Pion DA in AMBER. A celebrity and a puzzle

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Perceiving the Emergence of Hadron Mass through AMBER@CERN-V

April 27-30, 2021

Overview

❶ **Pion distribution amplitude (DA) of twist two $\varphi_{\pi}^{(2)}$**

- Definition in QCD
- Expansion in Gegenbauer harmonics
- Models

❷ **Accessing the pion DA in experiments**

- $\gamma^*\gamma \to \pi^0$ transition form factor in single-tag experiments
  CELLO, CLEO, *BABAR*, Belle, BESIII (preliminary), Belle-II (forthcoming)
- Electromagnetic pion form factor
  JLab 12 at $Q^2 \sim 10(5)$ GeV^2 (in progress)
- Diffractive dijet production (E791 2001)
- Pion DAs within AMBER project at CERN (on the horizon)

❸ **Lattice calculations** (see graphics in ❷)

❹ **Cross relations** between $\rho^{\parallel}$ and π DAs (work in progress)

❺ **Conclusions and Outlook**

❶ Pion distribution amplitude

▶ **Pion DA at twist two**

- $$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5[z,0]u(0)|\pi(P)\rangle|_{z^2=0} = if_\pi P_\mu \int_0^1 dx e^{ix(z\cdot P)}\varphi_\pi^{(\text{tw-2})}(x,\mu^2)\ ,$$

$$[z,0] = \mathcal{P}\exp\left[ig\int_0^z t_a A_a^\mu(y)dy_\mu\right] \qquad (A^+ = 0 \Rightarrow [z,0] = 1)$$

- Q^2 dependence controlled by **ERBL** evolution equation

$$\frac{d\,\varphi_\pi(x;\mu^2)}{d\,\ln\mu^2} = V\left(x,u;a_s(\mu^2)\right)\underset{u}{\otimes}\varphi_\pi(u;\mu^2)$$

$$\tilde{\psi}_n(x)\underset{x}{\otimes}V_0(x,u)\underset{u}{\otimes}\psi_n(u) = -\gamma_n(a_s)$$

$$V(x,y;a_s) = a_s\ V_0(x,y) + a_s^2\ V_1(x,y) + \ldots;\quad a_s = \alpha_s/(4\pi)$$

$$\gamma_n(a_s) = \tfrac{1}{2}[a_s\gamma_0(n) + a_s^2\gamma_1(n) + \ldots],$$ Bakulev et al. PRD 67, 074012 (2003)

- $$\varphi_\pi^{(2)}(x;Q^2) \sim x(1-x)\sum_{n=0}^{\infty}a_n(\mu^2)C_n^{3/2}(2x-1)\left(\ln Q^2/\Lambda^2_{\rm QCD}\right)^{\gamma_n/2\beta_0}$$

Two-loop evolution with heavy-quark thresholds in NGS, PRD102, 034022 (2020)

$\boxed{a_n\text{: nonperturbative hadronic parameters}}$ QCD-based approaches, lattice, data fits

Conformal expansion of pion DA

- Expand $\varphi_\pi^{(2)}(x,\mu^2)$ over eigenfunctions of 1-loop ERBL Eq.: $\{\psi_n(x)\}$ on $x \in [0,1]$

$$\varphi_\pi^{(2)}(x,\mu^2) = \sum_{n=0,2,4,\ldots}^{\infty} a_n(\mu^2)\psi_n(x)\,; \quad \psi_n(x) = 6x\bar{x}C_n^{(3/2)}(2x-1)\,; \quad \varphi_\pi^{\text{asy}}(x) = 6x\bar{x}$$

