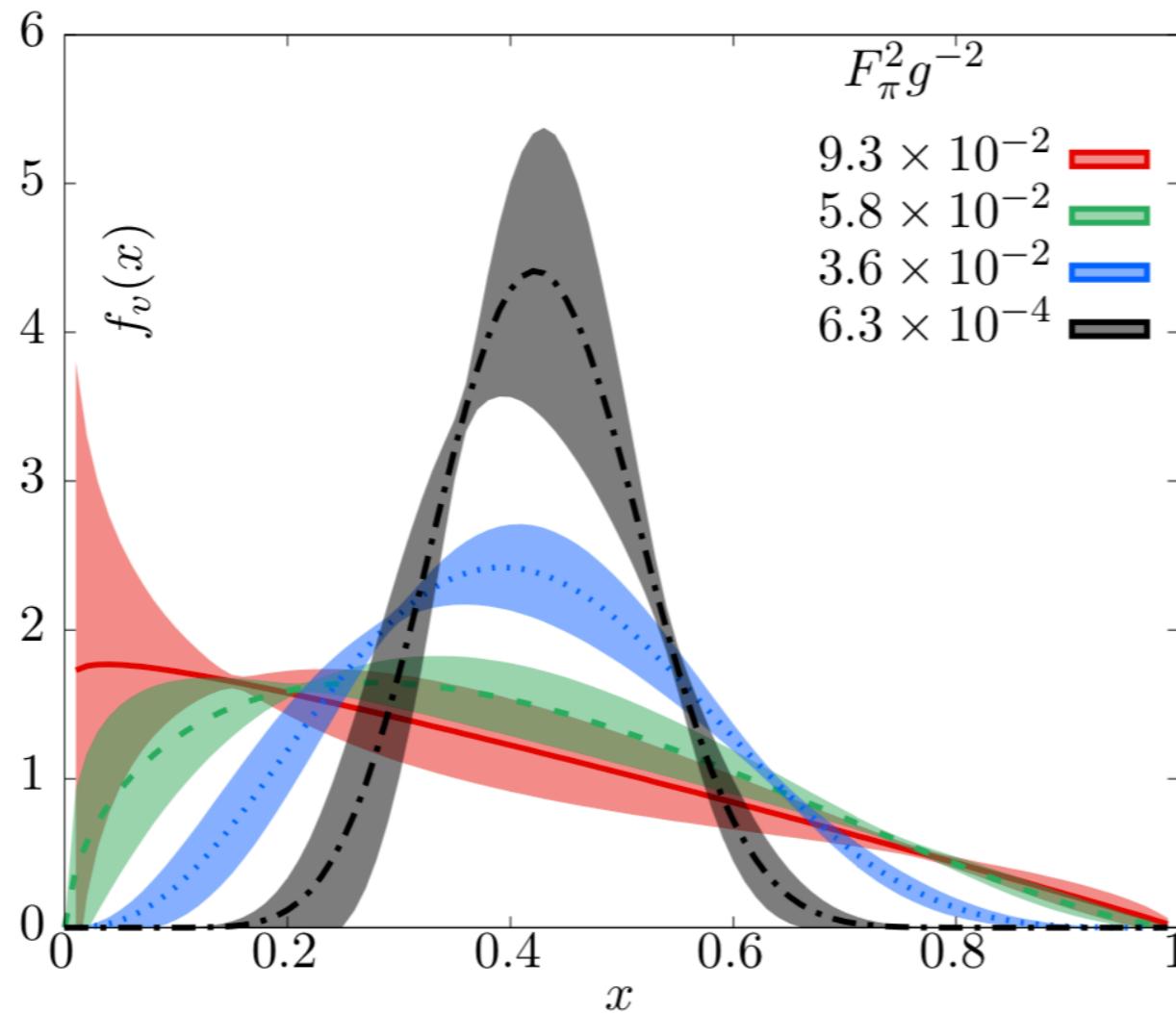
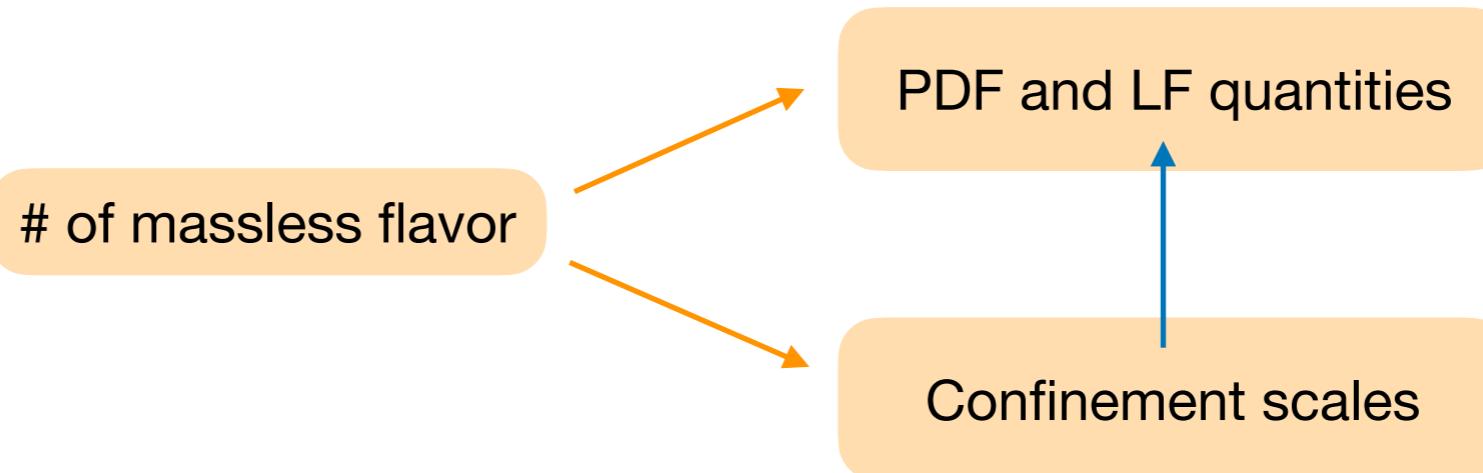
$$\kappa_n \equiv \frac{\partial^n}{\partial s^n} \log \left(\int_{-1}^1 f_{u-d}(x) e^{sx} dx \right) \Big|_{s=0}$$