- Determine conformal coefficients $a_n(\mu^2)$ via moments

$$\langle\xi^N\rangle_\pi \equiv \int_0^1 dx(2x-1)^N\varphi_\pi^{(2)}(x,\mu^2)\,, \qquad N = 2,4,\ldots$$

at typical hadronic scale $\mu^2 \gtrsim 1$ GeV^2 with $\bar{x} = 1-x$; $\xi = 2x-1 = x-\bar{x}$:

$$a_2 = \frac{7}{12}\left(5\left\langle\xi^2\right\rangle - 1\right)\,; \quad a_4 = \frac{77}{8}\left(\left\langle\xi^4\right\rangle - \frac{2}{3}\left\langle\xi^2\right\rangle + \frac{1}{21}\right)$$

$$a_6 = \frac{5}{64}\left(429\langle\xi^6\rangle - 495\langle\xi^4\rangle + 135\langle\xi^2\rangle - 5\right) \quad \ldots$$

a_6 influence discussed in NGS et al., PRD 87 (2013) 094025

- Conformal coefficients $a_n(Q^2 > \mu^2)$ multiplicatively renormalizable
  ★ **Advantage**: ERBL evolution easy to include
  ★ **Disadvantage**: Slow convergence if many harmonics included.

Gegenbauer-α representation

Chang et al., PRL110 (2013) 132001; Gao et al., PRD90 (2014) 014011

Use Gegenbauer polynomials of variable dimensionality $\alpha = \alpha_- + 1/2$:

- $\varphi_\pi^{(2)}(x,\mu^2) = f(\{\alpha, a_2^\alpha, ..., a_{j_s}^\alpha\}, x) = \psi_0^{(\alpha)}(x) \quad + \sum_{j=2,4,...}^{j_s} a_j^\alpha(\mu^2)\psi_n^{(\alpha)}(x)$
- Basis functions [α is an index not a power]

$$\psi_n^{(\alpha)}(x) = N_\alpha (x\bar{x})^{\alpha_-} C_n^{(\alpha)}(2x-1) \quad \text{[in general } \alpha \neq 3/2]$$

- $N_\alpha = 1/B(\alpha+1/2, \alpha+1/2)$; $\alpha_- = \alpha - 1/2$; [$B(x,y)$ Euler beta function]
  ★ **Advantage**: Sufficient to include only one coefficient $a_2^\alpha \Rightarrow$ fast convergence
- Pion DA expressible only in terms of two parameters: a_2^α and α_-:

$$\varphi_\pi^{(\alpha)}(x,\mu^2) = N_\alpha (x\bar{x})^{\alpha_-} [1 + a_2^\alpha C_2^{(\alpha)}(x - \bar{x})]$$

★ **Disadvantage**: Set $\{\psi_n^{(\alpha)}(x)\}$ **NOT** eigenfunctions of one-loop ERBL Eq.
To evolve $\varphi_\pi^{(2)}(x, \mu^2 \approx 4 \text{ GeV}^2)$ to $Q^2 > \mu^2$, one has to project it first onto conformal basis $\{\psi_n(x)\}$, evolve, and then determine α_- and a_j^α at the new scale anew.

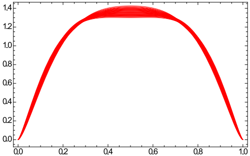
Platykurtic concept and pion DA [NGS, PLB738 (2014) 483]

Preamble

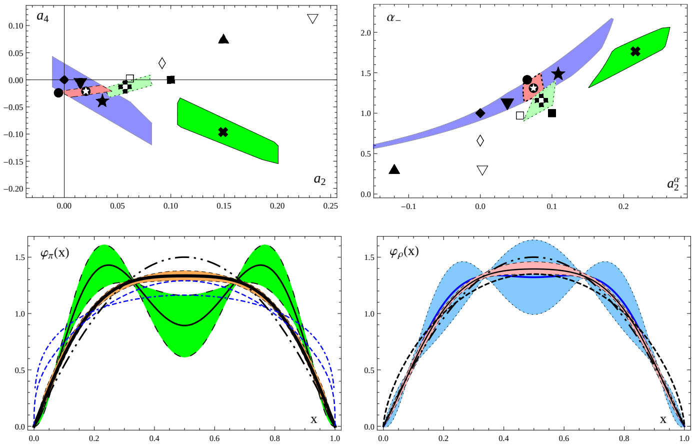
- In 2012 ATLAS and CMS experiments at Large Hadron Collider (LHC) at CERN discovered the Higgs boson which provides mass to some particles in the Standard Model via the Higgs mechanism. Peter Higgs and Francois Englert, were awarded the Nobel Prize in Physics in 2013 for their theoretical predictions.
- But is the Higgs mechanism enough to explain the hadron masses? I refer to C. D. Roberts, "On mass and Matter", arXiv:2101.08340 and talk here.
- Nontrivial structure of QCD vacuum generates correlations at scales $\propto 0.3$ fermi described in terms of nonlocal condensates (NLC) Mikhailov, Radyushkin (1986).

Big Question

- BMS π DAs obtained from **NLC-SRs** reflect nonlocality in bimodal profile and suppressed tails tuned by virtuality $\lambda_q^2 = 0.4 \pm 0.05$ GeV^2 of vacuum quarks.
- Mass generation induced by dynamical chiral symmetry breaking (**DCSB**) dresses confined quark and entails broadening of pion DA in whole $x \in [0, 1]$ interval.
- Is it possible to amalgamate these detrimental effects in a single pion DA? ✓



Model DAs for π and $\rho^{\parallel}$ in (a_2, a_4) (left) and (α_-, a_2^{α}) (right)



- **Large green strips**: BMS pion DAs [Bakulev, Mikhailov, NGS PLB508(2001)279]
- **Small strips**: platykurtic DA regions—π (green) [NGS, PLB738(2014)483], $\rho^{\parallel}$ (orange) [NGS, Pimikov NPA945 (2015) 248]
- **Blue strips** analogous results for $\rho^{\parallel}$ DAs

Functional details of various model DAs for π and $\rho^{\|}$

- QCD sum rules with nonlocal condensates [Mikhailov, Radyushkin JETP Lett. 43 (1986) 712; PRD45 (1992) 1754; Bakulev, Mikhailov, NGS, PLB508 (2001) 279]

$$\varphi_{\pi}^{\text{BMS/pk}}(x, \mu^2 \gtrsim 1 \text{ GeV}^2) = 6x\bar{x}\left[1 + a_2 C_2^{(3/2)}(x - \bar{x}) + a_4 C_4^{(3/2)}(x - \bar{x})\right]$$

✖ $a_2^{\text{BMS}}(x) = 0.2,\ a_4^{\text{BMS}} = -0.14,\ \lambda_q^2 = 0.4\ \text{GeV}^2$ [BMS (2001)]
✜ $a_2^{\text{pk}}(x) = 0.08,\ a_4^{\text{pk}} = -0.019,\ \lambda_q^2 = 0.45\ \text{GeV}^2$ [NGS, PLB738 (20014) 483]

- DSE [**Chang et al., PRL110 (2013) 132001; Gao et al., PRD90 (2014) 014011**]

$$\varphi_{\pi}^{(\alpha)}(x, \mu^2) = N_{\alpha}(x\bar{x})^{\alpha_-}[1 + a_2^{\alpha} C_2^{(\alpha_-)}(x - \bar{x})] \quad \alpha_- = \alpha - 1/2$$

π DSE-DB ▲ (- - -): $(N_\alpha = 0.181, \alpha_- = 0.31, a_2^{\alpha} = -0.12)$
π DSE-RL ▽ (- . -): $(N_\alpha = 0.174, \alpha_- = 0.29, a_2^{\alpha} = 0.0029)$
$\rho^{\|}$ DSE ♢ (- - -): $(N_\alpha = 3.37, \alpha_- = 0.66, a_2^{\alpha} = 0)$

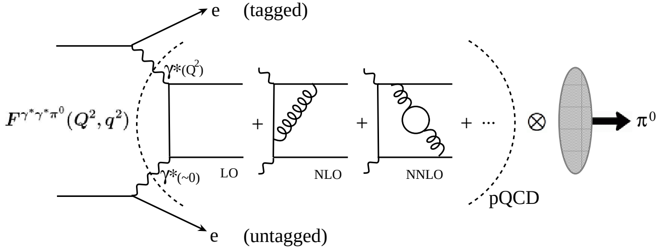
- AdS/QCD △ (unimodal) [Brodsky, de Teramond, PRD77 (2008) 056007]:
$\varphi_{\pi}^{\text{AdS/QCD}}(x, \mu^2 = 1\ \text{GeV}^2) = (8/\pi)(x\bar{x})^{1/2}$ (not shown)