$$\kappa_4 = \langle x^4 \rangle_v - 3\langle x^2 \rangle_v^2,$$

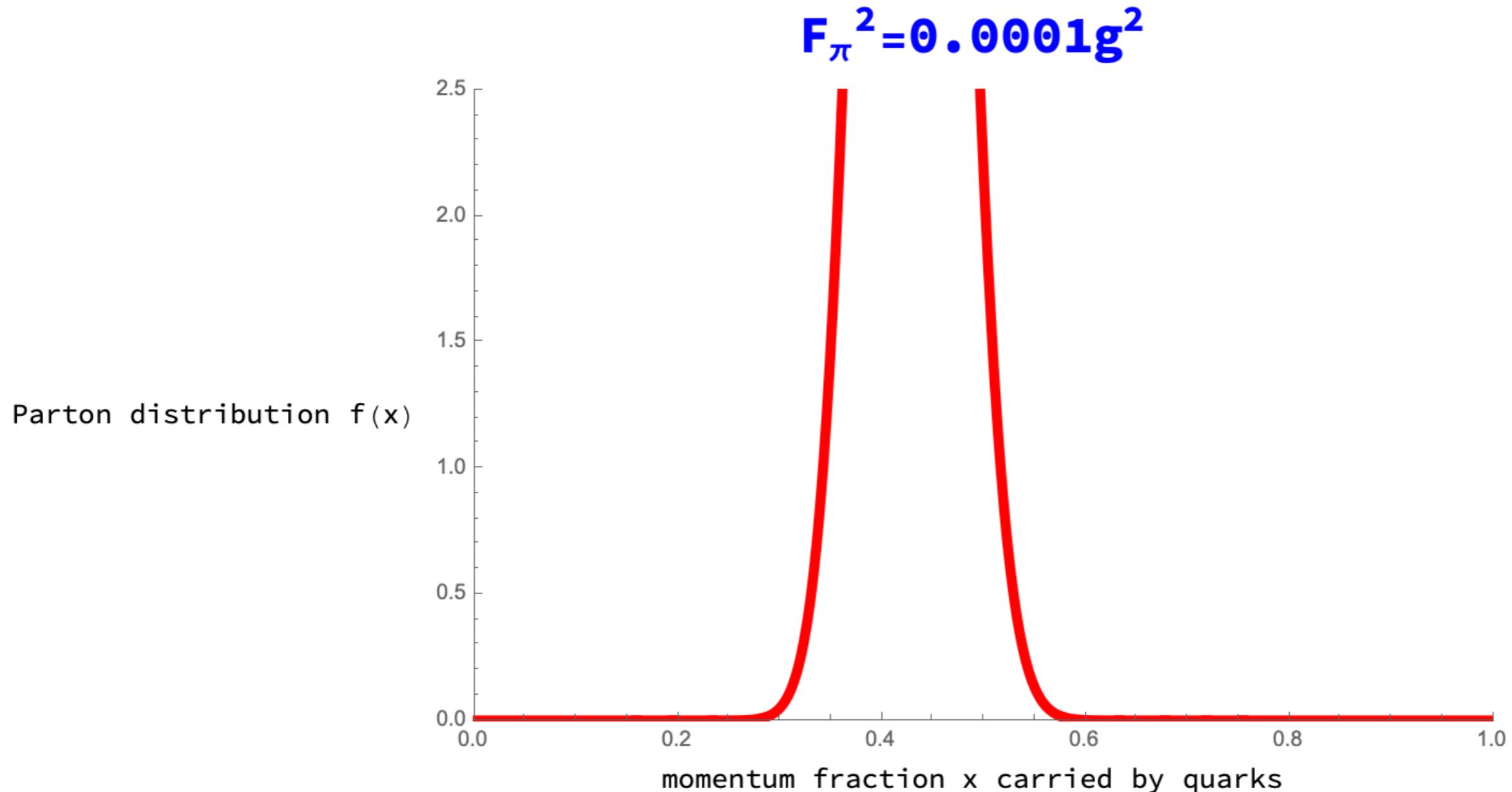
$$\kappa_6 = \langle x^6 \rangle_v - 15\langle x^2 \rangle_v \langle x^4 \rangle_v + 30\langle x^2 \rangle_v^3.$$



The main observation: broadening of pion PDF when the strength of symmetry-breaking is *dialed-up*



Animation to show how pion PDF evolves to its form as the underlying vacuum is changed from near-conformality



Summary and outlook

- Ab initio lattice determination of PDF using LaMET/SDF successful