- Asymptotic DA ♦ (- . . -): $\varphi^{\text{asy}}(x, \mu^2 \to \infty) = 6x\bar{x}$
  - BMS (bimodal), platykurtic (unimodal) both have endpoints suppressed
  - Light-front based $\rho^{\|}$ DA □ (unimodal) [Choi, Ji, PRD91 (2015) 014018]
  - AdS/QCD $\rho^{\|}$ DA ▼ (unimodal) [Ahmady et al., (2017)]

❷ Accessing the pion DA in single-tag experiments

- $e^+e^- \to e^+e^-\pi^0$ two-photon exclusive process using QCD factorization

$$F_{\text{QCD}}^{\gamma^*\gamma^*\pi^0}(Q^2, q^2, \mu_{\text{F}}^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx\, T\left(Q^2, q^2; \mu_{\text{F}}^2; x\right)\varphi_\pi^{(\text{tw-2})}\left(x, \mu_{\text{F}}^2\right) + \text{h.tw.}$$



Measurements of differential cross sections in different "single-tagged" experiments for $q_2^2 \lesssim 0$ giving access to $F^{\gamma^*\gamma\pi^0}(q_1^2 = -Q^2)$:

CELLO (1991): 0.70 $\div$ 2.20 GeV^2 H. J. Behrend et al., Z Phys C49 (1991) 401
CLEO (1998): 1.64 $\div$ 7.90 GeV^2 J. Gronberg et al., PRD57 (1998) 33
BaBar (2009): 4.24 $\div$ 34.36 GeV^2 B. Aubert et al., PRD80 (2009) 052002
Belle (2012): 4.46 $\div$ 34.46 GeV^2 S. Uehara et al., PRD86 (2012) 092007

Pion-photon TFF in state-of-the-art LCSRs—formalism

[NGS, PRD 102 (2020) 034022]

- $F^{\gamma^*\gamma^*\pi^0}(Q^2, q^2)$ with $Q^2 = -q_1^2$, $q^2 = -q_2^2$ defined as current correlator

$$\int d^4 z\, e^{-iq_1\cdot z}\langle\pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F^{\gamma^*\gamma^*\pi^0}(Q^2, q^2)$$

$j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$: quark electromagnetic current
For $Q^2 \gg q^2$ keeping Q^2 fixed, TFF expressed as a dispersion integral [Khodjamirian, EPJC6 (1999) 477; Balitsky, Braun, Kolesnichenko, NPB 312 (1989) 509]:

$$F_{\text{LCSR}}^{\gamma^*\gamma^*\pi^0}(Q^2, q^2) = N_{\text{T}}\int_0^\infty ds\frac{\rho(Q^2, s)}{q^2+s}, \quad \left(N_{\text{T}} = \frac{\sqrt{2}}{3}f_\pi\right)$$

Spectral density: $\rho(Q^2, s) = \rho^{\text{h}}(Q^2, s)\theta(s_0 - s) + \rho^{\text{pert}}(Q^2, s)\theta(s - s_0)\,,$

$$\rho^{\text{h}}(Q^2, s) = \sqrt{2}f_\rho F^{\gamma^*\rho\pi}(Q^2)\delta(s - m_\rho^2)\,,$$

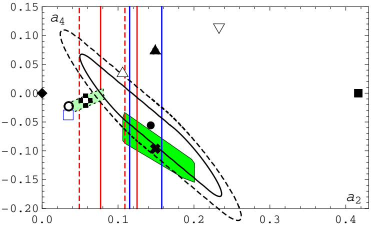
$$\rho^{\text{pert}}(Q^2, s) = \frac{1}{\pi}\textbf{Im}[F_{\text{QCD}}^{\gamma^*\gamma^*\pi^0}(Q^2, -s - i\epsilon)] = \rho_{\text{tw-2}} + \rho_{\text{tw-4}} + \rho_{\text{tw-6}} + \ldots\,,$$

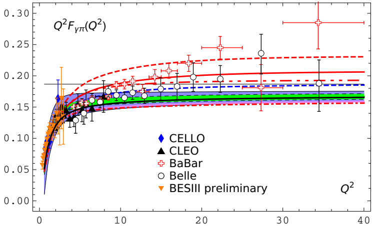
$$\rho_{\text{tw-2}} \sim \frac{1}{\pi}\textbf{Im}[T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}}\ldots] \otimes \varphi_\pi^{\text{tw-2}}(x, \mu^2)$$

For $\rho_{\text{tw-4}}$, $\rho_{\text{tw-6}}$, see [Mikhailov, NGS, NPB 821 (2009) 291; Agaev et al., PRD83 (2011) 054020].

Pion-photon TFF in state-of-the-art LCSRs—results

[NGS, PRD 102 (2020) 034022]





- **Left**. a) Large rectangle: BMS DAs; small rectangle: platykurtic range
  b) Vertical lines mark lattice constraints on a_2: Bali et al., JHEP 08(2019)065; NLO (dashed red lines), NNLO (solid red lines); Braun et al., PRD93(2015)014504 (solid blue lines). Bali et al., JHEP11(2020)037: N^{3}LO $a_2(2\text{ GeV}) = 0.116^{19}_{-20}$ (not shown)
  c) 1σ (solid line) and 2σ (dashed line) data [$BABAR$($\leq$ 9 GeV^2)] regions.
  d) Various DAs, see NGS, PRD 102 (2020) 034022.
- **Right**. Predictions for $Q^2F_{\gamma\pi}(Q^2)$ with various DAs.
  a) Green strip: BMS DAs; thick black line: platykurtic DA
  b) Dashed blue line: AdS/QCD Brodsky, Cao, de Terámond PRD84 (2011) 033001
  c) Red lines: Solid/DSE-DB, dashed/DSE-RL Chang et al., PRL110 (2013) 132001; Gao et al., PRD90 (2014) 014011
  d) BESIII preliminary data $0.3 \leq Q^2[\text{GeV}^2] \leq 3.1$ included, Redmer, 1810.00654

TFF calculation within LCSR approach [NGS, PRD 102 (2020) 034022]

PION-PHOTON TRANSITION FORM FACTOR IN LIGHT CONE … PHYS. REV. D **102,** 034022 (2020)

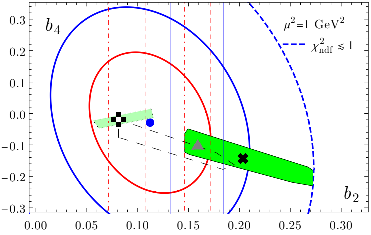
TABLE II. Theoretical ingredients entering the TFF calculation within the applied LCSR scheme using various pion DAs with conformal coefficients a_n ($a_0 = 1$) at the normalization scales $\mu_1 = 1$ GeV and $\mu_2 = 2$ GeV given in Table I. The question mark indicates that $\mathcal{T}_c$ is unknown. It is included as the main theoretical uncertainty in the TFF predictions obtained with the BMS/platykurtic DAs within FOPT, see Appendix B. The other NNLO terms and the NLO contribution are explained in Sec. (II A). $\mathcal{F}_\infty$ denotes the asymptotic limit given by Eq. (33).

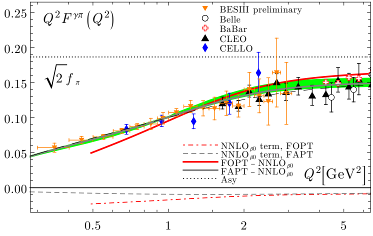
LCSR $[\rho_2, \rho_4, \rho_6]$ π DAs	LO + NLO $T_{\rm LO} + \alpha_s T_{\rm NLO}$	NNLO (α_s^2) $T_\beta, T_{\Delta V}, T_L, \mathcal{T}_c$	Error Range	ERBL App. A	$\mathcal{F}_\infty$ Fig. 4 (Left)
BMS [16] / pk [17]	$\{a_2, a_4\}_{\mu_1}$	$\{a_2, a_4\}_{\mu_1}$, a_0, 0, ?	$\mathcal{T}_c \sim T_\beta$	YES	below
DSE [56,57]$\{^{\rm DB}_{\rm RL}$	$\{a_2, a_4, \ldots, a_{12}\}_{\mu_2}$	$\{a_2, a_4, a_6\}_{\mu_2}$, a_0, 0, ?	NO	YES	above
AdS/QCD [59]	$\{a_2, a_4, \ldots, a_{12}\}_{\mu_1}$	$\{a_2, a_4, a_6\}_{\mu_1}$, a_0, 0, ?	NO	YES	below
Light-Front QM [62]	$\{a_2, a_4, a_6\}_{\mu_1}$	$\{a_2, a_4, a_6\}_{\mu_1}$, a_0, 0, ?	NO	YES	below
NLχ QM [63]	$\{a_2, a_4, a_6\}_{\mu_1}$	$\{a_2, a_4, a_6\}_{\mu_1}$, a_0, 0, ?	NO	YES	below

- Includes $\underbrace{T_{\mathbf{LO}}}_{(+)}$, $\underbrace{T_{\mathbf{NLO}}}_{(-)}$, $\underbrace{T_{\mathbf{NNLO}_{\beta_0}}}_{(-)}$, $\underbrace{T_{\mathbf{NNLO}_{\Delta V}}}_{(-)}$, $\underbrace{T_{\mathbf{NNLO}_{L}}}_{(\mathbf{0})}$, $\underbrace{T_{\mathbf{NNLO}_{c}}}_{(?)}$, $\underbrace{\textbf{Tw-4}}_{(-)}$, $\underbrace{\textbf{Tw-6}}_{(+)}$
- Hadronic content of real photon via Breit-Wigner form for ρ-meson resonance
- Various π DAs with large number of conformal coefficients can be included
- NLO (two-loop) ERBL evolution includes heavy-quark thresholds
- No tuning of hadronic parameters to data
- All available data included in comparison

Pion-photon TFF using FAPT/LCSRs [Mikhailov, Pimikov, NGS, 2101.12661, PRD in press]

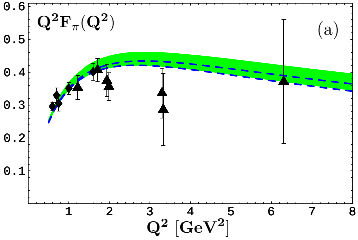
- **Goal**: Resum QCD radiative corrections by means of RGE
  Why? Avoid dominance of particular terms in fixed-order pQCD expansion
- Implement RG summation in LCSR using modified spectral density and analytic couplings $\Rightarrow$ FAPT/LCSR Bakulev, Mikailov, NGS, PRDD72(2005)074014; PRD75(2007) 056005.
- Compare TFF with high precision preliminary data of BESIII Redmer, 1810.00654



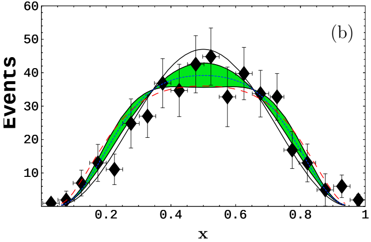


- **Left. Conformal coefficients b_2, b_4 at $\mu_1^2 = 1$ GeV^2**
  a) ▲: best-fit DA to data, lattice, NLC-SR: ($b_2 = 0.159, b_4 = -0.098$)
  b) Lattice constraints on a_2 [Bali et al., JHEP08(2019)065; JHEP11(2020)037]: NLO (dashed-dotted red); NNLO (dashed red); N^{3}LO (solid blue)
  c) 1σ (red) and 2σ (blue) ellipses [BESIII, CELLO, CLEO<3.1 GeV^2]
- **Right. Predictions for $Q^2 F_{\gamma\pi}(Q^2)$ in range $0.3 \leq Q^2 \leq 5.5$ GeV^2**
  a) Green strip: BMS DAs; black dashed line: platykurtic DA
  b) Grey line: Best-fit DA ▲

Other exclusive processes involving π DA



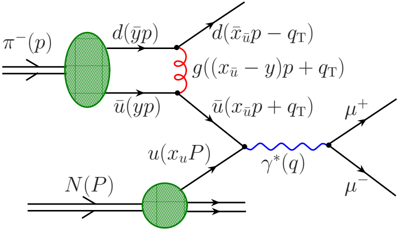
Electromagnetic pion form factor

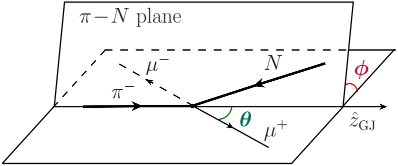


Diffractive di-jet production in E791

- **(Left)** $F_\pi(Q^2)$ calculated with BMS DAs (green strip) using analytic coupling in NLO pQCD + soft contribution from local duality, Bakulev et al. PRD70 (2004) 033014 see Richards' talk.
- **(Right)** Predictions for diffractive di-jet events at Fermilab experiment **E791**, Aitala et al., PRL 86 (2001) 4768; calculations in convolution scheme of Braun et al., NPB638 (2002) 111 — details in Bakulev, Mikhailov, NGS, PLB 578 (2004) 91
- Lines: **BMS** DAs ($\chi^2 = 10.96$), **CZ** DA ($\chi^2 = 14.15$), **Asy** ($\chi^2 = 12.56$)
- LCSR analysis of $F_\pi(Q^2)$ by Cheng et al, PRD102 (2020) 074022 of JLab data $Q^2 = 0.60 - 2.45$ GeV^2 Huber et al. PRC78 (2008) 045203 using various pion DAs

Drell-Yan process $\pi^- N \to \mu^+ \mu^- X$ [Bakulev, NGS, Teryaev, PRD76(2007)074032]



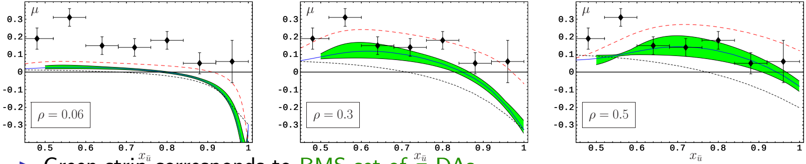


- Drell-Yan process $\pi^- N \to \mu^+ \mu^- X$ dominant mechanism for production of lepton pairs with large invariant mass Q^2
- FNAL experiment E615: $s = (p_\pi + P_N)^2 = 500$ GeV^2, $Q^2 = q^2 = 16 - 70$ GeV^2, $\rho \equiv Q_T/Q = 0 - 0.5$, $Q_T^2 = -q_T^2$
- As $x_{\bar{u}} \to 1$, $p_{\bar{u}}^2$ large and far spacelike $\Longrightarrow$ u-quark nearly free and on-shell, i.e., $x_u = x_N$ (no transverse momenta)

Drell-Yan process $\pi^- N \to \mu^+\mu^- X$ (results for angular parameter μ)

Angular distribution of μ^+ in pair rest frame of DY reaction with unpolarized target in terms of kinematic variables λ, μ, ν [Brandenburg et al., PRL73(1994)939]:

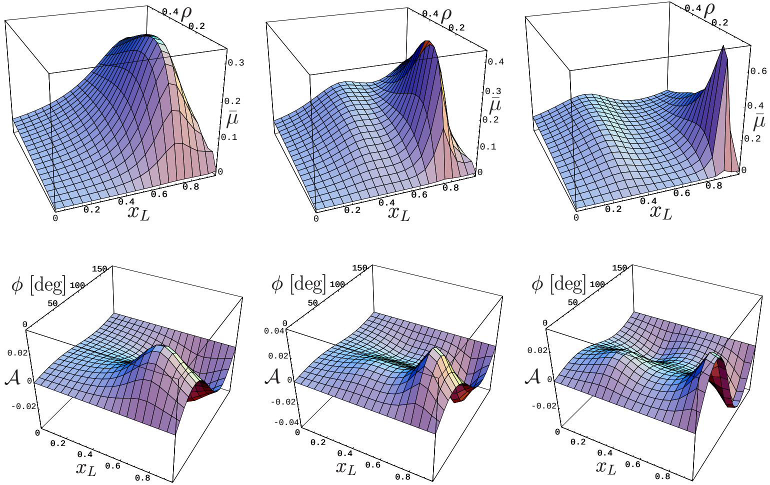
$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L\, d\cos\theta d\phi} \propto N(\tilde{x}, \rho)\Big(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\Big)$$



- Green strip corresponds to BMS set of π DAs
- Solid line shows result for asymptotic DA
- Dashed line denotes result for CZ π DA

More graphics in [Bakulev, NGS, Teryaev, PRD76(2007)074032]

DY process. 3D plots of angular parameters at $s = 400\ \text{GeV}^2$



- Upper row. Angular parameter $\bar{\mu}(x_L, \rho)$
- Lower row. Azimuthal asymmetry $\mathcal{A}(\phi, x_L, \rho = 0.3)$
- Left: φ_{as}; center: BMS DA; right: CZ DA

Cross-link relations between π and ρ-meson channels

- Polyakov and Son [PRD102(2020)114005] found by using dispersion relations, unitarity, and crossing symmetry that Gegenbauer moments of ρ-meson DA can be expressed via two-pion DA [Polyakov, NPB 555 (1999) 231].
- Derived relation between $a_2^{(\rho)}$ and $a_2^{(\pi)}$ involving second moment of pion PDF.
- [Mikhailov, Polyakov, Son, NGS, work in preparation] extended conformal expansion and obtained generalized relation between π (axial) and ρ-meson (vector) channels using NLC-SR's.
- Discussed implications for $\rho^{\parallel}$ and π DAs in a_2, a_4 parameter space.

What we know so far

π DA shape

- **NLC-SR "BMS laboratory"**: Description of QCD vacuum with NLC-SR induces correlations and endpoint suppression because $|a_4| \lesssim a_2$ but negative: Bimodal BMS DAs ($\lambda_q^2 = 0.40$ GeV^2); Platykurtic DA ($\lambda_q^2 = 0.45$ GeV^2).
- **DSE "Laboratory"**: EHM via DCSB: broad unimodal function in whole $x \in [0, 1]$ range within BMS error margin [Roberts et al. arXiv:2102.01765].
- **BLFQ scheme** yields similar DA [Quian et al., PRC102(2020)055207], Karthik talk.
- Lattice results (RQCD Coll.) at NLO, NNLO, N^{3}LO: $a_2^{(3)}(2GeV) = 0.116^{19}_{-20}$ exclude $\varphi_{\rm as}$ and CZ DA.
- **LaMET** [Xiangdong Ji et al., 2004.03543] favors unimodal pion DA but endpoint behavior unclear.

$\pi - \gamma$ TFF (within LCSR)

- All pion DA's expressed as a convergent Gegenbauer series (therefore vanishing at endpoints), unable to reproduce Q^2 dependence of *BABAR* data above 10 GeV^2.
- Low diagonal $\sim \frac{3}{\sqrt{2}f_\pi} Q^2 F_{\gamma^*\gamma\pi^0}(Q^2) = \langle x^{-1}\rangle_\pi = 3(1 + a_2 + a_4 + \ldots)$ agrees with 1σ and 2σ of $\pi - \gamma$-TFF data—nonlocal constraint on π DA [BMS (2001)].
- LCSR's at NNLO+Tw-4+Tw-6+NLO-ERBL and Breit-Wigner resonance form for real γ agrees with pQCD($Q^2 \to \infty$) and most data [NGS, PRD 102 (2020) 034022].
- RG-summation of radiative corrections improves TFF at $Q^2 \ll 1$ GeV^2 [Mikhailov, Pimikov, NGS, 2101.12661, PRD in press].

What we would like to know (better)

- Need observable sensitive to upper diagonal, $\propto (a_2 - a_4)$, e.g., heavy-to-light form factors $B\pi$ and hadronic coupling constants $g_{B^*B\pi}$, $g_{\rho\omega\pi}$ from vector mesons radiative decays.
- Electromagnetic pion form factor $F_\pi(Q^2)$ after JLab 12 upgrade may provide additional boundaries of π DA moments.
- Diffractive dijet production at E791 not able to single out a particular pion DA. Hard diffractive dissociation of mesons in the field of a heavy nucleus deserves attention in AMBER.
- COMPASS++/AMBER dedicated to the pion/kaon structure study—new results expected for moments of pion PDA and pion PDF.
- Lattice constraints on $\langle \xi^2 \rangle_\rho$ with π-level accuracy.
- This will improve understanding of different dynamical behavior between π and ρ-meson channels.
- Reliable determination of kinematical endpoint behavior of pion DA in LaMET.

C'mon, let's get it nailed down!